Question 1

a. The marginal costs are not equal and so the regulation violates the equi-marginal principle and does not reduce pollution in the least costly way. To reduce costs, pollution needs to be reduced more where it is cheaper to do so (in this case, new cars). For example, if we increase emissions by 1 for old cars, we save $10. If we decrease emissions by 1 for new cars, we pay $5. Overall emissions are unchanged, and thus so are damages. But we have saved $10 − $5 = $5.

b. Since it is cheaper to reduce pollution in new cars, we want to set up the vintage differentiated regulation so that new cars must pass a stricter emissions test.

c. Households will have an incentive to hang on to older cars longer. Since older cars are more polluting, overall pollution may rise even though new cars have a stricter test!

Question 2

The gas tax provides an incentive to reduce gas consumption. Therefore, the gas tax provides an incentive for any activity that reduces gas consumption. Further, households will choose whatever method is best for them, lowering the cost of reducing gas.

Conversely the CAFE standards require manufacturers to have a specific average fuel economy. In order to increase their average fuel economy, manufacturers must lower the price of fuel efficient cars and raise the price of gas guzzlers. Therefore, CAFE standards indirectly provide an incentive to purchase more fuel efficient new cars, and an incentive not to buy new gas guzzlers. It provides no incentive for other methods to reduce fuel consumption. To the extent that other options are cheaper for at least some people, CAFE standards a more expensive way to reduce gas consumption than the gas tax.

Question 3

a. For efficiency, we set demand equal to supply which equals marginal costs, taking care to include the costs of production and the costs to society of production:

\[ P = MC + MD, \]  \hspace{1cm} (1)

\[ 8 - Q = 2Q + 2, \]  \hspace{1cm} (2)

\[ Q^* = 2. \]  \hspace{1cm} (3)
b. The monopolist restricts production so as to increase the price. Mathematically, they consider not only that producing one more unit increases revenue by $P$, but also that the price falls, reducing revenue on all goods sold. The marginal revenue accounts for both effects. In the absence of regulation, firms do not consider the cost to society of polluting, so the monopolist considers only it’s own costs. Therefore the monopolist sets:

$$MR = MC, \quad (4)$$

$$8 - 2Q = 2Q, \quad (5)$$

$$Q^m = 2. \quad (6)$$

In a competitive market, the quantity would be found by setting $P = MC$, which gives $Q = 8/3$. The monopolist restricts production from $8/3$ to $2$ so as to increase the price.

c. The Pigouvian tax equals the marginal damage. The monopolist now considers that production will raise tax payments:

$$MR = MC + t, \quad (7)$$

$$MR = MC + MD. \quad (8)$$

By setting the tax equal to the marginal damage, we indirectly force the firm to consider the costs to society of production.

$$t = MD = 2, \quad (9)$$

$$8 - 2Q = 2Q + 2, \quad (10)$$

$$Q = \frac{3}{2}. \quad (11)$$

d. The monopolist, in seeking to increase profits by reducing production and therefore raising the price, creates a side benefit of reducing pollution. In this case, two wrongs make a right: the monopolist restricted production to the efficient level. Imposing the Pigouvian tax on the monopolist will therefore cause the monopolist to reduce production even further. So the Pigouvian tax results in too little production when
the producer is a monopolist.

**Question 4**

a. For efficiency, we set the value of producing one more unit \( P \) to equal the extra cost of producing one more unit \( MC \) plus the cost to society \( MD \). Producing more is not worth it: households do not value additional production enough to justify the cost. Therefore:

\[
P = MC + MD, \tag{12}
\]

\[
30 = 2C + 4 + 2C + 2, \tag{13}
\]

\[
C^* = 6. \tag{14}
\]

b. For the market provision, the firm in the absence of regulation does not account for the cost to society. Therefore:

\[
P = MC, \tag{15}
\]

\[
30 = 2C + 4, \tag{16}
\]

\[
C^{mn} = 13. \tag{17}
\]

Mining profits are therefore:

\[
\pi = \text{total revenue} - \text{total costs} \tag{18}
\]

\[
\pi = P \cdot C - TC, \tag{19}
\]

\[
\pi = 30 \cdot 13 - \left(13^2 + 2 \cdot 13 + 6\right) = 163. \tag{20}
\]

c. If the mining company owns the river, it can dump at will, but the household may pay the company to reduce production. The household is willing to pay \( P' = MD \) per unit reduced: at this point the household is just indifferent between paying to
reduce pollution and just living with the next unit of pollution. The company now
incurs an extra opportunity cost of lost payments from the household when it increases
production. Therefore the company sets:

\[ P = MC + P', \]  \tag{21}  

\[ P = MC + MD. \]  \tag{22}  

Therefore the company chooses the efficient quantity \( C^* = 6 \). Profits are therefore:

\[ \pi = 30 \cdot 6 - \left( 6^2 + 2 \cdot 6 + 6 \right) + P' \left( C^m - C^* \right). \]  \tag{23}  

Notice that the company includes payments from the household in it’s profits (the
amount of \( C \) reduced is \( C^m - C^* \)).

