LOBBYING AND ACCESS

A Dissertation

Presented to the Faculty of the Graduate School

of Cornell University

in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy

by

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August 2008
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There is substantial evidence that political contributions are often used to buy access to politicians. A special interest group that receives access may have the opportunity to present evidence and arguments in favor of a preferred position (or against a less-preferred position). This dissertation presents three game-theoretic analyses of decision making in which money may buy access. The underlying structure which is common to all three analyses is built on two fundamental assumptions. First, interest groups have verifiable private evidence that may influence a politician’s beliefs about his preferred decision. Second, the politician decides which interest groups receive access to present their evidence. Unlike other models of hard-information disclosure, interest groups must be granted “access” in order to disclose their evidence to the politician.

The dissertation focuses on two means by which the politician may award access to interest groups. One possibility is that the politician gives access to the interest groups that provide the largest contributions. The analysis considers such a competition for access model in detail. Another possibility is that the politician sets access fees and any interest group that pays its fee gains access to the politician. These two means of awarding access have different implications regarding the impact of contribution limits on policy choice and constituent welfare.
Christopher Cotton began his graduate studies in economics at Cornell University in August 2003. He completed the requirements for an M.A. in Economics in October 2006. His primary research focus at Cornell has been in Political Economics and Applied Game Theory. Stephen Coate is his advisor. His other committee members include Talia Bar, Kaushik Basu, and Ted O’Donoghue.

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In August 2008, he will begin employment as an Assistant Professor at the University of Miami. Both he and his wife Lydia are looking forward to the sunshine.
This dissertation is dedicated to my beautiful wife, Lydia.
ACKNOWLEDGEMENTS

In late 2005, I went to my advisor Stephen Coate to talk about a paper I wanted to write. The proposed work would combine a model in which political contributions buy access to a politician and a model in which contributions help those committed to one’s preferred policy win election. Steve suggested that I focus on the access model, as such a framework has largely been overlooked by the theoretical literature. A draft version of Chapter 2 first appeared in February 2006, and that paper as well as the subsequent papers that make up this dissertation have undergone numerous revisions during the past two-plus years. Each version of a paper was an improvement upon earlier versions thanks in large part to the questions, comments, and constructive criticism from numerous people who have talked with me about my work. Without these discussions, this research endeavor would have been much less successful (and less enjoyable) than it was.

Thank you to everyone who has provided me with advice, encouragement, and inspiration during my graduate studies at Cornell. Stephen Coate has been an outstanding advisor, who has read and re-read my papers, and provided invaluable guidance. Hopefully, through my interactions with him, I have gained some sense as to what separates a good paper from a great paper. My other committee members, Talia Bar, Kaushik Basu, and Ted O’Donoghue, have also been a great help through this process. In addition to generally providing excellent comments, I must thank Talia for pushing me to more formally define my equilibrium concept during my first presentation of the initial paper, Kaushik for encouraging me to think about additional applications of the access framework, and Ted for being justifiably picky about generalizations of my model including allowing for correlated interest group evidence quality. I also thank Ted for providing me with a semester-long research assistantship that required little work for him, and a lot of time to work on my
own research.

Josh Teitelbaum and Jayand Ganguli deserve special acknowledgment. As my office mates, they served as exceptional sounding boards for new ideas. A conversation with David Easley motivated me to develop the access fees model in Chapter 3. Additionally, I appreciate feedback and comments from Dan Benjamin, Larry Blume, Archishman Chakraborty, Ani Guerdjikova, Ben Ho, Justin Johnson, Joe Price, Ian Schmutte, and Michael Waldman, as well as seminar participants at Baruch College, Clemson University, Cornell University, the University of Iowa, the University of Louisville, the University of Miami, PET07, the 2006 Midwest Economic Theory Meeting, the 2006 North American Summer Meeting of the Econometric Society, and the 17th International Conference on Game Theory at Stony Brook. I’ve enjoyed discussions with Levon Barseghyan, Jessica Bean, Arnaud Dellis, Ram Dubey, Amanda Griffith, Jin-Hyuk Kim, Koralai Kirabaeva, Albert Liu, Mandar Oak, Jonathan Peterson, Aziz Simsir, Ben Suwankiri, Russell Toth, Mary-Louis Viero, and countless others not mentioned here. Eric Maroney was a great help with mailing job market packets. My parents Steve and Mary Cotton, my siblings Alex Cotton and Nicole Burns, and my other family and friends have provided me with priceless emotional support, even if most of them don’t really understand what it is I do.

Most importantly, I also thank my wife Lydia who endured all of my studying and stress, and who supported me throughout my time at Cornell. She moved with me to Ithaca and married me just before I began graduate school. Certainly, she had no idea how challenging life as a grad student’s wife would be. I could not be more happy that she put up with me for the past five years, and that she will continue to put up with me after we leave Ithaca.
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CHAPTER 1
INTRODUCTION

The economics and political science literatures focus on two motivations behind political contributions. First, one may contribute in the quid pro quo exchange for policy favors. Second, one may contribute to help a politician already in favor of one’s cause win election. The literature has largely overlooked a third reason for contributing: to secure access to a politician, where those with access can influence the drafting of legislation or the politician’s voting record through the provision of information or arguments in support of one’s preferred policy, or against a less-preferred option.

Although the first two motivations may help drive contributions, there is substantial evidence that the access motivation also has a significant (if not stronger) influence on interest group contributions.\footnote{See for example, Herndon (1982), Langbein (1986), Wright (1990), Hall and Wayman (1990), Milyo et al. (2000), Ansolabehere et al. (2002), Clawson et al. (1992), Schram (1995).} Despite this, few papers attempt to model the contributions-for-access story.\footnote{The exceptions include Austen-Smith (1995, 1998) and Lohmann (1995).} My thesis is intended to help fill this void in the theoretical literature.

The following chapters are made up of three game-theoretic analyses of access in politics. The analyses share similar underlying frameworks, and assumptions regarding how interest groups may influence policy by gaining access to the politician. In each analysis, a single politician must choose an action that influences both representative constituent welfare and interest group payoffs. The politician prefers to choose the action that is best for his constituent; however, he is ex ante uncertain about the welfare implications of different actions. Interest groups have private, verifiable evidence about the consequences of different actions that they
can share with the politician before the politician makes his decision. This framework differs from more-traditional games of hard information in that the politician controls which interest groups may disclose evidence. Unlike other models of hard information disclosure, access to the politician is not an inalienable interest group right; the politician must choose to listen to an interest group before the group can disclose its evidence.\(^3\)

Since the politician controls whether an interest group receives access, the analysis depends on how he awards access. Chapters 2 and 3 consider two mechanisms through which the politician may award access to an interest group. In Chapter 2 the politician awards access through a *competition for access*, in which interest groups compete for access by providing political contributions and the groups that provide the largest contributions win access. The competition takes the form of an all-pay auction, in which all interest groups pay their contributions, even if they do not receive access. Interest groups who have stronger evidence in their favor are willing to pay higher contributions to disclose their evidence than interest groups who have weaker evidence. This means that the politician can learn about an interest group’s evidence by observing its contribution, as well as by giving the group access. In the equilibrium of the competition for access game, the contributions are fully-revealing of interest group evidence, and the politician learns about the evidence quality of all interest groups even when he only gives access to some of the groups.

Chapter 3 considers an alternative to the competition for access mechanism. In it the politician commits to *access fees* at the beginning of the game, and any interest group that pays the access fee receives access and can disclose its evidence.

\(^3\)Austen-Smith (1998) is the only other paper that I am aware of in which the politician must grant an interest group access before the group can disclose hard evidence.
Just as in the earlier chapter, an interest group who has stronger evidence in its favor is willing to pay more to disclose its evidence than an interest group with weaker evidence. Therefore, when the politician observes that an interest group is unwilling to pay the access fee, he can correctly infer that the group had relatively low-quality evidence. However, in the access fee game (unlike in the competition for access game) the politician does not become fully informed about the evidence quality of the groups that do not receive access. The equilibrium access fee is higher for rich interest groups than for poor interest groups, and relatively low for interest groups involved with issues that are important to the politician. I also show that the politician sets the access fee strictly above the fee preferred by the representative constituent.

Both Chapter 2 and Chapter 3 consider the impact that limiting or banning contributions has on politician information and expected constituent welfare. In the competition for access game of Chapter 2, contributions allow the politician to become fully informed about interest group evidence, and in equilibrium he implements the welfare-maximizing policy. Any contribution limit or ban tends to distort the signaling power of the contributions, which results in a less-than-fully informed politician, and a worse policy outcome. The same is not true when the politician sells access using access fees, as he does in Chapter 3. In this case, a contribution limit can decrease the access fee charged by the politician, which tends to increase the likelihood that the politician becomes informed. This tends to increase constituent welfare. However, too low of a contribution limit may cause the politician to not grant any access, since for some issues he will not find it worth his time to do so when he cannot charge a sufficiently high price. In this way, a limit may decrease welfare by reducing the range of issue for which the politician sells access. Combined, Chapters 2 and 3 highlight that the way in which the politician
allocates access matters. When the politician awards access through a competition for access, a contribution limit has a strictly negative impact on expected welfare; while, if the politician awards access using access fees, a contribution limit can improve expected welfare.

Chapters 2 and 3 model the politician’s choice of policy from a continuous policy space. In Chapter 4, the politician wants to award a discrete, non-divisible prize to the most-qualified agent. This may include choosing a discrete policy in favor of the interest group that can make the strongest case in its favor, or it may include awarding a government contract to the most qualified applicant. I adapt both the competition for access and access fee frameworks to the prize allocation setting. Competition for access is an optimal mechanism by which the politician can award the prize: it maximizes total contributions, and it awards the prize to the most qualified applicant. Awarding the prize through an access fee mechanism results in lower contributions and does not necessarily award the prize to the most qualified applicant. This suggests that the politician prefers to allocate the access and the prize through a competition for access. However, it remains unclear whether politicians actually do so. Future empirical work should attempt to determine how politicians do award access.
CHAPTER 2
COMPETITION FOR ACCESS

2.1 Introduction

In the United States, interest groups and lobbyists provide political contributions in an effort to gain access to politicians. Access allows a contributor to present evidence and arguments in favor of its preferred policy. Contributions typically are not provided in a quid pro quo exchange for policy favors. These statements not only summarize the claims of interest groups and policy makers (e.g., Herndon 1982, Schram 1995), they are also supported by empirical evidence (e.g., Langbein 1986, Milyo et al. 2000, Ansolabehere et al. 2002). Even campaign finance reform advocates argue that the current system is bad for society because it favors wealthy interests who can more easily buy access to politicians compared with less-wealthy interest groups and individuals (Makinson 2003).

This paper analyzes the impact of contribution limits under the assumption that political contributions buy access to politicians. I begin by incorporating access into a model of informational lobbying. The model relies on three fundamental assumptions that are consistent with the lobbying process as described by

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1For example, in Herndon (1982) an anonymous interest group representative stated: “About all you get [in exchange for a contribution] is a chance to talk to them... If you have a good case you can win them over. But you have to be able to talk to them.” Former US Senator Howard Metzenbaum said “Those who contribute may have more ready access and may at least be able to present their arguments with you whether you agree with them or not,” and former US Senator Dennis DeConcini said “What they got out of me for that contribution is access to come in .. and to tell me why ... it’s good ‘for America’” (Schram 1995).

2See also Sabato (1984), Hall and Wayman (1990), Wright (1990), Clawson et al. (1992).

3This view of political contributions and access is not limited to the United States. For example, Lee Rhiannon, an Australian politician, and Norman Thompson argue the following about their country: “Access is power, and money buys access to politicians in our country. This means large donors can influence governmental decisions, which benefit them and their companies” (Rhiannon and Thompson 2006).
interest groups and politicians. First, interest groups have verifiable evidence that can influence a politician’s beliefs about the best course of action. If interest group evidence is completely unverifiable, or if the groups have no private evidence, then they do not have an incentive to gain access to the politician since access does not enable them to influence the politician’s beliefs. Second, the politician controls who receives access. In order for an interest group to present its information to the politician, the politician must grant the group access. Third, the politician is constrained in his ability to provide access to all interest groups. Although the analysis assumes that the politician simply cannot give access to all groups due to time constraints, the results continue to hold if the politician can give access to everyone but faces a cost of doing so.

Using the above assumptions as a foundation, Section 2.3 models competition for access in which interest groups compete for access to a politician by providing political contributions. The groups that provide the largest contributions win access, and can present evidence or arguments in support of their preferred policies. After observing the contributions from all groups and the evidence presented by groups with access, the politician then implements the policies he believes are best. The model shows how politicians may collect contributions from interest groups without sacrificing voter welfare. Interest groups provide money to politicians even when the money buys access, not explicit favors. Even more interesting is that in equilibrium interest groups with better evidence provide larger contributions than those with worse evidence. The strict monotonicity of the equilibrium contribution function means that the rational politician learns an interest group’s evidence strength by observing its contribution, even if he does not grant the group access. By observing contributions, the politician learns about the evidence of all interest
groups, even when he only provides access to some groups. When money buys access, allowing contributions moves policy closer to the position that is best for the representative citizen.

A contribution limit distorts the signaling power of the contributions, and tends to result in a less-informed politician and worse policy. This is the focus of Section 2.4, where, for the basic competition for access model, I show that a contribution limit strictly reduces expected citizen welfare. The main result is in contrast to much of the past lobbying literature in which money is given to buy policy favors or to help preferred candidates win election (e.g., Grossman and Helpman 1994, Coate 2004a). In these previous models, a contribution limit tends to reduce the influence interest groups have on policy, thereby improving expected citizen welfare.5

The competition for access model analyzed in Sections 2.3 and 2.4 presents an optimistic view of money in politics, and stark results about the impact of contribution limits. However, the real world is far more complicated than the stylized competition for access model developed in Section 2.3. Sections 2.5 through 2.8 generalize the model in a variety of ways. Although some generalizations weaken the results, it remains clear that selling access tends to improve the politician’s ability to choose policies that benefit his constituents. In certain cases—when there is unobserved interest group heterogeneity or the politician can choose whether to sell access or sell policy favors—a carefully set contribution limit can improve expected citizen welfare.

One way to generalize the model is to allow for interest group heterogeneity. For example, interest groups might differ in terms of wealth, opportunity costs

4However, it is the possibility of receiving access (in which case the politician learns one’s evidence quality for sure) that drives the monotonicity of the contribution function. Therefore, it is essential that the politician gives access to a positive number of groups.

5See Section 2.2 for a review of the past literature.
of money, preference intensity, or distribution of evidence strength. By allowing for interest group heterogeneity, the model can directly address one of the most popular arguments in favor of contribution limits: that they level the playing field between wealthy interest groups and less-wealthy groups. This is the focus of Section 2.5.

Section 2.5.1 considers the case when interest group differences are known by the politician. This is the case for gun control policy, for example. Politicians recognize that the interest group against gun control, the National Rifle Association (NRA), is well financed and that interest groups in favor of gun control are relatively poor. Given that interest group asymmetries are observed by the politician, he can take these differences into account when updating his beliefs about an interest group’s evidence strength. The politician recognizes that a wealthy group chooses to contribute more than a less-wealthy group with the same quality evidence. Similarly, an interest group that cares intensely about an issue will contribute more than a group that cares less, all else equal. When there is interest group heterogeneity, each interest group may have a unique equilibrium contribution function that is determined by its own, individual characteristics. The rational politician, who is fully aware of these characteristics, can correctly determine the contribution functions for all agents (which continue to be strictly increasing in evidence quality), and he can thereby also correctly infer an interest group’s evidence strength given its contribution. In this case, contribution limits unambiguously result in a less informed politician who is less capable of identifying and implementing the socially optimal policy.

In Section 2.5.2, I allow for unobserved interest group heterogeneity. The analysis focuses on wealth heterogeneity by assuming that some interest groups face
a binding budget constraint while others do not. In this section, the politician is aware that interest groups may differ in terms of their ability to pay, but he is uncertain as to the characteristics of individual interest groups. The analysis shows that rich interest groups tend to realize higher equilibrium payoffs compared to poor groups, and that wealth differences may result in an equilibrium policy profile that is biased in favor of rich groups. Contribution limits can eliminate the rich-group advantage. However, I show that just because the contribution limit levels the playing field between rich and poor groups, this does not imply that a limit improves citizen welfare; although it can do so under certain conditions. Just as in the case without wealth differences, a contribution limit tends to reduce the number of interest groups for which the politician is certain about evidence quality. This can result in a less-informed politician and a policy profile that is worse for society. Although campaign finance reform advocates are correct in their claim that the exchange of contributions for access tends to favor wealthy interest groups, they are incorrect in concluding that contribution limits will therefore improve citizen welfare. Contribution limits do level the playing field between rich and poor interests, but they may also reduce the politician’s ability to identify and implement the welfare maximizing policy.

The initial model assumes a simple information structure in order to maximize intuition for the results. Section 2.6 discusses a variety of more general information structures, including cases in which an interest group’s evidence strength is not independent from other groups’ evidence, and in which interest groups are uncertain as to how the politician will interpret their evidence. Section 2.7 endogenizes the politician’s choice of how many groups receive access.

Section 2.8 allows the politician to choose whether he sells access or sells policy
favors. I show that selling policy maximizes contributions, but selling access results in higher policy utility for the politician (he is able to identify and implement the best policies). When the politician does not care enough about policy relative to contributions, he prefers to engage in the quid pro quo exchange of money for policy favors. When he puts enough weight on the policy outcome relative to contributions, the politician prefers to sell access.\(^6\) In this case, a carefully set contribution limit can improve expected citizen welfare by making it more likely that the politician sells access rather than policy. However, too strict of a limit is not optimal because it reduces the politician’s ability to identify and implement the socially optimal policy.

The paper concludes with Section 2.9 where I discuss the results, policy implications, and additional extensions and applications of the model.

### 2.2 Literature

This paper incorporates access into a model of informational lobbying. Although empirical and survey evidence supports the idea that political contributions buy access,\(^7\) few theoretical papers incorporate access into their models. Past papers

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\(^6\)The revenue results are consistent with Ansolabehere et al. (2003) who argue that total political contributions are significantly less than would be expected if money was given in the direct exchange for policy favors.

\(^7\)For example, Langbein (1986) finds correlation between a politician’s contributions and the amount of time the politician spends meeting with constituents and interest groups. Ansolabehere et al. (2002) finds correlation between an organizations political contributions and their spending on lobbyists. This suggests that influencing policy requires both contributions to politicians (which secures access) and the careful presentation of information (which may require a lobbyist). Herndon (1982) surveys interest group representatives to determine their reason for contributing. All of the business interests in his survey emphasize access as a reason for contributing. Other interest groups, including labor unions, say that their main reason for contributing is to help their preferred candidates win election. Schram (1995) and Makinson (2003) interview retired politicians, as well as representatives from special interests and campaign finance reform advocacy groups. They offer substantial anecdotal support of the idea that money buys access to politicians.
that incorporate “access” do so in a way that is largely inconsistent with the access story told by interest groups and politicians. Both Austen-Smith (1995) and Lohmann (1995) develop models in which interest groups receive private, unverifiable signals about the impact of a certain policy. In these models, an interest group who favors the policy will always claim it received a positive signal, independent of its true signal. However, groups that do receive positive signals have a higher expected benefit from the policy being implemented, and a higher expected benefit from the politician taking their signals into account when updating his beliefs. In equilibrium, the politician acts as if an interest group received a signal in support of its own position only if the group provides a large-enough contribution.\footnote{In this way, these models are similar to the signaling models of Austen-Smith (1994) and Esteban and Ray (2006) in which interest groups provide contributions or engage in costly lobbying in order to provide a signal to a policy maker regarding their preferences.}

Although these models are considered “access” models, the possibility of face-time with politicians is completely irrelevant since interest groups will always claim that their signal was in favor of their own position.\footnote{In the basic model developed in Section 2.3 of this paper, interest groups do not care whether they win access in equilibrium since their evidence strength is fully revealed by their contributions. However, unlike in these other access models, it is essential that the politician grant access to a positive number of interest groups, otherwise, the interest groups have no incentive to contribute in equilibrium. Furthermore, when there is a contribution limit or unobserved interest group wealth differences, certain interest groups do receive additional benefit from gaining access to the politician.} Equilibrium behavior in these models is clearly inconsistent with the story told interest groups and politicians, in which access itself plays an important role.\footnote{These models also make the unrealistic prediction that contributions are increasing in the distance between an interest group’s preferences and the preferences of the politician. However, data show that interest groups tend to give more money to politicians with similar preferences. Austen-Smith (1995) shows how this limitation of the model is eliminated when he assumes that politicians do not know the policy preferences of the interest groups. He argues that if one finds such an assumption unreasonable (which I do), then he may have to reject the belief that interest groups seek access in order to share evidence. I show that this conclusion is incorrect. In this paper’s competition for access model, access is present even though the politician knows the policy preferences of the interest groups.}

Assuming that interest group information is verifiable results in a more realistic model of informational lobbying. To my knowledge, Austen-Smith (1998) is the only other existing
paper in which access allows one to present verifiable evidence to the politician. The primary difference between the present paper and Austen-Smith (1998) is the mechanism through which the politician allocates access. In this paper, the politician allocates access through an auction, and the resulting competition between interest groups is the driving force that results in full revelation of evidence quality.\(^\text{11}\) The competition for access structure developed in this paper is a reasonable starting point from which to consider the allocation of access, as the competition framework is similar to previous rent-seeking models in which a politician auctions off an explicit prize such as a government contract or the politician’s vote (e.g., Baye et al. 1993, 1996, Che and Gale 1998). Furthermore, I present a more simple evidentiary structure compared to Austen-Smith (1998) that allows the analysis to focus more directly on the implications regarding the sale of access.

There is a significant theoretical literature that considers the role of money in politics, but ignores the issue of access.\(^\text{12}\) Most of the literature focuses on one of two motivations behind political contributions. First, contributions may help one’s favorite candidate compete for election (e.g., Coate 2004b). For example, the National Riffle Association may give money to candidates who hunt, and Planned Parenthood may give money to candidates who have made public statements against stricter abortion legislation. A politician who requires money to fund his campaign may have an incentive to commit to policies that attract larger interest group contributions rather than the policies that are best for his constituents. Second, contributions may be intended to directly influence policy. In this case, politicians and interest groups engage in the quid pro quo exchange of contributions for favorable policy. In traditional rent-seeking models, interest

\(^{11}\)In Austen-Smith (1998), the politician sets prices for access, and any interest group that pays the access price gains access.

\(^{12}\)For an excellent overview of the lobbying literature, see Grossman and Helpman (2002).
groups compete (by providing contributions) for an explicit policy favor such as a government contract or favorable legislation (e.g., Tullock 1980, Baye et al. 1993, Che and Gale 1998). In the influential models of Bernheim and Whinston (1986) and Grossman and Helpman (1994, 1996), interest groups commit to a payment schedule that defines how much they contribute to a politician based on his policy choice. In Prat (2002a,b) and Coate (2004a), a politician who cares about winning election may provide policy favors to special interests in order to attract contributions to finance his campaign. In each of these models, the politician may choose policies that favor wealthy special interests in order to attract contributions. In this way, contributions tend to distort policy away from that which is best for constituents. In the current paper, contributions have the opposite impact on policy, as they tend to move policy closer to the position that is best for constituents.

There is also a significant literature on the disclosure of hard information; however, this literature ignores the issue of access all-together. Milgrom and Roberts (1986) consider conditions under which agents have an incentive to fully reveal their private information to a decision maker in a setting where all agents automatically receive access. Applied to the competition for access framework developed in this paper, their results establish that an interest group will always disclose its evidence once it receives access. Only an interest group with the worst possible evidence quality has the incentive not to reveal its evidence. Bennedsen and Feldmann (2002) consider the choice of interest groups to disclose information in a multiple-policy-maker setting. Bennedsen and Feldmann (2006) and Dahm and Porteiro

13See also other applications of an all-pay auction by Holt (1979), Holt and Sherman (1982), Baye et al. (1993, 1996), Anderson et al. (1998), Che and Gale (1998), and Moldovanu and Sela (2001). When applied to lobbying, these models require that a politician be willing to award policy favors to the highest contributors, even if the action hurts their constituents. In competition for access, interest groups bid for access rather than the prize itself, and the politician is free to choose his preferred policy at the end of the game. Interest groups are willing to pay for the opportunity to influence the politicians belief’s about the best policy.
(2006a,b) allow interest groups to influence policy through both the disclosure of hard evidence and a quid pro quo exchange of contributions and policy favors. Papers such as Green and Laffont (1986), Lipman and Seppi (1995), and Bull and Watson (2004, 2007) formalize the concept of evidence. More-formal evidentiary structures may be incorporated into the competition for access model without changing the results. In these models, and throughout this literature, agents do not require access to disclose their information.

After showing that contributions enable to politician to choose policy that is better for his constituents, I formally consider the impact of a contribution limit on citizen welfare. This is not the first paper to analyze contribution limits, but it is the first paper to do so in a setting where interest groups provide contributions in competition for access to present hard evidence. Much of the previous literature on contribution limits considers the impact of limits in models in which politicians sell policy favors. In such a setting, limits decrease the monetary incentive to sell policy favors, and increase the likelihood that the politician chooses the policies preferred by his constituents (e.g., Prat 2002a,b, Coate 2004a). In this way, limits can have a welfare-improving affect on policy. A similar case is made by Austen-Smith (1998) where a contribution limit can cause a politician to provide access to more-informed interest groups rather than groups with higher willingness to pay for access; this can result in a better-informed politician. However, not all papers suggest that contribution limits improve welfare. For example, in Drazen et al. (2007) limits can increase politically motivated government spending. Riezman and Wilson (1997) show how a politician may choose to sell additional policy favors in order to make up for a decrease in contributions that can result from a limit. In Wittman (2002) and Coate (2004b) limits decrease the amount of advertising,

\[14\] The evidentiary structure must meet the evidentiary normality condition from Bull and Watson (2007), equivalently, the full reports condition from Lipman and Seppi (1995).
which results in a less-informed electorate. Coate (2004b) shows that a limit tends to redistribute welfare from ordinary citizens to interest groups. Where Wittman (2002) and Coate (2004b) show how a limit may result in less-informed voters, I show how a limit may also result in a less-informed politician. Additionally, a variety of other papers consider the impact that bid caps have on bidder behavior in auctions. For example, Che and Gale (1998) and Gavious et al. (2002) focus on the effect contribution limits have on total revenue.

2.3 Informational Lobbying and Access Game

In this section, I develop a simple model of competition between interest groups for access to a decision maker. By starting with a simple model (i.e., one with a simple information structure and without interest group heterogeneity), the analysis can maximize intuition for the results. As I show in later sections, one can improve the descriptive ability of the model without changing the primary results.

2.3.1 Model

There are \( N \) independent policy issues. There is a risk-neutral politician who, for each issue, must choose a policy from a single-dimensional policy space defined by the interval \([-1, 1]\). There are a total of \( 2N \) interest groups, where for each of the \( N \) issues one interest group prefers policy \(-1\), and one group prefers policy \(1\). An interest group is denoted by the issue it is concerned with and its policy preference; therefore \((n, j)\) refers to the interest group concerned with issue \(n \in \{1, ..., N\}\) and policy preference \(j \in \{-1, 1\}\). Where it is clear which issue a group is concerned
with, I refer to it as group j.

At the beginning of the game, each interest group draws private evidence in support of its preferred policy. The quality of \((n, j)\)'s evidence is denoted by \(e^j_n\) and is the independent realization of a random variable distributed on the continuum \([0, 1]\). A higher \(e^j_n\) can be thought of as interest group \((n, j)\) having stronger evidence or a better argument in support of its preferred policy. The distribution of evidence quality is denoted by function \(F\), with density function \(f\), and is common knowledge.\(^{15}\)

After the interest groups realize their evidence qualities, they independently provide contributions to the politician. Group \((n, j)\) provides contribution \(b^j_n \geq 0\). Interest groups receive access if they provide one of the \(K\) largest contributions, where \(K \in \{1, \ldots, 2N - 1\}\).\(^{16}\) If the \(K\)th and \((K + 1)\)th largest contributions are equal, then all of the groups that provide this same contribution receive access with equal probability.

Interest groups with access present their evidence to the politician. When a group presents its evidence, the politician becomes fully informed of the group’s evidence quality. Assuming that interest groups with access must present their evidence greatly simplifies the description of the game. However, the results do

\(^{15}\)The body of the paper assumes that the distribution of evidence quality is the same for all interest groups. This does not have to be the case. Alternatively, \(F^j_n\) could define the distribution of group \((n, j)\)'s evidence. So long as the different distributions are common knowledge, the results of the analysis do not change.

Assuming that the evidence quality of the different interest groups is independently drawn simplifies the analysis. However, an alternative model can be developed in which the evidence quality of the two groups concerned with the same issue is correlated. For example, when one of the groups concerned with an issue has very strong evidence, it may be more likely that the other group concerned with the same issue has relatively low evidence. Under reasonable assumptions, such an alternative model would not change the results of the analysis.

\(^{16}\)If \(K = 0\) or \(K = N\), then interest groups provide no contributions in equilibrium. The basic model assumes that \(K\) is determined independently of the model. Section 6 discusses the case where \(K\) is endogenous. As Section 6 shows, the politician will commit to provide a positive amount of access \((K \geq 1)\) when \(K\) is endogenous.
not change if groups are allowed to reject access.\textsuperscript{17} Let $\Omega$ denote the vector of interest group evidence qualities revealed to the politician through access. After observing the contributions of all interest groups and the evidence quality of those with access, the politician chooses a policy for each of the $N$ issues (by maximizing a payoff function defined later). Let $p^*_n \in [-1, 1]$ denote the policy implemented by the politician for issue $n$, and let $p^* = \{p^*_1, ..., p^*_N\}$ denote the policy profile chosen for all issues.

The politician does not sell policy favors. Contributions determine whether an interest group receives access, but do not directly influence the policy choice of the politician. Contributions are non-refundable, and are not contingent on being granted access. Therefore, the exchange of access for political contributions is an \textit{all-pay auction}: all bidders (interest groups) pay their bids (contributions) before the prizes (access) are allocated to the highest bidders.\textsuperscript{18}

The politician is \textit{fully informed} about an issue if he is certain about the evidence quality of both interest groups involved with the issue. When a politician is fully informed about an issue, he can determine the welfare-maximizing policy choice for that issue. Let $p^o_n$ denote the \textit{socially optimal policy} for issue $n$, and the vector $p^o = \{p^o_1, ..., p^o_N\}$ define the vector of socially optimal policies across all issue. For simplicity, I assume $p^o_n$ takes the form $p^o_n = e^1_n - e^{-1}_n$. It is straightforward to

\textsuperscript{17}It can be shown that interest groups with access will always accept. If a group rejects access, the politician believes that the group had lower evidence quality than he expected. This causes him to update his beliefs and lower his expectation. This results in an unravelling of beliefs until the politician believes any interest group that rejects an offer of access has the lowest possible evidence quality.

\textsuperscript{18}I use an all-pay auction to model competition for access because it seems the most realistic framework. The results continue to hold so long as the probability of winning access is non-decreasing in the size of a group’s contribution, which results in an equilibrium contribution function that is strictly increasing in an interest group’s evidence quality. The model could alternatively assume that the politician allocates access through another type of auction, a lottery in which one’s probability of gaining access is proportional to the relative size of a group’s contribution, or even if all groups receive access with equal probability independent of their contributions.
incorporate a more complicated socially optimal policy function into the analysis, and reasonable changes to the function do not change the paper’s results.\footnote{For any issue \( n \), the analysis requires that \( p^n_\alpha (e_n^{-1}, e_n^1) \) be strictly decreasing in \( e_n^{-1} \), strictly increasing in \( e_n^1 \), additively separable, and such that \( -1 \leq p^n_\alpha (1, 0) \leq p^n_\alpha (0, 1) \leq 1 \). The function \( p^n_\alpha \) does not have to be linear in evidence quality. So long as the function is additively separable in terms of \( e^{-1} \) and \( e^1 \), the asymmetries between the impact of the two groups’ evidence on the optimal policy may be accounted for through a transformation of their evidence distribution functions \( F^n_j \). As stated previously, allowing for asymmetric distribution functions does not change the results of the analysis.}

**Payoffs**

Citizen welfare depends on the difference between the implemented policy and the socially optimal policy across all issues. The parameter \( \gamma_n > 0 \) represents the relative weight society places on issue \( n \). Citizen welfare is

\[
W (p^*, p^\alpha) = -\sum_{n=1}^N \gamma_n \times |p^*_n - p^\alpha_n|.
\] (2.1)

Welfare is maximized when the politician implements the socially optimal policy profile \( p^* = p^\alpha \). The politician can determine \( p^\alpha \) with certainty only when he is fully informed about all issues.

