Internet Appendix to

"Sovereign Risk Premia and Global Macroeconomic Conditions"

[Not for publication]

This Internet Appendix presents supplementary material and results not included in the main body of the paper.

A Theory

This Appendix provides details on the model derivation. We determine all claims and endogenous variables according to the state of the global economy, which can be in expansion $(s_t = H)$ or in recession $(s_t = L)$. The country i subscripts are dropped for convenience.

A.1 State-price density and equilibrium risk-free rate

This section closely follows Bhamra, Kuehn and Strebulaev (2010b) and describes the state-price density and the equilibrium risk-free rate. The state-price density is initially derived by Duffie and Skiadas (1994) for the general class of stochastic differential utility function proposed by Duffie and Epstein (1992). This utility function decouples the agent's risk aversion from her preference for intertemporal resolution of the uncertainty. The coefficient of relative risk aversion is γ, the elasticity of intertemporal substitution is ψ, and the subjective time discount factor is β.

The representative agent's state-price density π_t when $\psi \neq 1$ is given by

$$\pi_t = \left(\beta e^{-\beta t}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left(p_{C,s_t} e^{\int_0^t p_{C,s_u}^{-1} du}\right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}, \tag{A.1}$$

where C_t is the agent's consumption and p_{C,s_t} is the price-consumption ratio in state s_t. The latter satisfies the following system of implicit non-linear equations:

$$\begin{aligned} p_{C,s_t}^{-1} &= \bar{r}_{s_t} - \mu_{c,s_t} + \gamma\sigma_{c,s_t}^2 - \\ &\quad \left(1 - \frac{1}{\psi}\right) \lambda_{s_t} \left(\frac{\left(\frac{p_{C,j}}{p_{C,s_t}}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} - 1}{1-\gamma} \right), \quad s_t, j \in \{L, H\}, j \neq s_t \end{aligned} \tag{A.2}$$

with

$$\bar{r}_{s_t} = \beta + \frac{1}{\psi}\mu_{c,s_t} - \frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right)\sigma_{c,s_t}^2, \tag{A.3}$$

where μ_{c,s_t} and σ_{c,s_t} denote the first and second conditional moments of consumption growth.

The dynamics of the state-price density π_t follows the stochastic differential equation:

$$\frac{d\pi_t}{\pi_t} = -r_{s_t}dt - \frac{dM_t}{M_t} \tag{A.4}$$

$$= -r_{s_t}dt - \Theta^B_{s_t}dB_t - \Theta^P_{s_t}dN_{s_t,t}, \tag{A.5}$$

where M is a martingale under the physical measure, $N_{s_t,t}$ a Poisson process which jumps upwards by one whenever the state of the global economy $s_t = \{L, H\}$ switches, $\Theta^P_{s_t} = 1 - \Delta_{s_t}$ is the market price of risk due to Poisson shocks when the global economy switches out of state s_t, and $\Theta^B_{s_t} = \gamma\sigma_{c,s_t}$ is the market price of risk due to Brownian shocks in state s_t. The risk distortion factors are such that $\Delta_H = \Delta_L^{-1}$, where Δ_H is the solution to $G(\Delta_H) = 0$ with

$$G(x) = x^{-\frac{1-\frac{1}{\psi}}{\gamma-\frac{1}{\psi}}} - \frac{\overline{r}_H + \gamma\sigma^2_{c,H} - \mu_{c,H} + \lambda_H \frac{1-\frac{1}{\psi}}{\gamma-1}\left(x^{\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1\right)}{\overline{r}_L + \gamma\sigma^2_{c,L} - \mu_{c,L} + \lambda_L \frac{1-\frac{1}{\psi}}{\gamma-1}\left(x^{-\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1\right)}, \quad \psi \neq 1. \tag{A.6}$$

Finally, the equilibrium instantaneous risk-free rate r_{s_t} is given by

$$r_{s_t} = \begin{cases} \overline{r}_L + \lambda_L \left[\frac{\gamma-\frac{1}{\psi}}{1-\gamma}\left(\Delta_H^{-\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1\right) - \left(\Delta_H^{-1} - 1\right)\right], & s_t = L \\ \overline{r}_H + \lambda_H \left[\frac{\gamma-\frac{1}{\psi}}{1-\gamma}\left(\Delta_H^{\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1\right) - \left(\Delta_H - 1\right)\right], & s_t = H. \end{cases} \tag{A.7}$$

A.2 Sovereign bond valuation and credit spread

The sovereign bond value, denoted by $B_{s_t}(Y_t)$ when the current state is s_t, is determined by

$$B_{s_t}(Y_t) = \mathbb{E}_t\left[\int_t^{t_D} c\frac{\pi_u}{\pi_t}du \middle| s_t\right] + \mathbb{E}_t\left[\int_{t_D}^{\infty} (1-\kappa)\, c\frac{\pi_u}{\pi_t}du \middle| s_t\right] \tag{A.8}$$

$$= \mathbb{E}_t\left[\int_t^{\infty} c\frac{\pi_u}{\pi_t}du \middle| s_t\right] - \mathbb{E}_t\left[\frac{\pi_{t_D}}{\pi_t}\int_{t_D}^{\infty} \kappa c\frac{\pi_u}{\pi_{t_D}}du \middle| s_t\right], \tag{A.9}$$

where c is the perpetual debt coupon, κ is the debt haircut in default, and t_D is the unknown default time. The first term of Equation (A.9) represents a risk-free claim that delivers c in every period. It corresponds to the value of a perpetual risk-free bond which equals

$$\mathbb{E}_t\left[\int_t^{\infty} c\frac{\pi_u}{\pi_t}du \middle| s_t\right] = \frac{c}{r_{B,s_t}}, \tag{A.10}$$

where r_{B,s_t} is the discount rate for a riskless perpetuity when the current state is s_t and is given by

$$r_{B,s_t} = r_{s_t} + \frac{r_j - r_{s_t}}{\hat{p} + r_j}\hat{p}\hat{f}_j, \quad j \neq s_t; \quad j, s_t = \{L, H\}. \tag{A.11}$$

The discount rate $r_{B,H}$ is lower than the corresponding instantaneous risk-free rate r_H (and analogously $r_{B,L}$ is higher than r_L) because the risk-free rate is expected to change in the future with the state of the global economy.

The second part of Equation (A.9) is given by

$$\begin{aligned}
\mathbb{E}_t\left[\frac{\pi_{t_D}}{\pi_t}\int_{t_D}^{\infty}\kappa c\frac{\pi_u}{\pi_{t_D}}du\middle|\, s_t\right] &= \sum_{s_D}\mathbb{E}_t\left[Prob\left(s_D \mid s_t\right)\frac{\pi_{t_D}}{\pi_t}\int_{t_D}^{\infty}\kappa c\frac{\pi_u}{\pi_{t_D}}du\middle|\, s_t\right] && \text{(A.12)}\\
&= \sum_{s_D}\mathbb{E}_t\left[Prob\left(s_D \mid s_t\right)\frac{\pi_{t_D}}{\pi_t}\middle|\, s_t\right]\mathbb{E}_t\left[\int_{t_D}^{\infty}\kappa c\frac{\pi_u}{\pi_{t_D}}du\middle|\, s_{t_D}\right] && \text{(A.13)}\\
&= \sum_{s_D}\frac{\kappa c}{r_{B,s_D}}q_{s_t s_D}\left(Y_t\right), && \text{(A.14)}
\end{aligned}$$

where $s_D \in \{L, H\}$ is the state at the time of default and the summation over s_D indicates that a default can occur in state $s_D = L$ or state $s_D = H$. Given the state-price density is Markovian, Equation (A.12) can be separated into two parts, as shown in Equation (A.13). The first term of Equation (A.13) is equal to

$$\mathbb{E}_t\left[Prob\left(s_D \mid s_t\right)\frac{\pi_{t_D}}{\pi_t}\middle|\, s_t\right] = q_{s_t s_D}\left(Y_t\right), \tag{A.15}$$

which is the Arrow-Debreu claim paying one unit of consumption at the default time t_D when the current state is s_t, and denoted by $q_{s_t s_D}\left(Y_t\right)$. The second term of Equation (A.13) is the value, at default time, of a claim which pays κc in perpetuity and whose discount rate is r_{B,s_D}. It is thus equal to $\frac{\kappa c}{r_{B,s_D}}$. Combining Equations (A.9), (A.10), and (A.14), the sovereign bond value is equal to

$$B_{s_t}\left(Y_t\right) = \frac{c}{r_{B,s_t}} - \sum_{s_D}\frac{\kappa c}{r_{B,s_D}}q_{s_t s_D}\left(Y_t\right), \; s_t, s_D = \{L, H\}. \tag{A.16}$$

The sovereign credit spread that the agent requires for holding the country's government bond when the current state is s_t is determined as follows:

$$
\begin{aligned}
CS_{s_t}(Y_t) &= \frac{c}{B_{s_t}(Y_t)} - r_{B,s_t} && \text{(A.17)}\\
&= \frac{1}{\left[\frac{1}{r_{B,s_t}} - \sum\limits_{s_D} \frac{\kappa}{r_{B,s_D}} q_{s_t s_D}(Y_t)\right]} - r_{B,s_t} && \text{(A.18)}\\
&= r_{B,s_t}\left[\frac{1}{1-\sum\limits_{s_D} \kappa \frac{r_{B,s_t}}{r_{B,s_D}} q_{s_t s_D}(Y_t)} - 1\right], \quad s_t, s_D = \{L, H\}. && \text{(A.19)}
\end{aligned}
$$

The probability of sovereign default, over an horizon T and within a given state s_t, is given by:

$$
\begin{aligned}
P\left(\inf_{0\le t\le T} Y_t \le Y_{D,s_t} \middle| Y_0 > Y_{D,s_t}\right) &= \Phi\left(\frac{\ln\left(\frac{Y_{D,s_t}}{Y_0}\right) - \left(\mu_{Y,s_t} - \frac{\sigma^2_{Y,s_t}}{2}\right)T}{\sigma_{Y,s_t}\sqrt{T}}\right) && \text{(A.20)}\\
&\quad + \left(\frac{Y_{D,s_t}}{Y_0}\right)^{\frac{2\mu_{Y,s_t}}{\sigma^2_{Y,s_t}}-1} \Phi\left(\frac{\ln\left(\frac{Y_{D,s_t}}{Y_0}\right) + \left(\mu_{Y,s_t} - \frac{\sigma^2_{Y,s_t}}{2}\right)T}{\sigma_{Y,s_t}\sqrt{T}}\right),
\end{aligned}
$$

where the conditional expected growth rate of government revenue is μ_{Y,s_t}, the conditional volatility is σ_{Y,s_t}, and $\Phi(\cdot)$ is the cumulative density function of a standard normal distribution.

