Economics 685
Managerial Economics
Notes

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INTRODUCTION

I Managerial Economics

A What is Managerial Economics?

Definition 1 Managerial Economics is the application of economic theory to decisions made by managers and firms.

Definition 2 Economics is the study of the allocation of scarce resources.

Economics is the study of the allocation of scarce resources. Because all decisions are essentially about the allocation of scarce resources, economics is in fact the study of decision making and problem solving in general. So we will simply apply economic rules for decision making to problems faced by managers.

B Allocating scarce business resources

Some typical business decisions viewed as an allocation of scarce resources.

1. **Budget Allocation:** A marketing executive may use both traditional TV advertising and social media advertising. The advertising dollars in the executive’s budget are the scarce resource. *Exactly* how many dollars should be spent on each type?

2. **Going Green:** A trucking company can save fuel by reducing the speed at which the truckers drive. However, since truckers take longer to arrive at their destination, total labor costs rise (the trucking company must pay for more hours of labor by the truckers). So *exactly* what speed should the truckers drive? Or alternatively, how much labor and how much fuel produce truck miles traveled at the least possible cost?

   Both fuel and labor are scarce resources. Many companies have “green” initiatives, which involve reducing fuel or electricity consumption, but often at the cost of using more of other inputs.

3. **Market Entry:** In 2011, Yoplait (General Mills) entered the Greek yogurt market by launching Yoplait Greek, challenging the dominant brand Chobani. General Mills is a much larger company and has lower costs. However, Chobani was the first mover and had already established a loyal customer base. Should General Mills enter or invest its scarce capital in another business?
II  Decisions Studied in Managerial Economics

Although managerial economics principles can be applied to any decision, managerial economics typically focuses on three types of decisions.

1. **How much to produce?** How many cruise passenger spots should Royal Caribbean offer per day?

2. **What inputs to use?** How many ships, staff, entertainment workers, etc. should Royal Caribbean use?

3. **What price to charge?** Should Royal Caribbean charge a fixed markup over costs (“cost-plus” pricing) or some other method?

Notice that these three decisions are very high-level decisions, typically made by CEOs and senior management, not newly minted MBAs (unless they are entrepreneurs or work in a small firm). This is one reason why the most common major among CEOs is economics. Nonetheless, the examples illustrate that the economic decision making may be applied to virtually any decision. Further, some of you no doubt aspire to be CEOs or entrepreneurs, and so will eventually need this information.

III  Managerial Economics’ Advice for Decision Making

Economic decision making gives many general principles for making business decisions. MBA students who forget the math may years later still find the course helpful by remembering these principles.

Beyond the principles, managerial economics provides a data driven method for making business decisions.

A  General Principle Example: Airline Pricing

Airlines have customers which are both price sensitive (typically consumers) and customers which are not price sensitive (typically business travelers). How can the airline charge more to the business travelers? This is the concept of price discrimination, which we will look at in section III of the course. **Recognizing that ways do exist to charge customers different prices is the economic principle.**
B Data Driven Decisions: Airline Pricing

The more advanced question is exactly how much should the airline charge each type in order to get the highest possible sales revenue? This requires gathering data and then using spreadsheets and/or calculations. We will see many examples of how to make data driven decisions in this course.

Business decisions are becoming more data driven, and not only for airlines. Companies across the world are collecting vast customer databases. MBAs who know how to make use of such data are in high demand.

C Alternative Methods of Making Decisions

Alternative methods include relying on experience or expertise. One can actually do a pretty good job relying on experience, if you have it. Nonetheless, most MBA students would not be here if they had already 30 years of experience in their field.

The movie “Money Ball” comes to mind here. In that movie a young economics major tells a baseball executive that his analysis revealed that the team should recruit players who walk rather than players who hit home runs (we will learn exactly how to do this kind of analysis in the first part of the course). The team experts believed the opposite. The executive filled the team with players who walk and the strategy worked.

Both decision methods require common sense to work. The airline data may have been collected in a recession, resulting in numbers that are unlikely to be repeated. Changing prices of competitors, weather, and a host of other factors affect demand for air travel. Relying on experience also becomes more difficult when the business environment changes rapidly.

IV Theory of the Firm

A Objective of the firm

What is the objective or goal that a manager has in mind when making decisions? At the most general level, managerial economists suppose that managers try to maximize the value of a firm.

Definition 3 The value of a firm is the present value of the firm’s expected future profits.
Profits are revenues less costs. Thus:

\[ pv = \frac{\pi_1}{1 + i} + \frac{\pi_2}{(1 + i)^2} + \cdots + \frac{\pi_n}{(1 + i)^n} = \sum_{t=1}^{n} \frac{\pi_t}{(1 + i)^t} \]  

(1)

\[ \pi_t = TR_t - TC_t \]  

(2)

Here \( pv \) is the present value of the firm, \( n \) is the planning horizon, \( i \) is the appropriate interest rate (or the rate of return that could be earned if the profits were invested elsewhere). Also, \( \pi \) is profits and \( TR_t \) and \( TC_t \) are total revenues and total costs. The profits of the firm in the last period include revenues from the sale of any remaining assets (capital).

