Where’s the Kink? Disappointment Events in Consumption Growth and Equilibrium Asset Prices

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Abstract

I propose a consumption-based asset pricing model with disappointment aversion to investigate the link between downside consumption risk and expected returns across asset markets. I find that the disappointment model can explain 95% of the cross-sectional variation in size/book-to-market portfolios and more than 80% of the variation in the joint sample of stocks, bonds, and commodity futures. I also show that the performance of the disappointment model is comparable to that of the Fama-French three-factor specification, regardless of the sample frequency (annual, quarterly). Overall, my results indicate that disappointment aversion considerably improves the fit of consumption-based asset pricing models.

Keywords: asset pricing, cross-section, consumption growth, disappointment aversion, disappointment events, downside consumption risk

JEL classification: D51, D91, E21, G12
1 Introduction

A growing literature in economics and finance has argued that investors exhibit asymmetric preferences over gains and losses. Specifically, a number of studies at the micro-level conclude that downside risk is important because investors dislike losses more than they enjoy gains (e.g., Kahneman and Tversky 1979; Choi et al. 2007; Duncan 2010). Based on these results, recent work in asset pricing has shown that models of asymmetric or downside risk can explain the equity risk premium and the cross-section of expected returns better than traditional models with symmetric preferences.¹

Nevertheless, a number of important issues remain unresolved. First, reference points are not directly observable, and thus most of the downside capital asset pricing models (downside CAPMs) make additional assumptions to identify them. Specifically, the key factor in these downside CAPMs is usually an indicator of stock market returns being less than an arbitrary reference point. In certain cases, the reference point is (a multiple of) the risk-free rate, and in others, the reference point is a constant equal to zero or to a negative value.²

Second, only a few studies consider downside consumption-based asset pricing models (downside C-CAPMs). These studies tend to focus on the equity premium (e.g., Epstein and Zin 1990, 2001; Routledge and Zin 2010; Bonomo et al. 2011) and do not provide any results for the cross-section of expected returns. Third, many downside C-CAPMs (e.g., Barberis and Huang 2001) assume additional behavioral biases, such as probability weighting and narrow framing, and thus limit our understanding on the role of downside consumption risk alone.

Motivated by these observations, the goal of this paper is to investigate whether downside consumption risk can help explain the cross-section of expected returns across different asset classes and sample frequencies. To this end, I build an asset pricing model in which reference points are endogenously derived from investor preferences. Moreover, in this paper, I only consider downside consumption risk and do not allow for other behavioral biases or additional return-generated factors.

The starting point of my theoretical framework is Gul’s (1991) model of disappointment aversion. In his model, investor utility over stochastic consumption has three characteristics: (1) it is defined based on deviations from a reference point; (2) it is steeper for losses than for gains; and (3)

¹Epstein and Zin (1990), Benartzi and Thaler (1995), and Barberis, Huang, and Santos (2001).
the reference level for gains and losses is based on the certainty equivalent of consumption. These characteristics imply that disappointment aversion preferences are described by utility functions with a kink and that the location of this kink is the certainty equivalent of the random payoff. In this paper, I extend Gul’s static model to a dynamic setting following the methodology in Routledge and Zin (2010). In the dynamic model, preferences are nonseparable across time à la Epstein and Zin (1989), and the stochastic discount factor is a function of consumption growth, as well as unobservable lifetime utility.

The presence of unobservable lifetime utility in the discount factor implies that the disappointment model cannot be directly estimated. To circumvent this problem, I assume that consumption growth is predictable and homoscedastic. Based on this assumption, I can derive explicit solutions for lifetime utility and the disappointment aversion discount factor in terms of observable consumption growth. These solutions imply that the disappointment aversion pricing kernel is a nonlinear function of two factors: (1) consumption growth and (2) an indicator of consumption growth being less than its certainty equivalent. Using this solution for the disappointment aversion discount factor, I am able to estimate the disappointment model via the generalized method of moments (GMM), while conducting a number of empirical tests in the cross-section of returns.

I begin my empirical analysis by estimating a GMM system that jointly fits consumption growth moments and unconditional Euler equations for annual asset returns over the 1933-2012 period. The test assets consist of the risk-free asset, the stock market, and the six Fama-French portfolios sorted on size and book-to-market (size/bm). One of the main goals of this baseline empirical analysis is to obtain an estimate of the disappointment aversion coefficient. This coefficient measures the asymmetry in investor preferences over gains and losses, and is the key parameter in the disappointment model. My empirical results indicate that the estimates of the disappointment aversion parameter are positive and statistically significant, with values between 3.29 and 8.41. This finding implies that during disappointment periods, investors penalize losses 4.29 to 9.41 times more than losses during normal times.

In addition to estimating the disappointment model, I compare its cross-sectional fit to that of traditional consumption-based models (e.g., CRRA; Epstein and Zin 1989). For these tests, I compute several measures of fit: the cross-sectional $R^2$, the root-mean-square error, the $\chi^2$ test (Hansen 1982), and the $HJ$-statistic (Hansen and Jagannathan 1997). Based on these measures,
I find that the disappointment model has an $R^2$ of approximately 95% in the sample of size/bm portfolios, and its fit is comparable to that of the Fama and French (1993) three-factor model. Moreover, the disappointment model is the only consumption-based model in this sample not rejected by the $\chi^2$ test.

Next, I extend the annual sample of size/bm portfolios to include portfolios of corporate bonds sorted by their credit ratings as well as portfolios of commodity futures sorted by their basis (futures price over spot price). The results of these tests indicate that the disappointment model can explain expected returns for stocks, bonds, and commodities better than traditional consumption models or the three-factor Fama-French specification.

The differences between the disappointment aversion discount factor and the traditional consumption-based models become starker when I consider quarterly returns. Specifically, the fit of the disappointment model in the quarterly sample of size/bm portfolios is almost identical to that of the annual sample. In contrast, traditional consumption-based models cannot explain the cross-section of expected returns in the quarterly sample, exhibiting low or negative $R^2$s. These findings suggest that, unlike traditional consumption models, the cross-sectional fit of the disappointment model does not depend on the sample frequency.

Overall, the proposed disappointment model has such good performance because it can successfully identify important events in consumption growth that affect asset prices. These “disappointment events” take place when consumption growth drops below its certainty equivalent, which is located approximately one standard deviation below the conditional mean of consumption growth. I find that over the 1933-2012 period, disappointment events occur with 11.2% probability and typically take place before or during recessions.

Even though disappointment events do not always overlap recessions, investors are quite sensitive to these events and demand high risk premiums for holding assets that underperform during disappointment periods (e.g., small and value stocks, BAA-rated corporate bonds, and low-basis commodity futures). For instance, back-of-the-envelope calculations based on my estimation results suggest that the covariance between the stock market and the set of disappointment events scaled by the disappointment aversion parameter can explain 46% to 98% of the average stock market return, depending on the disappointment aversion estimates. Further, as noted above, the market price of risk during disappointment periods is 4.29 to 9.41 times higher than during normal times.
My results contribute to the asset pricing literature on disappointment aversion. This literature has considerably grown over the years following the works of Ang, Bekaert, and Liu (2005) and Routledge and Zin (2010). Specifically, Routledge and Zin (2010) and Bonomo et al. (2011) use consumption-based models with disappointment aversion to explain the equity premium but do not provide any results for the cross-section of expected stock returns. In contrast, Ostrovnaya, Routledge, and Zin (2006) and Faragó and Tédongap (2016) conduct cross-sectional tests of the disappointment aversion model using stock market returns as a proxy for returns on aggregate wealth. In this paper, I explicitly solve the value function of the disappointment averse investor in terms of consumption growth alone and use this solution to explain the cross-section of expected returns from a consumption-based perspective.

An explicit solution for the value function in terms of consumption growth is important for a number of reasons. To begin with, characterizing the pricing kernel in terms of aggregate consumption forces the disappointment model to confront asset pricing moments using macroeconomic data alone. This is consistent with the consumption-based paradigm of Breeden (1979), who shows that “expected excess return on any security should be proportional to its covariance with respect to aggregate consumption alone.” Further, unlike previous studies on disappointment aversion that rely on calibrations, the consumption-based solutions for the stochastic discount factor allow me to identify actual disappointment events in consumption growth, and relate these events to aggregate macroeconomic conditions. Finally, by jointly estimating Euler equations and consumption growth moments, I can map preference parameters directly into prices of risk.

An additional contribution of this paper is that it extends the literature on asymmetric asset pricing models by showing that the location of the reference point affects the cross-sectional performance of these models. Consistent with Gul’s disappointment framework, I find that to explain the cross-section of expected returns, the reference point in consumption-based models with asymmetric preferences should be equal to the certainty equivalent of consumption growth. In contrast, asymmetric models that specify alternative reference points for gains and losses in consumption growth (e.g., zero consumption growth, expected consumption growth, past consumption growth)
cannot explain risk premiums.\(^4\)

Collectively, the results in this paper reveal that the main weakness of the original C-CAPM stems from the assumption of a smooth utility function, which does not account for aversion to downside risk. In contrast, I find that disappointment risk is priced in the cross-section of expected returns, and that disappointment aversion considerably helps improve the cross-sectional fit of consumption-based asset pricing models.

2 Recursive Utility with Disappointment Aversion

In this section, I introduce the disappointment aversion (DA) discount factor and obtain explicit solutions for the DA pricing kernel in terms of observable consumption growth. Specifically, I consider a discrete-time, single-good, closed economy in which there is no production activity, while endowments are generated exogenously by “tree-assets” (Lucas 1978). Equity claims for these assets are traded in complete markets, free of transaction costs. Investors are fully rational, face no restrictions on asset holdings, and are characterized by identical disappointment aversion preferences which are nonseparable in time. Note that in comparison to alternative first-order risk-aversion models, disappointment aversion does not violate first-order stochastic dominance or transitivity of preferences (see Gul 1991).

Under the above assumptions, Routledge and Zin (2010) show that there exists a representative investor who chooses consumption \(C_t\) and portfolio weights \(\{w_{i,t}\}_{i=1}^n\) to maximize lifetime utility \(V_t\). The investor’s maximization problem is given by

\[
V_t = \max_{C_t, \{w_{i,t}\}_{i=1}^n} \left[(1 - \beta)C_t^\rho + \beta \mu_t(V_{t+1})^\rho\right]^{\frac{1}{\rho}},
\]

\[
\text{with } \mu_t(V_{t+1}) = \mathbb{E}_t \left[\frac{V_{t+1}^\alpha (1 + \theta \{V_{t+1} \leq \mu_t(V_{t+1})\})^{\frac{1}{\alpha}}}{1 + \theta \mathbb{E}_t \{V_{t+1} \leq \mu_t(V_{t+1})\}}\right],
\]

subject to the usual budget and transversality constraints. Lifetime utility \(V_t\) in equation (1) is a strictly increasing, strictly concave, and linear homogeneous function. Lifetime utility is also time consistent since \(V_t\) is increasing in \(V_{t+1}\). The operator \(\mu_t\) in equation (2) is the DA certainty

\(^4\)In Section G of the Online Appendix, I compare models with alternative reference points for gains and losses. The results for these tests show that when the reference point is based on the status quo assumption of Kahneman and Tversky (1979), the expectation model of K˝oszegi and Rabin (2006), or a quickly adapting reference point as implied by Arkes et al. (2008), then downside consumption risk is not priced in the cross-section of expected returns.
equivalent, and $1\{V_{t+1} \leq \mu_t(V_{t+1})\}$ is the disappointment indicator that takes the value of one when lifetime utility $V_{t+1}$ is less than its DA certainty equivalent.\(^5\)

The key parameter in the disappointment model is the DA coefficient $\theta$ ($\theta \geq 0$). This parameter measures the asymmetry in investor preferences over gains and losses. If $\theta$ is positive, a $1$ loss in consumption during disappointment periods hurts approximately $1 + \theta$ times more than a $1$ loss in consumption during normal times. When $\theta$ is zero, the effects of first-order risk aversion vanish, and the DA model reduces to the standard Epstein-Zin (1989) framework.

The constant $\alpha$ ($\alpha \leq 1$) is the coefficient of second-order risk-aversion that affects the piecewise curvature of the utility function, while the parameter $\beta$ ($\beta \in (0, 1)$) is the discount rate that measures how fast investors discount future utility. Finally, the coefficient $\rho$ ($\rho \leq 1$) determines the elasticity of intertemporal substitution ($\text{EIS} = 1/(1 - \rho)$), which shows the responsiveness of consumption growth to changes in the risk-free rate.

### 2.1 Disappointment aversion stochastic discount factor

From the consumption-portfolio problem in equation (1), the corresponding DA stochastic discount factor of the aggregate investor can be written as

$$M^{DA}_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{\rho - 1} \left[ \frac{V_{t+1}}{\mu_t(V_{t+1})} \right]^{\alpha - \rho} \left[ \frac{1 + \theta 1\{V_{t+1} \leq \mu_t(V_{t+1})\}}{1 + \theta E_t[1\{V_{t+1} \leq \mu_t(V_{t+1})\}]} \right].$$

The discount factor above adjusts expected values by taking into account investor preferences over the timing, risk, and disappointment of stochastic payoffs. The first term in equation (3) corrects for the timing of risky payoffs which occur at a future date (resolution of risk). The second term adjusts expected values for investors’ dislike of risk (second-order risk aversion). When preferences are time additive ($\alpha = \rho$), adjustments for time and risk are identical, and the second term is one.