A.3 Arrow-Debreu default claims

This section derives the two kinds of Arrow-Debreu default claims used to price securities. The first kind of Arrow-Debreu claims captures a default triggered by the country's government revenue continuously falling below a default threshold within a given state. It is given by

$$
q_{s_t s_D} = \mathbb{E}_t\left[\frac{\pi_{t_D}}{\pi_t} Prob\left(s_D \mid s_t\right) \middle| s_t\right]. \tag{A.21}
$$

The second kind of Arrow-Debreu claims accounts for a default arising from a sudden change in the state of the global economy, even if the level of the country's government revenue remains unchanged. This situation can occur when the global economy is in the economic state with the lower default threshold and switches to the other state, such that the default threshold instantaneously increases to a higher level. If the level of the country's government revenue was above the initial default threshold, but below the new default threshold, there is a sudden default. This second kind of Arrow-Debreu claims is given by

$$q'_{s_t s_D} \quad = \quad \mathbb{E}_t \left[\frac{\pi_{t_D}}{\pi_t} \frac{Y_{t_D}}{Y_{D,s_D}} Prob\left(s_D \mid s_t\right) \middle| s_t \right]. \tag{A.22}$$

A.3.1 First kind

The Arrow-Debreu default claim $q_{s_t s_D}$ is the time-t value of a security that pays one unit of consumption at the moment of default t_D, where s_t represents the current state of the global economy, and s_D the state at the default time. The time of default is the first time that the government revenue of the country falls to the threshold Y_{D,s_D}. By definition, this Arrow-Debreu claim is given by

$$q_{s_t s_D} \quad = \quad \mathbb{E}_t \left[\frac{\pi_{t_D}}{\pi_t} Prob\left(s_D \mid s_t\right) \middle| s_t \right], \tag{A.23}$$

which solves the pair of ordinary differential equations (ODE):

$$\frac{1}{2}\sigma^2_{Y,s_t} Y^2 \frac{d^2 q_{s_t s_D}}{dY^2} + \mu_{Y,s_t} Y \frac{d q_{s_t s_D}}{dY} + \hat{\lambda}_{s_t}\left(q_{j s_D} - q_{s_t s_D}\right) - r_{s_t} q_{s_t s_D} \quad = \quad 0, \tag{A.24}$$

where μ_{Y,s_t} and σ_{Y,s_t} denote the expected growth rate and the volatility of government revenue in state s_t and $\hat{\lambda}_{s_t}$ is the risk-neutral probability of leaving state s_t, with $j \neq s_t$ and $j, s_t = \{L, H\}$.
The above ODEs are obtained by applying Ito's Lemma to the classical non-arbitrage condition

$$\mathbb{E}^{\mathbb{Q}}_t \left[dq_{s_t s_D} - r_{s_t} q_{s_t s_D}\right] \quad = \quad 0. \tag{A.25}$$

The Arrow-Debreu claim payoffs are such that:

$$q_{s_t s_D}\left(Y\right) = \begin{cases} 1, & s_t = s_D, \quad Y \leq Y_{D,s_t} \\ 0, & s_t \neq s_D, \quad Y \leq Y_{D,s_t}. \end{cases}, \ s_t, s_D = \{L, H\} \tag{A.26}$$

Therefore, each state of the global economy is characterized by a specific default threshold. The Arrow-Debreu claims are derived in two distinct cases: $Y_{D,H} < Y_{D,L}$ or $Y_{D,H} > Y_{D,L}$.
In the first case, $Y_{D,H} < Y_{D,L}$, the default barriers are higher in recession and lower in expansion and each of the four Arrow-Debreu claims is determined over three separate intervals: $Y \geq Y_{D,L}$, $Y_{D,L} \geq Y \geq Y_{D,H}$, and $Y \leq Y_{D,H}$.

From the payoff equations we can infer the values of the four Arrow-Debreu claims in the interval

$Y \le Y_{D,H}$. For the interval $Y \ge Y_{D,L}$, we are looking for a solution of the following general form:

$$q_{s_t s_D}(Y) = h_{s_t s_D} Y^k, \tag{A.27}$$

which implies that k must be a root of the quartic equation

$$\left[\frac{1}{2}\sigma_{Y,L}^2 k(k-1) + \mu_{Y,L} k + \left(-\hat{\lambda}_L - r_L\right)\right]\left[\frac{1}{2}\sigma_{Y,H}^2 k(k-1) + \mu_{Y,H} k + \left(-\hat{\lambda}_H - r_H\right)\right] - \hat{\lambda}_L \hat{\lambda}_H = 0. \tag{A.28}$$

The Arrow-debreu claims can be written as

$$q_{s_t s_D}(Y) = \sum_{m=1}^{4} h_{s_t s_D, m} Y^{k_m} \tag{A.29}$$

with $k_1, k_2 < 0$ and $k_3, k_4 > 0$. However, when Y goes to infinity the Arrow-Debreu claims must be null, which indicates that we should have $h_{s_t s_D,3} = h_{s_t s_D,4} = 0$. We then obtain

$$q_{L s_D}(Y) = \sum_{m=1}^{2} h_{L s_D, m} Y^{k_m} \tag{A.30}$$

$$q_{H s_D}(Y) = \sum_{m=1}^{2} h_{H s_D, m} \varepsilon(k_m) Y^{k_m}, \tag{A.31}$$

where

$$\varepsilon(k_m) = -\frac{\hat{\lambda}_H}{\frac{1}{2}\sigma_{Y,H}^2 k(k-1) + \mu_{Y,H} k - \left(\hat{\lambda}_H + r_H\right)} = -\frac{\frac{1}{2}\sigma_{Y,L}^2 k(k-1) + \mu_{Y,L} k - \left(\hat{\lambda}_L + r_L\right)}{\hat{\lambda}_L}. \tag{A.32}$$

Finally, over the interval $Y_{D,L} \ge Y \ge Y_{D,H}$, both $q_{D,LL}$ and $q_{D,LH}$ are known from the payoffs equations and are respectively equal to 1 and 0. Then,

$$q_{HL}(Y) = \frac{\hat{\lambda}_H}{r_H + \hat{\lambda}_H} + \sum_{m=1}^{2} s_{L,m} Y^{j_m} \tag{A.33}$$

$$q_{HH}(Y) = \sum_{m=1}^{2} s_{H,m} Y^{j_m}, \tag{A.34}$$

where

$$\frac{1}{2}\sigma_{Y,H}^2 j\,(j-1)+\mu_{Y,H} j-\left(\hat{\lambda}_H+r_H\right)=0 \tag{A.35}$$

with $j_1 < j_2$.

To summarize, the four Arrow-Debreu claims can be written as follows

$$q_{LL} = \begin{cases} \sum_{m=1}^{2} h_{LL,m} Y^{k_m}, & Y \geq Y_{D,L} \\ 1, & Y_{D,L} \geq Y \geq Y_{D,H} \\ 1, & Y \leq Y_{D,H} \end{cases} \tag{A.36}$$

$$q_{LH} = \begin{cases} \sum_{m=1}^{2} h_{LH,m} Y^{k_m}, & Y \geq Y_{D,L} \\ 0, & Y_{D,L} \geq Y \geq Y_{D,H} \\ 0, & Y \leq Y_{D,H} \end{cases} \tag{A.37}$$

$$q_{HL} = \begin{cases} \sum_{m=1}^{2} h_{LL,m}\varepsilon\,(k_m)\, Y^{k_m}, & Y \geq Y_{D,L} \\ \frac{\hat{\lambda}_H}{r_H+\hat{\lambda}_H}+\sum_{m=1}^{2} s_{L,m} Y^{j_m}, & Y_{D,L} \geq Y \geq Y_{D,H} \\ 0, & Y \leq Y_{D,H} \end{cases} \tag{A.38}$$

$$q_{HH} = \begin{cases} \sum_{m=1}^{2} h_{LH,m}\varepsilon\,(k_m)\, Y^{k_m}, & Y \geq Y_{D,L} \\ \sum_{m=1}^{2} s_{H,m} Y^{j_m}, & Y_{D,L} \geq Y \geq Y_{D,H} \\ 1, & Y \leq Y_{D,H}. \end{cases} \tag{A.39}$$

The eight constants $(h_{LL,1}, h_{LL,2}, h_{LH,1}, h_{LH,2}, s_{L,1}, s_{L,2}, s_{H,1}, s_{H,2})$ are determined by eight threshold conditions, which are

$$\lim_{Y\to Y_{D,L}} q_{LL}=1, \quad \lim_{Y\to Y_{D,L}} q_{LH}=0$$

$$\lim_{Y\to Y_{D,L}^+} q_{HL}=\lim_{Y\to Y_{D,L}^-} q_{HL}, \quad \lim_{Y\to Y_{D,L}^+} q_{HH}=\lim_{Y\to Y_{D,L}^-} q_{HH}$$

$$\lim_{Y\to Y_{D,L}^+} \dot{q}_{HL}=\lim_{Y\to Y_{D,L}^-} \dot{q}_{HL}, \quad \lim_{Y\to Y_{D,L}^+} \dot{q}_{HH}=\lim_{Y\to Y_{D,L}^-} \dot{q}_{HH}$$

$$\lim_{Y\to Y_{D,H}} q_{HL}=0, \quad \lim_{Y\to Y_{D,H}} q_{HH}=1,$$

where $\dot{q}_{s_t s_D}$ denotes the derivative of $q_{s_t s_D}$ with respect to Y.

