Why Corporate Bonds May Disappoint: Disappointment
Aversion and the Credit Spread Puzzle\textsuperscript{1}

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Abstract

I propose a novel consumption-based asset pricing model with disappointment averse investors to address the credit spread puzzle. In the post-war sample, periods of high default rates and low recovery rates coincide with periods of worse than expected macroeconomic conditions (disappointment events). The disappointment model can therefore generate realistic credit spreads because investors penalize defaults more heavily during these disappointment periods than during normal times. Further, disappointment aversion is able to match credit spreads, equity premia, and the risk-free rate using risk and disappointment aversion coefficients that are consistent with experimental evidence at the micro-level. By applying disappointment theory to corporate bond pricing, I am also able to disentangle disappointment aversion from traditional risk aversion, while showing that the two cannot be separated if the stock market is the only test asset.

**keywords:** consumption-based asset pricing, credit spread puzzle, disappointment aversion, disappointment events, equity premia, risk-free rate

**JEL classification:** D51, D53, D81, D91, E21, E44, G11, G12, G33
You cannot receive anything from someone who has nothing

“Dialogues of the Dead”, Lucian (125 – 175 A.D.)

1 Introduction

The task of explaining observed Baa-Aaa credit spreads has proven especially challenging for the asset pricing literature. First, structural models of default are unable to generate realistic credit spreads when they are calibrated to actual default rates and losses given default (Huang and Huang 2012). Second, state-of-the-art consumption-based asset pricing models cannot rationalize Baa-Aaa credit spreads, even if they can successfully explain equity premia (Chen et al. 2009). Theoretically, a universal stochastic discount factor that can resolve the equity premium puzzle should also be able to fit prices in corporate bond markets.

In this paper, I address the credit spread puzzle by proposing a consumption-based asset pricing model with disappointment averse investors. While most behavioral asset pricing theories tend to focus on the equity premium puzzle, I also consider corporate spreads. This distinction is important as I show that if the stock market is the only test asset, then disappointment aversion and risk aversion cannot both be identified. In contrast, by jointly matching the equity risk premium and corporate bond prices, I can disentangle the effects of disappointment aversion and traditional risk aversion.

My results show that disappointment aversion can explain the credit spread puzzle generating expected Baa-Aaa credit spreads of 102 bps for four-year maturities. These spreads are identical to the historical average of 103 bps in Huang and Huang (2012), and almost twice as large as the spreads (58 bps) implied by a discrete-time version of Merton’s model (Merton 1974). Further, I show that the disappointment model is able to match equity premia, and the risk-free rate using risk and disappointment aversion coefficients that are consistent with experimental evidence at the

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In the post-war sample, periods of high default rates and low recovery rates coincide with periods of worse than expected macroeconomic conditions (disappointment events). The disappointment model can therefore generate realistic credit spreads because investors penalize defaults more heavily during these disappointment periods than during normal times. To better illustrate the disappointment aversion mechanism, Figure I shows Baa-Aaa credit spreads, Baa default rates, and NBER recessions for the 1946-2011 period. Two patterns become immediately clear: i) credit spreads are countercyclical; ii) with the exception of 1986, Baa default rates remain close to zero over long periods of time, and tend to spike up during and after recessions which, in turn, coincide with disappointment events (Delikouras 2013). Disappointment aversion is able to fit the Baa-Aaa credit spreads precisely because it amplifies the significance of these very small Baa default risks during disappointment events.

The disappointment framework in this paper follows the model proposed by Gul (1991). This model captures well documented patterns for risky choices, such as asymmetric marginal utility over gains and losses as well as reference-based evaluation of stochastic payoffs. Contrary to traditional preference specifications (e.g., CRRA or second-order risk aversion), disappointment averse investors are worried about losses much more than they enjoy gains, and disappointment aversion preferences are described by utility functions with kinks (first-order risk aversion).

I focus on disappointment aversion preferences because, unlike alternative first-order risk aversion models (e.g., loss aversion), disappointment theory provides clear guidelines on how reference levels for gains and losses (kinks in the utility function) are formed and dynamically updated. Specifically, investors feel disappointed whenever outcomes are worse than the certainty equivalent. Furthermore, disappointment aversion preferences are able to capture stylized facts about behavior under risk without sacrificing analytical tractability since they do not violate first-order

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2Almeida and Philipon (2007) also document that bankruptcies and distress costs are most likely to happen during times when marginal utility is high.
4Routledge and Zin (2010) generalize disappointment aversion. In their framework, investors feel disappointed whenever outcomes are worse than some multiple of the certainty equivalent.
stochastic dominance, transitivity of preferences, or aggregation of investors.

The disappointment framework is cast in a dynamic setting by taking the model of Routledge and Zin (2010), and explicitly defining its stochastic discount factor (sdf). The disappointment aversion sdf combines asymmetric marginal utility with piece-wise concave CRRA preferences to capture both first- and second-order risk aversion. Moreover, the stochastic discount factor is non-separable across time as in Epstein and Zin (1989) to allow for different preferences over the timing and risk of stochastic payoffs.

Due to non-separable preferences, the stochastic discount factor is a function of consumption growth and lifetime utility. However, lifetime utility is unobservable. Therefore, in order to analyze the asset pricing implications of the disappointment model, I obtain solutions for the disappointment aversion sdf when the elasticity of intertemporal substitution is equal to one, and consumption growth is conditionally heteroscedastic. These solutions prescribe that price-dividend ratios are functions of three state variables: consumption growth, consumption growth volatility, and consumption growth variance. I then use these solutions, and conduct a simulation exercise to examine the importance of disappointment aversion in explaining asset prices.

I find that the disappointment aversion model is quite flexible, and can fit credit spreads and equity premia with reasonable preference parameters. Specifically, the model fits key asset pricing moments with a risk aversion parameter of 1.900 and a disappointment aversion coefficient equal to 1.439. These results are consistent with experimental evidence at the micro-level (e.g., Choi et al. 2007). In contrast, traditional consumption models require risk aversion coefficients around 50 to match the annual equity premium (Cochrane 2001), while even the long-run risk model of Bansal and Yaron (2004) requires a risk aversion parameter of around 9.

Finally, I show that in order to disentangle disappointment from second-order risk aversion, the set of test assets must include corporate bonds. As in Dolmas (2014), I find that there can be different combinations of disappointment and risk aversion coefficients that generate almost identical moments for stock market returns. Therefore, if the stock market is the only test asset, then first- and second-order risk aversion cannot be disentangled. This result is important because
most first-order risk aversion models\textsuperscript{5} tend to focus on the equity premium or the cross-section of expected returns, and have ignored possible identification issues between first- and second-order risk aversion.

Overall, the findings of this paper contribute to the existing literature in three important ways. First, this paper compliments a growing body of results arguing that first-order risk aversion preferences might be able to address a variety of stylized facts in financial markets such as the equity premium puzzle (Routledge and Zin 2010, Bonomo et al. 2011), the cross-section of expected returns (Ostrovnaya et al. 2006, Delikouras 2013), and limited stock market participation (Ang et al. 2005, Khanapure 2012).

Second, this paper adds to recent results by Chen et al. (2009) and Chen (2010), who address the credit spread puzzle using a consumption-based stochastic discount factor, and take a stance on its functional form. Most of the existing credit spread literature\textsuperscript{6} has relied on risk-neutral probability measures to derive asset prices, while being silent on investor preferences. An explicit discount factor is important for two reasons: first, we can identify whether a particular set of preferences is able to generate plausible asset pricing moments across different markets; second, parameter estimates can be compared to experimental findings for choices under risk in order to assess their empirical plausibility.

Finally, the paper contributes to the broader asset pricing literature. Specifically, I propose a single consumption-based stochastic discount factor that can explain asset prices in different markets. This discount factor is based on recent experimental results for choices under risk that emphasize the importance of certainty equivalent-based reference-dependent utility.\textsuperscript{7} Although there are many asset pricing models that can efficiently explain stylized facts in financial markets, most models usually explain asset prices one market at a time. In this paper, I address the equity premium and the credit spread puzzles in a unified manner using a universal stochastic discount factor across bond and equity markets.


\textsuperscript{6}Leland (1994), Leland and Toft (1996), Goldstein et al. (2001), Bhamra et al. (2010).

\textsuperscript{7}Choi et al. (2007), Gill and Prowse (2012), Artstein-Avidan and Dillenberger (2011).
The remainder of the paper is organized as follows. Section 2 introduces the disappointment aversion discount factor. Section 3 revisits the credit spread puzzle, and discusses the asset pricing implications of the disappointment model. Section 4 reviews previous literature, and Section 5 concludes.

2 Recursive utility with disappointment aversion preferences

In this section, I introduce the disappointment aversion discount factor, and obtain explicit solutions for asset returns in terms of observable macroeconomic variables. I then use these solutions to simulate the economy, and test the asset pricing predictions of the disappointment model.

2.1 The disappointment aversion stochastic discount factor

Consider a discrete-time, single-good, closed, endowment economy in which there is no productive activity. At each point in time, the endowment of the economy is generated exogenously by \( n \) “tree-assets” as in Lucas (1978). Equity, debt, and claims on the total output of these “tree-assets” are traded in complete markets free of transaction costs. Disappointment averse investors are fully rational, face no restrictions on asset holdings, and are characterized by identical homothetic preferences.

Under these assumptions, there exists a representative agent whose stochastic discount factor is given by

\[
M_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{(\rho-1)} \frac{V_{t+1}}{\mu_t(V_{t+1})} \left[ \frac{1 + \theta 1 \{V_{t+1} < \delta \mu_t\}}{1 + \theta \delta^{-\alpha} \mathbb{E}_t[1\{V_{t+1} < \delta \mu_t\}]} \right].
\]

(1)

with

\[
\mu_t(V_{t+1}) = \mathbb{E}_t \left[ \frac{V_{t+1} - \alpha (1 + \theta 1 \{V_{t+1} < \delta \mu_t\})}{1 + \theta \delta^{-\alpha} \mathbb{E}_t[1\{V_{t+1} < \delta \mu_t\}]} \right]^{-\frac{1}{\alpha}}.
\]

(2)

\( ^8 \)See Appendix A.
The variable \( V_t \) in (1) denotes lifetime utility from time \( t \) onwards. \( \mu_t \) in equation (2) is the disappointment aversion certainty equivalent, and \( \mathbb{E}_t \) is the conditional expectation operator. The term \( 1\{V_{t+1} < \delta \mu_t\} \) is the disappointment indicator that overweighs bad states of the world (disappointment events), and shows when investors feel disappointed. The denominator in (2) is a normalization constant such that \( \mu_t(\mu_t) = \mu_t \).

The novel parameter in the disappointment model is the disappointment aversion parameter \( \theta \geq 0 \). This parameter characterizes the degree of asymmetry in marginal utility above and below the reference point. If \( \theta \) is positive,\(^9\) then an additional one-dollar-loss in consumption during disappointment periods hurts approximately \( 1 + \theta \) times more than an additional one-dollar-loss in consumption during normal times. When \( \theta \) is zero, investors have symmetric preferences, and the effects of first-order risk aversion vanish.

The parameter \( \delta \) is associated with the location of the reference point or the threshold for disappointment. According to (2), disappointment events happen whenever lifetime utility \( V_{t+1} \) is less than some multiple \( \delta \) of its certainty equivalent \( \mu_t \). In Gul (1991), \( \delta \) is 1, and disappointment happens when lifetime utility drops below the certainty equivalent. On the other hand, in Routledge and Zin (2010), disappointment events may happen below or above the certainty equivalent depending on whether the generalized disappointment aversion parameter \( \delta \) is lower or greater than one, respectively. Here, I follow Gul (1991), and set \( \delta \) equal to 1 in order to solve the model analytically.

The constant \( \alpha \geq -1 \) is the coefficient of second-order risk aversion that determines the piece-wise curvature of the utility function (Pratt 1964). The parameter \( \beta \in (0, 1) \) is the rate of time preference, and \( \frac{1}{1 - \beta} \geq 0 \) is the elasticity of intertemporal substitution (EIS). Here, I set the EIS equal to one to facilitate the derivation of analytical solutions.\(^10\) Cox, Ingersoll, and Ross (1985), Piazzesi and Schneider (2006), Hansen et al. (2007), Hansen and Heaton (2008) are a few examples in which the EIS is also one in order to obtain explicit solutions for non-separable models. When

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\(^9\)If \( \theta \) is negative, then investor preferences are characterized by convex utility functions, losses hurt less than gains give joy, and investors are usually referred to as "elation seekers".

\(^10\)Delikouras (2013) provides explicit solutions for the disappointment model when the EIS is a free parameter and consumption growth is homoscedastic.
both the EIS and $\delta$ are set equal to one, then the disappointment aversion stochastic discount factor in (1) becomes

$$M_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left( \frac{V_{t+1}}{\mu_t(V_{t+1})} \right)^{-\alpha} \frac{1 + \theta \mathbb{1}\{V_{t+1} < \mu_t(V_{t+1})\}}{\mathbb{E}_t[1 + \theta \mathbb{1}\{V_{t+1} < \mu_t(V_{t+1})\}]}.$$ (3)

The disappointment aversion sdf essentially decreases expected values by taking into account investor preferences over the timing and risk of stochastic payoffs. The first term in (1) and (3) corrects for the timing of uncertain payoffs that happen at a future date (resolution of risk). The second term adjusts future payoffs for investors’ dislike towards risk (second-order risk aversion). When investors’ preferences are time-additive, adjustments for time and risk are identical, and the second term is identically equal to one. The third term in (1) and (3) is the novel term due to disappointment aversion preferences, and it corrects future payoffs for investors’ dislike towards disappointment (first-order risk aversion). This term distorts probability weights, shifting more mass towards disappointment events, i.e., states of the world in which lifetime utility $V_{t+1}$ is less than some multiple $\delta$ of its certainty equivalent $\mu_t$. If investors are disappointment neutral, then the disappointment term is always equal to one.

### 2.2 Approximate solutions for the disappointment aversion discount factor

The stochastic discount factor in (3) is a function of consumption growth as well as lifetime utility which is unobservable. In order to facilitate the analysis and provide valuable intuition, I express lifetime utility as a function of observable state variables. I assume two state variables: consumption growth ($\Delta c_{t,t+1}$) which affects conditional expectations, and consumption growth volatility ($\sigma_t$) which drives macroeconomic risk. Based on these assumptions, the model economy can be fully
described by the following system of equations

\[
\Delta c_{t,t+1} = \mu_c + \phi_c \Delta c_{t-1,t} + \sigma_t \epsilon_{c,t+1}, \quad (4)
\]

\[
\sigma_{t+1} = \mu_\sigma + \phi_\sigma \sigma_t + \nu \epsilon_{\sigma,t+1}, \quad (5)
\]

\[
\Delta o_{m,t,t+1} = \mu_o + \phi_o \Delta c_{t-1,t} + \sigma_o \sigma_t \epsilon_{o,t+1}. \quad (6)
\]

Consumption growth in (4) and consumption growth volatility in (5) are AR(1) processes with i.i.d. normal shocks.\textsuperscript{11} Similar specifications are quite common in the asset pricing literature.\textsuperscript{12} However, I focus on consumption growth volatility, rather than consumption growth variance, due to the first-order risk aversion channel of disappointment aversion preferences. A drawback of the normality assumption in (5) is that the probability of negative volatility is positive. However, consumption growth variance is always positive.

The last equation in (6) describes the evolution of aggregate payout growth. The generic payout variable \(o_{m,t}\) stands for different kinds of cashflows depending on the asset we want to price. For example, if we want to price aggregate equity claims, then the relevant payout is dividends \((o_{m,t} = d_{m,t})\). On the other hand, if we want to price assets-in-place, the relevant payout is earnings \((o_{m,t} = e_{m,t})\). Expected payout growth depends on aggregate consumption growth through the multiplier \(\phi_o\). For instance, if \(\phi_o\) is greater than one, then aggregate payout is a levered claim to consumption as in Abel (1999). Finally, I assume that all macroeconomic shocks \((\epsilon_{c,t}, \epsilon_{\sigma,t}, \epsilon_{o,t})\) are mutually uncorrelated for analytical tractability.

Using the system of equations in (4) and (5), and the log-linear structure of investor’s preferences, I can derive analytical solutions for the stochastic discount factor in terms of the three state variables: consumption growth, \(\Delta c_{t-1,t}\), consumption growth volatility, \(\sigma_t\), and consumption growth variance, \(\sigma_t^2\).