The third term in equation (3) is the novel term in the DA discount factor. This term adjusts

\(^5\)Following the original disappointment model of Gul (1991), I assume that disappointment events occur whenever utility falls below its certainty equivalent, that is, $1\{V_{t+1} \leq \mu_t\}$. In the generalized disappointment aversion (GDA) model of Routledge and Zin (2010), the threshold for disappointment is a multiple of the certainty equivalent, i.e. $1\{V_{t+1} \leq \delta \mu_t\}$, and therefore it is also affected by the positive GDA parameter $\delta$. According to the GDA framework, disappointment events may happen above or below the certainty equivalent, depending on whether the GDA coefficient $\delta$ is greater or lower than one. In Section A of the Online Appendix, I solve and estimate a GDA model in which $\delta$ is unrestricted. The empirical results for this model suggest that my unconditional tests cannot reject the null hypothesis that $\delta$ is one. Similarly, in Bonomo et al. (2011), the calibrated value for the GDA parameter is 0.989. This value is very close to one, and my unconditional tests cannot statistically identify such a small difference. Therefore, in this paper, I adopt Gul’s (1991) DA framework for parsimony.
expected values for investors' aversion to disappointment (first-order risk aversion). The DA term distorts probability weights by shifting more mass to disappointment events, namely states of nature in which lifetime utility $V_{t+1}$ is less than its DA certainty equivalent $\mu_t$. When investors are disappointment neutral ($\theta = 0$), the disappointment term is one.

According to equation (3), the DA discount factor is a function of the observable consumption growth and the unobservable lifetime utility since households have nonseparable preferences. Epstein and Zin (1989) and Routledge and Zin (2010) use the first-order conditions of the optimal consumption problem to replace the unobservable lifetime utility with the return on aggregate wealth, which is also hard to measure. Instead, I impose additional structure on the consumption growth process, and use the price-dividend log-linearization of Campbell and Shiller (1988) to express returns on aggregate wealth as a function of the observable consumption growth.

### 2.2 Explicit solutions for the DA stochastic discount factor

To derive explicit solutions for the DA discount factor in terms of the observable consumption growth, I employ the standard assumption (e.g., Mehra and Prescott 1985; Routledge and Zin 2010) that consumption growth is an autoregressive process (AR(1)) with constant volatility and i.i.d. $N(0,1)$ shocks $\epsilon_{c,t}$:

$$\Delta c_{t+1} = \mu_c (1 - \phi_c) + \phi_c \Delta c_t + \sqrt{1 - \phi_c^2} \sigma_c \epsilon_{c,t+1}. \tag{4}$$

$\mu_c$, $\sigma_c^2$, and $\phi_c$ represent the unconditional mean, variance, and first-order autocorrelation for consumption growth, respectively. I assume that consumption growth is homoscedastic for two reasons. First, it allows me to derive explicit solutions for the DA discount factor in terms of the observable consumption growth. Second, under homoscedasticity, I can easily estimate Euler equations for asset returns together with consumption growth moments in a joint GMM system.

#### 2.2.1 Explicit solutions for the price-dividend ratio

Using the AR(1) assumption in equation (4), I obtain an empirically tractable version of the DA stochastic discount factor in terms of observable consumption growth.

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6In my empirical tests, I verify that consumption growth is predictable and show that the autocovariance of consumption growth is statistically significant.
Proposition 1: For the consumption growth dynamics in equation (4), the log price-dividend ratio \( \log \frac{P_{c,t}}{C_t} \) of the consumption claim is affine in consumption growth:

\[
\log \frac{P_{c,t}}{C_t} = \mu_v + \phi_v \Delta c_t, \quad \text{where} \quad \mu_v = \frac{1}{1 - \kappa_{c,1}} \left[ \log \beta + \kappa_{c,0} + \frac{\rho(1 - \phi_c)}{1 - \kappa_{c,1} \phi_c} \mu_c + \frac{\rho \phi}{1 - \kappa_{c,1} \phi_c} \sqrt{1 - \phi_c^2 \sigma_c} \right], \quad \phi_v = \frac{\rho \phi_c}{1 - \kappa_{c,1} \phi_c},
\]

and

\[
d_1 = \frac{\alpha}{2(1 - \kappa_{c,1} \phi_c)} \sqrt{1 - \phi_c^2 \sigma_c} + \frac{1}{1 - \kappa_{c,1} \phi_c} \log \left[ \frac{1 + \theta \mathbb{P}\{\Delta c_{t+1} \leq \mu_c(1 - \phi_c) + \phi_c \Delta c_t + d_1 \sqrt{1 - \phi_c^2 \sigma_c}\}}{1 + \theta \mathbb{E}\{\Delta c_{t+1} \leq \mu_c(1 - \phi_c) + \phi_c \Delta c_t + d_1 \sqrt{1 - \phi_c^2 \sigma_c}\}} \right]. \tag{5}
\]

Proof: See the Appendix.

\( \kappa_{c,0} \) and \( \kappa_{c,1} \) represent log-linearization constants described in the Appendix, and \( N(.) \) is the standard normal c.d.f.. The parameter \( \mu_v \) above determines the unconditional mean of the log price-dividend ratio. This parameter depends on the discount rate and the unconditional mean of consumption growth, adjusted for persistence and disappointment. The parameter \( \phi_v \) captures the sensitivity of the price-dividend ratio to consumption growth. The sign of this parameter depends on the magnitude of the EIS and the autocorrelation of consumption growth. For positively autocorrelated consumption growth process (\( \phi_c > 0 \)), if the EIS is greater than one (\( \rho > 0 \)), \( \phi_v \) is positive and the price-dividend ratio of the claim on aggregate consumption is procyclical. In contrast, if the EIS is less than one (\( \rho < 0 \)), the price-dividend ratio for the consumption claim is countercyclical.

Finally, the parameter \( d_1 \) determines the threshold for disappointment, which is equal to the DA certainty equivalent for consumption growth. According to equation (5), \( d_1 \) is the solution to a fixed-point problem that depends on the consumption growth moments \( \phi_c \) and \( \sigma_c \), and the preference parameters \( \alpha \) and \( \theta \). In the Appendix, I show that this fixed-point problem has a unique solution and that for \( \alpha \) negative and \( \theta \) positive, the coefficient \( d_1 \) is negative.

2.2.2 Explicit solutions for the DA stochastic discount factor

Based on Proposition 1, the DA stochastic discount factor can be expressed as

\[
M^D_{t+1} = \exp \left[ \log_{M} - \left( 1 - \rho + \frac{\rho - \alpha}{1 - \kappa_{c,1} \phi_c} \right) \Delta c_{t+1} + \frac{(\rho - \alpha) \phi_c}{1 - \kappa_{c,1} \phi_c} \Delta c_t \right] \\
\times \frac{1 + \theta \mathbb{P}\{\Delta c_{t+1} \leq \mu_c(1 - \phi_c) + \phi_c \Delta c_t + d_1 \sqrt{1 - \phi_c^2 \sigma_c}\}}{1 + \theta \mathbb{E}\{\Delta c_{t+1} \leq \mu_c(1 - \phi_c) + \phi_c \Delta c_t + d_1 \sqrt{1 - \phi_c^2 \sigma_c}\}}. \tag{6}
\]
where $\beta_M$ is a constant that depends on preference parameters and consumption growth moments (see equation (26)). The consumption-based DA discount factor above corrects expected future payoffs for time, risk, and disappointment, similar to the DA pricing kernel from equation (3). The difference between the two specifications is that in equation (6), unobservable lifetime utility is replaced by the observable consumption growth. Specifically, the DA pricing kernel above is a non-linear function of two factors: (1) consumption growth $\Delta c_{t+1}$ and (2) the disappointment indicator $1\{\Delta c_{t+1} \leq \mu_c(1 - \phi_c) + \phi_c \Delta c_t + d_1 \sqrt{1 - \phi_c^2} \sigma_c\}$, which takes the value of one when consumption growth is less than its DA certainty equivalent.

### 2.2.3 Disappointment events in consumption growth

According to the solution for the DA discount factor in equation (6), investors experience disappointment when consumption growth falls below its DA certainty equivalent. Formally, disappointment events in consumption growth happen when

$$\Delta c_{t+1} \leq \mu_c(1 - \phi_c) + \phi_c \Delta c_t + d_1 \sqrt{1 - \phi_c^2} \sigma_c.$$  \hfill (7)

The DA certainty equivalent in the RHS above is equal to the expected consumption growth $(\mu_c(1 - \phi_c) + \phi_c \Delta c_t)$ plus a disappointment correction term $(d_1 \sqrt{1 - \phi_c^2} \sigma_c < 0)$, which depends on the coefficient $d_1$ and the consumption growth volatility $(\sqrt{1 - \phi_c^2} \sigma_c)$. Alternatively, the parameter $d_1$ measures how many standard deviations below its conditional mean consumption growth must fall for investors to experience disappointment. According to my empirical results, $d_1$ is approximately $-0.8$. This means that disappointment events occur when consumption growth is less than 0.8 standard deviations below its conditional mean.

Even though the threshold for disappointment is time varying, the probability of disappointment events is constant due to the homoscedasticity assumption for consumption growth. Specifically, I can replace $\Delta c_{t+1}$ in the LHS of equation (7) using the AR(1) process from equation (4), and redefine disappointment events as

$$\epsilon_{c,t+1} \leq d_1,$$

where $\epsilon_{c,t+1}$ are the i.i.d. $N(0,1)$ shocks to consumption growth. The interpretation of the above
equation is that in an endowment economy, disappointment events in consumption growth happen when shocks to consumption growth are substantially negative and smaller than $d_1$. The conditional probability of disappointment events is then given by $P_t(\epsilon_{c,t+1} \leq d_1)$, which is constant for i.i.d. $\epsilon_{c,t+1}$ shocks.

### 2.2.4 The risk-free rate

Based on the solution for the DA discount factor in equation (6), I can express the one-period log risk-free rate $r_{f,t}$ as a function of consumption growth:

$$r_{f,t+1} = -\log \beta + (1 - \rho)\left[\mu_c (1 - \phi_c) + \phi_c \Delta c_t\right] - h_1(\sigma_c^2) - h_2(\sigma_c).$$

The functions $h_1$ and $h_2$ above capture precautionary savings due to risk and disappointment aversion, respectively. The traditional second-order precautionary savings term ($h_1(\sigma_c^2)$) depends on consumption growth variance, whereas the DA savings term ($h_2(\sigma_c)$) depends on consumption growth volatility due to first-order risk aversion.\(^7\)

### 3 Data and Estimation Methodology

In the previous section, I derived the DA discount factor in terms of observable consumption growth. Next, I describe the data and estimation methodology used to test the DA model.

#### 3.1 Data

For the empirical analysis, I use annual and quarterly data. The annual sample is from 1933 to 2012 for equity portfolios, from 1973 to 2012 for corporate bonds, and from 1974 to 2008 for commodity futures. The quarterly sample for equity returns is from 1947.Q2 to 2013.Q4. Aggregate consumption is defined on a per capita basis as services plus non-durables. Each component of aggregate consumption is deflated by its corresponding PCE price index (base year is 2009). Seasonally adjusted personal consumption expenditures (PCE) and the PCE price index data are from the BEA. Population data is from the U.S. Census Bureau and recession dates are from the NBER.

In matching consumption growth with asset returns, I follow the “beginning-of-period” convention

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\(^7\)In Section K of the Online Appendix, I use equation (8) to assess the ability of the DA model to conditionally fit the risk-free rate.
of Campbell (2003) and Yogo (2006), because beginning-of-period consumption growth is better aligned with asset returns.

Equity returns for the six equal-weighted size/book-to-market portfolios, factor returns, and the risk-free rate are from Kenneth French’s Web site. Returns for the Barclays AAA, AA, A, BAA, and aggregate investment-grade corporate bond indexes came from Datastream. Returns for the basis-sorted commodity futures portfolios were generously provided by Michael Weber. All returns have been adjusted for inflation by subtracting the growth rate of the PCE price index.

Table 1 shows summary statistics for consumption growth, the risk-free rate, and asset returns. For the annual sample, the average stock market return is 7.17% with a Sharpe ratio of 0.38, and the average return for the investment-grade corporate bond index is 4.38% with a Sharpe ratio of 0.31. The value premium for the 1933-2012 period is 4.58% and the size premium is 2.14%. The average spread between BAA and AAA bond returns in the 1973-2012 sample is 1%, and the basis premium in commodity futures for the 1974-2008 period is 9.31%.

3.2 Estimation methodology

A key feature of the disappointment model is that the coefficient $d_1$, which determines the disappointment threshold, is endogenous and depends on preferences and consumption dynamics. Therefore, to identify this coefficient, I jointly estimate preference parameters and consumption growth moments. Specifically, I obtain estimates for the discount rate $\beta$, the EIS parameter $\rho$, the risk-aversion coefficient $\alpha$, and the DA parameter $\theta$. I also estimate the mean, variance, and autocovariance of the consumption growth process.