B Economic Profits

Profits here are economic profits.

Definition 4 Economic Profits Revenues less opportunity costs.

Definition 5 Accounting Profits Revenues less accounting costs.

Definition 6 Opportunity Cost Accounting costs plus the value of using inputs in their next best alternative.

Accounting costs are costs that show up on an accounting statement. Economic profits are lower, since opportunity costs are larger than accounting costs.

1 Example 1

A caterer is considering investing $310,000 in the business to add a kitchen. Suppose:

- the planning horizon is (unrealistically) just one year \( (pv = \pi_1 / (1 + i)) \),
- the market rate of return is 10%.
- the owner values the additional time required to supervise the larger kitchen at $40,000 for the year (or suppose she could hire someone at this price, but prefers to do it herself),
- she estimates additional revenue of $70,000 and food costs of $10,000,
- all costs and revenues except the cost of the kitchen are at the end of the year,
and the kitchen can be resold for $310,000 at the end of the year.

So the question is: should she tie up her money in the kitchen for one year, or invest in the market?

The next best alternative is therefore to invest the $310,000 in the market. The economic profit (in future dollars) is:

\[
\text{Economic Profit} = \text{Revenue} - \text{Opportunity Costs}
\]

\[
\text{Economic Profit} = \text{Revenue} - \text{Accounting Costs} - \text{value next best alternative}
\]

\[
= $70,000 - $10,000 - ($31,000 + $40,000) = -$11,000.
\]

So present value of the economic profits are:

\[
pv = -$11,000 / (1 + 0.1) = -$10,000.
\]

Here the next best alternative use for the caterer’s capital generates $31,000. Since the caterer values her time at $40,000, the next best alternative for her time must be worth $40,000. The opportunity costs are $71,000.

In this case, the economic profit is negative. Although the larger kitchen would earn $60,000 in accounting profits, the owner would do better by investing the $310,000 in the market. Thus the optimal decision would be not to expand the kitchen. In fact, the rule is:

- One should choose the option for which economic profits are positive. That is, if the option is better than the next best alternative.

- Only one option can have positive economic profits, since the best option is the next best option for all other choices.

Clearly, accounting profits are good for maintaining records, but economic profits are needed to make decisions.

Notice that I did not include the investment capital of $310,000. We are considering profits, which is the dollars earned on the $310,000 investment capital.

I also did not subtract the $310,000 payment for the kitchen nor did I add the $310,000 revenue from selling kitchen at the end. This will not change the answer. Some find this easier to understand if the cost of the kitchen, paid up front, is first converted to end of the year dollars:

\[
\text{Future Cost of Kitchen} = $310,000 \cdot (1 + i) = $341,000.
\]
The economic profit is then:

\[
\text{Economic Profits} = 70,000 - 10,000 - 40,000 - 341,000 + 300,000 = -11,000. \tag{6}
\]

The present value of economic profits is still -$10,000. The reason why the $310,000 used to buy the kitchen is more valuable today than in a year is because of the opportunity cost.

2 Example 2

In many cases some inputs are already purchased. Suppose the owner chose to expand the kitchen and purchased the food. Suppose after the purchase an opportunity arises: another firm offers to rent the kitchen for $20,000. Assume further that the if the owner rents the kitchen, she does not have to supervise the kitchen. Finally, assume she can resell the food for $8,000.

Now the (future) economic profits of not renting the kitchen are:

\[
\text{Economic Profit (not rent)} = \text{Revenue} - \text{Accounting Costs} - \text{value next best alternative}
\]
\[
= 70,000 - 10,000 - (-10,000 + 20,000 + 8,000 + 40,000) = 2,000 \tag{7}
\]

The present value is \(pv = 2,000/1.1\), which is positive. So she should not rent the kitchen. Notice that, because the food is already purchased, the original cost of the food is a cost for both the option and the next best alternative. So costs from previous decisions cancel, and do not affect the current decision. Our convention is to omit such costs from the calculation. Costs which are incurred regardless of the action taken are not included in opportunity costs. Here the original cost of the food is incurred regardless, so it does not affect the decision to rent the kitchen.

\[
\text{Economic Profit (not rent)} = \text{Revenues} - \text{Opportunity Costs} =
\]
\[
70,000 - (20,000 + 8,000 + 40,000) = 2,000 \tag{8}
\]

So she should not rent the kitchen. Whether or not to resell the food is not a previous decision which is incurred regardless of action, and so is included.

Notice finally that one can calculate either decision. The economic profits of renting the
kitchen are:

\[
\text{Economic Profit (rent)} = \text{Revenues} - \text{Opportunity Costs} = \\
(\$20,000 + \$8,000) - (\$70,000 - \$40,000) = -\$2,000
\]  

(9)

In the long run, economic profits are generally driven to zero. If economic profits are negative, firms will drop out of the market and prices will rise, increasing profits. Conversely, if economic profits are positive firms will enter the market and competition will drive prices down. Thus unless the industry has barriers which prevent firms from entering and exiting (e.g. monopolies or patents) or regulations on price setting exist (e.g. electricity prices), economic profits tend to zero.