The politician is concerned with citizen welfare and collecting political contributions. The parameter \( \rho \geq 0 \) represents how much the politician cares about revenue generation relative to citizen welfare, and \( b \) represents the profile of contributions from all interest groups. His payoff is

\[
U^P (p^*, p^\alpha, b) = W (p^*, p^\alpha) + \rho \sum_{n=1}^N (b^1_n + b^{-1}_n).
\] (2.2)

An interest group’s payoff is decreasing in the size of its contribution and the distance between its preferred policy and the implemented policy for the issue it
cares about. Group \((n,j)\)'s payoff is

\[
U_n^j (p^*_n, b^j_n) = V (|p^*_n - j|) - b^j_n. 
\] (2.3)

The function \(V\) defines interest group policy utility where \(V'(\cdot) < 0\), and \(V(0) = 0\). Since interest group \((n,j)\) prefers policy \(j\), \(|p^*_n - j|\) denotes the distance between the implemented policy and the group’s preferred policy.\(^{20}\)

**States and Beliefs**

The realized state of the world is defined by the vector of realized evidence qualities \(\{e^j_n\}_{\forall (n,j)}\). Let \(S\) denote the set of all potential states of the world, and \(s \in S\) denote an arbitrary state within the state space \(S\). Each \(s\) assigns a value \(e \in [0, 1]\) to each interest group. Let \(e^j_n (s)\) denote the evidence quality of \((n,j)\) in state \(s\). The function \(\mu (\cdot | b, \Omega)\) defines the politician’s beliefs about the state of the world given the contribution vector \(b\) and the vector of evidence revealed through access \(\Omega\). These beliefs may be fully represented by the vector of all updated density functions \(\{f^j_n (\cdot | b, \Omega)\}_{\forall (n,j)}\), where \(\mu (s | b, \Omega) = \prod_{(n,j)} f^j_n (e^j_n (s) | b, \Omega)\). Also the operator \(E\) represents the expectations given the ex ante distribution of states, and \(E_\mu\) represents the politician’s expectations given his beliefs \(\mu\).

**Solution Concept**

The analysis solves for the symmetric Perfect Bayesian Equilibrium of the game, which I label the *contribution equilibrium*. A complete description of the equilibrium must include the strategy profiles for the interest groups and the politician, as well as the politician’s beliefs about the state of the world at the time he chooses a policy profile. The politician’s beliefs must be consistent with using Bayes’ Rule on the ex ante distribution of evidence quality given the strategies of the interest

\(^{20}\)All interest groups have the same policy utility function. The results do not change if the utility functions differ, so long as they are common knowledge.
groups. Each player’s strategy must be a best response to the strategies of the other players, given the player’s beliefs.

In the contribution equilibrium, all interest groups share the same contribution function \( B : [0, 1] \rightarrow b \), where \( B(e) \) defines the equilibrium contribution for an interest group with evidence quality \( e \). The value \( P^*_n(\mu) \) defines the politician’s equilibrium policy choice given his beliefs. A description of \( P^*_n(\mu) \) for all possible \( \mu \) and each \( n \) fully describes the politician’s equilibrium strategy.\(^{21}\)

### 2.3.2 Equilibrium

I first determine the politician’s policy choice at the conclusion of the game, then analyze the all-pay auction in which interest groups choose the size of their contributions, and the highest contributors receive access.

#### Policy Choice

\(^{21}\)A formal definition of a contribution equilibrium requires some additional notation. Let \( \Omega (b_n, s; B) \) define the vector of evidence qualities presented by interest groups with access in state \( s \) when group \( (n, j) \) contributes \( b_{nj} \), and all other groups contribute according to \( B \). Let \( \hat{\mu}(s \mid e) \) denote the probability that an interest group puts on the world being in state \( s \in S \) given that its own evidence quality is \( e \).

**Definition 1** The interest group contribution function \( B \), politician strategy \( \{P^*_n\}_{n=1}^N \), and politician beliefs \( \mu \) constitute a **contribution equilibrium** if

1. For all \( e_j^n \in [0, 1] \),
   
   \[ B(e_j^n) \in \arg\max_{b_{nj}} \int_{s \in S} \hat{\mu}(s \mid e_j^n) U_{nj}^j(P^*_n(\mu), b_{nj}) \, ds \]

2. For any possible \( b \) and \( \Omega \),
   
   \[ \{P^*_n(\mu)\}_{n=1}^N \in \arg\max_{p^*, \{p_{nj}(e_1^n(s), e_n^{-1}(s))\}_{\forall n}, \{B(e_j^n(s))\}_{\forall (n,j)}} \int_{s \in S} \mu(s \mid b, \Omega) U^P(p^*, \{p_{nj}(e_1^n(s), e_n^{-1}(s))\}_{\forall n}, \{B(e_j^n(s))\}_{\forall (n,j)}) \, ds \]

3. Beliefs \( \mu \) meet the requirements of Perfect Bayesian Equilibrium, given the equilibrium strategy profile.

For a detailed description of Perfect Bayesian Equilibrium belief requirements, see Fudenberg and Tirole (1991, pp. 324-326).
At the time the politician chooses policy, the interest groups have already given their contributions. This means the policy choice can only impact citizen welfare. The politician chooses the policy profile that maximizes expected citizen welfare given his beliefs. For each issue \( n \), his policy choice \( p^*_n \) is defined by the function
\[
P^*_n(\mu) = E_\mu p^*_n = E_\mu e^1_n - E_\mu e^{-1}_n,
\]
where \( E_\mu \) denotes the politician’s expectations given beliefs \( \mu \).
When the politician is fully informed, \( E_\mu e^j_n = e^j_n \), and \( E_\mu p^*_n = p^*_n \).
A politician who is fully informed about all issues implements the socially optimal policy profile and maximizes citizen welfare.

**Interest Group Contributions**

In equilibrium, all interest groups contribute according to the contribution function \( B \). I start with the assumption that the contribution function is strictly increasing in a group’s evidence quality. After solving for \( B \), I show that this assumption holds. Since \( B \) is strictly increasing, it is invertible, where \( e(b) = B^{-1}(e) \), and there is a one-to-one mapping between the group’s contribution and its evidence quality. It immediately follows that a rational agent can determine an interest group’s evidence quality if he observes its contribution.

To solve for the contribution function, the analysis considers the contribution decision of interest group \((n,j)\) assuming that all other groups contribute according to the equilibrium function. Because the other groups contribute according to \( B \), the politician is certain regarding all other groups’ evidence qualities. Let \( \Theta(e; e^{-j}_n) \) be the probability that fewer than \( K \) other interest groups have evidence quality greater than \( e \), given that group \((n,−j)\) has \( e^{-j}_n \). Therefore, \( \Theta(e(b); e^{-j}_n) \) denotes the probability that group \((n,j)\) receives access given contribution \( b \). Interest

\[22\] Also, \( E_\mu e^j_n = \int_{s \in S} \mu(s \mid b, \Omega) e^j_n(s) \, ds \).
group \((n, j)\) chooses its contribution \(b\) to maximize the expression:

\[
\int_0^1 f(e^{-j}) \left[ (1 - \Theta(e(b); e_{n}^{-j})) V(1 - e(b) + e_{n}^{-j}) + \Theta(e(b); e_{n}^{-j}) V(1 - e_{n}^{j} + e_{n}^{-j}) \right] de_{n}^{-j} - b. 
\]

(2.4)

With probability \(\Theta(e(b); e_{n}^{-j})\) group \((n, j)\) receives access and presents its evidence to the politician who then chooses policy \(e_{n}^{j} - e_{n}^{-j}\). This results in policy utility \(V(1 - e_{n}^{j} + e_{n}^{-j})\) for the interest group. Alternatively, with probability \((1 - \Theta(e(b); e_{n}^{-j}))\) the group does not receive access and the politician believes the interest group has evidence quality \(e(b)\) rather than its true evidence quality \(e_{n}^{j}\). This results in interest group policy utility \(V(1 - e(b) + e_{n}^{-j})\).

First order conditions for the interest groups’ problem are given by

\[
\int_0^1 f(e_{n}^{-j}) \left[ (1 - \Theta(e(b); e_{n}^{-j})) V'(1 - e(b) + e_{n}^{-j}) \frac{\partial e}{\partial b} (-1) + \Theta(e(b); e_{n}^{-j}) V(1 - e_{n}^{j} + e_{n}^{-j}) \right] de_{n}^{-j} - 1 = 0.
\]

(2.5)

The first row of notation represents the marginal impact of a change in an interest group’s contribution on the politician’s beliefs about the group’s evidence quality provided that it does not win access, and the corresponding change in the group’s policy utility. The second row represents the marginal impact of a change in a group’s contribution on the probability the group wins access.

In equilibrium, all interest groups contribute according to the function \(B\), which implies \(e(b) = e_{n}^{j}\) for all \((n, j)\). Strict monotonicity of the function means \((\frac{\partial e}{\partial b})^{-1} = \frac{\partial B}{\partial e}\). The first order conditions simplify to

\[
\frac{\partial B(e)}{\partial e} = - \int_0^1 f(e_{n}^{-j}) \left( 1 - \Theta(e_{n}^{j}, e_{n}^{-j}) \right) V'(1 - e_{n}^{j} + e_{n}^{-j}) \; de_{n}^{-j}.
\]

(2.6)

It is straightforward to show that \(\frac{\partial B}{\partial e}\) is positive.\(^{23}\) Therefore, the contribution

\(^{23}\)This follows because \(f(e) > 0, (1 - \Theta(e)) \geq 0\) (with strict inequality for some \(e\)), and \(V'(\cdot) < 0\).
function $B$ is strictly increasing in a group’s evidence quality. This also means that a group’s evidence quality is increasing in the size of its equilibrium contribution, and that the politician can correctly infer an interest group’s evidence quality by observing its contribution.

The closed-form solution for the contribution function is

$$B(e) = -\int_0^e \int_0^1 f(e_n^j) \left(1 - \Theta(y, e_n^j)\right) V'(1 - y + e_n^j) de_n^j dy. \quad (2.7)$$

In equilibrium, for any evidence quality $e$, the benefit an interest group receives from bidding more than $B(e)$ in an attempt to convey higher-quality evidence is completely offset by the cost of doing so.

**Game Equilibrium**

The above analysis derives the unique contribution equilibrium of the game. The first lemma summarizes the results.

**Lemma 1** In the contribution equilibrium,

1. $b_n^j = B(e_n^j)$ for all $(n, j)$

2. $P_n^*(\mu) = E_\mu \rho_n^0 = E_\mu e_n^1 - E_\mu e_n^{-1}$ for all $n \in \{1, ..., N\}$, and

3. the politician’s beliefs $\mu$ are such that for any $(n, j)$, $f_n^j(e_n^j | b, \Omega) = 1$ if group $(n, j)$ has access, and $f_n^j(e(b_n^j) | b, \Omega) = 1$ if group $(n, j)$ does not have access.

In the contribution equilibrium, all interest groups contribute according to the same function $B$, and the politician chooses the policy profile that he believes maximizes citizen welfare. Because the monotonicity of the contribution function allows the politician to learn the evidence quality of all interest groups, the
politician knows the socially optimal policy profile at the time he chooses policies. It immediately follows that the politician implements the socially optimal policy profile. This result is stated by the first proposition.

**Proposition 1** In the contribution equilibrium, \( p_n^* = p_n^o \) for all \( n \in \{1, \ldots, N\} \).

The first lemma and proposition follow directly from the above analysis.

Previous models of political contributions imply that contributions result in policies that benefit special interests, but decrease overall welfare (e.g., Grossman and Helpman 1994). This paper suggests that contributions can have the opposite impact on citizen welfare. In competition for access, political contributions can have a positive impact on overall welfare because they enable the politician to recognize and implement better policies.

**Importance of Access**

In equilibrium, the politician becomes fully informed about the evidence quality of all groups by observing their contributions alone. This does not imply that the politician can provide no access. If the politician does not provide access to any group, then the contributions become uninformative. Without access, all interest groups face the same incentives when choosing their contributions; groups with high qualifications are no longer willing to provide larger contributions than groups with low qualifications. The politician recognizes this and does not take the size of the contributions into account when updating his beliefs. This means that \( E_\mu e^\mu_n = E e^{ij}_n \) for all \((n, j)\), and the politician chooses \( p_n^* = 0 \) for all \( n \). Because contributions have no impact on the politician’s beliefs, the interest groups are unwilling to contribute anything, and the politician receives nothing. If the politician provides
no access, the interest groups provide no contributions and the politician does not
learn anything about the groups’ evidence. However, providing access to at least
one group allows the politician to become fully informed about the evidence quality
of all groups. Section 5 endogenizes the amount of access.

2.4 Contribution Limit

The previous section assumes that there are no limits to the maximum size of
interest group contributions. This section considers how the analysis changes if
contributions are constrained.\textsuperscript{24} The parameter $\bar{b}$ denotes the maximum allowed
size of a contribution, where $b_{ij} \in [0, \bar{b}]$ for all $(n, j)$. Assume $0 < \bar{b} < B(1)$, which
implies that the contribution limit is lower than the maximum possible contribution
in the game without contribution limits, and high enough such that contributions
exist.\textsuperscript{25} If $\bar{b} \geq B(1)$, the limit has no affect on interest group contributions when
the politician sells access.

When there is a contribution limit, the politician continues to implement the
policy profile he believes is best for society, or $P^*_n(\mu) = E_\mu e^1_n - E_\mu e^{-1}_n$ for each $n$.
However, because contribution limits change the politician’s beliefs $\mu$, they change
the resulting policy profile.

\textbf{Impact of Contribution Limits}

\textsuperscript{24}Che and Gale (1998) consider the impact of a contribution limit in a game where bidders
in an all-pay auction compete for a policy favor (e.g., a government contract) rather than for
access (as is the case in this paper). They show that even in the competition for policy favors,
contribution limits can have a negative impact on social surplus.

\textsuperscript{25}When more interest groups provide the maximum contribution than the politician provides
access to, I assume that the politician allocates access between each of the groups with equal
probability. Alternatively, he could provide access to the interest groups involved with the issues
that he cares the most about (those with the largest $\gamma_n$’s). This alternative assumption does not
change the results, although it complicates the analysis.
If there did not exist a contribution limit, then groups with high-enough evidence quality would contribute more than $\bar{b}$ in equilibrium. With a limit in place, groups with high $e$ prefer to provide the maximum contribution $\bar{b}$ compared to a lower amount; although some groups may prefer to contribute more than $\bar{b}$ if it was allowed. Groups with relatively low $e$ prefer to contribute less than $\bar{b}$.

The appendix formally derives the equilibrium of the game with a contribution limit. Here, I describe the results. The equilibrium contribution function $B_{CL} : [0, 1] \rightarrow b$ is a discontinuous function comprised of two parts: a continuous function $\tilde{B}$ for low enough $e$, and the constant $\bar{b}$ for higher $e$. Let $\bar{e}$ denote the evidence quality of the interest group that is indifferent between contributing according to $\tilde{B}$ and contributing amount $\bar{b}$. Therefore,

$$B_{CL}(e) = \begin{cases} 
\tilde{B}(e) & \text{for } e \in [0, \bar{e}) \\
\bar{b} & \text{for } e \in [\bar{e}, 1]. 
\end{cases}$$ (2.8)

The function $\tilde{B}$ is derived in the same way that $B$ was derived in the game without contribution limits (I formally derive $\tilde{B}$ in the appendix). $\tilde{B}$ is strictly increasing in a group’s evidence quality. Therefore $\tilde{B}$ is invertible and the politician becomes fully informed of the evidence quality of any group that provides a contribution according to this function. In contrast, when the politician observes contribution $\bar{b}$, he cannot infer which value of $e \in [\bar{e}, 1]$ resulted in such a contribution. The politician only learns with certainty the evidence quality of an interest group that provided the maximum contribution if he grants that group access. It is possible that more interest groups provide the maximum contribution than the politician grants access to. When this happens, the politician remains less than fully informed about the evidence quality of some groups, and cannot determine the policy profile that maximizes citizen welfare.\textsuperscript{26}

\textsuperscript{26}In the equilibrium of the no-limit game, all interest groups are indifferent between gaining
Figure 2.1: Example contribution functions with limit $\bar{b}$

An example of a contribution function $B_{CL}$ is provided by Figure 2.1.

The politician may no longer know each group’s evidence quality with certainty. The following proposition interprets this difference in terms of citizen welfare.

Expected citizen welfare is strictly lower when there is a contribution limit than when there is no limit.

**Proposition 2** *In the informational lobbying game, $EW(p^*, p^0)$ is strictly higher when there is no contribution limit than if there exists a contribution limit $\bar{b} \in [0, B(1))$.***

access and not gaining access to the politician after they submitted their contribution. This is because their contributions communicate their evidence quality to the politician, and gaining access does not allow them to further impact the politician’s beliefs. This is not the case in the game with a contribution limit. The politician acts as if all interest groups that provide the maximum contribution (and do not gain access) have the same expected evidence quality. The groups that have evidence quality above this expected level are made better off if they gain access, since access results in the politician learning that their evidence quality is higher than his expectations.
Since the politician provides access to fewer than the total number of interest groups \((K < 2N)\), there is a positive probability that the number of interest groups with \(e \geq \bar{e}\) (the number that contribute \(\bar{b}\)) is larger than the number of groups that receive access. Since the politician only learns the evidence quality of a group that provides \(\bar{b}\) if the group receives access, there is a positive probability that the politician is less than fully informed when he chooses a policy profile. A less than fully informed politician almost certainly chooses a policy profile that is different from the social optimal. Therefore, contribution limits strictly reduce expected welfare. This does not mean that the realized welfare is necessarily lower. Rather, contribution limits never improve realized citizen welfare, and they reduce realized welfare with positive probability.

Although a contribution limit reduces expected citizen welfare, it does not necessarily reduce total contribution revenue collected by the politician. Che and Gale (1998) establish that a bid cap in an all-pay auction can increase total revenue generation. This possibility is also present in the competition-for-access framework. Therefore, when the politician cares enough about contributions relative to citizen welfare, he may benefit from a contribution limit, even though the limit strictly reduces expected citizen welfare.

### 2.5 Interest Group Asymmetries

The previous sections assume that interest groups all have the same wealth, preference intensity, and distribution of evidence quality. In reality interest groups are not homogeneous. Section 2.5.1 allows for interest group asymmetries when these differences are observed by the politician. In many situations, assuming that
the politician is aware of interest group asymmetries seems the most reasonable
approach. However, it may not always be the case, and in Section 2.5.2 I consider
the case in which differences in interest group budget constraints are unobserved
by the politician.

Throughout Sections 2.5.1 and 2.5.2, I focus on wealth differences instead of
preference intensity, evidence distribution, or other possible heterogeneities be-
cause it allows me to address issues central to the policy debate. Campaign fi-
nance reform advocates often argue that the contributions-for-access system fa-
vors wealthy interest groups relative to less-wealthy interest groups and individ-
uals. They claim that limiting or banning contributions reduces the rich-group
advantage and therefore increases citizen welfare. The analysis illustrates the flaw
in this logic. In Section 2.5.1, limits never improve citizen welfare. In Section
2.5.2, a contribution limit can level the playing field between rich and poor inter-
est groups; however, I show that this does not imply that the limit also improves
citizen welfare.

2.5.1 Known differences

If the politician observes asymmetries between the interest groups, then the main
results of the model remain unchanged. Contributions continue to provide the
politician with information about interest group evidence strength, and contribu-
tion limits tend to reduce the accuracy of the politician’s beliefs.

There are two simple ways to incorporate wealth inequality into the model. So long as the politician observes the differences between the interest groups, it is not required that interest groups themselves are aware of the characteristics of other groups.

An implicit assumption here is that there are multiple rich interest groups. When this is the case, the politician may continue to grant access to the highest bidders. Alternatively, the
First, interest group utility functions may put different weights on contributions. In this case, rich group utility functions put a smaller weight on money compared with poor groups. Second, some interest groups may face binding budget constraints.

Consider the first way of incorporating wealth differences. Let $\beta^j_n$ denote the weight that interest group $(n, j)$’s utility function puts on its contribution, where $\beta^j_n \in \{\beta_R, \beta_P\}$ and $\beta_R > \beta_P$. Therefore, the interest group’s payoff is given by

$$U^j_n (p^*_n, b^*_j) = V(|p^*_n - j|) - \beta^j_n b^*_j.$$ (2.9)

The value $\beta^j_n$ is observed by group $(n, j)$ and the politician. One can show that, in equilibrium, a group’s contribution is decreasing in $b^*_j$, all else equal. The more costly an interest group finds providing a contribution, the less the group will contribute given the same quality evidence. Because the politician knows how much interest groups care about money (he knows each group’s utility function), he can account for these differences when updating his beliefs. When two groups give the same contribution in equilibrium, and the politician knows that one of the groups cares more about money than the other group, then the politician will correctly infer that the interest group that cares more about money must have higher-quality evidence compared to the group that cares less about money. The rational politician can correctly derive each group’s individual contribution function, and he can therefore also infer each group’s evidence quality. Without a contribution limit, the politician remains fully informed about interest group

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29 Incorporating wealth differences in such a way has similar implications as allowing groups to have different policy utility functions or ex ante distributions of evidence quality.

30 The analysis does not require that an interest group’s characteristics are observed (or unobserved) by the other groups, so long as the distribution of the characteristic is common knowledge.
evidence quality, and can choose the socially optimal policy in equilibrium. A contribution limit has a similar impact on each individual contribution function as it has on the common contribution function in Section 2.4.

The above explanation applies to cases when group asymmetries do not limit the ability of some groups to provide a contribution. Conceivably, interest groups might differ in terms of their budget constraints. In this alternative case, the politician may be less than fully informed about interest group evidence quality even when there is no limit. However, one can show that even though the politician may not be able to identify and implement the socially optimal policy, imposing a contribution limit tends to result in an even less informed politician and worse policy choice than when there is no limit.

In both of the cases considered here, a contribution limit reduces the ability of groups with relatively high-quality evidence to communicate their evidence strength to the politician. This effect is greatest for rich groups. Since poor groups contribute less than similar rich groups, the contribution limit is less likely to prevent a poor group from contributing its preferred amount than it is to prevent a rich group from doing so. Although a contribution limit can improve the expected payoff of poor groups relative to rich groups, a limit reduces expected citizen welfare. As in Section 2.4, a contribution limit makes the politician less informed about evidence quality.

2.5.2 Unknown wealth differences

In this section, some interest groups (the poor) face a binding budget constraint, while other groups (the rich) do not. The politician does not know which groups
are rich and which are poor, although he knows the distribution of types. Unlike in the case when asymmetries are known to the politician, here it is possible for a contribution limit to improve citizen welfare. However, this is not generally true, and a contribution limit often reduces citizen welfare even when it eliminates biases in favor of rich groups.\textsuperscript{31}

The model with unobserved wealth differences differs from the game presented in Section 2.3 as follows. Each interest group is rich with probability $\alpha$, and poor with probability $(1 - \alpha)$, where the parameter $\alpha \in (0, 1)$ is common knowledge. A poor group faces a binding budget constraint such that its contribution must be less than $\omega$.\textsuperscript{32} Each interest group knows its own wealth, but does not know the wealth of other interest groups. The rest of the model is unchanged.

The first subsection describes the equilibrium of the game; first for when there is no contribution limit, then for when there is a limit. The second subsection shows how rich groups tend to receive higher payoffs compared to similar poor groups, and how a contribution limit can eliminate this payoff inequality. In the third subsection, I show that contribution limits often reduce citizen welfare even when they eliminate the rich group advantage. The details regarding the analysis is provided in the appendix, Section A.2.

\textsuperscript{31}To keep things simple, I focus on the case when wealth differences are represented by differences in budget constraints. To consider the case with differences in utility function weight on contributions ($\beta$ from Section 2.5.1), one needs to formulate and solve a complex multi-dimensional mechanism design problem.

\textsuperscript{32}For the budget constraint to be binding, $\omega$ must be less than the amount an interest group with the highest quality evidence ($e_{jn} = 1$) would want contribute in equilibrium if there was no budget constraint. Formally, $\omega < B(1)$ where $B$ is the contribution function derived in Section 2.
Equilibrium Contribution Functions

No Contribution Limit

I consider the symmetric Perfect Bayesian Equilibrium of the game, which I label the *contribution equilibrium with wealth differences*. A description of the new equilibrium requires an explanation of two different contribution functions: one for rich groups, and one for poor groups. The following lemma describes the contribution functions of the game with wealth differences. The functions $B_P$ and $B_R$ respectively denote the poor and rich group contribution functions.

**Lemma 2** In the contribution equilibrium with wealth differences, there exists cut off values $\bar{e}_a \geq 0$ and $\bar{e}_b \in (\bar{e}_a, 1)$, and functions $B_a$ and $B_b$, where $B_a (0; \omega) = 0$, $B_b (\bar{e}_b; \omega) = 0$, $\frac{\partial B_a}{\partial e} > 0$, and $\frac{\partial B_b}{\partial e} > 0$ such that

\[
B_P (e; \omega) = \begin{cases} 
   B_a (e; \omega) & \text{for } e \in [0, \bar{e}_a) \\
   \omega & \text{for } e \in [\bar{e}_a, 1], \text{ and}
\end{cases}
\]

(2.10)

\[
B_R (e; \omega) = \begin{cases} 
   B_a (e; \omega) & \text{for } e \in [0, \bar{e}_a) \\
   \omega & \text{for } e \in [\bar{e}_a, \bar{e}_b) \\
   \omega + B_b (e; \omega) & \text{for } e \in [\bar{e}_b, 1].
\end{cases}
\]

(2.11)

Figure 2.2 illustrates example rich and poor group contribution functions.

The wealth constraint has a similar impact on the poor group contribution function $B_P$ as a contribution limit had on the contributions of all interest groups in Section 2.4. For low enough $e$, a wealth constrained interest group’s contribution is strictly increasing in its evidence quality, and for higher $e$ the group contributes
This means that when the politician observes a contribution not equal to \( \omega \), he can accurately infer the interest group’s \( e \). However, when he observes a contribution equal to \( \omega \) and does not grant the interest group access, he remains uncertain about the true evidence quality of the group and acts as if the group has \( e = E_\mu (e \mid \omega) \).

For \( e \leq E_\mu (e \mid \omega) \) rich groups provide the same contributions as poor groups with similar evidence in the symmetric equilibrium. This is because rich and

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33 The cutoff value \( \bar{e}_a \geq 0 \) represents the evidence quality at which an interest group is indifferent between providing a contribution according to function \( B_a \), and providing contribution \( \omega \). If the group provides contribution \( B_a (\bar{e}_a) \), then the politician can correctly infer the group’s evidence quality. If instead the group provides contribution \( \omega > B_a (\bar{e}_a) \), then the politician learns the group’s true evidence quality only if the group receives access. If the group does not receive access, which happens with positive probability, the politician overestimates the interest group’s evidence quality. For an interest group with evidence quality \( \bar{e}_a \), the benefit from contributing \( \omega \) is completely offset by the cost of doing so.

34 The paper generally assumes that the highest \( K \) contributors receive access. This assumption makes the analysis more straightforward. Similar results will follow from an analysis that allows interest groups who contribute \( \omega \) to receive access before groups who contribute more than \( \omega \).
poor groups only differ in terms of the budget constraint. Therefore, a group’s preferred contribution is independent of its wealth, and when a poor group prefers its chosen contribution to any other \( b \geq 0 \), a rich group with the same \( e \) prefers to contribute the same amount. Alternatively, for \( e > E_\mu (e | \omega) \) both rich and poor groups prefer to provide more than \( \omega \), but only the rich group can afford to do so. Therefore, rich interest groups with high enough \( e \) contribute more than similar poor groups. Poor interest groups cannot afford a higher contribution, therefore poor groups with \( e > E_\mu (e | \omega) \) provide \( \omega \) in equilibrium instead. It follows that \( \bar{e}_b = E_\mu (e | \omega) \).

**Contribution Limit**

In considering relative payoffs between the different wealth types, I focus on the case where contribution limit \( \bar{b} \) is no greater than \( \omega \). This ensures that all interest groups can provide the maximum contribution if they choose to do so. Focusing on this range of limits is most consistent with the policy debate in which limits are intended to eliminate the ability of wealthy interest groups to outspend less wealthy groups. The appendix describes the impact that a higher limit has on the contribution functions.

Since \( \bar{b} \leq \omega \), both rich and poor interest groups have the same equilibrium contribution constraint. This means that in the symmetric equilibrium, the contribution function is independent of wealth, and is the same as \( B_{CL} \) in Section 3. See Figure 1 for an illustration. There exists a cut off value \( \bar{e} \in [0, 1) \) such that an interest group with \( e < \bar{e} \) contributes \( \bar{B} (e) \), and an interest group with \( e \geq \bar{e} \) contributes \( \bar{b} \).
Rich Group Advantage

When there is no contribution limit, in equilibrium, poor interest groups with evidence quality greater than $\bar{e}_b$ prefer to contribute more than $\omega$ (just as the rich groups do), but they are prevented from doing so by their budget constraint. Therefore, rich groups with high-quality evidence have a higher probability of communicating their evidence to the politician compared with poor interest groups with similar quality evidence. This can result in policies that are on average biased in favor of the rich groups, and it does result in rich groups having higher expected payoffs than similar poor groups.

If a rich interest group and a poor interest group have opposite policy preferences involving the same issue, the politician will tend to choose a policy that favors the rich group, all else equal. This is stated by the following lemma.

Lemma 3 If $(n, j)$ is rich, $(n, -j)$ is poor, and $e_n^j = e_{-n}^{-j}$, then $Ep_n^* > 0$ when $j = 1$ and $Ep_n^* < 0$ when $j = -1$.

Furthermore, a group’s expected payoffs are higher when it is rich than when it is poor. This is because rich interest groups are always able to provide the contribution that maximizes their expected payoffs, but poor interest groups face a budget constraint that prevents them from doing so for some values of $e$. This result is stated by the following proposition.

Proposition 3 $EU_n^j$ is at least as great, and is strictly greater for some $e_n^j \in [0, 1]$, when $(n, j)$ is rich than when $(n, j)$ is poor.

A contribution limit can eliminate the bias in favor of rich groups. When there
is a contribution limit, an interest group’s contribution and the probability that the group receives access are independent of its wealth. This means that policies do not tend to favor rich groups, and that an interest group’s expected payoff is independent of its wealth. This is stated by Proposition 4.

**Proposition 4** If $\bar{b} \leq \omega$, then (1) $E\rho_n^* = 0$ for all $n$ and (2) $EU_j^n$ is independent of wealth for all $(n, j)$.

A contribution limit can completely eliminate the rich group advantage. This is consistent with the argument made by many campaign finance reform advocates. However, it is important to recognize that these results says nothing about citizen welfare. I discuss the welfare implications in the following section.

**Impact on Welfare**

This section considers the impact that a contribution limit has on expected citizen welfare when interest groups have different levels of wealth. I show that under certain parameter values, contribution limits can improve expected welfare. However, this is not generally the case. Just because a contribution limit eliminates the impact of wealth on interest group payoffs, this does not imply that the same limits improve citizen welfare.