In the second case ($Y_{D,H} > Y_{D,L}$), the default barriers are higher in expansion and lower in recession and each of the four Arrow-Debreu claims is determined over three separate intervals: $Y \geq Y_{D,H}$,

$Y_{D,H} \geq Y \geq Y_{D,L}$, and $Y \leq Y_{D,L}$. We then obtain

$$q_{LL} = \begin{cases} \sum_{m=1}^{2} h_{LL,m} Y^{k_m}, & Y \geq Y_{D,H} \\ \sum_{m=1}^{2} s_{L,m} Y^{j_m}, & Y_{D,H} \geq Y \geq Y_{D,L} \\ 1, & Y \leq Y_{D,L} \end{cases} \tag{A.40}$$

$$q_{LH} = \begin{cases} \sum_{m=1}^{2} h_{LH,m} Y^{k_m}, & Y \geq Y_{D,H} \\ \frac{\hat{\lambda}_L}{r_L + \hat{\lambda}_L} + \sum_{m=1}^{2} s_{H,m} Y^{j_m}, & Y_{D,H} \geq Y \geq Y_{D,L} \\ 0, & Y \leq Y_{D,L} \end{cases} \tag{A.41}$$

$$q_{HL} = \begin{cases} \sum_{m=1}^{2} h_{LL,m} \varepsilon\left(k_m\right) Y^{k_m}, & Y \geq Y_{D,H} \\ 0, & Y_{D,H} \geq Y \geq Y_{D,L} \\ 0, & Y \leq Y_{D,L} \end{cases} \tag{A.42}$$

$$q_{HH} = \begin{cases} \sum_{m=1}^{2} h_{LH,m} \varepsilon\left(k_m\right) Y^{k_m}, & Y \geq Y_{D,H} \\ 1, & Y_{D,H} \geq Y \geq Y_{D,L} \\ 1, & Y \leq Y_{D,L}. \end{cases} \tag{A.43}$$

The eight constants $(h_{LL,1}, h_{LL,2}, h_{LH,1}, h_{LH,2}, s_{L,1}, s_{L,2}, s_{H,1}, s_{H,2})$ are determined by the following eight threshold conditions:

$$\lim_{Y \to Y_{D,L}} q_{LL} = 1, \quad \lim_{Y \to Y_{D,L}} q_{LH} = 0$$

$$\lim_{Y \to Y_{D,H}^{+}} q_{LL} = \lim_{Y \to Y_{D,H}^{-}} q_{LL}, \quad \lim_{Y \to Y_{D,H}^{+}} q_{LH} = \lim_{Y \to Y_{D,H}^{-}} q_{LH}$$

$$\lim_{Y \to Y_{D,H}^{+}} \dot{q}_{LL} = \lim_{Y \to Y_{D,H}^{-}} \dot{q}_{LL}, \quad \lim_{Y \to Y_{D,H}^{+}} \dot{q}_{LH} = \lim_{Y \to Y_{D,H}^{-}} \dot{q}_{LH}$$

$$\lim_{Y \to Y_{D,H}} q_{HL} = 0, \quad \lim_{Y \to Y_{D,H}} q_{HH} = 1.$$

A.3.2 Second kind

We use the same approach to derive the second kind of Arrow-Debreu default claims, which accounts for the possibility that a default happens when the state of the global economy suddenly switches. When

$Y_{D,H} < Y_{D,L}$ the only claim that is different from that of the first kind is q_{HL} given by

$$q'_{HL} = \begin{cases} \sum_{m=1}^{2} h_{LL,m} \varepsilon\left(k_m\right) Y^{k_m}, & Y \geq Y_{D,L} \\ \frac{\hat{\lambda}_H}{r_H + \hat{\lambda}_H - \mu_{Y,H}} \frac{Y}{Y_{D,L}} + \sum_{m=1}^{2} s_{L,m} Y^{j_m}, & Y_{D,L} \geq Y \geq Y_{D,H} \\ 0, & Y \leq Y_{D,H}. \end{cases} \tag{A.44}$$

When $Y_{D,H} > Y_{D,L}$ the only claim that is different from that of the first kind is q_{LH} given by

$$q'_{LH} = \begin{cases} \sum_{m=1}^{2} h_{LH,m} Y^{k_m}, & Y \geq Y_{D,H} \\ \frac{\hat{\lambda}_L}{r_L + \hat{\lambda}_L - \mu_{Y,L}} \frac{Y}{Y_{D,H}} + \sum_{m=1}^{2} s_{H,m} Y^{j_m}, & Y_{D,H} \geq Y \geq Y_{D,L} \\ 0, & Y \leq Y_{D,L}. \end{cases} \tag{A.45}$$

A.4 Government

This section derives the debt issuance benefits, the present value of the country's government revenue, and the country's sovereign wealth.

A.4.1 Debt issuance benefits

The government's motivation for issuing debt is to invest internally the amount of capital raised at the time of debt issuance ($t = 0$). Financing public investments yields a return r_g. The government's incentives for issuing debt, denoted by $I_{s_t}(Y_t)$ when the state is s_t at time t, equals

$$I_{s_t}(Y_t) = \mathbb{E}_t \left[\int_t^{\infty} r_g \frac{\pi_u}{\pi_t} du \middle| s_t \right] B_{s_0}(Y_0) \tag{A.46}$$

$$= r_g \mathbb{E}_t \left[\int_t^{\infty} \frac{\pi_u}{\pi_t} du \middle| s_t \right] B_{s_0}(Y_0) \tag{A.47}$$

$$= \frac{r_g}{r_{B,s_t}} B_{s_0}(Y_0). \tag{A.48}$$

A.4.2 Discounted government revenue

The present value of the country's government revenue, denoted by $G_{s_t}(Y_t)$ when the current state is s_t, can be written as

$$G_{s_t}(Y_t) = \mathbb{E}_t\left[\int_t^{t_D} Y_u \frac{\pi_u}{\pi_t} du \,\middle|\, s_t\right] + \mathbb{E}_t\left[\int_{t_D}^{\infty} (1-\alpha) Y_u \frac{\pi_u}{\pi_t} du \,\middle|\, s_t\right] \tag{A.49}$$

$$= \mathbb{E}_t\left[\int_t^{\infty} Y_u \frac{\pi_u}{\pi_t} du \,\middle|\, s_t\right] - \alpha \mathbb{E}_t\left[\int_{t_D}^{\infty} Y_u \frac{\pi_u}{\pi_t} du \,\middle|\, s_t\right]. \tag{A.50}$$

The first term of Equation (A.50) can be written as

$$\mathbb{E}_t\left[\int_t^{\infty} Y_u \frac{\pi_u}{\pi_t} du \,\middle|\, s_t\right] = Y_t \mathbb{E}_t\left[\int_t^{\infty} \frac{\pi_u}{\pi_t} \frac{Y_u}{Y_t} du \,\middle|\, s_t\right] \tag{A.51}$$

$$= Y_t \frac{1}{r_{Y,s_t}}, \tag{A.52}$$

where r_{Y,s_t} is the discount rate applicable to risky government revenue, given by

$$r_{Y,s_t} = r_{s_t} - \widehat{\mu}_{Y,s_t} + \frac{(r_j - \widehat{\mu}_{Y,j}) - (r_{s_t} - \widehat{\mu}_{Y,s_t})}{\hat{p} + r_j - \widehat{\mu}_{Y,j}} \hat{p} \hat{f}_j, \quad j \neq s_t; \quad j,\ s_t = \{L, H\}, \tag{A.53}$$

with $\widehat{\mu}_{Y,s_t} = \mu_{Y,s_t} - \gamma \sigma_{c,s_t} \rho_{s_t} \sigma_{Y,s_t}$ denoting the expected growth rate under the risk-neutral measure.

