HW I
(Due on February 4, 2010; at the beginning of class)

Instructions:
• Please solve the questions in the order they are given.
• Show all the steps to your solution and make sure that your answers are clean and legible.
• Acknowledge the contribution of others by mentioning their name on your homework.
• Please staple all the pages. If you fail to do so, your overall grade from this homework will be reduced by 15%.
Thank you!

1. Find the first order derivative of the following functions:
   i. \( f(x) = -x^4 + 108x \)
   ii. \( f(x) = \frac{2x^2 + 5}{x - 6} \)
   iii. \( f(x) = (3x^3 - 2x)\left(\frac{4}{x + 3}\right) \)
   iv. \( f(x) = \ln\left(\frac{2x + 5}{x^2}\right) \)
   v. \( f(x) = (4x^4 + 3x^2 - x + 2)^3 \)
   vi. \( f(x) = -(2x - 1)^2 \)
   vii. \( f(x) = \ln(x) + \ln(5 - 2x) \)
   viii. \( f(x) = x^{1/2}(3x - x^2) \)

2. Solve the following equations in one unknown \( x \):
   i. \( 6x - 5 = x^2 \)
   ii. \( \frac{x - 2}{4} = \frac{2x}{5} \)
   iii. \( 2x^3 + 8x^2 - x - 4 = 0 \)
   iv. \( \frac{x(x - 1)}{2x^2 - 3x + 1} = \frac{1}{4} \)
   v. \( \frac{1}{2x + 1} = \frac{x}{x^2} \)
   vi. \( x^{1/3} = 1/2 \)
   vii. \( \frac{1}{x} - \frac{2}{5 - 2x} = 0 \)
3. A consumer has to choose between consuming slices of pizza (P) and burritos (B). He has $10 to spend per day. His preferences over these two goods are summarized by the following utility function:

\[ U(P, B) = 1.5 \ln P + \ln B \]

Price of a slice of pizza is $2 and a burrito is $4.

(i) What is the budget constraint for this consumer? Draw its graph. Make sure that the axes are labeled, and the graph is drawn according to a scale.

(ii) Find the optimal consumption bundle \((P^*, B^*)\) for this consumer. Please write down the consumer's problem and each step of the substitution method explicitly while solving the problem. (You don’t have to check for the second order condition.)

(iii) What would be the optimal choices of burritos and pizzas if his utility function was \( U(P, B) = \ln P + 1.5 \ln B \) and the prices of the two goods stayed the same? Explain the intuition behind your answer.

4. An individual has to decide how much leisure to have and how much to consume in one day. A day can be devoted either to work (L) or leisure, so let’s denote fraction of one day devoted to leisure as \((1-L)\). His preferences over leisure and consumption are represented by

\[ U(C, 1-L) = C - L^2. \]

One unit of consumption good is 2 dollars and the individual makes 4 dollars per unit of time he works.

(i) Write down the budget constraint for this individual.

(ii) Find the optimal amount of work and consumption for this individual. Please write down the consumer’s problem and each step of the substitution method explicitly while solving the problem. (Do not forget to check for the second order condition.)

5. Suppose that the production function for a firm is given by \( Y = \left(L^D\right)^{\frac{1}{2}} \), where \( L^D \) denotes the quantity of labor demanded by the firm and \( Y \) denotes the quantity produced.

(i) What is a production function? Draw the graph of the function given in this question.

(ii) Define the marginal product of labor and calculate it for the production function in this question. Is there diminishing marginal product of labor? If so, explain your answer.

(iii) Write down the firm’s profit maximization problem and solve for the labor-demand curve. Draw the graph of the labor demand function.

6. Now, consider a consumer as in question 4 and a firm as in question 5. In particular, the utility function and the firm’s production function are as given in these questions. Please answer the following questions:

(i) Solve for the labor market equilibrium, i.e, find the market clearing real wage and the equilibrium employment in the market.
(ii) Now suppose that the government imposes a 25% tax on nominal wage income, that is, for every dollar the worker earns, the government takes away 25 cents and the worker is left with 75 cents. Solve for the equilibrium in the labor market in this case.

(iii) For our example in this question, illustrate the effects of income tax on the market clearing real wage and employment by drawing graphs of equilibrium outcomes in (i) and (ii). Comment on your answer. What happened to real wage? What happened to employment in the economy? Why?