In the game with wealth differences, as in the game without differences, a contribution limit tends to reduce the number of interest groups for which the politician is certain about their $e$. Without a limit, the politician learns with certainty the $e$ of any rich group with high enough evidence, or any group (rich or poor) with $e < e_a$ even if these groups do not receive access. A contribution limit
can reduce the range of $e$ for which an interest group contributes according to a strictly increasing function, and for which the politician learns a group’s evidence quality regardless of access. This means that a limit can result in the politician being less informed when he chooses a policy profile, which tends to reduce citizen welfare.

A limit does not necessarily have this affect, and can improve welfare under certain parameter values. For example, a contribution limit at $\bar{b} = \omega$ can increase the probability that a group who gave $\omega$ in the no limit game receives access. Depending on the model parameters, the benefit of increasing the probability that the groups who had given $\omega$ receive access may be greater than the expected costs of not always learning the evidence quality of the rich groups that gave more than $\omega$ before the limit. This is because when there is no contribution limit, the politician may tend to have significantly inaccurate beliefs about the $e$ of those that give $\omega$ and do not receive access, and when there is a contribution limit, the politician may have relatively accurate beliefs about the $e$ of groups for whom he does not learn the $e$ with certainty but for whom he would have learned $e$ if contributions were not limited (the rich interest groups that give more than $\omega$ when there is no limit). When this is true, the contribution limit can result in the policeman having more accurate beliefs and choosing better policies, on average. Although this is possible, the conditions that must be met for a limit to improve citizen welfare may be very specific, and one should not conclude that contribution limits generally have a positive impact on welfare.

To illustrate the potential welfare impact of a contribution limit, I consider a very simple competition for access game that allows for the explicit solution of different equilibrium variables and payoffs. Suppose that evidence quality is
uniformly distributed on $[0, 1]$, and that interest group policy utility is linear, or $V(1 - e^j + e^{-j}) = -(1 - e^j + e^{-j})$. There is a single issue with two interest groups, and one groups receives access, or $K = 1$. The appendix provides details regarding the solution of this game. Here, I briefly describe the results.

In this simplified game, there exists cut off values $\alpha', \omega'_L(\alpha)$, and $\omega'_H(\alpha)$ such that contribution limit $\bar{b} \leq \omega$ improves expected citizen welfare if and only if $\alpha \in (\alpha', 1)$, $\omega \in [\omega'_L(\alpha), \omega'_H(\alpha))$, and $\bar{b} \in \left[\frac{1}{4}, \omega\right]$. The values $\omega'_L(\alpha)$ and $\omega'_H(\alpha)$ depend on the value of parameter $\alpha$, and determining the values $\alpha'$ and $\omega'_H(\alpha)$ require calculating the root of high-degree polynomials for which a non-numeric solution is not possible. To overcome this issue, I use Mathematica to numerically determine the cut off values. $\omega'_H(\alpha)$ is greater than $\omega'_L(\alpha)$ only when $\alpha$ is high enough, implying that $\alpha'$ is approximately 0.750427. This means that most interest groups must be poor in order for a contribution limit to potentially improve expected welfare. Additionally, given any $\alpha$, the range of $\omega$ for which a limit can have a positive impact is even more restrictive. For any $\alpha > \alpha'$, $\omega'_L(\alpha)$ takes on values between $\frac{1}{4}$ and 0.260199, and $\omega'_H(\alpha)$ takes on values between $\frac{1}{4}$ and 0.260751. At its maximum, the difference between $\omega'_H(\alpha)$ and $\omega'_L(\alpha)$ is approximately 0.00215, meaning that for any value $\alpha$, only a very narrow range of $\omega$ results in the contribution limit being beneficial. Furthermore, because $\omega'_H(\alpha)$ is close to $\frac{1}{4}$, even when the other conditions are met, there only exists a small range of contribution limits that benefit society. Whenever the above conditions do not hold, the contribution limit strictly reduces expected citizen welfare.

The results have two important implications. First, unlike in the game without wealth differences, a contribution limit can improve citizen welfare when there is wealth inequality. For the limit to actually do so may require that a large
enough portion of interest groups are poor, and that the contribution limit is high enough (among other possible restrictions). Imposing a low limit, or banning all contributions, has a strictly negative impact on expected citizen welfare in the simple example above. Second, the result that a contribution limit eliminates the advantage that rich groups tend to have over poor groups does not imply that the limit improves citizen welfare. This means that the logic behind the popular argument in support of contribution limits is flawed. As the above example illustrates, the contribution limit often reduces welfare even when it eliminates the bias in favor of wealthy groups.

2.6 Alternative Information Structures

Up to this point, the analysis assumes a simple information structure. Interest groups know their own evidence, and their evidence quality is the independent realization of a random variable. Although the information structure presented in Section 2.3 is simple, the model and analysis are robust to a variety of generalizations. Here, I discuss some of these refinements, which help improve the real-world representation of the model.

2.6.1 Comments on the Evidence Structure

An interest group’s evidence quality represents the impact that presenting its evidence has on the policy the politician believes is best. I have not modeled evidence itself, only the impact that evidence has on policy. However, one could incorporate a more formal model of evidence into the paper without changing the results of the
analysis. For example, one could assume that an interest group’s evidence is made up of a collection of verifiable documents or facts, and when groups receive access they choose which pieces of their evidence to reveal to the politician. Such a definition of hard evidence has been formalized in a variety of papers (e.g., Green and Laffont 1986, Lipman and Seppi 1995, Bull and Watson 2004, 2007), although it has not been applied to games in which revealing evidence requires access. Under general assumptions about the evidentiary structure, incorporating such a formal model of evidence into this paper will not change the results.\footnote{For the results of the analysis to hold, the evidentiary structure must meet Bull and Watson (2007)’s normal condition, or, equivalently, Lipman and Seppi (1995)’s full reports condition. When interest group $(n, j)$’s type is given by the variable $e_{jn}$ (for any $e_{jn} \in (0, 1]$), these conditions ensure that $(n,j)$’s evidence is sufficient to distinguish it from groups with lower types (i.e., $e < e_{jn}$).}

Additionally, it is important that each interest group knows all of the evidence in favor of its own preferred policy. Otherwise, the equilibrium outcome is not first-best in terms of citizen welfare. In equilibrium, an interest group with access has an incentive to reveal all of the evidence in favor of its preferred policy position, and none of the evidence against its preferred position. No interest group besides $(n, j)$ has an incentive to present evidence in favor of policy $j$ for issue $n$. If $(n, j)$ does not have all of the evidence in its favor to present, even if that evidence is known to some other group, then the politician will not learn about the evidence in equilibrium. This also implies, however, that the politician becomes fully informed about the socially optimal policy whenever each group has all of the evidence in favor of its own position, even if groups also have information against their own positions. Evidence against $(n, j)$’s preferred policy is evidence in favor of $(n, -j)$’s preferred policy. Therefore, although $(n, j)$ will not reveal evidence against its position, this evidence will be revealed by group $(n, -j)$ and the politician remains fully informed.
2.6.2 Correlated Evidence Quality

The main analysis assumes that an interest group’s evidence quality is independent of other group evidence quality. It is reasonable, however, that if one group has a strong case in favor of its preferred policy, then the opposite group concerned with the same issue may tend to have a weaker case. Incorporating correlation (or negative correlation, as the case may be) of evidence quality into the model does not change the results.

To see this, let the function $F(\cdot | e^j)$ denote the expected distribution of group $-j$’s evidence quality from the perspective of group $j$, given that $j$ realized its own evidence quality $e^j$. Denote the density of this distribution by function $f(\cdot | e^j)$. For any $e^j$ and $e \in [0,1]$, $f(e | e^j) > 0$. The density function is determined by the ex ante distribution of evidence quality, and the correlation between group $j$ and $-j$’s evidence quality.

This alteration of the model has minimal impact on the interest groups’ optimization problem. In the game without evidence correlation, interest group $(n,j)$ chooses a contribution to solve Eq. 2.4. When evidence is correlated, $(n,j)$ chooses a contribution to maximize an equation that is identical to Eq. 2.4, except that the ex ante density function $f$ is replaced by the revised density function $f(\cdot | e^j)$. Solving the problem in the same way that Section 2.3 solved Eq. 2.4, one can show that with correlated evidence, an interest group with information $e$ contributes according to the function

$$B(e) = -\int_0^e \int_0^1 f(e^{-j}_n | y) \left(1 - \Theta(y, e^{-j}_n)\right) V' \left(1 - y + e^{-j}_n\right) de^{-j}_n dy.$$  \hspace{1cm} (2.12)

Just as in Section 2.3, the contribution function is strictly increasing in an interest group’s evidence strength. Therefore, the results from the earlier analyses
continue to hold. The politician can correctly infer an interest group’s evidence strength from its contribution, and imposing contribution limits tends to reduce the information available to the politician and result in worse policy decisions.

2.6.3 Uncertain Evidence Quality

Up to this point, I assume that interest groups know exactly how the politician will react to their evidence. Alternatively, one may assume that interest groups observe a signal that is correlated with their evidence quality, but they remain uncertain as to its true value until they reveal their evidence to the politician. One can show that, in this case, an interest group’s contribution is strictly increasing in its signal; just as it was increasing in its true evidence quality in the previous sections. In equilibrium, the politician correctly learns the interest groups’ signals by observing their contributions, but he remains uncertain regarding a group’s actual evidence quality unless he provides the group access. When interest groups are uncertain regarding their true evidence quality, contributions lead to the politician having more accurate expectations about the socially optimal policy than he does without contributions or if contributions are limited.\(^{36}\) Similar to the case when groups know their\( e \) values with certainty, a contribution limit decreases the expected accuracy of the politician’s beliefs, and results in lower expected citizen welfare than when contributions are unconstrained.

\(^{36}\)Unlike in Section 2.3, citizen welfare is not independent of the level of access. In this case, citizen welfare is increasing in the number of groups that receive access. This is because access allows the politician to learn a group’s evidence with certainty, rather than only learn the signals received by the groups.
2.7 Endogenous Access Choice

The previous sections assume that the number of interest groups that receive access $K$ is determined exogenously. In this section, I relax this assumption, and allow the politician to choose the amount of access, where $K \in \{0, 1, \ldots, K_{\text{max}}\}$. The politician commits to $K$ at the beginning of the game, before the interest groups provide contributions.

First, I show that the politician always provides access to some groups ($K \geq 1$). If the politician provides no access ($K = 0$), there is no possibility an interest group gets caught if it signals higher quality evidence than it actually has. When there is no possibility of receiving access, interest groups with high quality evidence no longer have an incentive to provide larger contributions than groups with low quality evidence. Instead, contributions are independent of evidence quality and the politician can no longer infer anything about a group’s evidence quality by observing its contribution. This eliminates any incentive for the interest groups to provide contributions. Therefore, if the politician chooses no access, he learns nothing about the interest groups’ evidence, and receives no contributions. By choosing a positive $K$ he receives contributions and learns about group evidence quality (as I show in the previous sections). Thus, the politician will always commit to providing a positive amount of access.

When there are no contribution limits and interest groups are certain about

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37 Moldovanu and Sela (2001) consider the optimal choice of $K$ in a traditional all-pay auction in which bidders benefit from receiving a prize (not from the act of bidding, which is the case here).

38 If the politician chooses $K$ following the contribution decisions of the interest groups, there exists $K_{\text{max}} + 1$ contribution equilibria. In each of these equilibria, the politician becomes fully informed of all interest groups’ evidence quality, and implements the policy profile that maximizes citizen welfare. When there are contribution limits, the politician is less than fully informed with positive probability. This means that contribution limits strictly reduce expected citizen welfare, independent of which contribution equilibrium is achieved.
their evidence quality, the politician becomes fully informed about the evidence quality of all interest groups independent of the amount of access (see Section 2.3). In this case, the amount of access he chooses to provide only impacts total contributions since he becomes fully informed so long as $K \geq 1$. One can show that expected total contributions are strictly decreasing in the number of interest groups that receive access. Therefore the politician chooses the minimum, positive amount of access ($K = 1$), which results in the politician maximizing contributions while becoming fully informed about interest group evidence quality.

This result is inconsistent with what one observes in reality: politicians provide access to more than one interest group. The result is driven by the simplifying assumptions of the basic model, and is eliminated when there are contribution limits (Section 2.4), interest groups have unobserved wealth differences (Section 2.5.2), or interest groups are uncertain about their evidence quality (Section 2.6.3). When the model makes any of these more realistic assumptions, the politician does not necessarily choose the lowest amount of access. When the politician lowers $K$, he receives higher contributions, but also learns with certainty the evidence quality of fewer interest groups. The politician’s access decision trades off contributions for evidence, and it is not possible to determine the equilibrium choice of $K$ without making further assumptions regarding the weight the politician places on contributions relative to citizen welfare, or the ex ante distribution of evidence quality.

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Access can be interpreted as the politician’s monitoring of interest group contributions in order to limit the groups’ ability to overrepresent their true evidence quality. When an interest group increases its contribution, it benefits from the increase in the evidence quality the politician believes it has only when it does not win access. When the politician reduces the number of groups that receive access, the action increases the probability an interest group does not win access given any contribution, thereby increasing the potential benefit to an interest group from increasing its contribution. This is true for all groups independent of their evidence quality. Therefore, when the politician reduces the number of interest groups that receive access, all groups increase their contributions, except for those that already provide the maximum contribution amount (in the case of contribution limits). This is also the reason why revenue equivalency (of auctions) does not apply to a competition for access.
Although one does not know which $K$ the politician will choose in the game with contribution limits, it is likely more than one. Just as in the previous sections, it is straightforward to show that a contribution limit strictly reduces expected citizen welfare.

Furthermore, when politicians spend time meeting with a greater number of interest groups, they may do so at the cost of spending less time in their home districts meeting with constituents. When the politician can choose the amount of access to provide, he may choose to meet with a greater number of interest groups when there are contribution limits compared with when there are no limits. To the extent that this choice reduces the amount of time the politician spends meeting with constituents, contribution limits may further reduce citizen welfare. However, I do not explicitly model this trade off here.

### 2.8 Selling Access Versus Selling Policy

This section considers the conditions under which a politician prefers to sell access rather than sell policy favors. The results establish that selling access maximizes the politician’s policy utility but does not maximize expected revenue. Selling policy favors, on the other hand, maximizes revenue but not policy utility. Therefore, a politician who cares enough about the policy outcome relative to revenue prefers to sell access, otherwise he prefers to sell policy. This suggests that politicians are more likely to sell access to interest groups concerned with issues which constituents are passionate about (those with a high $\gamma$ in the citizen welfare function), and are more likely to explicitly sell policy favors regarding issues about which constituents care less. When the politician is free to choose whether to sell...
access or policy favors, a contribution limit can make selling access relatively more favorable.

Throughout this section, I make a couple of assumptions that simplify the analysis. Relaxing these assumptions will complicate the analysis, but will not change the results. First, interest group policy utility is linear, where $V(x) = -v x$. Therefore, $U_{jn}(p_n^*, b_n^j) = -v |p_n^* - j| - b_n^j$. Additionally, there is only one issue or $N = 1$.

In this section, the politician chooses whether to sell access, or to explicitly sell a policy position. He commits to the policy mechanism at the beginning of the game before interest groups submit bids or contributions. If he sells access, he does so using the competition for access mechanism developed earlier in the paper. Since $N = 1$, he awards access to a single interest group. Alternatively, if he sells policy favors, I assume that he does so using a winner-pay auction in which the high bidder gets to select the policy. Obviously, the high bidder chooses its own (extreme) preferred policy. One could incorporate a more complex money-for-prize mechanism such as a menu auction (e.g., Bernheim and Whinston 1986, Grossman and Helpman 1994), lottery (e.g., Tullock 1980), or all-pay auction (e.g., Che and Gale 1998) into the game in place of the winner-pay auction. Under these alternative mechanisms, the politician still prefers to sell access when he cares enough about the policy outcome relative to revenue. However, changing the way in which the politician sells policy will alter the parameter values for which the politician is indifferent between selling access and selling policy favors.
2.8.1 Selling Access

When the politician chooses to sell access, the game proceeds as it did in the earlier sections of the paper. In equilibrium, both interest groups contribute according to the function

\[ B(e) = v \int_0^e (1 - F(y)) dy \]  

(2.13)

which is a restatement of Eq. 2.7 given that there are only two interest groups and policy utility is linear. Because the contribution function is strictly increasing in \( e \), there exists a one-to-one mapping between each group’s contribution and its evidence quality. In equilibrium, the politician correctly infers the evidence quality of both interest groups, even though he only gives access to one of them. Therefore, the politician identifies and implements the socially optimal policy, where \( p^* = p^o \). No other mechanism can result in higher policy utility for the politician. However, this does not imply that selling access necessarily maximizes the politician’s utility since he also cares about collecting contributions.

The expected revenue from selling access is

\[ 2 \int_0^1 f(e) B(e) de. \]  

(2.14)

Given that the politician’s policy utility is maximized at 0 when \( p^* = p^o \), the politician’s expected utility when he sells access is equal to his expected revenue, or

\[ EU^P = 2v \rho \int_0^1 f(e) \int_0^e (1 - F(y)) dy de. \]  

(2.15)
2.8.2 Selling Policy

Now, consider the case when the politician sells a policy position rather than access. In the symmetric equilibrium of the winner-pay auction, each of the interest groups bid $2v$, which equals the benefit that an interest group receives when the politician implements its preferred policy compared to the preferred policy of the other group. The bids are independent of whether the auction is a first-price or second-price auction. In this case, the politician earns $2v$ in equilibrium.

When the politician sells policy, his expected revenue is higher than if he sells access. However, achieving this higher amount of revenue comes at the cost of no longer being able to identify or implement the socially optimal policy. Instead, the politician implements an extreme policy. Given that both interest groups share the same distribution of evidence quality and that $p^o = e^1 - e^{-1}$, the policy implemented when the politician sells a policy favor (either $-1$ or $1$) averages a distance of 1 from the socially optimal policy. This results in expected citizen welfare and politician policy payoff of $-\gamma$. In equilibrium, the politician’s expected utility when he sells a policy favor is $2v\rho - \gamma$.

---

40 Remember that in the setting analyzed here, interest groups differ in terms of their qualifications rather than their valuations. Because interest groups share the same benefit from winning the auction, and this benefit is known to both interest groups, the bids and expected payments are the same for both a first-price and second-price winner-pay auction.

41 In fact, choosing a policy by selling the policy choice to the highest bidder is an expected-revenue maximizing mechanism. For this framework, there does not exist a mechanism for choosing policy that results in higher expected revenue for the politician than $2v$. I leave the proof of this to the interested reader.

42 If the politician uses a different type of auction (such as a menu auction) to sell policy, then the expected distance between the socially optimal and implemented policies may be less than 1. However, the expected distance will be greater than 0, which means that selling policy will continue to result in lower policy utility for the politician.
2.8.3 Selling Access v. Selling Policy

Given the politician’s payoffs when he sells access and when he sells policy, it is straightforward to determine when he prefers each course of action. The politician prefers to sell access when

\[ 2v\rho \int_0^1 f(e) \int_0^e (1 - F(y)) dyde \geq 2v\rho - \gamma. \] (2.16)

This simplifies to

\[ \frac{\gamma}{\rho} \geq 2v \int_0^1 f(e) \left[ 1 - e + \int_0^e F(y) dy \right] de. \] (2.17)

This result is restated in the following proposition, where \( \bar{z} \) is the right hand side of Eq. 2.17.

**Lemma 4** Consider the game in which the politician chooses whether to sell access or policy favors. There exists a cut-off value \( \bar{z} > 0 \) such that

1. if \( \frac{\gamma}{\rho} \geq \bar{z} \), then the politician sells access, and
2. if \( \frac{\gamma}{\rho} < \bar{z} \), then the politician explicitly sells policy.

The fraction \( \frac{\gamma}{\rho} \) represents how much the politician cares about citizen welfare (\( \gamma \)) relative to how much he cares about political contributions (\( \rho \)). This means that the politician prefers to sell access rather than policy favors when he cares enough about citizen welfare (or the policy outcome) relative to contributions. If he does not care enough about policy, then he will choose to sell policy favors which results in higher revenue (and lower policy utility).

\[^{43}\text{I assume that the indifferent politician sells access.}\]
2.8.4 Contribution Limit

In Section 2.4, a contribution limit could never improve citizen welfare. When the politician can choose whether to sell access or sell policy, however, a carefully set contribution limit may be advantageous because it makes explicitly selling policy relatively less attractive to the politician. This analysis focuses on the contribution limit that maximizes citizen welfare.

If Eq. 2.17 is not met, then the politician will sell policy favors rather than access. When the politician gives the highest bidder the right to choose policy, the policy ends up being extreme. On average the distance between the implemented policy and the socially optimal policy is 1, and the expected citizen welfare from selling policy is $EW = -\gamma$. In contrast, if the politician sells access, he becomes fully informed and implements the socially optimal policy, and $W = 0$.

Given that expected citizen welfare is strictly lower when the politician sells policy rather than access, the policy question then becomes: does there exist a limit that entices the politician to sell access and improves citizen welfare? In short, the answer is yes. To see this, consider a complete contribution ban (i.e., $\bar{b} = 0$). In this case, expected citizen welfare is 

$$EW = -\gamma \int_0^1 f(e) \mid e - \int_0^1 f(e) e de \mid de$$

which is strictly higher than $-\gamma$. Because the politician collects zero contributions regardless of whether he sells policy or access, he will choose to sell access which results in strictly higher expected citizen welfare.\(^\text{44}\)

This does not, however, imply that banning contributions is ideal. When there are no contributions, the politician is unable to infer anything about the evidence\(^\text{44}\)

\(^{44}\)Although I use the term "sell", when contributions are banned the politician chooses between randomly assigning access to an interest group, or randomly choosing an interest group to win the policy favor.
quality of the interest group that does not receive access. If one can impose a strictly positive contribution limit such that the politician continues to sell access rather than policy, then the politician can infer something about interest group evidence quality from the contributions, even if the limit prevents contributions from being fully revealing. This results in the politician being better informed than when contributions are completely banned. The socially optimal limit for any single policy choice is the maximum limit under which the politician chooses to sell access.

When $\gamma > \rho$ is high enough (when Eq. 2.17 holds), no limit is needed to entice the politician to sell access, in which case any $\bar{b} \geq B(1)$ is optimal including no limit ($\bar{b} = \infty$), where $B(1)$ is the highest possible contribution when interest groups compete over access. Any $\bar{b} \geq B(1)$ has no impact on the contributions in the competition for access, and will not limit the politician’s ability to infer evidence quality from contributions.

When the value $\frac{\gamma}{\rho}$ is known at the time one chooses a contribution limit $\bar{b}$, then the limit $\bar{b}$ can be set just low enough to entice the politician to sell access instead of policy. The assumption that $\frac{\gamma}{\rho}$ is known, however, is largely unjustified in a world in which a contribution limit applies to all interest groups, not only those concerned with a single issue. To better represent a situation in which the politician must choose many policies, while maintaining the simplifying assumption that $N = 1$, I assume that $\frac{\gamma}{\rho}$ (or at least $\gamma$) is observed after $\bar{b}$ is set, and before the politician chooses whether to sell access or policy. In this case imposing (or decreasing) a contribution limit makes it more likely that the politician sells access, but also decreases the accuracy of the politician’s beliefs about the socially optimal policy.

\footnote{Technically, the optimal limit makes the politician indifferent between selling policy and selling access, and in equilibrium the politician chooses to sell access.}
when he does sell access. The socially optimal contribution limit is such that the expected welfare costs of increasing or decreasing the limit outweighs the expected benefits of doing so.

One cannot determine the value of the socially optimal contribution limit without additional assumptions regarding the distribution of $e$ and $\frac{\gamma}{\rho}$. Even without further assumptions, however, one can conclude that a carefully set contribution limit strictly improves expected citizen welfare in the case when $\frac{\gamma}{\rho}$ is unknown ex ante. When $\frac{\gamma}{\rho}$ is known, a carefully set limit can improve (and never reduces) expected citizen welfare.

For a number of different evidence quality distributions, including the Uniform$[0,1]$ distribution, setting $\bar{b} = B(1)$ guarantees that a politician will sell access. This is because, given the parameters, selling access results in higher expected revenue, as well as higher policy utility, compared with selling policy. When this is the case, $\bar{B} = B(1)$ results in the first-best citizen welfare outcome. For other distributions of evidence quality,\textsuperscript{46} whether $\bar{b} = B(1)$ is optimal depends on the distribution of $\frac{\gamma}{\rho}$. When a low $\frac{\gamma}{\rho}$ is sufficiently unlikely, $\bar{b} = B(1)$ is optimal. On the other hand, when the politician tends to care relatively little about citizen welfare (i.e., a low $\frac{\gamma}{\rho}$ is likely), setting the limit below $B(1)$ is optimal.

**Proposition 5** Consider the game in which the politician chooses whether to sell access or policy favors. There exists a $\bar{b}^* \in [0, B(1)]$ such that expected citizen welfare is maximized when $\bar{b} = \bar{b}^*$.

In contrast to the results from Section 2.4, this proposition formally establishes that, when the politician is free to choose whether he sells access or sells policy,\textsuperscript{46} Such as the Beta$[x,y]$ distribution where $x = 1/2$ and $y > 1$.  

46
contribution limits tend to improve citizen welfare. This is because a limit reduces the expected revenue from selling policy by more than it reduces the expected revenue from selling access, thereby making selling policy relatively less attractive to the politician. However, although a contribution limit tends to improve citizen welfare by making it more likely the politician sells access, a limit can also reduce the information value of contributions which results in an access-selling politician making less-informed policy decisions. Therefore, one must be cautious about setting an overly-strict limit.

2.9 Conclusion

This paper incorporates access into a model of informational lobbying. When the politician sells access to the highest bidders, contributions help him learn about the impact of different policies. In contrast to much of the previous literature, I show how political contributions may move policy closer to the platform that is best for constituents. The model illustrates how a contribution limit may reduce the amount of information available to the politician, and results in worse policy decisions.

In no way are these results irrefutable evidence in favor of allowing unlimited payments from special interests to politicians. Instead, the results should be viewed as evidence that political contributions may not be as bad as generally portrayed by the media and campaign finance reform groups. When the politician can sell access, he may choose better policies than if he could not sell access.

Certainly, the real world is much more complicated than the competition for access model developed in this paper. When I generalize the model to allow for
unobserved interest group wealth differences, or allow the politician to choose whether to sell access or sell explicit policy favors, contribution limits can improve welfare. However, even in these cases, setting too low of a limit has adverse effects on welfare.

Also, the competition for access framework likely applies to some issues better than others. Certain interest groups likely give to candidates because they want to help the candidate win election, not because they want to secure access. In this way, competition for access probably does not apply to an issue like abortion, for which politicians are already well informed or likely to publicly commit to a position. Instead, an access model is likely a better fit for an issue such as steel tariffs in which the domestic auto producers have arguments against a tariff, the domestic steel industry has arguments in favor of a tariff, and most politicians are not well informed as to the optimal level of tariff for their constituents.

This paper may serve as a starting point for future research on hard information and access. Future work may incorporate the competition for access model into more complete models of the political process. For example, interactions between the politician and interest groups are often repeated over time, policy is often chosen by a group of legislators rather than a single decision maker, and politicians use contributions to compete against other politicians for election. Additional work may also consider alternative mechanisms through which the politician may award access, or apply the competition for access model to non-political settings.\footnote{For example, instead of allocating access through an auction, the politician may set individual prices for access. If an interest group pays the price assigned to them, they receive access, otherwise they do not receive access. Chapter 3 develops such a model.}
3.1 Introduction

The economics and political science literatures focus on two motivations behind political contributions. First, one may contribute in the quid pro quo exchange for policy favors. Second, one may contribute to help a politician already in favor of one’s cause win election. The literature has largely overlooked a third reason for contributing: to secure access to a politician, where those with access can influence the drafting of legislation or the politician’s voting record through the provision of information or arguments in support of one’s preferred policy, or against a less-preferred option.

Although the first two motivations may help drive contributions, there is substantial evidence that the access motivation also has a significant (if not stronger) influence on interest group contributions.1 Despite this, few papers attempt to model the contributions-for-access story (I discuss those that do in the Literature Review). This paper presents a simple model of lobbying in which money buys access. I use the model to develop a better understanding of the interaction between politicians and interest groups, and to analyze the impact of a contribution limit on payments and policy outcomes.

A politician must choose a policy, but he is ex ante uncertain about the effects of different choices. An interest group has private evidence in favor of its own preferred policy, which it can verifiably reveal to the politician only if the politi-

1See for example, Herndon (1982), Langbein (1986), Wright (1990), Hall and Wayman (1990), Milyo et al. (2000), Ansolabehere et al. (2002), Clawson et al. (1992), Schram (1995).
cian grants the group access. Unlike other models of hard-information disclosure, the politician controls which interest groups receive access. He can therefore require than an interest group provide a political contribution in exchange for the opportunity to disclose its evidence. I refer to this required contribution as the access fee.

In this simple game, the politician sets an access fee, and an interest group decides whether or not to pay the fee. If the group pays the fee, the politician becomes fully informed of its private evidence in favor of its known position. When setting the access fee, the politician also has the option of granting access for free, or not selling access at any price.

The politician cares about choosing the policy that is best for a representative constituent; and the more he knows about the interest group’s evidence, the more accurate are his beliefs about this best policy. He also cares about collecting political contributions (which come from access fee payments), and he finds granting access costly. Expected representative constituent welfare is maximized when the politician has full information about the interest group’s evidence. This happens when the politician grants access for free, since then the interest group will always present its evidence. If the politician charges a positive access fee, the group only buys access if its evidence is of high-enough quality, otherwise it does not buy access (which happens with positive probability) and the politician remains less than fully informed.

The model yields the following insights. The equilibrium access fee, set by the politician, is strictly increasing in the interest group’s wealth and strictly decreasing in the importance of the issue. This means that the politician charges a wealthy interest group a higher price for access than an otherwise similar poor group,
and that access is relatively inexpensive for interest groups involved with issues about which the politician (or his constituents) cares intensely. However, for any group, the politician sets an access fee that is higher than the fee that would be preferred by the representative constituent. In equilibrium, the politician trades off constituent welfare in order to increase expected contributions. Interestingly, as interest groups become wealthier, the politician tends to become more informed about policy, which improves expected constituent welfare.

The analysis identifies competing positive and negative effects of a contribution limit. A contribution limit may improve expected constituent welfare by reducing the price of access, which tends to result in more access and a better-informed politician. However, a contribution limit also reduces the politician’s financial incentive to grant access. For some issues, a limit may result in the politician no longer finding it worthwhile to grant access. For these issues, the limit causes the politician to refuse access at any (allowed) price, which tends to result in him being less informed and choosing worse policy. When the contribution limit applies to many issues, I show that it is always optimal from the standpoint of constituent welfare to impose a contribution limit that is binding for some issues. Under the optimal limit, the politician will refuse to provide any access for some issues. Banning contributions is never optimal.

3.2 Literature Review

This paper develops a game theoretic model of lobbying in which interest groups provide political contributions to gain access to a politician. Access allows an interest group to present verifiable or hard evidence in favor of its preferred policy.
position. To my knowledge, only two other papers share this foundation. Chapter 2 models political contributions when the politician auctions off access to the highest bidders. Austen-Smith (1998) tells a similar story to the present paper, in which the politician sets implicit prices for access.

Chapter 2 shows how an interest group with more-persuasive evidence in favor of its preferred policy is willing to pay more to share its evidence with the politician compared to a similar group with less-persuasive evidence. The politician not only learns about the evidence of interest groups that win access; he also makes inferences about interest group evidence by observing the political contributions. In equilibrium, the politician learns about the evidence quality of all interest groups, even when he only gives access to some of the groups. A contribution limit distorts the signaling power of the contributions, which results in a less-informed politician. When the politician sells access to the highest bidder, a contribution limit has a strictly negative impact on expected constituent welfare.