**Proposition 1:** The log utility-consumption ratio for disappointment averse investors is approx-

\textsuperscript{11}Bloom (2009) and Bloom et al. (2012) propose a general equilibrium model in which stochastic second moments are responsible for business cycle fluctuations.

\textsuperscript{12}Bansal and Yaron (2004), Bansal et al. (2007), Lettau et al. (2007), and Bonomo et al. (2011).
approximately affine in consumption growth, consumption growth volatility, and consumption growth variance: \( \log \frac{V_t}{C_t} = v_t - c_t \approx A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma_t^2 \), with\(^\text{13}\)

- \( A_1 = \frac{\beta \phi_c}{1 - \beta \phi_c} \),
- \( A_2 \approx -\frac{\theta \beta n(\bar{x})(A_1+1)+2\beta A_3 \mu_\sigma \phi_\sigma}{1 - \beta \phi_\sigma} \),
- \( A_3 \approx -\frac{1}{2} \frac{\beta \alpha (A_1+1)^2}{1 - \beta \phi_\sigma^2} \),
- \( A_0 \approx \frac{\beta}{1 - \beta} [(A_1+1)\mu_c + A_2 \mu_\sigma + A_3 \mu_\sigma^2] \),

and \( n(.) \) is the standard normal p.d.f.\(^\text{13}\)

**Proof.** See Appendix D.3

The parameter \( A_1 \) is the consumption growth multiplier, and its sign depends on consumption growth autocorrelation \( \phi_c \).\(^\text{14}\) If consumption growth is i.i.d., then \( A_1 \) is zero, and the utility-consumption ratio does not depend on consumption growth. The constant \( A_3 \) is the variance multiplier. If the risk aversion coefficient, \( \alpha \), is positive, then \( A_3 \) is negative, and an increase in consumption growth variance will decrease the log utility-consumption ratio. Finally, the parameter \( A_2 \) is the novel coefficient in the stochastic discount factor due to disappointment aversion. \( A_2 \) captures first-order risk aversion through the \( \theta n(\bar{x}) \) term. For positive risk and disappointment aversion coefficients, then \( A_2 \) is negative, and an increase in consumption growth volatility will lead to a lower utility-consumption ratio.

An immediate consequence of Proposition 1 is that we can express the disappointment aversion

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\(^{13}\)To ease the notational burden, the values for \( A_2, A_3 \) and \( A_0 \) in Proposition 1 are approximations assuming that the variance for consumption growth volatility \( (\nu_\sigma^2) \) is a number close to zero. For the simulated economy, I use the exact solutions from Appendix D.5.

\(^{14}\)The denominator in \( A_1 \) is always positive because \( \phi_c \in (-1,1) \) and \( \beta \in (-1,1) \), and therefore \( \beta \phi_c < 1 \).
stochastic discount factor in (3) as a function of the three state variables

\[ M_{t,t+1} \approx \exp \left\{ \log \beta - \Delta c_{t,t+1} - \alpha A_0 \left( 1 - \frac{1}{\beta} \right) \right\} \times \exp \left\{ \alpha \left( - \left[ (A_1 + 1) \Delta c_{t,t+1} - \frac{1}{\beta} A_1 \Delta c_{t-1,t} \right] - A_2 (\sigma_{t+1} - \frac{1}{\beta} \sigma_t) - A_3 (\sigma_{t+1}^2 - \frac{1}{\beta} \sigma_t^2) \right) \right\} \times \frac{1 + \theta \{ A_0 + (A_1 + 1) \Delta c_{t,t+1} + A_2 \sigma_{t+1} + A_3 \sigma_{t+1}^2 < \frac{1}{\beta} (A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma_t^2) \}}{\mathbb{E}_t [1 + \theta \{ A_0 + (A_1 + 1) \Delta c_{t,t+1} + A_2 \sigma_{t+1} + A_3 \sigma_{t+1}^2 < \frac{1}{\beta} (A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma_t^2) \}]} \].

\( M_{t,t+1} \) in (7) adjusts expected future payoffs for timing, risk, and disappointment, just like the discount factor in (3). The crucial difference between the two expressions is that for the discount factor in (7), unobservable lifetime utility \( V_{t+1} \) has been replaced by the observable state variables.

Based on the solution for the disappointment aversion stochastic discount factor in (7), I can also derive an expression for the risk-free rate\(^{15}\)

\[ r_{f,t,t+1} \approx -\log \beta + \mu_c + \phi_c \Delta c_{t-1,t} - 0.5 \left[ 2 \alpha (A_1 + 1) + 1 \right] \sigma_t^2 - \theta n(\bar{x}) \sigma_t. \] (8)

The last two terms in (8) capture the precautionary motive for investors to save. Precautionary savings reflect both risk and disappointment aversion. Second-order risk aversion terms depend on consumption growth variance, while disappointment aversion terms depend on consumption growth volatility. For positive risk and disappointment aversion parameters, higher macroeconomic risk forces investors to save more in the risk-free technology, and leads to a decrease in interest rates.

### 2.3 Approximate solutions for asset returns

In the previous section, I solved the disappointment aversion sdf in terms of observable macroeconomic variables. In a similar way, I now obtain explicit solutions for aggregate price-dividend ratios and aggregate asset returns. I will then use these solutions to simulate asset pricing moments, and test the predictions of the disappointment model.

Let \( z_{m,t}^o = \log \frac{P_{m,t}}{O_{m,t}} \) be the log price-payout ratio for a stream of aggregate payments. Exploiting

\(^{15}\)See Appendix D.4.
the log-linear structure of the model, we can express $z_{m,t}$ as a linear function of the three state variables: $\Delta c_{t,t+1}$, $\sigma_t$, and $\sigma_t^2$.

**Proposition 2:** The log price-payout ratio for a claim on a stream of aggregate payments is approximately affine in the state variables: $z_{m,t} \approx A_{m,0}^o + A_{m,1}^o \Delta c_{t-1,t} + A_{m,2}^o \sigma_t + A_{m,3}^o \sigma_t^2$, with $^{16}$

- $A_{m,1}^o = \frac{\phi_o - \phi_c}{1 - \kappa_{m,1} \phi_c}$,
- $A_{m,2}^o \approx \frac{\theta n(\bar{x})(1 - \kappa_{m,1} A_{m,1}^o) + 2 \kappa_{m,1} A_{m,3}^o \mu_{\sigma} \phi_{\sigma}}{1 - \kappa_{m,1} \phi_{\sigma}}$,
- $A_{m,3}^o \approx \frac{1}{2} \frac{(1 - \kappa_{m,1} A_{m,1}^o)^2 + 2 \alpha (A_1 + 1)(1 - \kappa_{m,1} A_{m,1}^o) + \sigma^2_o}{1 - \kappa_{m,1} \phi_{\sigma}^2}$,
- $A_{m,0}^o \approx \frac{1}{1 - \kappa_{m,1}} \left[ \log \beta + \kappa_{m,0} + \mu_o + (\kappa_{m,1} A_{m,1}^o - 1) \mu_c + \kappa_{m,1} A_{m,2}^o \mu_{\sigma} + \kappa_{m,1} A_{m,3}^o \mu_{\sigma}^2 \right]$,

where $\kappa_{m,0}$ and $\kappa_{m,1}$ are log-linearisation constants.

**Proof.** See Appendix D.5

The multipliers $A_{m,1}^o$, $A_{m,2}^o$, $A_{m,3}^o$ and $A_{m,0}^o$ for the price-payout ratio in **Proposition 2** are closely related to the multipliers for the utility-consumption ratio in **Proposition 1**. $A_{m,1}$ is the consumption growth coefficient. For reasonable parameter values ($\phi_o > \phi_c$), $A_{m,1}$ is positive, and the price-dividend ratio is procyclical in consumption growth.$^{17}$ $A_{m,3}^o$ is the multiplier for the consumption growth variance $\sigma_t^2$. It depends on the risk aversion coefficient and on the persistence of the shocks to consumption growth variance. $A_3$ can be positive or negative based on the relative magnitudes of the risk aversion parameter $\alpha$ and the variance of payout growth $\sigma_o^2$.

The constant $A_{m,2}^o$ is the novel multiplier in the price-payout ratio due to disappointment aversion. $A_{m,2}^o$ captures disappointment aversion through the $\theta n(\bar{x})$ term. According to **Proposition 2**, the first-order risk aversion multiplier ($A_{2,m}^o$), and the second-order risk aversion parameter ($A_{3,m}^o$) can be of different signs. For instance, $A_{2,m}^o$ can be negative while $A_{3,m}^o$ is positive. In

$^{16}$To ease the notational burden, the values for $A_{m,2}^o$, $A_{m,3}^o$ and $A_{m,0}^o$ in **Proposition 2** are approximations assuming that the variance for consumption growth volatility ($\nu^2$) is a number close to zero. For the simulated economy, I use the exact solutions from Appendix D.5.

$^{17}$The denominator in $A_{m,1}$ is always positive because $\phi_c \in (-1, 1)$ and $\kappa_{m,1} < 1$, and therefore $1 - \kappa_{m,1} \phi_c > 0$.  

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this case, an increase in consumption growth volatility ($\sigma_t$) will have a negative impact on the price-payout ratio, while an increase in consumption growth variance ($\sigma^2_t$) will have a positive effect on the price-payout ratio. Hence, in consumption-based models that allow for first- and second-order risk aversion channels, consumption growth volatility and consumption growth variance might have opposing effects on asset prices. To our knowledge, this is the first paper to document this subtle but important difference between traditional second-order risk aversion models (e.g., Epstein and Zin 1989) and the disappointment aversion framework.

We can now combine the results in Proposition 2 with the Campbell-Shiller log-linearization (Campbell and Shiller 1988) to express asset log-returns as a linear function of the state variables

$$r_{m,t,t+1}^o \approx \kappa_{m,0} + \kappa_{m,1}z_{m,t+1}^o - z_{m,t}^o + \Delta o_{m,t,t+1}. \tag{9}$$

Asset returns in (9) correspond to aggregate claims. In order to describe firm-level asset returns, I introduce idiosyncratic shocks as follows

$$r_{i,t,t+1}^o \approx \kappa_{m,0} + \kappa_{m,1}z_{m,t+1}^o - z_{m,t}^o + \Delta o_{m,t,t+1} + \sigma_i^o \epsilon_i^o, \tag{10}$$

$$r_{i,t,t+1}^{o,ex} = z_{m,t+1}^o - z_{m,t}^o + \Delta o_{m,t,t+1} + \sigma_i^o \epsilon_i^o. \tag{11}$$

The variables $r_{i,t,t+1}^o$ and $r_{i,t,t+1}^{o,ex}$ are cum-payout and ex-payout asset returns, respectively. The constant $\sigma_i^o$ captures idiosyncratic volatility, and $\epsilon_i^o$ are idiosyncratic shocks, orthogonal to the macroeconomic shocks in (4)-(6). For equity returns, the relevant payouts in equations (9) - (11) are dividends, whereas for returns on assets-in-place, the relevant payouts are earnings. In the next section, I use the expression for the disappointment aversion sdf in (7) and the solutions for asset returns from (10) and (11) to price corporate bonds, the stock market, and the risk-free asset.
3 Simulation results for the disappointment aversion model

In the previous section, I expressed asset returns as functions of the three state variables. In this section, I use these solutions to test the asset pricing implications of the disappointment model.

3.1 Data

Before proceeding to the main results of the paper, I briefly describe the data. Average default rates during 1970-2012 and recovery rates for the 1982-2012 period are from the Moody’s annual report. Corporate bond yields are obtained from Datastream, the St. Louis Fed website, and Huang and Huang (2012).

Personal consumption expenditures (PCE), and the PCE price index are from the Bureau of Economic Analysis (BEA). Per capita consumption expenditures are services plus non-durables. Each component of consumption expenditures is deflated by its corresponding PCE price index (base year is 2009). All other variables have been adjusted for inflation using the general PCE price index. Population data are from the U.S. Census Bureau. For consumption data, I follow the “beginning-of-period” convention as in Campbell (2003) and Yogo (2006) because beginning-of-period consumption growth is better aligned with dividend growth. Recession dates are from the NBER. One-year interest rates are from Kenneth French’s (whom I kindly thank) website. Market returns, dividends, and price-dividend ratios are obtained through CRSP for the value-weighted S&P500 index.

Earnings before interest (EBIT minus TXT) from Compustat are calculated for firms whose fiscal year ends in December so that all flows (consumption, dividends, earnings) are aligned. Due to the growing Compustat sample, there is a time-trend in aggregate earnings. This trend is removed by subtracting the average growth rate of the Compustat sample during the postwar period from the growth rates of Compustat earnings. Finally, all variables are sampled annually from 1951

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18Moody’s calculates average default rates over three different periods: 1920-2012, 1970-2012, and 1983-2012. Default rates for the 1983-2012 sample are almost identical to the ones for the 1970-2012 period. However, average default rates for the 1920-2012 period are higher than the other two samples due to the Great Depression.
onwards because Compustat earnings start in 1950.

3.2 Preference parameters and state variable dynamics

The first step in calibrating the disappointment model is choosing values for preference parameters. These values are shown in Table I. The EIS and GDA coefficients are one in order to facilitate analytical results. An EIS of one implies that consumption growth moves one-for-one with interest rates, while a GDA of one means that the threshold for disappointment is exactly equal to the certainty equivalent. For the remaining parameters, I set the risk aversion coefficient $\alpha$ equal to 1.900, and the disappointment aversion parameter $\theta$ equal to 1.439. This value for $\theta$ implies that investors penalize consumption losses during disappointment periods 2.439 times more than losses during normal times. Finally, the rate of time preference $\beta$ is 0.997 which means that in the deterministic steady-state of the economy, an additional $1 of consumption tomorrow is worth $0.997 today.

The preference parameters in Table I are consistent with experimental evidence at the micro-level. For instance, in portfolio choice problems, Choi et al. (2007) find disappointment aversion coefficients that range from 0 to 1.876 with a mean of 0.390. They also estimate second-order risk aversion parameters that range from -0.952 to 2.871 with a mean of 1.448. Similarly, using experimental data on real effort provision, Gill and Prowse (2012) estimate disappointment aversion coefficients ranging from 1.260 to 2.070.

At the macro-level, disappointment aversion parameters depend on the assumption of homoscedastic consumption growth. Bonomo et al. (2011) consider a stochastic variance process, similar to the one in (5), and set $\theta$ equal to 2.33 and $\alpha$ equal to 1.5. These parameters are very similar to the ones used here. In contrast, disappointment models that do not account for time-variation in consumption growth volatility require high values for the disappointment aversion coefficient. For example, Routledge and Zin (2010) consider homoscedastic consumption growth, and set $\theta$ equal to 9 with $\alpha$ equal to -1 (second-order risk neutrality). Similar results are obtained in Delikouras (2013) who also assumes constant consumption growth volatility.
To close the model, Table I summarizes parameters for the state variable dynamics in (4)-(6). These parameters are chosen so that simulated moments for the macroeconomy match sample moments over the post-war period.\(^{19}\) Many of these parameters are similar to the ones used in previous studies. For instance, the volatility parameters \(\sigma_d\) and \(\sigma_e\) are larger than one as in Chen et al. (2009) because dividend and earnings growth are more volatile than consumption growth. The consumption growth multipliers \(\phi_d\) and \(\phi_e\) in (6) are also larger than one as in Bansal and Yaron (2004) because dividends and earnings are considered levered claims to consumption. Earnings are a levered claim to consumption because the endowment model ignores salaries, depreciation expenses, or taxes that need to be paid out before interest and dividends. Also, for uncorrelated macroeconomic shocks in (4)-(6), letting \(\phi_e\) and \(\phi_d\) be larger than one is the only way to obtain realistic correlations between earnings growth, dividend growth, and consumption growth.\(^{20}\)

Despite the above similarities with previous studies, there are also a few important differences. First, in Bansal and Yaron (2004) and Bansal et al. (2007), expected consumption growth is a very persistent process, whereas in Chen et al. (2009) and Bonomo et al. (2011) consumption growth is i.i.d.. Here, I set the autocorrelation parameter for consumption growth \(\phi_c\) equal to 0.464 in order to match the persistence of the BEA consumption data. Second, the volatility parameter \(\mu\) in Bansal and Yaron (2004) and Bonomo et al. (2011) implies that annual consumption growth volatility is approximately 3%. Yet, consumption growth volatility for the BEA sample is only 1.3%. In this study, I set \(\mu\) equal to a very small value so that consumption growth volatility remains low.