From the strong Markov property, we can solve for the second term of Equation (A.50), which yields

$$\mathbb{E}_t\left[\int_{t_D}^{\infty} Y_u \frac{\pi_u}{\pi_t} du \,\middle|\, s_t\right] = \sum_{s_D} q'_{s_t s_D}(Y_t) \frac{Y_{D,s_D}}{r_{Y,s_D}}. \tag{A.54}$$

Combining Equations (A.50), (A.52), and (A.54), the present value of the country's government revenue is given by

$$G_{s_t}(Y_t) = \frac{Y_t}{r_{Y,s_t}} - \alpha \sum_{s_D} \frac{Y_{D,s_D}}{r_{Y,s_D}} q'_{s_t s_D}(Y_t). \tag{A.55}$$

A.4.3 Sovereign wealth and smooth pasting conditions

Sovereign wealth is defined as the present value of government revenue plus the benefits of issuing debt. From the derivation above, sovereign wealth $W_{s_t}(Y_t)$ at time t and for current state s_t is given by

$$
\begin{aligned}
W_{s_t}(Y_t) &= G_{s_t}(Y_t) + I_{s_t}(Y_t) && \text{(A.56)} \\
&= \frac{Y_t}{r_{Y,s_t}} - \alpha \sum_{s_D} \frac{Y_{D,s_D}}{r_{Y,s_D}} q^{'}_{s_t s_D}(Y_t) + \frac{r_g}{r_{B,s_t}} B_{s_0}(Y_0). && \text{(A.57)}
\end{aligned}
$$

We now derive the smooth-pasting conditions that ensure continuity in the objective function at the time of default (see Merton, 1973; Dumas, 1991). For convenience, we denote the net value of sovereign wealth by $\overline{W}_{s_t}(Y_t) \equiv W_{s_t}(Y_t) - B_{s_t}(Y_t)$. Combining Equations (A.16) and (A.57), $\overline{W}_{s_t}(Y_t)$ is given by

$$
\begin{aligned}
\overline{W}_{s_t}(Y_t) &= \frac{Y_t}{r_{Y,s_t}} - \alpha \sum_{s_D} \frac{Y_{D,s_D}}{r_{Y,s_D}} q^{'}_{s_t s_D}(Y_t) + \frac{r_g}{r_{B,s_t}} B_{s_0}(Y_0) \\
&\quad - \left[\frac{c}{r_{B,s_t}} - \sum_{s_D} \frac{c\kappa}{r_{B,s_D}} q_{s_t s_D}(Y_t) \right]. && \text{(A.58)}
\end{aligned}
$$

The smooth-pasting conditions must satisfy the following equations:

$$
\left. \frac{\partial \overline{W}_{s_t}(Y_t)}{\partial Y_t} \right|_{Y_t = Y_{D,s_t}} = \frac{\partial}{\partial Y_{D,s_t}} \left(\overline{W}_{s_t}(Y_t) \big|_{Y_t = Y_{D,s_t}} \right), \qquad s_t = \{L, H\}. \tag{A.59}
$$

From the definition of the Arrow-Debreu claims (A.26), $\overline{W}_{s_t}(Y_t)$ at default time is

$$
\overline{W}_{s_t}(Y_t) \big|_{Y_t = Y_{D,s_t}} = Y_{D,s_t} \frac{1-\alpha}{r_{Y,s_t}} + \frac{r_g}{r_{B,s_t}} B_{s_0}(Y_0) - \frac{(1-\kappa)\, c}{r_{B,s_t}} \tag{A.60}
$$

and the right-hand side of Equation (A.59) is thus determined by

$$
\frac{\partial}{\partial Y_{D,s_t}} \left(\overline{W}_{s_t}(Y_t) \big|_{Y_t = Y_{D,s_t}} \right) = \frac{1-\alpha}{r_{Y,s_t}}. \tag{A.61}
$$

Hence, the smooth-pasting conditions satisfy the pair of equations given by

$$
\left. \frac{\partial \overline{W}_{s_t}(Y_t)}{\partial Y_t} \right|_{Y_t = Y_{D,s_t}} = \frac{(1-\alpha)}{r_{Y,s_t}}, \quad s_t = \{L, H\}. \tag{A.62}
$$

B Model calibration

This Appendix provides details about the model calibration.

B.1 Transition probabilities

This section describes the estimation of transition probabilities. Following Hamilton (1989), we estimate a two-state Markov regime-switching model for U.S. consumption growth over the 1994:Q1-2018:Q2 period. We constrain the long-run frequency of the state $s_t = L$ to correspond to the frequency of NBER recessions observed during the postwar period (1952Q1-2018Q2), that is, $f_L = 0.1353$ and $f_H = 1 - f_L$.

We denote the probability of switching from state i to state j by T_{ij}, such that the transition probability matrix is $T = \begin{bmatrix} T_{HH} & T_{HL} \\ T_{LH} & T_{LL} \end{bmatrix}$, with $T_{HH} = 1 - T_{HL}$ and $T_{LL} = 1 - T_{LH}$. As in Bhamra, Kuehn and Strebulaev (2010a, 2010b), the relation between the physical long-run frequency f_{s_t} and the transition probability matrix T_{ij} is $f_H = \left(1 + \frac{T_{HL}}{T_{LH}}\right)^{-1}$. Hence, for a given long-run frequency f_H, we must have $\frac{T_{HL}}{T_{LH}} = f_H^{-1} - 1$. From the constrained maximum likelihood estimation, we obtain the following transition probability matrix:

$$T = \begin{bmatrix} 0.9851 & 0.0149 \\ 0.0953 & 0.9047 \end{bmatrix}. \tag{A.63}$$

While the constraint $f_L = 0.1353$ ensures a reasonable long-run frequency of the state $s_t = L$, we verify that the constrained and the unconstrained estimations are not statistically different from each other: the Likelihood-ratio test has a *p*-value of 0.76. Finally, the probability λ_{s_t} that the global economy leaves the state $s_t \in \{L, H\}$ is given by $\lambda_L = pf_H$ and $\lambda_H = pf_L$, with $p = -4ln\left(1 - \frac{T_{HL}}{1-f_H}\right)$.

B.2 Conditional output growth moments

This section estimates the conditional output growth moments of the representative sovereign bond issuer. First, we determine business cycle dates based on the filtered probability of being in recession. Figure 2 in the paper displays the time series of U.S. consumption growth used in the estimation (Panel A) and the filtered conditional probability of being in the state $s_t = L$ (Panel B). Panel C displays the quarters when the economy is in the recession state ($s_t = L$). Based on this regime categorization, we compute the conditional moments of output growth (μ_{X,i,s_t} and σ_{X,i,s_t}) and the correlation with U.S. consumption growth (ρ_{i,s_t}) using GDP in constant U.S. dollars. Table A.1 reports the equally-weighted and GDP-weighted moments, as well as their median, standard deviation, and the interdecile range. The GDP-weights are computed using each country's average GDP in constant U.S. dollars. Both the median and the average country (using either equal or GDP weights) display higher output growth rate in expansion than in recession ($\mu_{X,i,H} > \mu_{X,i,L}$) and lower output growth volatility in expansion than in recession ($\sigma_{X,i,\ L} > \sigma_{X,i,H}$). The correlation between output growth and U.S. consumption growth (ρ_{i,s_t}) is small in all cases.

Table A.1 [about here]

B.3 Estimation of the leverage parameter

This section empirically assesses the leverage parameter η for a set of emerging economies. Based on data availability, we consider 10 countries with different sizes and levels of economic development. The countries are Bolivia, Brazil, Bulgaria, Chile, Colombia, India, Mexico, Philippines, Russia, and South Africa. We first construct each country's time series of government revenue ($Y_{i,t}$) by multiplying GDP in constant U.S. dollars (the same concept used in the output calibration) by Revenue Excluding Grants as a percentage of GDP. The latter data is from the World Bank's website. Then we compute country-level leverage proxies as the ratio of unconditional government revenue growth volatility ($\sigma_{Y,i}$) to unconditional output growth volatility ($\sigma_{X,i}$), as defined in Equation (6) of the model.

Table A.2 reports the results. The average volatility of government revenue growth is 8.57% and the average volatility of output growth is 2.18%. This implies a GDP-weighted (equally-weighted) average leverage of 3.80 (4.06) and a standard deviation of 1.07. The GDP-weights are computed using each country's average GDP in constant U.S. dollars.

Table A.2 [about here]

B.4 Investor preferences and the bond risk premium

This section discusses the role of investor preferences in determining the bond risk premium. Figure A.1 presents the bond risk premium for different levels of relative risk aversion (γ) and time preference (β), while Table A.3 reports corresponding predictions on the price of risk.

Figure A.1 and Table A.3 [about here]

The bond risk premium increases when the representative agent is more risk-averse (higher γ). Both the short- and long-run macro risk premium components increase. The short-run macro risk premium, given by $\gamma\sigma_{c,s_t}\rho_{i,s_t}\sigma^B_{i,s_t}$, directly increases with the risk aversion coefficient γ through the price of risk. The long-run macro risk component also increases with risk aversion because investors display a stronger preference for early resolution of uncertainty when the difference between γ and $\frac{1}{\psi}$ increases. In addition to these direct effects, higher risk aversion increases the precautionary motives and thus reduces the equilibrium risk-free rate. A lower risk-free rate increases the present value of the debt coupons that the government must service, thereby increasing default risk and the risk premium. Similarly, the bond risk premium decreases with time preference: less impatience (lower β) translates into a lower risk-free rate and higher default risk.

