Online Appendix

to accompany

Where’s the Kink? Disappointment Events in Consumption Growth and Equilibrium Asset Prices

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The Online Appendix provides additional material to support the results in the main text.

A. The GDA Stochastic Discount Factor

Following Gul’s original disappointment model (Gul 1991), the analysis in the paper assumes that the generalized disappointment aversion (GDA) coefficient $\delta$ of Routledge and Zin (2010) is one. In this section, I derive analytical solutions for the GDA model in which $\delta$ is unrestricted. The purpose of this exercise is to show that my unconditional tests in the cross-section of size/bm portfolios cannot reject the null hypothesis that $\delta$ is one.

Formally, with the introduction of the GDA parameter $\delta$, the disappointment aversion (DA) certainty equivalent in equation (2) of the main text becomes

$$
\mu_t(V_{t+1}) = \mathbb{E}_t \left[ \frac{V_t^\alpha (1 + \theta 1\{V_{t+1} \leq \delta \mu_t\})}{1 - \theta (\delta^\alpha - 1) 1\{\delta > 1\} + \theta \delta^\alpha \mathbb{E}_t[1\{V_{t+1} \leq \delta \mu_t]\}} \right].
$$

The GDA parameter $\delta$ is associated with the threshold for disappointment. In the DA model, $\delta$ is one and disappointment events occur when utility falls below its certainty equivalent. In contrast, in the GDA framework, disappointment events may happen above or below the certainty equivalent, depending on whether the GDA coefficient $\delta$ is greater or lower than one.

According to Routledge and Zin (2010), the GDA discount factor is given by

$$
M_{t+1}^{GDA} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{\rho - 1} \left[ \frac{V_{t+1}}{\mu_t(V_{t+1}; V_{t+1} \leq \delta \mu_t)} \right]^{\alpha - \rho} \left[ \frac{1 + \theta 1\{V_{t+1} \leq \delta \mu_t\}}{1 - \theta (\delta^\alpha - 1) 1\{\delta > 1\} + \theta \delta^\alpha \mathbb{E}_t[1\{V_{t+1} \leq \delta \mu_t]\}} \right].
$$

Starting from the above expression, I assume that consumption growth is an AR(1) process with constant volatility as in the main text. Then, I follow the proof in Appendix A of the paper and derive explicit solutions for the GDA model in terms of consumption growth. In particular, when the GDA coefficient $\delta$ is unrestricted, the consumption-based threshold for disappointment is equal to the certainty equivalent for consumption growth adjusted for $\delta$:

$$
\Delta c_{t+1} \leq (1 - \kappa_c,1 \phi_c) \log \delta + \mu_c (1 - \phi_c) + \phi_c \Delta c_t + d_1 \sqrt{1 - \phi_c^2} \sigma_c.
$$

Above, $\mu_c$, $\sigma_c$, and $\phi_c$ are the unconditional mean, variance, and autocorrelation coefficient of consumption growth, respectively, and $\kappa_c,1$ is a log-linearization constant. The parameter $d_1$ is the
solution to a fixed point problem similar to the one in equation (5) of the main text:

\[ d_1 = \frac{1}{2} \frac{\alpha}{1 - \kappa_c \phi_c} \sqrt{1 - \phi_c^2 \sigma_c} + \frac{\log \left( \frac{1 + \theta N \left( d_1 + \left(1 - \phi_c \kappa_c, 1 \right) \log \delta \frac{\phi_c}{1 - \phi_c^2 \sigma_c} \right)}{1 - \theta \left(1 - \kappa_c \phi_c \right)} + \frac{\alpha}{1 - \kappa_c \phi_c} \sqrt{1 - \phi_c^2 \sigma_c} \right)}{1 - \theta \left(1 - \kappa_c \phi_c \right)} \].

Additionally, the GDA discount factor expressed in terms of consumption growth alone becomes

\[ M_{t+1}^{GDA} = \exp \left[ \log \beta_M - \left( \frac{\rho - \alpha}{1 - \kappa_c \phi_c} + 1 - \rho \right) \Delta c_t + \frac{(\rho - \alpha) \phi_c}{1 - \kappa_c \phi_c} \Delta c_t \right] \]

\[ \times \frac{1 + \theta 1 \left\{ \Delta c_{t+1} \leq (1 - \kappa_c \phi_c) \log \delta + \mu_c \left(1 - \phi_c \right) + \phi_c \Delta c_t + d_1 \sqrt{1 - \phi_c^2 \sigma_c} \right\}}{1 - \theta \left(1 - \kappa_c \phi_c \right) \Delta c_t + \mu_c \left(1 - \phi_c \right) + \phi_c \Delta c_t + d_1 \sqrt{1 - \phi_c^2 \sigma_c}}. \]  

When \( \delta \) is one in equations (a.2) and (a.3) above, we recover the disappointment threshold and the DA discount factor from the paper (equations (6) and (7) in the main text).

The estimation methodology for the GDA discount factor is identical to the one in the main text, and the results are shown in Table A.1. According to these results, the DA coefficient \( \theta \) in the GDA model is economically and statistically significant with values between 2.344 and 8.519. These estimates are very similar to the ones in Table 2 of the paper. Thus, the introduction of the GDA parameter \( \delta \) does not affect the estimation of the DA coefficient \( \theta \).

The introduction of the GDA parameter does not affect the cross-sectional fit of the disappointment model either. Specifically, the \( R^2 \)s for the GDA model in Table A.1 are between 91\% and 98\%. These values are practically indistinguishable from the \( R^2 \)s for the DA model in Table 2 of the paper. Similar results also hold for the \( \chi^2 \)test and RMSE.\(^1\)

In addition to the above results, the focus of Table A.1 is the estimation of the GDA coefficient \( \delta \). Specifically, based on the 95\% confidence intervals for the GDA parameter, I cannot reject the hypothesis that \( \delta \) is one across all GDA specifications. Hence, my unconditional tests in the cross-section of size/bm portfolios cannot distinguish between the DA and GDA models.\(^2\)

Collectively, the previous findings confirm that the GDA parameter does not affect the estimation of the DA parameter \( \theta \) or the fit of the DA model in unconditional tests for the size/bm portfolios. Nevertheless, the GDA parameter \( \delta \) is important in explaining the time-series of asset

\(^1\)Table A.1 does not show results for the HJ-statistic of the GDA model due to insufficient degrees of freedom.

\(^2\)It should be noted that in Bonomo et al. (2011), the calibrated value for the GDA parameter is 0.989. This value is very close to one, and my unconditional tests cannot statistically identify such a small difference.
returns (Bonomo et al. 2011). For instance, as shown in Routledge and Zin (2010), the GDA parametrization can generate time variation in risk premiums within an endowment economy even if the underlying endowment process is homoscedastic. In contrast, the DA framework of the main text cannot generate time variation in risk premiums, unless the endowment process were heteroscedastic. In that case however, explicit solutions for the DA discount factor in terms of observable consumption growth would be difficult to derive.

B. Covariances of the DA Discount Factor with Asset Returns

One potential concern with the empirical estimation of the main text is that the covariances of the DA pricing kernel with excess returns may be weakly identified. Specifically, if the cross-sectional dispersion in covariances between the DA discount factor and asset returns is not statistically significant, then the DA model may spuriously fit the cross-section of expected returns. Burnside (2011) discusses this issue in cross-sectional tests of currency portfolios, while Bryzgalova (2015) finds that consumption growth betas are statistically insignificant in tests of linear asset pricing models.

To this end, I estimate the DA model using an augmented GMM system which is designed to directly test the statistical significance of covariances between the DA discount factor and asset returns. Specifically, in addition to the moment conditions of the GMM system in equation (9) of the main text, the augmented GMM system in this section includes two new ones. The first condition is the covariance of the DA discount factor with the excess return on the stock market:

$$\text{Cov}(M^D_t, R_{m,t} - R_{f,t}) - \text{cov}_1 = 0.$$  

This condition allows me to formally test whether the estimated covariance, $\text{cov}_1$, between the DA discount factor and the excess return on the stock market is significantly different than zero. The second condition is the covariance of the DA discount factor with the spread in returns of the small/value ($R_{s/v}$) minus the big/growth ($R_{b/g,t}$) portfolio:

$$\text{Cov}(M^D_t, R_{s/v,t} - R_{b/g,t}) - \text{cov}_2 = 0.$$  

This condition allows me to test whether the estimated covariance, $\text{cov}_2$, between the DA pricing kernel and the small/value minus the big/growth portfolio is significantly different from zero. In other words, this condition tests the statistical significance of the dispersion in covariances between the two extreme size/bm portfolios.
Formally, the augmented GMM system is given by

\[
\mathbb{E}[q(z_t, x)] = \begin{bmatrix}
\mathbb{E}[\Delta c_t] - \mu_c \\
\mathbb{E}[\Delta^2 c_t] - \mu_c^2 - \sigma_c^2 \\
\mathbb{E}[\Delta c_t \Delta c_{t-1}] - \mu_c^2 - \phi_c \sigma_c^2 \\
\mathbb{E}[(\log R_{f,t})^2] - \mathbb{E}[\log R_{f,t}]^2 - (1 - \rho)^2 \phi_c^2 \sigma_c^2 \\
\mathbb{E}[M_t^{DA} R_{f,t}] - 1 \\
\mathbb{E}[M_t^{DA} (R_{m,t} - R_{f,t})] \\
\text{Cov}(M_t^{DA}, R_{m,t} - R_{f,t}) - \text{cov}_1 \\
\text{Cov}(M_t^{DA}, R_{b/b}, R_{b/g,t}) - \text{cov}_2 \\
\mathbb{E}[M_t^{DA} (R_{i,t} - R_{f,t})] \quad \text{for} \quad i = 1, 2, \ldots, n
\end{bmatrix}.
\] (a.4)

The weighting matrix for the GMM system above overweights moment conditions for consumption growth, the risk-free rate, and the equity premium, as in the main text. The weighting matrix also overweights the new covariance conditions to increase the accuracy of the estimates of \text{cov}_1 and \text{cov}_2.\footnote{The weights for the consumption growth moments and the variance of the risk-free rate are 10^8 (10^{12} for the CRRA model). The weights for the mean of the risk-free rate, the equity premium, and the covariance restrictions are 10^5. Finally, the weights for the remaining test assets are one.}

The results for the augmented system are shown in Table A.2. According to these results, the estimated parameters and the fit of the DA model in the augmented GMM system are very similar to the findings in Table 2 of the main text. The most important finding in Table A.2 is the statistical significance of the covariance estimates \text{cov}_1 and \text{cov}_2. Based on the bootstrap confidence intervals, both \text{cov}_1 and \text{cov}_2 are statistically significant across all DA specifications. These results verify that the estimated covariance of the DA discount factor with the excess return of the stock market is statistically different from zero. Most importantly, these results indicate that the cross-sectional dispersion in the estimated covariances of the DA pricing kernel with the size/bm portfolios is statistically significant. Taken together, these findings alleviate the concerns that the covariances with the DA pricing kernel are weakly identified.\footnote{It should be noted that the choice of the weighting matrix for the GMM system in the main text also addresses the issue of spurious factors. Specifically, the GMM weighting matrix in the paper overweights the Euler equations for the risk-free rate and the stock market by a factor of 100,000:1 in relation to the Euler equations for the size/bm portfolios. This means that the disappointment parameter \theta and the set of disappointment events are mainly estimated from the equity premium, while the cross-sectional differences between portfolios are not particularly important for identification.}
C. Post-1945 Sample

The annual sample of the six size/bm portfolios in the main text starts in 1933. In this section, I repeat the estimation for the DA model and the six size/bm portfolios for an annual sample that starts in 1945. Estimation results for the post-1945 sample are shown in Table A.3. Based on these results, I find that the consumption growth process in the postwar period (1945-2012) is more persistent, less volatile, and has a lower unconditional mean than the consumption growth process of the full 1933-2012 sample in the paper.

