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Option Put-Call Parity Relations When the Underlying Security Pays Dividends

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Abstract

The original put-call parity relations hold under the premise that the underlying security does not pay dividends before the expiration of the options. Similar to Hull (2003), this paper relaxes the non-dividend-paying assumption. The underlying security price in the original European-style put-call parity relation is adjusted downwards by the present value of expected dividends before the option expires. The upper bound of the American-style put-call parity relation is adjusted upwards by the amount of the present value of expected dividends. The results provide theoretical boundaries of options prices and expand application of put-call parity relations to all options on currencies and dividend-paying stocks and stock indices, both European-style and American-style.

Key words: options, dividends, and put-call parity

JEL classification: G13: contingency pricing

1. Introduction

The option put-call parity condition quantifies the relations among the price of a call option, the price of an otherwise identical put option, the price of the underlying security of the call and put options, and the present value of the exercise price of the call and put options. The parity relations can be applied to both European-style and American-style options. They help to explain the intricate relations among prices of call and put options, prices of their underlying security, and the price of risk-free Treasury Bills.

Put-call parity relations for standard European-style and American-style options have been well accepted by the finance profession. Early studies in options, e.g., Merton (1973), Smith (1976), Cox and Ross (1976), and Cox and Rubinstein (1985), and essentially all options textbooks, e.g., Tucker (1991), Hull (2002, 2003), Jarrow

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and Turnbull (2000), and Chance (2003), among others, cover discussions on the put-call parity relations.

The original put-call parity relations are derived under the premise that the underlying security does not pay dividends before the expiration of the options. However, a large number of stocks and almost all stock indices pay dividends. Furthermore, all foreign currencies bear foreign risk-free rates. The foreign risk-free rates of interest can be viewed as dividends (or leakages) paid on the foreign currencies. Consequently, the non-dividend-paying assumption severely restricts the usage of the original put-call parity relations. The original put-call parity relations may not be applied to these heavily traded options on dividend-paying securities.

Similar to Hull (2003), this paper relaxes the non-dividend-paying assumption on the underlying security. It presents a variation of the relations when the underlying securities pay dividends. The underlying security price in the original put-call parity relation for European-style options is adjusted downwards by the present value of expected dividends before the option expires. The upper bound of the put-call parity relation for American-style options is adjusted upwards by the present value of expected dividends before the option expires.

This paper shows the theoretical boundary conditions to call and put option prices when the underlying security pays dividends. Our results expand the application of the powerful tool of put-call parity relation to a much wider range of options. It is particularly important to option traders who trade American-style options on stocks and stock indices (e.g., S&P 100 Index OEX), currencies, and over-the-counter non-standard options because most of the underlying securities generate either discrete cash dividends or a continuous dividend stream.

Inherited in the put-call parity relation, another important property of our results is that they are purely arbitrage-driven and totally model-free. The results are independent of any particular option pricing models, e.g., the Black-Scholes (1973) option pricing model. The only assumption needed for the conclusions to hold is that the financial markets are efficient, which is widely believed to be true.

This paper is organized as follows. The next section derives and proves the put-call parity relation for European-style options. The third section deals with the put-call parity relation for American-style options. The final section concludes.

2. Put-Call Parity for European-Style Options

If the underlying security does not pay dividends before the option expires, the original put-call parity relation for European-style options can be given by the following simple equation:

$$S_0 + P^E = C^E + Xe^{-rT}, \tag{1}$$

where C^E and P^E are European-style call and put option premiums, respectively, S_0 is the current price of the underlying security, X is the options' common exercise price, r is the annualized continuously compounded risk-free rate of interest, and T

is the time to options' maturity. Proof of the above relation can be found in most textbooks on options. The put-call parity relation for European-style options states that the sum of the current underlying security price and a put option premium equals to the sum of a call option premium and the present value (discounted at the risk-free rate) of the options' exercise price.

If the underlying security pays a dividend (or dividends) before the options expire, the put-call parity relation can be modified as:

$$S_0 - PV(Div) + P^E = C^E + Xe^{-rT} \text{ ,} \tag{2}$$

where $PV(Div)$ is the present value (discounted at the risk-free rate) of all expected cash dividend payments generated by the underlying security to be paid on or before the options expire. For example, if the underlying security is expected to pay a dividend D at time t where $0 < t < T$, then $PV(Div) = De^{-rt}$.