Table II shows simulated and sample moments for all macroeconomic variables. Simulated values have been generated according to the system of equations in (4)-(6) with parameters from

\(^{19}\)The constants \(\mu_c\), \(\mu_d\) and \(\mu_e\) have been chosen so that average growth rates for consumption, dividends, and earnings match the 1951-2012 sample. The value for \(\phi_c\) fits consumption growth persistence. \(\phi_d\) and \(\phi_e\) match the correlations between consumption growth, dividend growth and earnings growth over the 1951-2012 period. \(\sigma_d\) and \(\sigma_e\) fit dividend growth and earnings growth volatility. The value for \(\sigma_o\) matches the persistence of the price-dividend ratio. \(\mu_o\) is chosen so that consumption growth volatility is approximately 2%, and \(\mu_o\) is 0.0017 from Chen et al. (2009). I set idiosyncratic volatility \(\sigma_o\) equal to 0.190 to match the Sharpe ratio of 0.220 for the median Baa firm (Chen et al. 2009, p. 3377). Finally, the linearization constant \(\bar{z}_o\) is 3 to match the unconditional mean for the simulated log price-dividend ratio.\(^{20}\)Chen et al. (2009, p. 3404) set \(\phi_d\) equal to 3.5 and \(\phi_e\) equal to 2.7.
Table I. Simulated moments for aggregate variables are almost identical to actual ones. The only exception is the volatility of the simulated consumption growth process which is slightly larger than the volatility of the 1951-2012 sample. The simulated consumption growth process is more volatile than the actual one to prevent consumption growth volatility \((\sigma_t)\) from taking negative values during simulations.

The simulated consumption growth volatility is not compared to sample data because consumption growth volatility is not readily observable. In contrast, the simulated volatility process is compared to a discretized CIR process (Cox, Ingersoll, and Ross 1985) which is calibrated to volatility dynamics from Chen et al. (2006) and Bansal and Yaron (2004). There are two main differences between the stochastic volatility process here and the one in Chen et al. (2006). First, I assume a less volatile consumption growth process that matches consumption growth volatility in the BEA sample. Second, the volatility process here is more persistent than the one in Chen et al. (2006) so as to fit the persistence of the price-dividend ratio.

### 3.3 Baa-Aaa credit spreads for the simulated economy

The simulated results in the previous section are in line with sample moments for the U.S. economy during the 1951-2012 period. Matching macroeconomic dynamics (consumption, dividends, earnings, volatility) guarantees that asset prices for the simulated economy are not driven by abnormally persistent, excessively volatile, or extremely correlated state variables. Next, I test the asset pricing predictions of the disappointment model.

#### 3.3.1 Baa-Aaa credit spreads for the CRRA model

To better illustrate the magnitude of the credit spread puzzle, I first examine the predictions of a consumption-based model with CRRA investors. Towards this goal, I consider a \(T\)-period, zero-coupon bond written on a firm’s assets. This bond pays \$1 if the firm remains solvent at time

\[21\] Because the volatility dynamics in (5) admit negative values, if the simulated consumption growth volatility becomes negative, then the negative observation is replaced with the previous observation.
$t + T$, and $$(1 - L) < 1$$ otherwise. I focus on zero-coupon bonds because the inclusion of coupon payments does not really affect credit spreads (Chen et al. 2009, p. 3384). According to Appendix B, expected bond yields for the CRRA economy are given by\(^{22}\)

$$E[y_{i,t,t+T}] = r_f - \frac{1}{T} \log \left[ 1 - LN \left( N^{-1}(\pi_{i,T}^p) + \frac{\bar{\mu}_i - r_f}{\sigma_i} \sqrt{T} \right) \right].$$  \tag{12}

The variables $y_{i,t,t+T}$ and $r_f$ are the continuously compounded yield-to-maturity and risk-free rate, respectively. $L$ are losses given default, $N()$ is the standard normal c.d.f., and $N^{-1}()$ is its inverse. $\pi_{i,T}^p$ is the physical probability of default, $\bar{\mu}_i$ and $\sigma_i$ are the expected value and standard deviation for asset returns, and $\frac{\bar{\mu}_i - r_f}{\sigma_i}$ is the Sharpe ratio for returns on assets-in-place. One of the key assumptions for deriving the bond pricing equation in (12) is that asset returns are normally distributed under the risk-neutral measure. In Appendix D.1, I show that, in discrete-time models, normality under the risk-neutral measure is preserved if and only if investor preferences are described by CRRA power utility.\(^{23}\) Hence, the model in (12) is essentially a statement about investor preferences even if preference parameters are absent.

In order to generate Baa bond yields from the model in (12), I set the Sharpe ratio equal to 0.22 which is the Sharpe ratio for the median Baa firm in Chen et al. (2009).\(^{24}\) Losses given default ($L$) are set equal to 62.4% in order to match the average recovery rate of 37.6% for senior subordinate bonds in the 2013 Moody’s report.\(^{25}\) Finally, physical probabilities of default ($\pi_{i,T}^p$) are calibrated to the average default rates for Baa bonds during the 1970-2012 period. These rates are shown in Panel A of Table III. Expected yields for Aaa bonds are obtained using the same values for the Sharpe ratio and for losses given default as for Baa bonds. However, default probabilities for Aaa bonds are calibrated to Aaa default rates during the 1970-2012 period. These rates are also shown in Panel A of Table III. Following the credit spread literature, I focus on Baa-Aaa spreads instead

\(^{22}\)This expression is identical to the continuous-time one in Chen et al. (2009), p. 3377. However, Appendix B derives the expression in (12) for a discrete-time economy with CRRA investors.

\(^{23}\)See also Brennan (1979).

\(^{24}\)The Sharpe ratio in (12) is the Sharpe ratio for the firm’s assets-in-place, not the equity Sharpe ratio. However, because returns for assets-in-place are hard to measure, I follow Chen et al. (2009), p. 3375, who proxy asset Sharpe ratios with equity Sharpe ratios.

\(^{25}\)Chen et al. (2009) use an average recovery rate of 44.1%.
of Baa-$r_f$ spreads because Aaa yields encompass liquidity, callability, and tax spreads which are unrelated to default risk, and are ignored by the model in (12).\footnote{Longstaff, Mithal and Neis (2005) find evidence in favor of a liquidity component in corporate bond spreads over treasuries. Ericsson and Renault (2006) suggest that part of the credit spreads can be attributed to taxes.}

Table III illustrates the magnitude of the credit spread puzzle. According to the results in Panel C, the Baa-Aaa spreads implied by the CRRA model in (12) are not consistent with the actual ones. For instance, the sample credit spreads for 4yr bonds (103 bps) are twice as large as the credit spreads generated by the CRRA model (58 bps). Similar results, albeit less pronounced, hold for medium maturities (10yrs). Nevertheless, the CRRA model is able to match average credit spreads for long maturities (15yrs+) because the Sharpe ratio term $\frac{\mu_i - r_f}{\sigma_i} \sqrt{T}$ in (12) becomes larger as maturity $T$ increases.

The credit spread puzzle is also depicted in Figure II. The dotted line shows fitted credit spreads for different maturities according to the CRRA model, while the scattered dots are average Baa-Aaa spreads from Huang and Huang (2012) and the Barclays index. If the CRRA model were able to fit expected credits spread, then the scattered dots would coincide with the model-implied term structure of credit spreads. According to Figure II, the credit spread puzzle is particularly pronounced for short maturities up to 10 years, however, as maturity increases, the CRRA model is able to better fit credit spreads.

Besides the implicit assumption of CRRA preferences, the model in (12) imposes three additional restrictions that can explain its problematic empirical performance. First, asset returns are normally distributed with constant mean and variance.\footnote{Besides constant moments, the normal distribution also appears to be a restrictive assumption. Nevertheless, Huang and Huang (2012) and Chen et al. (2009) show that introducing jumps or relaxing the normality assumption cannot address the credit spread puzzle.} Second, recovery rates in (12) are constant, even though Panel B in Table III shows that recovery rates are most likely procyclical, and Figure III indicates that recovery rates decrease substantially during recessions.\footnote{Altman et al. (2005), and Acharya et al. (2007) also provide evidence in favor of procyclical recovery rates.} Third, default boundaries are constant. Although default boundaries cannot be observed, it seems that time-variation in these boundaries is an important ingredient for resolving the credit spread puzzle. In the next subsection, I relax all these assumptions, and study the predictions of the disappointment
aversion model.

### 3.3.2 Baa-Aaa credit spreads for the disappointment model

In this section, I obtain expected credit spreads for the disappointment aversion discount factor, and compare them to the results for the CRRA model in (12). Contrary to the CRRA model, I am not able to derive closed-form solutions for bond prices due to the complicated structure of disappointment aversion preferences. Instead, I test the asset pricing performance of the disappointment aversion sdf by simulating the unconditional Euler equation for zero-coupon corporate bonds

\[
\mathbb{E}[y_{Baa,t+1} - y_{Aaa,t+1}] = \\
\mathbb{E} \left[ -\frac{1}{T} \log \mathbb{E}_t \left( \prod_{j=1}^{T} M_{t+j-1,t+j} \right) \left( 1 - L_{t+T} 1 \{ r_{Baa,t+T}^{ex} < D_{Baa,t+T} \} \right) \right] - \\
\mathbb{E} \left[ -\frac{1}{T} \log \mathbb{E}_t \left( \prod_{j=1}^{T} M_{t+j-1,t+j} \right) \left( 1 - L_{t+T} 1 \{ r_{Aaa,t+T}^{ex} < D_{Aaa,t+T} \} \right) \right].
\]

\( M_{t+j} \) is the disappointment aversion sdf from (7), \( L_{t+T} \) are losses given default, \( r_{i,t+T}^{ex} \) are ex-payout log-returns for assets-in-place according to (11), and \( D_{i,t+T} \) is the default boundary. Corporate bond spreads in Table III are measured in nominal terms. However, the model economy is simulated in real terms, and thus, credit spreads are inflation-free. To the extend that inflation risk premia are approximately equal for Baa and Aaa bonds, then nominal credit spreads should be very similar to real spreads.

In order to address the shortcomings of the CRRA model, losses given default \( L_{t+T} \) and default boundaries \( D_{i,t+T} \) are allowed to be functions of the state variables \( \Delta c_{t,t+1} \) and \( \sigma_t \)

\[
1 - L_{t+T} = a_{rec,0} + a_{rec,c} \Delta c_{t+T-1,t+T}, \tag{14}
\]

\[
D_{i,t+T} = a_{i,def,0} + a_{def,c} \left( \Delta c_{t+T-1,t+T} - \frac{\mu_c}{1 - \phi_c} \right) + a_{def,\sigma} \left( \sigma_{t+T} - \frac{\mu_\sigma}{1 - \phi_\sigma} \right). \tag{15}
\]

Based on these specifications, I consider four different cases for the disappointment model: 1)
constant recovery rates and default boundaries, 2) procyclical recovery rates and constant default boundaries, 3) constant recovery rates and countercyclical default boundaries, and 4) procyclical recovery rates and countercyclical default boundaries. This setup will allow me to isolate the role of each risk component in generating realistic credit spreads.

Table IV shows the key results of this study. These results have been obtained by simulating equation (13) according to the methodology described in Appendix C. The first step in simulating credit spreads is to specify values for the default boundaries. The default boundaries in Panel A of Table IV are expressed in terms of returns for assets-in-place. For example, the 4yr constant Baa default boundary is equal to -0.998 which means that the value of assets-in-place must decrease to $e^{-0.998} = 36.861\%$ of initial value before a Baa firm defaults.\(^\text{29}\) The default boundaries in Table IV are such that the simulated default rates match the actual ones. Indeed, the simulated default probabilities in Panel B are practically indistinguishable from the default rates in the Moody’s report due to suitably selecting default boundaries.

Panel C in Table IV shows simulated credit spreads when the disappointment aversion discount factor in (7) is calibrated to the parameters from Table I. The first case of the disappointment model, Case 1, assumes constant default and recovery rates. Credit spreads for Case 1 of the disappointment model are larger than those for the CRRA model, yet the average increase is only 7 bps across maturities. Disappointment averse investors are particularly worried about periods during which lifetime utility is less than its certainty equivalent. During these periods, defaults for Baa firms happen more often than defaults for Aaa bonds which are fairly acyclical. Baa-rated bonds should be discounted at higher rates than Aaa-rated ones because Baa corporate bonds expose the aggregate investor to more disappointment risk than Aaa bonds.

Case 1 in Panel C differs from the model in Table III along two dimensions. First, as explained in Appendix D.1, the model in (12) implicitly assumes CRRA preferences. Even though concave CRRA utility functions overweigh unfavorable outcomes, they do not capture asymmetric marginal utility over gains and losses. In contrast, the disappointment model relies heavily on investors

\(^{29}\text{Chen et al. (2009), p. 3384, also assume similar default boundaries for 4 year Baa bonds.\)
penalizing losses that happen during disappointment periods $1 + \theta$ times more than they penalize losses during normal times. Second, the CRRA model assumes constant expected returns. On the other hand, disappointment aversion induces time-variation in expected returns due to time-variation in risk attitudes which is further amplified by stochastic consumption growth volatility.

Despite asymmetric marginal utility and time-variation in expected returns, disappointment aversion alone cannot fully rationalize expected Baa-Aaa credit spreads, especially for short maturities, since Case 1 of the disappointment model can only explain 65 out of 103 bps for 4yr bonds. This result should not cast any doubt on the explanatory power of disappointment aversion. According to Chen et al. (2009), neither the habit, nor the long-run risk models can explain credit spreads,\textsuperscript{30} unless we assume time-varying recovery rates or stochastic default boundaries.

Evidence from Table III suggests that recovery rates are procyclical. Therefore, the assumption of constant recovery rates in Case 1 ignores an important risk source for credit spreads: the procyclicality of recovery rates. Case 2 in Table IV relaxes this assumption, and postulates that recovery rates $(1 - L_{t+T})$ are a linear function of consumption growth as in (14). The parameter $a_{rec,0}$ is set equal to 0.292 and $a_{rec,c}$ is 4.822 to match the regression estimates from Panel B of Table III. According to Table IV, the addition of procyclical recovery rates increases Baa-Aaa by 28 bps on average across maturities relative to the spreads implied by the CRRA model in (12).

When recovery rates are procyclical, corporate bonds must compensate disappointment averse investors for two sources of systematic risk. First, during disappointment events, the default frequency of Baa bonds increases more than the default frequency of Aaa bonds. Second, during disappointment periods, recovery rates decrease. Moreover, disappointment aversion punishes the procyclicality of recovery rates more severely than second-order risk aversion. Nevertheless, even with countercyclical recovery rates, 20 bps in 4yr spreads and 12 bps in 10yr spreads remain unexplained by Case 2 of the disappointment model.

So far, default boundaries for the first two cases of the disappointment model are assumed constant. I relax this assumption in Cases 3 and 4 of the model. For these two cases, I follow Chen \textsuperscript{30}Chen et al. 2009, p. 3384 and p. 3405.
et al. (2009), and specify time-varying exogenous default boundaries that are functions of the state variables. Since default boundaries are unobservable, boundary parameters have been calibrated so that the unconditional default probabilities for the simulated economy match the default rates in the Moody’s sample. The values for $a_{def,c}$ and $a_{def,\sigma}$ in (15) imply that default boundaries are countercyclical. During bad times, when consumption growth (volatility) is lower (higher) than its unconditional mean, default boundaries are high, and managers can easily declare bankruptcy. In contrast, during good times, when consumption growth (volatility) is higher (lower) than its mean, default boundaries are low, and firms cannot default as easily as during bad times.

Countercyclical default boundaries lead to a larger number of defaults during economic downturns, and fewer number of defaults during good times than constant default boundaries. However, unconditionally, average default rates are equal to the ones observed in the data. Countercyclical default boundaries imply that defaults in Case 3 are more countercyclical than in Cases 1 and 2. Indeed, the combination of disappointment aversion preferences with countercyclical default boundaries improves the fit of the model, and increases model-implied credit spreads by 15 bps across maturities relative to the CRRA model in (12). Nevertheless, Case 3 still leaves 28 bps in 4yr spreads and 24 bps in 10yr spreads unexplained.