Nevertheless, the results for the DA model in the postwar sample are similar to the ones for the 1933-2012 period in Table 2 of the main text. Specifically, the estimates of the DA coefficient $\theta$ are economically and statistically significant, with magnitudes between 2.434 and 11.007. More importantly, the $R^2$s in Table A.3 indicate that the fit of the DA models in the postwar sample is better than the fit of the traditional consumption-based specifications (CRRA, EZ) or the performance of the three-factor Fama-French model. Similar results also hold for the remaining measures of fit: the $\chi^2$ test, the $HJ$-statistic, and the $RMSE$s. Overall, the cross-sectional performance of the DA model in the post-1945 period is similar to that for the 1933-2012 sample of the main text. This finding confirms that the fit of the DA model in the size/bm portfolios does not depend on the sample period.

D. Earnings/Price Portfolios

Fama and French (1993) use portfolios sorted on earnings/price (E/P) to test their three-factor model. For robustness, I also test the DA model of the main text using the sample of five E/P portfolios over the 1953-2012 period. Estimation results for these portfolios are shown in Table A.4. These results are similar to the ones in Table 2 of the main text for the size/bm portfolios.

To begin with, the estimates of the DA coefficient $\theta$ in Table A.4 are economically and statistically significant across all DA specifications. Second, the fit of the DA model in the E/P portfolios is better than the fit of the traditional consumption-based specification, and comparable to the performance of the Fama-French three-factor model. Overall, the results for the E/P portfolios provide additional evidence in support of the ability of the DA model to explain cross-sections of

\footnote{Table A.4 does not show results for the $HJ$-statistic due to insufficient degrees of freedom.}
portfolios that are related to the value premium.

E. Corporate Bonds and Commodity Futures

In this section, I conduct additional tests using the sample of corporate bonds and commodity futures of the main text. Unlike the tests in the paper, in which all assets are pooled together, in this section I consider each asset class separately.

E.1. Corporate bonds

Table A.5 shows estimation results for the DA discount factor in the sample of corporate bond portfolios. According to these results, the estimates of the disappointment parameter are economically and statistically significant across all DA specifications. In general, the $\theta$ estimates in the bond sample are lower than those for the size/bm portfolios in Table 2 of the paper. Nevertheless, disappointment events for the bond sample are almost identical to the disappointment events for the size/bm sample. This is because in both cases, disappointment events are mainly identified from the Euler equations for the aggregate stock market index and the risk-free rate since the GMM weighting matrix overweights the corresponding moment conditions for these two assets. Overall, according to the results in Table A.5, the cross-sectional performance of the DA discount factor in the sample of corporate bond portfolios is better than the fit of the traditional consumption models (CRRA, EZ), with $R^2$s ranging from 84% to 93% and $RMSE$s between 0.28% and 0.43%.

E.2. Commodity futures

The estimation results in the commodity sample are similar to the ones for the equity and bond portfolios. Specifically, according to the findings in Table A.6, the DA coefficient $\theta$ is statistically significant across all the DA specifications, and its magnitude ranges from 2.309 to 10.808. Further, the set of disappointment events for the commodity sample is quite similar to the set of disappointment events for the equity sample of the main text. However, the performance of the DA model in

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7Tables A.5 and A.6 do not show results for the HJ-statistic due to insufficient degrees of freedom.

the commodities sample is not as good as its fit in the equity and bond samples. This is probably because the commodities sample is short, and ignores disappointment events before 1974 and after 2008. Regardless, the performance of the DA discount factor in this sample is superior to that of the traditional consumption models or the Fama-French three-factor specification.

F. NBER-Based Stochastic Discount Factor

The analysis in Section 4 of the paper indicates that disappointment events in consumption growth are distinct from NBER recessions. In this section, I show that NBER recessions have little explanatory power over the cross-section of expected returns for the size/bm portfolios. To this end, I test a reduced-form stochastic discount factor which is based on an annual NBER recession indicator. The NBER discount factor is given by

\[ M_{t}^{NBER} = \beta (1 + \theta_{NBER} \mathbf{1}\{\text{at least } x \text{ NBER recession months in year } t\}) \], \quad \text{with } x \in \{5, 7, 9\}. \quad (a.5) \]

Because NBER recessions are defined on a monthly basis, I create an annual NBER indicator by considering an arbitrary cutoff for the number of NBER recession months in a year.\(^9\) I then estimate the NBER discount factor using a GMM system that consists of the Euler equations for the risk-free rate, the aggregate stock market, and the six size/bm portfolios. The results for the NBER indicator with at least 5, 7, and 9 NBER recession months in a year are shown in Table A.7.\(^10\) According to these results, the NBER-based pricing kernel cannot price the size/bm portfolios, and the corresponding pricing errors are much larger than the pricing errors of the DA models in Table 2 of the main text. Collectively, the findings in this section highlight the distinct asset pricing implications between disappointment events and NBER recession years.

G. Alternative First-Order Risk Aversion Models

First-order risk aversion models models are characterized by their assumptions regarding the location of the reference point. Gul’s (1991) disappointment theory takes a strong stance on this issue: outcomes are evaluated with respect to their certainty equivalent. In this section, I compare the

\(^9\)The number of recession months in a year are from the FRED.

\(^10\)The results are similar when I consider alternative cutoffs for NBER recession months in a year.
performance of the DA framework to a set of first-order risk aversion models that specify alternative reference points for gains and losses.

The most famous member of the first-order risk aversion class of preferences is probably the loss aversion model of Kahneman and Tversky (1979). Loss aversion shares a number of common features with disappointment aversion. For instance, the value function is defined on deviations from reference levels, and it is steeper for losses than for gains. However, the original loss aversion framework and the majority of its subsequent empirical applications do not explain how reference points are formed or dynamically updated. Towards the end of their paper, Kahneman and Tversky discuss time varying reference outcomes, but their entire analysis is based on the assumption that the reference point is the “status quo or one’s current assets.”

Contrary to Kahneman and Tversky, Kőszege and Rabin (2006) suggest that investors do not necessarily define gains and losses with respect to the status quo. Instead, they propose that expectations are a more suitable benchmark for evaluating outcomes. Therefore, I also consider a reference-based model in which the reference point is expected consumption growth. Finally, Arkes et al. (2008) show that reference points tend to adapt quickly to past experiences, and that reference-point adaptation is greater following gains than following losses of equivalent size. Based on the discussion in Arkes et al., I set the reference point equal to last period’s consumption growth in order to capture a quickly adapting reference point. An alternative motivation for using last period’s consumption growth as a reference point is a status quo model in growth rates.

Based on the discussion above, the general first-order risk aversion (FORA) discount factor is

\[
M_t^{\text{FORA}} = \exp \left[ \log \beta_M - \left( \frac{\rho - \alpha}{1 - \kappa_c,1 \phi_c} + 1 - \rho \right) \Delta c_t + \frac{(\rho - \alpha) \phi_c}{1 - \kappa_c,1 \phi_c} \Delta c_{t-1} \right]
\]

\[
\times \frac{1 + \theta 1 \{\Delta c_t \leq d\}}{1 + \theta \mathbb{E}_t \left[ 1 \{\Delta c_t \leq d - \frac{\alpha}{1 - \kappa_c,1 \phi_c} (1 - \phi_c^2) \sigma_c^2 \} \right]^2}.
\]

\[\text{A.8}\]

11 In addition to loss aversion, probability weighting is another ingredient in Kahneman and Tversky’s (1979) prospect theory. In this paper, I ignore probability weighting, and maintain the rational expectations assumption because my goal is to examine the explanatory power of asymmetric utility alone.

12 In the loss aversion framework, the value function is concave over gains and convex for losses. A number of recent results (e.g., Duncan 2010) have questioned the S-shaped utility function. When the utility function is S-shaped, the second order necessary conditions must also be checked. In contrast, the DA utility function is globally concave.

13 The quick adaptation framework here does not allow for asymmetric adaptation of the reference point following gains and losses as in Arkes et al. (2008). To account for asymmetric effects in reference-point adaptation, in unreported tests, I set the reference point equal to \(\max[\Delta c_t, 0]\). These results are similar to the ones shown here. When consumption growth is an AR(1) process with positive autocorrelation, the DA certainty equivalent from equation (7) of the main text is also adaptive. However, if the reference point is equal to last period’s consumption growth, then the reference point adapts faster than the DA reference point because the autocorrelation of consumption growth in the DA reference point is less than one.
The constant $d$ above is the parameter that determines the location of the reference point. For $d = 0$, the reference level is zero consumption growth based on the status quo reference point of Kahneman and Tversky (1979). For $d = \mathbb{E}_t[\Delta c_{t+1}]$, the reference level is expected consumption as in the expectation model of Kőszeği and Rabin (2006). Finally, for $d = \Delta c_t$, the reference level is last period’s consumption growth. For comparison, I also estimate two additional specifications: i) a model in which the location of the reference level is a free parameter ($d = d_0$) to be estimated by GMM and ii) the DA specification in which the reference level is equal to the certainty equivalent of consumption growth.

G.1. GMM results for the FORA discount factor

Table A.8 shows estimation results for the alternative first-order risk aversion models. The most important finding in Table A.8 is that the economic significance of the first-order risk aversion coefficient $\theta$ depends on the assumption regarding the location of the reference point. For instance, when the reference level is located at zero (FORA(1) model), the estimate of the first-order risk aversion coefficient is statistically and economically insignificant, whereas the estimate of the (second-order) risk aversion parameter is around $-39$. When the reference level is the expected consumption growth (FORA(2) model), $\theta$ is also insignificant and $\alpha$ is equal to $-39$. Similar results hold for the quick adaptation model (FORA(3) model).

Overall, the estimates for the first three specifications of the FORA model are almost identical to the results for the Epstein-Zin model from Table 2 of the main text, where downside consumption risk per se is not priced. In contrast, when the location of the reference level is a free parameter that is estimated by GMM (FORA(4) model), downside consumption risk is priced. In this case, the estimate of the first-order risk aversion parameter $\theta$ is 8.488, and the estimate of the loss threshold for consumption growth is 1.097%. Based on these estimates, when consumption growth is less than 1.097%, investors in the FORA(4) model penalize losses 9.488 times more than they do during normal times.

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14To calculate expected consumption growth, I maintain the assumption of an AR(1) process as in the main body of the paper.
G.2. Empirical fit of the FORA discount factor

The previous results show that the location of the reference point affects the statistical significance of the first-order risk aversion coefficient. In this section, I show that the location of the reference level also affects the empirical fit of the FORA models.

According to the results in Table A.8, the status quo, expectation, and adaptation models are rejected by the $\chi^2$ test, while their cross-sectional fit is identical to that of the Epstein-Zin model from Table 2 of the main text. This is because the price of downside consumption risk in these three FORA models is zero. The free-parameter model is also rejected by the $\chi^2$ test, and its fit is a minor improvement over the first three FORA models because the reference level for this model is constant. In contrast, the performance of the DA model (FORA(5)) is much better than that of the alternative FORA specifications because disappointment aversion can successfully describe the reference level for gains and losses in consumption growth.

G.3. Loss events in consumption growth

To explain the findings in Table A.8, note that the asset pricing performance of asymmetric preference models lies in their ability to correctly characterize periods of loss events in consumption growth, i.e., periods during which consumption growth is below the reference point. For the DA model, these periods are called disappointment events and happen whenever consumption growth is below its certainty equivalent. According to the results in Table A.8, disappointment events in consumption growth occur with 11.2% probability. In relation to disappointment events, loss events for the status quo reference point (FORA(1)) occur too rarely (6.2% probability), whereas loss events for the expectation, quick-adaptation, and free-parameter models (FORA(2)-FORA(4)) happen too often (50%, 51.2%, and 26.5% probability, respectively).