C Other objectives

1 Other possible objectives of firms

I claim that firms maximize profits. Firms that do not tend to go out of business. CEOs who do not maximize profits tend to be replaced, since shareholders want CEOs to maximize profits.

Many other objectives, such as maximizing market share, are just intermediate goals toward the final goal of maximizing profits (remember, by setting a price of zero one could always maximize market share so it is doubtful if any firms, despite their claims, really do this). Most firms that have goals with respect to corporate social responsibility believe that it creates reputation benefits which ultimately increase profits.

2 Other possible objectives of managers

“You get the behavior you measure and reward.” – Jack Welch, former GE CEO.

Individual managers or CEOs may have other objectives. A marketing executive trying to maximize sales generally has no incentive to return some of her budget unused if she figures that $1 of ads generates less than $1 of profits from extra sales. In general, incentive compensation for managers should focus on maximizing profits. Profit incentives include stock options and stock incentives, and bonuses paid relative to profitability.

However, motivating managers to maximize profits is difficult, even with profit incentives because:
1. Most managers make decisions that enter on the revenue or cost side, but not both.

2. In a large corporation, individual managers have little effect on profits. For example, a manager may use a company plane for personal use. Although profits could be increased by saving money on fuel and the pilot, overall profits (and therefore the manager’s bonus) are only marginally affected.

D Profit maximization, ethics, and welfare

Maximizing profits improves the welfare of society in several ways:

1. The firm provides products and services that consumers want (as evidenced by their willingness to pay for the products, and thus add to firm profits).

2. The firm minimizes the use of costly resources. Other firms are then free to use these resources to produce other goods that society values.

3. The firm stays in business, thus providing income to workers and stockholders.

Famous theorem in economics: under certain conditions, maximizing (economic) profits maximizes the welfare of society. If economic profits are negative, then either the good can be produced while using less resources at another firm, or another firm can use the resources to produce something consumers value more, or both.

For example, consider “price gouging” of gasoline after a hurricane. By charging high prices when supply is low, gas retailers insure that the consumers who most need the gas (as evidenced by their willingness to pay high prices) get it. Further, retailers have a greater incentive to invest in costly inventories of gas if a higher price may be charged after the hurricane. In addition, the retailer generates income in the form of wages and profits for stockholders. Charging a low price means that whoever gets the small supply of gas is whoever gets to the station first, which is unlikely to be the person who values the gas most.

Another example is corporate charities. Corporate giving may indirectly increase profits. If giving results in goodwill which increases sales by more than the cost of the donation, then the gift has positive economic profits and unambiguously increases welfare. However, if a manager has alternative objectives and gives money even though it causes profits to fall, then giving is not necessarily better for society. Since the firm is taking money from stockholders and giving it to charity, welfare depends on what the stockholders would have
done with the money. Perhaps stockholders really need the money, or are better at picking charities.
PRODUCTION THEORY

Production Theory helps managers decide what inputs to use. The book presents the material largely from a manufacturing prospective, for example whether or not to automate (reduce labor and add machines). But the material applies equally to services, where the decision is low versus high skilled labor. An example is a law firm which must decide how many JDS and how many paralegals to hire.

I The Production Function

A Definition

Definition 7 The Production Function is a graph, table, or equation showing the maximum output rate that can be achieved by any specified set of inputs.

For example, consider the production function in the book, for Thompson Machine Co. Thompson has five machines in a shop that produces machine parts. Let’s suppose the input “number of machines” is fixed in the short run. Let $Q$ be hundreds of parts produced per year, and let $L$ be the number of full time workers, again per year (fractions are OK: $L = 3.5$ corresponds to 3 full time and 1 half time worker).

- The production function in table form:

<table>
<thead>
<tr>
<th>Full time laborers ($L$)</th>
<th>Parts Produced ($Q$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>49</td>
</tr>
<tr>
<td>2</td>
<td>132</td>
</tr>
<tr>
<td>3</td>
<td>243</td>
</tr>
<tr>
<td>6</td>
<td>684</td>
</tr>
<tr>
<td>9</td>
<td>1161</td>
</tr>
<tr>
<td>12</td>
<td>1512</td>
</tr>
<tr>
<td>14</td>
<td>1596</td>
</tr>
<tr>
<td>15</td>
<td>1575</td>
</tr>
</tbody>
</table>

Table 1: Production data for Thompson Machine Co.
• Then the production function equation form:

\[ Q = 30L + 20L^2 - L^3 \]  

(10)

• The production function in graph form: see figure 1 below.

The production function tells us how much inputs to use.

1. **Choose inputs to produce a specified quantity of outputs.** If the manager expects 1161 orders this year, she knows to hire 9 workers.

2. **Choose inputs to meet the objective of maximizing production.** See section II.

3. **Choose inputs to maximize profits.** With information on wages, hire enough workers to meet the objective of maximizing profits. See section III

Where does the production function come from? We can use statistics to estimate a production function (see Section VI). It is also possible to get data from engineering. For example, gas is an input to a trucking firm which produces output “miles traveled.” The production function is computed using the truck’s MPG rating. For more examples of how to compute production functions see section VI.