Case 4 of the disappointment model combines disappointment aversion with procyclical recovery rates and countercyclical default boundaries. According to Table IV, this case can perfectly fit average credit spreads for short and medium maturities, but overestimates credit spreads at the long end of the term structure. Countercyclical default boundaries increase the frequency of defaults during bad times, while procyclical recovery rates imply that losses given default increase during periods of low economic growth. Case 4 of the disappointment model is able to fit Baa-Aaa credit spreads because periods of high default rates and low recoveries are also associated with periods of worse than expected macroeconomic conditions. These periods are heavily penalized by disappointment averse investors who require high yields for holding Baa bonds.

To illustrate why time-variation in recovery rates or default boundaries is a necessary condition
for explaining credit spreads, consider the approximation to the Euler equation for four-year spreads

\[ E[y_{Baa,t,t+4} - y_{Aaa,t,t+4}] \approx \frac{1}{4} E[1_{\{i,t+4,\text{def}\}} = 1] \left[ Cov(M_{t,t+4}, L_{t+4} | 1_{\{Baa,t+4,\text{def}\}} = 1) \right. \\
\left. + E[M_{t,t+4} | 1_{\{Baa,t+4,\text{def}\}} = 1] E[L_{t+4} | 1_{\{Baa,t+4,\text{def}\}} = 1] \right]. \] (16)

If recovery rates were constant, then the covariance term above would be zero, and credit spreads would only be driven by the term \( E[M_{t,t+4} | 1_{\{Baa,t+4,\text{def}\}} = 1] \). However, for reasonable degrees of curvature and asymmetry, this term is not large enough to generate realistic credit spreads. On the other hand, if losses given default are countercyclical, then the covariance in (16) is also positive, and the disappointment aversion discount factor can better fit credit spreads. The analysis is very similar for counter-cyclical default boundaries.

Collectively, the results in Table IV suggest that when losses given default and default boundaries are countercyclical, then disappointment aversion is able to address the credit spread puzzle with preference parameters that are consistent with recent experimental results on risky choices. However, as shown in Figure IV, by fitting average credit spreads for short and medium maturities, the disappointment model overestimates average credit spreads for long maturities. This is the first paper to provide results on long maturities since the credit spread literature has almost exclusively considered either 4yr or 10yr bonds.

### 3.4 Equity premia and the risk-free rate

We could possibly improve the ability of traditional consumption models to match credit spreads by assuming extremely high prices of risk. However, high risk prices would also imply unrealistic equity risk premia. In this subsection, I show that the disappointment aversion model can match key moments for the risk-free rate, the equity premium, and the price-dividend ratio with the same preference parameters and state variable dynamics from Table I.

Towards this goal, I use the expressions in (8), (9), and Proposition 2 to simulate key moments for the risk-free rate, the stock market, and the aggregate price-dividend ratio, respectively. Accord-
ing to the results in Table V, the mean, standard deviation, and autocorrelation for the simulated stock market are almost identical to the 1951-2012 sample. Moreover, the simulated risk-free rate is quite persistent and has a low mean similar to the risk-free rate during the post-war period. However, the volatility of the simulated risk-free rate is lower than the sample volatility because the EIS in the disappointment model is set equal to one. \(^{31}\) Finally, even though the simulated persistence of the price-dividend ratio is accurate, the simulated price-dividend ratio has a lower mean, and is less volatile than the one obtained through CRSP.

Traditional consumption-based asset pricing models with time-separable power utility require high values for the risk aversion parameter in order to generate realistic equity premia: around 50 for annual data, \(^{32}\) and around 150 for quarterly data. \(^{33}\) Moreover, large risk aversion parameters lead to an extremely volatile risk-free rate (Weil 1989). Non-separable Epstein-Zin preferences (Epstein and Zin 1989) also require large risk aversion coefficients, around 30 (Routildege and Zin 2010), to match the equity premium, unless we assume a very persistent process for expected consumption growth. \(^{34}\) These inconsistencies between model parameters at the micro- and macro-levels are concealed by the CRRA model in (12), or any other model that directly uses risk-neutral pricing, because these models do not explicitly account for investor preferences.

Collectively, results in the previous subsections suggest that the disappointment aversion discount factor can generate plausible asset pricing moments across bond and equity markets using parameters that are consistent with experimental results for choices under risk. The next subsection quantifies the importance of the disappointment and second-order risk aversion channels in generating these results.

\(^{31}\) See also Delikouras 2013.
\(^{33}\) Aït-Sahalia et al. (2004), Yogo (2004).
\(^{34}\) Bonomo et al. (2011), Beeler and Campbell (2012), Delikouras (2013)
3.5 Alternative parameters for the disappointment model

This subsection performs a sensitivity analysis on the disappointment aversion discount factor. The analysis focuses on the two parameters that affect risky choices: the risk aversion parameter $\alpha$, and the disappointment aversion coefficient $\theta$. The rest of the parameters in Table I are the same as before. The purpose of this sensitivity analysis is threefold. First, alternative parameters need to be within the range of experimental estimates. Second, the price-payout multipliers $\{A_0, A_3\}$ and $\{A_{m,0}, A_{m,3}\}$ in Propositions 1 and 2, respectively, must be real. Third, alternative parameters should be able to identify the marginal importance of the first- and second-order risk aversion channels.

For the first alternative scenario, the risk aversion parameter is set equal to -1 (second-order risk neutrality), and the disappointment aversion parameter is 1.985. By setting $\alpha$ equal to -1, we are essentially downgrading the importance of consumption growth variance $\sigma^2_t$ as a state variable. This is done through the parameter $A_3$ which significantly decreases in magnitude. According to Table VI, if we turn off the risk aversion channel, and slightly increase the magnitude of disappointment aversion, then the expected risk-free rate decreases relative to the baseline scenario because now the first-order precautionary savings motive in (8) intensifies. In contrast, the risk-free rate volatility for the first alternative scenario is almost identical to the baseline scenario because the risk-free rate volatility is mainly affected by the EIS, which is kept constant across scenarios and equal to 1.

Turning our attention to risky assets, I find that the equity premium for the first alternative scenario is identical to the baseline model: 5.670% for the first alternative scenario versus 5.677% for the baseline disappointment model. Similar results hold for stock market volatility which remains constant across scenarios. On the other hand, expected credit spreads for the first alternative scenario decrease by approximately 23 bps for 4yr maturities across all four cases. Collectively, the results for the first alternative scenario in Table VI suggest that even though equity risk premia are not affected by second-order risk neutrality, shutting down second-order risk aversion has an important impact on credit-spreads.
For the second alternative scenario, \( \theta \) is zero, and the disappointment aversion channel is turned off (first-order risk neutrality), while the risk aversion parameter is set equal to 5. If we shut down disappointment aversion, then there is a decrease in model-implied credit spreads relative to the baseline calibration by approximately 33 bps for 4yr maturities across all four cases. Moreover, when investors are disappointment neutral, the equity risk premium is almost zero because second-order risk aversion with reasonable curvature cannot map consumption growth risk into expected stock returns. Finally, the mean of the risk-free rate for the second alternative scenario increases relative to the baseline case because, without disappointment aversion, the precautionary savings motive in (8) attenuates.

Comparative results in Table VI suggest that disappointment aversion and second-order risk aversion are substitutes when it comes to equity pricing. Specifically, there can be different combinations of disappointment and second-order risk aversion coefficients that generate almost identical moments for equity risk premia. Here, I only focus on two sets of parameters, but it is easy to extend these results to multiple combinations. On the other hand, credit spreads are more sensitive to changes in risk and disappointment aversion parameters. Therefore, in order to disentangle the effects of disappointment and second-order risk aversion, test assets should include corporate bonds. This result is interesting because most behavioral asset pricing models tend to focus on equity premia or the cross-section of expected returns, and have ignored the possible identification issues between first- and second-order risk aversion.

For the second alternative scenario, the risk aversion parameter \( \alpha \) is set equal to 5 which is considered a reasonable value for risk aversion by the asset pricing literature. Nevertheless, experimental results imply that \( \alpha \) cannot be greater than 2.8 (Choi et al. 2007). Moreover, for very high values of \( \alpha \), the multipliers \( A_3 \) and \( A_{m,3} \) in Propositions 1 and 2 are not real numbers, unless we decrease the volatility of consumption growth volatility \( \nu_\sigma \). Therefore, the dynamics for consumption growth volatility in (5) impose restrictions on the magnitude of the risk aversion parameter. These restrictions imply that we cannot resolve asset pricing puzzles simply by increasing the magnitude of the risk aversion parameter.
To illustrate why both disappointment as well as second-order risk aversion are needed to match credit spreads, consider again the credit spread approximation for four-year spreads

\[
\mathbb{E}[y_{Baa,t+4} - y_{Aaa,t+4}] \approx \frac{1}{4} \mathbb{E}[\mathbb{1}_{\{i,t+4,def\}} = 1] \left[ \text{Cov}(M_{t,t+4}, L_{t+4}|\mathbb{1}_{\{Baa,t+4,def\}} = 1) \right]
\]

\[
+ \mathbb{E}[M_{t,t+4}|\mathbb{1}_{\{Baa,t+4,def\}} = 1] \mathbb{E}[L_{t+4}|\mathbb{1}_{\{Baa,t+4,def\}} = 1].
\]

If disappointment and default states almost coincide, i.e., \( \mathbb{E}[\mathbb{1}_{\{dis,t+4\}}|\mathbb{1}_{\{i,t+4,def\}} = 1] \approx 1 \), then

\[
\mathbb{E}[M_{t,t+4}|\mathbb{1}_{\{Baa,t+4,def\}} = 1] \geq \frac{1 + \theta}{1 + \theta \mathbb{E}[M_{t,t+4}|\mathbb{1}_{\{Baa,t+4,def\}} = 1]} \approx 2.439 \text{ for } \theta = 1.439,
\]

and therefore disappointment aversion amplifies the term \( \mathbb{E}[L_{t+4}|\mathbb{1}_{\{Baa,t+4,def\}} = 1] \) more than a CRRA utility function of reasonable curvature. However, when disappointment and default states coincide, first-order risk aversion doesn’t affect \( \text{Cov}(M_{t,t+4}, L_{t+4}|\mathbb{1}_{\{Baa,t+4,def\}} = 1) \), which is mainly influenced by second-order risk aversion.\(^{35}\) The previous analysis suggests that, in order to obtain plausible credit spreads, the disappointment aversion self inflates the term \( \mathbb{E}[L_{t+4}|\mathbb{1}_{\{Baa,t+4,def\}} = 1] \) through first-order risk aversion, and amplifies the term \( \text{Cov}(M_{t,t+4}, L_{t+4}|\mathbb{1}_{\{Baa,t+4,def\}} = 1) \) through second-order risk aversion. If either channel is turned off, then the disappointment model cannot fit credit spreads. This analysis also explains why applying disappointment aversion to corporate bond pricing can help us disentangle the effects of disappointment aversion and traditional risk aversion.

4 Related literature

Before concluding, I briefly relate the disappointment framework to some key results from the corporate securities literature. These results are also summarized in Table VII. Previous evidence on the inability of the Merton model to match credit spreads date back to Jones et al. (1984). Huang and Huang (2012) also show that the credit puzzle is robust to a variety of specifications for the risk-neutral dynamics of asset returns.

\(^{35}\) I would like to thank Jim Dolmas for this suggestion.
In Merton’s early framework, there are no taxes, no bankruptcy costs, and capital structure was irrelevant. Leland (1994) and Leland and Toft (1996), extend Merton’s model to account for tax benefits of debt, bankruptcy costs, and optimal leverage decisions. Goldstein et al. (2001) propose an asset pricing model in which bondholders, equityholders, and the government have stakes in the firm’s EBIT-generating process. Moreover, in Goldstein et al. (2001), setting the value of bond coupons, declaring default, and choosing the optimum amount of leverage are all endogenous decisions. However, these papers focus on risk-neutral dynamics, and do not examine the empirical implications of their model across different security markets.

In contrast, Bhamra et al. (2010) propose a unified framework to explain the equity premium and the credit-spread puzzles simultaneously. Even though the aggregate investor is characterized by Epstein-Zin preferences, the authors use risk-neutral dynamics to construct a model with endogenous capital structure and default decisions. Nevertheless, their model generates average credit spreads of 45 bps for 5yr maturities and 75 bps for 10yr bonds, while the model-implied equity risk premium is only 3.190%.

Chen et al. (2009) compare the habit model of Campbell and Cochrane (1999) to the long-run risk model of Bansal and Yaron (2004). Although, both models can explain the equity premium puzzle, the long-run risk model is not able to generate realistic credit-spreads, while the habit-model requires countercyclical default boundaries or procyclical recovery rates in order to fit Baa-Aaa spreads. Finally, Chen (2010) addresses the credit spread and underleverage puzzles simultaneously, while matching moments for equity risk premia. However, he only focuses on 10yr maturities. Most of these papers tend to focus on 4yr or 10yr maturities. It would be useful to re-visit all these models, and test whether fitting credit spreads for the short end of the term structure implies overestimating the spreads at the long end.
5 Conclusion

This paper examines whether disappointment aversion can address the credit spread puzzle within a consumption-based asset pricing framework of an endowment economy. When recovery rates are calibrated to realistic procyclical patterns, and default boundaries are countercyclical, then the disappointment model is able to fit expected credit spreads while matching key moments for stock market returns, the risk-free rate, and the persistence of the price-dividend ratio.

More importantly, the disappointment model can successfully fit asset moments using risk and disappointment aversion parameters whose values are consistent with the empirical findings at the micro-level. Finally, the disappointment model is based on macroeconomic dynamics that do not exaggerate the persistence, the comovement, or the volatility of aggregate state variables. Directions for future research include using disappointment aversion to fit the entire term structure of credit spreads and interest rates, or introducing disappointment aversion in an economy where capital structure matters so as to endogenize default decisions and the underleverage puzzle.
References


6 Figures

Figure I  Baa-Aaa credit spreads and default rates for Baa bonds (1951-2012)

Figure I shows Baa-Aaa credit spreads and Baa default rates during the 1951-2012 period. The solid line shows Baa-Aaa credit spreads for the Moody’s corporate bond index, and the dashed line shows annual Baa default rates from the Moody’s 2013 annual report. Shaded areas are NBER recessions.
Figure II  Sample and fitted credit spreads for the CRRA model

Figure II shows sample and fitted credit spreads for Baa and Aaa bonds. The scattered points are average Baa-Aaa credit spreads for the Huang and Huang (2012) sample (4yr and 10yr maturities) and for the Barclays long-term corporate bond index (15yrs+). The dotted line shows fitted Baa-Aaa credit spreads for maturities from 1 up to 20 years according to the CRRA model in (12).
Figure III  Recovery rates and aggregate consumption growth (1982-2012)

Figure III shows recovery rates and consumption growth during the 1982-2012 period. The solid line shows recovery rates for senior subordinate bonds from the Moody’s 2013 report, and the dotted line shows BEA aggregate consumption growth. Shaded areas are NBER recessions.
Figure IV  Sample and fitted credit spreads for the disappointment model

Figure IV shows sample and fitted expected credit spreads for Baa and Aaa bonds. Sample spreads are from the Huang and Huang (2012) sample (4yr and 10yr maturities) and from the Barclays long-term corporate bond index (15yrs+). Fitted credit spreads are according to the disappointment model in (13) and the CRRA model in (12).
### Tables

#### Table I  Baseline parameters for the disappointment model

Table I shows model parameters for the state variable dynamics in (4)-(6) and the disappointment aversion stochastic discount factor in (7).