To shed additional light on the relation between the location of the reference point and the fit of the FORA models, Figure A.1 plots the dynamics of the alternative reference points for gains and losses. The solid line in Figure A.1 is consumption growth, and the dotted line denotes the zero reference level in the FORA(1) model (status quo model). In this model, the unconditional probability of consumption growth crossing the zero threshold is only 6.2%. Thus, the status quo specification cannot explain asset prices because it ignores important loss events in consumption growth (e.g., loss events in 1948, 1979, 1990, 1999, 2007).
The dashed line in Figure A.1 is the expected consumption growth threshold for the FORA(2) model. When the reference level is equal to expected consumption growth and consumption growth is a normal random variable, then loss events should occur with 50% probability. Indeed, according to the results in Table A.8, the empirical probability of loss events for the FORA(2) model is exactly 50%. However, in this case, loss events happen so often that they overlap periods of economic growth and high equity returns. These periods are not particularly important for asset prices, and the first-order risk aversion parameter becomes irrelevant for fitting expected returns. The results are similar when the reference point is either last period’s consumption growth or a constant parameter (FORA(3) and FORA(4) models). In both of these models, loss events also happen too often.

Overall, the results in this section show that the location of the reference point determines the cross-sectional performance of the FORA models. Specifically, when the reference point is based on either the status quo assumption of Kahneman and Tversky (1979), the expectation model of Köszegi and Rabin (2006), or a quickly adapting reference point as implied by Arkes et al. (2008), then the estimate of the price of downside consumption risk is insignificant. In this case, the asymmetric utility model is equivalent to the traditional consumption-based framework with symmetric preferences.

H. Willingness-to-Pay and Time Premium for the DA Investor

In this section, I use the willingness-to-pay calculations of Epstein and Zin (2001) and the time premium framework in Epstein, Farhi, and Strzalecki (2014) to assess the plausibility of the DA estimates from Table 2 of the main text. First, I discuss the willingness-to-pay of DA investors.

H.1. Willingness-to-pay

The effective risk aversion of DA investors depends on the risk and disappointment aversion coefficients. To assess the effective risk aversion of DA investors, Table A.9 reports willingness-to-pay estimates as in Epstein and Zin (2001). These estimates denote the amount of money an investor is willing to pay to avoid a gamble that pays $100,000 ± $ with equal probability, and simply receive $100,000. In other words, I compute the difference between the mean and the DA certainty equivalent of a $100,000 ± $ gamble for different values of the random variable $. Large numbers

A.11
for the willingness-to-pay calculations in Table A.9 imply a very risk averse behavior. For these tests, I use the estimated preference parameters from Table 2 in the main paper.

Based on the results in Table A.9, for small gambles ($\epsilon = 1,000$), DA investors are more risk averse than Epstein-Zin individuals due to first-order risk aversion. However, for medium and large gambles, the DA(1) specification from Table 2 of the main text, implies a less risk averse behavior than the Epstein-Zin model. In contrast, the effective risk aversion for the remaining DA specifications (DA(2) - DA(5)) is comparable to the effective risk aversion of the Epstein-Zin model. Overall, the results in Table A.9 show that the level of effective risk aversion across the various DA specifications is roughly the same.

### H.2. Time premium

Epstein, Farhi, and Strzalecki (2014) define the time premium as the fraction of lifetime utility that investors are willing to sacrifice at time 0 in order to have all risk resolved in period one. Formally, the time premium is equal to $1 - V_0/V^*_0$ where

$$V^*_0 = [(1 - \beta)C_0^\rho + \beta \mu_0 \left( (1 - \beta)^{1/\rho} C_1^\rho + \beta^2 C_2^\rho + \beta^3 C_3^\rho + \ldots \right)^{1/\rho}], \quad (a.7)$$

$$V_0 = [(1 - \beta)C_0^\rho + \beta \mu_0 (V_1)^\rho]. \quad (a.8)$$

Above, $\mu_0$ is the DA certainty equivalent from equation (2) of the paper. The variable $V^*_0$ denotes lifetime utility when all risk is resolved in period one, and $V_0$ is the traditional measure of lifetime utility from equation (1) of the main text when risk is resolved gradually.

Following Epstein, Farhi, and Strzalecki (2014), I calculate $V^*_0$ in equation (a.7) by simulating 100,000 trials for 2,500 years of the consumption growth process in equation (4) of the paper. To calculate lifetime utility $V_0$ in equation (a.8), I use: i) the ex-dividend price of the consumption claim ($P_{c,0} = W_0 - C_0$); ii) the homotheticity of DA preferences ($V_0 = J_0 W_0$); iii) the marginal utility of wealth in equation (A7) of the main text ($J_0^\rho = (1 - \beta) \left( \frac{C_1}{W_0} \right)^{\rho - 1}$); iv) the log price-dividend approximation $\left( \log \left( e^{\log P_{c,0}/C_0} + 1 \right) = \tilde{\kappa}_{c,0} + \tilde{\kappa}_{c,1} \log \frac{P_{c,0}}{C_0} \right)$; and v) the results for the log price-dividend ratio from Proposition 1 of the main text ($\log \frac{P_{c,0}}{C_0} = \mu + \phi \Delta c_0$) to conclude that

$$V_0 = (1 - \beta)^{1/\rho} C_0 e^{\rho \left( \tilde{\kappa}_{c,0} + \tilde{\kappa}_{c,1} (\mu + \phi \Delta c_0) \right)}. \quad 15$$

15In the simulations, I set the terminal value for $V^*_0$ equal to $V_0$.
The parameters $\tilde{\kappa}_{c,1} = \frac{\tilde{p}c}{e^{\tilde{p}c} + 1}$ and $\tilde{\kappa}_{c,0} = \log(1 + e^{\tilde{p}c}) - \kappa_{c,1}\tilde{p}c$ above are log-linearization constants that depend on the parameter $\tilde{p}c$. In turn, the parameter $\tilde{p}c$ is the unconditional average of the simulated $\log P_{c,t}^C$ for the above values of $\tilde{\kappa}_{c,0}$ and $\tilde{\kappa}_{c,1}$ and the consumption growth dynamics from equation (4) of the main text.

Following Epstein, Farhi, and Strzalecki (2014), I set the value for $C_0$ equal to one for both $V^*_0$ and $V_0$. Moreover, I set $\Delta c_0$ equal to the unconditional mean of consumption growth based on the estimates from Table 1 of the paper ($\Delta c_0 = \mu_c = 7.16\%$). Finally, the preference parameters for the time premium calculations are calibrated to the estimates from Table 2 of the main text, with the exception of the discount rate $\beta$, which is set equal to 0.998 (the $\beta$ estimate for the DA(1) model) across all models, to avoid the complications of discount rates that are greater than one.

Based on the results in Table A.9, the time premium for the DA model ranges from 2.76% to 14.09%. This means that DA investors are willing to pay between 2.76% and 14.09% of their lifetime utility to have all risk resolved in the next period. These numbers are higher than the ones for the Epstein-Zin model (0.16%), but lower than the results in Epstein, Farhi, and Strzalecki (2014) for the long-run risk model of Bansal and Yaron (2004), where the time premium is 30%. This is because in the DA model of the main text, the EIS is lower than one and consumption growth is not very persistent. In contrast, in the long-run risk framework, the EIS is greater than one and the consumption growth process is quite persistent. Overall, the time premium calculations highlight the fact that the DA model does not rely on the persistence of consumption growth to amplify consumption risk.

I. Hypothesis Testing for Nondifferentiable GMM Estimators

Consistency and asymptotic normality of GMM estimators require differentiability of the GMM objective function. However, continuity and differentiability break down when the GMM moment conditions include indicator functions such as the DA indicator. In this section, I use the results in Andrews (1994) and Newey and McFadden (1994) to show consistency and asymptotic normality of the DA estimates, even if the DA discount factor is not continuous.
I.1. Identification, consistency, and asymptotic normality for nondifferentiable
GMM estimators

Let $z_t$ be a vector of random variables, $x$ a vector of parameters, $q(z_t, x)$ a vector-valued function, and $W$ a symmetric, positive-definite matrix. For the DA model, $x = [\mu_c, \phi_c, \sigma_c^2, \beta, \alpha, \rho, \theta]$, $z_t = [\Delta c_t, \Delta c_{t-1}, R_{f,t}, R_{m,t}, \{R_{i,t}\}_{i=1}^n]$, and

$$q(z_t, x) = \begin{bmatrix}
\Delta c_t - \mu_c \\
\Delta c_t^2 - \mu_c^2 - \sigma_c^2 \\
\Delta c_t \Delta c_{t-1} - \mu_c^2 - \phi_c \sigma_c^2 \\
(\log R_{f,t})^2 - \mathbb{E}[\log R_{f,t}]^2 - (1 - \rho)^2 \phi_c^2 \sigma_c^2 \\
M_t^D A R_{f,t} - 1 \\
M_t^D A (R_{m,t} - R_{f,t}) \\
M_t^D A (R_{i,t} - R_{f,t}) \text{ for } i = 1, 2, \ldots, n
\end{bmatrix}, \quad (a.9)$$

where $M_t^D A$ is the DA discount factor from equation (6) of the paper, and $n$ is the number of risky assets. Consider the GMM objective function for the DA model

$$Q_0 = \mathbb{E}[q(z_t, x)]'W \mathbb{E}[q(z_t, x)], \quad (a.10)$$

and its sample analog

$$\hat{Q}_T = \left[\frac{1}{T} \sum_{t=1}^T q(z_t, x)\right]' \hat{W} \left[\frac{1}{T} \sum_{t=1}^T q(z_t, x)\right]. \quad (a.11)$$

Let $x_0$ be the minimizer in (a.10) and $\hat{x}_T$ the minimizer in (a.11).

Economic theory suggests that for DA investors, $\beta \in (0, 1)$, while $\theta \geq 0$, $\alpha \leq 1$, and $\rho \leq 1$ are bounded away from infinity by real numbers $B_\theta > 0$, $B_\alpha < 0$, and $B_\rho < 0$ respectively. I also assume that $\Delta c_t$ is stationary and therefore $\phi_c$ takes values in $(-1, 1)$, $\mu_c$ is a real bounded number, and $\sigma_c^2$ is a positive bounded number. All these restrictions imply that the vector of parameters takes values in a compact space $X \in \mathbb{R}^7$. Finally, I assume that $z_t$ is characterized by a continuous probability distribution function and a well-defined moment generating function $\forall x \in X$.

**Identification:** I assume that the GMM objective function for the DA model in equation
(a.10) satisfies the conditions of Lemma 2.3 in Newey and McFadden (1994) so that $x_0$ is globally identified. These conditions are:

1. $W$ is positive definite
2. $\mathbb{E}[q(z_t, x_0)] = 0$
3. $W\mathbb{E}[q(z_t, x_0)] \neq 0$ for $x \neq x_0$.

For the empirical tests in the paper, I use the variance of the risk-free rate (equation (11) in the main text) to identify $\rho$ and address the issue of joint identification of $\alpha$ and $\rho$. I also restrict the value of the risk aversion coefficient $\alpha$ to address any concerns that $\theta$ and $\alpha$ cannot be jointly identified. Finally, I consider a grid of initial values for the minimization of the GMM objective function in equation (a.11) to ensure that the simplex algorithm does not lead to local minima.

**Consistency:** Regarding the consistency of estimators when the GMM objective function is not continuous, the reader is referred to Theorem 2.6 in Newey and McFadden (1994). According to this theorem, $\hat{x}_T$ converges in probability to $x_0$ if the following conditions are met:

1. $z_t$ is stationary and ergodic
2. $\hat{W} \overset{p}{\to} W$, $W$ is positive definite
3. $W\mathbb{E}[g(z_t, x_0)] = 0$ only if $x = x_0$
4. $x_0 \in X$, which is compact
5. $q(z_t, x)$ is continuous $\forall x \in X$ with probability one
6. $\mathbb{E}[\sup_{x \in X} ||q(z_t, x)||] < +\infty$.