3.2.1 GMM moment conditions

To estimate the DA model, I use first-stage GMM (Hansen and Singleton 1982), where the moment conditions are the unconditional moments for consumption growth and the unconditional Euler
equations for asset returns. Formally, the set of GMM moment conditions is given by

\[
\mathbb{E}[q(z_t, x)] = \begin{bmatrix}
\mathbb{E}[\Delta c_t] - \mu_c \\
\mathbb{E}[\Delta c_t^2] - \mu_c^2 - \sigma_c^2 \\
\mathbb{E}[\Delta c_t \Delta c_{t-1}] - \mu_c^2 - \phi_c \sigma_c^2 \\
\mathbb{E}[(\log R_{f,t})^2] - \mathbb{E}[\log R_{f,t}]^2 - (1 - \rho)^2 \phi_c^2 \sigma_c^2 \\
\mathbb{E}[M_t^{DA} R_{f,t}] - 1 \\
\mathbb{E}[M_t^{DA} (R_{m,t} - R_{f,t})] \\
\mathbb{E}[M_t^{DA} (R_{i,t} - R_{f,t})] \text{ for } i = 1, 2, \ldots, n
\end{bmatrix}.
\] (9)

The vector-valued function \( q(z_t, x) \) above denotes the set of moment conditions, \( z_t \) is the vector of consumption growth and asset returns, and \( x \) is the vector of parameters. \( R_{f,t} \) is the risk-free rate; \( R_{m,t} \) denotes stock market returns; and \( R_{i,t} \) is the return for asset \( i \). The GMM system in equation (9) jointly fits consumption growth moments (mean, variance, and autocovariance) with the unconditional Euler equations for asset returns, which are derived in the Appendix.\(^8\)

The GMM system in equation (9) is weighted by a prespecified matrix \( W \) with constant elements. Specifically, I use a diagonal weighting matrix where the first six diagonal elements are large numbers and the remaining diagonal terms are one.\(^9\) This weighting matrix overweights the moment conditions for consumption growth, the risk-free rate, and the equity premium. There are two main reasons for choosing this prespecified weighting matrix. First, the GMM moment conditions for consumption growth and expected returns have different scales. Therefore, consumption growth moments need to be weighted accordingly. For example, the autovariance of consumption growth is approximately \( 1/1000 \)th of the market risk premium (see Table 1). Thus, if all moment conditions were equally weighted, the GMM minimization would focus on matching the Euler equations for asset returns and completely ignore the moment conditions for consumption growth.

\(^{8}\)When \( \theta \) is positive, the DA discount factor is not continuous and the GMM function in equation (9) is not differentiable. However, Andrews (1994) and Newey and McFadden (1994) show that in establishing consistency and asymptotic normality of noncontinuous estimators, continuity and differentiability of the GMM objective function can be replaced by the less stringent conditions of continuity with probability one and stochastic differentiability. In Section I of the Online Appendix, I show that both of these conditions are satisfied by the DA stochastic discount factor provided that consumption growth and asset returns are continuous random variables.

\(^{9}\)The GMM weights for the consumption growth moments and the variance of the risk-free rate are \( 10^8 \) (\( 10^{12} \) for the CRRA model; this is necessary to fit the consumption growth moments under this model). The weights for the mean of the risk-free rate and the equity premium are \( 10^9 \), and the weights for the remaining test assets are one. For the Fama and French (1993) model, the weighting matrix is the identity matrix, because the GMM system for this model does not include consumption growth moments.
Second, the proposed weighting matrix allows me to test whether the various consumption models can explain asset returns while overweighting key moment restrictions for the risk-free rate and the stock market. In other words, although the DA model is tested using the full cross-section of asset returns, the DA coefficient $\theta$ and the set of disappointment events are mainly identified from the moment conditions for the equity premium and the risk-free rate. Overall, my prespecified weighting matrix does not allow the GMM estimation to fit the cross-section of expected returns at the cost of errors in the equity premium, the risk-free rate, or the consumption growth moments.

Finally, to assess the cross-sectional performance of the DA model, I use the Euler equations to generate fitted expected returns

$$
E[R_{i,t}] = E[R_{f,t}] - \text{Cov}(R_{i,t} - R_{f,t}, M_{t}^{DA}) / E[M_{t}^{DA}].
$$

(10)

Then I use the fitted returns to generate pricing errors and compute various measures of fit such as the cross-sectional $R^2$, the root-mean-square error ($RMSE$), the $\chi^2$ test (Hansen 1982), and the $HJ$-statistic (Hansen and Jagannathan 1997).\(^{10}\)

### 3.2.2 Identification of preference parameters

Identification in the DA model is quite challenging. For example, Delikouras (2014) and Dolmas (2014) show that joint identification of the risk and disappointment aversion parameters is difficult, especially in cases where the stock market is the only test asset. To address this concern, I follow the methodology in Ostrovnya et al. (2006) and estimate the DA model for a set of prespecified risk-aversion parameters: $\alpha \in \{1, -3, -9, -19\}$.

I set $\alpha$ equal to one to obtain a simplified DA model in which investors are disappointment averse, but (second-order) risk neutral. The remaining prespecified values for the risk-aversion coefficient are motivated by recent studies at the micro- and macro-levels. For example, using portfolio choice experiments at the individual level, Choi et al. (2007) estimate $\alpha$ to be, at most, $-3$. On the macro-side, Bansal and Yaron (2004) fit their long-run risk model to risk-aversion parameters around $-9$, and Mehra and Prescott (1985) show that standard consumption-based models require large, in absolute value, risk-aversion coefficients ($\alpha > -19$) to explain the equity

\(^{10}\)Due to the relatively short time-series sample, error covariances, confidence intervals, and test statistics are estimated using Kunsch’s (1989) block bootstrap method, which is described in Section J of the Online Appendix. In that section, I also describe the methodology for deriving the $p$-values associated with the $HJ$-statistic. Hypothesis testing for model parameters is based on 95% confidence intervals because the bootstrap distribution of the estimated parameters is skewed.
premium.

The parameters $\alpha$ and $\rho$ in the DA discount factor of equation (6) appear in an additive manner. Hence, the risk aversion and EIS coefficients cannot be identified using Euler equations only. To address this issue, the GMM system in equation (9) estimates the EIS parameter $\rho$ by fitting the variance of the risk-free rate. Specifically, according to equation (8), the variance of the risk-free rate depends only on the EIS parameter since

$$Var(r_{f,t+1}) = (1 - \rho)^2 \phi_c^2 Var(\Delta c_t).$$

(11)

Although this condition is quadratic in $\rho$, I use the restriction $\rho < 1$ to identify a unique solution for the EIS parameter.

Finally, because the price of a claim on aggregate consumption is unobservable, I assume that the linearization constant $\kappa_{c,1}$ in equation (5) is equal to the discount rate $\beta$. This assumption is true when $\rho$ is zero (log-time preferences). However, setting $\kappa_{c,1}$ equal to $\beta$ does not affect the empirical results even if $\rho$ is nonzero. This is because for reasonable consumption growth persistence ($\phi_c = 0.2 - 0.5$), the disappointment threshold $d_1$ in equation (5) is minimally affected by the linearization constant $\kappa_{c,1}$. Next, I use the GMM framework to estimate the DA model.

### 4 GMM Results for the Consumption-Based DA Discount Factor

For the first set of tests, I estimate the DA discount factor using moment restrictions for the mean, variance, and autocovariance of consumption growth, the mean and variance of the risk-free rate, and the Euler equations for the stock market and the six size/bm portfolios over the 1933-2012 period.\(^{11}\) For comparison, I also estimate the CRRA (Mehra and Prescott 1985) and Epstein-Zin models (Epstein and Zin 1989). These models are nested by the disappointment model. In particular, Epstein-Zin investors are disappointment neutral ($\theta = 0$), while CRRA investors are disappointment neutral ($\theta = 0$) and the risk-aversion parameter is equal to the inverse of the EIS ($\alpha = \rho$). The results for the first set of tests are shown in Table 2.

Following Ostrovnaya, Routledge, and Zin (2006), in Table 2, I estimate four different versions of the DA model (DA(1) - DA(4)) based on prespecified values for the risk-aversion parameter:

\(^{11}\)In Section C of the Online Appendix, I also test the fit of the DA model in the postwar sample of the size/book-to-market portfolios, and in Section D, I test the fit in the cross-section of earnings/price portfolios.
\( \alpha \in \{1, -3, -9, -19\} \). The simplest DA specification in Table 2 is the DA(1) model in which \( \alpha = \rho = 1 \). In this case, preferences are described by a piecewise linear utility function with asymmetric slopes around the DA reference point. The DA(1) specification allows me to isolate the effect of disappointment risk since in this model, investors are disappointment averse but second-order risk neutral. In addition to the four prespecified values for the risk-aversion coefficient, I consider a DA specification (DA(5) model) where risk and disappointment aversion coefficients are jointly estimated.

### 4.1 Risk preferences

The most important parameter in the DA discount factor is the disappointment aversion coefficient \( \theta \). This parameter measures the asymmetry in investor preferences over gains and losses. The estimates of \( \theta \) in Table 2 are positive, with values ranging from 3.291 to 8.414. According to the 95% confidence intervals, the estimates of the disappointment aversion coefficient are statistically significant across all DA models. Hence, I can reject the hypothesis that the aggregate investor is disappointment neutral \((\theta = 0)\).

The estimated magnitudes for the DA parameter \( \theta \) are also of interest. For example, in the DA(1) model, \( \theta \) is 8.414, implying that in this model, investors penalize losses during disappointment events 9.414 times more than losses during normal times. Moreover, the results in Table 2 suggest that there is a degree of substitutability between disappointment aversion and risk aversion. Specifically, as I decrease the prespecified risk-aversion parameter from 1 to \(-19\), the estimates of the DA coefficient decrease monotonically from 8.414 in the DA(1) model to 3.291 for the DA(4) specification. These findings indicate in the presence of second-order risk aversion, a lower degree of first-order risk-aversion is required to price assets. When risk and disappointment aversion parameters are jointly estimated, \( \theta \) is 6.050 and \( \alpha \) is \(-5.891\). Collectively, the results in Table 2 suggest that the disappointment aversion coefficient is economically and statistically significant across all DA specifications, and that its magnitude depends on the prespecified values for the risk-aversion parameter.\(^{12}\)

Another important parameter in the DA model is the coefficient \( d_1 \). This parameter determines

\(^{12}\)In Section H of the Online Appendix, I use the willingness-to-pay calculations of Epstein and Zin (2001) and the time premium framework in Epstein, Farhi, and Strzalecki (2014) to assess the plausibility of the DA estimates from Table 2. These calculations indicate that the time premium implied by the DA model is substantially larger than the one obtained by conventional calculations for the cost of business cycles (e.g., Lucas 1987).
the disappointment threshold, and is a function of preference parameters and consumption growth moments (see equation (5)). The estimates of $d_1$ in Table 2 range from $-0.878$ to $-0.763$. According to equation (7), these values imply that investors feel disappointed whenever consumption growth is less than approximately 0.8 standard deviations below its mean. This finding corresponds nicely with the assumption in Lettau, Maggiori, and Weber (2014) that the loss threshold for the aggregate stock market index is one standard deviation below its average return. Finally, based on the estimates of $d_1$ and the consumption growth moments in Table 2, the probability of disappointment events in consumption growth during the 1933-2012 period ranges from to 0.1 to 0.187.\footnote{The disappointment years for the DA(1) model are 1937, 1946, 1973, 1979, 1990, 1999, and 2007-2008. The disappointment years for the DA(2) and DA(5) models consist of 1937, 1946, 1948, 1973, 1979, 1990, 1999, and 2007-2008. The disappointment years for the DA(3) model consist of 1937, 1946, 1948, 1956, 1973, 1979, 1990, 1999, and 2007-2008. Finally, the disappointment years for the DA(4) model are 1937, 1946, 1948, 1953, 1956, 1973, 1979, 1980, 1990, 1999, 2006-2008, and 2011-2012.}

### 4.2 Pricing errors for the consumption-based DA discount factor

In addition to parameter estimates, Table 2 reports results for the cross-sectional fit of the various DA specifications and the traditional consumption models (CRRA, Epstein-Zin). In particular, the negative $R^2$ and high $RMSE$ for the CRRA model suggest that time-additive preferences cannot simultaneously explain the low volatility of the risk-free rate and the cross-section of expected returns. This is because the variance of the risk-free rate restricts the magnitude of the EIS, and in the CRRA case, it also determines the value of the risk-aversion coefficient. The fit of the CRRA model can be improved by disentangling risk attitudes from time preferences with the Epstein-Zin specification. In this case, the $R^2$ increases to 71% and the $RMSE$ decreases to 1.59%. Even though the $R^2$ for the Epstein-Zin model is relatively high, this model is rejected by the $\chi^2$-test, and the $HJ$-statistic is significant.

Contrary to the CRRA and Epstein-Zin specifications, none of the DA models are rejected by the $\chi^2$-test. Specifically, the $\chi^2$-test for the DA(1) model is 1.819 ($p$-value = 0.93), and the $HJ$-statistic is insignificant. Additionally, the $R^2$ for the DA(1) specification is 95%, and the $RMSE$ is low ($RMSE = 0.62\%$), rendering its fit comparable to the performance of the Fama and French (1993) three-factor model ($R^2 = 90\%, RMSE = 0.91\%$).

The performance of the DA(1) model is important for two reasons. First, Lewellen, Nagel, and Shanken (2010) show that multifactor models may mechanically generate high $R^2$s in the sample
of size/bm portfolios, even if these models “explain little of the cross-sectional variation in true expected returns”. This critique is less relevant for the DA(1) specification because it is a single-factor model. Second, the results for this specification suggest that a consumption-based model with disappointment aversion alone can explain the level and the cross-sectional dispersion of the size/bm portfolios as accurately as the three-factor Fama-French model.

The fit of the remaining DA models (DA(2) - DA(5)) is similar to that of the DA(1) specification. In particular, all the $R^2$s are greater than 90%, and the RMSEs are low relative to the average portfolio returns from Table 1. Similarly, the $\chi^2$ tests for all the DA models are insignificant, and the corresponding $HJ$-statistics are much lower than the ones for the traditional consumption and three-factor Fama-French models.

An important issue with the $\chi^2$ test is that it may fail to reject the null because the denominator in the statistic is large (the model is not accurate), and not because the numerator is small (the average error is zero). To examine whether disappointment aversion truly improves the fit of consumption models, Table 3 reports the average pricing errors (alphas) for the six size/bm portfolios across all consumption models. According to the findings in Table 3, the pricing errors of the DA models are economically and statistically insignificant across all portfolios, and their magnitude is similar to that of the Fama-French errors. In contrast, the pricing errors of the CRRA model are large and statistically significant across all portfolios, while the Epstein-Zin model can only price two out of the six size/bm portfolios: the small/medium and big/growth portfolios.