<table>
<thead>
<tr>
<th>variable</th>
<th>variable description</th>
<th>variable value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>elasticity of intertemporal substitution</td>
<td>0</td>
</tr>
<tr>
<td>$\delta$</td>
<td>generalized disappointment aversion</td>
<td>1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>rate of time preference</td>
<td>0.997</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>risk aversion</td>
<td>1.900</td>
</tr>
<tr>
<td>$\theta$</td>
<td>disappointment aversion</td>
<td>1.439</td>
</tr>
<tr>
<td>$\mu_c$</td>
<td>consumption growth constant</td>
<td>0.0106</td>
</tr>
<tr>
<td>$\phi_c$</td>
<td>consumption growth autocorrelation</td>
<td>0.4646</td>
</tr>
<tr>
<td>$\mu_\sigma$</td>
<td>volatility constant</td>
<td>0.0005</td>
</tr>
<tr>
<td>$\phi_\sigma$</td>
<td>volatility autocorrelation</td>
<td>0.9680</td>
</tr>
<tr>
<td>$\nu_\sigma$</td>
<td>volatility of volatility</td>
<td>0.0017</td>
</tr>
<tr>
<td>$\mu_d$</td>
<td>dividend growth constant</td>
<td>-0.0574</td>
</tr>
<tr>
<td>$\phi_d$</td>
<td>consumption growth sensitivity</td>
<td>4</td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>volatility parameter</td>
<td>6.3690</td>
</tr>
<tr>
<td>$\mu_e$</td>
<td>earnings growth constant</td>
<td>-0.0523</td>
</tr>
<tr>
<td>$\phi_e$</td>
<td>consumption growth sensitivity</td>
<td>3.8</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>volatility parameter</td>
<td>5.3560</td>
</tr>
<tr>
<td>$\sigma_i^\alpha$</td>
<td>idiosyncratic volatility</td>
<td>0.1900</td>
</tr>
<tr>
<td>$z_m^\alpha$</td>
<td>idiosyncratic volatility</td>
<td>3</td>
</tr>
<tr>
<td>$\hat{x}$</td>
<td>linearization constant for the normal c.d.f. in (24)</td>
<td>0</td>
</tr>
</tbody>
</table>
Table II  Simulated moments for the aggregate state variables

Table II shows sample and simulated moments for the aggregate state variables. $\mathbb{E}[]$ is expected value, $\text{Vol}()$ is volatility, and $\rho()$ is the correlation coefficient. $\Delta c_{t-1,t}$, $\Delta d_{m,t-1,t}$, and $\Delta e_{m,t-1,t}$ are real consumption, dividend, and earnings growth, respectively. $\sigma_t$ is consumption growth volatility. Simulated moments for the state variables are generated according to the system in (4)-(6) with parameters from Table I. Sample results for consumption, dividend, and earnings growth are according to the BEA, CRSP, and Compustat datasets, respectively. Sample results for consumption growth volatility are based on the volatility process from Chen et al. (2006, p. 31). All variables have been simulated for 100,000 years.

<table>
<thead>
<tr>
<th></th>
<th>1951-2012 sample</th>
<th>simulated economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{E}[\Delta c_{t+1}]$</td>
<td>1.982%</td>
<td>1.982%</td>
</tr>
<tr>
<td>$\text{Vol}(\Delta c_{t+1})$</td>
<td>1.237%</td>
<td>1.981%</td>
</tr>
<tr>
<td>$\rho(\Delta c_{t-1,t}, \Delta c_{t+1})$</td>
<td>0.464</td>
<td>0.461</td>
</tr>
<tr>
<td>$\mathbb{E}[\Delta d_{m,t+1}]$</td>
<td>1.915%</td>
<td>1.938%</td>
</tr>
<tr>
<td>$\text{Vol}(\Delta d_{m,t+1})$</td>
<td>13.705%</td>
<td>13.654%</td>
</tr>
<tr>
<td>$\rho(\Delta d_{m,t+1}, \Delta e_{m,t+1})$</td>
<td>0.265</td>
<td>0.264</td>
</tr>
<tr>
<td>$\mathbb{E}[\Delta e_{m,t+1}]$</td>
<td>1.870%</td>
<td>1.831%</td>
</tr>
<tr>
<td>$\text{Vol}(\Delta e_{m,t+1})$</td>
<td>12.046%</td>
<td>12.042%</td>
</tr>
<tr>
<td>$\rho(\Delta e_{m,t+1}, \Delta e_{m,t+1})$</td>
<td>0.291</td>
<td>0.292</td>
</tr>
<tr>
<td>$\mathbb{E}[\sigma_t]$</td>
<td>2.697%</td>
<td>1.622%</td>
</tr>
<tr>
<td>$\text{Vol}(\sigma_t)$</td>
<td>0.458%</td>
<td>0.664%</td>
</tr>
<tr>
<td>$\rho(\sigma_t)$</td>
<td>0.922</td>
<td>0.964</td>
</tr>
</tbody>
</table>
Table III  Default rates, recovery rates, and Baa-Aaa credit spreads (1970-2012)

Table III shows default rates, recovery rates, as well as Baa-Aaa credit spreads for the 1970-2012 period. In Panel A, average default rates for Baa and Aaa firms are from the Moody’s 2013 annual report. Panel B shows OLS regression results of recovery rates \((1 - L_t)\) on aggregate consumption growth \((\Delta c_{t-1,t})\) with \(t\)-statistics in parenthesis. Panel C summarizes sample and fitted credit spreads. Sample spreads are from the Huang and Huang (2012) sample (4yr and 10yr maturities) and from the Barclays long-term corporate bond index (15yrs+). Fitted credit spreads are according to the CRRA model in (12).

### Panel A: average default rates (1970-2012)

<table>
<thead>
<tr>
<th></th>
<th>1 year</th>
<th>4 year</th>
<th>10 year</th>
<th>15 year</th>
<th>20 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>0.000%</td>
<td>0.037%</td>
<td>0.503%</td>
<td>0.935%</td>
<td>1.104%</td>
</tr>
<tr>
<td>Baa</td>
<td>0.177%</td>
<td>1.369%</td>
<td>4.740%</td>
<td>8.628%</td>
<td>12.483%</td>
</tr>
</tbody>
</table>

### Panel B: recovery rates and consumption growth (1982-2012)

<table>
<thead>
<tr>
<th></th>
<th>(1 - L_t)</th>
<th>cons.</th>
<th>(\Delta c_{t-1,t})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta c_{t-1,t})</td>
<td>4.822 (3.353)</td>
<td>0.292 (9.543)</td>
<td>0.279</td>
</tr>
</tbody>
</table>

### Panel C: average Baa-Aaa credit spreads (bps)

<table>
<thead>
<tr>
<th>maturity</th>
<th>sample period</th>
<th>short</th>
<th>medium</th>
<th>long</th>
</tr>
</thead>
<tbody>
<tr>
<td>Huang and Huang (2012)</td>
<td>1973-1993</td>
<td>103</td>
<td>131</td>
<td></td>
</tr>
<tr>
<td>CRRA model in (12)</td>
<td></td>
<td>58</td>
<td>89</td>
<td>111</td>
</tr>
</tbody>
</table>
Table IV  Default rates and credit spreads for the disappointment model

Table IV shows fitted credit spreads for the four cases of the disappointment model. These cases are: 1) constant recovery rates and default boundaries, 2) procyclical recovery rates as in (14) and constant default boundaries, 3) constant recovery rates and countercyclical default boundaries as in (15), and 4) procyclical recovery rates and countercyclical default boundaries. In Panel A, default boundaries for the simulated economy are expressed in terms of asset log-returns. The constants $a_{i,def,0}$, $a_{def,c}$, and $a_{def,\sigma}$ are the default boundary parameters from equation (15). Panel B shows average default rates for the simulated economy and the Moody’s sample. Panel C shows simulated Baa-Aaa credit spreads according to the four cases of the disappointment model in (13), and the CRRA model in (12). Sample spreads are from the Huang and Huang (2012) sample (4yr and 10yr maturities), and from the Barclays long-term corporate bond index (15yrs+).

### Panel A: default boundaries

<table>
<thead>
<tr>
<th></th>
<th>4 yr</th>
<th>10 yr</th>
<th>15 yr</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baa</td>
<td>Aaa</td>
<td>Baa</td>
</tr>
<tr>
<td>$a_{i,def,0}$</td>
<td>-1.053</td>
<td>-1.781</td>
<td>-1.140</td>
</tr>
<tr>
<td>$a_{def,c}$</td>
<td>-4</td>
<td>-4</td>
<td>-4</td>
</tr>
<tr>
<td>$a_{def,\sigma}$</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

### Panel B: average default rates

<table>
<thead>
<tr>
<th></th>
<th>4 year</th>
<th></th>
<th></th>
<th>15 year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baa</td>
<td>Aaa</td>
<td>Baa</td>
<td>Aaa</td>
</tr>
<tr>
<td>Case 1</td>
<td>1.365%</td>
<td>0.036%</td>
<td>4.728%</td>
<td>0.503%</td>
</tr>
<tr>
<td>Case 2</td>
<td>1.366%</td>
<td>0.035%</td>
<td>4.718%</td>
<td>0.505%</td>
</tr>
<tr>
<td>Case 3</td>
<td>1.382%</td>
<td>0.035%</td>
<td>4.742%</td>
<td>0.502%</td>
</tr>
<tr>
<td>Case 4</td>
<td>1.378%</td>
<td>0.036%</td>
<td>4.744%</td>
<td>0.503%</td>
</tr>
<tr>
<td>1970-2012 sample</td>
<td>1.369%</td>
<td>0.037%</td>
<td>4.740%</td>
<td>0.503%</td>
</tr>
</tbody>
</table>

### Panel C: credit spreads for the disappointment model

<table>
<thead>
<tr>
<th></th>
<th>4 year</th>
<th></th>
<th></th>
<th>10 year</th>
<th></th>
<th></th>
<th>15 year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baa-$r_f$</td>
<td>Aaa-$r_f$</td>
<td>Baa-Aaa</td>
<td>Baa-$r_f$</td>
<td>Aaa-$r_f$</td>
<td>Baa-Aaa</td>
<td>Baa-$r_f$</td>
</tr>
<tr>
<td>Case 1</td>
<td>68</td>
<td>3</td>
<td>65</td>
<td>126</td>
<td>28</td>
<td>98</td>
<td>160</td>
</tr>
<tr>
<td>Case 2</td>
<td>88</td>
<td>5</td>
<td>83</td>
<td>156</td>
<td>37</td>
<td>119</td>
<td>196</td>
</tr>
<tr>
<td>Case 3</td>
<td>79</td>
<td>4</td>
<td>75</td>
<td>139</td>
<td>32</td>
<td>107</td>
<td>170</td>
</tr>
<tr>
<td>Case 4</td>
<td>108</td>
<td>6</td>
<td>102</td>
<td>177</td>
<td>44</td>
<td>133</td>
<td>216</td>
</tr>
<tr>
<td>CRRA sample</td>
<td>58</td>
<td></td>
<td>89</td>
<td></td>
<td></td>
<td>111</td>
<td></td>
</tr>
<tr>
<td></td>
<td>103</td>
<td></td>
<td>131</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table V  Equity premium and risk-free rate for the disappointment model

Table V shows sample and simulated moments for the stock market and the risk-free rate according to the disappointment aversion sdf in (7). $r^d_{m,t:t+1}$ are real stock market returns, $r^d_{f,t:t+1}$ is the real risk-free rate, and $z^d_{m,t}$ is the aggregate price-dividend ratio. The firm-level Sharpe ratio is the equity Sharpe ratio for the median firm. Asset moments for the risk-free rate, stock market returns, and the aggregate price-dividend ratio are simulated based on the expressions in (8), (9), and Proposition 2, respectively.

<table>
<thead>
<tr>
<th></th>
<th>1951-2012</th>
<th>simulated economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[r^d_{m,t:t+1}]$</td>
<td>6.842%</td>
<td>6.904%</td>
</tr>
<tr>
<td>$\text{Vol}(r^d_{m,t:t+1})$</td>
<td>17.426%</td>
<td>16.345%</td>
</tr>
<tr>
<td>$\rho(r^d_{m,t-1:t}, r^d_{m,t:t+1})$</td>
<td>-0.050</td>
<td>0.031</td>
</tr>
<tr>
<td>Market Sharpe ratio</td>
<td>0.321</td>
<td>0.347</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[r^d_{f,t:t+1}]$</td>
<td>1.232%</td>
<td>1.226%</td>
</tr>
<tr>
<td>$\text{Vol}(r^d_{f,t:t+1})$</td>
<td>2.265%</td>
<td>1.034%</td>
</tr>
<tr>
<td>$\rho(r^d_{f,t-1:t}, r^d_{f,t:t+1})$</td>
<td>0.749</td>
<td>0.565</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[z^d_{m,t}]$</td>
<td>3.419</td>
<td>2.977</td>
</tr>
<tr>
<td>$\text{Vol}(z^d_{m,t})$</td>
<td>0.438</td>
<td>0.239</td>
</tr>
<tr>
<td>$\rho(z^d_{m,t}, z^d_{m,t-1})$</td>
<td>0.820</td>
<td>0.824</td>
</tr>
<tr>
<td>firm-level Sharpe ratio</td>
<td>0.220</td>
<td>0.227</td>
</tr>
</tbody>
</table>
Table VI  Alternative parameters for the disappointment model

Table VI shows simulation results for Baa-Aaa credits spreads, stock market returns, and the risk-free rate when the disappointment aversion discount factor in (7) is calibrated to alternative preference parameters. In the baseline scenario, $\theta$ is 1.439 and $\alpha$ is 1.900. For the first alternative scenario, $\theta$ is 1.985 and $\alpha$ is -1 (second-order risk neutrality). For the second alternative scenario, $\theta$ is zero (first-order risk neutrality), and $\alpha$ is 5. $E[r_{m,t+1}^{d} - r_{f,t+1}]$ is the stock market risk premium.

<table>
<thead>
<tr>
<th>Case</th>
<th>Baa-Aaa 4yr</th>
<th>baseline</th>
<th>scenario I</th>
<th>scenario II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta = 1.439, \alpha = 1.900$</td>
<td>$\theta = 1.985, \alpha = -1$</td>
<td>$\theta = 0, \alpha = 5$</td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>65</td>
<td>47</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>Case 2</td>
<td>83</td>
<td>59</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>Case 3</td>
<td>75</td>
<td>52</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>Case 4</td>
<td>102</td>
<td>69</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>$E[r_{m,t,t+1}^{d} - r_{f,t,t+1}]$</td>
<td>5.677%</td>
<td>5.670%</td>
<td>0.764%</td>
<td></td>
</tr>
<tr>
<td>$Vol(r_{m,t,t+1}^{d})$</td>
<td>16.345%</td>
<td>16.707%</td>
<td>16.459%</td>
<td></td>
</tr>
<tr>
<td>$E[r_{f,t,t+1}]$</td>
<td>1.226%</td>
<td>1.039%</td>
<td>1.980%</td>
<td></td>
</tr>
<tr>
<td>$Vol(r_{f,t,t+1})$</td>
<td>1.034%</td>
<td>1.045%</td>
<td>0.947%</td>
<td></td>
</tr>
</tbody>
</table>
Table VII  Credit spreads and equity premia in the literature

Table VII shows fitted credit spreads and equity risk premia in the credit spread literature. The constant $\alpha$ is the risk aversion coefficient, EIS is the elasticity of intertemporal substitution, and $\theta$ is the disappointment aversion parameter. $\mathbb{E}[r_{m,t,t+1}^d - r_{f,t,t+1}]$ is the stock market risk premium.

<table>
<thead>
<tr>
<th>Model characteristics</th>
<th>Maturity</th>
<th>$\mathbb{E}[r_{m,t,t+1}^d - r_{f,t,t+1}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chen et al. (2009)</td>
<td>4yr</td>
<td>107</td>
</tr>
<tr>
<td></td>
<td>10yr</td>
<td>124</td>
</tr>
<tr>
<td></td>
<td>15yr</td>
<td>7.300%</td>
</tr>
<tr>
<td>Chen et al. (2009)</td>
<td>countercyclical boundaries &amp; losses given default</td>
<td>52</td>
</tr>
<tr>
<td>Chen (2010)</td>
<td>endogenous default, EIS=1.500, $\alpha = 6.500$</td>
<td>104.5</td>
</tr>
<tr>
<td>Bhamra et al. (2010)</td>
<td>risk-neutral measure</td>
<td>45 (5yr)</td>
</tr>
<tr>
<td>Huang &amp; Huang (2012)</td>
<td>Goldstein et al. (2001) model</td>
<td>31</td>
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<td>Case 4 in Table IV</td>
<td>EIS=1, $\alpha = 1.900$, $\theta = 1.439$, countercyclical boundaries &amp; losses given default</td>
<td>102</td>
</tr>
</tbody>
</table>

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Appendix

Appendix A  Stochastic discount factor for the disappointment model

In this section, I derive the disappointment aversion stochastic discount factor in (1). The starting point is the disappointment model in Routledge and Zin (2010). According to this model, along an optimal consumption path, lifetime utility $V_t$ for disappointment averse investors satisfies

$$V_t = [(1 - \beta)C_t^\rho + \beta \mu_t(V_{t+1})^\rho]^{\frac{1}{\rho}},$$

in which $\mu_t$ is the disappointment aversion certainty equivalent from (2).