\[ P' = MD = 2C^* + 2 = 2 \cdot 6 + 2 = 14, \]  \tag{24}  

\[ \pi = 114 + 14 \left( 13 - 6 \right) = 212. \]  \tag{25}  

Household total damages and surplus are thus:

\[ TD = 6^2 + 2 \cdot 6 - 60 = -12, \]  \tag{26}  

\[ TS = -TD - \text{payments} = 12 - 14 \left( 13 - 6 \right) = -86. \]  \tag{27}  

Notice that total economy wide surplus is: \( \pi + TS = 212 - 86 = 126. \)

d. If the households own the river, the company must pay each time it dumps in the river.
At \( P' = MD \), the household is just compensated enough to offset the damage from
the last unit of pollution. The firm incurs an extra cost of the fee for dumping, and so
now the firms problem solves:

\[ P = MC + P', \]  \tag{28}  

\[ P = MC + MD. \]  \tag{29}  

So the efficient \( C^* = 6 \) results and the household charges \( P' = 14 \). Mining profits are
now:

\[ \pi = 114 - 14 \cdot 6 = 30. \quad (30) \]

The mining company subtracts payments to the household from profits. Household damage and surplus are:

\[ TD = 6^2 + 2 \cdot 6 - 60 = -12, \quad (31) \]

\[ TS = -TD + \text{payments} = 12 + 14 \cdot 6 = 96. \quad (32) \]

The household adds payments from the company to its surplus. Total economy wide surplus is: \( \pi + TS = 30 + 96 = 126. \)

e. The Pigouvian tax is \( t = MD. \) The firm must add tax payments to it’s costs, and therefore produces according to:

\[ P = MC + t = MC + MD. \quad (33) \]

So the efficient production results and \( t = MD = 2 \cdot 6 + 2 = 14. \) Mining profits must subtract the tax payments:

\[ \pi = 114 - 14 \cdot 6 = 30. \quad (34) \]

Households do not get any payments, and so:

\[ TD = 6^2 + 2 \cdot 6 - 60 = -12, \quad (35) \]

\[ TS = -TD = 12. \quad (36) \]

Finally, government tax revenue is \( 14 \cdot 6 = 84. \) This counts as part of the total surplus for the economy as the government can use the tax revenue for some spending or rebate it back to taxpayers. Total surplus is therefore \( \pi + TS + \text{tax revenue} = 30 + 12 + 84 = 126. \)

f. The Coase theorem states that it doesn’t matter who owns the river, the mining company, the household, or the government, the efficient quantity results and total surplus is maximized (126 in all three cases). The Coase theorem says nothing about fairness, and whoever owns the property right gets most of the surplus.

**Question 5**

a. We have:
b. For efficient emissions we first calculate total marginal damages:

\[
MD = MD_1 + MD_2 = \frac{1}{2} + \frac{1}{2} + \frac{Ar}{10} = 1 + \frac{Ar}{10}.
\]  

(37)

Next we calculate total emissions. For the CA firm:

\[
MC_{CA} = 3 - \frac{Ar_{CA}}{10}.
\]  

(38)

\[
\frac{Ar_{CA}}{10} = 3 - MC_{CA}
\]  

(39)

\[
Ar_{CA} = 10 \cdot (3 - MC_{CA}) = 30 - 10MC_{CA}.
\]  

(40)

For the AZ firm:

\[
MC_{AZ} = 4 - \frac{Ar_{AZ}}{20}.
\]  

(41)

\[
\frac{Ar_{AZ}}{20} = 4 - MC_{AZ}
\]  

(42)
\[ Ar_{AZ} = 20 \cdot (4 - MC_{AZ}) = 80 - 20MC_{AZ}. \]  

(43)

Adding the two together anticipating that the marginal costs will be equal across the two sources:

\[ Ar = Ar_{AZ} + Ar_{CA} = 80 - 20MC + 30 - 10MC = 110 - 30MC. \]  

(44)

Next, since \( MC = MD \), we can combine equations (37) and (44):

\[ Ar = 110 - 30 \left( 1 + \frac{Ar}{10} \right). \]  

(45)

Solving for \( Ar \) gives:

\[ Ar = 110 - 30 - 3Ar, \quad \rightarrow \quad Ar^* = 20. \]  

(46)

The efficient \( Ar \) level is 20. The Pigouvian tax equals the marginal damages:

\[ t^* = MD (Ar^*) = 1 + \frac{Ar^*}{10} = 1 + \frac{20}{10} = 3. \]  

(47)

The efficient tax is \( t^* = 3. \) Since each firm sets the tax equal to marginal costs, we have:

\[ t^* = 3 = MC_{AZ} = MC_{CA}. \]  

(48)

Plugging into the marginal cost equations gives:

\[ MC_{CA} = 3 = 3 - \frac{Ar_{CA}}{10}, \quad \rightarrow \quad Ar_{CA} = 0. \]  

(49)