The results in Chapter 2 depend on the assumption that the politician allocates access through some form of auction mechanism, in which the probability an interest group wins access is strictly increasing in the group’s contribution. It is unclear, however, that the politician allocates access in such a way. For example, an interest group that attends a $1000 per plate fundraiser for a politician may expect some minimal amount of access. But the results in Chapter 2 rely on some uncertainty regarding whether the fundraiser attendee receives access, which may not be the case. As I show in this paper, when the politician commits to access fees before collecting contributions, a contribution limit no longer has a strictly negative impact on representative constituent welfare. Furthermore, the model in Chapter 2 suggests that total political contributions are decreasing in the num-
ber of interest groups the politician provides access. This conclusion is not only counterintuitive, but also not supported by the empirical evidence. For example, Langbein (1986) finds that political contributions are increasing in the time spent by politicians meeting with constituents and interest groups. In the present paper, given any access fee, contributions are increasing in the amount of access.

In Austen-Smith (1998), the politician sets access fees, similar to in this paper. The primary difference between Austen-Smith (1998) and my paper involves the underlying information structure. The earlier paper develops a model in which there are multiple interest groups involved with an issue, and interest groups differ in terms of their ex ante policy preferences relative to those of the politician. The paper considers how access fees depend on whether an interest group has similar policy preferences to the politician. In the current paper, I fix interest group policy positions and assume there is only one interest group per issue. Although these assumptions may result in a less realistic information structure, the resulting model allows for an intuitive analysis with greater focus on interest group wealth differences, issue asymmetries, and the impact of contribution limits. This alternative focus allows me to better address questions central to the current policy debate on campaign finance reform; particularly questions involving contribution limits.

Other “access” models, including Austen-Smith (1995) and Lohmann (1995), assume that information is completely unverifiable. Therefore, the presentation of information by itself can have no impact on the politicians beliefs, and the impact that any piece of information has on the politician depends on who provides it and how much money they attach to the information. This paper, as well as Chapter 2 and Austen-Smith (1998), make the alternative assumption that evidence can have an impact on the politician’s beliefs independent of who provides it, or the
size of the contribution attached to it. In other words, interest groups have hard
evidence that they can disclose to the politician.

The political-access framework differs from other models of hard information
in that the politician has control over which interest groups have access to present
information. Typically in the hard information literature, an agent with private
information can disclose its information whenever it chooses to do so (e.g., Milgrom
2007). In the political-access framework, the politician determines which interest
groups receive access, and he is able to grant access based on political contributions.
Once an interest group receives access, it behaves as if it is in a more traditional
game of hard information disclosure, and will always present its evidence. As
Milgrom and Roberts (1986) establishes, only an interest group with the worst
possible evidence will refuse to present when given access.

3.3 Model

There are two players: a politician and an interest group. The politician must
choose a policy $p$ from a single-dimensional policy space on the real line. The
politician prefers to set $p$ as close to the socially optimal policy $p_o$ as possible;
however, he is ex ante uncertain about the identity of $p_o$. The interest group
prefers strictly higher $p$.

At the beginning of the game, the interest group draws private, verifiable ev-

dence regarding the identity of $p_o$. Consistent with the information structure
developed in Chapter 2, the interest group’s evidence consists of both evidence in
favor of a higher policy choice, and evidence in favor of a lower policy choice (or
against a higher $p$). Let $e_h$ denote the strength of the evidence in favor of a high $p$, and let $e_l$ denote the strength of the evidence in favor of a low $p$, where both variables are the realization of a random variable $\hat{e}$ uniformly distributed on the unit interval $[0, 1]$. The socially optimal policy depends on both $e_h$ and $e_l$, where $p_o \equiv e_h - e_l$.

Before choosing a policy $p$, the politician can grant the interest group access. If the politician grants the group access, the interest group can send a message to the politician communicating its evidence. Similar to the evidence itself, the message consists of two parts: $m_h$ which communicates evidence in favor of a higher $p$, and $m_l$ which communicates evidence in favor of a lower $p$. The interest group can downplay or ignore evidence, but cannot exaggerate it; therefore, $m_h \in [0, e_h]$ and $m_l \in [0, e_l]$. It is straightforward to show that the interest group with access will reveal the maximum amount of evidence in its favor, and the minimum amount of evidence against its position. Thus, $m_h = e_h$ and $m_l = 0$. Therefore, the politician learns about the evidence in favor of a higher $p$ by giving the interest group access. Although the politician does not learn anything about the evidence against a higher $p$, he is more informed about the socially optimal policy when he grants the group access than when he does not grant the group access.

Although giving access to the interest group enables the politician to become better informed about the socially optimal policy, granting access is costly for the politician, imposing on him a utility cost of $\tau$. The politician may require that the interest group pay a political contribution in order to receive access. Let $c \geq 0$ denote the access fee set by the politician. The politician commits to give the

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2This paper’s evidentiary structure is consistent with Bull and Watson (2007)’s normality condition, or, equivalently, Lipman and Seppi (1995)’s full-reports condition. As these authors have shown, the interest group has an incentive to reveal the maximum amount of favorable evidence, and the minimum amount of unfavorable evidence.
interest group access if it provides contribution $c$. If $c = 0$, then the politician grants access to the interest group for sure. If $c$ is higher than the interest group would ever be willing to pay, the politician is said to “not grant access."

Let $a \in \{0, 1\}$ denote the interest group’s contribution decision, where $a = 1$ if the group pays price $c$. Let $m$ denote the evidence revealed by the interest group. If the interest group does not pay the access fee, then $m = \emptyset$. If the group does pay the access fee, then it reveals all of the evidence in favor of a higher $p$, and $m = e_h$. The realization of $m$ given access choice $a$ can be written $m(a)$, where $m(1) = e_h$ and $m(0) = \emptyset$. (If $c = 0$, the politician grants access to the group and learns $e_h$ for sure.)

The social welfare function is $W(p; p_o, \gamma) = -|p_o - p|\gamma$, where the variable $\gamma$ represents the relative importance of the politician’s policy decision. $\gamma$ may be thought of as how much the representative constituent cares about the issue for which the policy choice is made. $\gamma$ is the realization of a random variable that is continuously distributed on $\mathbb{R}^{++}$ with distribution $G$ and density $g$. Let $\mathcal{G}$ denote the set of all continuous distributions on $\mathbb{R}^{++}$, where $G \in \mathcal{G}$. The variable $\gamma$ is realized at the beginning of the game, before the politician sets a price of access.

**Game Order**

The game takes place as follows:

1. The politician observes $\gamma$. The interest group learns $\gamma$, $e_h$, and $e_l$. The politician then announces access fee $c$.

2. The interest group chooses whether to pay $c$. This decision is denoted by $a \in \{0, 1\}$. If the interest group pays $c$, then the politician becomes fully informed about $e_h$. 

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3. The politician chooses policy $p$.

**States and Beliefs**

Although the state of the world is technically given by the realization of $e_h$, $e_l$, and $\gamma$, the value $e_l$ does not influence play during the game.\(^3\) Therefore, the formal consideration of states and beliefs can ignore $e_l$, and focus instead on the realization of favorable evidence $e_h$, and the importance of the issue for social welfare $\gamma$.

The interest group knows both $e_h$ and $\gamma$ with certainty. The politician knows $\gamma$, but does not observe the draw of $e_h$. At the time the politician sets the price for access $c$, the politician’s beliefs about $e_h$ are given by the ex ante distribution of evidence quality. When the politician chooses a policy, his beliefs about $e_h$ are consistent with Bayes Rule given the ex ante distribution of evidence quality, the interest group’s choice of whether to pay for access, and the revelation of evidence if the interest group does receive access. Denote these updated beliefs by $\mu(a, m)$, where $\mu(a, m)$ may be fully represented by an updated density function $f_{\mu}$. The value $f_{\mu}(e_h)$ gives the probability that politician puts on the interest group having evidence $e_h$, given his beliefs $e_h$.

The politician is a Bayesian. Therefore, if the interest group pays for access (or if $c = 0$), the politician fully learns $e_h$; so $f_{\mu}(e_h) = 1$ and $f_{\mu}(e) = 0$ for all $e \neq e_h$.

Let $E$ denote the ex ante expectations about the state of the world, and let $E_{\mu}$ denote the politician’s expectations about the state of the world given beliefs $\mu$.

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\(^3\)The politician does not observe $e_l$ until the end of the game after the policy is implemented. The interest group, although it observes $e_l$ at the beginning of the game, will never reveal evidence against its preferred position. Therefore, $e_l$ has no impact on the interest group’s strategy.
Payoffs

The politician cares about the welfare of a representative constituent (or citizen), which is maximized when he implements the socially optimal policy \( p_o \), and about collecting political contributions. The variable \( \gamma \) represents how much the representative constituent cares about the issue. Representative citizen welfare, and politician policy payoff is given by \( W(p; p_o, \gamma) = -|p_o - p|\gamma \). Letting the parameter \( \phi \) represent how much the politician cares about political contributions, the politician's utility is

\[
U_P(p, c; p_o, \gamma, a) = W(p; p_o, \gamma) + (\phi c - \tau)a \\
= -|p_o - p|\gamma + (\phi c - \tau)a.
\]

The analysis assumes that \( \tau < \frac{v}{2}\phi \), which implies that the cost of providing access is less than the maximum possible financial incentive from doing so. As I show in the analysis, \( \frac{v}{2} \) is the maximum possible access fee. Any greater fee always results in no interest group buying access.

The interest group strictly prefers a higher policy choice \( p \), and paying a lower contribution. Let the parameter \( v \) denote how much the group cares about policy relative to money. Therefore, the interest group's utility is

\[
U_{IG}(a; p, c) = vp - ca.
\]

3.4 Contribution Equilibrium

The analysis solves for the pure-strategy Perfect Bayesian Equilibrium of the game, which I call the contribution equilibrium. A complete description of the equilibrium must include the strategy profiles for the interest group and the politician, as well
as the politician’s beliefs about the state of the world at the time he chooses policy. The politician’s beliefs must be consistent with using Bayes’ Rule on the ex ante distribution of evidence quality given the strategies of the interest group. Each player’s strategy must be a best response to the strategies of the other players, given their beliefs.

Let the function \( C^* \) denote the politician’s equilibrium choice of access fee, where \( C^*(\gamma) \) is the access price when the issue is of \( \gamma \) importance. Let the function \( P^* \) denote the politician’s equilibrium policy choice, where \( P^*(a, m; \mu) \) describes his choice given \( a \) and \( m \). Similarly, let the function \( A^* \) define the interest group’s equilibrium strategy, where \( A^*(c, e_h) \) is the group’s choice of whether to pay for access given access fee \( c \) and the realized evidence quality \( e_h \).

**Definition 2** Strategy profile \( \{(C^*, P^*), A^*\} \), and beliefs \( \mu \) constitute a **contribution equilibrium** if

1. For all \( \gamma \in \mathbb{R}_+ \),
   \[
   C^*(\gamma) \in \arg\max_c \int_0^1 \int_0^1 f_\mu(e_h) \times U_P(P^*(A^*(c, e_h), m(A^*(c, e_h)); \mu), c; p_o(e_h, e_l), \gamma, A^*(c, e_h)) \, de_h \, de_l,
   \]
2. For any \( e_h \in [0, 1] \),
   \[
   A^*(C^*, e_h) \in \arg\max_a U_{IG}(a; P^*(a, m(a); \mu), C^*),
   \]
3. \( P^*(A^*(C^*, e_h), m(A^*(C^*, e_h)); \mu) \in \arg\max_p \int_0^1 \int_0^1 f_\mu(e_h) U_P(p, C^*; p_o(e_h, e_l), \gamma, A^*(C^*, e_h)) \, de_h \, de_l, \) and
4. beliefs \( \mu \) meet the requirements of Perfect Bayesian Equilibrium, given \( \{(C^*, P^*), A^*\} \).
I solve for the equilibrium of the game using backward induction, first solving for the politician’s policy choice, then for the interest group’s choice of whether or not to pay the access fee, then finally for the politician’s choice of access fee. The equilibrium is described in Proposition 6.

**Proposition 6** In the contribution equilibrium of the access fee game:

1. The politician chooses the expected socially optimal policy given his beliefs $\mu$, where

\[
P^*(a, m; \mu) = \int_0^1 \mu(e_h | a, m, c)e_h de_h - \frac{1}{2}.
\]  \hspace{1cm} (3.1)

2. The interest group buys access iff its favorable evidence is strong enough,

\[
A^*(c, e_h) = \begin{cases} 
0 & \text{when } e_h \leq \frac{2c}{v} \\
1 & \text{when } e_h > \frac{2c}{v}.
\end{cases}
\]  \hspace{1cm} (3.2)

3. The politician sets a positive access fee,

\[
C^*(\gamma) = -\frac{2v^2\phi + v\sqrt{2}v\sqrt{v\gamma\phi + 2v^2\phi^2 + 2\gamma\tau}}{2\gamma}.
\]  \hspace{1cm} (3.3)

4. In the contribution equilibrium, the politician’s beliefs $\mu$ are such that

(a) if $a = 1$, then

\[
f_\mu(e) = \begin{cases} 
1 & \text{for } e = e_h \\
0 & \text{for all other } e \neq e_h; \text{ and}
\end{cases}
\]

(b) if $a = 0$, then

\[
f_\mu(e) = \begin{cases} 
\frac{1}{\bar{e}(c)} & \text{for } e \in [0, \bar{e}(c)] \\
0 & \text{for all other } e \notin [0, \bar{e}(c)].
\end{cases}
\]
I will discuss the equilibrium strategies, starting with the policy choice at the end of the game.

**Policy Choice**

At the time the politician chooses policy, the interest group has already chosen whether to pay the access fee, and all evidence revelation has already taken place. At this point, the politician’s choice of policy cannot influence contributions. This means the policy choice can only impact the policy portion of his utility function, and he will choose the policy that maximizes expected constituent welfare given his beliefs. The politician’s equilibrium policy choice is therefore \( P^* (a, m; \mu) = E\mu p^\circ = E\mu e_h - E\mu e_l \). Since \( e_l \) is uniformly distributed on the unit interval, this simplifies to \( P^* (a, m; \mu) = E\mu e_h - \frac{1}{2} \).

Notice that the equilibrium policy choice depends only on the politician’s beliefs about \( e_h \). Therefore, earlier actions only impact policy through their influence on \( \mu \).

**Interest Group Behavior**

The interest group chooses whether to pay access fee \( c \). If it pays fee \( c \), the politician fully learns the value of \( e_h \) and will choose policy \( P^* = e_h - \frac{1}{2} \). Paying \( c \) therefore results in interest group payoff \( (e_h - \frac{1}{2}) v - c \). If the group does not pay \( c \), then the politician relies on his expectations regarding \( e_h \) and chooses policy \( P^* = E\mu e_h - \frac{1}{2} \). In this case, the interest group receives payoff \( (E\mu e_h - \frac{1}{2}) v \). The interest group prefers to pay the fee iff

\[
\left( E\mu e_h - \frac{1}{2} \right) v < \left( e_h - \frac{1}{2} \right) v - c.
\]  

(3.4)

For any \( E\mu e_h \), if the interest group prefers to pay the fee for some evidence \( e_h \), then it will also prefer to pay the fee for any higher evidence quality \( e'_h \geq e_h \). Similarly,
if the group prefers not to pay the fee for some \( e_h \), then it will not pay the fee for any lower evidence quality \( e_h' \leq e_h \). Since this holds for all \( e_h \in [0, 1] \), there must exist some cut-off value \( \bar{e}(c) \) such that for all \( e_h < \bar{e}(c) \) the interest group does not pay access fee \( c \), and for all \( e_h > \bar{e}(c) \) the interest group pays \( c \) for access.\(^4\)

Given the existence of \( \bar{e} \) and the uniform distribution of \( e_h \), it follows that when the group does not buy access \( E_\mu e_h = \frac{\bar{e}}{2}\).\(^5\)

When \( e_h = \frac{2c}{v} \), the interest group is indifferent between buying access and not buying it at fee \( c \). Therefore, \( \bar{e} = \frac{2c}{v} \). When \( e_h \) is higher than this, the benefits of disclosing evidence strictly outweigh the cost imposed by the fee. When \( e_h \) is lower than this, the interest group prefers the politician to act as if it has evidence quality \( \frac{\bar{e}}{2} \) than to pay the fee and disclose its actual evidence quality.

**Access Fee**

Determining the equilibrium access fee requires solving a straightforward optimization problem, given interest group behavior and policy choice at later stages in the game. The politician’s choice of access fee \( c \) must maximize his expected payoff

\[
\int_0^1 \int_0^1 \left( - |p^\circ(e_h, e_i) - P^*(A^*(c, e_h), m(A^*(c, e_h)); \mu)| \gamma + (c\phi - \tau)A^*(c, e_h) \right) de_h de_i.
\]

(3.5)

As I show in the proof to Proposition 6 in the appendix, after substituting in for \( P^* \) and \( A^* \) this expression simplifies to

\[
EU_P = \left[ - \left( \frac{1}{4} + \frac{2c^3}{3v^3} \right) \gamma \right] + \left[ (c\phi - \tau) \left( 1 - \frac{2c}{v} \right) \right]
\]

(3.6)

\(^4\)The value \( \bar{e} \) is not restricted to be positive. If \( \bar{e} < 0 \), then the politician will always buy access independent of the realized \( e_h \). Similarly, if \( \bar{e} > 1 \), then the politician will never buy access independent of the realized \( e_h \).

\(^5\)When the group does buy access, the politician learns \( e_h \) with certainty; therefore, \( E_\mu e_h = e_h \).
when $c \in \left[0, \frac{v}{4} + \frac{\tau}{2\phi}\right]$. (I later show that the politician always chooses $c$ from this range of values.) The term inside the first set of brackets is the politician’s expected policy utility (and expected constituent welfare) given access fee $c$. When $c = 0$, the interest group buys access for sure and policy utility is maximized at $-\gamma$. As $c$ increases, policy utility strictly decreases. The term inside the second set of brackets is expected revenue given $c$. Inside this second term, $(c\phi - \tau)$ denotes contribution utility minus the cost of providing access, and $(1 - \frac{2c}{v})$ denotes the probability that the interest group draws high-enough $e_h$ that it buys access at fee $c$.

The derivative of expression 3.6 with respect to $c$ simplifies to

$$-\gamma 2c^2 v^{-3} + \phi + (\tau - 2\phi c)2 v^{-1}.$$  (3.7)

The term inside the first set of brackets represents the impact that increasing the access fee has on the politician’s expected policy utility (and constituent welfare). Notice that this term is strictly negative for all positive $c$. The term inside the second set of brackets represents the impact that increasing the access fee has on expected revenue. When $c = \frac{v}{4} + \frac{\tau}{2\phi}$, this second term is maximized, and for any access fee, moving the fee closer to this amount strictly increases expected revenue. The politician will never prefer $c$ greater than $\frac{v}{4} + \frac{\tau}{2\phi}$ since increasing $c$ above this value results in both lower policy utility and lower revenue.

Setting the expression 3.7 equal to 0 gives the first order conditions for the politician’s maximization problem. Solving for $c$ provides a closed-form solution for the equilibrium access fee $C^\ast$. The solution is given in Proposition 6 by equation 3.3. Section 3.4.1 considers the characteristics of $C^\ast$ in more detail.
3.4.1 Characteristics of Equilibrium Access Fee

Proposition 7 describes the notable characteristics of the equilibrium access fee function $C^*$. 

**Proposition 7** The equilibrium access fee $C^*$ is

1. strictly increasing in the cost of providing access \( \left( \frac{\partial C^*}{\partial \tau} > 0 \right) \),
2. strictly increasing in interest group wealth \( \left( \frac{\partial C^*}{\partial v} > 0 \right) \),
3. strictly decreasing in issue importance \( \left( \frac{\partial C^*}{\partial \gamma} < 0 \text{ for all } \gamma > 0 \right) \), where $C^* \to 0$ as $\gamma \to \infty$, and $C^* \to \left( \frac{v}{4} + \frac{\tau}{2v} \right)$ as $\gamma \to 0$, and
4. strictly positive \( \left( C^* > 0 \text{ for all } \gamma \right) \).

These results make intuitive sense. As the politician’s cost of providing access $\tau$ increases, he increases the access fee to help offset this increase. The parameter $v$ represents how much the interest group cares about the policy choice relative to money. All else equal, as the interest group becomes more wealthy, or as it becomes more concerned about the policy choice, the value $v$ increases. As the proposition shows, in equilibrium the politician charges a higher price of access to relatively wealthy interest groups compared with less wealthy groups.

The variable $\gamma$ represents how important the policy choice is to the representative constituent. The proposition says that the politician charges a lower price for access to an interest group that is involved with an issue that he considers important. As the importance of the issue increases, the access fee falls, with the fee approaching (but never reaching) 0 in the limit. Conversely, for issues not considered important by the constituents, the politician charges a relatively high price.
for access. As the importance of the issue approaches 0, the access fee approaches $\frac{v}{4} + \frac{\tau}{2\phi}$, which is the access fee that maximizes the politician’s expected payment from the interest group minus the cost of providing access.\(^6\) An illustration of $C^*$ as a function of $\gamma$ is provided by Figure 3.1.

Although the politician’s policy utility (and constituent welfare) is maximized if he sets an access fee equal to 0, he will always set a positive access fee. This is because the politician cares about collecting contributions as well as choosing the best policy. At a low-enough $c$, the marginal benefit to contributions (the right-hand portion of expression 3.7) will exceed the marginal cost to the policy choice (the left-hand portion of expression 3.7). Compared to the case of $c = 0$, a marginally positive fee has essentially no effect on the policy choice. A marginally positive access fee means that only a group with the lowest possible evidence quality

\(^6\)This is the access fee the politician would choose if his utility function did not incorporate constituent welfare, and only included contribution utility (so, $U_P = (c\phi - \tau)a$). When the politician cares—even just a little—about constituent welfare, he trades off at least some contribution revenue in order to increase social welfare. Notice that $\frac{v}{4} + \frac{\tau}{2\phi} < 1$ is assured by the assumption that the cost of providing access is less than the benefit from the maximum possible access fee, or $\tau < \frac{v\phi}{2}$.
will not buy access. When the politician sees that a group does not buy access, he correctly infers that the group must have the lowest possible $e_h$, and therefore essentially remains fully informed about $e_h$. Therefore, compared to $c = 0$, a marginally positive access fee has strictly positive effect on expected politician payoffs since it has a positive influence on expected contributions, and essentially no effect on the policy utility.\footnote{This does not imply that the optimal limit is 0. Furthermore, this same argument does not hold starting from an already positive access fee. At a higher access fee, there is a range of $e_h$ for which the interest group does not buy access. Therefore, the politician is less than certain about the evidence quality of a group that does not buy access, and a marginal increase in the access fee has a negative effect on the accuracy of the politician’s beliefs about $e_h$.}

### 3.4.2 Constituent Welfare

When the politician chooses an access fee, he is concerned with constituent welfare and political contributions. His expected utility was given by equation 3.6. The term within the first set of brackets of this expression represents expected constituent welfare given access fee $c$. Given access fee function $c$, expected welfare for an issue of $\gamma$ importance is

$$EW = -\left(\frac{1}{4} + \frac{2}{3} \frac{c^3}{v^3}\right) \gamma. \quad (3.8)$$

In equilibrium, this becomes,

$$EW(\gamma) = -\left(\frac{1}{4} + \frac{2}{3} \left(\frac{C^*(\gamma)}{v^3}\right)^3\right) \gamma. \quad (3.9)$$

### 3.5 Contribution Limit

The previous section assumes that there are no limits to the maximum size of the interest group’s payment to the politician. This section considers how the analysis
changes if political contributions are constrained. In particular, I am interested in the impact that a contribution limit has on social welfare.

The analysis first determines the equilibrium access fee given any limit $\bar{c}$. I then describe the impact a limit has on politician information and expected constituent welfare. Depending on issue importance $\gamma$, the limit may have a positive or negative effect on politician information. The potential positive effect results because the limit decreases the equilibrium access fee for a range of $\gamma$. A lower fee means that the interest group is more likely to buy access and share its information with the politician. The potential negative effect results because the politician may not find it worth his time to provide access when he can only charge a fee up to the limit. When the politician does not offer access at any allowed price, he remains uninformed with probability 1. I show that there exists a limit that improves constituent welfare, compared to the case when contributions are unlimited. However, too strict of a limit decreases welfare.

Denote the contribution limit by $\bar{c} \geq 0$. Under the contribution limit, the politician still chooses the policy he believes is best at the final period of the game. Furthermore, the limit does not influence the interest group’s willingness to pay for access. The limit only influences the politician’s ability to set the access fee. Any limit greater than the maximum equilibrium contribution has no impact on behavior; therefore, I limit the analysis to the case when $\bar{c} \in \left[0, \frac{v}{4} + \frac{r}{20}\right)$. A contribution ban implies that $\bar{c} = 0$. Throughout this section, $C^*$, $A^*$, and $P^*$ refer to the equilibrium strategies for the game without a contribution limit, and $C_{\bar{c}}^*$, $A_{\bar{c}}^*$, and $P_{\bar{c}}^*$ refer to the strategies under limit $\bar{c}$. 
### 3.5.1 Access Fees Under the Limit

The equilibrium access fee for the game with contribution limit \( \bar{c} \) is described by the following proposition. The proposition also says that the interest group’s access strategy and the politician’s policy strategy are unchanged by the imposition of a limit.

**Proposition 8** In the contribution equilibrium of the game with contribution limit \( \bar{c} \in \left[ 0, \frac{v}{4} + \frac{\tau}{2v} \right) \),

1. the politician sets access fee

\[
C^*_\bar{c} = \begin{cases} 
C^*(\gamma) & \text{if } \gamma \geq \gamma^*(\bar{c}) \\
\bar{c} & \text{if } \gamma \in [\bar{\gamma}(\bar{c}), \gamma^*(\bar{c})] \\
\emptyset \text{ (no access)} & \text{if } \gamma < \bar{\gamma}(\bar{c})
\end{cases}
\]

where \( \gamma^*(\bar{c}) = \frac{(2\tau - 4c\phi + v\phi)v^2}{2c^2} \) and \( \bar{\gamma}(\bar{c}) = \frac{12\tau^2(\tau - c\phi)}{4c^2 + 2cv + v^2} \), and

2. \( A^*_\bar{c} = A^*, P^*_\bar{c} = P^*, \) and \( \mu_{\bar{c}} = \mu \).

The function \( \gamma^* \) denotes the inverse function of \( C^* \) (i.e., \( \gamma^*(\bar{c}) \equiv C^{*-1}(\gamma) \)); therefore, \( \gamma^*(\bar{c}) \) is the value of \( \gamma \) that solves \( C^*(\gamma) = \bar{c} \). To deal with the case of contribution bans, note that \( \gamma^*(0) = \infty \). When the realized value of \( \gamma \) is greater than \( \gamma^*(\bar{c}) \), the politician prefers to charge an access fee less than the maximum contribution, and the limit has no effect on the access fee.\(^8\)

The value \( \bar{\gamma}(\bar{c}) \) is the value of \( \gamma \) at which the politician is indifferent between selling access at price \( \bar{c} \), and not granting any access. For \( \gamma < \bar{\gamma}(\bar{c}) \), the politician does not find it worth his time to grant the interest group access when he can only

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\(^8\)In the case of a contribution ban, there is no \( \gamma \) for which this condition holds. Therefore, a ban influences behavior for all potential issues.
charge an access fee up to $\bar{c}$. For this range of $\gamma$, the politician does not sell any access and remains fully uninformed about interest group evidence.

For $\gamma$ between $\bar{\gamma}$ and $\gamma^*$, the politician is willing to provide access at a price equal to the contribution limit; although he would prefer to set the fee above the limit. For this range of issue importance, the politician sets the fee at $\bar{c}$.

The value $\bar{\gamma}(\bar{c})$ is strictly decreasing in $\bar{c}$, and for large enough $\bar{c}$ it will be the case that $\bar{\gamma}(\bar{c}) \leq 0$. When this is the case, the cost to the politician of providing access is sufficiently low such that, independent of how important the issue is, he still finds it worthwhile to sell access at price $\bar{c}$.

An illustration of the access fee is provided by Figure 2 for the case when $\bar{\gamma}(\bar{c}) > 0$. For $\gamma < \bar{\gamma}(\bar{c})$, the politician does not sell access; therefore, the access fee function does not exist over that range of values.

---

9This will always be the case as $\bar{c}$ approaches $\frac{\psi_1}{4} + \frac{\phi_2}{2}$, the highest possible equilibrium access fee without the limit. This is because, by assumption, the cost of providing access is sufficiently low such that for some feasible access fee the politician finds granting access to the interest group worthwhile, even as the importance of the issue approaches 0.
3.6 Welfare Effects of Limit for One Issue

In this section, I consider the effects that a limit has on politician information and welfare for a single issue. I then identify the socially optimal limit, which maximizes expected constituent welfare. Throughout this section, I assume that the limit only applies to a single issue with known \( \gamma \). The more realistic case, when a limit applies across multiple issues, is addressed in Section 3.7.

Expected constituent welfare under limit \( \bar{c} \) depends on how important the issue is, and the access fee the politician charges given the limit. Proposition 8 describes the equilibrium contribution function. When the politician charges an access fee \( c \), expected welfare is given by expression 3.8. When he does not sell any access, expected welfare is simply \(-\frac{\gamma}{3}\), which is expected welfare when the politician is completely uninformed about interest group evidence.

Lemma 5  For any issue \( \gamma \), expected constituent welfare given contribution limit \( \bar{c} \) is

\[
EW_{\bar{c}}(\gamma) = \begin{cases} 
-\left(\frac{1}{4} + \frac{2}{3} \frac{(C^*(\gamma))^3}{\delta^4}\right) \gamma & \text{if } \gamma \geq \gamma^*(\bar{c}) \\
-\left(\frac{1}{4} + \frac{2}{3} \frac{c^3}{\delta^4}\right) \gamma & \text{if } \gamma \in [\bar{\gamma}(\bar{c}), \gamma^*(\bar{c})] \\
-\frac{\gamma}{3} & \text{if } \gamma < \bar{\gamma}(\bar{c}).
\end{cases}
\]

The following proposition comes from comparing equilibrium expected constituent welfare under the limit (lemma 5) with equilibrium expected constituent welfare when there is no limit (equation 3.9).

Proposition 9  For any issue \( \gamma \), compared to the case of no limit, contribution limit \( \bar{c} \) has
• **no effect on politician information and expected constituent welfare if** $\gamma \geq \gamma^*(\bar{c})$,

• **a positive effect on politician information and expected constituent welfare if** $\gamma \in (\gamma(\bar{c}), \gamma^*(\bar{c}))$, and

• **a negative effect on politician information and expected constituent welfare if** $\gamma \leq \gamma(\bar{c})$.

Below, I provide intuition for these three possible effects: *no effect, positive effect,* and *negative effect.*

**No Effect** – When the realized value of $\gamma$ is sufficiently high (i.e., greater than $\gamma^*(\bar{c})$), the politician prefers to set an access fee below the contribution limit. For issue $\gamma$, imposing such a contribution limit therefore does not affect his ability to set his desired access fee. Under the limit, the politician sets fee $C^*(\gamma)$; just as he would if there was no limit. The limit has no effect on politician information, his policy choice, or expected constituent welfare.

**Positive Effect** – For moderate realizations of $\gamma$ (i.e., when $\gamma$ is between $\gamma(\bar{c})$ and $\gamma^*(\bar{c})$), the politician prefers to charge an access fee greater than the contribution limit $\bar{c}$; however, he is willing to grant access even if he can only charge a fee equal to the contribution limit. For these issues, the contribution limit causes the politician to set a lower price for access than he otherwise would. The lower access fee means a higher probability that the interest group buys access, as well as more accurate beliefs about $e_h$ when the group does not buy access.\(^{10}\) This tends to result in a more-informed politician, who is better able to identify and implement the socially optimal policy, and higher expected constituent welfare.