The expression for the stochastic discount factor is given by

$$M_{t,t+1} = \frac{\partial V_t/\partial C_{t+1}}{\partial V_t/\partial C_t},$$

in which

$$\frac{\partial V_t}{\partial C_t} = \frac{1}{\rho}V_t^{1-\rho}(1 - \beta)\rho C_t^{\rho-1},$$

and

$$\frac{\partial V_t}{\partial C_{t+1}} = \frac{1}{\rho}V_t^{1-\rho}\beta \rho \mu_t(V_{t+1})^{\rho-1} \left(-\frac{1}{\alpha}\right) \mathbb{E}_t\left[\frac{V_{t+1}^{-\alpha} (1 + \theta \mathbb{1}\{V_{t+1} < \delta \mu_t\})}{1 + \theta \delta - \alpha \mathbb{E}_t[\mathbb{1}\{V_{t+1} < \delta \mu_t\}]}\right]^{-\frac{1}{\alpha} - 1} \times$$

$$(-\alpha)V_t^{-\alpha-1} \frac{1 + \theta \mathbb{1}\{V_{t+1} < \delta \mu_t\}}{1 + \theta \delta - \alpha \mathbb{E}_t[\mathbb{1}\{V_{t+1} < \delta \mu_t\}]} \frac{1}{\rho}V_{t+1}^{1-\rho}(1 - \beta)\rho C_{t+1}^{\rho-1},$$

to conclude that

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{(\rho-1)} \left[\frac{V_{t+1}}{\mu_t(V_{t+1})}\right]^{-\alpha-\rho} \left[\frac{1 + \theta \mathbb{1}\{V_{t+1} < \delta \mu_t\}}{1 + \theta \delta - \alpha \mathbb{E}_t[\mathbb{1}\{V_{t+1} < \delta \mu_t\}]}\right].$$
Appendix B  Bond prices according to the CRRA model

In this section, I derive the bond pricing formula for a CRRA economy. The proof relies on the fact that if asset returns are normally distributed under the physical measure, and investors are characterized by CRRA preferences, then, under the risk-neutral measure, asset returns are also normally distributed with mean equal to the risk free rate.

Suppose that one-period, cum-payout, asset log-returns for firm \( i \) \((r_{e,i,t,t+1}^e)\) are i.i.d. normal random variables with constant mean \( \tilde{\mu}_i - \frac{1}{2}\sigma_i^2 \), and volatility \( \sigma_i \). Suppose also that \( \Delta_i \) is the constant log-payout yield, \( \Delta_i = \log(1 + \frac{E_{i,t+1}}{P_{i,t+1}}) \), in which \( E_{i,t+1} \) are earnings before interest, and \( P_{i,t+1}^e \) is the price for assets-in-place.\(^{36}\) Then, one-period, ex-payout log-returns, \( r_{e,ex,i,t,t+1}^e = r_{e,i,t,t+1}^e - \Delta_i \), are also normal random variables, and, in a discrete-time setting, these returns are given by

\[
r_{e,ex,i,t,t+1}^e = \tilde{\mu}_i - \Delta_i - \frac{1}{2}\sigma_i^2 + \sigma_i\epsilon_{i,t+1},
\]

with \( \epsilon_{i,t+1} \) i.i.d. \( N(0,1) \) shocks. Consequently, \( T \)-period ex-payout returns are also i.i.d. normal random variables with mean \((\tilde{\mu}_i - \Delta_i - \frac{1}{2}\sigma_i^2)T\) and volatility \( \sigma_i\sqrt{T} \).

Suppose now that the log risk-free rate \( (r_f) \), the default boundary \( (D_{i,T}) \), as well as losses given default \( (L) \) are all constants. Suppose also that there are no taxes or transaction costs, and let \( \pi_{i,t,t+T}^P \) be the physical probability of default for a \( T \)-period zero-coupon bond

\[
\pi_{i,t,t+T}^P = \mathbb{P}_t(P_{i,t,T}^e < D_{i,T}).
\]

If we normalize current period firm value \( P_{i,t}^e \) to one, then the physical probability of default can be expressed in terms of asset log-returns as follows

\[
\pi_{i,t,t+T}^P = N\left(\frac{\log D_{i,T} - (\tilde{\mu}_i - \Delta_i - \frac{1}{2}\sigma_i^2)T}{\sigma_i\sqrt{T}}\right),
\]

(17)

where \( N() \) is the standard normal c.d.f.. From (17), we conclude that \( \pi_{i,t,t+T}^P \) depends only on

\(^{36}\)If consumption growth is i.i.d. in the CRRA economy, then the price-payout ratio is also constant.
maturity $T$, $\pi_{i,t,t+T}^P = \pi_{i,T}^P$. Finally, we can use the inverse of the normal c.d.f., $N^{-1}(\cdot)$, to express the unobservable log-default boundary ($\log D_{i,T}$) as a function of the physical probability of default ($\pi_{i,T}^P$), expected returns for assets-in-place ($\mu_i$), and asset return volatility ($\sigma_i$)

$$\log D_{i,T} = (\mu_i - \Delta_i - \frac{1}{2} \sigma_i^2)T + N^{-1}(\pi_{i,T}^P)\sigma_i \sqrt{T}. \quad (18)$$

The continuous-time framework in Black and Scholes (1973) allows for frictionless trading and hedging between underlying and derivative securities. Therefore, if asset returns under the physical measure are normally distributed with constant mean and variance, then asset returns under the risk-neutral measure are also normally distributed with the same variance, and mean equal to the risk-free rate.

Continuous trading is not possible in a discrete-time setting. However, according to Lemma 1 in Appendix D.1, if asset returns are normally distributed under the physical measure, preferences are described by a CRRA utility function, and aggregate consumption growth is a normal random variable, then asset returns are also normally distributed under the risk-neutral measure with the same variance, and mean equal to the risk-free rate. Therefore, if all conditions in Lemma 1 hold, then $T$-period log-returns under the risk-neutral measure are normally distributed with mean $(r_f - \Delta_i - \frac{1}{2} \sigma_i^2)T$ and volatility $\sigma_i \sqrt{T}$.

Now, let $y_{i,t,t+T}$ be the continuously compounded yield to maturity for a $T$-period, zero-coupon bond written on firm $i$ at time $t$. Then, under the risk-neutral measure

$$e^{-Ty_{i,t,t+T}} = e^{-Tr_f} \left(1 - LN\left(\frac{\log D_{i,T} - (r_f - \Delta_i - \frac{1}{2} \sigma_i^2)T}{\sigma_i \sqrt{T}}\right)\right). \quad (19)$$

Taking logs in (19), and substituting $\log D_{i,T}$ with the expression from (18), we get that

$$y_{i,t,t+T} - r_f = -\frac{1}{T} \log \left[1 - LN\left(N^{-1}(\pi_{i,T}^P) + \frac{\mu_i - r_f}{\sigma_i} \sqrt{T}\right)\right].$$
Since the right-hand side and the risk-free rate are constants, we conclude that

$$E_t[y_{i,t,t+T} - r_f] = -\frac{1}{T} \log \left[ 1 - LN\left(N^{-1}(\pi^{P}_{i,T}) + \frac{\hat{\mu}_i - r_f}{\sigma_i} \sqrt{T}\right) \right].$$

Appendix C Simulations

Appendix C.1 Simulation methodology

In this section, I describe the simulation methodology for the disappointment aversion model. According to (13), the consumption-Euler equation for a $T$-period zero-coupon bond reads

$$e^{-Ty_{i,t,t+T}} = E_t\left[ \prod_{j=1}^{T} M_{t+j-1,t+j} \left( \mathbb{1}\{r_{i,t,t+T}^{e,ex} \geq D_{i,t} + T\} + (1 - L_{t+T})\mathbb{1}\{r_{i,t,t+T}^{e,ex} < D_{i,t} + T\} \right) \right].$$

The first step in simulating bond yields is to discretize the consumption growth and consumption growth volatility space into $N_{\Delta c} = 20$ and $N_{\sigma} = 20$ equidistant points with a pace of $d_{\Delta c}$ and $d_{\sigma}$, respectively. The consumption growth space is truncated by $\hat{E}[\Delta c_{t-1,t}] \pm 3\hat{\text{Vol}}(\Delta c_{t-1,t})$, whereas the volatility space is truncated by $\hat{E}[\sigma_{t}] \pm 2\hat{\text{Vol}}(\sigma_{t})$. $\hat{E}[\cdot]$ and $\hat{\text{Vol}}(\cdot)$ are the simulated unconditional means and standard deviations from Table II. The tighter bound for the volatility space guarantees that the initial values for volatility are always positive.

The second step is to choose starting values for consumption growth and consumption growth volatility. To do so, I iterate though all possible pairs of $\{\Delta c_{l}, \sigma_{k}\}$ for $l = 1, 2, ..., N_{\Delta c}$ and $k = 1, 2, ..., N_{\sigma}$ as starting values. Then, for each pair of starting values, I simulate $N = 10,000^{37}$ paths for consumption growth, consumption growth volatility, and payout growth according to the system of equations in (4)-(6). I also simulate idiosyncratic volatility shocks. Each simulated path contains $T$ nodes, as many nodes as the life of of the zero-coupon security. Negative volatility observations are replaced by the lowest positive observation from the volatility grid.

At each node of the simulated path, I obtain values for the disappointment aversion stochastic

\[37\]Simulation results are not affected by the number of simulation paths $N$ or the number of grid points $(N_{\Delta c}, N_{\sigma})$, provided of course that these numbers are relatively large.
discount factor $M_{t+j-1,t+j}$ from (7), for the price-payout ratio from Proposition 2, and for returns on assets-in-place from (11). I also obtain values for losses given default and default boundaries from (14) and (15). For each simulated path, the discounted cashflow of a zero-coupon, corporate bond is given by

$$\left(\prod_{j=1}^{T} M_{t+j-1,t+j}\right) \left(1\{r_{i,t,t+T}^{e,ex} \geq D_{i,t+T}\} + (1 - L_{t+T})1\{r_{i,t,t+T}^{e,ex} < D_{i,t+T}\}\right).$$

where $T$-period log-returns are simply the sum of single-period returns $r_{i,t,t+T}^{e,ex} = \sum_{j=1}^{T} r_{i,t}^{e,ex} - 1, t+1$.

Finally, averaging across all $N$ simulated paths, we obtain the yield to maturity as a function of the initial values for $\Delta c_{t-1,t}$ and $\sigma_t$

$$\hat{y}_{i,t,t+T}(\Delta c_l, \sigma_k) \approx -\frac{1}{T} \log \left[ \frac{1}{N} \sum_{n=1}^{N} \left(\prod_{j=1}^{T} M_{t+j-1,t+j}^{(n)}\right) \left(1\{r_{i,t,t+T}^{e,ex} \geq D_{i,t+T}^{(n)}\} + (1 - L_{t+T})^{(n)}1\{r_{i,t,t+T}^{e,ex} < D_{i,t+T}^{(n)}\}\right) \right].$$

The objective now is to match unconditional first moments for credit spreads. Therefore, I calculate unconditional expected values for credit spreads over the grid of starting values for consumption growth and consumption growth volatility as follows

$$\mathbb{E}[\hat{y}_{i,t,t+T}(\Delta c_l, \sigma_k)] \approx \sum_{j=1}^{N_o} \left\{ \sum_{k=1}^{N_o} \sum_{l=1}^{N_{\Delta c}} \hat{y}_{i,t,t+T}(\Delta c_l, \sigma_k) f(\Delta c_l | \sigma_k, \sigma_j) d_{\Delta c}^l \right\} f(\sigma_j) d_{\sigma}^l,$$

where $f(\Delta c_l | \sigma_k, \sigma_j)$, $f(\sigma_k | \sigma_j)$, and $f(\sigma_j)$ are the p.d.f.’s for $\Delta c_{t-1,t}$, $\sigma_t$, and $\sigma_{t-1}$, respectively. The derivation of these p.d.f.’s is described in Appendix C.2. Finally, $d_{\Delta c}^l$, $d_{\sigma}^l$, and $d_{\sigma}^o$ are constants such that $\sum_{l=1}^{N_{\Delta c}} f(\Delta c_l | \sigma_k, \sigma_j) d_{\Delta c}^l = 1$, $\sum_{k=1}^{N_o} f(\sigma_k | \sigma_j) d_{\sigma}^l = 1$, and $\sum_{j=1}^{N_o} f(\sigma_j) d_{\sigma}^o = 1$.

**Appendix C.2 Unconditional density functions for $\Delta c_{t-1,t}$, $\sigma_{t-1}$, and $\sigma_t$**

This section derives the p.d.f.’s for consumption growth and consumption growth volatility used in simulations. From (5), consumption growth volatility ($\sigma_{t-1}$) is unconditionally normally distributed
with mean $\mu_{\sigma}/(1 - \phi_{\sigma})$ and variance $\nu_{\sigma}^2/(1 - \phi_{\sigma}^2)$. Further, according to the expression in (4), conditional on $\sigma_{t-1}$, consumption growth is also normally distributed with mean

$$\mathbb{E}[\Delta c_{t-1,t}|\sigma_{t-1}] = \frac{\mu_c}{1 - \phi_c},$$

and variance

$$\text{Var}(\Delta c_{t-1,t}|\sigma_{t-1}) = \frac{\sigma_{t-1}^2}{1 - \phi_c^2}.$$

From equations (4)-(5), we know that the p.d.f. for $\sigma_{t-1}$ is

$$f(\sigma_{t-1}) = \frac{1}{\sqrt{2\pi} \nu_{\sigma}/\sqrt{1 - \phi_{\sigma}^2}} e^{-\frac{(\sigma_{t-1} - \mu_{\sigma})^2}{2\nu_{\sigma}^2/(1 - \phi_{\sigma}^2)}},$$

the p.d.f. for $\sigma_t|\sigma_{t-1}$ is given by

$$f(\sigma_t|\sigma_{t-1}) = \frac{1}{\sqrt{2\pi} \nu_{\sigma}} e^{-\frac{(\sigma_t - \mu_{\sigma} - \phi_{\sigma} \sigma_{t-1})^2}{2\nu_{\sigma}^2}},$$

and the p.d.f for $\Delta c_{t-1,t}$ conditional on $\sigma_t$ and $\sigma_{t-1}$ is

$$f(\Delta c_{t-1,t}|\sigma_t, \sigma_{t-1}) = \frac{1}{\sqrt{2\pi} (\sigma_{t-1}/\sqrt{1 - \phi_{\sigma}^2})} e^{-\frac{(\Delta c_{t-1,t} - \mu_c/(1 - \phi_c))^2}{2\sigma_{t-1}^2/(1 - \phi_c^2)}}.$$

Therefore, the joint p.d.f. for $\Delta c_{t-1,t}$, $\sigma_t$ and $\sigma_{t-1}$ can be written as

$$f(\Delta c_{t-1,t}, \sigma_t, \sigma_{t-1}) = \frac{1}{\sqrt{2\pi} (\nu_{\sigma}/\sqrt{1 - \phi_{\sigma}^2})} e^{\frac{(\sigma_t - \mu_{\sigma} - \phi_{\sigma} \sigma_{t-1})^2}{2\nu_{\sigma}^2}} \times \frac{1}{\sqrt{2\pi} \nu_{\sigma}} e^{-\frac{(\Delta c_{t-1,t} - \mu_{\sigma}/(1 - \phi_{\sigma}))^2}{2\nu_{\sigma}^2/(1 - \phi_{\sigma}^2)}} \times \frac{1}{\sqrt{2\pi} (\sigma_{t-1}/\sqrt{1 - \phi_{\sigma}^2})} e^{-\frac{(\Delta c_{t-1,t} - \mu_c/(1 - \phi_c))^2}{2\sigma_{t-1}^2/(1 - \phi_c^2)}}.$$
Appendix D  Proofs

Appendix D.1  Lemma 1

This lemma shows that if aggregate preferences are characterized by a CRRA utility function, and asset returns are normally distributed under the physical measure with constant mean and variance, then, under the risk-neutral measure, asset returns are also normally distributed with the same variance, and mean equal to the risk-free rate.

