1. Saving and Investment in a closed economy

Consider a closed, representative agent economy in which the household lives for two periods, youth and old age. The household has 20 units of endowment when young and 0 units when old. Suppose that there is a firm in this economy, which produces output in the second period from investment done in the second period according to the technology $Y = 10I^{1/2}$.

Household’s preferences are represented by the utility function $U(C_1, C_2) = \ln C_1 + \frac{1}{3} \ln C_2$, where $C_1$ is the first period consumption and $C_2$ is the second period consumption.

(a) Find the optimal consumption levels as well as the optimal savings for this household.
(b) Write down the investment demand function for the firm.
(c) Find the equilibrium in the loanable funds market.
   Hint: You need to find the equilibrium interest rate, equilibrium investment, and savings in this economy.

2. Saving and Investment in a Small Open Economy

Consider a small open economy, where households and firms take the world interest rate $r_w = 0.2$ (20%) as given. The investment demand function for the firm is $I = 5 - (r_w)^2$.

Household’s preferences over the first- and second period consumption are given by $U(C_1, C_2) = \ln C_1 + \beta \ln C_2$, where $C_1$ is the first period consumption, $C_2$ is the second period consumption, and $\beta$ indicates how much the household values second period consumption relative to the first period consumption. The household has 10 units of income in the first period and 5 units in the second period.

For parts (a), (b), and (c) below assume $\beta = 0.5$.

(a) Write down the budget constraints for the household.
(b) Calculate the amount of consumption and savings in each period in equilibrium (You can use any of the two methods you learned in class.)
(c) What is the trade deficit/surplus of this country?
(d) Now suppose that $\beta = 0.9$. What is the optimal amount of savings and consumption for the household? Compare your results to part (b) and explain the relationship between $\beta$, equilibrium consumption, and savings.

3. The Effects of Taxes on Savings and Consumption

Consider a closed economy in which the household lives for two periods, youth and old age. The household earns 20 units of commodities when young and 0 units when old. The real interest rate is 2%. Preferences of the household are represented by the utility function $U(C_1, C_2) = \ln C_1 + \ln C_2$, where $C_1$ is the first period consumption and $C_2$ is the second period consumption.

(a) Write down the budget constraints of the household by using the specific numbers given above.
(b) Find the optimal consumption levels as well as the optimal savings for this household.
(c) Suppose that the household pays 4 units of taxes out of his income when young and receives 6 units in benefits when old. What is the new lifetime budget constraint?
(d) In part (c) how much will the household save?

4. Solve Question 5 on page 168 of Farmer.