Stationarity and ergodicity are reasonable properties for $z_t = [\Delta c_t, \Delta c_{t-1}, R_{f,t}, R_{m,t}, \{R_{i,t}\}_{i=1}^n]$ at the annual and quarterly frequencies. The second condition is satisfied by the GMM objective function in equation (a.11) because the first-stage GMM weighting matrix is constant, and equal to a diagonal matrix with prespecified elements. The third condition is satisfied because, according to the identification assumption above, the GMM objective function in equation (a.10) has a unique minimizer $x_0$ which can be identified. The fourth condition is satisfied because preference

---

\(^{16}\)See also the discussion in Newey and McFadden (1994, p. 2127) on the Hansen and Singleton (1982) model.
parameters are bounded and consumption growth is stationary. The fifth condition is also satisfied for the GMM system in equation (a.9) because the only point of discontinuity in the DA discount factor is

$$\Delta c_{t+1} = \mu_c(1 - \phi_c) + \phi_c \Delta c_t + d_1 \sqrt{1 - \phi_c^2} \sigma_c.$$ 

This discontinuity a zero-probability event $\forall x \in X$ because consumption growth is a continuous random variable. Finally, the last condition for consistency is satisfied because I assume that $X$ is compact and the distribution of $z_t$ has a well-defined moment generating function $\forall x \in X$.

**Asymptotic normality:** Theorem 7.2 in Newey and McFadden (1994) provides conditions for asymptotic normality of the GMM estimates when the GMM objective function is not continuous. These conditions are:

1. $[\frac{1}{T} \sum_{t=1}^{T} q(z_t, \hat{x}_T)]^T \hat{W} [\frac{1}{T} \sum_{t=1}^{T} q(z_t, \hat{x}_T)] \leq \inf_{x \in X} [\frac{1}{T} \sum_{t=1}^{T} q(z_t, x)]^T \hat{W} [\frac{1}{T} \sum_{t=1}^{T} q(z_t, x)] + o_p(T^{-1})$

2. $\hat{W} \overset{p}{\to} W$, $W$ is positive definite

3. $\hat{x}_T \overset{p}{\to} x_0$

4. $\mathbb{E}[g(z_t, x_0)] = 0$

5. $x_0$ is in the interior of $X$

6. $\sqrt{T}[\frac{1}{T} \sum_{t=1}^{T} q(z_t, x_0)] \overset{d}{\to} N(0, \Sigma)$

7. $\mathbb{E}[g(z_t, x)]$ is differentiable at $x_0$ with derivative $G$, and $G'WG$ is nonsingular

8. for $\delta_T \to 0$, then

$$\sup_{||x-x_0|| \leq \delta_T} \frac{\sqrt{T} \left[ \left| \frac{1}{T} \sum_{t=1}^{T} q(z_t, x) - \frac{1}{T} \sum_{t=1}^{T} q(z_t, x_0) \right| - \mathbb{E}[g(z_t, x)] \right|}{1 + \sqrt{T} ||x-x_0||} \overset{p}{\to} 0.$$

If the above conditions are met, then

$$\sqrt{T}(\hat{x}_T - x_0) \overset{d}{\to} N\left(0, (G'WG)^{-1}G'W\Sigma WG(G'WG)^{-1}\right),$$

where $G$ is the gradient of the GMM objective function and $\Sigma$ is the spectral density matrix.
The first condition above is related to identification. The second condition is satisfied by the GMM objective function for the DA model in equation (a.11) because $\hat{W}$ is a diagonal matrix with prespecified elements. The third condition is satisfied by the consistency theorem above. Conditions 4, 5, and 6 are standard GMM assumptions. The seventh condition is satisfied by the GMM system in equation (a.9) provided that the joint probability density function for asset returns and consumption growth is continuous, and the moment generating function is well-defined. The critical condition for asymptotic normality is condition 8, the stochastic equicontinuity condition, which is discussed in detail below.

I.2. The stochastic equicontinuity condition

Theorem 1 of Andrews (1994) provides primitive conditions to verify stochastic equicontinuity. These conditions are related to Pollard’s entropy condition (Pollard 1984). Specifically, according to Theorem 2 in Andrews, indicator functions, which are called “type I” functions, satisfy Pollard’s condition. A second class of functions that satisfy Pollard’s condition are functions that depend on a finite number of parameters and are Lipschitz-continuous with respect to these parameters. These functions are called “type II” functions.\(^{17}\)

The GMM system in equation (a.9) consists of linear and exponential terms, which are functions of a finite number of preference parameters. Exponential functions are only locally Lipschitz-continuous. However, the GMM system in equation (a.9) is Lipschitz-continuous because the parameter space $X$ is compact. Therefore, the rate of change of all the exponential functions in equation (a.9) remains bounded $\forall x \in X$. To conclude, according to Theorem 2 in Andrews (1994), the GMM system for the DA model includes terms that individually satisfy Pollard’s entropy condition. Further, according to Theorem 3 in Andrews, elementary operations between “type I” and “type II” functions result in functions that also satisfy Pollard’s entropy condition. Thus, the GMM system in equation (a.9) satisfies the stochastic equicontinuity condition because it is a product of “type I” and “type II” functions.

The previous discussion confirms that even though the GMM system in equation (a.9) is not continuous when $\theta$ is positive, I can still apply standard asymptotic results for hypothesis testing provided that certain regularity conditions are satisfied. These conditions are associated with\(^{17}\)

\(^{17}\)Lipschitz continuity is also exploited in Theorem 7.3 of Newey and McFadden (1994) as a primitive condition to show stochastic equicontinuity.
continuity” and “differentiability” of the function $E[q(z_t, x)]$ rather than the function $q(z_t, x)$.

I.3. Approximating the gradient of the GMM objective functions

Even if $q(z_t, x)$ in equation (a.9) is not continuously differentiable, I can still proceed with hypothesis testing by replacing derivatives with finite differences. Theorem 7.4 in Newey and McFadden (1994) suggests that the numerical derivative estimator of $\frac{1}{T} \sum_{t=1}^{T} q(z_t, \hat{x}_T)$ will asymptotically converge in probability to the derivative of $E[q(z_t, x_0)]$.

Specifically, let $e_i$ be the $i$th unit vector whose elements are all zeros except for the $i$th element which is 1. Also, let $h_{i,T}$ a small positive constant that depends on sample size $T$. If $h_{i,T} \to 0$ and $h_{i,T}\sqrt{T} \to +\infty$ as $T \to +\infty$, and the conditions in Theorem 7.2 of Newey and McFadden (1994) are satisfied, then

$$\frac{1}{T} \sum_{t=1}^{T} \frac{q(z_t, \hat{x}_T) - q(z_t, \hat{x}_T + e_i h_{i,T})}{h_{i,T}} \to G_i,$$

where $G_i$ is the $i$th element of the gradient of $E[q(z_t, x_0)]$. According to this theorem, I can obtain asymptotically consistent estimators of the GMM gradient using finite differences. However, a practical problem with numerical derivatives is the choice of the perturbation parameter $h_{i,T}$ used in the denominator of the expression in equation (a.12). To address this issue, I use the bootstrap method to estimate the gradient of $E[q(z_t, x_0)]$. Bootstrap gradients are less sensitive to the choice of the perturbation parameter $h_{i,T}$ due to the large bootstrap sample.

J. Bootstrap Test Statistics for the DA Discount Factor

In this section, I discuss the bootstrap methodology used to obtain confidence intervals and test statistics for the DA model. The original sample consists of $\{z_1, ..., z_T\}$ observations, where $z_t = [\Delta c_t, \Delta c_{t-1}, R_{f,t}, R_{m,t}, \{R_{i,t}\}_{i=1}^{n}]$. As in Hall and Horowitz (1996), I assume that $z_t$ is stationary, ergodic, and that $E[z_t'z_{t+s}] = 0$ for some $s < +\infty$. Stationarity, ergodicity, and weak time-series dependence are reasonable properties for consumption growth, the risk-free rate, and asset returns at the annual and quarterly frequencies.
J.1. Bootstrap sample

Following Kunsch (1989), I create \( m \) blocs of observations from the original sample in order to preserve the autocorrelation and covariance structures. Each block has length equal to \( l \) such that \( k \cdot l = T \), where \( k \) is an integer and \( T \) is the number of time-series observations in the original sample. I set the length of each bootstrap block equal to \( T/2 \) observations because long blocks of observations are required to accurately estimate autocorrelations and covariances. Horowitz (2001) discusses alternative methods of resampling dependent data and the optimal selection for \( m \) and \( l \).

Let \( Z_i, i \in \{1, .., m\} \) be a block of observations from the original data, and in particular let

\[
Z_1 = \{z_1, .., z_l\} \\
Z_2 = \{z_2, .., z_{l+1}\} \\
\vdots \\
Z_m = \{z_{T-l+1}, .., z_T\},
\]

with \( l = T/2 \). Let \( \{Z_1^*, ..., Z_B^*\} \) be a collection of \( B \) bootstrap samples generated by randomly drawing blocks of observations with replacement from the set of the available blocks \( Z_1, ..., Z_m \). Because I set \( l \) equal to \( T/2 \), each bootstrap sample, \( Z_b^* \), is the union of two blocks of observations, \( Z_b^* = Z_i \cup Z_j \), and has length \( T \). According to Efron and Tibshirani (1993, p. 162), 1000 replications are usually enough to compute standard errors and percentiles.

J.2. GMM in the bootstrap sample

Let \( z_t^{(b)} \) be the \( t^{th} \) observation of the bootstrap sample \( b \). For each bootstrap sample, we minimize the GMM objective function, and obtain parameter estimates \( \tilde{x}_T^{(b)} \) according to

\[
\tilde{x}_T^{(b)} = \underset{x}{\text{argmin}} \left[ \frac{1}{T} \sum_{t=1}^{T} q(z_t^{(b)}, x) - \frac{1}{T} \sum_{t=1}^{T} q(z_t, \tilde{x}_T) \right] \hat{W} \left[ \frac{1}{T} \sum_{t=1}^{T} q(z_t^{(b)}, x) - \frac{1}{T} \sum_{t=1}^{T} q(z_t, \tilde{x}_T) \right]. \tag{a.13}
\]

The vector-valued function \( q(z_t^{(b)}, x) \) is the vector of GMM moment conditions from equation (a.9) for the bootstrap observation \( z_t^{(b)} \), and \( \hat{W} \) is the first-stage weighting matrix. The vector \( \tilde{x}_T \) is the vector of the GMM estimates in the original sample. Following Hall and Horowitz (1996), the bootstrap version of the GMM objective function in equation (a.13) is recentered relative to the
population version to make sure that the bootstrap implements moment conditions that hold in
the population. By minimizing the expression in equation (a.13) for each bootstrap sample, the
bootstrap process results in a collection of parameter estimates \( \hat{x}(1)_T, \ldots, \hat{x}(1000)_T \) and GMM errors
\( \left( \frac{1}{T} \sum_{t=1}^{T} q(z_t, \hat{x}_T(1)), \ldots, \frac{1}{T} \sum_{t=1}^{T} q(z_t, \hat{x}_T(1000)) \right) \).

To show asymptotic normality of noncontinuous GMM estimators, in the previous section we
assumed that \( \sqrt{T} \left[ \frac{1}{T} \sum_{t=1}^{T} q(z_t, x_0) \right] \overset{d}{\rightarrow} N(0, \Sigma) \) or equivalently that
\[
T \left[ \frac{1}{T} \sum_{t=1}^{T} q(z_t, x_0) \right] \Sigma^{-1} \left[ \frac{1}{T} \sum_{t=1}^{T} q(z_t, x_0) \right] \overset{d}{\rightarrow} \chi^2(K),
\]
where \( \chi^2(K) \) is the chi-square distribution with \( K \) degrees of freedom. \( \Sigma \) is the asymptotic variance-
covariance matrix of \( \sqrt{T} \left[ \frac{1}{T} \sum_{t=1}^{T} q(z_t, x_0) \right] \), if we did not estimate any parameters (Cochrane, p.
204), and the parameter \( K \) is the number of GMM moment conditions. For the GMM system
in equation (a.9), \( K = n + 6 \), where \( n + 1 \) is the number of risky assets. The remaining five
GMM restrictions are the three moments of the consumption growth process (mean, variance,
autocovariance) and the two moments of the risk-free rate (mean and variance).