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\(^{10}\) The more accurate beliefs in this case result from there being a smaller range of $e_h$ for which the group chooses not to buy access.
**Negative Effect** – When the realized value of $\gamma$ is sufficiently low (i.e., less than $\bar{\gamma}(\bar{c})$), the politician prefers to charge an access fee above the limit, and he is *not* willing to sell access at a fee equal to the limit. Although the politician would be willing to offer access at a high-enough fee, the contribution limit prevents him from being able to charge a sufficiently high amount.\(^{11}\) The politician therefore does not grant any access, he learns nothing about the interest group’s evidence quality, and with probability 1 he remains fully uninformed about the socially optimal policy. This is in contrast to when there is no contribution limit, and the politician becomes fully informed with positive probability.\(^{12}\) This tends to result in a less-informed politician, who is less able to identify and implement the socially optimal policy, and lower expected constituent welfare.

The ranges of $\gamma$ for which a contribution limit has positive, negative, or no effects depends on how strict the limit is. Remember that $\bar{\gamma}$ and $\gamma^*$ are both decreasing in $\bar{c}$. Increasing the limit decreases the range of $\gamma$ for which there is a negative impact on politician information, and increases the range of $\gamma$ for which there is no impact on information. A high enough limit results in $\bar{\gamma} < 0$, which means that any limit will have an unambiguously non-negative effect on expected constituent welfare. At the opposite extreme, a contribution ban means that $\gamma^*(0) = \infty$, which means that the limit will always have either a positive or negative effect on welfare. Furthermore, a contribution ban results in the largest range of issues for which there is a negative effect.

\(^{11}\)For these values of $\gamma$, the costs of providing access $\tau$ outweigh the expected informational and monetary benefits when the fee cannot exceed $\bar{c}$.

\(^{12}\)Also note that when the politician grants no access, he is also less informed about the interest group’s evidence quality compared to the situation when there is no limit and the group does not buy access. When the group does not buy access in the no-limit case, the politician can still infer that the group has sufficiently-low evidence quality such that buying access was not worthwhile.
3.6.1 Optimal Limit for One Issue

This section is concerned with the contribution limit that maximizes expected constituent welfare. The *optimal limit for issue* $\gamma$ is denoted $\bar{c}_o(\gamma)$. Given the realization of $\gamma$, the limit $\bar{c}_o(\gamma)$ maximizes expected constituent welfare. If a contribution ban is optimal, then $\bar{c}_o(\gamma) = 0$. If it is optimal to impose no limit, then $\bar{c}_o(\gamma) = \emptyset$.

An overly-strict limit results in the politician not selling any access. A limit that is not strict enough results in the interest group being willing to buy access for a smaller range of evidence quality than might otherwise be possible. The optimal limit is defined by the following proposition.

**Proposition 10**  
Let $\bar{c}'$ solve $\bar{\gamma}(\bar{c}') = \gamma$. Then $\bar{c}_o(\gamma) = \max\{0, \bar{c}'\}$.

The optimal limit for the issue $\bar{c}_o(\gamma)$ equals either the lowest possible limit at which the politician is willing to sell access or 0, whichever is greater. The lowest limit at which the politician is willing to sell access results in the politician being indifferent between selling access at a fee equal to $\bar{c}_o$, and not selling any access. Any lower limit means that $\bar{\gamma}(\bar{c}) > \gamma$, and the politician chooses not to sell access. In this case, the politician will remain completely uninformed about interest group evidence, and increasing the limit to $\bar{c}_o(\gamma)$ tends to result in a more fully informed politician. Any limit higher than $\bar{c}_o(\gamma)$ results in $\bar{\gamma}(\bar{c}) < \gamma$; for which case the politician charges a higher access fee than he needs to cover the costs of providing access. If the limit is reduced to $\bar{c}_o(\gamma)$, the politician continues to sell access, but at a lower price. This tends for him to be more informed, since the interest group is more likely to buy access.
When the optimal limit is imposed, the politician charges an access fee equal to the limit, which is less than the fee he would charge if there was no limit. If the limit is too high, then the positive effect of the limit is not as high as it otherwise could be. If the limit is too low, then the negative effect of the limit is present, and the negative effect can be decreased by increasing the limit.

3.7 Many Issues

When there are multiple issues, the optimal contribution limit for any individual issue is unchanged from the above analysis. However, it is reasonable that a single contribution limit applies across multiple issues. If the optimal limit for one issue is implemented, there will be other issues for which the limit is too low or too high. This section considers the optimal limit when there are multiple issues. Denote this optimal limit across all issues by $\bar{c}_o$.

To incorporate multiple issues into this paper’s framework, I assume that the contribution limit is set prior to the realization of $\gamma$. Remember that $\gamma$ is the realization of a random variable drawn from a continuous distribution on $\mathbb{R}_{++}$. After the realization of $\gamma$, the game proceeds as it did in section 3.3. Although the game involves only a single policy choice for the politician, one may interpret the continuum from which $\gamma$ is drawn as representing many issues for which the politician chooses policy.
3.7.1 Expected Constituent Welfare

For any $\gamma$, equation 3.8 gives expected constituent welfare when there is no contribution limit. From an ex ante perspective, before the realization of $\gamma$, expected constituent welfare is

$$EW_{\text{no limit}} = -\int_{0}^{\infty} g(\gamma) \left( \frac{1}{4} + \frac{2}{3} \frac{(C^*(\gamma))^3}{v^3} \right) \gamma d\gamma. \quad (3.10)$$

When the politician chooses policy for a continuum of issues, $EW_{\text{no limit}}$ represents the average constituent welfare across all issues for the case when there is no limit.

Under limit $\bar{c}$, expected constituent welfare across all issues is

$$EW_{\bar{c}} = -\int_{0}^{\gamma_{\bar{c}}} g(\gamma) \frac{\gamma^2}{3} d\gamma - \int_{\gamma_{\bar{c}}}^{\gamma^*_{\bar{c}}} g(\gamma) \left( \frac{1}{4} + \frac{2}{3} \frac{\bar{c}^3}{v^3} \right) \gamma d\gamma - \int_{\gamma^*_{\bar{c}}}^{\infty} g(\gamma) \left( \frac{1}{4} + \frac{2}{3} \frac{(C^*(\gamma))^3}{v^3} \right) \gamma d\gamma. \quad (3.11)$$

Similar the $EW_{\text{no limit}}$, $EW_{\bar{c}}$ represents average constituent welfare across all $\gamma$.

For important enough issues (i.e., when $\gamma$ is greater than $\gamma_{\bar{c}}$), the politician prefers to charge an access fee below the limit, and the limit does not influence the equilibrium access fee. For this range of $\gamma$, the limit has neither a positive nor a negative effect on politician information, and expected constituent welfare is the same as it was without a limit (3.8). For any positive limit, there exists a range of $\gamma$ for which this is the case.\(^{13}\)

When issue importance is sufficiently low (i.e., when $\gamma$ is less than $\gamma_{\bar{c}}$), the politician is unwilling to sell access for a fee that cannot exceed the limit (although he would offer to sell access if he could set a high-enough fee). For these issues the politician remains fully uninformed about interest group evidence. The limit has

\(^{13}\)When contributions are banned (i.e., $\bar{c} = 0$), all issues are affected. The politician always sets a positive access fee when he is able to do so; therefore banning contributions affects his behavior on all issues. Formally, this is because $\gamma^*_{\bar{c}} \to \infty$ when $\bar{c} \to 0$. 
a negative effect on politician information, decreasing welfare to $-\frac{\gamma}{3}$. This range of $\gamma$ has positive weight so long as $\bar{\gamma}(\bar{c})$ is positive.\footnote{If $\bar{\gamma}(\bar{c}) \leq 0$, then the politician is willing to sell access for any issue at a fee equal to the contribution limit. In this case, there will not exist any $\gamma < \bar{\gamma}(\bar{c})$.}

For $\gamma$ between $\bar{\gamma}(\bar{c})$ and $\gamma^*(\bar{c})$ the politician is willing to sell access at a fee equal to the contribution limit, although he would prefer to set the access fee above the limit. For these issues, the limit causes the politician to set a lower access fee than he otherwise would have. By decreasing the price of access, the limit has a positive effect on politician information, thereby improving expected constituent welfare.\footnote{Since $\bar{\epsilon} < C^*(\gamma)$ when $\gamma \in [\bar{\gamma}(\bar{c}), \gamma^*(\bar{c})]$, it follows that $-\left(\frac{1}{4} + \frac{2}{3} \left(\frac{\gamma}{v^3}\right)^3\right) \gamma > -\left(\frac{1}{4} + \frac{2}{3} \left(\frac{C^*(\gamma)}{v^3}\right)^3\right) \gamma$ for this range of $\gamma$.} It is always the case that $\gamma^*(\bar{c}) > \bar{\gamma}(\bar{c})$ and $\gamma^*(\bar{c}) > 0$; therefore, there always exists a range of $\gamma$ for which the positive effect exists.

Depending on the model parameters including the distribution of $\hat{\gamma}$, a contribution limit $\bar{c}$ may either increase or decrease ex ante expected constituent welfare. Compared to the case of no limit, imposing limit $\bar{c}$ increases expected constituent welfare when $\gamma$ is between $\bar{\gamma}(\bar{c})$ and $\gamma^*(\bar{c})$, and the limit decreases expected constituent welfare when $\gamma$ is less than $\bar{\gamma}(\bar{c})$. Whether the limit increases or decreases ex ante expected constituent welfare depend on the distribution of $\gamma$, $G$.

### 3.7.2 Optimal Limit

The optimal limit maximizes $EW_{\bar{c}}$, which was given by equation 3.11. The derivative of $EW_{\bar{c}}$ with respect to $\bar{c}$ simplifies to

$$\frac{\partial EW_{\bar{c}}}{\partial \bar{c}} = -g(\bar{\gamma}(\bar{c})) \left[ \frac{1}{12} - \frac{2}{3} \bar{c}^3 \right] \bar{\gamma}(\bar{c}) \gamma'(\bar{c}) - \int_{\bar{\gamma}(\bar{c})}^{\gamma^*(\bar{c})} g(\gamma) \frac{2\bar{c}^2}{v^3} \gamma d\gamma. \quad (3.12)$$
To make the discussion more intuitive, I rewrite expression 3.12 to give the marginal effect of a decrease in $c$. As the limit becomes stricter, expected constituent welfare changes according to

$$
-\frac{\partial EW_c}{\partial c} = Z1 + Z2.
$$

(3.13)

where $Z1 \equiv g(\bar{\gamma}(\bar{c})) \left[ \frac{1}{12} - \frac{2 \varepsilon_1}{3 v^3} \right] \bar{\gamma}(\bar{c}) \bar{\gamma}'(\bar{c})$ and $Z2 \equiv \int_{\bar{\gamma}(\bar{c})}^{\gamma^*(\bar{c})} g(\gamma) \frac{2\varepsilon_2}{v^3} \gamma d\gamma$.

Imposing a stricter limit increases the range of $\gamma$ for which the politician does not sell any access, and for which the limit has a negative effect on information. This effect is represented by $Z1$, which is strictly negative.\(^{16}\)

At the same time, a stricter limit also reduces the average access fee on issues for which the politician does sell access. This means that the interest group is more likely to buy access and the politician become informed. $Z2$ represents this positive effect of a stricter limit on information and welfare that is present for some range of $\gamma$. $Z2$ is strictly positive.

Without making assumptions regarding the distribution of $\gamma$, one cannot find a closed-form solution for the optimal contribution limit $\bar{c}_o$. One can, however, conclude the following.

**Proposition 11** There exists some $\bar{c}_o \in (0, \frac{\tau}{\phi})$ such that

1. $EW_{\bar{c}_o} > EW_{\text{no limit}}$, and
2. $EW_{\bar{c}_o} \geq EW_{\bar{c}}$ for all other $\bar{c} \geq 0$.

Not only does Proposition 11 establish that an optimal limit exists, it also provides a range of values within which the optimal limit is located. Corollary 1 follows from this range.

\(^{16}\)Since $\bar{\gamma}'(\cdot) < 0$, $Z1 < 0$. 84
Corollary 1

1. It is never optimal (for expected constituent welfare) to ban contributions.

2. It is always optimal (for expected constituent welfare) to set a contribution limit. Under the optimal limit, the politician sells access for some, but not all, issues.

If contributions are banned, the politician only grants access for issues where the expected policy benefit of learning the interest group’s evidence exceeds the time costs of granting access. For any issue that the politician does grant access, the politician learns \( e_h \) for sure. Now, consider the impact of a marginally positive limit compared to a contribution ban. \( \frac{\partial E_W}{\partial \bar{c}} \) is strictly positive at \( \bar{c} = 0 \), which means that increasing the limit above 0 improves constituent welfare. The intuition for this follows. A positive limit increases the range of \( \gamma \) for which the politician chooses to sell policy—a welfare benefit. A positive limit also decreases the probability that the interest group buys access, which tends to decrease welfare since a politician is less likely to fully learn \( e_h \). This negative impact on welfare is minimized when considering a marginally positive fee. This is because when the politician sets a marginally positive access fee, only an interest group with the lowest possible evidence quality does not buy access. When the group does not pay for access, the politician correctly infers that the interest group has the lowest-possible \( e_h \), and the politician remains fully informed about \( e_h \). A marginally positive limit causes the politician to grant access for a larger range of issues (a benefit) and does not result in him being less informed regarding any of the issues. Therefore, a marginally positive limit is strictly better for expected constituent welfare than a contribution ban.
If $\bar{c} = \frac{\tau}{\varphi}$, then $\bar{\gamma}(\bar{c}) = 0$ and the politician sells access at some fee for all issues. Any higher contribution limit results in a higher access fee for some issues, but does not increase the range of issues for which the politician does sell access (since he is already offering to sell access for all issues). Therefore, setting the limit equal to $\frac{\tau}{\varphi}$ is better than setting a higher limit, or no limit at all. Now, consider the impact of a limit that is marginally less than $\frac{\tau}{\varphi}$ compared to one equal to $\frac{\tau}{\varphi}$. Evaluated at $\bar{c} = \frac{\tau}{\varphi}$, $-\frac{\partial E W}{\partial \bar{c}} > 0$; therefore, a stricter limit improves constituent welfare. The intuition is as follows. Imposing a stricter limit reduces the access fee associated with some issues, which tends to have a positive impact on welfare. A stricter limit also means that for low-enough $\gamma$ the politician will not sell any access, which tends to decrease expected welfare. For a limit just below $\frac{\tau}{\varphi}$, the politician sells access for all issues except the least-important ones (those with the smallest possible $\gamma$). Such a limit causes the politician to become less informed about the least important issues, and to become more informed about relatively important ones. The net effect of such a limit on welfare is positive compared to a limit equal to $\frac{\tau}{\varphi}$.

Under the optimal limit, the politician sells access for some, but not all issues. That he does not sell access for every issue follows from $\bar{c}_o < \frac{\tau}{\varphi}$ which means $\bar{\gamma}(\bar{c}_o) > 0$. That the politician always gives access for some issues follows because he prefers to grant access to interest groups concerned with issues for which he cares passionately (those with high-enough $\gamma$), even when he does not have a monetary incentive to do so.
3.8 Conclusion

I develop a simple model of access fees in politics, and use it to analyze the impact of contribution limits on policy choice and constituent welfare. The model, adapted from the evidence model in Chapter 2, has some significant advantages. By fixing the policy preferences of the interest group, and using a relatively simple evidence structure, I am able to consider in detail the impact of interest group wealth, the politician’s cost of providing access, and the importance of the issue for which the politician must choose a policy. This is in contrast to the earlier work by Austen-Smith (1998) in which there is a more-complex evidence structure and interest groups differ in how similar their preferences are to the politician. The focus of Austen-Smith (1998) is on how extreme or moderate the interest groups that buy access are, and the paper says relatively little about the relationship between access fees and interest group wealth or issue importance, or about the impact of contribution limits (although there is a brief discussion).

This paper predicts that politicians charge higher access fees to more wealthy interest groups relative to poor groups, and lower access fees to groups involved with relatively important issues. Both predictions have strong intuitive appeal, and supporting empirical evidence. Furthermore, I show that the politician tends to become more informed about interest group evidence and make better policy decisions when the issue is more important. Interestingly, increasing interest group wealth can improve expected constituent welfare.

The analysis identifies positive and negative effects of a contribution limit. A limit has a positive effect because it decreases the average access fee, which increases the probability that an interest group buys access. A limit has a negative effect because it may decrease the number of issues for which the politician is willing
to sell any access. When the politician cannot charge more than the limit for access, he may not find selling access for certain issues worth his time. (Although he will always find it worth his time to sell access for issues about which he cares passionately enough.)

The paper shows that, when the limit applies across many issues, it is always optimal to set a contribution limit that results in the politician refusing to sell any access for some issues. I also show that a contribution ban is never optimal. This result is in contrast to Chapter 2 in which limits have a strictly negative impact on politician information and constituent welfare. It is also in contrast to the number of models in which the politician knows the socially optimal policy ex ante, and allowing contributions enables the politician to trade policy favors that decrease constituent welfare for contributions. Clearly the mechanism by which the politician allocates access has a significant impact on the welfare implications of a contribution limit. Future empirical work should attempt to better understand the process by which politician’s award access.
CHAPTER 4
DISCRETE PRIZE ALLOCATION AND ACCESS

4.1 Introduction

A politician may want to choose the policy that is best for his constituents from a set of proposed legislation. A bureaucrat may want to award a government contract to the most-competent service provider. A judge may want to award custody of a child to the most-stable parent in a divorce case. A firm manager may want to promote the most-capable employee or hire the most-competent applicant. In each of these examples, a principal wants to award a non-divisible prize to the “most-qualified” agent. However, the principal might not know the qualifications of each agent ex ante.

This paper is concerned with such settings, in which a principal must award a non-divisible prize when he cares at least somewhat about the qualifications of the prize winner. (The principal may also care about collecting monetary payments from the agents.) Each agent knows her own qualifications, but the principal does not. The principal can, however, pay a price to learn an agent’s qualifications. The costs to learning an agent’s qualifications may represent the time costs of conducting research or of allowing an agent to present verifiable evidence regarding her qualifications.

The primary contributions of this chapter are twofold. First, it incorporates access into a framework in which a decision maker must award a non-divisible prize. This setting is differs from the previous two chapters in which the principal chooses a policy from a continuum of possible options. Where the earlier framework was a
good representation of a policy choice when a politician was responsible for drafting and implementing the legislation, this chapter’s discrete-prize setting is a more-reasonable representation of choosing whether to vote for or against an already established piece of legislation, or choosing between different well-established policy positions. As I suggest above, it is also a better representation of a number of non-policy decisions.

Second, the non-divisible prize setting offers a promising framework for comparing alternative access and prize allocation mechanisms. The analysis considers traditional models of selling the prize (i.e., through a posted-price, auction, or lottery) with two models of awarding access (i.e., the competition for access mechanism from Chapter 2 and the access fee mechanism from Chapter 3).

The analysis shows that a competition for access mechanism is an optimal means of awarding a non-divisible prize from the standpoint of the principal. That is, it (i) awards the prize to the most-qualified agent with probability 1, (ii) maximizes expected revenue, and (iii) gives access to the fewest number of agents. Even when the principal does not care about revenue, he still prefers to award the prize through competition for access since it ensures that the prize goes to the most-qualified agent, and minimizes the time-costs involved with doing so.

The access fee mechanism results in neither the prize going to the most qualified agent for sure, nor the maximum revenue. Similarly, mechanisms that involve explicitly selling the prize can maximize revenue, but do not ensure that the prize goes to the most-qualified agent.

The results require that the prize be non-divisible and that agents share a common valuation of the prize. As Chapter 2 showed, the competition for access
mechanism does not maximize revenue when there is a continuous policy space (which is equivalent to the prize being divisible). Similarly, when agents differ in terms of their valuations for the prize, competition for access will not maximize revenue.\footnote{When agents differ in terms of their valuations, expected revenue is maximized by setting a take-it-or-leave-it offer at a price equal to the highest agent valuation.} In either of these cases, when the principal cares at all about revenue, competition for access will not be a first-best optimal mechanism. However, the principal may still prefer competition for access to other mechanism so long as he cares enough about choosing the best policy.

In Section 4.2 I review the related literature. Section 4.3 describes the setting in which the principal must awarding the prize. The following sections then consider alternative means of awarding the prize given this setting. Section 4.4 considers traditional prize allocation mechanism in which the politician explicitly sells the prize. Section 4.5 applies the competition for access framework developed in Chapter 2 to the discrete prize setting, and presents results regarding the optimality and efficiency of the mechanism. Section 4.6 applies the access fee framework developed in Chapter 3 to the discrete prize setting, and shows that it results in both lower revenue and less-than-optimal prize allocation. I conclude the paper in Section 4.7.

4.2 Literature

This paper differs from most of the literature by assuming that the agents have hard (or verifiable) information, and that they must be granted access to share this information with the principal. Few papers make both of these assumptions. Those that do include Chapters 2 and 3 of this dissertation, and Austen-Smith
(1998). Both Chapter 3 and Austen-Smith (1998) assume that a principal set an access fee, and interest groups have the option of buying access at the known fee or not buying it. Interest groups with access can disclose evidence in favor of their preferred policies. Chapter 2 applies a competition for access model (similar to the one considered in this paper) to a setting where a politician must choose policies for a number of different issues, and in which interest groups require access to present arguments in favor of their preferred policies. The paper shows how contribution limits tend to reduce the amount of information available to the politician when he chooses policy, and result in the politician choosing policies that are worse for his constituents. In Chapter 2, the politician chooses policy from a continuous policy space, and selling access rather than a policy position is preferred by the politician so long as he cares enough about choosing the best policy compared with raising revenue. In the present paper, an agent either wins the prize or does not. In this alternative setting, I show that selling access is an optimal (and efficient) mechanism so long as the principal cares (even an arbitrarily small amount) about prize-winner qualifications.

With the exception of these three papers, I am aware of no other paper that assumes that information is verifiable, and that agents require access to present the information to the decision maker. There are, however, a variety of papers that incorporate one of the two assumptions.

The disclosure of hard information has been widely studied; however, the existing models tend to ignore the need for access in order to present one’s evidence to a decision maker (e.g., Green and Laffont 1986, Lipman and Seppi 1995). Milgrom and Roberts (1986) consider the decision of interest groups to disclose private verifiable information to a decision maker. Bennedsen and Feldmann (2002) extend
Milgrom and Roberts (1986) to a multiple-policy maker setting. Bennedsen and Feldmann (2006) and Dahm and Porteiro (2006a,b) combine a simple verifiable information disclosure framework with a quid pro quo exchange of contributions and policy favors. Bull and Watson (2007) incorporate hard evidence into a traditional mechanism design framework. The primary difference between this paper’s setting and the setting in these earlier papers is that the earlier papers assume that agents have an inalienable right to present their evidence to the principal, and I assume that agents require access to the principal in order to reveal their evidence.

Other papers model political access in the case of soft information, or cheap talk. The few papers that do incorporate access (Austen-Smith 1995, Lohmann 1995) assume that interest group information is completely unverifiable. In these models, presenting unverifiable information to a politician does nothing to the politician’s beliefs about the best policy or the most qualified interest group. The only way the interest groups can influence a politician is by providing contributions to signal the intensity of their preferences. This paper incorporates both hard information and access.

When agents require access to present hard information, I show that the principal finds it optimal to sell access to the highest bidder, then award the prize to the agent he believes has the highest qualifications. This result is consistent

\textsuperscript{2}The majority of informational lobbying models assume that interest group information is completely unverifiable. See Grossman and Helpman (2002) for an excellent overview.

\textsuperscript{3}In this way, these “access” models are similar to the signaling models of Austen-Smith (1994) and Esteban and Ray (2006) in which agents provide contributions or engage in costly lobbying in order to provide a signal to a principal regarding their preferences. In these signaling models information is unverifiable, and access is ignored. Austen-Smith (1995) and Lohmann (1995) both assume interest groups have private, unverifiable information in support of or against their favorite policy, and that the groups can use contributions to send signals to a politician about their information. However, because information is \textit{completely unverifiable}, revealing it to the politician cannot influence the politician’s beliefs about the best policy, and therefore access itself does nothing in these models. In reality, interest groups expect their information (not just their contributions) to influence the decision maker’s beliefs.
with the existing empirical and survey evidence regarding informational lobbying in politics. Herndon (1982), Schram (1995), and Makinson (2003) survey current and former interest group representatives and politicians. They present substantial evidence that interest groups give money to politicians for two reasons: (1) to support candidates who share their policy preferences, and (2) to secure access to politicians. Based on these surveys, there is little reason to believe that contributions are typically provided in the quid pro quo exchange for policy favors. It is generally difficult to empirically determine the reason why money is given to politicians. This difficulty stems from the fact that US politicians are not required to report the amount of time they spend meeting with interest groups; nor can we be certain whether a politician adopts a specific policy stance because it is consistent with his prior convictions, because he was exposed to persuasive evidence in favor of that position, or because he wants to attract contributions from the interest groups that support the position. A few papers have attempted to get around these issues. For example, Langbein (1986) uses data from an (one-time) internal audit by the US House of Representatives to show a significant correlation between political contributions and time spent meeting with representatives. Ansolabehere et al. (2002) consider business expenditures and finds a strong correlation between an organization’s political contributions and additional spending on lobbying activities. This suggests that contributions by themselves are not enough to influence policy, and that a group must also present their information to the politician.

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4For example, former US Congressman Thomas Downey said "Money doesn’t buy...a position. But it will definitely buy you some access so you can make your case" (Schram 1995). Talking about campaign contributors, former US Senator Dennis DeConcini said "What they got our of me for that contribution is access to come in .. and to tell me why ... it’s good for America" (Schram 1995). In Herndon (1982) an anonymous interest group official said "About all you get [in exchange for a contribution] is a chance to talk to them... If you have a good case you can win them over. But you have to be able to talk to them."

5Although lobbyists play a significant role in the presentation of information to politicians, the theoretical model presented in this paper generally ignores lobbyists, and instead assumes that agents with access can costlessly present their information to the politician.
4.3 Setting

There are $N$ agents indexed by $i \in \{1, ..., N\}$ where $N \geq 2$. A single principal (agent 0) must choose which one of these individuals receives a valuable prize. When the principal is a politician or bureaucrat and the agents are special interest groups, the prize may be a government contract, permit, or policy favor. When the principal is an employer and agents are employees, the prize may be a promotion.

The key difference between this paper and a traditional contest is that here the principal cares about the qualifications of the agent who receives the prize. A politician or bureaucrat wants to award a contract or permit to the agent who will provide citizens with the highest quality service; employers want to promote the most qualified employees. Each agent privately knows her own qualifications, but her qualifications are not observed by the other agents or the principal.

The principal can verifiably learn an agent’s qualifications, but only if he grants the agent access. Let $\phi_i = 1$ if agent $i$ receives access, and $\phi_i = 0$ otherwise, where $\phi = (\phi_1, ..., \phi_N)$. The principal faces a cost of providing access, determined by function $c$, where $c(m)$ is the cost to the principal of providing access to $m$ agents. The greater number of interest groups the principal grants access to, the higher his costs; therefore, $c'(m) > 0$ for all $m \geq 1$. For analytical convenience, I assume that $c(0) = c(1) = 0$. However, all of the results in the paper will continue to hold so long as the cost of providing access to a single agent is sufficiently small.\(^6\)

Formally, $q_i$ represents the qualifications of agent $i$, where $q_i$ is the independent realization of a random variable with common distribution $F$ and pdf $f$. Without

\(^6\)Alternatively, one may assume that the principal is time constrained and simply cannot give access to all agents. Under such an assumption, the politician does not need to face positive costs of providing access. What is important is that granting access to all agents is more difficult than granting access to a single agent.
loss of generality, we can assume that the support for qualifications is the unit interval, \( q_i \in [0, 1] \) for all \( i \). A relatively high \( q_i \) means that agent \( i \) has relatively strong qualifications. The vector of all agent qualifications is \( q = (q_1, \ldots, q_N) \), and the vector of qualifications of all agents except agent \( i \) is denoted \( q_{-i} \). The function \( f_q \) denotes the pdf of \( q \), and \( f_{-i} \) denotes the pdf of \( q_{-i} \).\(^7\) To allow the analysis to focus on the differences in agent qualifications, I assume that all agents share the same known valuation for the prize, \( v > 0 \). An agent’s type is therefore fully defined by her qualifications. A type-\( q_i \) agent is one with qualifications \( q_i \).

Agents may make payments to the principal during the game, where \( t_i \in \mathbb{R} \) represents the payment made by agent \( i \), and \( t = (t_1, \ldots, t_N) \). The value \( \omega_i \) indicates whether agent \( i \) receives the prize, where \( \omega_i = 1 \) if \( i \) wins the prize, and \( \omega_i = 0 \) otherwise, where \( \omega = (\omega_1, \ldots, \omega_N) \). Agent utility is linear, where \( U_i(\omega_i, t_i) = \omega_i v - t_i \). If agent \( i \) wins the prize, she receives payoff \( U_i(1, t_i) = v - t_i \). If agent \( i \) does not win the prize, her payoff is \( U_i(0, t_i) = -t_i \).

The principal’s payoff is increasing in both the qualifications of the agent who receives the prize, and the total amount of money he collects from the agents. The parameter \( \delta \geq 0 \) describes how much weight the principal puts on agent qualifications relative to revenue. The principal receives payoffs

\[
U_0(q, t, \omega, m) = \sum_{j=1}^{N} \omega_j q_j + \delta \sum_{j=1}^{N} t_j + C(m). \tag{4.1}
\]

The analysis is concerned with how the principal chooses an agent to receive the prize. The game takes place in the following order.

1. The principal chooses the rules (the *mechanism*) through which he will select an agent to receive the prize.

\(^7\) \( f_{-i}(q) = f_{-j}(q) \) for any \( i \) and \( j \) and for all \( q \) because \( f \) is common across all agents.
2. Given the rules chosen by the principal, the agents compete for the prize.

After the principal chooses the mechanism through which the prize is awarded, the agents compete in the mechanism subgame. For example, if the mechanism involves the principal auctioning off the prize to the highest bidder, then in the subgame agents submit bids to the principal and the highest bidder wins the prize.

Section C.1 in the appendix incorporates access into a formal mechanism design framework, and establishes that the revelation principal holds in such a setting. For the purposes of this paper, however, such a formal approach is not needed. The analysis is concerned with specific mechanisms, which may be compared in a more intuitive setting without transforming each mechanism into its truthfully-revealing equivalent.

### 4.3.1 Revenue maximization

The prize winner will value the prize at the common value $v$. Each agent will not participate in the process if it expects to pay more than the expected benefit of participating ($v$ times the probability that it expects to win the good). In aggregate, across all agents this implies that there is no feasible mechanism by which the principal can earn more than $v$ in expected revenue.

**Lemma 6** A revenue-maximizing mechanism for allocating a prize with common value $v$ results in expected revenue for the principal of $v$. 
4.3.2 First-Best Optimal Mechanisms

A mechanism is first-best optimal for the principal if it (i) is a maximizes expected revenue, (ii) awards the prize to the most qualified agent with probability 1, and (iii) grants access to no more than one agent. Such a mechanism results in the highest-possible expected utility for the principal.

If the principal does not care about revenue, then a first-best optimal mechanism for awarding the prize must still satisfy requirements (ii) and (iii) above. This may be the more reasonable case if the principal is an honest judge or bureaucrat who has no interest in extracting payment from the agents for the purpose of collecting revenue.

4.4 Explicitly Selling the Prize

Traditional models of prize allocation assume that a principal awards a prize through a lottery (e.g., Tullock 1980), an all-pay or winner pay auction (e.g., Che and Gale 1998, Baye et al. 1993, 1996, 1999, Grossman and Helpman 1994, Bernheim and Whinston 1986), or a posted-price mechanism.

Given the framework presented in section 4.3, each of these means of awarding the prize are revenue equivalent for the principal. In the equilibrium of each of these traditional models, the principal expects to earn revenue of \( v \), and each agent expects a payoff of zero. Because the agents share a common, known valuation, the principal is able to fully extract the expected rent of the agents. Here, I briefly describe the equilibria of these models.
**Posted price.** If the principal relies on a simple posted price mechanism to award the prize, he can maximize his revenue. In a posted price game, the principal announces a take-it-or-leave-it price for the prize. The agents accept or reject the price. Then one of the agents who accepted the price is randomly selected to receive the prize, and that agent must pay the announced price. When agents share a common valuation for the prize, there exists an equilibrium of the game in which the principal announces a price of $v$, and one or more agents accept the price.