It is much cheaper to reduce in \( CA \), so that firm reduces to zero. For the \( AZ \) firm:

\[ MC_{AZ} = 3 = 4 - \frac{Ar_{AZ}}{20}, \quad \rightarrow \quad Ar_{AZ} = 20. \]  

(50)

Notice that emissions by each firm add up to the total emissions.

c. See the graph in part (a).

d. If only one firm is in the market, total emissions equal the \( CA \) firm’s emissions, and we can just set \( MC_{CA} \) equal to marginal damages:

\[ MD = 1 + \frac{Ar}{10} = MC_{CA} = 3 - \frac{Ar}{10}. \]  

(51)
\[ \frac{2}{10} Ar = 2, \quad \rightarrow \quad Ar = 10. \] (52)

e. A standard of 10 ppb in AZ is inefficient first because the efficient emissions is 20 and the standard is 10. The standard calls for too much emissions reduction. Second, the standard does not equalize marginal costs. It is much more costly for AZ to reduce to 10 ppb than it is for CA. Too much emissions reduction is done by AZ water and too little by CA water in the AZ town.

f. Clearly CA is happy with this regulation. The efficient \( Ar \) results for the CA town, and since only one firm exists there is no issue about equalizing marginal costs. Arizona is against the law because it calls for too much reduction and does not achieve reductions in the least costly way. In fact, the EPA did pass such a standard over the objections of Western states with naturally high \( Ar \) levels. East and west coast states supported the EPA action because it was easy for them to comply.

**Question 6**

a. In the absence of regulation, both firms set price equal to marginal cost. For Firestone:

\[ P = 60 = MC_F = 4Q_F, \] (53)

\[ 60 = 4Q_F, \quad \rightarrow \quad Q_F = 15. \] (54)

Profits are:

\[ \pi = P \cdot Q_F - \left( 300 + 2Q_F^2 \right), \] (55)

\[ \pi = 60 \cdot 15 - 300 - 2 \cdot 15^2 = 150. \] (56)

For Goodyear:

\[ P = 60 = MC_G = 2Q_G, \] (57)

\[ 60 = 2Q_G, \quad \rightarrow \quad Q_G = 30. \] (58)

Profits are:

\[ \pi = P \cdot Q_G - \left( 500 + Q_G^2 \right), \] (59)
\[ \pi = 60 \cdot 30 - 500 - 30^2 = 400. \]  \hspace{2cm} (60)

b. The marginal damage is $12, so set the tax there. The firms now set price equal to \( MC \) plus the tax costs (it costs the firms an extra $12 to produce each good):

\[ P = 60 = MC_F + MD = 4Q_F + 12, \]  \hspace{2cm} (61)

\[ 48 = 4Q_F, \quad \rightarrow \quad Q_F = 12. \]  \hspace{2cm} (62)

Notice we must subtract the tax payments from profits:

\[ \pi = P \cdot Q_F - (300 + 2Q_F^2) - 12Q_F, \]  \hspace{2cm} (63)

\[ \pi = 60 \cdot 12 - 300 \cdot -2 \cdot 12^2 - 12 \cdot 12 = -12. \]  \hspace{2cm} (64)

For Goodyear:

\[ P = 60 = MC_G + MD = 2Q_G + 12, \]  \hspace{2cm} (65)

\[ 48 = 2Q_G, \quad \rightarrow \quad Q_G = 24. \]  \hspace{2cm} (66)

Profits are:

\[ \pi = P \cdot Q_G - (500 + Q_G^2) - 12Q_G, \]  \hspace{2cm} (67)

\[ \pi = 60 \cdot 24 - 500 - 24^2 - 12 \cdot 24 = 76. \]  \hspace{2cm} (68)

c. We have:
d. For the subsidy, for each unit of rubber produced, the firms give up $12 of lost subsidy revenue. This is an opportunity cost of producing to the firms. Thus:

\[ P = MC + 12. \]  \hfill (69)

So we will get the same production as with the tax: \( Q_G = 24 \) and \( Q_F = 12 \). Profits are different, however. Rather than subtract a tax payment, we must add a subsidy payment of \( 12 (Q_M - Q) \), where \( Q_M \) is the production in the absence of regulation determined in part (a). So:

\[ \pi = P \cdot Q_F - \left(300 + 2Q_F^2\right) + 12 (Q_M - Q_F), \]  \hfill (70)

\[ \pi = 60 \cdot 12 - 300 - 2 \cdot 12^2 + 12 (15 - 12) = 168. \]  \hfill (71)

For Goodyear:

\[ \pi = P \cdot Q_G - \left(500 + Q_G^2\right) + 12 (Q_M - Q_G), \]  \hfill (72)

\[ \pi = 60 \cdot 24 - 500 - 24^2 + 12 \cdot (30 - 24) = 436. \]  \hfill (73)
e. Although the tax and subsidy have the same short run production, in the long run they will be different. From equation (64), Fireston earns negative profits and exits the market with the tax. Goodyear will have the entire market and produce 24, significantly less than the total production of $24 + 12 = 36$ given the subsidy. The tax is efficient: the capital at Firestone cannot generate zero profits, so it’s capital is better employed producing something else.