B Properties

Properties of the production function.

1. **Zero inputs implies zero output.** Production cannot take place without inputs.

2. **Positive marginal product.** Up to a point, adding additional workers increases output.

3. **Definition 8 Law of diminishing marginal returns (diminishing marginal product):** if all other inputs are held constant, then the additional output from increasing one input eventually falls.

Hold the number of machines constant at five. Then going from four to five workers is no problem: each can use one machine. Adding one more worker has a lower marginal product: the sixth worker can only assist one of the machine operators. At some point (from figure 1 below, approximately the 14th worker), adding a worker results in zero
additional output: that worker can only watch. Adding still more workers decreases output: extra workers now get in the way.

The graph below illustrates these concepts.

Figure 1: Typical production function using one input.

II Application 1: Maximizing Production

Suppose you a manager assigned to a particular shop with five machines. A shortage of machine parts exists. The orders are to crank out as many parts as possible.

- **Objective:** The objective is to maximize production of parts.
- **Decision:** The decision is how many workers to hire.
- **Determine the optimal decision** using the graph, table, or equation.
A  Solution 1: read the graph

By looking at figure 1, we can see that output is maximized when adding an additional worker adds zero output. This occurs at about 14 workers (1596 parts).

B  Solution 2: use the table

Definition 9 The Marginal Product is the additional output from an additional unit of an input.

Therefore, the marginal product is the slope of the production function. For production data in table form, such as table 1, the marginal product is approximately:

\[ MP = \text{slope} \approx \frac{\Delta Q}{\Delta L}, \]  

The marginal product is the slope of the production function. From figure 1, we want to find where the slope or marginal product is zero. So we can compute the maximum production from table 2:

<table>
<thead>
<tr>
<th>Full time laborers ((L))</th>
<th>Parts Produced ((Q))</th>
<th>Marginal Product ((MP))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>49</td>
<td>(= (49-0)/(1-0) = 49)</td>
</tr>
<tr>
<td>2</td>
<td>132</td>
<td>(= (132-49)/(2-1) = 83)</td>
</tr>
<tr>
<td>3</td>
<td>243</td>
<td>111</td>
</tr>
<tr>
<td>6</td>
<td>684</td>
<td>147</td>
</tr>
<tr>
<td>9</td>
<td>1161</td>
<td>(= (1161 - 684)/(9-6) = 159)</td>
</tr>
<tr>
<td>12</td>
<td>1512</td>
<td>117</td>
</tr>
<tr>
<td>14</td>
<td>1596</td>
<td>42</td>
</tr>
<tr>
<td>15</td>
<td>1575</td>
<td>-21</td>
</tr>
</tbody>
</table>

Table 2: Production function and marginal product.

To maximize production, set \(MP = 0\). This occurs somewhere between 14 and 15 workers.

C  Solution 3: use calculus

Alternatively, we could use calculus and the equation form. Since the actual production function is available \((Q = 30L + 20L^2 - L^3)\) we can compute the actual marginal product
using calculus:

\[ MP = \frac{dQ}{dL} \]  

(12)

From the graph, when output is maximized, the marginal product which is the slope or derivative is zero. Thus:

\[ \text{marginal product of labor} = \text{slope} = \text{derivative} = \frac{dQ}{dL} = 0 \]  

(13)

The derivative is:

\[ \frac{dQ}{dL} = 30 + 40L - 3L^2 = 0 \]  

(14)

The solution is given by the quadratic formula:

\[ L = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-40 \pm \sqrt{1600 - 4(-3)30}}{2 \cdot (-3)} = 14.05 \]  

(15)

Notice I have discarded the negative solution. So in fact (if possible) the manager should pay some overtime so that the equivalent of 14.05 full time workers are employed to maximize production. To use calculus, it must be possible for \( L \) to be a fraction (part time workers are possible).

It is clear that the easiest way to solve this problem is to look at the graph. Which method we actually use depends on the exact problem we are solving and which data is available, however. Most often in business, data comes in the form of a table.

### III Application 2: Maximizing Profits

Suppose now the manager is paid a bonus based on the profits of the company. To maximize profits we need to know how output affects total revenues and how inputs affect total costs. Suppose the parts can be sold for $500 each (per hundred parts) and workers earn a salary of $45,000 per year.

- **Objective:** maximize profits.
- **Decision:** number of full time employees.
- **Determine the optimal decision** using equation, table, or graph.
A  Solution using equation/calculus

Total revenues are:

\[ TR = \$500 \cdot Q = \$500 \cdot (30L + 20L^2 - L^3) \]  \hspace{1cm} (16)

Total costs are:

\[ TC = \$45,000L \]  \hspace{1cm} (17)

Profits are:

\[ \pi = TR - TC = \$500 \cdot (30L + 20L^2 - L^3) - \$45,000L \]  \hspace{1cm} (18)

We have:

\[ \max_L \pi = \$500 \cdot (30L + 20L^2 - L^3) - \$45,000L \]  \hspace{1cm} (19)

\[ \max_L \pi = \$15,000L + \$10,000L^2 - \$500L^3 - \$45,000L \]  \hspace{1cm} (20)

Set the slope or derivative equal to zero:

\[ \frac{d\pi}{dL} = 15,000 + 20,000L - 1,500 \cdot L^2 - 45,000 = 0 \]  \hspace{1cm} (21)

Divide by 1000:

\[ 20L - 1.5 \cdot L^2 - 30 = 0 \]  \hspace{1cm} (22)

\[ L = \frac{-20 \pm \sqrt{20^2 - 4 \cdot (-1.5) \cdot (-30)}}{2 \cdot (-1.5)} = 11.6 \]  \hspace{1cm} (23)

Either answer works, I have chosen the larger. So to maximize profits, we should hire 11.6 workers.