Table A.3 indicates that the price of long-run macro risk is $\Delta_H = \hat{\lambda}_H/\lambda_H = 1.674$, which implies that investors overweight the probability of switching from expansion to recession by 67.4%. Investors thus price bonds as if recessions are more likely than in reality. The ratio of the unconditional risk-neutral default probability ($\mathbb{Q}$) over the unconditional physical default probability ($\mathbb{P}$), both computed at a 5-year horizon, reflects how much investors overweight the increase in default probability during recession. We compute the unconditional risk-neutral default probability ($\mathbb{Q}$) using the long-term risk-neutral distribution ($\hat{f}_L = 0.3048, \hat{f}_H = 0.6952$) to weight the default probability in each state $s_t = \{L, H\}$. Correspondingly, we use the real-world distribution ($f_L = 0.1353, f_H = 0.8647$) to compute the physical default probability ($\mathbb{P}$). The ratio of probabilities equals 1.94 in the baseline calibration, which indicates that investors price sovereign bonds as if the unconditional level of default risk were 94% greater than in reality, mostly because of long-run macro risk. Macroeconomic risk thus entails a substantial price of risk.

C Equity risk premium

This Appendix derives the equity risk premium in the economy. As in Abel (1999), among others, we assume that dividends D_t lever up consumption such that

$$D_t = \exp(-\beta_d t) C_t^{\eta_d}, \tag{A.64}$$

where $\eta_d \geq 1$ is the leverage parameter and $\beta_d > 0$ is an adjustment parameter determining the unconditional expected growth rate of dividends (Andrei, Hasler and Jeanneret, 2019). Applying Ito's

Lemma, the dividend process is:

$$\frac{dD_t}{D_t} = \underbrace{\left(-\beta_d + \eta_d \mu_{c,s_t} + \frac{1}{2}\eta_d\left(\eta_d - 1\right)\sigma^2_{c,s_t}\right)}_{\mu_{d,s_t}} dt + \eta_d \sigma_{c,s_t} dZ_{c,t}. \tag{A.65}$$

The stock price, denoted by $S_{s_t}(D_t)$ when the current state is s_t, is the claim to the dividend process and given by

$$S_{s_t}(D_t) = \mathbb{E}_t\left[\int_t^\infty D_u \frac{\pi_u}{\pi_t} du \middle| s_t\right] \tag{A.66}$$

$$= \frac{D_t}{r_{d,s_t}}, \tag{A.67}$$

where r_{d,s_t} is the discount rate applicable to the dividend dynamics, which is given by

$$r_{d,s_t} = r_{s_t} - \widehat{\mu}_{d,s_t} + \frac{(r_j - \widehat{\mu}_{d,j}) - (r_{s_t} - \widehat{\mu}_{d,s_t})}{\hat{p} + r_j - \widehat{\mu}_{d,j}} \hat{p}\hat{f}_j, \quad j \neq s_t; \quad j,\ s_t = \{L, H\}, \tag{A.68}$$

where $\widehat{\mu}_{d,s_t} = \mu_{d,s_t} - \gamma\eta_d\sigma^2_{c,s_t}$ is the expected growth rate of dividend under the risk-neutral measure.

The equity risk premium EP_{s_t} in state s_t, defined as the instantaneous expected return on the stock in excess of the risk-free rate, is equal to

$$EP_{s_t} = \gamma\eta_d\sigma^2_{c,s_t} + \lambda_{s_t}\Theta^P_{s_t} R^S_{i,s_t}, \quad s_t = \{L, H\}. \tag{A.69}$$

The first term of Equation (A.69) captures compensation for instantaneous dividend innovations. The second term captures compensation for changes in global macroeconomic conditions. The latter risk premium component is determined by the probability λ_{s_t} of leaving state s_t, the price of risk associated with this change of state $\Theta^P_{s_t} = 1 - \Delta_{s_t}$, and the change in stock valuation caused by the change of state, given by $R^S_{i,s_t} = \frac{S_{i,j}}{S_{i,s_t}} - 1,\ s_t \neq j = \{L, H\}$. The conditional stock return volatility is given by $\sigma^S_{s_t} = \sqrt{(\sigma_{c,s_t})^2 + \lambda_{s_t}\left(R^S_{i,s_t}\right)^2}$.

We calibrate the η_d and β_d parameters to generate theoretical price-dividend ratio and stock return volatility matching their empirical counterparts. Our calibration reproduces the log price-dividend ratio of 6.4 and stock return volatility of 19.8% per year reported in Schorfheide, Song and Yaron (2018).[18] Furthermore, we obtain a dividend growth volatility of 12% per year, in line with the 11.1% over the

[18]Similarly, Beeler and Campbell (2012) report a log price-dividend ratio of 6.36, a stock return volatility of 20.2%.

1930-2008 period reported by Beeler and Campbell (2012). Using such parameters and those on Table 1, Equation (A.69) implies conditional equity premia equal to 12.83% per year in recession and 2.88% in expansion, leading to an unconditional equity risk premium equal to 4.22% per year.

D Model simulation

This Appendix describes the simulation procedure discussed in Section 4.2.4. We generate 500 artificial datasets consisting of 40 countries over 98 quarters, corresponding to the 1994:Q1-2018:Q2 period. Each sovereign experiences two types of common shocks: a systematic component in output growth shocks ($dZ_{i,t}$) and synchronous changes in the state of the global economy s_t. To model such dependencies, we draw quarterly shocks to a sovereign's output from a distribution with conditional correlation ρ_{i,s_t} with quarterly shocks on U.S. consumption growth ($dZ_{c,t}$). In each period, the state of the global economy may also switch, and we use the business cycle estimated over the 1994:Q1-2018:Q2 period to determine whether the economy is in expansion or recession. The state of the global economy determines not only the correlation between output and global consumption shocks (ρ_{i,s_t}) but also the first and second moments of global consumption and output growth, as well as the equilibrium risk-free rate. While the business cycle is common across all countries, the exposure of their bond prices to a change in the state of the global economy can differ based on their current default risk level. The model simulation is thus useful to study how bond excess return and credit spreads vary over time and across countries in the model in a way that can be compared to the data.

At the start of the simulation, all sovereigns choose a debt coupon and default policies that correspond to those discussed in Section 4.1. To generate an initial cross-section of countries, each sovereign starts at date $t = 0$ with a level of government revenue drawn randomly between 0.6 and 1, such that the initial distance-to-default varies across sovereigns. The initial cross-country distribution of government revenue is calibrated so that simulated default frequencies match those observed empirically (more details below). At the start of every period, the state of the global economy over the previous quarter is determined. Then, each sovereign observes its own state-dependent revenue dynamics over the quarter. If the level of government revenue crosses a state-dependent boundary ($Y_{i,t} \leq Y_{D,i,s_t}$ in state s_t), the sovereign defaults. We replace a defaulted sovereign with another sovereign at optimal indebtedness level and government revenue equal to $Y_{i,t} = 1$, such that the number of countries in the economy remains constant over time. Similarly, if government revenue reaches an upper threshold ($Y_{i,t} \geq 3$), we replace the sovereign by a new one with government revenue equal to $Y_{i,t} = 1$, thus resetting its indebtedness ratio to the optimal level. This adjustment prevents that the economy becomes dominated by a few disproportionally large countries, and prevents government indebtedness (debt value to output) from vanishing over time. Next, we extract the quarterly value of the bond and its credit spread. We compute the annualized bond excess return of country i in quarter t when the current state is s_t as $R^e_{B,i,t,s_t} = \frac{1}{\Delta t}\frac{B_{i,t,s_t}+c\Delta t}{B_{i,t-1,s_{t-1}}} - r_{B,s_t}$, where B_{i,t,s_t} is the value of the bond, c is the debt coupon (common across countries), r_{B,s_t} is the risk-free discount rate, and $\Delta t = \frac{1}{4}$ is the discretized time increment.

We repeat each simulation 500 times. In total, the simulation thus consists of 500 economies of 40 countries each, which implies almost 2 million quarterly observations. We first compute the statistics for bond excess returns and credit spreads for each simulated economy exactly as in the data. We then average these statistics across economies and compute their 5th and 95th percentiles. This approach allows us to study the sampling distribution for statistics of interest produced for each economy.

Table A.4 presents the simulation results. Before comparing credit spreads and sovereign risk premia in the simulation to their empirical counterparts, we must check that the model generates reasonable default frequencies at different horizons. Panel A of Table A.4 shows that the level and the term structure of default probability in the simulation closely match their empirical counterparts. The average cumulative 1-, 5-, and 10-year default probabilities are 2.23%, 10.63%, and 20.08% across the simulated economies, while they are respectively 2.88%, 11.29%, and 19.11% for speculative-grade foreign currency sovereign bonds, as reported by Standard and Poor's (2020). These empirical default rates are well within the confidence intervals of our simulations. The model parameters and the initial cross-country distribution of government revenue are therefore properly calibrated.

Table A.4 [about here]

Panel B of Table A.4 compares bond excess returns and credit spreads in the simulation to those in the data. We report two sets of numbers for the empirical counterparts: 'raw' and 'AAA-adjusted', depending on the proxy for the risk-free rate. We compute the raw excess returns using the one-month T-bill from Ken French's website as the risk-free return, while the raw credit spreads as reported directly by JP Morgan for EMBI Global indices and thus based off the U.S. Treasury yield curve. The AAA-adjusted calculations use the ICE Bank of America AAA U.S. Corporate Index as a proxy for the risk-free asset, addressing the potential critique that Treasury bonds may not be appropriate proxies for the default-free borrowing rate because they are valued at a premium due to their extreme safety and liquidity (e.g., Krishnamurthy and Vissing-Jorgensen, 2012). Accordingly, we use the AAA Corporate Index return as the risk-free return for calculating bond excess returns and subtract the AAA Index credit spreads (relative to the U.S. Treasury curve) from the EMBI credit spreads to compute the sovereign bond credit spreads.