Lemma 1: Suppose that cum-payout asset returns \( r_{i,t+1} \) are i.i.d. normal random variables with constant mean \( \tilde{\mu}_i - \frac{1}{2} \sigma_i^2 \) and volatility \( \sigma_i \). Suppose also that financial markets are complete, that there exists a representative investor with CRRA power utility defined over consumption,\(^{38}\) that log-consumption growth \( \Delta c_{t+1} \) is normally distributed with constant mean \( \tilde{\mu}_c \) and volatility \( \sigma_c \), and that the correlation coefficient between \( r_{i,t+1} \) and \( \Delta c_{t+1} \) is \( \rho_{i,c} \). Then, the log risk-free rate \( r_f \) is constant, and asset returns under the risk-neutral measure \( Q \) are i.i.d. normal random variables with constant mean \( r_f - \frac{1}{2} \sigma_i^2 \) and volatility \( \sigma_i \).

Proof:

In equilibrium, the consumption-Euler equation for asset returns implies that

\[
\tilde{\mu}_i + \log \beta - \alpha \tilde{\mu}_c + \frac{1}{2} \alpha^2 \sigma_c^2 - \alpha \rho_{i,c} \sigma_c \sigma_1 = 0.
\]

(20)

where \( \beta \in (0, 1) \) is the rate of time-preference, and \( \alpha \geq -1 \) is the second-order risk aversion parameter. Similarly, the Euler equation for the log risk-free rate reads

\[
r_f + \log \beta - \alpha \tilde{\mu}_c + \frac{1}{2} \alpha^2 \sigma_c^2 = 0.
\]

(21)

Hence, the risk-free rate is a constant because \( \mu_c \) and \( \sigma_c \) are also constants.

We can rewrite the consumption-Euler equation in (20) using the p.d.f. for \( \Delta c_{t+1} \) conditional

\(^{38}\)See Chapter 1 of Duffie (2000), and Chapter 5 in Huang and Litzenberger (1989).
on \( r_{i,t,t+1} \)

\[
\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_i} e^{r_{i,t,t+1} - \frac{(r_{i,t,t+1} - \hat{\mu}_i + 0.5\sigma_i^2)^2}{2\sigma_i^2}} e^{-\alpha[i(\hat{\mu}_i + \rho_{i,c}\sigma_c)(r_{i,t,t+1} - \hat{\mu}_i + 0.5\sigma_i^2)] + \frac{1}{2}\alpha^2(1 - \rho_{i,c}^-)\sigma_i^2} dr_{i,t,t+1} = 1.
\]

Exploiting the consumption-Euler conditions in (20) and (21), we obtain

\[
e^{-rf} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_i} e^{r_{i,t,t+1} - \frac{(r_{i,t,t+1} - r_f + 0.5\sigma_i^2)^2}{2\sigma_i^2}} e^{-\alpha[i(\hat{\mu}_i + \rho_{i,c}\sigma_c)(r_{i,t,t+1} - r_f + 0.5\sigma_i^2)] + \frac{1}{2}\alpha^2(1 - \rho_{i,c}^-)\sigma_i^2} dr_{i,t,t+1} = 1.
\]

Further algebra yields

\[
e^{-rf} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_i} e^{r_{i,t,t+1} - \frac{(r_{i,t,t+1} - r_f + 0.5\sigma_i^2)^2}{2\sigma_i^2}} e^{-\alpha[i(\hat{\mu}_i + \rho_{i,c}\sigma_c)(r_{i,t,t+1} - r_f + 0.5\sigma_i^2)] + \frac{1}{2}\alpha^2(1 - \rho_{i,c}^-)\sigma_i^2} dr_{i,t,t+1} = 1.
\]

Canceling out terms, we conclude that

\[
e^{-rf} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_i} e^{r_{i,t,t+1} - \frac{(r_{i,t,t+1} - r_f + 0.5\sigma_i^2)^2}{2\sigma_i^2}} e^{-\alpha[i(\hat{\mu}_i + \rho_{i,c}\sigma_c)(r_{i,t,t+1} - r_f + 0.5\sigma_i^2)] + \frac{1}{2}\alpha^2(1 - \rho_{i,c}^-)\sigma_i^2} dr_{i,t,t+1} = 1.
\]

**Appendix D.2  Lemma 2**

In this section, I prove a property for the quadratic exponential of a normal random variable. This property facilitates the derivation of analytical solutions for the disappointment aversion sdf and asset returns in Propositions 1 and 2.

**Lemma 2:** Let \( x \) be a normal random variable with mean \( \mu \) and volatility \( \sigma \). Let \( A \) and \( B \) two real numbers with \( B > -\frac{1}{2\sigma^2} \), then

\[
\mathbb{E}
\left[
\begin{array}{c}
e^{-Ax-Bx^2} \\
\end{array}
\right]
= e^{rac{0.5A^2\sigma^2 - A\mu - B\mu^2}{1 + 2B\sigma^2}} \frac{1}{\sqrt{1 + 2B\sigma^2}}.
\]
Proof:

\[
E[e^{-Ax-Bx^2}] = \frac{1}{\sqrt{2\pi \sigma}} \int_{-\infty}^{+\infty} e^{-\frac{2A\sigma^2x^2-2\sigma^2Bx^2-x^2-\mu^2+2\mu x}{2\sigma^2}} dx.
\]

Completing the square in the right-hand side,

\[
E[e^{-Ax-Bx^2}] = e^{\frac{\left(\frac{\mu-A\sigma^2}{\sqrt{1+2B\sigma^2}}\right)^2-\mu^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi \sigma}} \int_{-\infty}^{+\infty} e^{-\frac{(1+2B\sigma^2)x^2+2\frac{\mu-A\sigma^2}{\sqrt{1+2B\sigma^2}}\sqrt{1+2B\sigma^2}x-\left(\frac{\mu-A\sigma^2}{\sqrt{1+2B\sigma^2}}\right)^2}{2\sigma^2}} dx.
\]

After a change of variables, \( \tilde{x} = \sqrt{1+2B\sigma^2} x \), we conclude that

\[
E[e^{-Ax-Bx^2}] = e^{\frac{0.5A^2\sigma^2_{x,-A\mu-B\mu^2}}{1+2B\sigma^2}} \frac{1}{\sqrt{1+2B\sigma^2}}.
\]

Appendix D.3 Proof of Proposition 1

In order to prove Proposition 1, I exploit the linear homogeneity of disappointment aversion, the assumptions that the EIS and the GDA parameters are one, and the linear structure of consumption growth and consumption growth volatility. This proof also makes use of Lemma 2 and the method of undetermined coefficients.

When the EIS is equal to one, the Bellman equation for the disappointment averse investor becomes

\[
V_t = C_t^{1-\beta} \mu_t (V_{t+1})^\beta,
\]

where \( \mu_t \) is the disappointment aversion certainty equivalent from (2) with \( \delta = 1 \). Suppose that the log utility-wealth ratio is linear in the state variables, \( v_t - c_t = A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma_t^2 \),
\[
\text{then the Bellman equation reads}
\]

\[
\exp\left[\frac{1}{\beta}(A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma_t^2)\right] = \mathbb{E}_t\left\{\exp\left[\frac{1}{\beta}(A_0 + (A_1 + 1) \Delta c_{t,t+1} + A_2 \sigma_{t+1} + A_3 \sigma_{t+1}^2)\right]\right\}
\]

\[
1 + \theta \mathbb{E}_t\left\{A_0 + (A_1 + 1) \Delta c_{t,t+1} + A_2 \sigma_{t+1} + A_3 \sigma_{t+1}^2 < \frac{1}{\beta}(A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma_t^2) \right\}^{-\frac{1}{\theta}}
\]

Dividing both parts by the left-hand side, we obtain

\[
\mathbb{E}_t\left\{\exp\left[\frac{1}{\beta}(A_0(1 - \frac{1}{\beta}) + A_1((1 + \frac{1}{A_1}) \Delta c_{t-1,t} - \frac{1}{\beta} \Delta c_{t-1,t}) + A_2(\sigma_{t+1} - \frac{1}{\beta} \sigma_t) + A_3(\sigma_{t+1}^2 - \frac{1}{\beta} \sigma_t^2))\right]\right\}
\]

\[
1 + \theta \mathbb{E}_t\left\{A_0 + (A_1 + 1) \Delta c_{t,t+1} + A_2 \sigma_{t+1} + A_3 \sigma_{t+1}^2 < \frac{1}{\beta}(A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma_t^2) \right\}^{-\frac{1}{\theta}} = 1.
\]

Recall that consumption growth and volatility shocks are independent. Therefore, we can rewrite the above condition as

\[
\mathbb{E}_t\left\{\exp\left[\frac{1}{\beta}(A_0(1 - \frac{1}{\beta}) + A_1((1 + \frac{1}{A_1}) \Delta c_{t-1,t} - \frac{1}{\beta} \Delta c_{t-1,t}) + A_2(\sigma_{t+1} - \frac{1}{\beta} \sigma_t) + A_3(\sigma_{t+1}^2 - \frac{1}{\beta} \sigma_t^2))\right]\right\}
\]

\[
1 + \theta \mathbb{E}_t\left\{A_0 + (A_1 + 1) \Delta c_{t,t+1} + A_2 \sigma_{t+1} + A_3 \sigma_{t+1}^2 < \frac{1}{\beta}(A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma_t^2) \right\}^{-\frac{1}{\theta}} = 1.
\]

Using the consumption growth dynamics in (4) and the partial moments for the normal distribution, the above expression becomes

\[
\mathbb{E}_t\left\{\exp\left[\frac{1}{\beta}(A_0(1 - \frac{1}{\beta}) + (A_1 + 1)(\mu_c + \phi_c \Delta c_{t-1,t}) - \frac{1}{\beta} A_1 \Delta c_{t-1,t} - \frac{1}{2} \alpha (A_1 + 1)^2 \sigma_t^2 + A_2(\sigma_{t+1} - \frac{1}{\beta} \sigma_t) + A_3(\sigma_{t+1}^2 - \frac{1}{\beta} \sigma_t^2))\right]\right\}
\]

\[
1 + \theta N\left(X_t + \alpha(A_1 + 1) \sigma_t\right)\right\}^{-\frac{1}{\theta}} = 1,
\]

where \(N()\) is the standard normal c.d.f., and

\[
X_t = \frac{\frac{1}{\beta}(A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma_t^2) - A_0 - (A_1 + 1) \mu_c - (A_1 + 1) \phi_c \Delta c_{t-1,t} - A_2 \sigma_{t+1} - A_3 \sigma_{t+1}^2}{(A_1 + 1) \sigma_t},
\]

is the reference level for disappointment.
For $\theta$ and $N(X_t)$ small numbers such that $\theta N(X_t) \approx 0$,$^{39}$ we can use the approximation $1 + \theta N(y) \approx e^{\theta N(y)}$ to get

$$
\mathbb{E}_t \left\{ \exp \left[ -\alpha \left( A_0(1 - \frac{1}{\beta}) + (A_1 + 1)(\mu_c + \phi_c \Delta c_{t-1,t}) - \frac{1}{\beta} A_1 \Delta c_{t-1,t} - \frac{1}{2} \alpha (A_1 + 1)^2 \sigma_t^2 \right.ight.
\left. + A_2(\sigma_{t+1} - \frac{1}{\beta} \sigma_t) + A_3(\sigma_{t+1}^2 - \frac{1}{\beta} \sigma_t^2) \right] \exp \left[ \theta \left( N(X_t + \alpha(A_1 + 1) \sigma_t) - N(X_t) \right) \right] \right\} = 1.
$$

Further, we can use the first-order approximation for the difference of the two standard normal c.d.f.'s, $N(x) - N(y) \approx n(\bar{x})(x - y)$,$^{40}$ to obtain

$$
\exp \left[ -\alpha A_0(1 - \frac{1}{\beta}) - \alpha [(A_1 + 1)(\mu_c + \phi_c \Delta c_{t-1,t}) - \frac{1}{\beta} A_1 \Delta c_{t-1,t}] + \frac{1}{2} \alpha^2 (A_1 + 1)^2 \sigma_t^2 \right.
\left. + A_2(\alpha A_1 + 1) \sigma_t + \alpha \frac{1}{\beta} A_2 \sigma_t + \alpha \frac{1}{\beta} A_3 \sigma_t^2 \right] \mathbb{E}_t \left\{ \exp \left[ -\alpha A_2 \sigma_{t+1} - \alpha A_3 \sigma_{t+1}^2 \right] \right\} = 1,
$$

where $n()$ is the standard normal p.d.f.. Finally, combining the volatility dynamics from (5) with Lemma 2 from Appendix D.2, the Bellman equation becomes

$$
\exp \left[ -\alpha A_0(1 - \frac{1}{\beta}) - \alpha [(A_1 + 1)(\mu_c + \phi_c \Delta c_{t-1,t}) - \frac{1}{\beta} A_1 \Delta c_{t-1,t}] + \frac{1}{2} \alpha^2 (A_1 + 1)^2 \sigma_t^2 \right.
\left. + A_2(\alpha A_1 + 1) \sigma_t + \alpha \frac{1}{\beta} A_2 \sigma_t + \alpha \frac{1}{\beta} A_3 \sigma_t^2 \right] \frac{1}{\sqrt{1 + 2\alpha A_3 \nu^2_\sigma}}
\times \exp \left[ \frac{0.5 \alpha^2 A_2^2 \nu^2_\sigma - \alpha A_2 \mu_c - \alpha A_2 \phi_c \sigma_t - \alpha A_3 \mu^2_\sigma - \alpha A_3 \phi^2_\sigma \sigma_t^2 - 2\alpha A_3 \mu_c \phi_c \sigma_t}{1 + 2\alpha A_3 \nu^2_\sigma} \right] = 1.
$$

We can now solve for the utility-consumption ratio parameters $A_0$, $A_1$, $A_2$, and $A_3$ using the method of undetermined coefficients. We first collect $\Delta c_{t-1,t}$ terms from (25) to get

$$
A_1 = \frac{\beta \phi_c}{1 - \beta \phi_c}.
$$

Note that since $\beta \in (0, 1)$ and $\phi_c \in (-1, 1)$, $A_1 + 1$ is positive. Also, for $\beta \in (0, 1)$, the sign of $A_1$ depends only on the sign of $\phi_c$.

$^{39}$In simulations, the probability of disappointment events is less than 0.5.

$^{40}$For this approximation to be valid, we require that $\frac{\alpha}{1 - \beta \phi_c} \sigma_t$ to be small.
Collecting $\sigma_t^2$ terms in (25), $A_3$ must satisfy the following quadratic equation

$$2\alpha \nu_\sigma^2 A_3 + [1 - \beta \phi_\sigma^2 + \beta \alpha^2 (A_1 + 1)^2 \nu_\sigma^2] A_3 + \frac{1}{2} \beta \alpha (A_1 + 1)^2 = 0. \quad (27)$$

The ratio of the constant term over the quadratic coefficient in the quadratic equation (27) is a positive number $(\beta (A_1 + 1)^2 / 4 \nu_\sigma^2)$. Therefore, the roots of the quadratic equation are of the same sign. Furthermore, since $\beta \in (0, 1)$ and $\phi_\sigma \in (-1, 1)$, then $1 - \beta \phi_\sigma^2$ is positive, $- [1 - \beta \phi_\sigma^2 + \beta \alpha^2 (A_1 + 1)^2 \nu_\sigma^2]$ is negative, and the solutions to the quadratic equation are negative. For $\alpha \neq 0$, these solutions are

$$A_3 = - [1 - \beta \phi_\sigma^2 + \beta \alpha^2 (A_1 + 1)^2 \nu_\sigma^2] \pm \sqrt{[1 - \beta \phi_\sigma^2 + \beta \alpha^2 (A_1 + 1)^2 \nu_\sigma^2]^2 - 4 \beta \alpha^2 (A_1 + 1)^2 \nu_\sigma^2} \over 4 \alpha \nu_\sigma^2. \quad (28)$$

I choose the largest negative root so that the quadratic solution in (28) is similar to the linear approximation in (29) below.