The results in Table 3 are also confirmed by Figure 1, which shows fitted and sample expected returns for the risk-free asset, the stock market, and the six size/bm portfolios. The graphical evidence in Figure 1 suggests that fitted expected returns according to the DA model are aligned with sample average returns across all portfolios. This alignment indicates that the DA model can fully explain the equity, size, and value premiums as well as the mean and volatility of the risk-free rate.\footnote{The moment conditions for the DA(1) specification do not include the volatility of the risk-free rate because in this model, $\rho$ is restricted to one, and the model-implied risk-free rate is constant.} In contrast, the CRRA model can explain either the volatility of the risk-free rate or the cross-section of expected stock returns, but not both. Finally, even though the Epstein-Zin model is a major improvement over the CRRA specification, the fit of this model is also problematic, especially for the small/value and small/growth portfolios. Overall, the evidence in Tables 2 and 3 and Figure 1 overwhelmingly indicates that disappointment risk is priced in the cross-section of
expected returns.

4.3 Additional test assets: Corporate bonds and commodity futures

Theoretically, a consumption-based model should explain asset prices across all markets, not just equity returns. Additionally, from an empirical standpoint, expanding the set of test assets should address the critique in Lewellen, Nagel, and Shanken (2010) regarding the factor structure in size/bm portfolios. Therefore, in this section, I estimate the DA discount factor in an extended cross-section of test assets. Specifically, in addition to the stock market and the six size/bm portfolios, I consider the aggregate index for investment-grade corporate bonds and a cross-section of four corporate bond portfolios sorted by credit rating (AAA - BAA). I focus on investment-grade corporate bond portfolios due to the availability of time-series observations.

In the extended sample of test assets, I also include the cross-section of five commodity futures portfolios sorted by their basis (the ratio of futures price to spot price). The choice of these portfolios is motivated by recent studies on the performance of asset pricing models in commodity markets. For instance, Yang (2013) sorts commodity futures by their basis, and shows that a basis factor can explain the cross-section of risk premiums in basis-sorted portfolios of commodity futures. Similarly, Lettau, Maggiori, and Weber (2014) also use portfolios of commodity futures sorted by basis in their empirical tests of asymmetric CAPMs. The GMM results for the extended cross-section of test assets are reported in Table 4.\(^{15}\)

Based on these results, the estimates for the disappointment parameter are economically and statistically significant across all DA specifications, and comparable to the ones from the equity sample. Similarly, the set of disappointment events for the extended sample is almost identical to the one from the equity sample.\(^{16}\) This is because identification of disappointment events mainly comes from the Euler equations for the stock market and the risk-free rate due to the choice of the GMM weighting matrix, which overweights the corresponding moment conditions. Thus, the set of

\(^{15}\)The estimation methodology for the extended sample is identical to the one used for the equity portfolios. The sample for the joint tests runs from 1974 to 2008 due to the short commodities sample. Because of the relatively large number of test assets and the limited time-series observations, in the extended sample, the covariance matrices of both asset returns and GMM errors are estimated imposing zero off-diagonal elements. In Section E of the Online Appendix, I test the DA model in each asset class separately. The fit of the DA model in the separate bond and commodity cross-sections is similar to the results for the combined sample.

In terms of model fit, Table 4 shows that the Epstein-Zin specification can explain expected returns for the joint cross-section quite well ($R^2 = 76\%$). Nevertheless, the DA discount factor is able to improve the fit of the Epstein-Zin model with $R^2$s ranging from 82\% to 87\%. Similar results also hold for the $RMSE$ and the $HJ$-statistic. Additionally, the $R^2$ and $RMSE$ for the DA(1) model ($R^2 = 87\%, \ RMSE = 1.58\%$) indicate a better fit than the Fama-French model ($R^2 = 55\%, \ RMSE = 2.99\%$). These findings are also confirmed in Figure 2, which plots fitted against sample expected returns across the equity, bond, and commodity portfolios. According to the visual evidence from Figure 2, the DA specification can fit the cross-section of expected returns better than the traditional CRRA and three-factor Fama-French models.

Collectively, the results in this section show that the DA model can explain expected returns for equities, corporate bonds, and commodity futures. This is quite surprising given the short time-series sample and the relatively small number of disappointment events. These findings also indicate that disappointment risk can explain the basis premium in commodity futures documented by Yang (2013). Finally, in addition to the above cross-sectional implications, the results from the joint tests, which run on a different sample (1974-2008) than the equity tests (1933-2012), suggest that the performance of the DA model does not depend on the time period.

5 Disappointment Events and NBER Recessions

An important advantage to estimating the disappointment model is that I can identify disappointment events in consumption growth, and relate these events to aggregate macroeconomic conditions (e.g., recessions). According to the results from Table 2, the number of disappointment periods is different across the various DA specifications, because the estimates of the DA coefficient $\theta$ and the disappointment threshold $d_1$ depend on the prespecified value for the risk-aversion parameter. Nevertheless, the number of disappointment periods does not significantly affect the fit of the various DA specifications, since all these models are able to identify a common set of disappointment events that is important for asset prices.

To study the relation between disappointment events and the macroeconomy, panel A in Figure 3 plots consumption growth, disappointment events, and NBER recession dates. Disappointment events are highlighted by ellipses, and are estimated for the DA(4) model of Table 2. Based on the
evidence in Figure 3, disappointment events are linked to real economic activity, and usually take place right before or during a recession. However, disappointment events do not always overlap recessions. According to the NBER, “a recession is a significant decline in economic activity spread across the economy”.\textsuperscript{17} In contrast, based on equation (7), disappointment events are periods of much worse-than-expected economic activity rather than periods of low economic activity. In some cases, the two events coincide (e.g., 1937, 1953, 1980, 1990, and 2008), but in others, disappointment events occur before recessions (e.g., 1946, 1948, 1956, 1973, 1979, 1999, and 2006-2007).

Even if disappointment periods do not always overlap recessions, it seems that investors demand high risk premiums for holding assets that perform poorly during disappointment events. For instance, the market risk premium is large because the stock market exposes investors to disappointment risk. Similarly, value firms command positive risk premiums over growth firms because value firms covary more, in absolute terms, with disappointment events than growth firms. This finding is also illustrated in panel B of Figure 3 that plots the time-series of the HML factor and consumption growth. The two time-series are positively correlated, and the time-series correlation between consumption growth and HML is 0.21. Comparable results also hold for small and large firms and the SMB factor. Overall, these findings indicate that, in terms of disappointment risk, value is riskier than growth and small firms are riskier than large ones.

6 Robustness Tests

In this section, I conduct additional tests to examine the robustness of my main findings. Specifically, I consider an alternative definition of disappointment events (disappointment events in the stock market) and a different sample frequency (quarterly data). Finally, I provide a brief discussion to address the concern that the covariances of the DA discount factor and asset returns are not statistically significant.

6.1 Disappointment events in the stock market

My results thus far suggest that a consumption-based model with disappointment aversion can explain both the level and the cross-sectional dispersion of expected returns across various asset

\textsuperscript{17}In Section F of the Online Appendix, I show estimation results for a reduced-form discount factor that depends on an NBER indicator. These results show that NBER recessions cannot explain the cross-section of expected returns.
classes. In this section, I show that the cross-sectional fit of the consumption-based DA model is more accurate than the performance of a DA model in which stock market returns are used as a proxy for returns on aggregate wealth (market-based DA model).

6.1.1 The market-based DA discount factor

The analysis in this section follows Routledge and Zin (2010), who show that using returns on aggregate wealth, the DA discount factor from equation (3) can be written as

\[ M_{t+1}^{DA} = Z_t^\alpha R_{W,t+1}^{-1} \frac{1 + \theta 1 \{ Z_t \leq \mu_t(Z_t) \}}{E_t \left[ \frac{1 + \theta 1 \{ Z_t \leq \mu_t(Z_t) \}}{} \right]} \]

with \( Z_t = \beta^\frac{1}{\rho} \left( \frac{C_{t+1}}{C_t} \right)^{\frac{\rho-1}{\rho}} R_{W,t+1}^{\frac{1}{\rho}} \).

Above, \( R_{W,t+1} \) denotes returns to aggregate wealth, and \( \mu_t \) is the disappointment aversion certainty equivalent from equation (2).

6.1.2 Differences with the consumption-based DA model

Although the market-based DA specification in equation (12) is equivalent to the DA discount factor from equation (6), there are two key differences. First, the disappointment threshold in the consumption-based DA pricing kernel of equation (6) is time varying, whereas the disappointment threshold in the market-based DA discount factor of equation (12) is constant. Specifically, Routledge and Zin (2010) showed that when the DA discount factor is expressed in terms of wealth returns, the DA certainty equivalent \( \mu_t \) in equation (12) is identified by the following equilibrium condition

\[ \mu_t \left( \beta^\frac{1}{\rho} \left( \frac{C_{t+1}}{C_t} \right)^{\frac{\rho-1}{\rho}} R_{W,t+1}^{\frac{1}{\rho}} \right)^\rho = 1. \]

The above condition implies that for the market-based DA discount factor of equation (12), the DA certainty equivalent \( \mu_t \) is constant and disappointment events happen when

\[ \beta^\frac{1}{\rho} \left( \frac{C_{t+1}}{C_t} \right)^{\frac{\rho-1}{\rho}} R_{W,t+1}^{\frac{1}{\rho}} < 1. \]

In contrast, the consumption-based DA discount factor from equation (6) is expressed in terms
of the price-dividend ratio of the consumption claim. In this case, using the price-dividend identity for returns on wealth (see equation (19) in the Appendix), I can rewrite the equilibrium condition (13) as

\[ \mu_t \left( \beta^\frac{1}{\rho} \frac{C_{t+1}}{C_t} \left( \frac{P_{c,t+1}}{C_{t+1}} + 1 \right)^\frac{1}{\rho} \right)^\rho = \frac{P_{c,t}}{C_t}. \]

According to Proposition 1, the price-consumption ratio \( \frac{P_{c,t}}{C_t} \) is time varying because it is a function of consumption growth. Therefore, the above condition implies that the disappointment threshold in the consumption-based DA model is time varying, and disappointment events happen when

\[ \beta^\frac{1}{\rho} \frac{C_{t+1}}{C_t} \left( \frac{P_{c,t+1}}{C_{t+1}} + 1 \right)^\frac{1}{\rho} < \left( \frac{P_{c,t}}{C_t} \right)^\frac{1}{\rho}. \]

The second difference between the two DA models in equations (6) and (12) is related to their empirical estimation. Specifically, by estimating the consumption-based DA discount factor via the GMM system in equation (9), I implicitly assume that asset prices are claims to aggregate consumption. In other words, in estimating the consumption-based DA model, I do not allow for imperfect correlation between consumption and dividends because I do not explicitly model asset cash flows (dividends, etc.) as levered claims to aggregate consumption similar to Bansal and Yaron (2004). Conversely, \( R_{W,t+1} \) in equation (12) denotes returns on aggregation wealth, that is, returns on a claim that pays aggregate consumption as its dividend. Such a claim is not traded in financial markets, and returns on aggregate wealth are unobservable. Therefore, in estimating the market-based DA model of equation (12), I follow Ostrovnaya, Routledge, and Zin (2006) and use stock market returns as a proxy for total wealth returns \( R_{m,t} \approx R_{W,t} \).

### 6.1.3 Estimating the market-based DA model

One drawback of the market-based approach is that I cannot use the variance of the risk-free rate to identify the EIS parameter \( \rho \), because the risk-free rate in the market-based DA model is a function of consumption growth and unobservable wealth returns. Therefore, to estimate the market-based disappointment model, I set \( \rho \) equal to one, as in Routledge and Zin (2010). Also, according to equation (12), by setting \( \rho \) equal to one, I can test the fit of a disappointment model that depends

\[ \text{Based on the discussion in Section 2.2.3, even though the disappointment threshold w.r.t consumption growth (\( \Delta c_t \)) is time varying, the disappointment threshold w.r.t consumption growth shocks (\( \epsilon_{c,t} \)) is constant.} \]
on stock market returns alone. Based on this assumption and the use of stock market returns as a
proxy for total wealth returns, the market-based DA discount factor from equation (12) becomes
\[ M_{t+1}^{DA} = \beta R_{m,t+1}^{\alpha-1} \frac{1 + \theta 1 \{ \beta R_{m,t+1} \leq \mu_t(\beta R_{m,t+1}) \}}{\mathbb{E}_t[1 + \theta 1 \{ \beta R_{m,t+1} \leq \mu_t(\beta R_{m,t+1}) \}]}, \]  
with \( \mu_t(\beta R_{m,t+1}) = 1 \).

Above, the DA certainty equivalent \( \mu_t \) is identified from the equilibrium condition (13) for \( \rho \) equal to one. According to equation (14), the disappointment threshold in the market-based DA discount factor above is constant, and disappointment events take place when stock market returns are less than the inverse of the discount rate: \( R_{m,t+1} \leq 1/\beta \).

To estimate the market-based DA model in equation (14), I use prespecified values for the risk-aversion parameter \( \alpha \in \{1, 0, -3\} \) as in Ostrovnaya, Routledge, and Zin (2006) due to identification concerns regarding the joint estimation of \( \alpha \) and \( \theta \). I also estimate a market-based DA model in which the risk-aversion coefficient is a free parameter. Finally, for this set of tests, I use an identity GMM weighting matrix, since there are no consumption moments to estimate, and the GMM system consists of the Euler equations for the risk-free asset, the stock market, and the six size/bm portfolios.