According to Lemma 4.1 in Hansen (1982) and page 210 in Cochrane (2001), the covariance
matrix for the first-stage GMM errors is given by
\[
\Sigma_{1st\ GMM} = \left( I - G(G'WG)G'W \right) \Sigma \left( I - G(G'WG)G'W \right)',
\]
where \( G \) is the gradient of \( E[q(z_t, x_0)] \). Based on the above, if \( \hat{\Sigma} \) and \( \hat{G} \) are consistent estimates of
\( \Sigma \) and \( G \) respectively, then
\[
\hat{\Sigma}_{1st\ GMM} = \left( I - \hat{G}(\hat{G}'\hat{W}\hat{G})\hat{G}'W \right) \hat{\Sigma} \left( I - \hat{G}(\hat{G}'\hat{W}\hat{G})\hat{G}'W \right)',
\]
\[
\sqrt{T} \left[ \frac{1}{T} \sum_{t=1}^{T} q(z_t, \hat{x}_T)_{1st\ GMM} \right] \overset{d}{\rightarrow} N(0, \hat{\Sigma}_{1st\ GMM}), \text{ and}
\]
\[
T \left[ \frac{1}{T} \sum_{t=1}^{T} q(z_t, \hat{x}_T)_{1st\ GMM} \right]' \left( \hat{\Sigma}_{1st\ GMM} \right)^{-1} \left[ \frac{1}{T} \sum_{t=1}^{T} q(z_t, \hat{x}_T)_{1st\ GMM} \right] \overset{d}{\rightarrow} \chi^2(K - L),
\]
where \( L \) is the number of parameters to be estimated.

When the number of observations is limited, asymptotic arguments fail. For instance, estimates
of the covariance matrix $\Sigma^{1st\ GMM}$ could be biased, and $\hat{\Sigma}^{1st\ GMM}$ could be singular. Furthermore, the GMM function $q(z_t, x_0)$ in equation (a.9) is not differentiable. Therefore, as shown in equation (a.12) of the previous section, $\hat{\Sigma}^{1st\ GMM}$ will depend on the choice of the perturbation parameter $h_{i,T}$ in the finite difference approximation of the GMM gradient $\hat{G}$. To a large extend all these issues can be addressed by the bootstrap methodology.

### J.3. Bootstrap test statistics

The bootstrap method consists of the following steps. First, I obtain an estimate of $\Sigma$ from the bootstrap sample

$$\hat{\Sigma}_{\text{boot}} = T \text{Cov} \left[ \frac{1}{T} \sum_{t=1}^{T} q(z_t^{(b)}, \hat{x}_T^{(b)})^{1st\ GMM} \right].$$

Then, I estimate $\hat{\Sigma}_{\text{boot}}^{1st\ GMM}$ as

$$\hat{\Sigma}_{\text{boot}}^{1st\ GMM} = \left( I - \hat{G}_{\text{boot}} \left( \hat{G}_{\text{boot}}' \hat{W} \hat{G}_{\text{boot}} \right) \right) \hat{\Sigma}_{\text{boot}} \left( I - \hat{G}_{\text{boot}} \left( \hat{G}_{\text{boot}}' \hat{W} \hat{G}_{\text{boot}} \right) \right)' \hat{G}_{\text{boot}}' \hat{W},$$

in which $\hat{G}_{\text{boot}}$ is the bootstrap estimate of the gradient of $E[q(z_t, x_0)]$. Finally, I proceed with hypothesis testing using

1. the entire set of the moment conditions:

$$T \left[ \frac{1}{T} \sum_{t=1}^{T} q(z_t, \hat{x}_T)^{1st\ GMM} \right]' \left( \hat{\Sigma}_{\text{boot}}^{1st\ GMM} \right)^{-1} \left[ \frac{1}{T} \sum_{t=1}^{T} q(z_t, \hat{x}_T)^{1st\ GMM} \right] \overset{d}{\to} \chi^2(K-L),$$

under $H_0: E[q(z_t, x_0)] = 0$

2. subsets of the moment conditions:

$$\sqrt{T} A \left[ \frac{1}{T} \sum_{t=1}^{T} q(z_t, \hat{x}_T)^{1st\ GMM} \right] \overset{d}{\to} N(0, A \hat{\Sigma}_{\text{boot}}^{1st\ GMM} A'),$$

under $H_0: AE[q(z_t, x_0)] = 0$

with $A$ an $M \times K$ matrix,
3. individual moment conditions:

$$\sqrt{T}\left[\frac{1}{T} \sum_{t=1}^{T} q_i(z_t, \hat{x}_T)^{1st \ GMM}\right] \xrightarrow{d} N(0, [\hat{\Sigma}^{1st \ GMM}_{boot}]_{ii}), \text{ under } H_0 : \mathbb{E}[q_i(z_t, x_0)] = 0.$$ 

Above, the exponent $* - 1*$ denotes the Moore-Penrose pseudo-inverse. This inverse adjusts the bootstrap covariance estimator for recentering in the bootstrap GMM objective function (equation (a.13)), and for the fact that the bootstrap blocks may not exactly replicate the dependence structure of the data (Hall and Horowitz 1996 and Chou and Zhou 2006). The random variable $q_i(z_t, x)$ above is the $i$-th element of the GMM system $q(z_t, x)$ (equation (a.9)), and $[\hat{\Sigma}^{1st \ GMM}_{boot}]_{ii}$ is the $i$-th diagonal element of the covariance matrix $\hat{\Sigma}^{1st \ GMM}_{boot}$.

J.4. The Hansen-Jagannathan statistic in the bootstrap sample

I also use the bootstrap methodology to estimate the $p$-values of the $HJ$-statistic shown in the main text. This statistic is based on the Hansen and Jagannathan (1997) distance. Specifically, the $HJ$-statistic in the paper is equal to $T \times Distance^2$, where $Distance$ is the Hansen and Jagannathan distance. To calculate the $p$-values for this statistic, let $B = [0 \ \Lambda]$ be a $(n + 1) \times (n + 6)$ matrix, where $0$ is a $(n + 1) \times 5$ matrix of zeros that neutralizes the five moment conditions for the consumption growth process and the risk-free rate in the GMM system of equation (a.9). Further, the matrix $\Lambda$ is the upper triangular Cholesky decomposition of the matrix $\hat{\Sigma}^{-1}[(\mathbf{R}_t - \mathbf{R}_f)(\mathbf{R}_t - \mathbf{R}_f)']$, which is the inverse of the $(n + 1) \times (n + 1)$ second moments matrix for the set of risky assets. Then, it follows that the expression

$$T\left[\frac{1}{T} \sum_{t=1}^{T} q(z_t, \hat{x}_T)^{1st \ GMM}\right]'B'B\left[\frac{1}{T} \sum_{t=1}^{T} q(z_t, \hat{x}_T)^{1st \ GMM}\right]$$

is the square of the Hansen-Jagannathan distance for the set of risky assets multiplied by $T$, i.e, the $HJ$-statistic of the paper:

$$T\left[\frac{1}{T} \sum_{t=1}^{T} g(z_t, \hat{x}_T)^{1st \ GMM}\right]'\hat{\Sigma}^{-1}[(\mathbf{R}_t - \mathbf{R}_f)(\mathbf{R}_t - \mathbf{R}_f)']^{-1}\left[\frac{1}{T} \sum_{t=1}^{T} g(z_t, \hat{x}_T)^{1st \ GMM}\right].$$
Above, \( g(z_t, \hat{x}_T) \) is the vector of moment conditions for the set of risky assets only. Further, the statistic
\[
T \left[ \frac{1}{T} \sum_{t=1}^{T} q(z_t, \hat{x}_T) \right]^{1st \ GMM} B' \left( B \sum_{boot}^{1st \ GMM} B' \right)^{-1} B' \left[ \frac{1}{T} \sum_{t=1}^{T} q(z_t, \hat{x}_T) \right]^{1st \ GMM}
\]
has a \( \chi^2_{(K-L-5)} \) distribution.

**J.5. Hypothesis testing for model parameters with the bootstrap sample**

Finally, the bootstrap methodology can be used to estimate the distribution of the point estimates of the preference parameters. Specifically, if \( \hat{x}_T^{(b)} \) is a bootstrap estimate of the parameters in the DA model, then, as long as \( \hat{x}_T - x_0 \sim \hat{x}_T^{(b)} - \hat{x}_T \), we can test the significance of the GMM estimates of \( x_0 \) using confidence intervals from the distribution of the bootstrap estimates \( \hat{x}_T^{(b)} \). In particular, the \( 1-a\% \) bootstrap confidence interval is given by
\[
\{ \hat{x}_{T,a\%}, \hat{x}_{T,1-a\%} \} \ s.t. \ \hat{P}(x_0 \in [\hat{x}_{T,a\%}, \hat{x}_{T,1-a\%}])_{boot} = 1 - 2a\%.
\]

**K. The Risk-Free Rate and the Conditional Performance of the DA Discount Factor**

In equation (A12) of the main text, I show that the DA stochastic discount factor is given by
\[
M_{t+1}^{DA} = e^{\log \beta + (\rho - 1) \Delta c_{t+1} + \frac{\rho - \alpha}{1 - \kappa_c \phi_c} \sigma_c (1 - \phi_c) + \frac{\rho - \alpha}{1 - \kappa_c \phi_c} \Delta c_t + \frac{(\rho - \alpha) \phi_c}{1 - \kappa_c \phi_c} \Delta c_t} \times \frac{1 + \theta \left[ \Delta c_{t+1} \leq \mu_c (1 - \phi_c) + \phi_c \Delta c_t + d_1 \sqrt{1 - \phi_c^2 \sigma_c} \right]}{1 + \theta \mathbb{E}_t [\Delta c_{t+1} \leq \mu_c (1 - \phi_c) + \phi_c \Delta c_t + d_1 \sqrt{1 - \phi_c^2 \sigma_c}]},
\]
where \( d_1 \) is the solution to the fixed point problem from equation (5) of the paper:
\[
d_1 = \frac{\alpha}{2(1 - \kappa_c \phi_c)} \sqrt{1 - \phi_c^2 \sigma_c} + \frac{1}{1 - \kappa_c \phi_c} \sqrt{1 - \phi_c^2 \sigma_c} \log \left[ \frac{1 + \theta N \left( d_1 - \frac{\alpha}{1 - \kappa_c \phi_c} \sqrt{1 - \phi_c^2 \sigma_c} \right)}{1 + \theta N \left( d_1 \right)} \right].
\]

Combining the above expressions with the AR(1) dynamics for consumption growth in equation (4) of the paper and the conditional Euler equation for the risk-free rate (\( \mathbb{E}_t[M_{t+1}^{DA}] R_{f,t+1} = 1 \)), I
can write the one-period real log risk-free rate as

\[
\begin{align*}
    r_{f,t+1} &= -\log \beta + (1 - \rho) \mu_c (1 - \phi_c) + (1 - \rho) \phi_c \Delta c_t \\
    &- \frac{1}{2} \left[ \left( \frac{\rho - \alpha}{1 - \kappa_c \phi_c} + 1 - \rho \right)^2 + \frac{\alpha (\rho - \alpha)}{(1 - \kappa_c \phi_c)^2} \right] (1 - \phi_c^2) \sigma_c^2 \\
    &- \frac{\rho - \alpha}{\alpha} \log \frac{1 + \theta N \left( d_1 - \frac{\alpha}{1 - \kappa_c \phi_c} \sqrt{1 - \phi_c^2} \sigma_c \right)}{1 + \theta N (d_1)} - \log \frac{1 + \theta N \left( d_1 + \left( \frac{\rho - \alpha}{1 - \kappa_c \phi_c} + 1 - \rho \right) \sqrt{1 - \phi_c^2} \sigma_c \right)}{1 + \theta N (d_1)}
\end{align*}
\]  

(a.15)

Using the above expression and the estimates from Table 2 of the main text, I can assess the conditional performance of the DA model in fitting the observed risk-free rate. To this end, Figure A.2 shows the realized and fitted values for the annual real log risk-free rate during the 1933-2012 period. The model-implied risk-free rate is generated using equation (a.15) and the estimated parameters for the DA(2) model from Table 2 of the main text. The results are very similar for the remaining DA models of Table 2 of the paper with the exception of the DA(1) model which implies a constant risk-free rate.