B  Marginal Revenue Product

To help with the intuition, note that:

\[ \pi = TR - TC \rightarrow \frac{d\pi}{d\text{input}} = \frac{dTR}{d\text{input}} - \frac{dTC}{d\text{input}} = 0 \]  \hspace{1cm} (24)
\[
\frac{dTR}{d\text{input}} = \frac{dTC}{d\text{input}} \tag{25}
\]

We call the first term the marginal revenue product (MRP).

**Definition 10 Marginal Revenue Product** is the amount of additional revenue from an additional unit of an input.

The second term is the marginal expenditure (ME).

**Definition 11** The Marginal Expenditure is the amount of additional costs from an additional unit of input.

So we hire additional inputs until marginal revenue product equals marginal expenditure. Ignoring the fancy jargon, we see that **we should hire a worker if the worker produces more revenue than the cost of hiring that person**. Such a worker adds to profits.

Finally, notice that:

\[
MRP = \frac{dTR}{d\text{input}} = \frac{dTR}{dQ} \frac{dQ}{d\text{input}} = MR \cdot MP \tag{26}
\]

Thus the additional revenue from an additional unit of input is equal to the marginal revenue (MR) times the marginal product. **For profit maximization, therefore:**

\[
MRP = MR \cdot MP = ME. \tag{27}
\]

C Solution using table and marginal revenue product

We can compute the profit maximizing amount of labor from table 2 using the concept of marginal revenue product and marginal expenditure. First, compute the marginal revenue product:
Table 3: Production function and marginal revenue product.

The marginal expenditure is the cost of one additional worker, $ME = $45,000. So profits are maximized where marginal revenue product is $45,000, between 12 and 14. Now the true answer is 11.6, so the table gives an answer that is slightly off. This is because the formula for marginal product in the table is an approximation:

$$\frac{\Delta Q}{\Delta L} \approx MP = \frac{dQ}{dL}.$$ \hspace{1cm} (28)

IV Multiple Inputs

A Isoquant

We now suppose there is more than one input to choose from. Everything will work the same, but there are more things to keep track of. Let $L$ and $K$ denote the two types of inputs, for example workers and machines (capital), or workers and managers. The price of capital is $P_K$ and the price of labor is $P_L$. **One can think of this section as determining whether or not to automate some worker tasks.** For example, should the airline replace some ticket agents with kiosks? If so, how many?

**Definition 12** An **Isoquant** is a curve which shows all possible input levels capable of producing a given output level.
Figure 2 gives typical isoquants.

Figure 2: Isoquants for a typical production function corresponding to $Q = 10, 20, 30$.

Looking at figure 2, we see that isoquants have some properties:

1. **Moving to the upper right moves to an isoquant with more production (isoquants do not cross).** More inputs means more production. Crossing isoquants implies more can be produced with less.

2. **The isoquant is convex.** Consider moving from the red point $K_0, L_0$ to the blue point $K_1, L_1$. Because at $K_0, L_0$ already a fair amount of capital is being used, we have diminishing marginal product. Units of capital around $K_0$ add very little output. Therefore, a small amount of labor ($L_0$ to $L_1$) replaces a lot of capital ($K_0$ to $K_1$) without changing production ($Q = 10$).

3. **The isoquant is downward sloping.** If we increase one input, the other must decrease to keep production constant.
B Marginal Rate of Technical Substitution

The slope of the isoquant is critical: it represents the amount of capital required to replace one unit of labor.

Definition 13 The Marginal Rate of Technical Substitution (MRTS) is the rate at which one input is substituted for another while keeping production constant. It is the negative of the slope of the isoquant.

The slope is negative: if we add more of one input, we must subtract some of another input to keep production constant. However, the MRTS is positive and is the quantity of one input needed to replace one unit of another input.

It is best to think of the MRTS as the ratio of marginal products:

\[
MRTS = -\frac{dK}{dL} = \frac{dQ}{dK} = \frac{MP_L}{MP_K}.
\] (29)

That is, if we decrease \( L \) by 1 unit, \( MP_L \) tells us how much production is lost. Then \( 1/MP_K \) tells us how much \( K \) produces a unit of \( Q \). Therefore, \( MP_L \cdot 1/MP_K \) units of \( K \) are required to replace all the production lost when \( L \) decreased by one unit.

Notice from the graph, if \( K \) is on the y-axis the MRTS is \( MP_L/MP_K \), not the reverse.