### 6.1.4 GMM results for the market-based DA model

Table 5 shows estimation results for the different specifications of the market-based DA model of equation (14). In terms of model fit, the \( R^2 \) of the traditional market-based model with second-order risk aversion and disappointment neutrality (market DA(1)) is low (\( R^2 = 38\% \)). In contrast, the \( R^2 \) of the market-based DA model with disappointment aversion and risk neutrality (market DA(2)) increases to 56\%. Similar results hold for the cross-sectional fit of the market-based DA specifications (3) and (5). In contrast, the fit of the market-based DA(4) model is weak because the prespecified risk-aversion parameter \( \alpha = -3 \) is large for a market-based model. In this model, the large risk-aversion coefficient also leads to a negative disappointment aversion estimate. Overall, the findings in Table 5 suggest that even within the CAPM framework, the cross-sectional performance of the asymmetric DA model is better than the fit of the smooth reference specification with second-order risk aversion.
The positive estimates of the DA coefficient $\theta$ in Table 5 range from 1.482 to 3.116. These estimates are similar, albeit larger, to the estimated prices of downside market risk from Lettau, Maggiori, and Weber (2014).\textsuperscript{19} Nevertheless, my market-based DA framework is not directly comparable to the downside CAPM in Lettau, Maggiori, and Weber (2014). One of the key differences between the two models is that in Lettau, Maggiori, and Weber (2014), bad states of nature occur when the stock market return is less than one standard deviation below the unconditional mean: $R_{m,t} \leq E[R_{m,t}] - Vol(R_{m,t})$. In contrast, in the structural market-based DA model of equation (14), bad states happen when the stock market return is less than the inverse of the discount rate: $R_{m,t} \leq 1/\beta$. This means that in the market-based DA model, bad states happen much more frequently than in the asymmetric CAPM of Lettau, Maggiori, and Weber (2014) since, based on the evidence from Tables 1 and 5, the estimates of $1/\beta$ are greater than $E[R_{m,t}] - Vol(R_{m,t})$.

Collectively, the results in this section indicate that the market-based DA model fits the cross-section of size/bm portfolios better than the traditional CAPM specification with smooth preferences. This is consistent with the findings in Lettau, Maggiori, and Weber (2014) regarding the performance of asymmetric CAPMs. Nevertheless, the performance of the market-based DA model is not as impressive as the fit of the consumption-based DA model in the first part of the paper.

### 6.1.5 Disappointment events for the market-based DA model

Figure 4 shows disappointment events for the market-based DA model when the risk aversion parameter is one (market DA(2) model).\textsuperscript{20} A quick comparison between Figures 3 and 4 shows that disappointment events in the stock market happen more often than disappointment events in consumption growth, even though the two sets partially overlap. Specifically, the set of disappointment events in the stock market typically includes periods that are characterized by market underperformance, such as the LTCM crisis of 1987 or the burst of the technology bubble from 2000 to 2002. However, during these years, disappointment events in the stock market do not spill over to the real economy, which is characterized by macroeconomic expansion and absence of disappointment events in consumption growth.

Overall, by comparing the results in Tables 2 and 5, I conclude that disappointment events in

\textsuperscript{19}Table 5 in Lettau, Maggiori, and Weber (2014).

consumption growth are more important for asset prices than disappointment events in the stock market. This outcome could be due to the high reference point in the structural market-based DA specification, which yields a large number of disappointment events, or because the stock market portfolio is an imperfect proxy of total wealth, as indicated by the critique in Roll (1977) and the evidence in Lustig, Van Nieuwerburgh, and Verdelhan (2013).

6.2 Quarterly returns

In the preceding sections, the sample frequency is annual, and disappointment events last for a year. This might overstate the importance of disappointment risk and the empirical success of the DA discount factor. Therefore, in this section, I study the performance of the consumption-based DA model using quarterly returns.

Table 6 shows GMM results for the DA discount factor from equation (6) at the quarterly frequency. Following the methodology from the annual tests, I estimate the DA coefficient using both prespecified values for the risk aversion parameter as well as joint identification of $\theta$ and $\alpha$.\(^{21}\) When the risk aversion parameter is prespecified ($\alpha \in \{1, -3, -9, -19\}$), the estimate of the DA parameter $\theta$ ranges from 13.599 to 9.281. When $\alpha$ and $\theta$ are jointly estimated, $\theta$ is 13.017 and the estimate of the risk aversion coefficient is economically insignificant ($\alpha = -0.570$). This finding suggests that at the quarterly frequency, disappointment aversion is more important than second-order risk aversion.

Based on the results in Table 6, the probability of disappointment events at the quarterly frequency does not change across the various DA specifications.\(^{22}\) In other words, the disappointment probability in the quarterly sample is not affected by the prespecified values for the risk aversion parameter. This is because the variance of the quarterly consumption growth process is very low ($\sigma_c^2 = 0.00002$). Thus, the prespecified risk aversion coefficients are not large enough to offset the low consumption variance and affect the disappointment threshold.

For the same reason, the equity premium puzzle in the quarterly sample is more pronounced than in the annual sample. For example, the risk aversion estimate for the Epstein-Zin model in

\(^{21}\)The GMM weights for the consumption growth moments and the variance of the risk-free rate are $10^8$. The weights for the mean of the risk-free rate and the equity premium are $10^5$, and the weights for the remaining test assets are one.

Table 6 ($\alpha = -175.173$) indicates that traditional consumption models require an abnormally large, in absolute value, risk aversion coefficient to explain quarterly expected returns. In contrast, the quarterly estimates of the DA parameter $\theta$, albeit large, are of the same order of magnitude as the estimates in the annual sample.

The most important finding in Table 6 is that in the quarterly sample, the differences in the cross-sectional fit between the DA discount factor and the traditional consumption models are more pronounced than in the annual sample. For instance, the quarterly $R^2$ of the Epstein-Zin model is only 20% while the corresponding $\chi^2$ test and $HJ$-statistic are large. In contrast, the $R^2$s for the DA models in the quarterly sample are all greater than 90% and the corresponding $RMSE$s range between 0.10% and 0.12%. Also, none of the DA specifications in Table 6 can be rejected based on the $\chi^2$ tests, while the $HJ$-statistics are much lower than those for the traditional consumption and three-factor Fama-French models. Collectively, the results from the quarterly sample provide additional evidence that the DA model is a significant improvement over the traditional consumption-based specification.

6.3 Are DA events a weak asset pricing factor?

One potential concern with my empirical estimation is that the covariances of the DA discount factor with excess returns are weakly identified. In particular, if the cross-sectional dispersion in the covariances between the DA discount factor and asset returns is not statistically significant, then the DA model may spuriously fit the cross-section of expected returns (e.g., Burnside 2011 and Bryzgalova 2015). To directly test the statistically significance of the covariances with the DA discount factor, in Section B of the Online Appendix, I estimate an augmented GMM system. Specifically, in addition to the moment conditions of equation (9), the augmented GMM system of the Online Appendix includes two extra conditions: (1) $\text{Cov}(M_t^{DA}, R_{mt,t} - R_{f,t}) - \text{cov}_1 = 0$ and (2) $\text{Cov}(M_t^{DA}, R_{s/v,t} - R_{b/g,t}) - \text{cov}_2 = 0$.

The first condition formally tests whether the estimated covariance, $\text{cov}_1$, between the DA discount factor and the excess return of the stock market is statistically different than zero. The second condition allows me to test whether the estimated covariance, $\text{cov}_2$, between the DA pricing kernel and the small/value minus the big/growth portfolio ($R_{s/v,t} - R_{b/g,t}$) is statistically significant. In other words, the second condition tests the statistical significance of the dispersion in covariances
between the two extreme size/bm portfolios. The results for the augmented GMM system in the Online Appendix indicate that both $\text{cov}_1$ and $\text{cov}_2$ are statistically significant across all DA specifications, thus alleviating the concerns that the covariances with the DA pricing kernel are weakly identified.

7 Conclusion

This paper studies the relation between downside macroeconomic risk and the cross-section of expected returns from the viewpoint of disappointment averse investors. To this end, I derive explicit solutions for the disappointment aversion discount factor when consumption growth is a continuous random variable with constant volatility. From a theoretical perspective, these solutions result in an alternative consumption-based asset pricing model that highlights the importance of downside consumption risk. The novel factor in the proposed model is an indicator for disappointment events in consumption growth, that is, periods during which aggregate consumption growth is less than its certainty equivalent. This certainty equivalent, which is endogenously derived from disappointment aversion, is located approximately one standard deviation below the consumption growth mean.

From an empirical standpoint, my explicit solutions for the disappointment aversion discount factor facilitate the identification of disappointment events in consumption growth and allow for a wide set of comparative tests across various test assets (equities, corporate bonds, commodity futures) and sample frequencies (annual, quarterly). In my empirical analysis, I find that a consumption-based discount factor with disappointment aversion can explain the cross-section of expected returns as accurately as the Fama-French (1993) three-factor specification. I also find that unlike traditional consumption models, the cross-sectional performance of the disappointment aversion discount factor does not depend on the frequency of returns.

Overall, my results challenge the established preference specifications which assume first-order risk neutrality. In contrast, I show that disappointment risk is priced in the cross-section of expected returns and that disappointment aversion considerably helps in improving the fit of consumption-based models.
Appendix

Proof of Proposition 1

To prove Proposition 1, I combine the linear structure of disappointment aversion with the AR(1) dynamics for consumption growth. The proof consists of four steps. First, I express the price-dividend ratio of the claim on aggregate consumption as a linear function of consumption growth. Second, I solve the disappointment aversion discount factor in terms of consumption growth. Then, I obtain the unconditional Euler equations for asset returns. Third, I show that the fixed-point problem for the disappointment reference point has a unique solution.

A.1 Price-Dividend Ratio of a Claim on Aggregate Consumption

Let $W_t$ denote total wealth and $R_{W,t+1}$ be total wealth returns. Using the linear homogeneity of the objective function and the budget constraint ($W_{t+1} = (W_t - C_t)R_{W,t+1}$), for $\rho \neq 0$, equation (1) can be written as

$$J_tW_t = \max_{C_t, \{w_{i,t}\}_{i=1}^n} \left[ (1 - \beta)C_t^\rho + \beta(W_t - C_t)^\rho \mu_t(J_{t+1}R_{W,t+1})^\rho \right]^{\frac{1}{\rho}} ,$$

where $J_t$ is marginal utility. The first-order condition for $C_t$ reads

$$(1 - \beta)C_t^{\rho-1} - \beta(W_t - C_t)^{\rho-1} \mu_t(J_{t+1}R_{W,t+1})^\rho = 0.$$

Dividing by $W_t^{\rho-1}$, we obtain

$$(1 - \beta)\left(\frac{C_t}{W_t}\right)^{\rho-1} - \beta\left(1 - \frac{C_t}{W_t}\right)^{\rho-1} \mu_t(J_{t+1}R_{W,t+1})^\rho = 0. \quad (15)$$

Along an optimal consumption path, the following holds:

$$J_t^\rho W_t^\rho = (1 - \beta)C_t^\rho + \beta(W_t - C_t)^\rho \mu_t(J_{t+1}R_{W,t+1})^\rho .$$

Dividing by $W_t^\rho$, we obtain

$$J_t^\rho = (1 - \beta)\left(\frac{C_t}{W_t}\right)^\rho + \beta\left(1 - \frac{C_t}{W_t}\right) \mu_t(J_{t+1}R_{W,t+1})^\rho . \quad (16)$$

Equations (15) and (16) imply that

$$J_t^\rho = (1 - \beta)\left(\frac{C_t}{W_t}\right)^{\rho-1} . \quad (17)$$

We can substitute the above relation into equation (15) to obtain

$$(1 - \beta)\left(\frac{C_t}{W_t}\right)^{\rho-1} - \beta(1 - \beta)\left(1 - \frac{C_t}{W_t}\right)^{\rho-1} \mu_t\left[\left(\frac{C_{t+1}}{W_{t+1}}\right)^{(\rho-1)/\rho} R_{W,t+1}^1\right]^\rho = 0.$$

Using the budget constraint once more, the first-order conditions for consumption simplify into

$$\beta \mu_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{(\rho-1)/\rho} R_{W,t+1}^{1/\rho}\right]^\rho = 1 . \quad (18)$$
Let $P_{c,t} = W_t - C_t$ be the price for a claim on aggregate consumption. We can use the price-dividend identity in Campbell and Shiller (1988),

$$R_{W,t+1} = \frac{C_{t+1}}{C_t} \frac{P_{c,t+1}/C_{t+1} + 1}{P_{c,t}/C_t},$$

(19)

to recast equation (18) as

$$\frac{1}{\beta} \left( \frac{C_{t+1}}{P_{c,t}} \right)^{\frac{1}{\beta}} = \mu_t \left[ \frac{C_{t+1}}{C_t} \left( \frac{P_{c,t+1}}{P_{c,t}/C_t} + 1 \right)^{1/\beta} \right].$$

(20)

A log-linear approximation to the price-dividend identity in equation (19) around the point $P_c$ is given by

$$r_{W,t+1} = \kappa_{c,0} + \kappa_{c,1} P_{c,t+1} - P_{c,t} + \Delta c_{t+1},$$

(21)

where $p_{c,t} = \log \frac{P_{c,t}}{C_t}$, and $\kappa_{c,1} = \frac{e^{P_c}}{1 + e^{P_c}} \in (0, 1)$ and $\kappa_{c,0} = \log(1 + e^{P_c}) - \kappa_{c,1} P_c$ are log-linearization constants.