For $A_3$ to be real, we require that

$$[1 - \beta \phi_\sigma^2 + \beta \alpha^2 (A_1 + 1)^2 \nu_\sigma^2]^2 - 4 \beta \alpha^2 (A_1 + 1)^2 \nu_\sigma^2 > 0.$$  

For $\nu_\sigma^2 \approx 0$ the determinant in (28) is approximately equal to

$$\lim_{\nu_\sigma^2 \downarrow 0} [1 - \beta \phi_\sigma^2 + \beta \alpha^2 (A_1 + 1)^2 \nu_\sigma^2]^2 - 4 \beta \alpha^2 (A_1 + 1)^2 \nu_\sigma^2 \approx [1 - \beta \phi_\sigma^2]^2 > 0.$$  

Therefore, for very small $\nu_\sigma^2$, $A_3$ is real. The restriction that $\nu_\sigma^2$ is a very small number implies that higher moments for consumption growth are well defined. Finally, for $\nu_\sigma^2 \approx 0$, equation (27) becomes linear, and an approximate solution for $A_3$ is

$$A_3 \approx - \frac{1}{2} \frac{\beta \alpha (A_1 + 1)^2}{1 - \beta \phi_\sigma^2}. \quad (29)$$
Collecting $\sigma_t$ terms in (25), we obtain the following solution for $A_2$

$$A_2 = \frac{-\theta n(\bar{x})(A_1 + 1)(1 + 2\alpha A_3 \nu^2_\sigma) + 2\beta A_3 \mu_\sigma \phi_\sigma}{1 + 2\alpha A_3 \nu^2_\sigma - \beta \phi_\sigma}. \quad (30)$$

It is easy to verify that if $A_3$ is negative and $\theta$ is positive, then $A_2$ is also negative. Furthermore, as $\nu^2_\sigma \downarrow 0$, an approximate solution for $A_2$ reads

$$A_2 \approx \frac{-\theta n(\bar{x})(A_1 + 1) + 2\beta A_3 \mu_\sigma \phi_\sigma}{1 - \beta \phi_\sigma}. \quad (31)$$

Finally, the remaining constant terms in (25) give the solution for the constant term $A_0$

$$A_0 = \frac{\beta}{1 - \beta}[(A_1 + 1)\mu_c + \frac{1}{1 + 2\alpha A_3 \nu^2_\sigma}(A_2 \mu_\sigma + A_3 \mu^2_\sigma - 0.5\alpha A^2_2 \nu^2_\sigma) + \frac{1}{2\alpha} \log(1 + 2\alpha A^2_3 \nu^2_\sigma)], \quad (32)$$

with the approximation for $\nu^2_\sigma \downarrow 0$

$$A_0 \approx \frac{\beta}{1 - \beta}[(A_1 + 1)\mu_c + A_2 \mu_\sigma + A_3 \mu^2_\sigma]. \quad (33)$$

**Appendix D.4 The log risk-free rate**

In this section, I derive an approximate solution for the risk-free rate based on the disappointment aversion discount factor from (7). The proof is almost identical to the one for Proposition 1.

The Euler condition for the log risk-free rate reads

$$e^{-r_{f,t,t+1}} = \mathbb{E}_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left( \frac{V_{t+1}}{\mu_t(V_{t+1})} \right)^{-\frac{1}{2}} \text{E}_t[1 + \theta \mathbb{1}\{V_{t+1} < \mu_t\}]. \right]$$

Repeating all the steps that lead to equation (24) in Appendix D.3, the Euler equation becomes

$$e^{-r_{f,t,t+1}} = \exp \left[ \log \beta - \alpha A_0 (1 - \frac{1}{\beta}) - \left[ \alpha (A_1 + 1) \right] \mu_c + \phi_c \Delta c_{t-1,t} - \frac{1}{\beta} A_1 \Delta c_t \right] + \frac{1}{2} \left[ \alpha (A_1 + 1) + 1 \right]^2 \sigma^2_t + \theta n(\bar{x}) \left[ \alpha (A_1 + 1) + 1 \right] \sigma_t + \alpha \frac{1}{\beta} A_2 \sigma_t + \alpha \frac{1}{\beta} A_3 \sigma^2_t \right] \mathbb{E}_t \left\{ \exp \left[ -\alpha A_2 \sigma_{t+1} - \alpha A_3 \sigma^2_{t+1} \right] \right\}.$$
But from (24) we know that
\[
\exp\left[ -\alpha A_0 (1 - \frac{1}{\beta}) - \alpha (A_1 + 1)(\mu_c + \phi_c \Delta c_{t-1,t}) - \frac{1}{\beta} A_1 \Delta c_{t-1,t} \right] + \frac{1}{2} \alpha^2 (A_1 + 1)^2 \sigma_t^2 + \\
\alpha \theta n(\bar{x}) (A_1 + 1) \sigma_t + \alpha \frac{1}{\beta} A_2 \sigma_t + \alpha \frac{1}{\beta} A_3 \sigma_t^2 \right] E_t \left\{ \exp\left[ -\alpha A_{t+1} - \alpha A_3 \sigma_t^2 \right] \right\} = 1.
\]

Therefore, the log risk-free rate must be approximately equal to
\[
r_{f,t,t+1} \approx -\log \beta + \mu_c + \phi_c \Delta c_{t-1,t} - \frac{1}{2} [2\alpha (A_1 + 1) + 1] \sigma_t^2 - \theta n(\bar{x}) \sigma_t.
\]

Appendix D.5 Proof of Proposition 2

The proof of this proposition is similar to one for Proposition 1. I first conjecture that the log price-payout ratio \( z_{m,t} \) for a claim on a stream of aggregate payments is an affine function\(^ {41} \) of the three state variables: \( \Delta c_{t-1,t}, \sigma_t, \sigma_t^2 \)

\[
z_{m,t}^o = A_{m,0} + A_{m,1} \Delta c_{t-1,t} + A_{m,1} \sigma_t + A_{m,2} \sigma_t^2. \tag{34}
\]

Then, I use the Euler equation, and the method of undetermined coefficients to endogenously solve for the set of parameters \( A_{m,0} - A_{m,3} \).

We start by combining the price-dividend ratio identity in (9) with our conjecture about \( z_{m,t}^o \) in (34) to re-write the Euler equation for asset returns as

\[
E_t \left[ M_{t,t+1} \cdot \exp \left\{ \kappa_{m,0} + \kappa_{m,1}(A_{m,0} + A_{m,1} \Delta c_{t,t+1} + A_{m,2} \sigma_{t+1} + A_{m,3} \sigma_t^2) \right. \right. \\
- \left( A_{m,0} + A_{m,1} \Delta c_{t-1,t} + A_{m,2} \sigma_t + A_{m,3} \sigma_t^2 \right) + \Delta o_{m,t,t+1} \right] = 1.
\]

Substituting the the disappointment aversion discount factor \( M_{t,t+1} \) from (7), we can further re-

\(^ {41}\text{I suppress the superscript} o \text{ from} A_{m,0}^o, A_{m,1}^o, A_{m,2}^o, \text{ and} A_{m,3}^o \text{ to ease notation.} \)
write the Euler equation as

\[
E_t \left\{ \exp \left( \log \beta - \Delta c_{t,t+1} \right) \right. \\
\left. - \alpha \left[ A_0 (1 - \frac{1}{\beta}) + [(A_1 + 1) \Delta c_{t,t+1} - \frac{1}{\beta} A_1 \Delta c_{t-1,t}] + A_2 (\sigma_{t+1} - \frac{1}{\beta} \sigma_t) + A_3 (\sigma^2_{t+1} - \frac{1}{\beta} \sigma^2_t) \right] \right\} \\
\times 1 + \theta \left\{ A_0 + (A_1 + 1) \Delta c_{t,t+1} + A_2 \sigma_{t+1} + A_3 \sigma^2_{t+1} < \frac{1}{2} (A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma^2_t) \right\} \\
\times \exp \left\{ \kappa_{m,0} + \kappa_{m,1} (A_{m,0} + A_{m,1} \Delta c_{t,t+1} + A_{m,2} \sigma_{t+1} + A_{m,3} \sigma^2_{t+1}) \right\} \\
- (A_{m,0} + A_{m,1} \Delta c_{t-1,t} + A_{m,2} \sigma_t + A_{m,3} \sigma^2_t) + \Delta \phi_{m,t,t+1} \right\} = 1.
\]

Following the same line of arguments as in Appendix D.3, the Euler equation becomes

\[
\exp \left[ \log(\beta) - \alpha (A_0 - \frac{1}{\beta} A_0) - \left[ [\alpha (A_1 + 1) + 1 - \kappa_{m,1} A_{m,1}] (\mu_c + \phi_c \Delta c_{t-1,t}) + \alpha \frac{1}{\beta} A_1 \Delta c_{t-1,t}] \right] \\
+ \frac{1}{2} \left[ \alpha (A_1 + 1) + 1 - \kappa_{m,1} A_{m,1} \right]^2 \sigma^2_t \theta n(\bar{x}) \left[ \alpha (A_1 + 1) + 1 - \kappa_{m,1} A_{m,1} \right] \sigma_t + \alpha \frac{1}{\beta} A_2 \sigma_t + \alpha \frac{1}{\beta} A_3 \sigma^2_t \\
+ \kappa_{m,0} + A_{m,0} (\kappa_{m,1} - 1) - A_{m,1} \Delta c_{t-1,t} - A_{m,2} \sigma_t - A_{m,3} \sigma^2_t + \mu_m + \phi_m \Delta c_{t-1,t} + \frac{1}{2} \sigma^2_t \sigma^2_t \right] \\
\times \exp \left[ \frac{0.5 (\alpha A_2 - \kappa_{m,1} A_{m,2}) \nu^2_{\sigma}}{1 + 2(\alpha A_3 - \kappa_{m,1} A_{m,3}) \nu^2_{\sigma}} - (\alpha A_2 - \kappa_{m,1} A_{m,2}) \mu_{\sigma} - (\alpha A_2 - \kappa_{m,1} A_{m,2}) \phi_{\sigma} \sigma_t \right] \\
\times \exp \left[ \frac{- (\alpha A_3 - \kappa_{m,1} A_{m,3}) \nu^2_{\sigma} - (\alpha A_3 - \kappa_{m,1} A_{m,3}) \phi^2_{\sigma} \sigma^2_t - 2 (\alpha A_3 - \kappa_{m,1} A_{m,3}) \mu_{\sigma} \phi_{\sigma} \sigma_t}{1 + 2(\alpha A_3 - \kappa_{m,1} A_{m,3}) \nu^2_{\sigma}} \right] \\
\times \frac{1}{\sqrt{1 + 2(\alpha A_3 - \kappa_{m,1} A_{m,3}) \nu^2_{\sigma}}} = 1.
\]

We are now able to solve for the price-dividend ratio parameters \( A_{m,0}, A_{m,1}, A_{m,2}, \) and \( A_{m,3} \) using the method of undetermined coefficients. Specifically, collecting consumption growth terms in (35), \( A_{m,1} \) must satisfy the following equation

\[- \phi_c - \alpha (A_1 + 1) \phi_c + \frac{1}{\beta} \alpha A_1 + \kappa_{m,1} A_{m,1} \phi_c - A_{m,1} + \phi_m = 0.\]

Using the expression for \( A_1 \) from (26), we conclude that

\[ A_{m,1} = \frac{\phi_m - \phi_c}{1 - \kappa_{m,1} \phi_c}. \]
Collecting $\sigma^2_t$ terms in (35), $A_{m,3}$ must satisfy the quadratic equation

$$
\frac{1}{2} \beta \left[ (\alpha (A_1 + 1) + 1 - \kappa_{m,1} A_{m,1})^2 + \sigma^2_m \right] \left[ 1 + 2(\alpha A_3 - \kappa_{m,1} A_{m,3}) \nu^2_\sigma \right] + \alpha A_3 \left( 1 + 2 \alpha A_3 \nu^2_\sigma - \beta \phi^2_\sigma \right) - 2 \alpha A_3 \kappa_{m,1} A_{m,3} \nu^2_\sigma - \beta A_{m,3} - 2 \alpha A_3 \nu^2_\sigma \beta A_{m,3} + 2 \beta \kappa_{m,1} \nu^2_\sigma A^2_{m,3} + \beta \kappa_{m,1} \phi^2_\sigma A_{m,3} = 0.
$$

After tedious algebra, the solution for $A_{m,3}$ is equal to

$$
A_{m,3} = \frac{-\tilde{b} \pm \sqrt{\tilde{b}^2 - 4 \tilde{a} \tilde{c}}}{2 \tilde{a}},
$$

with

$$
\tilde{a} = 2 \beta \kappa_{m,1} \nu^2_\sigma, \\
\tilde{b} = -\beta + \beta \kappa_{m,1} \phi^2_\sigma - 2 \alpha A_3 \kappa_{m,1} \nu^2_\sigma - 2 \alpha A_3 \nu^2_\sigma, \\
\tilde{c} = \frac{1}{2} \beta \left[ (\alpha (A_1 + 1) + 1 - \kappa_{m,1} A_{m,1})^2 + \sigma^2_m \right] \left( 1 + 2 \alpha A_3 \nu^2_\sigma \right) + \alpha A_3 \left( 1 + 2 \alpha A_3 \nu^2_\sigma - \beta \phi^2_\sigma \right).
$$

I choose the largest negative root so that the quadratic solution in (37) is similar to its linear approximation in (38) below. For $A_{m,3}$ to be real, the determinant in (37) also needs to be real. This conditions is satisfied for very small $\nu^2_\sigma$, and reasonable values for the risk aversion coefficient $\alpha$. Finally, since $\nu^2_\sigma$ is a small number close to zero, we can replace $A_3$ from equation (29) to obtain an approximate solution for $A_{m,3}$

$$
A_{m,3} \approx \frac{1}{2} \left[ \frac{\alpha (A_1 + 1) + 1 - \kappa_{m,1} A_{m,1}}{1 - \kappa_{m,1} \phi^2_\sigma} \right]^2.
$$

After collecting $\sigma_t$ terms in (35), the solution for $A_{2,m}$ is given by

$$
A_{m,2} = \frac{\theta \beta n(\bar{x}) \left[ \alpha (A_1 + 1) + 1 - \kappa_{m,1} A_{m,1} \right] \left[ 1 + 2 (\alpha A_3 - \kappa_{m,1} A_{m,3}) \nu^2_\sigma \right]}{\beta + 2 \beta \left( \alpha A_3 - \kappa_{m,1} A_{m,3} \right) \nu^2_\sigma - \beta \kappa_{m,1} \phi_\sigma} + \frac{\alpha A_2 \left[ 1 + 2 (\alpha A_3 - \kappa_{m,1} A_{m,3}) \nu^2_\sigma - \beta \phi_\sigma \right] - 2 \beta (\alpha A_3 - \kappa_{m,1} A_{m,3}) \mu_\sigma \phi_\sigma}{\beta + 2 \beta (\alpha A_3 - \kappa_{m,1} A_{m,3}) \nu^2_\sigma - \beta \kappa_{m,1} \phi_\sigma}.
$$
For $\nu_\sigma^2 \approx 0$, and the approximate expressions for $A_3$ and $A_2$ in (29) and (31), respectively, we conclude that

$$A_{m,2} \approx \frac{\theta n(\bar{x})(1 - \kappa_{m,1}A_{m,1}) + 2\kappa_{m,1}A_{m,3}\mu_\sigma \phi_\sigma}{1 - \kappa_{m,1}\phi_\sigma}.$$ 

Finally, collecting all the constant terms in (35), we obtain the value for $A_{m,0}$

$$A_{m,0} = \frac{1}{1 - \kappa_{m,1}} \left[ \log \beta + \kappa_{m,0} + \mu_m - \alpha A_0 \frac{\beta - 1}{\beta} - \left[ \alpha (A_1 + 1) + 1 - \kappa_{m,1}A_{m,1}\right] \mu_c 
\right.

\left. - (\alpha A_2 - \kappa_{m,1}A_{m,2})\mu_\sigma + (\alpha A_3 - \kappa_{m,1}A_{m,3})\mu_\sigma^2 - 0.5(\alpha A_2 - \kappa_{m,1}A_{m,2})^2 \nu_\sigma^2 
\right. 

\left. \frac{1 + 2(\alpha A_3 - \kappa_{m,1}A_{m,3})\nu_\sigma^2}{2(\alpha A_3 - \kappa_{m,1}A_{m,3})\nu_\sigma^2}
\right]

-0.5\log \left(1 + 2(\alpha A_3 - \kappa_{m,1}A_{m,3})\nu_\sigma^2 \right).$$

Exploiting the fact that $\nu_\sigma^2 \approx 0$, and using the expression for $A_0$ in (33), $A_{m,0}$ can be approximated as

$$A_{m,0} \approx \frac{1}{1 - \kappa_{m,1}} \left[ \log \beta + \kappa_{m,0} + \mu_m + (\kappa_{m,1}A_{m,1} - 1) \mu_c + \kappa_{m,1}A_{m,2}\mu_\sigma + \kappa_{m,1}A_{m,3}\mu_\sigma^2 \right].$$