Now we know one critical piece of information: how many machines are needed to replace one worker. But to determine which inputs to use, we need to know also how much each input costs. It may be that we can replace many workers with a machine, but if the machine is expensive enough, it is not worth it.

C Isocost

Suppose now the manager has a budget of \( M \) dollars. What are all possible ways to spend the budget?

Definition 14 The Isocost curve shows all input combinations that have the same total budget.

The budget is:

\[
M = p_KK + p_LL.
\] (30)
Holding the total budget fixed, we add capital by reducing labor. Solving for $K$:

$$K = \frac{M}{p_K} - \frac{p_L}{p_K}L.$$  \hspace{1cm} (31)

So if we add one unit of $L$, then $K$ must decrease by $p_L/p_K$ to keep production constant. Figure 3 graphs equation (31):

![Figure 3: Isocost for a budget of $M$.](image)

The slope of the isocost is the price ratio $-p_L/p_K$, which gives the cost of replacing one unit of labor with capital. We now have enough information to get the optimal input choices.

D Optimal Input Choice

I claim that optimal input choice requires setting the MRTS equal to the price ratio:

$$MRTS = \frac{MP_L}{MP_K} = \frac{p_L}{p_K},$$  \hspace{1cm} (32)

1. **Suppose instead that** $MRTS < \frac{p_L}{p_K}$. Then:

   (a) we can replace one unit of labor with $p_L/p_K$ units of capital. Costs are constant, but production increases. We needed only MRTS units of capital to keep production constant with the loss of one unit of labor but we got more than that since $p_L/p_K > MRTS$.

   (b) we can replace one unit of labor with $MRTS$ units of capital. Production is constant, but costs fall. We could replace with $p_L/p_K$ units of capital and keep
costs constant, but we actually used less: $MRTS < p_L/p_K$. So costs fall.

2. **Suppose** $MRTS > \frac{p_L}{p_K}$. Reversing the arguments:

(a) we can replace $p_L/p_K$ units of capital with 1 unit of labor. Costs are again unchanged. Production rises since we could remove $MRTS$ units of capital while adding 1 unit of labor and keep production constant, but we actually removed less since $p_L/p_K < MRTS$. So production must rise.

(b) we can replace $MRTS$ units of capital with one unit of labor. Production is unchanged, but costs fall. We could remove $p_L/p_K$ units of capital and add 1 unit of labor and keep costs constant, but we actually removed more capital since $MRTS > p_L/p_K$. Thus costs fall.

Graphically (note cases 1a, 1b, 2a, and 2b above match the cases on the graph),

![Diagram showing efficient and inefficient input combinations](image)

Figure 4: Efficient and inefficient input combinations.

Optimal input choice means the slopes of the isocost and isoquant are equal, which means the curves are tangent. In figure 4, the curves are tangent at $K^*$ and $L^*$. Any movement
away from $K^*$ and $L^*$ results in less production ($Q < 10$).

Using numbers sometimes helps:

1. Suppose $\frac{3}{4} = \text{MRTS} < \frac{L}{PK} = \frac{3}{2}$. A unit of $L$ produces $3/4$ as much as a unit of $K$ and costs 1.5 times as much. Clearly we want to reduce labor and increase capital (automate).

2. Suppose $\frac{3}{2} = \frac{L}{PK} < \text{MRTS} = \frac{2}{1}$. A unit of $L$ costs 1.5 times as much as a unit of $K$, but produces twice as much. We want to increase labor.

E  Example: Use the MRTS to Create Your Team

Suppose an engineering analysis firm uses a team of engineers and technicians to do their consulting. Engineers are paid $4,000 per month and technicians $2,000. The estimated production function is:

$$Q = 20E - E^2 + 12T - 0.5 \cdot T^2$$

(33)

The firm charges $1,000 to do an engineering analysis. This is a very common situation, engineers are more productive, but also cost more than technicians. The same problem comes up with doctors and nurses, research faculty and lecturers, financial planners and assistants, lawyers and paralegals, etc.

1. **How many engineers and technicians should be hired if the manager is given a maximum salary budget of $28,000?**

We know that the ratio of marginal products equals the price ratio at the optimal input choice:

$$\frac{MP_E}{MP_T} = \frac{\frac{dQ(T,E)}{dE}}{\frac{dQ(T,E)}{dT}} = \frac{P_E}{P_T}$$

(34)

Or:

$$\frac{20 - 2E}{12 - T} = \frac{\$4000}{\$2000} = 2$$

(35)

$$20 - 2E = 2 (12 - T)$$

(36)

$$-2E = 4 - 2T$$

(37)
\[ E = T - 2 \]  

The team should always have two fewer engineers than technicians.