**Lottery.** In Tullock’s (1980) influential rent-seeking game, agents submit payments to the principal, and the probability an agent wins the prize equals the ratio of her payment over the sum of total payments. In the symmetric equilibrium of the lottery model as applied to this paper’s framework, each agent submits a payment of $v/N$ and each wins the prize with probability $1/N$. There are various non-symmetric equilibrium that results in similar payoffs, including one in which $M \leq N$ agents submit payments equal to $v/M$ and each agent that submits a payment wins the prize with probability $1/M$.

**Winner-pay auction.** In the equilibrium of a first-price auction, at least two of the agents bid their valuation, and the prize again sells for $v$.

\[ \text{If only one agent bid } v, \text{ and the other players bid less than } v, \text{ then the high bidder would have an incentive to deviate and announce a lower bid. If the high bidder does not bid } v, \text{ then the other agents have an incentive to bid more than the high bid but less than their valuation.} \]
All-pay auction. One of the most common models of rent seeking is the all-pay auction in which all agents pay their bids regardless of whether they win, and the principal awards the prize to the high bidder. See for example the papers by Holt and Sherman (1982), Che and Gale (1998), and Baye et al. (1993, 1996, 1999).

One can verify the equilibria of this game for the case when agents share a common $v$. There exists mixed-strategy equilibria in which $n \in \{2, ..., N\}$ agents bid and the remaining $N - n$ agents do not submit bids. The $n$ bidders independently draw their bids from a continuous distribution with cdf $(b/v)^{1/n-1}$ along support $[0, v]$.

It is straightforward to verify that in the equilibria of each of these traditional mechanisms for awarding the prize, the principal expects to earn revenue equal to $v$, and the agents each expect to earn a payoff of zero. The principal can fully extract the expected rent of the agents, and in the equilibria, the agents are indifferent between participating and not participating. However, the principal cares about choosing the most qualified agent as well as collecting revenue. These means of awarding the prize do not take into account agent qualifications. Therefore, such a means of awarding the prize is not a first-best optimal mechanism. The principal unambiguously expect to be better off under a mechanism that maximizes revenue and awards the prize to the most qualified agents.

---

The second-price auction does not necessarily result in the principal earning $v$. There exists an equilibrium in which at least two agents bid $v$, in which case the principal does earn $v$. However, an equilibrium exists when one of the agents bids $v$, and the other agents bid less than $v$. This results in a payment of less than $v$. 

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4.5 Competition for Access

This section applies the competition for access mechanism first developed in Chapter 2 to the non-divisible prize setting considered in this chapter. Here, the principal sells access to an agent through an all-pay auction before awarding the prize to the agent he believes is most qualified. All agents submit payments to the principal, and the agent that submits the highest payment wins access. After learning the qualifications of the agent with access, the principal awards the prize to the agent he believes has the highest qualifications.

Similar to the earlier chapter, the principal will always award the prize to the agent he believes is most qualified. An agent can influence whether they receive the prize by influencing the principal’s beliefs about their qualifications. Furthermore, I consider the symmetric perfect Bayesian equilibrium, in which agents share a common contribution function $T : [0, 1] \rightarrow \mathbb{R}_+$, where $T(q_i)$ denotes the equilibrium contribution for an agent with qualifications $q_i$.

In equilibrium, all agents contribute according to the contribution function $t$. I start the analysis with the assumption that the contribution function is strictly increasing in an agent’s qualifications. After solving for $T$, I show that the function is indeed strictly increasing; furthermore, one can rule out the possibility of a non-increasing function. Since $T$ is strictly monotone, the contribution function is invertible, where $Q(t) = T^{-1}(q)$. When an agent contributes according to $T$, there is a one-to-one mapping between the agent’s contribution and its qualifications. It immediately follows that a rational principal can determine an agent’s qualifications if he observes its contribution. When an agent contributes $t$, the principal infers that the agent has qualifications $Q(t)$.,
Since $T$ is the equilibrium contribution function, when all other agents contribute according to the function, no individual agent $i$ has an incentive to deviate from $t_i = T(q_i)$. Consider the contribution decision of agent $i$. Since all other agents contribute according to the equilibrium contribution function, the principal becomes fully informed about their qualifications. Let $\Omega(q_i, t_i)$ be the ex ante probability that an agent with qualifications $q_i$ and contribution $t_i$ wins the prize when all other agents contribute according to $T$.

\[
\Omega(q_i, t_i) = \begin{cases} 
F(q_i)^{N-1} & \text{if } t_i \geq T(q_i) \\
F(Q(t_i))^{N-1} & \text{if } t_i \leq T(q_i).
\end{cases} \tag{4.2}
\]

This is because of the following. If $i$ bids more for access than she should in equilibrium and she receives access, then the politician still learns her true qualifications and she does not receive the prize unless no other agent has qualifications greater than her true qualifications. If she does not win access, then some other agent has qualifications greater than her exaggerated qualifications, and she will not win the prize no matter what she bid. Alternatively, if $i$ bids less for access than she should in equilibrium, she wins the prize if and only if she wins access. If she does not win access, then the principal believes that some other agent has higher qualifications (even if they do not).

Agent $i$’s expected payoff from bidding $t_i$ is therefore

\[
\Omega(q_i, t_i)v - t_i
\]

It is straightforward to show that an agent will never prefer to contribute more than $T(q_i)$. Doing so does not increase the probability that one wins the prize, while it does increase one’s payment to the principal. Starting from any higher
contribution, an agent has an incentive to decrease its payment to $T(q_i)$. However, for $T$ to be an equilibrium contribution function, agent $i$ must also prefer to pay $T(q_i)$ than any lower contribution. For any $t_i < T(q_i)$, expected agent payoff is

$$F(Q(t_i))^{N-1}v - t_i.$$ 

The first order conditions with respect to $t_i$ give

$$(N - 1)F(Q(t_i))^{N-2}f(Q(t_i))\frac{\partial Q}{\partial t_i}v - 1 = 0.$$ 

In equilibrium, agent $i$ contributes $T(q_i)$, which implies $(N - 1)F(q_i)^{N-2}f(q_i)\frac{\partial Q}{\partial q_i} v = 1$. Solving this expression for $\frac{\partial T}{\partial q_i}$ gives us

$$\frac{\partial T}{\partial q_i} = (N - 1)F(q_i)^{N-2}f(q_i)v.$$ (4.3)

It should be clear that $\frac{\partial T}{\partial q_i} > 0$, satisfying the initial condition that that contribution function must be strictly increasing in $q_i$. Integrating equation 4.3 gives a closed form solution for the contribution function $T$:

$$T(q_i) = F(q_i)^{N-1}v.$$ (4.4)

The above analysis derives the symmetric Perfect Bayesian Equilibrium of the game. Lemma 7 summarizes the results.

**Lemma 7** In equilibrium of the competition for access game, $t_i = F(q_i)^{N-1}$ for all $i$.

### 4.5.1 Optimality of Competition for Access

Proposition 12 establishes that competition for access is a first-best optimal mechanism for awarding the prize.
Proposition 12 (Optimality of competition for access) \textit{Competition for access is a first-best optimal mechanism for awarding the prize, in that it}

1. \textit{Awards the prize to the most qualified agent with probability 1,}

2. \textit{Maximizes expected revenue, and}

3. \textit{Provides access to no more than one agent.}

\textbf{Choosing the most-qualified agent.} In equilibrium, the competition for auction for access results in the principal awarding the prize to the most qualified agent. Because the contribution function is strictly increasing in an agent’s qualifications, the principal can infer an agent’s qualifications after observing its contribution. In equilibrium, these inferences are correct, and the principal becomes fully informed about the qualifications of all agents, even when he only gives access to the highest contributor. This allows him to confidently identify and award the prize to the most qualified candidate.

The principal provides access to the agent who announces the highest qualifications. In equilibrium, all agents truthfully announce their qualifications, so the agent who receives access also wins the prize. She wins the prize not because she has access (or because she gives the most money), but because informing the principal about her qualifications reinforces the principal’s beliefs that she is the most qualified agent.

Because competition for access results in the prize going to the most qualified candidate, the mechanism maximizes the portion of the politician’s utility function that depends on the prize winner’s qualifications, \( \sum_{j=1}^{N} \omega_j q_j \). This portion of his utility function cannot be higher than it is under competition for access.
Furthermore, since the mechanism only awards access to one agent, it minimizes the costs associated with providing access, $c(m) = 0$.

**Maximizing revenue.** It is straightforward to show that competition for access maximizes the principal’s expected revenue. From any agent, the principal expects to collect revenue equal to $\int_0^1 f(q_i) F(q_i)^{N-1} v dq_i$. Total expected revenue is $N$ times this amount. Therefore, total expected revenue equals

$$N \int_0^1 f(q_i) F(q_i)^{N-1} v dq_i = v N \left[ \frac{1}{N} F(q_i)^N \right]_0^1 = v.$$

As Lemma 6 established, there does not exist a means of allocating the prize that results in higher expected revenue for the principal.

### 4.5.2 Efficiency of Competition for Access

In addition to competition for access being optimal for the politician, it is also an efficient mechanism. One cannot improve the expected payoffs to an interest group without making the politician or other interest groups worse off.

**Proposition 13 (Efficiency of competition for access)** Competition for access mechanism is an efficient means of awarding the prize.

### 4.6 Access Fees

There are a variety of ways in which the principal can award access and the prize to an agent. Where in the previous section the principal awards access to the highest bidder, in this section he awards access to any interest group that pays some fixed
access fee, $\bar{T}$. Such an means of awarding access was proposed by Austen-Smith (1998) and by the model in Chapter 3. With an access fee mechanism, just as with competition for access, the principal awards the prize to the agent he believes is most qualified at the end of the game.

Suppose that the principal sets the access fee equal to $T \geq 0$. I assume that $\bar{T} \leq v \frac{N-1}{N}$, since any greater fee results in no agent ever being willing to pay the fee. Given any access fee $T$, there exists an qualification cut-off value $\bar{q}(T)$ such that any agent with $q_i > \bar{q}$ pays the access fee, and any agent with $q_i \leq \bar{q}$ does not pay the fee. I explain why such a cut-off value exists below.

If agent $i$ pays the fee, she wins the prize if no other agent has higher qualifications. Her expected utility from paying the fee is $EU_i = F(q_i)^{N-1}v - \bar{T}$. If she does not pay the fee, then she can win the prize only if all other agents also do not buy access. When no agent buys access, each agent wins the prize with probability $\frac{1}{N}$. Her expected utility from not paying the fee is $EU_i = F(\bar{q}(\bar{T}))^{N-1}\frac{v}{N}$.

Agent $i$ prefers to pay $\bar{T}$ than to not pay $\bar{T}$ if

$$F(q_i)^{N-1}v - \bar{T} > F(\bar{q}(\bar{T}))^{N-1}\frac{v}{N}.$$  

The benefit of paying the fee is therefore $F(q_i)^{N-1}v - F(\bar{q}(\bar{T}))^{N-1}\frac{v}{N} - \bar{T}$, which is strictly increasing in $q_i$. So long as $\bar{T} \leq v \frac{N-1}{N}$, there exists some cut off value $\bar{q} \in [0, 1]$ such that an agent buys access only if $q_i > \bar{q}$.

An agent with qualifications equal to the cut-off value is indifferent between buying access and not buying access. Therefore, $\bar{q}$ solves

$$F(\bar{q}(\bar{T}))^{N-1}v - \bar{T} = F(\bar{q}(\bar{T}))^{N-1}\frac{v}{N}.$$  

This can be further simplified, the results of which is given by Lemma 8.
Lemma 8 In the equilibrium of the access fee game with fee $\bar{T}$, for each $i$,

$$t_i = \begin{cases} \bar{T} & \text{if } q_i > \bar{q} \\ 0 & \text{if } q_i \leq \bar{q}. \end{cases}$$

where $\bar{q}$ solves

$$F(\bar{q})^{N-1} = \frac{\bar{T} N}{vN - 1}. \quad (4.5)$$

Proposition 14 establishes that selling access at a fixed fee is not an optimal mechanism.

Proposition 14 The access fee mechanism is not an optimal mechanism for awarding the prize. If the principal allocates access and the prize through an access fee mechanism, then

1. for any $\bar{T}$, total expected revenue is strictly less than $v$, and
2. for any $\bar{T} > 0$, the prize is allocated to the most qualified agent with probability less than 1.

Awarding the prize through a competition for access maximizes utility and results in the prize always being allocated to the most qualified agent. Therefore, if the principal sells access at any fixed access fee, he expects to be strictly worse off than if he had allocated access and the prize through a competition for access mechanism.

Notice that awarding access for free is a special case of setting a fixed access fee, for which $\bar{T} = 0$. In such a case, the principal gives access to all agents and can award the prize to the most qualified agent. However, granting access for free results in both (i) no expected revenue, and (ii) the maximum costs $c(N)$ incurred
from granting access to all $N$ agents. Because of the costs involved with granting access, even if the principal does not care about revenue, he is worse off than if he awarded access through competition for access, which results in him awarding the prize to the most qualified agent, while only giving access to a single agent.

4.7 Discussion

This chapter considers a framework in which a principal must award a non-divisible prize to an agent. The principal cares about the qualifications of the agent that wins the prize; however, learning an agent’s qualifications is costly. If the principal directly sells the prize through a traditional prize-allocation mechanism such as an auction or posted-price game, he will maximize his revenue, but he will not necessarily award the prize to the most-qualified agent.

In this framework, however, the principal controls which agents receive access. This means that the principal can choose to sell access to the agents. He may do so in a variety of ways, including giving access to the highest bidder in an auction, or setting a fixed price for access (i.e., and access fee) and awarding access to any agent that pays the set fee. I show that awarding access to the highest bidder, then awarding the prize to the agent he expects is most qualified, maximizes the principal’s expected utility. Awarding access and the prize in such a way maximizes expected revenue, gives access to only one agent, and awards the prize to the most-qualified agent with probability 1.

Alternatively, the principal may award access using an access fee. I show that doing so results in lower expected revenue for the principal. Furthermore, it either results in awarding access to all agents (which is more costly to the principal than
only awarding access to one agent), or results in not awarding the prize to the most-qualified agent with positive probability. Therefore, the principal expects to be better off if he allocates access and the prize through a competition for access than using access fees. Competition for access also dominates traditional mechanisms through which the principal ignores access and explicitly sells the prize to an agent.

The analysis depends on the assumption that agents share a common valuation for the prize. If agents have different valuations of the prize, then competition for access does not result in maximum revenue. For example, suppose one agent values the prize at $2v$ and all other agents value the prize at $v$. To maximize revenue, the principal may award the prize through a posted-price game in which he commits to a price of $2v$ for the prize, and in equilibrium the high-valuation buys the prize. Such a means of awarding the prize results in the prize going to the agent with the highest valuation, regardless of whether that agent is most qualified. The competition for access game may be adapted to the case of different valuations, such that the mechanism still awards the prize to the most qualified agent. However, it will not longer result in maximum revenue. Therefore, it is not a first-best optimal mechanism so long as $\delta > 0$, and it may not even be preferred by the principal if he cares enough about revenue (e.g., if he has a large-enough $\delta$). If the principal cares enough about the qualifications of the prize winner rather than revenue (e.g., for low-enough $\delta$), he will still prefer awarding the prize following a competition for access than through one of the alternative mechanisms studied in this paper.

The analysis shows that the principal prefers to sell access through a competition for access, rather than sell access using access fees or explicitly sell the prize.
This may be evidence in favor of the competition for access story, but it does not prove that the politicians sell interest groups access in such a way. The other prize-allocation mechanism may have other advantages that are not addressed by this analysis. For example, setting implicit access fees may be more feasible for a politician than selling access through an auction. Furthermore, if agents differ in terms of their valuation, or when there is a continuous policy space, the principal may prefer to explicitly sell the prize than to sell access. Therefore, I cannot theoretically rule out these other means of awarding the prize as empirical possibilities. Future work may attempt to collect data on politician time use, and test how politicians sell access. As Chapters 2 and 3 show, the means by which a politician sells access can substantially alter the welfare implications of campaign finance reform.
APPENDIX A
APPENDIX FOR CHAPTER 2

A.1 Contribution Equilibrium with Limits

Here, I derive the details regarding the contribution equilibrium with limit \( \bar{b} < B(1) \), where \( B(1) \) is the contribution of an interest group with the highest-possible quality evidence in the equilibrium without a limit. Let \( B_{CL}(e;\bar{b}) \) describe the equilibrium contribution of an interest group with evidence quality \( e \), when there is a contribution limit \( \bar{b} \). Since \( \bar{b} < B(1) \), a group with high-enough \( e \) prefers to contribute more than \( \bar{b} \) but cannot do so. Instead, it provides the maximum contribution \( \bar{b} \). There exists a cut off value \( \bar{e}(\bar{b}) \in [0,1) \) such that for any \( e \in [\bar{e}(\bar{b}),1] \) an interest group provides contribution \( \bar{b} \) in equilibrium. For any \( e \in [0,\bar{e}(\bar{b})] \), an interest group’s contribution is strictly increasing in \( e \). There exists a function \( \tilde{B} \), where \( \tilde{B}(0;\bar{b}) = 0 \) and \( \frac{\partial \tilde{B}}{\partial e} > 0 \), such that in the contribution equilibrium with limit \( \bar{b} \)

\[
B_{CL}(e;\bar{b}) = \begin{cases} 
\tilde{B}(e;\bar{b}) & \text{for } e \in [0,\bar{e}(\bar{b})] \\
\bar{b} & \text{for } e \in [\bar{e}(\bar{b}),1] 
\end{cases} \tag{A.1}
\]

For the rest of the analysis, I assume \( \bar{b} \) is fixed, and ignore its value when writing the functions. So, \( B_{CL}(e;\bar{b}) = B_{CL}(e) \) and \( \tilde{B}(e;\bar{b}) = \tilde{B}(e) \). Let \( \bar{e}(b) = \tilde{B}^{-1}(e) \).

In equilibrium, for every issue the politician chooses the policy he expects will maximize citizen welfare, or \( p^*_n = E_{e} e^1_n - E_{e} e^{-1}_n \) for every \( n \). A complete description of the equilibrium must also describe the politician’s beliefs \( \mu \). In equilibrium, if interest group \((n,j)\) receives access, then \( E_{\mu} e^j_n = e^j_n \). If \((n,j)\) does not receive access and provides contribution \( \bar{b} \), then \( E_{\mu} e^j_n = \frac{1}{1-F(\bar{e}(\bar{b}))} \int_{\bar{e}(\bar{b})}^{1} f(y) y dy \). If \((n,j)\) does not receive access and provides contribution \( b^j_n < \bar{b} \), then \( E_{\mu} e^j_n = \bar{e} \left( b^j_n \right) \).

\(^1\)Such a belief system is required by the concept of Perfect Bayesian Equilibrium for all values of \( b \) that are on the path of play for any \( e \in [0,1] \) (Fudenberg and Tirole,1991, pp. 324-326). The concept of Perfect Bayesian Equilibrium does not restrict beliefs off
The function $\tilde{B}$ is derived just as $B$ was derived in the no-limit game. Interest group $(n,j)$ chooses a contribution $b$ to maximize the expression

$$f_0^1 f_{CL} \left( e_n^{-j} \right) \left[ (1 - \Theta_{CL} \left( \tilde{e} (b); e_n^{-j} \right)) V \left( 1 - \tilde{e} (b) + e_n^{-j} \right) + \Theta_{CL} \left( \tilde{e} (b); e_n^{-j} \right) V \left( 1 - \tilde{e}_n^{-j} + e_n^{-j} \right) \right] de_n^{-j} - b. \quad (A.2)$$

This differs from a group’s no-limit maximization problem in that $b$ must now be on the interval $[0, \tilde{b}]$, the functions $\Theta_{CL}$ and $f_{CL}$ take into account the fact that all other interest groups with $e \geq \tilde{e} (\tilde{b})$ provide the same contribution $\tilde{b}$. The function $\Theta_{CL} (e; e^{-j})$ defines the probability that $(n,j)$ is granted access in equilibrium given that it contributes $b$. I leave the formal derivation of $\Theta_{CL}$ to the reader, as it is not required for the analysis. Note that $\Theta_{CL} (e; e^{-j})$ is increasing in $e$ for all $e \leq \tilde{e}$. In equilibrium, all groups with $e \in [\tilde{e}, 1]$ have the same ex ante probability of being offered access. I denote this probability by $\bar{\Theta}_{CL}$ when nothing is assumed about the evidence quality of another interest group, and by $\bar{\Theta}_{CL} (e^{-j})$ when one other group has evidence quality $e^{-j}$. The value $f_{CL} (e)$ denotes the ex ante probability that $E_{\mu e_n^{-j}} = e$. Therefore,

$$f_{CL} (e) = \begin{cases} f (e) & \text{for } e \in [0, \tilde{e}) \\ \Theta_{CL} f (e) & \text{for } e \in [\tilde{e}, 1] \cap e \neq \int_0^1 f (y) y dy \\ (1 - \tilde{\Theta}_{CL}) \int_{\tilde{e}}^1 f (y) dy & \text{for } e = \int_0^1 f (y) y dy. \end{cases} \quad (A.3)$$

Using the interest group’s above maximization problem, one can solve for $\tilde{B}$ using the technique from Section 3. It follows that

$$\tilde{B} (e) = - \int_0^e \int_0^1 f_{CL} \left( e_n^{-j} \right) \left( 1 - \Theta_{CL} (y; e_n^{-j}) \right) V' (1 - y + e_n^{-j}) de_n^{-j} dy. \quad (A.4)$$

The cut-off evidence quality $\tilde{e} (\tilde{b})$ is the evidence quality at which an interest group is indifferent between contributing according to $\tilde{B}$ and contributing $\tilde{b}$, and solves the
following equation for $\bar{e}$

$$
\int_0^1 f_{CL} (e_n^{-j}) V \left( 1 - \bar{e} + e_n^{-j} \right) de_n^{-j} - \tilde{B} (\bar{e})
= \int_0^1 f_{CL} (e_n^{-j}) \left[ \left( 1 - \tilde{\Theta}_{CL} (e_n^{-j}) \right) V \left( 1 - \frac{1}{1 - F(\bar{e})} \int_{\bar{e}}^1 f(y) \, dy + e_n^{-j} \right) \right]
+ \tilde{\Theta}_{CL} (e_n^{-j}) V \left( 1 - \bar{e} + e_n^{-j} \right) de_n^{-j} - \tilde{b} \tag{A.5}
$$

For an interest group with evidence quality $\bar{e}$, the left hand side of the equality denotes the group’s expected utility from providing a contribution according to the increasing equilibrium function $\tilde{B}$, and the right hand side of the equality denotes the expected utility from providing the maximum contribution. It follows that $\tilde{B} (\bar{e} (\bar{b})) < \bar{b}$. In equilibrium, the group with the cut-off evidence quality is indifferent between the two actions. If the solution to this equality is negative, then $\bar{e} (\bar{b}) = 0$ and all interest groups contribute $\bar{b}$ independent of their evidence qualities.

Next, I show that interest groups that do not have an incentive to deviate from the contribution function $B_{CL}$. For an interest group with $e < \bar{e} (\bar{b})$, providing $\bar{b}$ rather than $b < \bar{b}$ gives an expected benefit of

$$
\int_0^1 f_{CL} (e_n^{-j}) \left[ \left( 1 - \tilde{\Theta}_{CL} (e_n^{-j}) \right) V \left( 1 - \frac{1}{1 - F(\bar{e})} \int_{\bar{e}}^1 f(y) \, dy + e_n^{-j} \right) \right]
+ \tilde{\Theta}_{CL} (e_n^{-j}) V \left( 1 - e + e_n^{-j} \right) de_n^{-j} - \tilde{b} - \int_0^1 f_{CL} (e_n^{-j}) V \left( 1 - e + e_n^{-j} \right) de_n^{-j} + \tilde{B} (e) \tag{A.6}
$$

which is strictly increasing in an interest groups evidence quality. To see this, take the derivative of the benefit with respect to $e$.

$$
\int_0^1 f_{CL} (e_n^{-j}) (1 - \Theta_{CL} (e_n^{-j})) V' (1 - e + e_n^{-j}) de_n^{-j} + \frac{\partial \tilde{B} (e)}{\partial e} \tag{A.7}
$$

Since $\tilde{\Theta}_{CL} (e_n^{-j}) - \Theta_{CL} (e; e_n^{-j})$ for any $e < \bar{e} (\bar{b})$, simplifying gives

$$
- \int_0^1 f_{CL} (e_n^{-j}) \left[ \Theta_{CL} (e_n^{-j}) - \Theta_{CL} (e; e_n^{-j}) \right] V' (1 - e + e_n^{-j}) de_n^{-j} > 0. \tag{A.8}
$$

From the derivation of $\tilde{B}$, one already knows that an interest group with $e < \bar{e} (\bar{b})$ prefers to provide $\tilde{B} (e)$ rather than any other $b < \bar{b}$.
From the derivation of $\tilde{B}$, it also follows that any group with $e \geq \tilde{e}(\bar{b})$ prefers to provide $b \rightarrow \bar{b}$ instead of any other $b < \bar{b}$. The expected benefit to an interest group with $e > \tilde{e}(\bar{b})$ from giving $\bar{b}$ instead of $b < \bar{b}$ is

$$\lim_{b \rightarrow \bar{b}} \int_{0}^{1} f_{CL} \left( e_{-j}^{-j} \right) \left[ \left( 1 - \Theta_{CL} \left( e_{-j}^{-j} \right) \right) V \left( 1 - \frac{1}{1 - F(e)} \int_{0}^{1} f(y) \, dy + e_{-j}^{-j} \right) + \Theta_{CL} \left( e_{-j}^{-j} \right) V \left( 1 - e + e_{-j}^{-j} \right) \right] \, de_{-j}^{-j} - \bar{b}$$

which is strictly increasing in the group’s $e$. To see this, take the derivative with respect to $e$:

$$\lim_{b \rightarrow \bar{b}} - \int_{0}^{1} f_{CL} \left( e_{-j}^{-j} \right) \left[ \Theta_{CL} \left( e_{-j}^{-j} \right) - \Theta_{CL} \left( \tilde{e}(\bar{b}) ; e_{-j}^{-j} \right) \right] V' \left( 1 - e + e_{-j}^{-j} \right) \, de_{-j}^{-j} > 0. \quad (A.10)$$

Since the benefit of providing $\bar{b}$ rather than $b < \bar{b}$ is strictly increasing in a group’s evidence quality, and $\tilde{e}(\bar{b})$ is the evidence quality at which a group is indifferent between the two actions, it follows that interest groups with $e < \tilde{e}(\bar{b})$ strictly prefer to contribute according to $\tilde{B}$, and groups with $e > \tilde{e}(\bar{b})$ strictly prefer to contribute $\bar{b}$.

### A.2 Contribution Equilibrium with Wealth Differences

In this section, I derive the contribution equilibrium of the model presented in Section 2.5.2. Poor interest groups have wealth constraint such that their contributions must be no greater than $\omega$, and rich group contributions are unconstrained. Assume $\omega < B(1)$. A poor group with high-enough $e$ prefers to contribute more than $\omega$ but cannot do so. Instead, it provides $\omega$.

The functions $B_P$ and $B_R$ respectively denote the equilibrium poor and rich group contribution functions. There exists cut off values $\bar{e}_a \geq 0$ and $\bar{e}_b \in (\bar{e}_a, 1)$, and functions
$B_a$ and $B_b$, where $B_a (0; \omega) = 0$, $B_b (\bar{e}_b; \omega) = 0$, $\frac{\partial B_a}{\partial e} > 0$, and $\frac{\partial B_b}{\partial e} > 0$ such that

\[
B_P (e; \omega) = \begin{cases} 
B_a (e; \omega) & \text{for } e \in [0, \bar{e}_b), \\
\omega & \text{for } e \in [\bar{e}_a, 1), \text{ and} \\
B_a (e; \omega) & \text{for } e \in [0, \bar{e}_a), \\
\omega & \text{for } e \in [\bar{e}_a, \bar{e}_b) \\
\omega + B_b (e; \omega) & \text{for } e \in [\bar{e}_b, 1]. 
\end{cases} \tag{A.11}
\]

(A.11)

The cut off values are dependent on poor group wealth $\omega$, and the probability that a group is poor, $\alpha$. Also, through the rest of the analysis, I take $\omega$ as fixed and ignore it in writing the contribution functions $B_P (e)$, $B_R (e)$, $B_a (e)$, and $B_b (e)$. Let $e_a (b) = B_a^{-1} (e)$ and $e_b (b) = B_b^{-1} (e)$.

In equilibrium, the politician chooses the policy profile he expects will maximize citizen welfare, or $p^*_n = E \mu e_n^1 - E \mu e_n^{-1}$ for every $n$. A complete description of the equilibrium must also describe the politician’s beliefs $\mu$. In equilibrium, if interest group $(n,j)$ receives access, then $E \mu e_n^j = e_n^j$. If $(n,j)$ does not receive access and provides contribution $\omega$, then

\[
E \mu e_n^j = E (e | \omega) = \frac{(1 - \alpha) \int_{\bar{e}_a}^{1} f (y) y dy + \alpha \int_{\bar{e}_a}^{\bar{e}_b} f (y) y dy}{(1 - \alpha) \left(1 - F (\bar{e}_a)\right) + \alpha \left(F (\bar{e}_b) - F (\bar{e}_a)\right)}. \tag{A.12}
\]

If $(n,j)$ does not receive access and provides contribution then $E \mu e_n^j = e_a \left(b_n^j\right)$ if $b_n^j < \omega$, and $E \mu e_n^j = e_b \left(b_n^j\right)$ if $b_n^j > \omega$.\(^2\)

The function $B_a$ is derived just as $B$ was derived in the no-limit game. Interest group $(n,j)$ chooses a contribution $b$ to maximize

\[
\int_{0}^{1} f_{WL} \left(e_n^{-j}\right) \left[ \left(1 - \Theta_{WL} \left(e_a (b); e_n^{-j}\right)\right) V \left(1 - e_a (b) + e_n^{-j}\right) \\
+ \Theta_{WL} \left(e_a (b); e_n^{-j}\right) V \left(1 - e_n + e_n^{-j}\right) \right] d \omega_n^{-j} - b. \tag{A.13}
\]

\(^2\)Such a belief system is required by the concept of Perfect Bayesian Equilibrium for all values of $b$ that are on the path of play for any $e \in [0, 1]$. For those $b$ that are never played in equilibrium, such assumptions regarding the politician beliefs guarantees that a pure strategy equilibrium exists; other less restrictive beliefs produce the same results.
This differs from the maximization problem in Section 2 since \( \Theta_{WL} \) and \( f_{WL} \) take into account the fact that all other interest groups contribute according to the contribution function described above. The value \( \Theta_{WL} (e(b); e_n^{-j}) \) defines the probability that \((n, j)\) is granted access in equilibrium given that it contributes \(b\). In equilibrium, all groups that provide contribution \(\omega\) have the same ex ante probability of being offered access. I denote this probability by \(\bar{\Theta}_{WL}\) when nothing is assumed about the evidence quality of another interest group, and by \(\bar{\Theta}_{WL} (e^{-j})\) when one other group has evidence quality \(e^{-j}\). The value \(f_{WL}(e)\) denotes the ex ante probability that \(E_{\mu}e_n^{-j} = e\).