We conjecture that the log price-dividend ratio is linear in consumption growth:

$$p_{c,t} = \mu_v + \phi_v \Delta c_t$$

with $1 + \frac{1}{\rho} \kappa_{c,1} \phi_v > 0$. Using the definition of the DA certainty equivalent from equation (2), equation (20) becomes

$$-\frac{\alpha}{\rho} \left( \log \beta - p_{c,t} \right) = \log E_t \left[ e^{\alpha \Delta c_{t+1} + \frac{\phi_v}{\rho} \left( \kappa_{c,0} + \kappa_{c,1} P_{c,t+1} \right)} \right] \times$$

$$1 + \theta \left\{ \frac{C_{t+1}}{C_t} \left( \frac{P_{c,t+1}}{P_{c,t}/C_t} + 1 \right)^{\frac{1}{\beta}} \leq \mu_t \left[ \frac{C_{t+1}}{C_t} \left( \frac{P_{c,t+1}}{P_{c,t}/C_t} + 1 \right)^{1/\beta} \right] \right\} \frac{1}{1 + \theta \mathbb{E}_t \left[ \left\{ \frac{C_{t+1}}{C_t} \left( \frac{P_{c,t+1}}{P_{c,t}/C_t} + 1 \right)^{\frac{1}{\beta}} \leq \mu_t \left[ \frac{C_{t+1}}{C_t} \left( \frac{P_{c,t+1}}{P_{c,t}/C_t} + 1 \right)^{1/\beta} \right] \right\} \right]}.$$
where \( N() \) is the standard normal c.d.f., and \( d_1 \) is the threshold for disappointment defined as

\[
d_1 = -\log(\beta) - \mu_v + \phi_v \Delta c_t - \kappa_{c,0} - \kappa_{c,1} \mu_v - (\kappa_{c,1} \phi_v + \rho) \mu_c(1 - \phi_c) + \phi_c \Delta c_t \over (\kappa_{c,1} \phi_v + \rho) \sqrt{1 - \phi_c^2} \sigma_c. \tag{23}
\]

We can now use the method of undetermined coefficients to find the values for \( \mu_v \) and \( \phi_v \). First, we collect consumption growth terms ignoring the terms \( \log\left(1 + \theta N\left(d_1 - \left(\frac{\rho}{\phi} \kappa_{c,1} \phi_v + \alpha\right) \sqrt{1 - \phi_c^2} \sigma_c\right)\right) \)
and \( \log\left(1 + \theta N(\phi_v)\right) \) in equation (22). Then, we solve for \( \phi_v \) to obtain

\[
\phi_v = \frac{\rho \phi_c}{1 - \kappa_{c,1} \phi_c}. \tag{24}
\]

For the above value of \( \phi_v \), all \( \Delta c_t \) terms in equation (23) vanish, and \( d_1 \) becomes a function of constant terms alone. Also, for the above value of \( \phi_v \) and stationary consumption growth process, that is, \(-1 < \phi_c < 1\), our conjecture \( 1 + \frac{1}{\rho} \kappa_{c,1} \phi_v > 0 \) is satisfied since \( \kappa_{c,1} = \frac{\rho \phi_c}{1 + \phi_c^2} < 1 \).

Collecting constant terms in equation (22), the solution for \( \mu_v \) is given by

\[
\mu_v = \frac{1}{1 - \kappa_{c,1}} \left[\log(\beta) + \kappa_{c,0} + (\kappa_{c,1} \phi_v + \rho) \mu_c(1 - \phi_c) + \frac{1}{2} \frac{\alpha}{\rho} \left((\kappa_{c,1} \phi_v + \rho) \sqrt{1 - \phi_c^2} \sigma_c\right)^2 + \frac{\rho}{\alpha} \log\left(1 + \theta N\left(d_1 + \frac{\alpha}{\rho} (\kappa_{c,1} \phi_v + \rho) \sqrt{1 - \phi_c^2} \sigma_c\right)\right) - \frac{\rho}{\alpha} \log\left(1 + \theta N(\phi_v)\right)\right],
\]

and \( d_1 \) in equation (23) becomes the solution to the fixed-point problem

\[
d_1 = \frac{1}{2} \frac{\alpha}{\rho} (\kappa_{c,1} \phi_v + \rho) \sqrt{1 - \phi_c^2} \sigma_c + \frac{\log\left(1 + \theta N\left(d_1 + \frac{\alpha}{\rho} (\kappa_{c,1} \phi_v + \rho) \sqrt{1 - \phi_c^2} \sigma_c\right)\right)}{\frac{\alpha}{\rho} (\kappa_{c,1} \phi_v + \rho) \sqrt{1 - \phi_c^2} \sigma_c}.
\]

Using the solution for \( \phi_v \) in equation (24), the fixed-point problem for \( d_1 \) does not depend on \( \rho \) since

\[
d_1 = \frac{1}{2} \frac{\alpha}{1 - \kappa_{c,1} \phi_c} \sqrt{1 - \phi_c^2} \sigma_c + \frac{\log\left(1 + \theta N\left(d_1 + \frac{\alpha}{1 - \kappa_{c,1} \phi_c} \sqrt{1 - \phi_c^2} \sigma_c\right)\right)}{\frac{\alpha}{1 - \kappa_{c,1} \phi_c} \sqrt{1 - \phi_c^2} \sigma_c}, \tag{25}
\]

and we can rewrite \( \mu_v \) as

\[
\mu_v = \frac{1}{1 - \kappa_{c,1}} \left[\log(\beta) + \kappa_{c,0} + (\kappa_{c,1} \phi_v + \rho) \mu_c(1 - \phi_c) + \frac{1}{2} \frac{\alpha}{\rho} \left((\kappa_{c,1} \phi_v + \rho) \sqrt{1 - \phi_c^2} \sigma_c\right)^2 + \frac{\rho}{\alpha} \log\left(1 + \theta N\left(d_1 + \frac{\alpha}{\rho} (\kappa_{c,1} \phi_v + \rho) \sqrt{1 - \phi_c^2} \sigma_c\right)\right) - \frac{\rho}{\alpha} \log\left(1 + \theta N(\phi_v)\right)\right].
\]

### A.2 Explicit Solutions for the Disappointment Aversion Stochastic Discount Factor

From Routledge and Zin (2010), the DA stochastic discount factor can be written as

\[
M_{t+1} = \beta^\frac{\alpha}{\rho} \left(\frac{C_{t+1}}{C_t}\right)^{\frac{\alpha-1}{\rho}} R_{t+1}^{\alpha/\rho-1} \frac{1 + \theta \{\beta^\frac{1}{\rho} \left(\frac{C_{t+1}}{C_t}\right)^{\frac{\alpha-1}{\rho}} R_{W,t+1}^{1/\rho} \leq 1\}}{1 + \Theta \{\beta^\frac{1}{\rho} \left(\frac{C_{t+1}}{C_t}\right)^{\frac{\alpha-1}{\rho}} R_{W,t+1}^{1/\rho} \leq 1\}}.
\]
Using the solutions for $\phi$ and $\mu$, it follows that order risk aversion $A$ with the stochastic discount factor can be further expressed as

$$M_{t+1} = e^{\frac{\alpha}{p_{1}}\log \beta + \frac{\alpha}{p_{1}}(\rho - 1)\Delta c_{t+1} + \frac{\alpha}{p_{1}}[\kappa_{c,0} + \kappa_{c,1}(\mu_{c} + \phi_{c}\Delta c_{t+1}) - (\mu_{c} + \phi_{c}\Delta c_{t}) + \Delta c_{t+1}]}$$

$$1 + \theta\mathbb{E}_{t}[\{\frac{1}{p_{1}}\log \beta + \frac{1}{p_{1}}(\rho - 1)\Delta c_{t+1} + \frac{1}{p_{1}}[\kappa_{c,0} + \kappa_{c,1}(\mu_{c} + \phi_{c}\Delta c_{t+1}) - (\mu_{c} + \phi_{c}\Delta c_{t}) + \Delta c_{t+1}] \leq 0\}]$$

Using the solutions for $\phi_{v}$ and $\mu_{v}$, we conclude that

$$M_{t+1} = e^{\frac{\alpha}{p_{1}}(\rho - 1)\Delta c_{t+1} + \frac{\alpha}{p_{1}}[\kappa_{c,0} + \kappa_{c,1}(\mu_{c} + \phi_{c}\Delta c_{t+1}) - (\mu_{c} + \phi_{c}\Delta c_{t}) + \Delta c_{t+1}]}$$

$$1 + \theta\mathbb{E}_{t}[\{\frac{1}{\rho_{1}}(\rho - 1)\Delta c_{t+1} + \frac{1}{\rho_{1}}[\kappa_{c,0} + \kappa_{c,1}(\mu_{c} + \phi_{c}\Delta c_{t+1}) - (\mu_{c} + \phi_{c}\Delta c_{t}) + \Delta c_{t+1}] \leq 0\})].$$

(A.3) **Unconditional Euler Equations**

We can write the Euler equation for the risk-free rate as

$$\mathbb{E}_{t}[\tilde{M}_{t+1} R_{f,t+1}] = 1 + \theta\mathbb{E}_{t}[\{\Delta c_{t+1} \leq \mu_{c}(1 - \phi_{c}) + \phi_{c}\Delta c_{t} + d_{1}\sqrt{1 - \phi_{c}^{2}\sigma_{c}}\}],$$

with

$$\tilde{M}_{t+1} = e^{\frac{\alpha}{p_{1}}(\rho - 1)\Delta c_{t+1} + \frac{\alpha}{p_{1}}[\kappa_{c,0} + \kappa_{c,1}(\mu_{c} + \phi_{c}\Delta c_{t+1}) - (\mu_{c} + \phi_{c}\Delta c_{t}) + \Delta c_{t+1}]}$$

$$\times [1 + \theta\mathbb{E}_{t}[\{\Delta c_{t+1} \leq (1 - \phi_{c})\mu_{c} - \phi_{c}\Delta c_{t} + d_{1}\sqrt{1 - \phi_{c}^{2}\sigma_{c}}\}].$$

Taking unconditional expectations in both sides of the Euler equation above and rearranging, we obtain

$$\mathbb{E}[M_{t+1} R_{f,t+1}] = 1,$$

where

$$M_{t+1}^{DA} = e^{\frac{\alpha}{p_{1}}(\rho - 1)\Delta c_{t+1} + \frac{\alpha}{p_{1}}[\kappa_{c,0} + \kappa_{c,1}(\mu_{c} + \phi_{c}\Delta c_{t+1}) - (\mu_{c} + \phi_{c}\Delta c_{t}) + \Delta c_{t+1}]}$$

$$\times [1 + \theta\mathbb{E}[\Delta c_{t+1} \leq (1 - \phi_{c})\mu_{c} - \phi_{c}\Delta c_{t} + d_{1}\sqrt{1 - \phi_{c}^{2}\sigma_{c}}\}]]$$

(A.4) **Existence and Uniqueness of the Solution to the Fixed-Point Problem for $d_{1}$**

To show that the fixed-point problem in equation (25) has a unique negative solution, consider the function $h : \mathbb{R} \to \mathbb{R}$ such that

$$h(x) = x + 0.5A + \frac{1}{A}\log \left[\frac{1 + \theta N(x + A)}{1 + \theta N(x)}\right].$$

with $A = -\frac{\alpha}{1 - \kappa_{c,1}\phi_{c}}\sqrt{1 - \phi_{c}^{2}\sigma_{c}} > 0$. The condition $A > 0$ implies that the coefficient of second-order risk aversion $\alpha$ is negative since the rest of the terms in $A$ are always positive. Specifically, for a stationary consumption growth process ($\sigma_{c} > 0$, $\phi_{c} \in (-1, 1)$) and $\kappa_{c,1} \in (0, 1)$ in equation (21), it follows that $1 - \phi_{c}^{2} > 0$ and $1 - \kappa_{c,1}\phi_{c} > 0$.

Now, for the function $h(x)$, we have that

$$h(0) = 0.5A + \frac{1}{A}\log \left[\frac{1 + \theta N(A)}{1 + \theta N(0)}\right] > 0 \quad \text{and} \quad \lim_{x \downarrow -\infty} h(x) = -\infty < 0.$$
Since the function $h(x)$ is continuous in $\mathbb{R}$, there exists at least one solution to the equation $h(x) = 0$ in $(-\infty, 0)$. Basically, because $\theta$ and $A$ are positive, $0.5A + \frac{1}{A} \log \left[ \frac{1 + \theta N(x+A)}{1 + \theta N(x)} \right]$ is also positive, and the solution to the equation $h(x) = 0$ has to be negative. Finally, even though monotonicity of $h(x)$ is hard to show algebraically, Figure A1 shows that $h(x)$ is strictly increasing, and, therefore, the solution to the equation $h(x) = 0$ is unique in $(-\infty, 0)$.