The proof of the dividend-adjusted put-call parity for European-style options is straightforward. For simplicity and without loss of generality, assume there is only one cash dividend payment *D*, occurring at time t during the life of the option, where $0 < t < T$. We treat continuous dividend yields at the end of the next section.

Consider that an investor holds the following two portfolios from today until the options expire: $S_0 - De^{-rt} + P^E$ and $C^E + Xe^{-rT}$. Note that the first portfolio contains a share of an underlying security that pays dividend D at time t. The terminal value of the first portfolio at time T is $S_T - De^{-rt}e^{rT} + P_T^E + De^{r(T-t)} = S_T + P_T^E$ because the future value of the dividend received at time t cancels the future value of $PV(Div)$.

It's now easy to see that when the options expire, the two portfolios have exactly the same terminal value $S_T + P_T^E = C_T^E + X$. This occurs because if $S_T \geq X$, then $S_T + P_T^E = S_T$ and $C_T^E + X = S_T$, so that $S_T + P_T^E = C_T^E + X$. Alternatively, if $S_T \leq X$, then $S_T + P_T^E = X$ and $C_T^E + X = X$, so that again $S_T + P_T^E = C_T^E + X$.

Because the two portfolios always have the same terminal value, they must have exactly the same present value in an efficient market. Consequently, we must have $S_0 - PV(Div) + P^E = C^E + Xe^{-rT}$. The put-call parity relation for European-style options is thus proved.

3. Put-Call Parity for American-Style Options

Under the assumption of no dividends, the original put-call parity relation for American-style options can be given by the following chain of inequalities:

$$C^A + Xe^{-rT} \leq P^A + S_0 \leq C^A + X \text{ ,} \tag{3}$$

where C^A and P^A are American-style call and put option premiums, respectively. Instead of one simple equation for European-style options, the put-call parity relation for American-style options is a chain of inequalities, where the difference between the upper and lower bounds, i.e., the width of the interval, is $(C^A + X) - (C^A + Xe^{-rT}) = X(1 - e^{-rT})$. For a reasonable exercise price, risk-free rate of interest, and time to maturity, the original put-call parity relation for American-style options provides a tight interval that brackets the put option premium and underlying security price.

If the underlying security pays a dividend (or dividends) before the options expire, then the American-style put-call parity relation can be modified as:

$$C^A + Xe^{-rT} \le P^A + S_0 \le C^A + X + PV(Div). \tag{4}$$

The proof can be shown by "proof by contradiction". For simplicity and without loss of generality, assume there is only one cash dividend payment D, occurring at time t during the life of the options, where $0 < t < T$. The case of a continuous dividend yield is addressed below.

Note that, due to the early exercise feature of American-style options, an option holder can choose to exercise her option early when doing so is optimal. Early exercise of a call option occurs when the underlying security pays a significantly large dividend, the amount of which exceeds the remaining time (or speculative) value. Early exercise of a put option occurs when the price of the underlying security is sufficiently low that interest income earned on the intrinsic value of the put option is greater than the remaining time value.

The proof of expression (4) is divided into two pieces:

$$C^A + Xe^{-rT} \le P^A + S_0, \tag{4a}$$

$$P^A + S_0 \le C^A + X + PV(Div). \tag{4b}$$

We prove both pieces by proof by contradiction. First, suppose that inequality (4a) does not hold for all securities and their options. Then there exists at least one underlying security and its options which satisfy $C^A + Xe^{-rT} > P^A + S_0$. We show that if this case happens, there must be an arbitrage opportunity.

An arbitrageur buys the American-style put option and buys a share of the underlying security. At the same time, she writes the American-style call option and sells a risk-free bond with a face value of the exercise price of the options. The initial cash flow is positive because $(C^A + Xe^{-rT}) - (P^A + S_0) > 0$. Consequently, the position $P^A + S - C^A - Xe^{-rT}$ is held.