Using the interest group’s above maximization problem, one can solve for \(\tilde{B}\) using the technique from Section 2. It follows that

\[
B_a(e) = -\int_0^e \int_0^1 f_{WL}(e_n^{-j}) (1 - \Theta_{WL}(y; e_n^{-j})) V' (1 - y + e_n^{-j}) \, de_n^{-j} \, dy. \quad (A.14)
\]

Similarly, \(B_b\) is derived from the maximization problem of an interest group with \(e > \bar{e}_b\).

\[
B_b(e) = -\int_{\bar{e}_b}^e \int_0^1 f_{WL}(e_n^{-j}) (1 - \Theta_{WL}(y; e_n^{-j})) V' (1 - y + e_n^{-j}) \, de_n^{-j} \, dy. \quad (A.15)
\]

The cut-off evidence quality \(\bar{e}_a\) is the evidence quality at which an interest group is indifferent between contributing according to \(B_a\) and contributing \(\omega\). The equilibrium value of \(\bar{e}_a\) solves the following equation for \(\bar{e}\)

\[
f_{WL}(e_n^{-j}) V \left(1 - \bar{e} + e_n^{-j}\right) \, de_n^{-j} - B_a(\bar{e})
= f_{WL}(e_n^{-j}) \left[ \left(1 - \bar{\Theta}_{WL}(e_n^{-j})\right) V \left(1 - e_{WL}(\omega) + e_n^{-j}\right) + \bar{\Theta}_{WL}(e_n^{-j}) V \left(1 - \bar{e} + e_n^{-j}\right) \right] \, de_n^{-j} - \omega \quad (A.16)
\]

where \(e_{WL}(\omega)\) denotes the politicians equilibrium expectations about the evidence quality of an interest group that provides \(\omega\) and does not receive access. If the solution to the equality is negative, then \(\bar{e}_a = 0\) and all poor interest groups contribute \(\omega\) independent of their evidence qualities.
The cut-off evidence quality \( \bar{e}_b \) is the evidence quality at which an interest group is indifferent between contributing according to \( B_b \) and contributing \( \omega \). In equilibrium,

\[
\bar{e}_b = E(e | \omega) = \frac{(1 - \alpha) \int_{\bar{e}_a}^{1} f(y) y dy + \alpha \int_{\bar{e}_a}^{\bar{e}_b} f(y) y dy}{(1 - \alpha) (1 - F(\bar{e}_a)) + \alpha (F(\bar{e}_b) - F(\bar{e}_a))}.
\] (A.17)

Just as I do in the previous section of the appendix where I derive the equilibrium for the case with contribution limits, it is straightforward to show that none of the interest groups have an incentive to deviate in the equilibrium. Groups with evidence quality \( e > \bar{e}_b \) strictly prefer to contribute \( \omega + B_b(e) \) compared to \( \omega \), and \( \omega \) compared to any amount less than \( \omega \). Groups with \( e > \bar{e}_a \) strictly prefer to provide \( \omega \) than any amount less than \( \omega \), and those with \( e \in (\bar{e}_a, \bar{e}_b) \) strictly prefer to provide \( \omega \) compared with any amount greater than or less than \( \omega \). Similarly, groups with \( e < \bar{e}_a \) strictly prefer to provide \( B_a(e) \) to any other contribution.

**With Contribution Limits**

Using the techniques developed earlier in this paper, one can incorporate contribution limits into this game. I describe the equilibrium contribution function here, but leave the details of the derivation to the reader.

The paper focuses on the case where \( \bar{b} \leq \omega \), which is consistent with the contribution limits proposed in the policy debate. In this case, the contribution limit has the same impact on the equilibrium contribution functions as the limit would have in the game without wealth differences. See Sections 2.3 and the first section of the appendix for a derivation and illustration.

Although the body of the paper does not discuss the other cases, I describe them here. When \( \bar{b} > \omega \), there are two cases. First, if the limit is high-enough, then the contribution limit impacts function \( B_b \) in the same manner that the limit impacted \( B \) in the case without wealth differences. In this case, rich interest groups with \( e > E(e | \omega) = \bar{e}_b \) contribute \( \omega + B_b(e) \) for relatively low \( e \) (as \( e \to \bar{e}_b \)) and contribute \( \bar{b} \) for higher \( e \) (as \( e \to 1 \)). The contributions of the poor interest groups are not impacted. This case is
Figure A.1: More example contribution functions with unknown wealth differences

illustrated in Figure A.1. Second, if $\bar{b} > \omega$ and the limit is relatively close to $\omega$, then there exists a function $\tilde{B}_a$ that is strictly increasing in $e$ and where $\tilde{B}(0) = 0$, and values $\bar{e}, \bar{e}_b \in (0, 1)$ such that when $(n, j)$ is rich, $b_n^j = \tilde{B}\left(e_n^j\right)$ for $e_n^j \in [0, \bar{e})$, $b_n^j = \omega$ for $e_n^j \in [\bar{e}, \bar{e}_b)$, and $b_n^j = \bar{b}$ for $e_n^j \in [\bar{e}_b, 1]$, and when $(n, j)$ is poor, $b_n^j = \tilde{B}\left(e_n^j\right)$ for $e_n^j \in [0, \bar{e}_a)$, $b_n^j = \omega$ for $e_n^j \in [\bar{e}_a, \bar{e}_b)$, and $b_n^j = \bar{b}$ for $e_n^j \in [\bar{e}_b, 1]$. It can be shown that $\bar{e} < \bar{e}_b < \bar{e}_b < E(e | \omega)$, where $\bar{e}_a$ and $\bar{e}_b$ are the cutoff values when there is no contribution limit, and $E(e | \omega)$ is the politician’s expectations about an interest group’s evidence quality when that group contributes $\omega$ but does not receive access. This case is illustrated by Figure A.2.

**Impact of limits on welfare in a simple game**

Here, I solve the simple example from Section 2.5.2. Evidence quality is uniformly distributed on $[0, 1]$, interest group policy utility is linear, or $V\left(1-e^{-e}+e^{-j}\right) = -\left(1-e^{-e}+e^{-j}\right)$, and both $N = 1$ and $K = 1$. Using the methods described previously, it is straightforward to solve for the game’s equilibrium.

For the case when there is no contribution limit. First, one can find that $B_a(e) = e - \frac{1}{2}e^2$, and calculate the expected payoff to an interest group from contributing less
than \( \omega \), which equals \( \frac{1}{2}e^2 - \frac{3}{2} \). Note that \( \omega \) must be less than \( \frac{1}{2} \) to have an impact on contributions. If the same group contributed \( \omega \) instead, its expected payoff equals

\[
- \int_{\bar{e}_b}^{\bar{e}_a} ((1 - \alpha)(1 - \bar{e}_b + e') + \alpha \left( \frac{1}{2}(1 - e + \bar{e}_b) + \frac{1}{2}(1 - \bar{e}_b + e') \right)) \, de' - \int_{\bar{e}_a}^{\bar{e}_b} \left( \frac{1}{2}(1 - e + \bar{e}_b) + \frac{1}{2}(1 - \bar{e}_b + e') \right) \, de' - \int_{0}^{\bar{e}_a} (1 - e + e') \, de' - \omega. \tag{A.18}
\]

This simplifies to

\[
- \frac{3}{2} - \frac{1}{4} \bar{e}_a^2 + (1 - \alpha) \left( \bar{e}_b - \frac{3}{4} \bar{e}_b^2 \right) + \frac{1}{4} \alpha + \frac{1}{2} e (\bar{e}_a + \alpha + \bar{e}_b (1 - \alpha)) - \omega. \tag{A.19}
\]

Because \( \bar{e}_b = E_\mu (e \mid \omega) \) it follows that \( \bar{e}_b = \frac{1 + \bar{e}_a}{1 + \alpha} \). When an interest group has \( e = \bar{e}_a \), these expected payoffs from contributing \( \omega \) equals the expected payoff from giving \( B_a (e) \).

Therefore, one can solve for \( \bar{e}_a \), where

\[
\bar{e}_a = 1 - \frac{(1 + \alpha) \sqrt{2(1 - 2\omega)}}{\sqrt{2 - \alpha + 3\alpha^2}} \tag{A.20}
\]

when \( 1 - (1 + \alpha) \sqrt{2(1 - 2\omega)} / \sqrt{2 - \alpha + 3\alpha^2} \geq 0 \). If the expression is negative, \( \bar{e}_a = 0 \).

Solving for \( B_b (e) \) is not required for the results.

It holds that \( \tilde{B} (e) = B_a (e) \) when there is no contribution limit.\(^3\) Since rich and poor groups give \( b \) for the same range of \( e \), it follows that \( \bar{e}_b' = \frac{1 + \bar{e}}{2} \), where \( \bar{e}_b' = E (e \mid \bar{b}) \).

\(^3\)This is true because \( V \) is linear.
Solving for \( \bar{e} \) in the same way that \( \bar{e}_a \) was found above gives

\[
\bar{e} = 1 - \frac{2\sqrt{1-b}}{\sqrt{3}} \tag{A.21}
\]

when \( 1 - \left(2\sqrt{1-b}\right)/\sqrt{3} \geq 0 \), and \( \bar{e} = 0 \) otherwise.

When there is no contribution limit, expected citizen welfare is

\[
\begin{align*}
-\alpha^2 \int_{\bar{e}_a}^1 \int_{\bar{e}_a}^1 |e_1 - \bar{e}_b| \, de_1 \, de_2 \\
-2\alpha \left(1 - \alpha\right) \int_{\bar{e}_a}^1 \int_{\bar{e}_a}^1 \left( \frac{1}{2} |e_1 - \bar{e}_b| + \frac{1}{2} (|\bar{e}_b - e_2|) \right) \, de_1 \, de_2 \\
-2\alpha \left(1 - \alpha\right) \int_{\bar{e}_b}^1 \int_{\bar{e}_a}^1 |e_1 - \bar{e}_b| \, de_1 \, de_2 \\
-\left(1 - \alpha\right)^2 \int_{\bar{e}_a}^1 \left( \int_{\bar{e}_a}^1 (\bar{e}_b - e_1) \, de_1 + \int_{\bar{e}_b}^1 (\bar{e}_b - e_2) \, de_1 \right) \, de_2 \\
-\left(1 - \alpha\right)^2 \int_{\bar{e}_b}^1 \int_{\bar{e}_a}^1 (\bar{e}_b - e_1) \, de_1 \, de_2.
\end{align*}
\] (A.22)

Substituting in for \( \bar{e}_b \) and \( \bar{e}_a \), when \( 1 - (1 + \alpha) \sqrt{2(1 - 2\omega)}/\sqrt{2 - \alpha + 3\alpha^2} > 0 \) expected citizen welfare simplifies to

\[
EW_{\text{no limit}} = -\frac{2\alpha \left(1 - 2\omega\right) \left(1 + \alpha\right) \sqrt{2(2 - \alpha + 3\alpha^2)(1 - 2\omega)}}{\left(2 - \alpha + 3\alpha^2\right)^2}. \tag{A.23}
\]

If \( 1 - (1 + \alpha) \sqrt{2(1 - 2\omega)}/\sqrt{2 - \alpha + 3\alpha^2} \leq 0 \), then \( EW_{\text{no limit}} = -\frac{\alpha}{(1+\alpha)^3} \).

When there is a contribution limit, expected citizen welfare is

\[
-\int_{\bar{e}}^1 \int_{\bar{e}}^1 |e_1 - \bar{e}_b| \, de_1 \, de_2.
\]

When \( 1 - \left(2\sqrt{1-b}\right)/\sqrt{3} > 0 \),

\[
EW_{\text{limit}} = -\frac{2(1 - \omega)^{2/3}}{3\sqrt{3}}. \tag{A.24}
\]

If \( 1 - \left(2\sqrt{1-b}\right)/\sqrt{3} \leq 0 \), then \( EW_{\text{limit}} = -\frac{1}{4} \).

When both \( \bar{e}_a \) and \( \bar{e} \) are 0, the contribution limit never improves expected citizen welfare since \( EW_{\text{no limit}} = -\frac{\alpha}{(1+\alpha)^3} > EW_{\text{limit}} = -\frac{1}{4} \) for \( \alpha \in (0, 1) \). Similarly, one can show that when \( \bar{e}_a > 0 \) and \( \bar{e} = 0 \), the contribution limit cannot improve welfare since

\[
-\frac{2\alpha(1-2\omega)(1+\alpha)\sqrt{2(2-\alpha+3\alpha^2)(1-2\omega)}}{(2-\alpha+3\alpha^2)^2} > -\frac{1}{4}.
\]

There are no parameter values for which \( \bar{e}_a = 0 \) and \( \bar{e} > 0 \).

When \( 1 - (1 + \alpha) \sqrt{2(1 - 2\omega)}/\sqrt{2 - \alpha + 3\alpha^2} > 0 \) and \( 1 - \left(2\sqrt{1-b}\right)/\sqrt{3} > 0 \), both \( \bar{e}_a \) and \( \bar{e} \) are positive. In this case the contribution limit improves expected citizen welfare
when
\[- \frac{2(1-\omega)^{2/3}}{3\sqrt{3}} > - \frac{2\alpha (1-2\omega)(1+\alpha)\sqrt{2(2-\alpha+3\alpha^2)(1-2\omega)} }{(2-\alpha+3\alpha^2)^2}\] (A.25)

Using Mathematica, I solve for the numerical values for which all three of these conditions as well as the priors on \( \alpha, \omega, \) and \( \bar{b} \) are simultaneously met. There exists cut off values \( \alpha', \omega_L'(\alpha), \) and \( \omega_H'(\alpha) \) such that contribution limit \( \bar{b} \leq \omega \) improves expected citizen welfare if and only if \( \alpha \in (\alpha',1), \omega \in [\omega_L'(\alpha),\omega_H'(\alpha)] \), and \( \bar{b} \in [\frac{1}{4},\omega] \). The values \( \omega_L'(\alpha) \) and \( \omega_H'(\alpha) \) depend on the value of parameter \( \alpha \), and determining the values \( \alpha' \) and \( \omega_H'(\alpha) \) require the calculating the root of high-degree polynomials for which a non-numeric solution is not possible. To overcome this issue, I use Mathematica to numerically determine the cut off values. \( \omega_H'(\alpha) \) is greater than \( \omega_L'(\alpha) \) only when \( \alpha \) is high enough, implying that \( \alpha' \) is approximately 0.750427. This means that most interest groups must be poor in order for a contribution limit to potentially improve expected welfare. Additionally, given any \( \alpha \), the range of \( \omega \) for which a limit can have a positive impact is even more restrictive. For any \( \alpha > \alpha' \), \( \omega_L'(\alpha) \) takes on values between \( \frac{1}{4} \) and 0.260199, and \( \omega_H'(\alpha) \) takes on values between \( \frac{1}{4} \) and 0.260751. At its maximum, the difference between \( \omega_H'(\alpha) \) and \( \omega_L'(\alpha) \) is approximately 0.00215, meaning that for any value \( \alpha \), only very specific values of \( \omega \) result in the contribution limit being beneficial. Furthermore, because \( \omega_H'(\alpha) \) is close to \( \frac{1}{4} \), even when the other conditions are met, there only exists a small range of contribution limits that benefit society. Whenever the above conditions do not hold, the contribution limit strictly reduces expected citizen welfare.

A.3 Proofs

Proof (Lemma 1). Follows immediately from analysis in paper. □

Proof (Proposition 1). Follows immediately from analysis in paper and Lemma 1. □
Proof (Proposition 2). Where \( W(p^*, p^o) = -\sum_{n=1}^{N} \gamma_n \times |p_n^* - p_n^o| \), let \( w_n = -\gamma_n \times |p_n^* - p_n^o| \) for each \( n \). When there are no contribution limits, the politician chooses \( p_n^* = p_n^o \) for all \( n \), and social welfare \( W = 0 \), and \( w_n = 0 \) for all \( n \).

Consider the case when there are contribution limits. The parameter \( M \) denotes the realized number of interest groups that contribute \( \bar{b} \) in equilibrium. The ex ante probability of any \( M \in \{0, 1, ..., 2N\} \) equals \( \varphi(M) = \frac{2N^1}{2N-M}F(e)^{2N-M}(1-F(e))^M \). Therefore, \( M > K \) with probability \( \sum_{m=K+1}^{2N} \varphi(m) \). For each \( m \in \{K+1, ..., 2N\} \), \( \varphi(m) > 0 \) since \( \bar{e} \in (0, 1) \) and \( F(e) \in (0, 1) \) for any \( \bar{b} \in (0, B(1)) \). Thus, \( \sum_{m=K+1}^{2N} \varphi(m) > 0 \), the politician is less than fully informed with positive probability. With probability \( \sum_{m=K+1}^{2N} \varphi(m) > 0 \) there exists at least one interest group for which the politician knows \( e^1 \in [\bar{e}, 1] \), but does not know \( e^1 \) when he chooses a policy profile. Without loss of generality assume this is group \( (1, 1) \). The politician implements policy \( p_1^* = E_{\mu}e_1^1 - E_{\mu}e_1^{-1} \), where \( E_{\mu}e_1^1 = \int_{\bar{e}}^{1} e \frac{f(e)}{1-F(e)} \, de \). Given the continuous distribution of \( e \), both \( E_{\mu}e_1^1 \neq e_1^1 \) and \( E_{\mu}e_1^{-1} - E_{\mu}e_1^{-1} \neq e_1^{-1} - e_1^{-1} \) with probability one. Citizen welfare attributable to issue 1 is \( w_1 = -\gamma_n \times |E_{\mu}e_1^1 - E_{\mu}e_1^{-1} - e_1^1 - e_1^{-1}| \). Since \( E_{\mu}e_1^1 - E_{\mu}e_1^{-1} \neq e_1^1 - e_1^{-1} \) with probability one, it follows that \( w_1 < 0 \). For all other issues \( m \in \{2, ..., N\} \), \( w_m \leq 0 \), and \( W(p^*, p^o) = \sum_{n=1}^{N} w_n < 0 \).

When there are contribution limits, \( W(p^*, p^o) < 0 \) with probability \( \sum_{m=K+1}^{2N} \varphi(m) > 0 \), and \( W(p^*, p^o) = 0 \) with probability \( 1 - \sum_{m=K+1}^{2N} \varphi(m) \). Therefore, expected citizen welfare is strictly negative and therefore strictly less than welfare when there are no contribution limits. \( \blacksquare \)

Proof (Lemma 2). Follows immediately from Section A.2. \( \blacksquare \)

Proof (Lemma 3). See Section A.2 for derivation of the equilibrium. Assume \( j = 1 \), and \( e = e^1 = e^{-1} \). Consider three cases: \( e \in [0, \bar{e}_a) \), \( e \in [\bar{e}_a, \bar{e}_b) \), and \( e \in (\bar{e}_b, 1] \). If \( e \in [0, \bar{e}_a) \), then \( b_n^1 = b_n^{-1} = B_a(e) \), and \( p_n^* = e - e = 0 \). So, when \( e \in [0, \bar{e}_a) \), \( E_{p_n^*} = 0 \).
If $e \in [\bar{e}_a, \bar{e}_b]$, then $b_n^1 = b_n^{-1} = \omega$. Let $\bar{\Theta}_2$ be the equilibrium probability an interest group that provides $\omega$ receives access when $b_n^1 = b_n^{-1} = \omega$. Therefore, when $e \in [\bar{e}_a, \bar{e}_b]$, $E_p_n^* = \bar{\Theta}_2 e + (1 - \bar{\Theta}_2) \bar{e}_b - \bar{\Theta}_2 e - (1 - \bar{\Theta}_2) \bar{e}_b = 0$.

If $e \in (\bar{e}_b, 1]$, then $b_n^1 > \omega$ and $b_n^{-1} = \omega$. Let $\bar{\Theta}_1$ be the equilibrium probability an interest group that provides $\omega$ receives access when $b_n^1 > \omega$ and $b_n^{-1} = \omega$. Therefore, when $e \in (\bar{e}_b, 1]$, $E_p_n^* = e - \bar{\Theta}_1 e - (1 - \bar{\Theta}_1) \bar{e}_b = (e - \bar{e}_b)(1 - \bar{\Theta}_1) > 0$.

For all values of $e$, $E_p_n^* = (1 - F(\bar{e}_b)) (e - \bar{e}_b) (1 - \bar{\Theta}_1) > 0$. If $j = -1$, the same method shows that $E_p_n^* < 0$.

**Proof (Proposition 3).** See Section A.2 for derivation of the equilibrium. Because $B_P(e) = B_R(e)$ for any $e \leq \bar{e}_b$, it follows that $b_n^j$ is independent of $(n, j)$’s wealth when $e_n^j \in [0, \bar{e}_b]$. Given $b_n^j$, the probability $\Theta_{WL}(e(b_n^j), e_n^j)$ is independent of $(n, j)$’s wealth. Therefore, if $e_n^j \in [0, \bar{e}_b]$, then $(n, j)$ faces the same expected payoff $EU_n^j$ independent its wealth.

Alternatively, if $e \in (\bar{e}_b, 1]$, then $B_R(e) > B_P(e)$. Since $B_R(e_n^j) = \arg \max_b EU_n^j (b)$ and $B_P(e_n^j) = \omega$ is a feasible contribution for a rich group, it follows that $EU_n^j (B_R(e_n^j)) > EU_n^j (B_P(e_n^j))$. Therefore, when $(n, j)$ is rich, $EU_n^j$ is the same as if he was poor when $e_n^j \in [0, \bar{e}_b]$, and $EU_n^j$ is strictly greater than if he was poor when $e \in (\bar{e}_b, 1]$.

**Proof (Proposition 4).** Notation is consistent with Section A.2. Let $\bar{\Theta}_2$ be the equilibrium probability that $(n, j)$ receives access if $b_n^j = b_n^{-j} = \bar{b}$, and let $\bar{\Theta}_1$ be the
equilibrium probability that \((n,j)\) receives access if \(b_{n}^{j} = \bar{b}\) and \(b_{n}^{-j} < \bar{b}\). Therefore,

\[
E p_{n}^{*} = \int_{0}^{\bar{e}} \int_{0}^{\bar{e}} (e^{1} - e^{-1}) d e^{-1} d e \\
+ \int_{0}^{\bar{e}} \int_{0}^{\bar{e}} \left(1 - \bar{\Theta}_{1} e^{-1} - (1 - \bar{\Theta}_{1}) f_{\bar{e}} f(e) e d e\right) d e^{-1} d e \\
+ \int_{\bar{e}}^{1} \int_{0}^{\bar{e}} \left(\bar{\Theta}_{1} e^{-1} + (1 - \bar{\Theta}_{1}) f_{\bar{e}} f(e) e d e - e^{-1}\right) d e^{-1} d e \\
+ \int_{\bar{e}}^{1} \int_{0}^{\bar{e}} \left(\bar{\Theta}_{2} e^{-1} + (1 - \bar{\Theta}_{2}) f_{\bar{e}} f(e) e d eight) \\
\quad - \bar{\Theta}_{2} e^{-1} - (1 - \bar{\Theta}_{2}) f_{\bar{e}} f(e) e d e \right) d e^{-1} d e
\]

which reduces to \(E p_{n}^{*} = 0\).

Rich and poor groups both contribute to \(B_{CL}\). Because a group’s contribution is independent of wealth for any \(e\), it follows that the realization of \(\Theta_{CL}\) is independent of wealth. Therefore \(EU_{n}^{j}\) is independent of wealth. \(\blacksquare\)

**Proof (Lemma 4).** Follows immediately from analysis in the paper, and that when \(f(e) > 0\) for all \(e \in [0,1]\),

\[
\bar{e} = 2v \int_{0}^{1} f(e) \left(1 - e + \int_{0}^{e} F(y) d y\right) d e > 0.
\]

\(\blacksquare\)

**Proof (Proposition 5).** It is sufficient to prove that \(\bar{b} = B(1)\) results in at least as high of expected citizen welfare than any other \(\bar{b} > B(1)\), including \(\bar{b} = \infty\) (no limit). Consider any contribution limit \(\bar{b} \geq B(1)\). If the politician sells policy, then citizen welfare is \(-\gamma\), and the politician’s payoff is \(v \rho \bar{b} - \gamma\). If the politician sells access, then expected citizen welfare is 0 since the politician can choose \(p^{*} = p^{0}\), and the politician’s expected payoff is \(2v \rho \int_{0}^{1} f(e) \int_{0}^{e} (1 - F(y)) d y d e\) which equals his expected total contributions when he sells access. For any \(\bar{b} \geq B(1)\), expected citizen welfare from selling policy and expected citizen welfare from selling access are independent of \(barb\). Therefore, \(\bar{b}\) only impacts citizen welfare through its affect on the probability the politician sells access rather than policy.
The politician chooses to sell access rather than policy when
\[ 2v\rho \int_0^1 f(e) \int_0^e (1 - F(y)) \, dy \, de \geq v\rho \bar{b} - \gamma. \]  
(A.28)

This simplifies to
\[ \frac{\gamma}{\rho} \geq v \left( \bar{b} - \int_0^1 f(e) \int_0^e (1 - F(y)) \, dy \, de \right). \]  
(A.29)

Define \( \bar{z}' \) equal to the right hand side of Eq. A.29, and let \( \bar{z}'(\bar{b}) \) denote the value of \( \bar{z}' \) when the contribution limit is \( \bar{b} \). \( \bar{z}' \) is strictly strictly increasing in limit \( \bar{b} \). Let the function \( H \) represent the ex ante distribution of \( \frac{\gamma}{\rho} \). Therefore, expected citizen welfare from contribution limit \( \bar{b} \) is \( EW(\bar{b}) = -\gamma H(\bar{z}'(\bar{b})) \). \( \frac{\partial EW(\bar{b})}{\partial \bar{b}} < 0 \) for all \( \bar{b} \geq B(1) \). Therefore, \( \bar{b} = B(1) \) results in strictly higher expected citizen welfare than any other \( \bar{b} > B(1) \).
APPENDIX B
APPENDIX FOR CHAPTER 3

B.1 Proofs

Proof. (Prop. 6) Most of the proof for Proposition 6 is provided in the body of the paper in Section 3.4. Here, I provide the analysis that is not included in the body of the paper. The body of the paper fully describes the derivation of $P^*$. $A^*$ is fully derived, except for explicitly stating that $\bar{e} = \frac{2c}{v}$ is the value of $e_h$ that solves expression 3.4 with equality. When $e_h = \frac{2c}{v}$, the interest group is indifferent between buying access at fee $c$ and not buying access.

The derivation of $C^*$ requires the simplification of the politician’s expected utility function, given $P^*$ and $A^*$. Expression 3.5 states the interest groups expected utility, given his uncertainty regarding $e_l$ and $e_h$. To simplify this expression, first note that since $A^* = 1$ iff $e_h \leq \frac{2c}{v}$,

$$\int_0^1 \int_0^1 \int_0^1 (c\phi - \tau)A^*(c,e_h)d_e_hd_e_l = (c\phi - \tau)(1 - \bar{e}). \quad (B.1)$$

The policy utility part of the expression 3.5 can be rewritten

$$-\gamma \int_0^1 \int_0^1 |p^0 - P^*|d_e_hd_e_l = -\gamma \int_0^1 \int_0^\bar{e} |p^0 - P^*|d_e_hd_e_l - \gamma \int_0^1 \int_{\bar{e}}^1 |p^0 - P^*|d_e_hd_e_l. \quad (B.2)$$

When $e_h > \bar{e}$, the group buys access and the politician becomes fully informed about $e_h$, and chooses $P^* = e_h - \frac{1}{2}$. When $e_h \leq \bar{e}$, the group does not buy access and the politician chooses policy $E_{\mu}e_h - \frac{1}{2} = \bar{e} - \frac{1}{2}$. The optimal policy is defined as $p_o \equiv e_h - e_l$.

Expression B.2 can therefore be written

$$-\gamma \int_0^1 \int_0^{\bar{e}} |(e_h - e_l) - \left(\frac{\bar{e}}{2} - \frac{1}{2}\right)|d_e_hd_e_l - \gamma \int_0^1 \int_{\bar{e}}^1 |(e_h - e_l) - \left(e_h - \frac{1}{2}\right)|d_e_hd_e_l. \quad (B.3)$$

Given the uniform distribution of $e$, the second part of this expression simplifies to

$$-\gamma \int_0^1 \int_0^1 |(e_h - e_l) - (e_h - \frac{1}{2})|d_e_hd_e_l = -\gamma \int_0^1 \int_{\bar{e}}^1 |\frac{1}{2} - e_l|d_e_hd_e_l = -\gamma \int_{\bar{e}}^1 \int_0^1 |\frac{1}{2} - e_l|d_e_hd_e_l$$

$$= -(1 - \bar{e})^{\frac{1}{2}}. \quad (B.4)$$
Now consider the first part of expression B.3. Let \( H \) denote the distribution of \( p_o \) given that \( e_l \in [0, 1] \) and \( e_h \in [0, \bar{e}] \). The density of \( H \) is denoted by \( h \), where \( h(p_o) = 1 - \frac{p_o + \bar{e}}{\bar{e}} \) or \( p_o \in [0, \bar{e}] \); \( h(p_o) = 1 \) or \( p_o \in [\bar{e} - 1, 0] \); \( h(p_o) = \frac{1}{\bar{e}} - \frac{p_o}{\bar{e}} \) or \( p_o \in [-1, \bar{e} - 1] \); and \( h(p_o) = 0 \) otherwise. The function \( h(\cdot) \) is symmetric around \( (\bar{e} - 1)/2 \) (which is the implemented policy \( P^* \) when the group does not buy access). One can therefore rewrite the first part of expression B.3,

\[
-\gamma \int_0^1 \int_0^{\bar{e}} |e_h - e_l - \frac{\bar{e} - 1}{2}| de_h de_l = -2\bar{e} \gamma \left[ \int_{\frac{\bar{e} - 1}{2}}^{0} \left( p_o - \frac{\bar{e} - 1}{2} \right) dp_o + \int_{\frac{\bar{e} - 1}{2}}^0 \frac{1 + p_o}{\bar{e}} \left( p_o - \frac{\bar{e} - 1}{2} \right) dp_o \right] = -\left( \frac{1}{4} + \frac{\bar{e}^2}{12} \right) \gamma \bar{e}.
\]

(B.5)

Taken together, expressions B.1, B.4, and B.5 imply

\[
EU_P(P^*, c; p_o, \gamma, A^*) = -\left( \frac{1}{4} + \frac{\bar{e}^2}{12} \right) \gamma \bar{e} - (1 - \bar{e}) \gamma \left( c\phi - \tau \right) (1 - \bar{e})
= \left[ -\left( \frac{1}{4} + \frac{\bar{e}^2}{3 \bar{v}} \right) \gamma \right] + \left[ (c\phi - \tau) \left( 1 - \frac{2\bar{e}}{\bar{v}} \right) \right] .
\]

After this is established, the body of the paper describes the rest of the process to derive \( C^* \).

The derivation of equilibrium also requires the derivation of politician beliefs, \( \mu \). Beliefs must be consistent with Bayes’ Rule given equilibrium strategies. If the interest group pays the access fee, the politician becomes fully informed about \( e_h \), and his beliefs must be such that \( f_\mu(e_h) = 1 \). If the group does not buy access, the politician can infer that \( e_h \leq \bar{e} \). Given the ex ante uniform distribution of \( e \), it is equally likely that the interest group has any \( e_h \in [0, e_h] \). Therefore, \( f_\mu(e) = \frac{1}{\bar{e}} \) for all \( e \in [0, e_h] \), and \( f_\mu(e) = 0 \) for \( e \) not in this range.

**Proof.** (Prop. 7) Straightforward. ■

**Proof.** (Prop. 8) Imposing a contribution limit \( \bar{c} \) constrains the politician’s choice of access fee, but does not influence the politician’s or interest group’s preferences. Given any \( \mu \), the politician prefers to choose the policy he believes is best for his constituents, \( P^*_c = E_\mu e_h - \frac{1}{2} \). Thus, \( P^*_c = P^* \). Given any fee, the interest group prefers to buy access
whenever \( c_h > \frac{2c}{\nu} \). Thus \( A^*_c = A^* \). Given that the interest group’s access decision does not change for any given fee \( c \), the politician’s beliefs about the interest group’s evidence quality also will not change. Thus, \( \mu_c = \mu \).