The mean of the model-implied risk-free rate is 0.25% and its standard deviation is 3.71%. These number are very similar to the sample moments for the risk-free rate during the 1933-2012 period, which are equal to 0.17% and 3.41%, respectively (see Table 1 of the main text). This is because the GMM system in equation (9) of the paper forces the DA model to almost perfectly fit the mean and volatility of the risk-free rate. However, Figure A.2 shows that the fitted and realized risk-free rate do not always coincide. This is particularly true around the WWII period, when consumption growth was high but the risk-free rate was low, and during the early 80’s, when consumption growth was low but the real risk-free was high. The conditional performance of the DA model in fitting the risk-free rate is driven by the assumption that consumption growth follows a very simple AR(1) process. By introducing additional state variables, such as stochastic volatility or MA terms, I could probably improve the fit of the DA model in explaining the risk-free rate on a period-by-period basis.
References


Figures

Figure A.1  Consumption growth, loss events, and NBER recessions

Figure A.1 shows annual loss events in consumption growth for the FORA discount factor of equation (a.6) with alternative reference points for gains and losses. The solid line is consumption growth. The dotted horizontal line shows the zero reference point according to the status quo model (FORA(1) model in Table A.8). The time varying dashed line shows the expected consumption growth reference point according to the expectation model (FORA(2) model in Table A.8). Finally, the horizontal dashed-dotted line shows the constant reference point estimated by GMM (FORA(4) model in Table A.8). Shaded areas are NBER recession dates. The sample period is 1933-2012.

A.27
Figure A.2  Realized and fitted real log risk-free rate

Figure A.2 shows the realized (solid line) and fitted (dashed line) annual real log risk-free rate according to the DA model. The expression for the fitted risk-free rate is given in equation (a.15). To generate the fitted risk-free rate, I use equation (a.15) and the estimated parameters for the DA(2) model in Table 2 of the main text. The sample period is 1933-2012.
Tables

Table A.1  GMM results for the consumption-based GDA discount factor

Table A.1 shows estimation results for annual returns and the GDA discount factor of equation (a.3) with different values for the risk aversion parameter $\alpha$. Unlike the analysis in the main text where the GDA parameter $\delta$ is one, in this table, $\delta$ is unrestricted and estimated by GMM. The GMM moment conditions are the consumption growth mean, the consumption growth variance, the consumption growth autocovariance, the variance of the risk-free rate, and the unconditional Euler equations for the risk-free asset, the aggregate stock market index, and the Fama-French six size/bm portfolios. The moment conditions for the GDA(1) model exclude the variance of the risk-free rate. Table A.1 does not report the results for the consumption growth moments. The parameter $\beta$ is the discount rate, $\alpha$ is the risk aversion parameter, $\rho$ is equal to $1 - 1/EIS$, $\theta$ is the disappointment aversion coefficient, and $\delta$ is the GDA parameter. The constant $d_1$ is the disappointment threshold from equation (a.2), and $\mathbb{P}(\text{disap.})$ is the probability of disappointment events. Bootstrap 95% confidence intervals are shown in brackets. $\chi^2$, d.o.f., and $p$ are the bootstrap first-stage $\chi^2$ test (Hansen (1982), Lemma 4.1), degrees of freedom, and $p$-value that all moment conditions are jointly zero. RMSE and $R^2$ are the cross-sectional root mean square error ($\times100$) and $R$-square ($\times100$), respectively, for the set of risky assets. The sample period is 1933-2012.

<table>
<thead>
<tr>
<th>Generalized disappointment aversion (GDA)</th>
<th>GDA(1)</th>
<th>GDA(2)</th>
<th>GDA(3)</th>
<th>GDA(4)</th>
<th>GDA(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = \rho = 1$</td>
<td>0.996</td>
<td>1.093</td>
<td>1.112</td>
<td>1.098</td>
<td>1.112</td>
</tr>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\alpha = -3$</td>
<td>-6.380</td>
<td>-14.96</td>
<td>-1.75</td>
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<td></td>
</tr>
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<td>$[-10.34, -2.76]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\alpha = -9$</td>
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<td>-5.921</td>
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<td>-12.62</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$\alpha = -19$</td>
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<td>7.309</td>
<td>4.570</td>
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<td>5.732</td>
</tr>
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<td>$[6.79, 12.94]$</td>
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<td></td>
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</tr>
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<td>$\theta$</td>
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<td>0.992</td>
<td>0.993</td>
<td>1.000</td>
<td>0.995</td>
</tr>
<tr>
<td>$[0.95, 1.00]$</td>
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<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_1$</td>
<td>-0.779</td>
<td>-0.680</td>
<td>-0.618</td>
<td>-0.669</td>
<td>-0.703</td>
</tr>
<tr>
<td>$\mathbb{P}(\text{disap.})$</td>
<td>0.112</td>
<td>0.100</td>
<td>0.100</td>
<td>0.200</td>
<td>0.100</td>
</tr>
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<td>$\chi^2$</td>
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<td>1.008</td>
<td>0.957</td>
<td>2.812</td>
<td>0.787</td>
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<td>4</td>
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</tr>
<tr>
<td>$p$</td>
<td>0.992</td>
<td>0.961</td>
<td>0.965</td>
<td>0.728</td>
<td>0.940</td>
</tr>
<tr>
<td>$RMSE$</td>
<td>0.388</td>
<td>0.725</td>
<td>0.886</td>
<td>0.662</td>
<td>0.813</td>
</tr>
<tr>
<td>$R^2$</td>
<td>98.2</td>
<td>93.9</td>
<td>91.0</td>
<td>94.9</td>
<td>92.4</td>
</tr>
</tbody>
</table>
Table A.2  GMM results for the consumption-based DA discount factor: Covariance restrictions

Table A.2 shows estimation results for annual returns and the DA discount factor from equation (6) of the main paper with different values for the risk aversion parameter $\alpha$. Table A.2 augments the set of GMM moment conditions of the main text by including two additional restrictions. The first restriction estimates the covariance of the DA discount factor with the excess return on the stock market ($\text{Cov}(M_t, R_{m,t} - R_{f,t}) - \text{cov}_1 = 0$). The second restriction estimates the covariance of the DA discount factor with the spread in returns of the small/value ($R_{s/v,t}$) minus the big/growth ($R_{b/g,t}$) portfolio ($\text{Cov}(M_t, R_{s/v,t} - R_{b/g,t}) - \text{cov}_2 = 0$). The remaining GMM moment conditions are the consumption growth mean, the consumption growth variance, the consumption growth autocovariance, the variance of the risk-free rate, and the unconditional Euler equations for the risk-free asset, the aggregate stock market index, and the Fama-French six size/bm portfolios. The moment conditions for the DA(1) model exclude the variance of the risk-free rate. Table A.2 does not report the results for the consumption growth moments. $\beta$ is the discount rate, $\alpha$ is the risk aversion parameter, $\rho$ is equal to $1 - 1/EIS$, and $\theta$ is the disappointment aversion coefficient. $\text{cov}_1$ is the estimated covariance of the discount factor and the market excess return, and $\text{cov}_2$ is the estimated covariance of the discount factor and the spread in returns of the small/value minus the big/growth portfolio. The constant $d_1$ is the disappointment threshold from equation (5) of the main text, and $\mathbb{P}(\text{disap.})$ is the probability of disappointment events. Bootstrap 95% confidence intervals are shown in brackets. $\chi^2$, d.o.f., and $p$ are the bootstrap first-stage $\chi^2$ test, degrees of freedom, and $p$-value that all moment conditions are jointly zero. $\text{RMSE}$ and $R^2$ are the cross-sectional root mean square error and $R$-square, respectively, for the set of risky assets. The sample period is 1933-2012.

<table>
<thead>
<tr>
<th>Disappointment aversion (DA)</th>
<th>CRRA $\alpha = \rho, \theta = 0$</th>
<th>EZ $\theta = 0$</th>
<th>DA(1) $\alpha = \rho = 1$</th>
<th>DA(2) $\alpha = -3$</th>
<th>DA(3) $\alpha = -9$</th>
<th>DA(4) $\alpha = -19$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.217 [1.06, 1.51]</td>
<td>1.163 [0.87, 1.42]</td>
<td>0.998 [0.98, 1.01]</td>
<td>1.070 [1.02, 1.12]</td>
<td>1.080 [1.01, 1.12]</td>
<td>1.067 [0.99, 1.20]</td>
</tr>
<tr>
<td>$\text{cov}_1$</td>
<td>-0.006 [-0.02, 0.00]</td>
<td>-0.089 [-0.09, -0.05]</td>
<td>-0.089 [-0.09, -0.05]</td>
<td>-0.088 [-0.09, -0.05]</td>
<td>-0.088 [-0.09, -0.05]</td>
<td>-0.088 [-0.09, -0.05]</td>
</tr>
<tr>
<td>$\text{cov}_2$</td>
<td>-0.013 [-0.03, 0.00]</td>
<td>-0.053 [-0.06, -0.02]</td>
<td>-0.076 [-0.10, -0.02]</td>
<td>-0.073 [-0.08, -0.02]</td>
<td>-0.077 [-0.08, -0.03]</td>
<td>-0.063 [-0.08, -0.03]</td>
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<td>$d_1$</td>
<td>-0.884</td>
<td>-0.862</td>
<td>-0.780</td>
<td>-0.676</td>
<td>-0.676</td>
<td>-0.676</td>
</tr>
<tr>
<td>$\mathbb{P}(\text{disap.})$</td>
<td>0.100</td>
<td>0.100</td>
<td>0.137</td>
<td>0.212</td>
<td>0.212</td>
<td>0.212</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>738.363 [114.194]</td>
<td>0.557</td>
<td>2.153</td>
<td>6.949</td>
<td>37.805</td>
<td>1.071</td>
</tr>
<tr>
<td>d.o.f.</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>$p$</td>
<td>0.000</td>
<td>0.997</td>
<td>0.905</td>
<td>0.325</td>
<td>0.000</td>
<td>0.956</td>
</tr>
<tr>
<td>$\text{RMSE}$</td>
<td>9.906</td>
<td>1.590</td>
<td>0.545</td>
<td>0.730</td>
<td>0.528</td>
<td>1.133</td>
</tr>
<tr>
<td>$R^2$</td>
<td>-1038.3</td>
<td>71.2</td>
<td>96.6</td>
<td>93.8</td>
<td>96.8</td>
<td>85.4</td>
</tr>
</tbody>
</table>
Table A.3  GMM results for the consumption-based DA discount factor: Post-1945 sample

Table A.3 shows estimation results for annual returns and the DA discount factor from equation (6) of the main paper with different values for the risk aversion parameter $\alpha$. Table A.3 focuses on the post-1945 sample. The GMM moment conditions for the consumption-based models are the consumption growth mean, the consumption growth variance, the consumption growth autocovariance, the variance of the risk-free rate, and the unconditional Euler equations for the risk-free asset, the aggregate stock market index, and the Fama-French six size/bm portfolios. The moment conditions for the DA(1) model exclude the variance of the risk-free rate. The moment conditions for the Fama-French model exclude the consumption growth moments and the variance of the risk-free rate. The constant $\mu_c$ is the consumption growth mean ($\times 100$), $\sigma^2_c$ is consumption growth variance ($\times 100$), and $\phi_c \sigma^2_c$ is consumption growth autocovariance ($\times 100$). The parameter $\beta$ is the discount rate, $\alpha$ is the risk aversion parameter, $\rho$ is equal to $1 - 1/EIS$, and $\theta$ is the disappointment aversion coefficient. The constant $d_1$ is the disappointment threshold from equation (5) of the main text, and $P(\text{disap.})$ is the probability of disappointment events. Bootstrap 95% confidence intervals are shown in brackets. Bootstrap $\chi^2$, d.o.f., and $p$ are the bootstrap first-stage $\chi^2$ test, degrees of freedom, and $p$-value that all moment conditions are jointly zero. $HJ$ is equal to $T \times Distance^2$, where $Distance$ is the Hansen-Jagannathan distance (Hansen and Jagannathan (1997)) for excess returns in the set of risky assets and $T$ is the number of time-series observations. RMSE and $R^2$ are the cross-sectional root mean square error and $R$-square, respectively, for the set of risky assets. The sample period is 1945-2012.