**Figure A1. Uniqueness of the solution for $d_1$**

Figure A1 shows that the function $h(x) = x + 0.5A + \frac{1}{A} \log \left[ \frac{1 + \theta N(x+A)}{1 + \theta N(x)} \right]$ is monotonically increasing in $x$, and that for positive $A$, the problem $h(x) = 0$ has a unique, negative solution in $\mathbb{R}$. 
References


Figure 1. Fitted expected returns for the DA discount factor

Figure 1 shows sample and fitted annual expected returns for the risk-free asset, the aggregate stock market index, and the six size/bm equity portfolios. Fitted expected returns are estimated according to equation (10) for the consumption-based DA discount factor of equation (6) with different values for the risk-aversion parameter $\alpha$. The fitted value for the volatility of the risk-free rate is given by the square root of equation (11). Figure 1 also shows fitted expected returns for the CRRA and Epstein-Zin (EZ) models. Estimation results are shown in Table 2. The sample period is 1933-2012.
Figure 2. Fitted expected returns for the DA discount factor: Corporate bonds and commodity futures

Figure 2 shows sample and fitted annual expected returns for the risk-free asset, the aggregate stock market index, the six size/bm equity portfolios, the five corporate bond portfolios sorted on credit rating, and the five commodity futures portfolios sorted on basis. Fitted expected returns are estimated according to equation (10) for the consumption-based DA discount factor of equation (6) with different values for the risk-aversion parameter $\alpha$. The fitted value for the volatility of the risk-free rate is given by the square root of equation (11). Figure 2 also shows fitted expected returns for the CRRA and Fama-French (FF3) models. Estimation results are shown in Table 4. The sample period is 1974-2008.
Figure 3. Consumption growth, disappointment events, and NBER recessions

Panel A in Figure 3 shows annual disappointment events in consumption growth for the DA discount factor of equation (6) when the risk-aversion parameter is equal to -19 (DA(4) model in Table 2). The solid line is consumption growth, and the dashed line is the time varying DA certainty equivalent of consumption growth according to equation (7). Disappointment events are highlighted by ellipses, and shaded areas are NBER recessions. Estimation results for the consumption growth moments and the disappointment threshold are shown in Table 2. Panel B plots the time-series of consumption growth (solid line) and the HML (dashed line). The sample period is 1933-2012.
Figure 4. Disappointment events for the market-based DA model

Figure 4 shows annual disappointment events for the market-based DA discount factor of equation (14) when the risk-aversion parameter is equal to one (market DA(2) model in Table 5). The solid line shows log-returns for the stock market index. The dashed-dotted line is the market-based threshold for disappointment, which is equal to $-\log \beta$, where $\beta$ is the discount rate. Estimation results for the market-based disappointment threshold are reported in Table 5. Shaded areas are NBER recessions. The sample period is 1933-2012.
Table 1. Summary statistics for consumption growth and real asset returns

Table 1 shows summary statistics for consumption growth ($\Delta c_t$), the log risk-free rate ($r_{f,t}$), and asset log-returns. In panels A and B, equity returns are for the aggregate stock market portfolio and the Fama-French six portfolios sorted on size and book-to-market. Corporate bond returns in panel C are for the aggregate investment-grade corporate bond index and the four portfolios of investment-grade corporate bonds sorted by credit rating. Commodity returns in panel D are for five portfolios of commodity futures sorted by the basis (futures price over spot price). All variables are expressed in real terms.

**Panel A: Equity portfolios (annual, 1933-2012)**

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**Panel C: Corporate bonds (annual, 1973-2012)**

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<tr>
<td>Autocorrelation</td>
<td>0.54</td>
<td>0.76</td>
<td>0.02</td>
<td>0.17</td>
<td>0.15</td>
<td>0.19</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.22</td>
<td>0.31</td>
<td>0.27</td>
<td>0.29</td>
<td>0.29</td>
<td>0.32</td>
</tr>
</tbody>
</table>

**Panel D: Commodity futures (annual, 1974-2008)**

<table>
<thead>
<tr>
<th></th>
<th>Stock market</th>
<th>Low basis</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.84</td>
<td>2.09</td>
<td>6.09</td>
<td>5.54</td>
<td>2.06</td>
<td>-0.04</td>
</tr>
<tr>
<td>Volatility</td>
<td>1.25</td>
<td>2.42</td>
<td>18.50</td>
<td>21.74</td>
<td>13.37</td>
<td>16.08</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.56</td>
<td>0.73</td>
<td>-0.06</td>
<td>-0.24</td>
<td>0.10</td>
<td>-0.00</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.21</td>
<td>0.15</td>
<td>-0.00</td>
<td>-0.13</td>
<td>-0.31</td>
<td>-0.35</td>
</tr>
</tbody>
</table>
Table 2. GMM results for the consumption-based DA discount factor

Table 2 shows estimation results for annual returns and the DA discount factor of Equation (6) with different values for the risk-aversion parameter \( \alpha \). Table 2 also shows estimation results for the CRRA, Epstein-Zin (EZ), and three-factor Fama-French (FF3) models. The GMM moment conditions for the consumption-based models are the consumption growth mean, the consumption growth variance, the consumption growth autocovariance, the variance of the risk-free rate, and the unconditional Euler equations for the risk-free asset, the aggregate stock market index, and the Fama-French six size/bm portfolios. The moment conditions for the DA(1) model exclude the variance of the risk-free rate. The moment conditions for the Fama-French model exclude the consumption growth moments and the variance of the risk-free rate. The constant \( \mu_c \) is the consumption growth mean (×100), \( \sigma^2_c \) is consumption growth variance (×100), and \( \phi_c \sigma^2 \) is consumption growth autocovariance (×100). The parameter \( \beta \) is the discount rate, \( \alpha \) is the risk-aversion parameter, \( \rho \) is equal to 1 − 1/EIS, and \( \theta \) is the disappointment aversion coefficient. The constant \( d_1 \) is the disappointment threshold in Equation (7), and \( P(\text{disap.}) \) is the probability of disappointment events. Bootstrap 95% confidence intervals are shown in brackets. \( \chi^2 \), d.o.f., and \( p \) are the bootstrap first-stage \( \chi^2 \) test (Hansen 1982; Lemma 4.1), degrees of freedom, and \( p \)-value that all moment conditions are jointly zero. \( HJ \) is equal to \( T \times \text{Distance}^2 \), where \( \text{Distance} \) is the Hansen-Jagannathan distance (Hansen and Jagannathan 1997) for excess returns in the set of risky assets, and \( T \) is the number of time-series observations. \( RMSE \) and \( R^2 \) are the cross-sectional root-mean-square error (×100) and R-square (×100), respectively, for the set of risky assets. The sample is 1933-2012.

<table>
<thead>
<tr>
<th></th>
<th>CRRA ( \alpha = \rho ), ( \theta = 0 )</th>
<th>EZ ( \theta = 0 )</th>
<th>DA(1) ( \alpha = -3 )</th>
<th>DA(2) ( \alpha = -9 )</th>
<th>DA(3) ( \alpha = -19 )</th>
<th>DA(4) ( \alpha = -38.369 )</th>
<th>DA(5) ( \alpha = -5.891 )</th>
<th>FF3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_c )</td>
<td>2.167 [1.91, 2.45]</td>
<td>2.169 [1.91, 2.45]</td>
<td>2.169 [1.91, 2.45]</td>
<td>2.169 [1.91, 2.45]</td>
<td>2.169 [1.91, 2.45]</td>
<td>2.169 [1.91, 2.45]</td>
<td>2.169 [1.91, 2.45]</td>
<td></td>
</tr>
<tr>
<td>( \sigma^2_c )</td>
<td>0.023 [0.012, 0.024]</td>
<td>0.023 [0.012, 0.024]</td>
<td>0.024 [0.012, 0.024]</td>
<td>0.023 [0.012, 0.024]</td>
<td>0.023 [0.012, 0.024]</td>
<td>0.022 [0.010, 0.030]</td>
<td>0.021 [0.011, 0.030]</td>
<td></td>
</tr>
<tr>
<td>( \phi_c \sigma^2 )</td>
<td>0.005 [0.002, 0.008]</td>
<td>0.005 [0.001, 0.009]</td>
<td>0.006 [0.001, 0.009]</td>
<td>0.006 [0.003, 0.009]</td>
<td>0.006 [0.002, 0.011]</td>
<td>0.005 [0.001, 0.008]</td>
<td>0.001 [0.001, 0.007]</td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>1.217 [1.05, 1.61]</td>
<td>1.159 [0.92, 1.34]</td>
<td>0.998 [0.98, 1.02]</td>
<td>1.066 [1.02, 1.12]</td>
<td>1.093 [1.01, 1.14]</td>
<td>1.107 [1.02, 1.16]</td>
<td>1.104 [1.02, 1.16]</td>
<td></td>
</tr>
<tr>
<td>( P(\text{disap.}) )</td>
<td>-0.878 [-0.866, -0.794]</td>
<td>-0.866 [-0.866, -0.794]</td>
<td>-0.794 [-0.763, -0.763]</td>
<td>-0.763 [-0.763, -0.763]</td>
<td>-0.763 [-0.763, -0.763]</td>
<td>-0.763 [-0.763, -0.763]</td>
<td>-0.763 [-0.763, -0.763]</td>
<td></td>
</tr>
<tr>
<td>( \chi^2 )</td>
<td>630.883 [48.64, 7.04]</td>
<td>156.795 [16.64, 2.54]</td>
<td>1.819 [1.64, 2.34]</td>
<td>4.748 [1.447, 1.447]</td>
<td>8.147 [0.45, 0.45]</td>
<td>8.147 [0.45, 0.45]</td>
<td>8.147 [0.45, 0.45]</td>
<td>81.024</td>
</tr>
<tr>
<td>d.o.f.</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.000 [0.935, 0.780]</td>
<td>0.000 [0.935, 0.780]</td>
<td>0.000 [0.935, 0.780]</td>
<td>0.000 [0.935, 0.780]</td>
<td>0.000 [0.935, 0.780]</td>
<td>0.000 [0.935, 0.780]</td>
<td>0.000 [0.935, 0.780]</td>
<td>0</td>
</tr>
<tr>
<td>d.o.f.</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>( p )</td>
<td>0.000 [0.403, 0.037]</td>
<td>0.000 [0.403, 0.037]</td>
<td>0.000 [0.403, 0.037]</td>
<td>0.000 [0.403, 0.037]</td>
<td>0.000 [0.403, 0.037]</td>
<td>0.000 [0.403, 0.037]</td>
<td>0.000 [0.403, 0.037]</td>
<td>0</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>-108.9 [71.1, 95.5]</td>
<td>97.0 [97.0, 95.7]</td>
<td>97.2 [97.2, 95.7]</td>
<td>97.7 [97.7, 95.4]</td>
<td>97.7 [97.7, 95.4]</td>
<td>97.7 [97.7, 95.4]</td>
<td>97.7 [97.7, 95.4]</td>
<td>90.4</td>
</tr>
</tbody>
</table>
Table 3. GMM pricing errors for the consumption-based DA discount factor

Table 3 shows pricing errors ($\times 100$) for the consumption-based DA discount factor of equation (6) with different values for the risk-aversion parameter $\alpha$. Table 3 also shows pricing errors for the two nested consumption-based specifications (CRRA, Epstein-Zin) and the three-factor Fama-French model. The test assets are the Fama-French six size/bm portfolios. Table 3 does not report pricing errors for the consumption growth moments, the risk-free rate, or the aggregate stock market index because these moments are perfectly fitted by all models, except for the CRRA one, due to the choice of the GMM weighting matrix. Bootstrap $t$-statistics are in parenthesis. Estimation results are shown in Table 2. The sample period is 1933-2012.

<table>
<thead>
<tr>
<th></th>
<th>Small/growth</th>
<th>Small/medium</th>
<th>Small/value</th>
<th>Big/growth</th>
<th>Big/medium</th>
<th>Big/value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA $\alpha = \rho, \theta = 0$</td>
<td>8.838</td>
<td>11.948</td>
<td>14.277</td>
<td>6.604</td>
<td>8.190</td>
<td>10.367</td>
</tr>
<tr>
<td>$\alpha = \rho, \theta = 0$</td>
<td>(6.15)</td>
<td>(15.64)</td>
<td>(13.86)</td>
<td>(12.47)</td>
<td>(11.26)</td>
<td>(8.19)</td>
</tr>
<tr>
<td>EZ $\theta = 0$</td>
<td>-1.628</td>
<td>1.560</td>
<td>3.099</td>
<td>-0.415</td>
<td>1.241</td>
<td>1.170</td>
</tr>
<tr>
<td>$\theta = 0$</td>
<td>(-3.38)</td>
<td>(1.55)</td>
<td>(4.15)</td>
<td>(-0.97)</td>
<td>(2.33)</td>
<td>(2.95)</td>
</tr>
<tr>
<td>DA(1) $\alpha = \rho = 1$</td>
<td>0.365</td>
<td>1.082</td>
<td>1.185</td>
<td>-0.070</td>
<td>-0.058</td>
<td>0.166</td>
</tr>
<tr>
<td>$\alpha = \rho = 1$</td>
<td>(0.22)</td>
<td>(0.38)</td>
<td>(0.34)</td>
<td>(-0.03)</td>
<td>(-0.02)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>DA(2) $\alpha = -3$</td>
<td>-0.216</td>
<td>0.657</td>
<td>1.051</td>
<td>0.026</td>
<td>-0.007</td>
<td>0.439</td>
</tr>
<tr>
<td>$\alpha = -3$</td>
<td>(-0.09)</td>
<td>(0.28)</td>
<td>(0.80)</td>
<td>(0.03)</td>
<td>(-0.00)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>DA(3) $\alpha = -9$</td>
<td>-0.599</td>
<td>0.655</td>
<td>1.227</td>
<td>-0.121</td>
<td>0.402</td>
<td>0.347</td>
</tr>
<tr>
<td>$\alpha = -9$</td>
<td>(-0.05)</td>
<td>(0.16)</td>
<td>(0.10)</td>
<td>(-0.01)</td>
<td>(0.02)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>DA(4) $\alpha = -19$</td>
<td>-0.959</td>
<td>0.192</td>
<td>0.481</td>
<td>-0.080</td>
<td>0.690</td>
<td>-0.180</td>
</tr>
<tr>
<td>$\alpha = -19$</td>
<td>(-0.17)</td>
<td>(0.01)</td>
<td>(0.06)</td>
<td>(-0.13)</td>
<td>(0.22)</td>
<td>(-0.07)</td>
</tr>
<tr>
<td>DA(5) $\alpha = -5.891$</td>
<td>-0.359</td>
<td>0.751</td>
<td>1.270</td>
<td>-0.009</td>
<td>0.089</td>
<td>0.534</td>
</tr>
<tr>
<td>$\alpha = -5.891$</td>
<td>(-0.13)</td>
<td>(0.22)</td>
<td>(0.90)</td>
<td>(-0.00)</td>
<td>(0.02)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>FF3 $\alpha = -5.891$</td>
<td>-1.100</td>
<td>1.202</td>
<td>0.612</td>
<td>0.948</td>
<td>-0.165</td>
<td>-1.352</td>
</tr>
<tr>
<td>$\alpha = -5.891$</td>
<td>(-4.26)</td>
<td>(6.63)</td>
<td>(2.39)</td>
<td>(2.67)</td>
<td>(-0.41)</td>
<td>(-5.13)</td>
</tr>
</tbody>
</table>
Table 4. GMM results for the consumption-based DA discount factor: Corporate bonds and commodity futures