Thus, since the wage bill is $28,000:

\[ 28,000 = 4000E + 2000T \]  

\[ 14 = 2E + T \]  

\[ 14 = 2(T - 2) + T \]  

\[ 18 = 3T \rightarrow T = 6 \rightarrow E = 4 \]  

2. How many engineers and technicians should be hired to maximize profits?

We know already that we need two fewer engineers than technicians. Further, we know that for any input the marginal revenue product equals the marginal expenditure. For example, for engineers:

\[ TR = 1000Q, \]  

\[ MR = \frac{dTR}{dQ} = 1000, \]  

\[ MRP_E = MR \cdot MP_E = 1000 \cdot (20 - 2E). \]  

\[ TC = 4000E + 2000T. \]  

\[ ME_E = \frac{dTC}{dE} = 4000. \]  

Therefore:

\[ MRP_E = 1000 \cdot (20 - 2E) = ME_E = 4000, \]  

\[ 20 - 2E = 4, \rightarrow E = 8. \]
Finally since we always have two fewer engineers than technicians, we have:

\[ T = E + 2 = 10. \]  

(50)

3. **How many engineers and technicians should be hired if the firm needs to perform an output of 166 engineering analysis?**

We need

\[ Q = 166 = 20E - E^2 + 12T - 0.5 \cdot T^2 \]  

(51)

Next we use that we always want two fewer engineers than technicians:

\[ 166 = 20 (T - 2) - (T - 2)^2 + 12T - 0.5 \cdot T^2 \]  

(52)

\[ 166 = 20T - 40 - T^2 + 4T - 4 + 12T - 0.5T^2 \]  

(53)

\[ 166 = 36T - 44 - 1.5T^2 \]  

(54)

\[ 0 = 1.5T^2 - 36T + 210 \]  

(55)

\[ T = \frac{36 + \sqrt{36^2 - 4 \cdot (210) \cdot 1.5}}{2 \cdot 1.5} = \frac{42}{3} = 14 \rightarrow E = 12 \]  

(56)

V  **Mergers and Spinoffs**

A  **Returns to Scale**

Here we think about the size of our production processes. Should the firm expand (say through a merger) or contract (say through a spin-off). Should the firm build a second factory or increase the size of the current factory? Should the firm close one plant and expand operations at another? Should the firm outsource some production processes? The production function tells us the answer.

**Definition 15** The production function exhibits **Increasing (decreasing, constant) returns to scale** if increasing all inputs by a proportion increases output by more than (less than, exactly) the same proportion.
It is easiest to think of returns to scale by setting the proportional increase to 100%, that is, double all inputs and see if output more than, less than, or exactly doubles. Doubling all inputs can be thought of as building an identical factory or merging with an identical firm. Suppose the firm is considering merging with an identical firm.

- **Increasing returns to scale**: the merger is beneficial, since the total costs do not change (the costs are the same as the costs of the two firms acting separately), but output is greater than if the two firms are separate.

- **Decreasing returns to scale**: Costs are the same but output is smaller than if the two firms are separate. Do not merge. Instead, make the firm smaller. If the firm is split into two identical firms through a spin-off, then costs are the same as with the single firm but output is greater with two firms.

- **Constant returns to scale**: No benefits to either a merger or spin-off. The firm is at an optimal size.

Consider our previous production function where engineers and technicians produce engineering analyses, and suppose we have an opportunity to merge with an identical firm. At the merged firm, inputs double:

\[
Q_{\text{merged}} = Q(2E, 2T) = 20(2E) - (2E)^2 + 12(2T) - 0.5(2T)^2
\]  

\[
= 40E - 4E^2 + 24T - 2T^2
\]  

Total output at the two separate firms is:

\[
2Q = 2 \cdot (20E - E^2 + 12T - 0.5T^2)
\]  

\[
2Q = 40E - 2E^2 + 24T - T^2
\]  

So \(Q_{\text{merge}} < 2Q\), doubling all inputs resulted in less than double the output. This firm has decreasing returns. Costs are the same, but output and therefore profits are larger when the two firms are separate. Do not merge. The firm should actually get smaller, not larger.

Reasons for decreasing returns to scale:

1. **Extra layers of management** are required in a large organization which do not directly contribute to overall production.
2. **Information may not flow well** to all employees in a large organization.

3. **Individual workers have little impact on profits**, and therefore have little incentive to engage in profit maximization.

4. **Regulation costs** may increase in larger firms. For example, many regulations exempt firms with less than 50 employees.

Reasons for increasing returns to scale:

1. **Indivisibilities.** Suppose two identical firms, each with one accountant, merge. The combined firm may not need two accountants. One may be able to handle the extra work. This usually arises because the accountant was underutilized at the single firm, because it may be difficult to hire a part time accountant. Similarly, it is difficult to purchase part of a machine. However, small firms may avoid indivisibility costs by outsourcing. In this case, doubling the size of the firm would not affect costs.

2. **Engineering Reasons.** It may be the case that doubling the size of the warehouse might not require double the steel, electricity, etc.

3. **Specialization.** Large firms can have employees become more efficient by specializing. A larger firm can hire an accountant, rather than have the head sales guy also do the accounting. Outsourcing can help small firms stay specialized as well.

B A special production function and output elasticity

**Definition 16** The **output elasticity** is the percentage increase in output from a one percent increase in inputs.

The firm has increasing (decreasing, constant) returns if and only if the output elasticity is greater than one (less than one, equal to one).