Panel B of Table A.4 shows that the average (annualized) bond excess return in the simulations is 1.90%, while the average credit spread is 162 bps per year. Both the average risk premium and average credit spread are significantly below their empirical counterparts in the 1994Q1-2018Q2 period. The average excess return of sovereign bonds over one-month T-bills is 2.13% per quarter in the data, which corresponds to 8.53% per year. It is 5.37% per year above AAA Corporate bonds. The average EMBI credit spread is 471 bps per year above U.S. Treasuries and 376 bps above AAA-rated corporates, also significantly larger than the 162 bps of the simulations.[19] The medians of excess returns and credit spreads in the data are also larger, and their distributions more positively skewed, than those in the simulations. However, the standard deviation of bond excess returns in the model closely matches the data.

Hence, while the simulated economy successfully captures the level of default risk in the data, as measured by the term structure of default probability, and the standard deviation of bond returns, it cannot generate a risk compensation as large as what is observed in emerging bond markets. While long-run macro risk significantly increases risk-premia and credit spreads compared to a canonical model with short-run risk only, additional mechanisms generate excess returns in the data, as discussed in Section 4.2.4.

E Additional empirical results

This Appendix presents additional empirical results.

E.1 Portfolio allocation and rebalancing

This section reports the country allocation and rebalancing frequency for the baseline double-sort procedure used in Panel A.I of Table 6. We refer to 'portfolio allocation' as the fraction of time a country belongs to each of the three portfolios conditional on being on the sample. 'Rebalancing frequency' is the fraction of time a country switches portfolios conditional on being on the sample. Results in Table A.5 show that countries have a dominant allocation, suggesting there are intrinsic, long-lasting

[19] Credit spreads are lower in the model than in the data because model-implied spreads are for perpetual bonds while the observed spreads are for finite maturity bonds. The term structure of spreads is severely negatively sloped when default risk is high (Augustin, 2018). Yet, the difference in average credit spreads is probably too significant to be attributed to such maturity mismatch effect only.

cross-country differences in terms of long-run macro risk. However, countries do shift portfolios over the sample period, i.e., the portfolio constituents vary over time. On average, the average rebalancing frequency across countries is 15%, which means that conditional on being on the sample, a country switches portfolios once every 7 quarters. Each of the three portfolios of Panel A.I in Table 6 contains 9 countries on average, such that the HML portfolio has 18 countries at a time, and its composition changes by approximately 3 countries every quarter.

Table A.5 [about here]

E.2 Regime identification based on economic turning points

This section provides additional evidence that bond excess returns vary in the cross-section with exposure to shifts in macroeconomic regimes. The results here complement those in Section 5.3. While we previously focused on a single event (the sudden worsening of the Global Financial Crisis in 2008:Q4), we now expand the analysis to additional regime change events. We use the Harding and Pagan (2006) procedure to define 'turning points' in U.S. consumption growth and, as such, identify regime changes based on realized consumption data.

The starting point of the Harding and Pagan (2006) methodology is a plot of the time series of annual consumption growth, as illustrated in Figure A.2. The procedure seeks local maxima and minima on the plot. During our sample period – starting in 1994:Q1 – there are seven turning points, identified by the Peaks and Troughs on the plot. There are four Peaks and three Troughs in our sample, corresponding to a total of seven regime changes.

Figure A.2 [about here]

If market participants could recognize in real-time that Peaks are peaks, that is, consumption growth will be declining in the following quarters, then sovereign bond excess returns should be negative during Peak quarters, on average. Analogously, they should be positive during Trough quarters. Indeed, the average bond excess return is -0.3% during the Peak quarters, but 7.4% during the Trough quarters. Using the same rationale as in Section 5.3, countries whose bonds fall more when consumption switches from High to Low growth (i.e., at Peaks), or increase more when consumption switches from Low to High growth (i.e., at Troughs), are highly sensitive to regime shifts in the global economy. Our theory predicts these countries are riskier and should have average excess returns over the entire sample.

To pool data from Peaks and Troughs, we define the signed returns variable $R_{Signed} = R \times \mathbf{1}$, where R is the sovereign bond return at a turning point quarter and $\mathbf{1}$ is an indicator function that equals -1 if it is a Trough and $+1$ if it is a Peak. Under this definition, countries that are highly sensitive to regime shifts will have relative low signed returns. For each country, we average R_{Signed} across the 7 turning points in our sample. Figure A.3 shows the results.

Figure A.3 [about here]

As in Figure 4 in the paper, there are four scatter plots, each of them corresponding to a different time subsample and number of countries dictated by data availability. All four scatter plots display a negative relationship between average quarterly excess returns and average signed returns. This result confirms the model prediction that countries whose bonds are more sensitive to regime changes in global consumption are riskier and thus offer a higher risk premium.

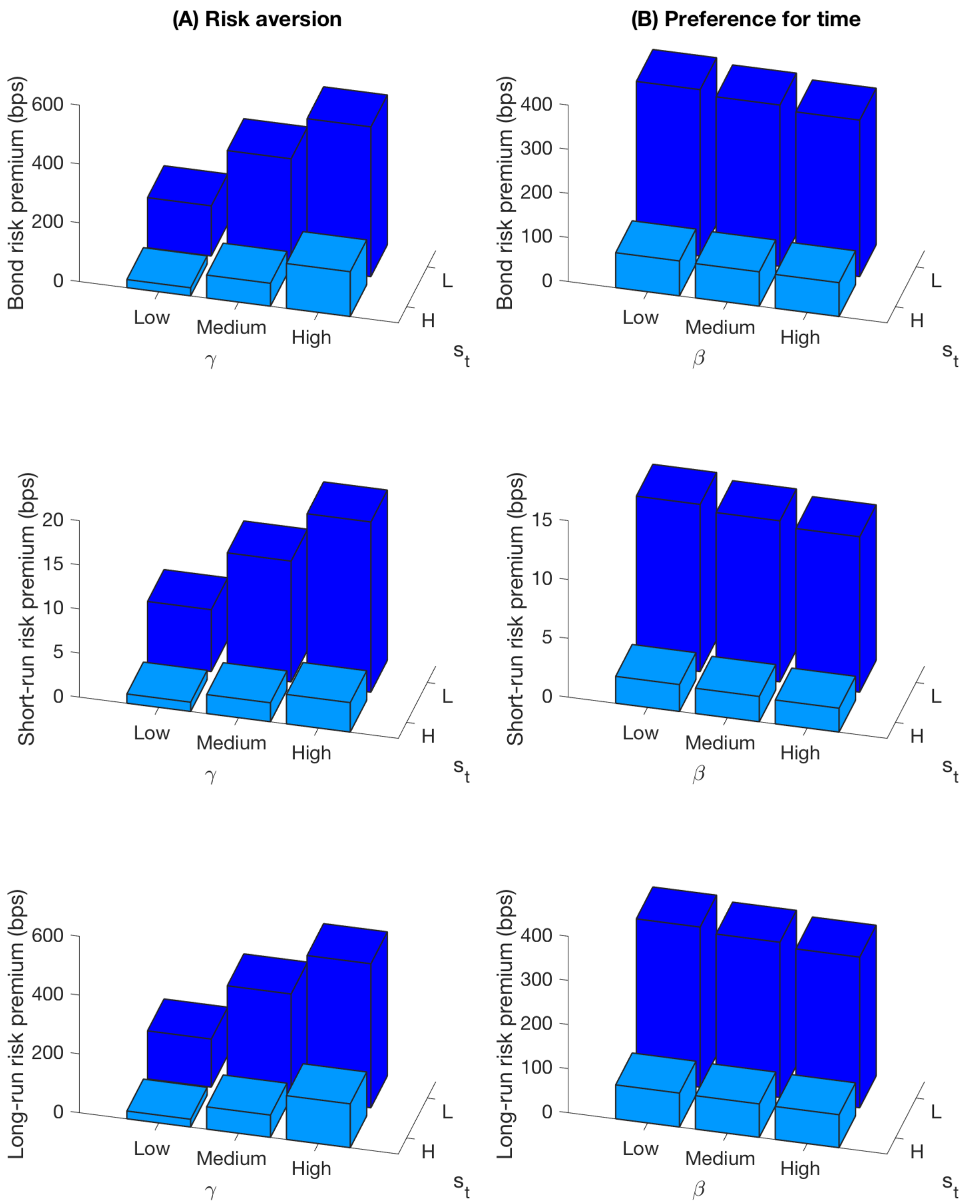

Figure A.1: **Sovereign risk premium by investor preferences.**
This figure illustrates how the bond risk premium varies with investor preferences. Panel A reports predictions for different levels of relative risk aversion, while Panel B reports predictions for different levels of preference for time. Low, medium, and high risk aversion corresponds to $\gamma = 5$, $\gamma = 10$, and $\gamma = 15$, while low, medium, and high preference for time corresponds to $\beta = 0.03$, $\beta = 0.04$, and $\beta = 0.05$, respectively. The figure compares predictions when the current state s_t is in recession (L) or expansion (H). Bond risk premium is in basis points (bps) per annum. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1) and report predictions for levels of government revenue observed at issuance time $(Y = 1)$.