Table 4 shows estimation results for annual returns and the DA discount factor from equation (6) with different values for the risk-aversion parameter \( \alpha \). Table 4 also shows estimation results for the CRRA, Epstein-Zin, and three-factor Fama-French models. The GMM moment conditions for the consumption models are the consumption growth mean, the consumption growth variance, the consumption growth autocovariance, the variance of the risk-free rate, and the unconditional Euler equations for the risk-free asset, the aggregate stock market index, the Fama-French six size/bm portfolios, the aggregate investment-grade corporate bond index, the four corporate bond portfolios sorted on credit rating, and the five commodity futures portfolios sorted by the basis. The moment conditions for the DA(1) model exclude the variance of the risk-free rate. The moment conditions for the Fama-French model exclude the consumption growth moments and the variance of the risk-free rate. Table 4 does not report results for the consumption growth moments. The parameter \( \beta \) is the discount rate, \( \alpha \) is the risk-aversion parameter, \( \rho \) is equal to \( 1 - \frac{1}{EIS} \), and \( \theta \) is the disappointment aversion coefficient. The constant \( d_1 \) is the disappointment threshold in equation (7), and \( P(\text{disap.}) \) is the probability of disappointment events. Bootstrap 95% confidence intervals are shown in brackets. \( \chi^2 \), d.o.f., and \( p \) are the bootstrap first-stage \( \chi^2 \) test, degrees of freedom, and \( p \)-value that all moment conditions are jointly zero. \( HJ \) is equal to \( T \times \text{Distance}^2 \), where Distance is the Hansen-Jagannathan distance for excess returns in the set of risky assets, and \( T \) is the number of time-series observations. The covariance matrices for the \( \chi^2 \) test and \( HJ \)-statistic in Table 4 are calculated imposing zero off-diagonal elements due to the short time-series sample and the relatively large number of tests assets. \( \text{RMSE} \) and \( R^2 \) are the cross-sectional root-mean-square error and \( R \)-square, respectively, for the set of risky assets. The sample is 1974-2008.

<table>
<thead>
<tr>
<th>Disappointment aversion (DA)</th>
<th>CRRA ( \alpha = \rho ), ( \theta = 0 )</th>
<th>EZ ( \theta = 0 )</th>
<th>DA(1) ( \alpha = \rho = 1 )</th>
<th>DA(2) ( \alpha = -3 )</th>
<th>DA(3) ( \alpha = -9 )</th>
<th>DA(4) ( \alpha = -19 )</th>
<th>DA(5)</th>
<th>FF3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>1.042 [1.01, 1.28]</td>
<td>1.000 [0.99, 1.00]</td>
<td>1.003 [0.93, 1.11]</td>
<td>1.012 [0.73, 1.11]</td>
<td>1.008 [0.49, 1.47]</td>
<td>0.992 [0.55, 1.66]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha )</td>
<td>-3.553 [-336.09, -3.73]</td>
<td>-15.129 [-82.28, -4.10]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho )</td>
<td>-2.615 [-12.43, -1.20]</td>
<td>-2.605 [-9.15, 0.14]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( d_1 )</td>
<td>-0.831 [-0.814, -0.776]</td>
<td>-0.871 [-0.814, -0.776]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d.o.f.</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>( p )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>d.o.f.</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>( p )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>55.5</td>
<td>55.5</td>
<td>55.5</td>
<td>55.5</td>
<td>55.5</td>
<td>55.5</td>
<td>55.5</td>
<td>55.5</td>
</tr>
</tbody>
</table>
Table 5. GMM results for the market-based DA discount factor

Table 5 shows estimation results for annual returns and the market-based DA discount factor of equation (14), in which stock market returns are used as a proxy for returns on aggregate wealth. The GMM moment conditions are the unconditional Euler equations for the risk-free asset, the aggregate stock market index, and the Fama-French six size/bm portfolios. The parameter \( \beta \) is the discount rate, \( \alpha \) is the risk-aversion parameter, \( \rho \) is equal to \( 1 - 1/EIS \), \( \theta \) is the disappointment aversion coefficient. Bootstrap 95% confidence intervals are shown in brackets. \( P(disap.) \) is the probability of disappointment events. \( \chi^2 \), d.o.f., and \( p \) are the bootstrap first-stage \( \chi^2 \)test, degrees of freedom, and \( p \)-value that all moment conditions are jointly zero, \( HJ \) is equal to \( T \times Distance^2 \), where \( Distance \) is the Hansen-Jagannathan distance for excess returns in the set of risky assets, and \( T \) is the number of time-series observations. The GMM weighting matrix for these tests is the identify matrix. \( RMSE \) and \( R^2 \) are the cross-sectional root-mean-square error and \( R \)-square, respectively, for the set of risky assets. The sample period is 1933-2012.

<table>
<thead>
<tr>
<th>Market-based disappointment aversion</th>
<th>Market DA(1) ( \theta = 0, \rho = 1 )</th>
<th>Market DA(2) ( \alpha = 1, \rho = 1 )</th>
<th>Market DA(3) ( \alpha = 0, \rho = 1 )</th>
<th>Market DA(4) ( \alpha = -3, \rho = 1 )</th>
<th>Market DA(5) ( \rho = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.970 [0.96, 0.98]</td>
<td>0.973 [0.95, 0.98]</td>
<td>0.944 [0.80, 1.24]</td>
<td>0.945 [0.93, 0.96]</td>
<td>0.973 [0.77, 1.49]</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>-1.886 [-2.21, -1.14]</td>
<td>0.657 [0.08, 2.00]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta )</td>
<td>3.116 [1.76, 3.27]</td>
<td>1.482 [-5.09, 13.95]</td>
<td>-0.465 [-0.60, -0.30]</td>
<td>2.492 [-6.84, 75.18]</td>
<td></td>
</tr>
<tr>
<td>( P(disap.) )</td>
<td>0.375</td>
<td>0.400</td>
<td>0.400</td>
<td>0.375</td>
<td></td>
</tr>
<tr>
<td>( \chi^2 )</td>
<td>139.679</td>
<td>138.604</td>
<td>42.447</td>
<td>144.176</td>
<td>48.988</td>
</tr>
<tr>
<td>d.o.f.</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>( p )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( HJ )</td>
<td>26.089</td>
<td>24.831</td>
<td>26.067</td>
<td>26.691</td>
<td>25.819</td>
</tr>
<tr>
<td>d.o.f.</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>( p )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( RMSE )</td>
<td>2.308</td>
<td>1.910</td>
<td>2.114</td>
<td>2.446</td>
<td>1.992</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>38.5</td>
<td>56.0</td>
<td>50.1</td>
<td>30.9</td>
<td>53.8</td>
</tr>
</tbody>
</table>
Table 6. GMM results for the consumption-based DA discount factor: Quarterly returns

Table 6 shows estimation results for quarterly returns and the DA discount factor of equation (6) with different values for the risk-aversion parameter $\alpha$. Table 6 also shows results for the CRRA, Epstein-Zin, and three-factor Fama-French models. The GMM moment conditions for the consumption models are the consumption growth mean, the consumption growth variance, the consumption growth autocovariance, the variance of the risk-free rate, and the unconditional Euler equations for the risk-free asset, the aggregate stock market index, and the Fama-French six size/bm portfolios. The moment conditions for the DA(1) model exclude the variance of the risk-free rate. The moment conditions for the Fama-French model exclude the consumption growth moments and the variance of the risk-free rate. Table 6 does not report results for the consumption growth moments. The parameter $\beta$ is the discount rate, $\alpha$ is the risk-aversion parameter, $\rho$ is equal to $1 - 1/EIS$, and $\theta$ is the disappointment aversion coefficient. The constant $d_1$ is the disappointment threshold in equation (7) and $P(\text{disap.})$ is the probability of disappointment events. Bootstrap 95% confidence intervals are shown in brackets. $\chi^2$, d.o.f., and $p$ are the bootstrap first-stage $\chi^2$ test, degrees of freedom, and $p$-value that all moment conditions are jointly zero. $HJ$ is equal to $T \times \text{Distance}^2$, where $\text{Distance}$ is the Hansen-Jagannathan distance for excess returns in the set of risky assets, and $T$ is the number of time-series observations. RMSE and $R^2$ are the cross-sectional root-mean-square error and $R$-square, respectively, for the set of risky assets. The sample period is 1947.Q2-2012.Q4.

<table>
<thead>
<tr>
<th>Disappointment aversion (DA)</th>
<th>CRRA $\alpha = \rho, \theta = 0$</th>
<th>EZ $\theta = 0$</th>
<th>DA(1) $\alpha = 0, \theta = 0$</th>
<th>DA(2) $\alpha = -3, \theta = 0$</th>
<th>DA(3) $\alpha = -9, \theta = 0$</th>
<th>DA(4) $\alpha = -19, \theta = 0$</th>
<th>DA(5)</th>
<th>FF3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.017 [0.98, 1.03]</td>
<td>0.926 [0.56, 1.21]</td>
<td>0.999 [0.99, 0.99]</td>
<td>0.996 [0.96, 1.00]</td>
<td>1.004 [0.96, 1.08]</td>
<td>0.996 [0.99, 1.00]</td>
<td>0.996 [0.99, 1.00]</td>
<td>0.996 [0.99, 1.00]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-175.173 [-247.83, -62.80]</td>
<td>-4.513 [-10.20, -0.92]</td>
<td>-2.734 [-4.41, 0.46]</td>
<td>-2.692 [-4.40, 0.14]</td>
<td>-2.299 [-3.77, -0.17]</td>
<td>-2.728 [-4.74, 0.53]</td>
<td>-0.570 [-1.13, 0.08]</td>
<td>-0.570 [-1.13, 0.08]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-3.408 [-6.19, -1.52]</td>
<td>-4.513 [-10.20, -0.92]</td>
<td>-2.734 [-4.41, 0.46]</td>
<td>-2.692 [-4.40, 0.14]</td>
<td>-2.299 [-3.77, -0.17]</td>
<td>-2.728 [-4.74, 0.53]</td>
<td>-0.570 [-1.13, 0.08]</td>
<td>-0.570 [-1.13, 0.08]</td>
</tr>
<tr>
<td>$\theta$</td>
<td>13.599 [5.38, 17.51]</td>
<td>12.426 [5.24, 16.16]</td>
<td>11.089 [4.66, 14.13]</td>
<td>9.281 [4.54, 12.34]</td>
<td>13.017 [5.31, 17.48]</td>
<td>0.093 [0.03, 0.16]</td>
<td>0.093 [0.03, 0.16]</td>
<td>0.093 [0.03, 0.16]</td>
</tr>
<tr>
<td>$d_1$</td>
<td>-1.043 [-1.94, -0.14]</td>
<td>-1.023 [-1.72, -0.33]</td>
<td>-1.007 [-1.69, -0.32]</td>
<td>-0.990 [-1.58, -0.40]</td>
<td>-1.030 [-1.92, -0.12]</td>
<td>-0.570 [-1.13, 0.08]</td>
<td>-0.570 [-1.13, 0.08]</td>
<td>-0.570 [-1.13, 0.08]</td>
</tr>
<tr>
<td>$P(\text{disap.})$</td>
<td>0.093 [0.03, 0.16]</td>
<td>0.093 [0.03, 0.16]</td>
<td>0.093 [0.03, 0.16]</td>
<td>0.093 [0.03, 0.16]</td>
<td>0.093 [0.03, 0.16]</td>
<td>0.093 [0.03, 0.16]</td>
<td>0.093 [0.03, 0.16]</td>
<td>0.093 [0.03, 0.16]</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>525.902 [37.219, 538.202]</td>
<td>37.219 [6.276, 73.165]</td>
<td>2.655 [0.946, 4.334]</td>
<td>4.334 [0.000, 8.667]</td>
<td>5.547 [0.000, 11.091]</td>
<td>1.179 [0.000, 3.048]</td>
<td>63.272 [0.000, 11.091]</td>
<td>63.272 [0.000, 11.091]</td>
</tr>
<tr>
<td>d.o.f.</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>$p$</td>
<td>0</td>
<td>0</td>
<td>0.850 [0.050, 0.770]</td>
<td>0.744 [0.440, 1.040]</td>
<td>0.633 [0.300, 0.966]</td>
<td>0.475 [0.213, 0.738]</td>
<td>0.946 [0.300, 0.966]</td>
<td>0.946 [0.300, 0.966]</td>
</tr>
<tr>
<td>$HJ$</td>
<td>49.785 [26.266, 76.308]</td>
<td>37.219 [26.266, 538.202]</td>
<td>3.546 [0.000, 7.092]</td>
<td>3.546 [0.000, 7.092]</td>
<td>3.546 [0.000, 7.092]</td>
<td>4.676 [0.000, 9.352]</td>
<td>3.422 [0.000, 6.844]</td>
<td>24.180 [0.000, 48.360]</td>
</tr>
<tr>
<td>d.o.f.</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>$p$</td>
<td>0</td>
<td>0</td>
<td>0.264 [0.014, 0.514]</td>
<td>0.050 [0.000, 0.204]</td>
<td>0.047 [0.000, 0.192]</td>
<td>0.032 [0.000, 0.192]</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>RMSE</td>
<td>2.395 [0.484, 0.109]</td>
<td>0.114 [0.014, 0.214]</td>
<td>0.117 [0.014, 0.214]</td>
<td>0.126 [0.014, 0.214]</td>
<td>0.115 [0.014, 0.214]</td>
<td>0.222</td>
<td>0.222</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
</tr>
</tbody>
</table>