Because the arbitrageur wrote an American-style call option, the buyer of the call option may choose to early exercise the call option in order to capture a significantly large cash dividend payment from the underlying security. At the instant before the underlying security goes ex-dividend, the call option buyer may exercise the call option early by paying X dollars in exchange for a share of the underlying security. In this case, the arbitrageur loses the underlying security, and the call option buyer captures the underlying security and its forthcoming dividend. The value of the arbitrageur's portfolio becomes:

$$P^A + S_t - S_t + X - Xe^{-r(T-t)} = P^A + X - Xe^{-r(T-t)} > 0.$$

If the cash dividend D is not large enough to trigger early exercise of the call option, the arbitrageur collects the dividend on the ex-dividend date and holds the portfolio to the option's date of maturity. The value of this portfolio becomes $P_T^A + S_T + De^{r(T-t)} - C_T^A - X = De^{r(T-t)} > 0$. It's therefore clear that the assumption $C^A + Xe^{-rT} > P^A + S_0$ induces an arbitrage opportunity. Thus, in an efficient financial market, the inequality $C^A + Xe^{-rT} \le P^A + S_0$ must hold for all securities and at all times.

Next, suppose that inequality (4b) does not hold for all securities and options. Then there exists at least one underlying security and its options which satisfy $P^A + S_0 > C^A + X + PV(Div)$. Again we argue that an arbitrage opportunity must result.

An arbitrageur buys the American-style call option, buys a risk-free bond with a face value in the amount of the expected cash dividend, and buys a risk-free bond priced at X dollars. At the same time, she writes an American-style put option and short sells a share of the underlying security. The initial cash flow is positive because $(P^A + S_0) - (C^A + X + PV(Div)) > 0$. The arbitrageur now holds the position $C^A + X + PV(Div) - P^A - S$.

Because the arbitrageur short sold a share of the underlying security, she is responsible for paying any dividends that the underlying security generates. The component $PV(Div)$ in her portfolio allows her to exactly meet this obligation. Consequently, the remaining discussion assumes that the component $PV(Div)$ neutralizes dividend obligations from the underlying security when sold short.

At any time before the option expires, if the put option holder exercises the option by submitting the underlying security in exchange for the exercise price, the value of the arbitrageur's portfolio is $C^A + Xe^{rt} + S_t - X - S_t = C^A + Xe^{rt} - X > 0$. If the put option holder does not exercise the option early, then the arbitrageur holds this portfolio until the option expires. The value of this portfolio becomes:

$$C_T^A + Xe^{rT} - P_T^A - S_T = Xe^{rT} - X > 0 .$$

Again, the assumption $P^A + S_0 > C^A + X + PV(Div)$ induces an arbitrage opportunity. Thus, in an efficient financial market, the inequality $P^A + S_0 \le C^A + X + PV(Div)$ must hold for all securities.

The dividend-adjusted put-call parity relation for American-style options implies a wider interval width $(C^A + X + PV(Div)) - (C^A + Xe^{-rT}) = X - Xe^{-rT} + PV(Div)$. The wider interval is due to the additional uncertainty of early exercise before the underlying security goes ex-dividend.

In summary, the dividend-adjusted put-call parity relations for European-style and American-style options are given by expressions (2) and (4):

European-style options: $S_0 - PV\left(Div\right) + P^E = C^E + Xe^{-rT}$,

American-style options: $C^A + Xe^{-rT} \le P^A + S_0 \le C^A + X + PV\left(Div\right)$.

In some cases, the underlying security does not pay discrete cash dividends. Instead, it generates a continuous stream of dividends. Typical examples include all foreign currencies which bear foreign risk-free interest rates as dividend yields, most stock indices, and custom-designed over-the-counter products. In some cases, the term leakage is used instead of dividends. If the underlying security generates a continuously compounded dividend yield d, then the present value of all dividends before the option expires is $PV(Div) = S_0(1-e^{-dT})$. Substituting the new $PV(Div)$ expression into equations (2) and (4), we conclude that the put-call parity relations under the continuous dividend yield case are:

European-style options: $S_0 e^{-dT} + P^E = C^E + Xe^{-rT}$, (5)

American-style options: $C^A + Xe^{-rT} \leq P^A + S_0 \leq C^A + X + S_0(1-e^{-dT})$. (6)

4. Conclusion

This paper derives put-call parity relations for European-style and American-style options when the underlying security pays dividends before the options expire. The dividend-adjusted put-call parity relation provides theoretical boundary conditions for call and put option prices when the underlying security pays dividends. Our results expand the scope and application of the original put-call parity relation to all European-style and American-style options on foreign currencies and dividend-paying stocks and stock indices.

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