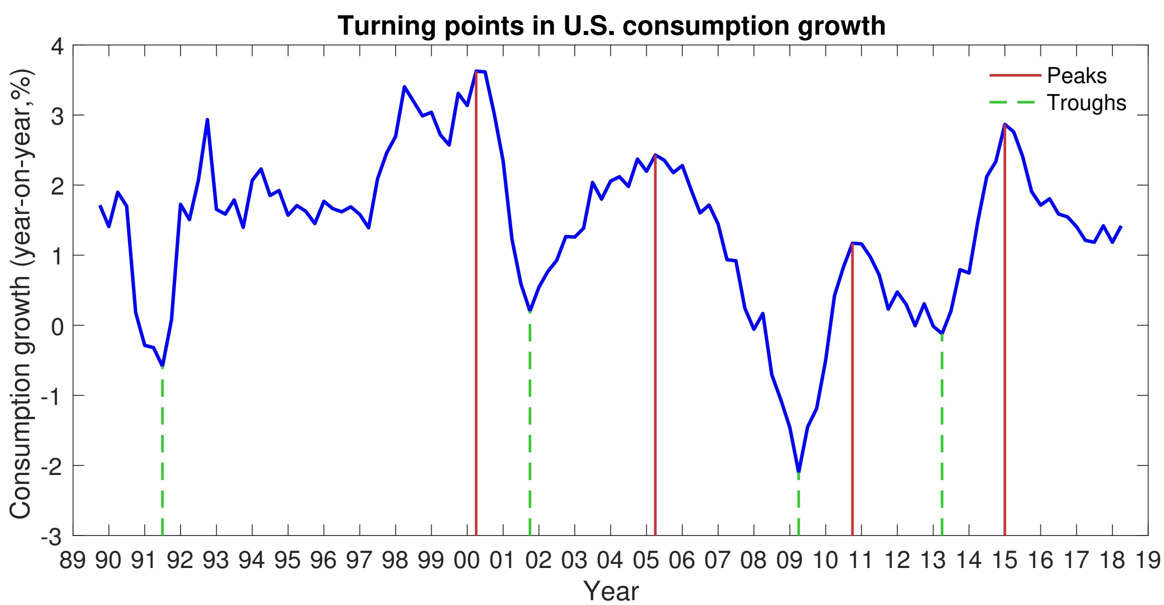

Figure A.2: **Regime changes based on economic turning points.**
This figure plots economic turning points, following the Harding and Pagan (2006)'s procedure to determine regime changes in realized consumption data. The plot illustrates the identified local maxima (peaks) and minima (troughs) of year-over-year U.S. consumption growth, expressed in percentage. Data span the 1990:Q1-2018:Q2 period.

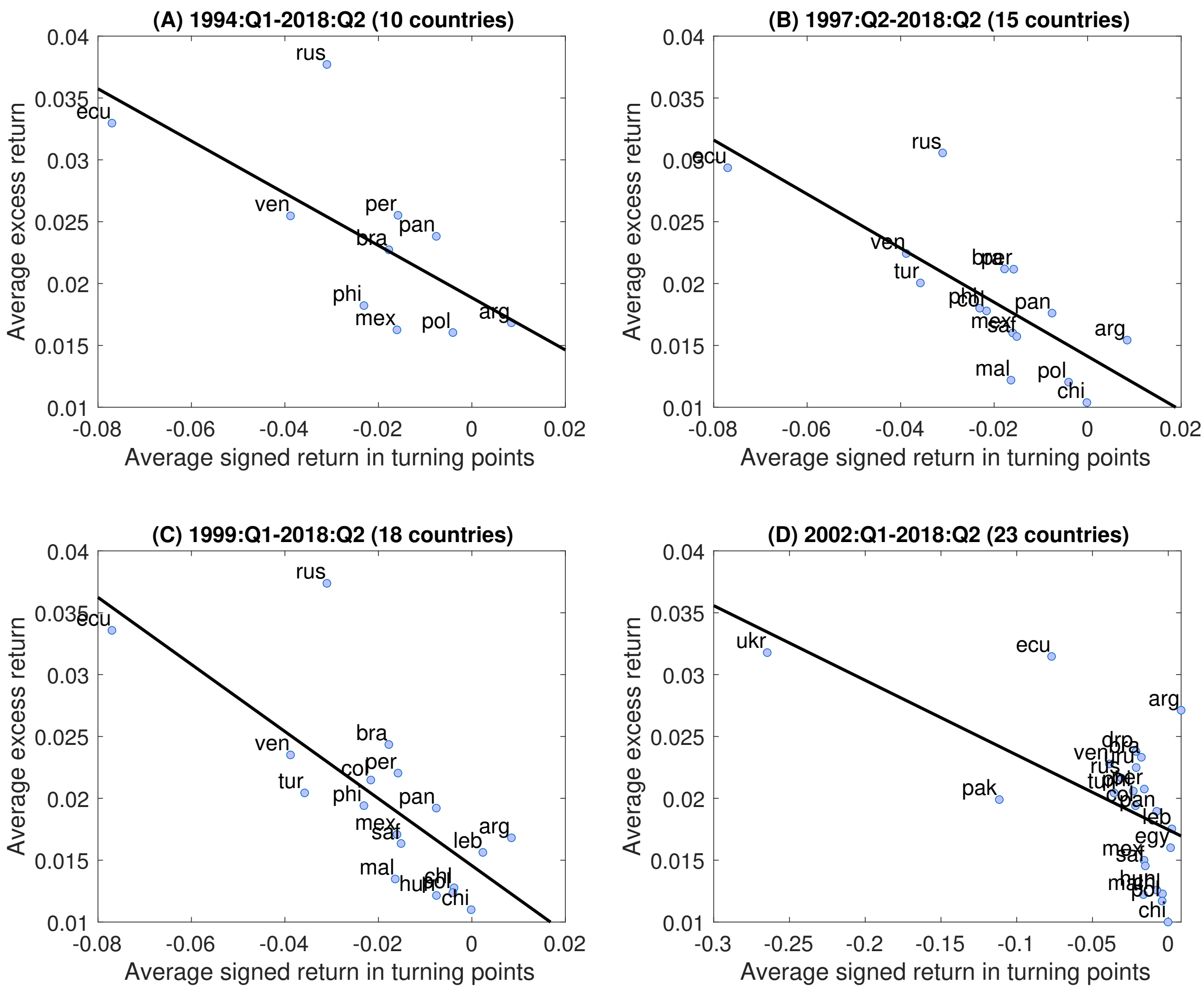

Figure A.3: **Average bond excess returns and economic turning points.**
This figure plots average quarterly bond excess returns versus the exposure to economic turning points. The exposure to turning points is computed, for each country, as the average signed return in the turning points identified in Figure A.2. Average quarterly excess returns are calculated over four different samples, each of them with a different number of countries with data available throughout the entire the sample. Best fit lines are displayed. Data span the 1994:Q1-2018:Q2 period.

Table A.1 : **Conditional output growth moments by country**.
The table displays the annualized mean and volatility of output growth for the emerging countries used in the model calibration. Conditional output growth moments are computed with quarterly real GDP data and use business cycle dating estimated with U.S. consumption data. The last rows report aggregate equally and GDP-weighted descriptive statistics using each country's average GDP in constant U.S. dollars, as well as details on the distribution. The Internet Appendix B.2 details the construction of the conditional moments. Data are from Datastream and span the 1994:Q1-2018:Q2 period.

Country	**Output growth rate (%)**		**Output growth volatility (%)**		**Correlation with consumption**	
	Recession	Expansion	Recession	Expansion	Recession	Expansion
Argentina	0.11	4.12	5.55	3.01	-0.03	0.05
Bahrain	5.17	2.89	5.35	4.32	0.09	0.02
Bolivia	5.41	3.90	1.56	2.14	0.55	-0.15
Brazil	2.46	2.25	3.49	2.23	0.00	0.10
Bulgaria	0.30	4.23	3.81	1.70	-0.49	0.22
Chile	2.46	4.00	2.34	2.14	-0.12	0.02
Colombia	2.59	4.41	2.51	1.56	-0.18	0.09
Croatia	-2.45	2.78	3.50	2.32	-0.19	0.12
Czech Republic	-1.50	3.11	2.45	1.27	-0.15	0.10
Dominican Republic	2.13	5.66	3.59	3.85	0.05	-0.04
Ecuador	3.50	3.75	2.13	2.16	-0.25	0.39
Estonia	-5.80	5.49	7.12	3.29	-0.12	-0.09
Greece	-3.80	1.59	3.50	2.66	0.13	0.41
Hungary	-1.71	2.99	3.05	1.29	0.16	0.13
India	5.16	6.82	3.64	2.32	0.57	-0.16
Kazakhstan	6.06	5.11	14.46	9.87	-0.06	0.08
Latvia	-5.86	5.39	5.49	3.13	0.34	0.07
Lithuania	-3.31	4.82	8.24	1.92	-0.23	-0.14
Malta	1.83	3.91	2.90	3.52	-0.11	-0.18
Malaysia	2.97	5.40	3.89	1.00	-0.20	0.04

Continued.