For a special production function, the output elasticity is constant and very easy to calculate. This production function is the **Cobb-Douglas production function** and takes the form:

\[ Q = AK^aL^b. \]  

(61)

Here \( A \), \( a \), and \( b \) are parameters which vary across industries.
Consider the telephone industry in Canada. The production function was found to be:

\[ Q = 0.70L^{0.70}K^{-0.41} \]  

(62)

A one percent increase in inputs gives:

\[ Q' = 0.70 (1.01L)^{0.70} (1.01K)^{-0.41} \]

(63)

\[ = 0.70 (1.01)^{0.70} L^{0.70} (1.01)^{-0.41} K^{-0.41} \]

(64)

\[ = (1.01)^{0.70} (1.01)^{-0.41} 0.70L^{0.70} K^{-0.41} \]

(65)

\[ = (1.01)^{0.70-0.41} Q \]

(66)

\[ = (1.01)^{1.11} Q = 1.0111Q \]

(67)

Percent change \[ \frac{Q' - Q}{Q} \cdot 100\% = \frac{1.0111Q - Q}{Q} \cdot 100 = 1.11\% \]

(68)

Thus if inputs increase by 1%, output increases by 1.11%, so the output elasticity is 1.11. The Cobb-Douglas production function has the special feature that the output elasticity is constant. Further, for the Cobb-Douglas production function the output elasticity is simply the sum of the exponents:

\[ \text{Output Elasticity} = a + b = 0.71 + 0.40 = 1.11 \]

(69)

VI Appendix: How Do We Find the Production Function?

We can do many great thing with the production function. How do we obtain the production function? Several ways exist, one of which involves gathering company data (or even competitors data) and then using statistics (regressions). We need data on inputs used and how much output was obtained.

A Obtain input-output data

The first step is to obtain some data. Here are several possibilities.
1. Time Series Data. Get historical data of the firm’s inputs and outputs.

2. Cross Section Data. Get data of all plants (or factories) owned by the firm in a single time frame. Or get data on all of the firms in the industry.

3. Use technical information supplied by engineers.

4. Conduct a randomized study. Select a random factory or factories and change the inputs.

5. Benchmarking. Observe firms outside the industry that specialize in this type of production.

Each has advantages and disadvantages. Time series data is often easy to come by. The manager does not need to request data from other managers or look up data on other firms. However, things change over time. Suppose the time series data is something like:

<table>
<thead>
<tr>
<th>Date</th>
<th>Full time laborers (L)</th>
<th>Parts Produced (Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 3</td>
<td>6</td>
<td>684</td>
</tr>
<tr>
<td>April 4</td>
<td>7</td>
<td>681</td>
</tr>
</tbody>
</table>

Table 4: Time series production data.

One might look at this data and think that the MP becomes negative after 6 workers, that is, the firm should never hire more than six workers. But it is also possible that something odd happened on April 4 that did not happen on April 3. For example, it could have been someone’s birthday on April 4, and the firm wasted a few hours giving out cake.

Cross sectional use data from multiple plants at a single time. To a lesser degree, cross sectional data has the same problem. There might be something special about one factory that the manager does not know about. With this data, we would again conclude that hiring more than 6 workers is a mistake. But it could be that the Miami plant uses older equipment, and so the workers are not as productive.

<table>
<thead>
<tr>
<th>Plant</th>
<th>Full time laborers (L)</th>
<th>Parts Produced (Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>West Palm Beach</td>
<td>6</td>
<td>684</td>
</tr>
<tr>
<td>Miami</td>
<td>7</td>
<td>681</td>
</tr>
</tbody>
</table>

Table 5: Cross sectional production data.
The randomized study, as is done in medicine, does not suffer from these problems, but is the most expensive way to obtain data. Because the plant selected is random, differences between plants are random and tend to average out. But you have to make the plant use odd combinations of inputs, possibly resulting in low production, just to gather data.

B Choosing a production function

The second step is to choose a production function. Suppose we have two inputs, $K$ and $L$ again. We would like our function to have all of the properties listed above like diminishing marginal products. Here are two that have these properties.

$$Q = aLK + bL^2K + cLK^2 - dL^3K - eLK^3$$  \hspace{1cm} (70)

$$Q = aL^bK^c$$  \hspace{1cm} (71)

C Regression

We now use the data to find values for $a$, $b$, $c$, $d$, and $e$. No production function is exactly like the above two. What the regression tells us is what values of $a$-$e$ make the above production function ‘closest’ to the data (ie closest to the real world).

Excel has a simple function “linest” which will be sufficient. On the website is an example Excel file that demonstrates how to do a linear regression. Statistics can tell us many things besides the values $a$ - $e$. We will focus on two other things the Excel file tells us.

1. T-stat. The T-stat tells us if $K$, $L$, $K^2$, etc. likely have any effect on $Q$. If the T-stat for $d$ is small, then we may want to drop the term $L^3K$ from the production function.

2. $R^2$. This tells us the percentage of the variation in output that is explained by variation in the inputs. If the number is close to one, the production function is doing a good job describing the real production process. If the number is close to zero, there are some aspects of production being missed. Perhaps another input.