<table>
<thead>
<tr>
<th>Disappointment aversion (DA)</th>
<th>CRRA</th>
<th>EZ</th>
<th>DA(1)</th>
<th>DA(2)</th>
<th>DA(3)</th>
<th>DA(4)</th>
<th>DA(5)</th>
<th>FF3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = \rho, \theta = 0$</td>
<td>$\mu_c$</td>
<td>1.922</td>
<td>1.922</td>
<td>1.922</td>
<td>1.922</td>
<td>1.922</td>
<td>1.922</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$[1.79, 2.36]$</td>
<td>$[1.79, 2.36]$</td>
<td>$[1.79, 2.36]$</td>
<td>$[1.79, 2.36]$</td>
<td>$[1.79, 2.36]$</td>
<td>$[1.79, 2.36]$</td>
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</tr>
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<td>$\sigma^2_c$</td>
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<td>0.018</td>
<td>0.018</td>
<td>0.018</td>
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<td>0.018</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$[0.011, 0.016]$</td>
<td>$[0.011, 0.016]$</td>
<td>$[0.006, 0.022]$</td>
<td>$[0.009, 0.021]$</td>
<td>$[0.010, 0.020]$</td>
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<td>$[0.010, 0.019]$</td>
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</tr>
<tr>
<td>$\phi_c \sigma^2_c$</td>
<td>0.008</td>
<td>0.008</td>
<td>0.008</td>
<td>0.008</td>
<td>0.007</td>
<td>0.005</td>
<td>0.008</td>
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<td>$[0.002, 0.010]$</td>
<td>$[0.003, 0.011]$</td>
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<td>$[0.005, 0.010]$</td>
<td>$[0.004, 0.010]$</td>
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</tr>
<tr>
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<td>1.075</td>
<td>0.944</td>
<td>0.997</td>
<td>1.032</td>
<td>1.040</td>
<td>1.029</td>
<td>1.027</td>
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<tr>
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<tr>
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<td>-0.694</td>
<td>-0.905</td>
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</tr>
<tr>
<td>$P(\text{disap.})$</td>
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<td>0.102</td>
<td>0.102</td>
<td>0.102</td>
<td>0.102</td>
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<tr>
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<td>2.027</td>
<td>7.588</td>
<td>0.804</td>
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</tr>
<tr>
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<td>$HJ$</td>
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<td>3.068</td>
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<tr>
<td>RMSE</td>
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<td>0.363</td>
<td>0.477</td>
<td>0.797</td>
<td>0.278</td>
<td>0.969</td>
</tr>
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<td>97.4</td>
<td>97.7</td>
<td>96.1</td>
<td>89.3</td>
<td>98.6</td>
<td>83.8</td>
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</table>

A.31
Table A.4  GMM results for the consumption-based DA discount factor: Earnings/price portfolios

Table A.4 shows estimation results for annual returns and the DA discount factor from equation (6) of the main text with different values for the risk aversion parameter $\alpha$. The GMM moment conditions for the consumption-based models are the consumption growth mean, the consumption growth variance, the consumption growth autocovariance, the variance of the risk-free rate, and the unconditional Euler equations for the risk-free asset, the aggregate stock market index, and the Fama-French five earnings/price portfolios. The moment conditions for the DA(1) model exclude the variance of the risk-free rate. The moment conditions for the Fama-French model exclude the consumption growth moments and the variance of the risk-free rate. Table A.4 does not report the results for the consumption growth moments. The parameter $\beta$ is the discount rate, $\alpha$ is the risk aversion parameter, $\rho$ is equal to $1 - 1/EIS$, and $\theta$ is the disappointment aversion coefficient. The constant $d_1$ is the disappointment threshold from equation (5) of the main text, and $P(\text{disap.})$ is the probability of disappointment events. Bootstrap 95% confidence intervals are shown in brackets. $\chi^2$, $d.o.f.$, and $p$ are the bootstrap first-stage $\chi^2$ test, degrees of freedom, and $p$-value that all moment conditions are jointly zero. RMSE and $R^2$ are the cross-sectional root mean square error and $R$-square, respectively, for the set of risky assets. The sample period is 1953-2012.

<table>
<thead>
<tr>
<th>Disappointment aversion (DA)</th>
<th>CRRRA $\alpha = \rho$, $\theta = 0$</th>
<th>EZ $\theta = 0$</th>
<th>DA(1) $\alpha = \rho = 1$</th>
<th>DA(2) $\alpha = -3$</th>
<th>DA(3) $\alpha = -9$</th>
<th>DA(4) $\alpha = -19$</th>
<th>DA(5)</th>
<th>FF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.054 [1.02, 1.23]</td>
<td>1.026 [0.47, 1.17]</td>
<td>0.989 [0.97, 0.99]</td>
<td>1.032 [0.94, 1.08]</td>
<td>1.046 [0.97, 1.19]</td>
<td>1.047 [0.93, 1.16]</td>
<td>1.020 [0.90, 1.11]</td>
<td></td>
</tr>
<tr>
<td>$d_1$</td>
<td>-0.977 [0.133]</td>
<td>-0.893 [0.100]</td>
<td>-0.945 [0.116]</td>
<td>-0.857 [0.150]</td>
<td>-0.960 [0.150]</td>
<td>0.100 [0.100]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P(\text{disap.})$</td>
<td>0.133</td>
<td>0.100</td>
<td>0.116</td>
<td>0.150</td>
<td>0.150</td>
<td>0.100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d.o.f.$</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>0</td>
<td>0.013</td>
<td>0.148</td>
<td>0.999</td>
<td>0.973</td>
<td>0.754</td>
<td>0.996</td>
<td>0.175</td>
</tr>
<tr>
<td>RMSE</td>
<td>8.326</td>
<td>1.737</td>
<td>0.708</td>
<td>0.475</td>
<td>0.276</td>
<td>0.572</td>
<td>0.344</td>
<td>0.244</td>
</tr>
<tr>
<td>$R^2$</td>
<td>-1048.9</td>
<td>50.6</td>
<td>91.8</td>
<td>96.3</td>
<td>98.7</td>
<td>94.6</td>
<td>98.0</td>
<td>97.6</td>
</tr>
</tbody>
</table>

A.32
Table A.5 GMM results for the DA discount factor: Corporate bonds

Table A.5 shows estimation results for annual returns and the DA discount factor from equation (6) of the main text with different values for the risk aversion parameter $\alpha$. The GMM moment conditions for the consumption-based models are the consumption growth mean, the consumption growth variance, the consumption growth autocovariance, the variance of the risk-free rate, and the unconditional Euler equations for the risk-free asset, the aggregate stock market index, the aggregate corporate bond market index, and four investment-grade bond portfolios sorted by credit rating. The moment conditions for the DA(1) model exclude the variance of the risk-free rate. The moment conditions for the Fama-French model exclude the consumption growth moments and the variance of the risk-free rate. Table A.5 does not report the results for the consumption growth moments. The parameter $\beta$ is the discount rate, $\alpha$ is the risk aversion parameter, $\rho$ is equal to $1 - \frac{1}{EIS}$, and $\theta$ is the disappointment aversion coefficient. The constant $d_1$ is the disappointment threshold from equation (5) of the paper, and $\mathbb{P}(\text{disap.})$ is the probability of disappointment events. Bootstrap 95% confidence intervals are shown in brackets. $\chi^2$, $d.o.f.$, and $p$ are the bootstrap first-stage $\chi^2$ test, degrees of freedom, and $p$-value that all moment conditions are jointly zero. $RMSE$ and $R^2$ are the cross-sectional root mean square error and $R$-square, respectively, for the set of risky assets. The sample period is 1973-2012.

<table>
<thead>
<tr>
<th>Disappointment aversion (DA)</th>
<th>CRRA $\alpha = \rho$, $\theta = 0$</th>
<th>EZ $\theta = 0$</th>
<th>DA(1) $\alpha = \rho = 1$</th>
<th>DA(2) $\alpha = -3$</th>
<th>DA(3) $\alpha = -9$</th>
<th>DA(4) $\alpha = -19$</th>
<th>DA(5)</th>
<th>FF3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.046</td>
<td>0.993</td>
<td>0.986</td>
<td>1.024</td>
<td>1.011</td>
<td>1.011</td>
<td>1.011</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[1.01, 1.16]</td>
<td>[0.18, 3.90]</td>
<td>[0.89, 1.08]</td>
<td>[0.81, 1.13]</td>
<td>[0.84, 1.42]</td>
<td>[0.66, 1.43]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>-2.891</td>
<td>-2.878</td>
<td>-3.391</td>
<td>-2.840</td>
<td>-2.879</td>
<td>-2.879</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>5.644</td>
<td>4.549</td>
<td>3.089</td>
<td>1.096</td>
<td>2.567</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[5.26, 54.37]</td>
<td>[3.68, 32.63]</td>
<td>[2.36, 27.82]</td>
<td>[0.42, 7.65]</td>
<td>[1.84, 19.74]</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$d_1$</td>
<td>-0.745</td>
<td>-0.708</td>
<td>-0.661</td>
<td>-0.510</td>
<td>-0.627</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathbb{P}(\text{disap.})$</td>
<td>0.150</td>
<td>0.150</td>
<td>0.225</td>
<td>0.250</td>
<td>0.225</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>139.007</td>
<td>0.398</td>
<td>93.164</td>
<td>1.486</td>
<td>1.155</td>
<td>0.998</td>
<td>1.188</td>
<td>2.709</td>
</tr>
<tr>
<td>$d.o.f.$</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>0</td>
<td>0.995</td>
<td>0</td>
<td>0.922</td>
<td>0.949</td>
<td>0.962</td>
<td>0.879</td>
<td>0.438</td>
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<tr>
<td>$RMSE$</td>
<td>3.566</td>
<td>0.466</td>
<td>0.430</td>
<td>0.315</td>
<td>0.352</td>
<td>0.289</td>
<td>0.320</td>
<td>0.132</td>
</tr>
<tr>
<td>$R^2$</td>
<td>-964.7</td>
<td>82.0</td>
<td>84.6</td>
<td>91.7</td>
<td>89.7</td>
<td>93.0</td>
<td>91.5</td>
<td>98.5</td>
</tr>
</tbody>
</table>
Table A.6  GMM results for the DA discount factor: Commodity futures

Table A.6 shows estimation results for annual returns and the DA discount factor from equation (6) of the main text with different values for the risk aversion parameter $\alpha$. Table A.6 also shows results for the CRRA, Epstein-Zin, and Fama-French models. The GMM moment conditions for the consumption-based models are the consumption growth mean, the consumption growth variance, the consumption growth autocovariance, the variance of the risk-free rate, and the unconditional Euler equations for the risk-free asset, the aggregate stock market index, and five portfolios of commodity futures sorted by the basis. The moment conditions for the DA(1) model exclude the variance of the risk-free rate. The moment conditions for the Fama-French model exclude the consumption growth moments and the variance of the risk-free rate. The moment conditions for the Fama-French model exclude the consumption growth moments and the variance of the risk-free rate. Table A.6 does not report the results for the consumption growth moments. The parameter $\beta$ is the discount rate, $\alpha$ is the risk aversion parameter, $\rho$ is equal to $1 - 1/EIS$, and $\theta$ is the disappointment aversion coefficient. The constant $d_1$ is the disappointment threshold from equation (5) of the paper, and $P(\text{disap.})$ is the probability of disappointment events. Bootstrap 95% confidence intervals are shown in brackets. $\chi^2$, d.o.f., and $p$ are the bootstrap first-stage $\chi^2$ test, degrees of freedom, and $p$-value that all moment conditions are jointly zero. RMSE and $R^2$ are the cross-sectional root mean square error and $R$-square, respectively, for the set of risky assets. The sample period is 1974-2008.