Country	**Output growth rate (%)**		**Output growth volatility (%)**		**Correlation with consumption**	
	Recession	Expansion	Recession	Expansion	Recession	Expansion
Mexico	0.21	2.73	4.05	2.42	0.63	0.10
Morocco	4.78	3.34	2.87	3.58	0.02	0.16
Mozambique	7.52	5.72	4.28	3.60	0.23	-0.02
Namibia	-0.81	4.54	7.64	7.06	-0.30	0.09
Peru	4.26	4.81	2.43	2.75	0.48	-0.22
Philippines	4.36	4.91	2.62	1.61	0.29	-0.16
Poland	2.20	4.27	1.44	1.86	0.01	0.01
Romania	2.04	3.10	5.53	3.12	0.34	0.21
Russia	-0.11	3.64	4.96	2.21	-0.26	0.16
Slovakia	0.58	4.39	6.88	2.33	0.02	0.03
Slovenia	-3.24	3.59	3.49	1.48	0.30	0.20
South Africa	1.24	2.94	1.56	1.12	0.67	0.09
South Korea	2.33	4.75	2.81	2.67	0.24	-0.06
Taiwan	-0.35	4.82	3.61	3.09	0.43	0.21
Tanzania	5.62	6.75	4.25	3.16	-0.28	-0.22
Thailand	2.70	3.62	3.68	3.97	0.41	-0.04
Turkey	2.87	4.73	6.50	4.29	-0.39	-0.01
Uganda	6.23	5.24	3.64	2.82	0.04	-0.15
Venezuela	0.47	2.18	2.72	8.34	-0.08	-0.12
Vietnam	8.48	5.80	19.54	6.62	-0.05	0.01
Average (equal-weighted)	1.68	4.21	5.66	3.62	0.06	0.03
Average (GDP-weighted)	1.87	3.92	4.45	3.05	0.16	0.04
Median	2.17	4.25	3.62	2.66	0.01	0.03
Standard deviation	3.44	1.21	8.13	4.40	0.29	0.15
10th percentile	-3.27	2.76	2.23	1.39	-0.27	-0.16
90th percentile	5.84	5.69	7.39	5.59	0.52	0.21

Table A.2 : **Leverage estimates for emerging economies**.

The table reports estimates of the leverage parameter η for 10 countries. Leverage is computed as the ratio of government revenue growth volatility to output growth volatility. The last rows report aggregate equally and GDP-weighted descriptive statistics using each country's average GDP in constant U.S. dollars, as well as the median. The Internet Appendix B.3 details the construction of the leverage estimates. Data are from Datastream and the World Bank and span the 1994:Q1-2018:Q2 period.

Country	**Output growth volatility (%)**	**Government revenue growth volatility (%)**	**Leverage**
	(A)	(B)	(B) / (A)
Bolivia	1.95	7.89	4.03
Brazil	2.42	8.14	3.37
Bulgaria	2.31	9.34	4.05
Chile	2.09	9.20	4.41
Colombia	1.32	8.89	6.73
India	2.54	8.45	3.33
Mexico	3.10	8.22	2.65
Philippines	1.76	6.46	3.68
Russia	3.05	14.82	4.86
South Africa	1.23	4.29	3.50
Average (equal-weighted)	2.18	8.57	4.06
Average (GDP-weighted)	2.55	9.61	3.80
Median	2.22	8.33	3.86

Table A.3 : **Sovereign risk premium, price of risk, and investor preferences.**
The table presents the model predictions for different preference parameters. Column A reports the bond risk premium, Column B the credit spread, Column C the ratio of the risk-neutral default probability ($\mathbb{Q}$) over the physical default probability ($\mathbb{P}$), and Column D the price of macro risk Δ_H. Panel A reports predictions for the baseline calibration, Panel B reports predictions for different levels of relative risk aversion γ, while Panel C reports predictions for different levels of preference for time β. Bond risk premium and credit spread are in basis points (bps) per annum. Unless otherwise specified, we use the baseline calibration parameters on Table 1 and report predictions for levels of government revenue observed at issuance time ($Y = 1$).

	Bond risk premium (bps)	**Credit spread (bps)**	**Risk-neutral over physical default probability ($\mathbb{Q}/\mathbb{P}$)**	**Price of macro risk (Δ_H)**
	(A)	(B)	(C)	(D)
Panel A: Baseline case				
	117	154	1.94	1.67
Panel B: Relative risk aversion				
High ($\gamma = 15$)	201	209	2.94	2.35
Low ($\gamma = 5$)	47	117	1.31	1.25
Panel C: Preference for time				
High ($\beta = 0.05$)	112	158	2.02	1.70
Low ($\beta = 0.03$)	122	146	1.86	1.65

Table A.4 : **Model simulation vs. data.**
The table reports results of models simulations which are compared to the data over the 1994:Q1-2018:Q2 period. We generate 500 artificial datasets consisting of 40 countries and 98 quarters each using the parameters of the baseline calibration on Table 1. Panel A has the sovereign default probability computed at the 1-, 5-, and 10-year horizons. The empirical counterparts are the cumulative default rates for foreign-currency speculative-grade sovereign bonds from Standard and Poor's (2020). Panel B has bond pricing moments, averaged across simulations and the data. Bond excess returns are in percentage points per year and credit spreads are basis points (bps) per year. The empirical sovereign bond returns and credit spreads are from country-level JP Morgan EMBI Global indices for the countries in Table 5. 'Raw' denotes excess returns over one-month T-bills and credit spreads based off the U.S. Treasury curve, while 'AAA-adjusted' denotes excess returns and credit spreads over the ICE BofA AAA Corporate Bond index. Values in squared parentheses denote the 5th and 95th percentiles across the 500 artificial datasets. The Internet Appendix D explains the simulation procedure.

Panel A: Default probability at different horizons

	1 year (A)	5 years (B)	10 years (C)
Default rate (%)			
Simulation	2.23	10.63	20.08
	[1.53, 2.96]	[7.42, 13.95]	[14.29, 25.95]
Data	2.88	11.29	19.11

Panel B: Bond price moments

	(A) Mean	(B) Median	(C) Std. dev.	(D) 5th	(E) 95th
Bond excess return (%)					
Simulation	1.90	1.70	17.97	-48.03	46.52
	[1.28, 2.48]	[1.45, 1.97]	[16.24, 19.92]	[-56.48, -40.35]	[40.09, 54.15]
Data (raw)	8.53	7.11	18.82	-36.35	61.12
Data (AAA-adjusted)	5.36	3.20	18.24	-36.35	54.00
Credit spread (bps)					
Simulation	162	112	175	36	459
	[146, 181]	[101, 125]	[155, 194]	[32, 40]	[387, 535]
Data (raw)	471	323	535	90	1271
Data (AAA-adjusted)	376	234	518	16	1144

Table A.5 : **Country allocation by portfolios and rebalancing frequency.**
The table reports the country allocation and rebalancing frequency for the baseline double-sort procedure in Panel A.I of Table 6. Allocation is the fraction of time a country belongs to each of the three portfolios conditional on being on the sample. Rebalancing frequency is the fraction of time a country switches portfolios conditional on being on the sample. The sample period is 1994:Q1 to 2018:Q2 but countries enter and exit the sample at different times as reported in Table 5.

Country	**Portfolio allocation (%)**			**Rebalancing frequency (%)**	**Country**	**Portfolio allocation (%)**			**Rebalancing frequency (%)**
	Low	Medium	High			Low	Medium	High	
Argentina	10.81	9.46	79.73	12.16	Lebanon	52.63	24.56	22.81	21.05
Belize	0.00	0.00	100.00	0.00	Lithuania	0.00	0.00	100.00	0.00
Brazil	40.54	45.95	13.51	24.32	Malaysia	66.67	33.33	0.00	14.29
Bulgaria	21.43	76.79	1.79	25.00	Mexico	40.54	41.89	17.57	14.86
Chile	75.47	9.43	15.09	16.98	Morocco	8.89	26.67	64.44	11.11
China	65.75	20.55	13.70	9.59	Nigeria	43.40	56.60	0.00	11.32
Colombia	29.03	58.06	12.90	11.29	Pakistan	6.82	20.45	72.73	11.36
Cote d'Ivoire	11.11	46.67	42.22	8.89	Panama	28.38	21.62	50.00	17.57
Croatia	63.16	36.84	0.00	5.26	Peru	45.95	45.95	8.11	21.62
Dominican Rep.	4.65	46.51	48.84	18.60	Philippines	51.35	17.57	31.08	8.11
Ecuador	2.70	51.35	45.95	13.51	Poland	48.65	43.24	8.11	24.32
Egypt	65.91	31.82	2.27	13.64	Russia	2.70	28.38	68.92	24.32
El Salvador	0.00	39.02	60.98	14.63	Serbia	0.00	39.29	60.71	10.71
Gabon	0.00	33.33	66.67	22.22	South Africa	71.43	28.57	0.00	14.29
Georgia	0.00	68.75	31.25	18.75	Sri Lanka	63.16	5.26	31.58	10.53
Ghana	0.00	10.53	89.47	10.53	Thailand	50.00	50.00	0.00	8.33
Hungary	66.67	33.33	0.00	25.93	Turkey	32.81	57.81	9.38	15.63
Indonesia	27.27	12.12	60.61	6.06	Ukraine	22.45	28.57	48.98	24.49
Iraq	36.00	16.00	48.00	8.00	Uruguay	33.33	17.78	48.89	24.44
Jamaica	0.00	0.00	100.00	0.00	Venezuela	2.70	37.84	59.46	12.16
Kazakhstan	0.00	40.00	60.00	25.00	Vietnam	3.70	51.85	44.44	22.22
Korea	47.06	52.94	0.00	17.65					