Similarly, for any \( \gamma \) the politician prefers to choose the same access fee as he did in the game without a limit. Function \( C^* \), defined in Section 3.4, gives the politician’s preferred access fee for any issue, \( \gamma \). When \( C^*(\gamma) \leq \bar{c} \), the politician chooses access fee. Since \( C^* \) is strictly decreasing in \( \gamma \), this will be true for all \( \gamma \geq \gamma^*(\bar{c}) \), where \( \gamma^*(c) \equiv C^*-1(\gamma) \). It is straightforward to solve for

\[
\gamma^*(c) = \frac{(2\tau - 4c\phi + v\phi) v^2}{2c^2}
\]

When \( C^*(\gamma) > \bar{c} \) (or equivalently \( \gamma < \gamma^*(\bar{c}) \)), the politician is unable to set his preferred access fee. When this is the case, he can choose to set the access fee at some value less than \( \bar{c} \), set the fee equal to \( \bar{c} \), or not grant any access.

First, I establish that for \( \gamma < \gamma^*(\bar{c}) \), the politician sets either \( c = \bar{c} \), or \( c = \emptyset \). To establish this, it is sufficient to show that \( EU_P \) is strictly increasing in \( c \) for all \( c \leq \bar{c} \); which means that \( c = \bar{c} \) results in higher \( EU_P \) than any \( c < \bar{c} \), and which rules out any fee less than the limit. Expression 3.6 gives the equation for \( EU_P \), which is concave in \( c \). The derivative of \( EU_P \) with respect to \( c \) is given by expression 3.7, the second derivative equals \( \frac{\partial^2 EU_P}{\partial c^2} = -\gamma \frac{4c}{\nu^3} - \frac{4\phi}{\nu} \), which is clearly negative. Access fee \( c = C^*(\gamma) \) solves \( \frac{\partial EU_P}{\partial c} = 0 \). Given the concavity of \( EU_P \), for any \( c < C^* \), \( EU_P \) is increasing in \( c \). Since \( \bar{c} < C^* \) and \( EU_P \) is increasing in \( c \) for \( c < C^* \), \( EU_P \) is also increasing in \( c \) for all \( c \leq \bar{c} \). Therefore, for \( \gamma < \gamma^*(\bar{c}) \), the politician prefers to set \( c = \bar{c} \) than any lower fee.

For \( \gamma < \gamma^*(\bar{c}) \), the politician sets \( c = \bar{c} \) or chooses not to sell access. Setting \( c = \bar{c} \) results in

\[
EU_P(\bar{c}) = -\left( \frac{1}{4} + \frac{2\bar{c}^3}{3v^3} \right) \gamma + \left( 1 - \frac{2\bar{c}}{\nu} \right) (\bar{c}\rho - \tau).
\]

Not selling access (i.e., \( c = \emptyset \)) results in \( EU_P(\emptyset) = -\frac{\gamma}{4} \). Let \( \bar{\gamma} \) denote the issue importance
for which the politician is indifferent between selling access at price $\bar{c}$ and not selling access. $\bar{\gamma}$ solves

$$-\frac{\bar{\gamma}}{3} = -\left(\frac{1}{4} + \frac{2}{3} \frac{c^3}{v^3}\right) \bar{\gamma} + \left(1 - \frac{2\bar{c}}{v}\right) (\bar{c}p - \tau).$$

Solving this expression for $\bar{\gamma}$ gives

$$\bar{\gamma} = \frac{12v^2(\tau - c\phi)}{4c^2 + 2cv + v^2}.$$

Note that $\gamma^* < \gamma^*(\bar{c})$. As I’ve already established, for $\gamma \geq \gamma^*(\bar{c})$, the politician sets access fee equal to $C^*(\gamma)$, which is less than $\bar{c}$ for this range of $\gamma$. For $\gamma \in [\bar{\gamma}(\bar{c}), \gamma^*(\bar{c})]$, $EU_P(\emptyset) < EU_P(\bar{c})$ and the politician will choose to sell access at $c = \bar{c}$ rather than not sell access. For $\gamma < \bar{\gamma}$, $EU_P(\bar{c}) < EU_P(\emptyset)$ and the politician will choose to not sell any access than to sell access at fee $c = \bar{c}$.

**Proof. (Lemma 5)** Expected constituent welfare given any access fee $c$ equals

$$-\left(\frac{1}{4} + \frac{1}{12} (\bar{e}(c))^3\right) \gamma,$$

where $\bar{e}(c)$ is the cutoff evidence quality associated with fee $c$, such the interest group buys access iff $e_h > \bar{e}(c)$. When the politician sets access fee $c$, I’ve already established that $\bar{e}(c) = \frac{2c}{v}$. Therefore, expected welfare given access fee $c$ is

$$-\left(\frac{1}{4} + \frac{2}{3} \frac{c^3}{v^3}\right) \gamma$$

(B.7)

When $\gamma \geq \gamma^*(\bar{c})$, the politician sells access at fee $C^*(\gamma)$. When $\gamma \in [\bar{\gamma}(\bar{c}), \gamma^*(\bar{c})]$, the politician sells access at fee $\bar{c}$. In both of these cases, it is straightforward to calculate $EW_c$ given expression B.7. When $\gamma < \bar{\gamma}(\bar{c})$ the politician does not sell access, no interest group buys access, which is represented by $\bar{e}(\emptyset) = 1$. For this case, it is straightforward to calculate $EW_c$ given expression B.6.

**Proof. (Prop. 9)** The lower is $\bar{e}$, the more informed is the politician. As I have already established, $\bar{e} = \frac{2c}{v}$ when the politician sells access at fee $c$, and $\bar{e} = 1$ when the politician does not sell access ($\bar{e} = 1$ means no interest group buys access). Lemma 5 gives $EW_c$
for each \( \gamma \). Also, it has already been established that when there is no contribution limit, 
\[ EW(\gamma) = -\left(\frac{1}{4} + \frac{2}{3} \frac{(C^*(\gamma))^2}{v^*}\right) \gamma. \]
Since \( C^*(\gamma) \in \left(0, \frac{v^*}{2\phi}\right) \), \( EW(\gamma) \in (-\frac{\gamma}{3}, -\frac{\gamma}{4}) \).

For \( \gamma \geq \gamma^*(\bar{c}) \), the politician sets fee \( c = C^*(\gamma) \). This is the same access fee he sets when there is not contribution limit. Therefore, \( \bar{e}(C^*(\gamma)) = \bar{e}(C^*(\gamma)) \) (i.e., the politician receives the same information), and \( EW(\gamma) = EW_c(\gamma) \) (i.e., expected constituent welfare is unchanged). For \( \gamma \in [\bar{\gamma}(\bar{c}), \gamma^*(\bar{c})] \), the politician sets fee \( c = \bar{c} \), where \( \bar{c} < C^*(\gamma) \). Given that \( \bar{c} < C^*(\gamma) \), it is straightforward to show that \( \bar{e}(\bar{c}) < \bar{e}(C^*(\gamma)) \) (i.e., the politician is better informed), and \( EW(\gamma) < EW_c(\gamma) \) (i.e., expected constituent welfare increases). For \( \gamma < \bar{\gamma}(\bar{c}) \), the politician does not sell access. Therefore, \( \bar{e}(\emptyset) > \bar{e}(C^*(\gamma)) \) (i.e., the politician is less informed), and \( EW(\gamma) > EW_c(\gamma) \) (i.e., expected constituent welfare decreases).

**Proof. (Prop. 10)** Let \( \bar{c}' \) solve \( \bar{\gamma}(\bar{c}') = \gamma \). Prop. 8 establishes that at \( \bar{c}' \) the politician sets access fee equal to \( \bar{c}' \). Also, \( \bar{e}(\bar{c}') \in (0, 1) \).

First, I establish that \( \bar{c}' \) results in higher \( EW_c \) compared to any lower \( \bar{c} \). For all \( \bar{c} < \bar{c}' \), \( \bar{\gamma}(\bar{c}) > \gamma \); therefore, the politician does not grant access and \( EW_c = -\frac{\gamma}{3} \). At \( \bar{c}' \), \( \bar{e} \in (0, 1) \); therefore, \( EW_{c'} \in (-\frac{\gamma}{3}, -\frac{\gamma}{4}) \) (see the proof to Prop. 9), and \( EW_c(\gamma) < EW_{c'}(\gamma) \) for all \( \bar{c} < \bar{c}' \).

Second, I establish that \( \bar{c}' \) results in higher \( EW_c \) compared to any higher \( \bar{c} \). Note that since \( \gamma < \gamma^*(\bar{c}) \), \( \bar{c}' < C^*(\gamma) \). If the new \( \bar{c} \in (\bar{c}', C^*(\gamma)] \), then the politician sets the fee equal to \( \bar{c} \). If the new \( \bar{c} \geq C^*(\gamma) \), then the politician sets access fee equal to \( C^*(\gamma) \), which is strictly greater than \( \bar{c}' \). Either way, increasing the contribution limit above \( \bar{c}' \) results in a strictly higher access fee. As I’ve already established, a strictly higher access fee increases \( \bar{e} \), and decreases expected constituent welfare. \( EW_c(\gamma) < EW_{c'}(\gamma) \) for all \( \bar{c} > \bar{c}' \).

Third, I establish that \( \bar{c}' \) results in higher expected constituent welfare compared to setting no limit. If there is no limit, the politician sets access fee equal to \( C^*(\gamma) \), which
I’ve shown is strictly higher than $\bar{c}$. As I’ve already established, a strictly higher access fee increases $\bar{c}$, and decreases expected constituent welfare. $EW(\gamma) < EW(\gamma)$. ■

**Proof. (Prop. 11)** First, I establish that $\bar{c} = \frac{\tau}{\phi}$ results in higher expected constituent welfare than no limit or any $\bar{c} > \frac{\tau}{\phi}$. Note that $\gamma(\tau/\phi) = 0$. Therefore,

$$EW(\tau/\phi) = -\int_0^{\gamma(\tau/\phi)} g(\gamma) \left( \frac{1}{4} + \frac{2}{3} \frac{(\tau/\phi)^3}{v^3} \right) \gamma d\gamma - \int_{\gamma(\tau/\phi)}^{\infty} g(\gamma) \left( \frac{1}{4} + \frac{2}{3} \frac{(C^*(\gamma))^3}{v^3} \right) \gamma d\gamma.$$  

For any $\bar{c} > \frac{\tau}{\phi}$, $\gamma(\bar{c}) < 0$. Therefore,

$$EW_{\bar{c} > (\tau/\phi)} = -\int_0^{\gamma(\bar{c})} g(\gamma) \left( \frac{1}{4} + \frac{2}{3} \frac{\bar{c}^3}{v^3} \right) \gamma d\gamma - \int_{\gamma(\bar{c})}^{\infty} g(\gamma) \left( \frac{1}{4} + \frac{2}{3} \frac{(C^*(\gamma))^3}{v^3} \right) \gamma d\gamma.$$  

Note that $EW(\tau/\phi) > EW_{\bar{c} > (\tau/\phi)}$ iff $EW(\tau/\phi) - EW_{\bar{c} > (\tau/\phi)} > 0$. For any $\bar{c} > \tau/\phi$,

$$EW(\tau/\phi) - EW_{\bar{c} > (\tau/\phi)} = -\int_0^{\gamma(\tau/\phi)} g(\gamma) \left( \frac{1}{4} + \frac{2}{3} \frac{(\tau/\phi)^3 - \bar{c}^3}{v^3} \right) \gamma d\gamma - \int_{\gamma(\tau/\phi)}^{\gamma(\bar{c})} g(\gamma) \left( \frac{1}{4} + \frac{2}{3} \frac{(C^*(\gamma))^3 - (C^*(\bar{c}))^3}{v^3} \right) \gamma d\gamma.$$  

Since $\frac{\tau}{\phi} < \bar{c}$ and $\frac{\tau}{\phi} < C^*(\gamma)$ for $\gamma \in (\gamma(\bar{c}), \gamma(\tau/\phi))$, it follows that $EW(\tau/\phi) - EW_{\bar{c} > (\tau/\phi)} > 0$. Thus, a contribution limit equal to $\frac{\tau}{\phi}$ results in higher expected constituent welfare than any higher limit.

Setting no limit results in

$$EW_{\text{no limit}} = -\int_0^{\infty} g(\gamma) \left( \frac{1}{4} + \frac{2}{3} \frac{(C^*(\gamma))^3}{v^3} \right) \gamma d\gamma.$$  

Note that $EW_{(\tau/\phi)} > EW_{\text{no limit}}$ iff $EW_{(\tau/\phi)} - EW_{\text{no limit}} > 0$.

$$EW_{(\tau/\phi)} - EW_{\text{no limit}} = -\int_0^{\gamma(\tau/\phi)} g(\gamma) \left( \frac{1}{4} + \frac{2}{3} \frac{(\tau/\phi)^3 - (C^*(\gamma))^3}{v^3} \right) \gamma d\gamma.$$  

Since $\frac{\tau}{\phi} < C^*(\gamma)$ for $\gamma \in (0, \gamma(\tau/\phi))$, it follows that $EW_{(\tau/\phi)} - EW_{\text{no limit}} > 0$. Thus, a contribution limit equal to $\frac{\tau}{\phi}$ results in higher expected constituent welfare than imposing no limit. Therefore, the optimal limit $\bar{c}_o \in [0, \tau/\phi]$.

I will now show that $\bar{c}_o \neq \frac{\tau}{\phi}$ and $\bar{c}_o \neq 0$. To do so, it is sufficient to show that $\frac{\partial EW_{\bar{c}}}{\partial \bar{c}}|_{\bar{c}=0} > 0$, and $\frac{\partial EW_{\bar{c}}}{\partial \bar{c}}|_{\bar{c}=(\tau/\phi)} < 0$. That $EW_{\bar{c}}$ is a continuous, smooth function between 0 and $\tau/\phi$ assures that the function achieves a maximum on this interval.
The derivative of expected constituent welfare with respect to $\bar{c}$ is given by equation 3.12. It follows from this expression that

$$\frac{\partial EW_{\bar{c}}}{\partial \bar{c}} \bigg|_{\bar{c}=0} = -g(\bar{\gamma}(0)) \left[ \frac{1}{12} \bar{\gamma}(0)\bar{\gamma}'(0) \right].$$

Note that $\bar{\gamma}(0) = 12\tau > 0$; $g(\gamma) > 0$ for all positive $\gamma$ including $\gamma = 12\tau$; and $\bar{\gamma}'(0) < 0$. Therefore, $\frac{\partial EW_{\bar{c}}}{\partial \bar{c}} \bigg|_{\bar{c}=0} > 0$. This means that a marginally positive contribution limit results in higher expected constituent welfare than banning contributions.

Similarly,

$$\frac{\partial EW_{\bar{c}}}{\partial \bar{c}} \bigg|_{\bar{c}=(\tau/\phi)} = -g(\bar{\gamma}(\tau/\phi)) \left[ \frac{1}{12} - \frac{2}{3} (\tau/\phi)^3 \right] \bar{\gamma}(\tau/\phi)\bar{\gamma}'(\tau/\phi) - \int_{\bar{\gamma}(\tau/\phi)}^{\gamma^*(\tau/\phi)} g(\gamma) \frac{2(\tau/\phi)^2}{v^3} \gamma d\gamma,$$

Since $\bar{\gamma}(\tau/\phi) = 0$, this simplifies to Similarly,

$$\frac{\partial EW_{\bar{c}}}{\partial \bar{c}} \bigg|_{\bar{c}=(\tau/\phi)} = - \int_{0}^{\gamma^*(\tau/\phi)} g(\gamma) \frac{2(\tau/\phi)^2}{v^3} \gamma d\gamma,$$

which is strictly negative. Therefore, setting a contribution limit just below $\tau/\phi$ results in strictly higher expected constituent welfare than setting a contribution limit equal to $\tau/\phi$.

Thus, $\bar{c}_o \neq \frac{\tau}{\phi}$ and $\bar{c}_o \neq 0$. Together, these results establish that $\bar{c}_o \in (0, \frac{\tau}{\phi})$.

**Proof. (Cor. 1)** Proposition 11 directly established that a contribution ban was never optimal, and that it was always optimal to impose a contribution limit $\bar{c}_0 \in (0, \frac{\tau}{\phi})$. It remains to be shown that any limit $\bar{c} \in (0, \frac{\tau}{\phi})$ results in the politician giving access for some, but not all, issues. To establish this, it is sufficient to show that $\bar{\gamma}\bar{c} > 0$ for all $\bar{c}$ in this range. Since $\bar{\gamma}'(\bar{c}) < 0$ for all possible limits, it is sufficient to show that $\bar{\gamma}(0) > 0$ which implies that for any $\bar{c} > 0$, $\bar{\gamma}(\bar{c})$ will also be positive. $\bar{\gamma}(0) = 12\tau > 0$, thus $\bar{\gamma}(\bar{c}) > 12\tau$ for all $\bar{c} > 0$. ■
This paper is concerned with the principal’s choice of mechanism to assign the prize to an agent. Here I structure the framework presented above as a mechanism design problem. The structure is similar to Bull and Watson (2007), except that here the communication of evidence is not an inalienable right: the principal controls which agents present their information to him.

The principal must award the prize to one of the agents. At the beginning of the game, the principal determines the rules that dictate which agent receives the prize, conditional on the strategy profile of the agents. When the agents choose their strategies, they know that the prize will be allocated based on these rules. These rules are known as the mechanism, and the principal’s choice of the rules to maximize his expected payoff is known as mechanism design.

After the principal chooses the mechanism, the agents independently make announcements, and given these announcements the mechanism determines (1) which agents receive access, (2) which agent receives the prize, and (3) the transfer payments each agent must make to the principal. The value $a_i$ denotes an arbitrary announcement by agent $i$, and the vector $a = (a_1, ..., a_N)$ denotes arbitrary announcements of all players. An agent $i$ must choose her announcement from a mechanism-defined strategy space $A_i$.

The function $\Theta(\cdot|a)$ denotes the pdf for $\phi$ given $a$. Therefore, $\Theta(\phi|a)$ is the probability that each agent $i$ with $\phi_i = 1$ receives access given all of the announcement. Given $\Theta$, it is straightforward to determine the probability that any agent $i$ receives access given $a$, which I denote by $\theta_i(a)$. Let $\theta = (\theta_1, ..., \theta_N)$. The mechanism determines $\Theta$, which directly determines $\theta$. If agent $i$ receives access, the principal directly observes $q_i$. Let
\( \tilde{q}(\phi; q) \) denote the vector of qualifications revealed through access, where \( \tilde{q} = (\tilde{q}_1, ..., \tilde{q}_N) \).

For each \( i \), \( \tilde{q}_i = q_i \) if \( \phi_i = 1 \) and \( \tilde{q}_i = \emptyset \) if \( \phi_i = 0 \).

The identity of the prize winner may depend on whether an agent received access (given by \( \phi \)) and the qualifications that are directly revealed through access (given by \( \tilde{q} \)), as well as the announcements. The function \( X(\cdot|a; \phi; \tilde{q}) \) denotes the pdf for \( \omega \), where \( X(\omega|a; \phi; \tilde{q}) \) is the probability that the prize is awarded according to \( \omega \). Given \( X \) it is straightforward to determine the probability that any agent \( i \) wins the prize, which I denote by \( x_i(a; \phi; \tilde{q}) \). The mechanism determines \( X \), which directly determines \( x = (x_1, ..., x_N) \).

The value \( t_i(a; \phi; \omega; \tilde{q}) \) is the required transfer payment for agent \( i \). Let \( t = (t_1, ..., t_N) \).

I allow the transfer payment to depend on which agents receive access and which agent receives the prize.

For each agent, a mechanism must define the strategy space, the access decision, the prize award, and the required transfer payments. Denote a mechanism in the prize allocation with access framework by \( \Gamma = (A, \Theta, X, t) \). Such a mechanism differs from a more traditional prize allocation mechanism in that it assigns access (there is typically no \( \Theta \)), \( X \) and \( t \) may depend on which agents receive access, and \( X \) may depend on revealed qualifications.

Given \( \Gamma \), each agent chooses some equilibrium strategy \( s_i(q_i) \in A_i \). The strategy may depend on the agent’s type, \( q_i \). Let \( s_{-i}(q_{-i}) \) denote the vector of strategies played by all other agent’s besides agent \( i \). When agent \( i \) makes some arbitrary announcement \( a_i \) and all other agents play according to \( s_{-i} \),

\[
E_{q_{-i}}U_i(a_i, s_{-i}; \Gamma) = \int_{q_{-i}} \int_{\phi} \int_{\omega} f_{-i}(q_{-i}) \Theta(\phi|a_i, s_{-i}(q_{-i})) X(\omega|a_i, s_{-i}(q_{-i}); \phi; \tilde{q}(\phi; q)) \times \left[ \omega_i v - t_i(a_i, s_{-i}(q_{-i}); \phi; \omega; \tilde{q}(\phi; q)) \right] d\omega d\phi dq_{-i}
\]

(C.1)

For \( s = (s_1, ..., s_N) \) to be a Bayesian Nash Equilibrium of the mechanism \( \Gamma \), the
following conditions must be met.

**Incentive compatibility.** Each agent must not have an incentive to announce an untruthful type. For all \( i = 1, \ldots, N \), every \( a_i \in A_i \), and any realization of \( q_i \),

\[
E_{q_{-i}} U_i(s; \Gamma) \geq E_{q_{-i}} U_i(a_i, s_{-i}; \Gamma).
\]

(C.2)

**Individual rationality.** Each agent must prefer to participate in the mechanism game than not participate. By not participating, an agent is certain to receive a payoff of 0. Therefore, for all \( i = 1, \ldots, N \), and any realization of \( q_i \),

\[
E_{q_{-i}} U_i(s; \Gamma) \geq 0.
\]

(C.3)

**Feasibility.** The mechanism must be feasible. For all \( a \), \( \int_\phi \Theta(\phi|a) d\phi = 1 \) and for each \( \phi \), \( \Theta(\phi|a) \in [0, 1] \). For all \( a \), \( \phi \) and \( \tilde{q} \), \( \int_\omega X(\omega|a; \phi; \tilde{q}) d\omega = 1 \). Since there is only one prize, \( X(\omega|a; \phi; \tilde{q}) \in [0, 1] \) for any \( \omega \) such that \( \sum_{i=1}^N \omega_i \leq 1 \), and \( X(\omega|a; \phi; \tilde{q}) = 0 \) for any \( \omega \) such that \( \sum_{i=1}^N \omega_i > 1 \) (such mechanisms require more than one prize).

### C.1.1 Revelation Principle

In an all-pay auction framework, an agent’s strategy may describe how much she bids. Because agents choose their bids, this is the most straightforward type of mechanism to represent an auction. However, the same auction may be represented by any number of alternative mechanisms that transform an agent’s alternative announcement into its desired bid. For example, in the equilibrium of an equivalent mechanism, an agent may make an announcement equal to half of her desired bid, and the mechanism would double her announcement to determine her required transfer. Or, agents may announce their qualifications and the mechanism would map their qualifications into their bids. The multiplicity of outcome-equivalent mechanisms can make for a burdensome analysis. To overcome this difficulty, I establish that the well-known revelation principle holds in the
prize allocation and access framework.\footnote{See Myerson (1981) for detailed discussion.} The revelation principle establishes that for any feasible mechanism, there exists an outcome-equivalent truthfully-revealing mechanism. A truthfully-revealing mechanism assigns to each agent a strategy space equal to the agent’s type space, and in equilibrium each agent announces \( s_i(q_i) = q_i \). Therefore, one can conduct a thorough analysis by focusing only on the class of truthfully revealing mechanisms.

**Lemma 9** If there exists a feasible mechanism \( \Gamma' = (A', \Theta', X', t') \) with equilibrium \( s' \), then there also exists a feasible truthfully-revealing mechanism \( \Gamma = (A, \Theta, X, t) \) such that in equilibrium \( s_i(q_i) = q_i \), and \( A = ([0, 1], ..., [0, 1]) \), \( \Theta(\cdot | q) = \Theta' (\cdot | s') \), \( X(\cdot | q; \phi; \tilde{q}(\phi, q)) = X'(\cdot | s'; \phi; \tilde{q}(\phi, q)) \), and \( t(q; \phi; \omega) = t'(s'; \phi; \omega) \).

### C.2 Formal Competition for Access Mechanism

Although the agents technically choose the payment they provide to the principal (their announcements equal the amount they pay), one can reformulate the all-pay auction for access as an equivalent truthfully revealing mechanism. In this formulation, the agents announce their qualifications (potentially untruthfully) and the mechanism assigns to them the payment amount they would have selected in the original formulation. If \( t_i \) is the payment function assigned to \( i \) by the truthfully revealing mechanism, then in equilibrium \( t_i(q_i) \) is the amount \( i \) chooses to pay the principal in the all-pay auction for access when she has qualifications \( q_i \).

There exists a truthfully-revealing mechanism representing competition for access may be denoted by \( \Gamma^* = (A^*, x^*, t^*, \theta^*) \), where

- \( A^* = ([0, 1], ..., [0, 1]) \),
• $x^* = (x_1^*, ..., x_N^*)$ is such that when agent $j$ wins access and $\tilde{q} = q_j$,

$$x_j^*(a; q_j) = 1(q_j \geq \max(a_{-j})),$$

and

$$x_i^*(a; q_j) = 1(q_j < \max(a_{-j})) \frac{1(a_i \in \max(a_{-j}))}{\#(k \mid a_k \in \max(a_{-j}))} \quad \text{for all } i \neq j,$$

• $t^* = (t_1^*, ..., t_N^*)$ is such that for each $i$,

$$t_i(q_i) = v(F(q_i))^{N-1},$$

• $\theta^* = (\theta_1^*, ..., \theta_N^*)$ is such that for each $i$

$$\theta_i^*(a) = \frac{1(a_i \in \max_{j=1}^{N}(a_j))}{\#(k \mid a_k \in \max_{j=1}^{N}(a_j))}.$$

C.3 Proofs

Proof. (Lemma 6) This proof relies on the mechanism design framework presented in Section C.1. Total expected revenue is given by $\sum_{i=1}^{N} \int_{q_i} f_i(q_i) \int_{q_{-i}} f_{-i}(q_{-i}) t_i(\cdot) dq_{-i} dq_i$. For each agent $i$ and given any $q_i$, total expected revenue is strictly increasing in $\int_{q_{-i}} f_{-i}(q_{-i}) t_i(\cdot) dq_{-i}$. The individual rationality constraint for each $i$ implies $v \int_{q_{-i}} f_{-i}(q_{-i}) x_i(\cdot) dq_{-i} \geq \int_{q_{-i}} f_{-i}(q_{-i}) t_i(\cdot) dq_{-i}$ for each $i$. Setting $\int_{q_{-i}} f_{-i}(q_{-i}) t_i(\cdot) dq_{-i}$ as large as possible implies that $\int_{q_{-i}} f_{-i}(q_{-i}) t_i(\cdot) dq_{-i} = v \int_{q_{-i}} f_{-i}(q_{-i}) x_i(\cdot) dq_{-i}$. Total expected revenue can therefore not exceed $v \sum_{i=1}^{N} f_i(q_i) \int_{q_{-i}} f_{-i}(q_{-i}) x_i(\cdot) dq_{-i} dq_i$. This may be rewritten $v \int_{q} f(q) \sum_{i=1}^{N} x_i(\cdot) dq$.

The mechanism can award the prize to at most one agent. Feasibility implies that for any variables, $\sum_{i=1}^{N} x_i(\cdot) \leq 1$. Total expected revenue is strictly increasing in $\sum_{i=1}^{N} x_i(\cdot)$ for any $q$, therefore total expected revenue is maximized at the maximum value when $\sum_{i=1}^{N} x_i(\cdot) = 1$. Therefore, total expected revenue from any feasible mechanism cannot exceed $v \int_{q} f(q) dq = v$. Such as mechanism does indeed exist, as is evident from the analysis in Sections 4.4 and 4.5. ■
**Proof. (Lemma 7)** The derivation of the equilibrium contribution function follows from the analysis in the body of the paper. It is straightforward to verify that the contribution function makes up the equilibrium of the game. Each agent $i$ is indifferent between contributing $F(q_i)^{N-1}$ and any contribution $t_i \leq F(q_i)^{N-1}$; and she strictly prefers to contribute $F(q_i)^{N-1}$ than any greater amount. Hence, no agent has an incentive to deviate. ■

**Proof. (Prop. 12)** Follows directly from analysis in the body of the paper. ■

**Proof. (Prop. 13)** Suppose that competition for access is not an efficient mechanism for awarding the prize. Then there exists another mechanism that improves the expected utility of an agent or the principal, while not making any of the other players worse off. I have already shown that the principal cannot be made better off. Therefore, there must exist a mechanism that improves the expected utility of an agent without making either the principal or another agent worse off.

To improve the expected utility of some agent $i$, the new mechanism must increase the probability that the agent wins the prize or decrease its expected payment such that

$$v \int_{q_i} f_{-i}(q_{-i})x_i(\cdot) dq_{-i} > \int_{q_{-i}} f_{-i}(q_{-i})t_i(\cdot) dq_{-i}. \quad (C.4)$$

From the analysis in the proof to Proposition 6, total expected revenue for the principal is given by $\sum_{i=1}^{N} \int_{q_i} f_i(q_i) \int_{q_{-i}} f_{-i}(q_{-i}) t_i(\cdot) dq_{-i} dq_i$. Given expression C.4, it is clear that

$$\sum_{i=1}^{N} \int_{q_i} f_i(q_i) \int_{q_{-i}} f_{-i}(q_{-i})t_i(\cdot) dq_{-i} dq_i < v \sum_{i=1}^{N} \int_{q_i} f_i(q_i) \int_{q_{-i}} f_{-i}(q_{-i})x_i(\cdot) dq_{-i} dq_i.$$ 

The right hand side of this expression simplifies to $v \int_{q} f(q) \sum_{i=1}^{N} x_i(\cdot) dq$, which mechanism feasibility guarantees is less than or equal to $v$. Therefore,

$$\sum_{i=1}^{N} \int_{q_i} f_i(q_i) \int_{q_{-i}} f_{-i}(q_{-i})t_i(\cdot) dq_{-i} dq_i < v.$$ 

This means that total expected revenue must decrease. Thus, improving the expected utility of an agent results in lower revenue and lower expected utility for the principal: a
contradiction. Therefore, competition for access is an efficient mechanism for awarding the prize.

**Proof. (Lemma 8)** Follows immediately from the analysis in the body of the paper.

**Proof. (Prop. 14)** Follows immediately from the analysis in the body of the paper.

**Proof. (Lemma 9)** Consider feasible mechanism $\Gamma' = (A', \Theta', X', t')$ with equilibrium $s'$, where $s'_i(q_i)$ denotes the equilibrium action of agent $i$ when she has qualifications $q_i$. For each $i$, the relevant components of the mechanism are $\Theta'_i(\cdot|a), X'_i(\cdot|a; \phi; \tilde{q}(\phi; q))$, and $t'_i(a; \phi; \omega; \tilde{q}(\phi; q))$. $A'_i$ is an arbitrary strategy space.

Let $\Gamma = (A, \Theta, X, t)$ denote an alternative mechanism, such that for each $i$, $A_i = [0, 1]$, $\Theta_i = \Theta'_i(\cdot|s'_1(a_1), ..., s'_N(a_N))$, $X_i = X'_i(\cdot|s'_1(a_1), ..., s'_N(a_N); \phi; \tilde{q}(\phi; q))$, and $t_i = t'_i(s'_1(a_1), ..., s'_N(a_N); \phi; \omega; \tilde{q}(\phi; q))$. Since announcing $s'_i(q_i)$ was an equilibrium under $\Gamma'$, announcing $q_i$ will be an equilibrium under mechanism $\Gamma$. Furthermore, for any $q$, the equilibrium outcomes for each player are equivalent under both mechanisms. For any $q_i$, the mechanism $\Gamma$ transforms an announcement of $a_i = q_i$ into the announcement that player $i$ would have made under mechanism $\Gamma'$. Since all agents now prefer to announce their qualifications, $\Gamma$ is a truthfully revealing mechanism. $\Gamma$ may be defined similarly relative to any $\Gamma'$, and therefore such a truthfully revealing mechanism exists generally.