<table>
<thead>
<tr>
<th></th>
<th>CRRA $\alpha = \rho$, $\theta = 0$</th>
<th>EZ $\theta = 0$</th>
<th>DA(1) $\alpha = \rho = 1$</th>
<th>DA(2) $\alpha = -3$</th>
<th>DA(3) $\alpha = -9$</th>
<th>DA(4) $\alpha = -19$</th>
<th>DA(5)</th>
<th>FF3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.042</td>
<td>1.033</td>
<td>0.980</td>
<td>1.033</td>
<td>1.012</td>
<td>1.008</td>
<td>1.010</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[1.01, 1.29]</td>
<td>[0.19, 3.70]</td>
<td>[0.95, 0.98]</td>
<td>[0.83, 1.12]</td>
<td>[0.84, 1.17]</td>
<td>[0.72, 1.14]</td>
<td>[0.80, 1.29]</td>
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</tr>
<tr>
<td>$\alpha$</td>
<td></td>
<td>-35.260</td>
<td>0.962</td>
<td>0</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>[-320.185, -14.33]</td>
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</tr>
<tr>
<td>$\rho$</td>
<td>2.615</td>
<td>2.603</td>
<td>-3.027</td>
<td>-2.836</td>
<td>-2.603</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td></td>
<td>10.178</td>
<td>7.675</td>
<td>4.612</td>
<td>2.309</td>
<td>10.808</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[5.43, 29.30]</td>
<td>[7.32, 96.22]</td>
<td>[4.07, 35.08]</td>
<td>[2.09, 20.94]</td>
<td>[7.53, 64.73]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_1$</td>
<td>-0.961</td>
<td>-0.881</td>
<td>-0.776</td>
<td>-0.678</td>
<td>-0.952</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P(\text{disap.})$</td>
<td>0.142</td>
<td>0.142</td>
<td>0.142</td>
<td>0.171</td>
<td>0.142</td>
<td></td>
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</tr>
<tr>
<td>$\chi^2$</td>
<td>140.346</td>
<td>2.004</td>
<td>10.343</td>
<td>4.206</td>
<td>12.884</td>
<td>13.687</td>
<td>0.419</td>
<td>11.557</td>
</tr>
<tr>
<td>d.o.f.</td>
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<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>0.848</td>
<td>0</td>
<td>0.520</td>
<td>0.024</td>
<td>0.017</td>
<td>0.980</td>
<td>0.009</td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>4.146</td>
<td>2.956</td>
<td>2.027</td>
<td>2.051</td>
<td>2.157</td>
<td>2.280</td>
<td>2.030</td>
<td>3.401</td>
</tr>
<tr>
<td>$R^2$</td>
<td>1.6</td>
<td>50.3</td>
<td>76.6</td>
<td>76.1</td>
<td>73.6</td>
<td>70.5</td>
<td>76.6</td>
<td>34.2</td>
</tr>
</tbody>
</table>
Table A.7  GMM results for the NBER-based discount factor

Table A.7 shows estimation results for annual returns and the NBER discount factor from equation (a.5) with different cutoffs for the number of NBER recession months in a year. The annual NBER recession indicator takes the value one when there are at least 5, 7, or 9 recession months in a year. The results are similar when I consider alternative cutoffs for the number of NBER months. The GMM moment conditions are the unconditional Euler equations for the risk-free asset, the aggregate stock market index, and the Fama-French six size/bm portfolios. The parameter $\beta$ is the discount rate, $\theta_{NBER}$ is the “recession aversion” coefficient, and $P(NBER)$ is the observed probability of NBER recession events. Bootstrap 95% confidence intervals are shown in brackets. $\chi^2$, d.o.f., and $p$ are the bootstrap first-stage $\chi^2$ test, degrees of freedom, and $p$-value that all moment conditions are jointly zero. RMSE and $R^2$ are the cross-sectional root mean square error and $R$-square, respectively, for the set of risky assets. The sample period is 1933-2012.

<table>
<thead>
<tr>
<th>NBER discount factor</th>
<th>NBER(1) 5-month cutoff</th>
<th>NBER(2) 7-month cutoff</th>
<th>NBER(1) 9-month cutoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.041 [0.01, 0.72]</td>
<td>0.444 [0.03, 0.70]</td>
<td>0.516 [0.18, 0.70]</td>
</tr>
<tr>
<td>$\theta_{NBER}$</td>
<td>113.54 [1.86, 351.45]</td>
<td>10.975 [3.63, 197.30]</td>
<td>14.682 [4.59, 72.82]</td>
</tr>
<tr>
<td>$P(NBER)$</td>
<td>0.200</td>
<td>0.112</td>
<td>0.087</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>978.28</td>
<td>82.728</td>
<td>485.86</td>
</tr>
<tr>
<td>d.o.f.</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>$p$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>RMSE</td>
<td>4.394</td>
<td>4.015</td>
<td>3.689</td>
</tr>
<tr>
<td>$R^2$</td>
<td>-127.86</td>
<td>-86.64</td>
<td>-58.53</td>
</tr>
</tbody>
</table>

A.35
Table A.8 GMM results for the FORA discount factor

Table A.8 shows estimation results for annual returns and the FORA discount factor from equation (a.6) with alternative reference points for gains and losses. These reference points are: i) zero consumption growth ($d = 0$), ii) expected consumption growth ($d = E_t(\Delta c_{t+1})$), iii) last period’s consumption growth ($d = \Delta c_t$), iv) a constant parameter estimated by GMM ($d = d_0$), and v) the DA certainty equivalent ($d = E_t(\Delta c_{t+1}) + d_1V o t_r(\Delta c_{t+1})$). The GMM moment conditions are the consumption growth mean, the consumption growth variance, the consumption growth autocovariance, the variance of risk-free rate, and the unconditional Euler equations for the risk-free asset, the aggregate stock market index, and the Fama-French six size/bm portfolios. Table A.8 does not report the results for the consumption growth moments. The parameter $\beta$ is the discount rate, $\alpha$ is the (second-order) risk aversion parameter, $\rho$ is equal to $1 - 1/EIS$, $\theta$ is the first-order risk aversion parameter, and $d_0(\times 100)$ is the constant reference point for gains and losses estimated by GMM. The constant $d_1$ is the disappointment threshold from equation (5) of the paper. $\mathbb{P}(\text{loss})$ is the probability that consumption growth is below the reference point. Bootstrap 95% confidence intervals are shown in brackets. $\chi^2$, $d.o.f.$, and $p$ are the bootstrap first-stage $\chi^2$ test, degrees of freedom, and $p$-value that the Euler equations for the test assets are jointly zero. $RMSE$ and $R^2$ are the cross-sectional root mean square error and $R$-square, respectively, for the set of risky assets. The sample period is 1933-2012.

<table>
<thead>
<tr>
<th>Reference Level</th>
<th>FORA(1)</th>
<th>FORA(2)</th>
<th>FORA(3)</th>
<th>FORA(4)</th>
<th>FORA(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>zero cons. growth</td>
<td>expected cons. growth</td>
<td>past cons. growth</td>
<td>unknown constant</td>
<td>certainty equiv. of cons. growth</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.170</td>
<td>1.170</td>
<td>1.171</td>
<td>1.134</td>
<td>1.094</td>
</tr>
<tr>
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<td>[0.91, 1.29]</td>
<td>[0.86, 1.31]</td>
<td>[0.84, 1.34]</td>
<td>[0.95, 1.34]</td>
<td>[1.02, 1.16]</td>
</tr>
<tr>
<td></td>
<td>[-73.51, -21.29]</td>
<td>[-80.89, -17.25]</td>
<td>[-79.75, -11.97]</td>
<td>[-10.10, -1.23]</td>
<td>[-17.69, -0.49]</td>
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<td>$\theta$</td>
<td>-0.007</td>
<td>-0.003</td>
<td>0.002</td>
<td>8.488</td>
<td>6.050</td>
</tr>
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<td>[-0.018, 0.001]</td>
<td>[-0.007, -0.000]</td>
<td>[-0.000, 0.007]</td>
<td>[3.25, 17.64]</td>
<td>[3.80, 16.11]</td>
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<tr>
<td>$\rho$</td>
<td>-8.566</td>
<td>-8.564</td>
<td>-8.564</td>
<td>-8.564</td>
<td>-8.522</td>
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<td>$d_0$</td>
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<td>1.097</td>
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<td>[1.06, 1.35]</td>
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<td></td>
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<td></td>
<td>-0.827</td>
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<td>$\mathbb{P}(\text{loss})$</td>
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<td>0.500</td>
<td>0.512</td>
<td>0.265</td>
<td>0.112</td>
</tr>
<tr>
<td>$\chi^2$</td>
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<td>60.648</td>
<td>64.783</td>
<td>47.647</td>
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<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>$p$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.999</td>
</tr>
<tr>
<td>$RMSE$</td>
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<td>1.594</td>
<td>1.594</td>
<td>1.436</td>
<td>0.609</td>
</tr>
<tr>
<td>$R^2$</td>
<td>71.1</td>
<td>71.1</td>
<td>71.1</td>
<td>76.2</td>
<td>95.7</td>
</tr>
</tbody>
</table>
Table A.9  Willingness-to-pay and time premium for the consumption-based DA discount factor

Table A.9 assesses the effective risk aversion and time preferences of the DA investor for different values of the risk and disappointment aversion parameters. Willingness-to-pay is defined as the difference between the mean and the disappointment aversion certainty equivalent of a gamble with an expected value of $100,000 and two equally likely outcomes ($100,000 ± ϵ). Table A.9 shows willingness-to-pay calculations for different values of the lottery parameter ϵ and different specifications of the DA certainty equivalent from equation (2) of the paper. Time premium is the fraction of lifetime utility that DA investors are willing to sacrifice at time 0 to have all risk resolved next period. The preference parameters in this table are calibrated to the estimates from Table 2 of the main text, with the exception of the discount rate β which is set equal to 0.998 (the β estimate for the DA(1) model in Table 2 of the paper) across all models to avoid the complications of discount rates that are greater than one.

<table>
<thead>
<tr>
<th>Preference parameters</th>
<th>Epstein-Zin</th>
<th>DA(1)</th>
<th>DA(2)</th>
<th>DA(3)</th>
<th>DA(4)</th>
<th>DA(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Willingness-to-pay (in $)</td>
<td>α = −38.369</td>
<td>α = 1</td>
<td>α = −3</td>
<td>α = −9</td>
<td>α = −19</td>
<td>α = −5.891</td>
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<tr>
<td>ϵ</td>
<td>ρ = −7.999</td>
<td>ρ = 1</td>
<td>ρ = −6.436</td>
<td>ρ = −7.271</td>
<td>ρ = −8.280</td>
<td>ρ = −8.522</td>
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<tr>
<td></td>
<td>θ = 0</td>
<td>θ = 8.414</td>
<td>θ = 7.386</td>
<td>θ = 4.924</td>
<td>θ = 3.291</td>
<td>θ = 6.050</td>
</tr>
<tr>
<td>1,000</td>
<td>192</td>
<td>807</td>
<td>794</td>
<td>734</td>
<td>678</td>
<td>766</td>
</tr>
<tr>
<td>10,000</td>
<td>8,360</td>
<td>8,079</td>
<td>8,506</td>
<td>8,704</td>
<td>9,026</td>
<td>8,613</td>
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<tr>
<td>25,000</td>
<td>23,633</td>
<td>20,199</td>
<td>22,787</td>
<td>23,703</td>
<td>24,169</td>
<td>23,383</td>
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<tr>
<td>50,000</td>
<td>49,088</td>
<td>40,398</td>
<td>48,163</td>
<td>49,126</td>
<td>49,445</td>
<td>48,863</td>
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<tr>
<td>75,000</td>
<td>74,544</td>
<td>60,598</td>
<td>74,046</td>
<td>74,563</td>
<td>74,723</td>
<td>74,431</td>
</tr>
</tbody>
</table>

| Time premium | 0.16% | 14.09% | 7.57% | 2.76% | 7.79% |