

Running Out of Time: Limited Unemployment Benefits and Wage Dispersion

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February 19, 2010

Abstract

We study unemployment insurance (UI) in a general equilibrium environment in which unemployed workers only receive benefits for a finite length of time. Although all workers have identical productivity and leisure value, the random arrival of job offers creates ex-post differences with respect to their time remaining until benefit expiration. Firms, which are also homogenous, can exploit these differences, leading to an endogenous wage distribution.

We analyze the equilibrium effect of both the length and size of unemployment benefits on wages, duration of unemployment, unemployment rate, and welfare. When calibrated to match key US labor market statistics, our model predicts that a one month extension of benefits is welfare-improving, increasing the average wages by 0.01 percent and unemployment rate by 1.4 percent. On the other hand, a 10 percent increase in the size of benefits is welfare-reducing, causing a 0.03 percent decrease in wages and an increase in unemployment rate.

1 Introduction

A common feature of practically all unemployment insurance schemes is that benefits are only offered for a limited duration. For instance, unemployed workers in the United States typically have been eligible for no more than 26 weeks of payments.¹

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¹The size of the weekly benefit is based on prior wages, but hits a maximum benefit well below the median wage. This computation, including the maximum benefit, varies across the US states.

Of course, the intent is to discourage moral hazard among workers, who might turn down reasonable job offers if supported by an open-ended benefit.

Indeed, it is not surprising that both the generosity and duration of unemployment benefits would influence employment search decisions. These decisions are often characterized in terms of the worker's *reservation wage*, the lowest wage offer which a worker is willing to accept. A larger benefit would typically increase a worker's reservation wage, as he can afford to be more selective in which job he accepts. As he nears the expiration of his benefits, however, his reservation wage will fall. This result was identified by Mortensen (1977) in an environment where the distribution of offered wages is exogenous. Quite significantly, this creates heterogeneity among otherwise similar individuals; workers who are closer to losing their benefit will be more desperate for a job.

As a consequence, the design of the unemployment insurance system will also affect the firms' incentives in making wage offers. By offering a particular wage, a firm effectively targets workers at a particular point in the unemployment spell. A higher wage will increase the likelihood of acceptance, but will decrease the realized profit ex-post. If these effects precisely cancel one another, identical firms can rationally offer different wages in equilibrium.

In this paper our aim is to characterize the general equilibrium effects of time limits on benefits on the unemployment rate, wage offer distribution, and the reservation wage distribution. In our model, we work with a continuous time environment with infinitely-lived workers who differ only in their realized employment shocks. There is no savings technology or a recall of past wage offers.

While unemployed, workers receive a constant benefit for up to T periods; they also derive constant utility from leisure (or home production). Job offers arrive randomly, with wages drawn from a known distribution $F(w)$. Unemployed workers formulate a time-dependant reservation wage, taking F as given; meanwhile, firms choose wages to maximize profits, taking the reservation wages and distribution of unemployed workers as given.

There are two main contributions of the paper. First, we show that limited benefits can generate wage dispersion, even though workers and firms are homogenous. The wage dispersion literature has sought to explain why wages vary dramatically among workers, even after controlling for all observable compensating differentials in firm or worker productivity. Burdett and Mortensen (1998) propose three potential sources

of wage dispersion. Two of these involve unobserved heterogeneity: workers could have unobserved differences in their value of leisure, or firms could have unobserved differences in their productivity. The remaining approach generates dispersion by allowing workers to receive additional wage offers while currently employed.²

Our work offers an additional (and simpler) explanation for dispersed wages: the impending loss of unemployment benefits. To isolate the impact of limited benefits, we do not permit on-the-job search or unobserved heterogeneity; certainly these could be integrated into a combined model. Two previous papers have made similar attempts in simpler environments, chosen due to concerns for tractability of the non-stationary search problem, which we overcome.

In Albrecht and Vroman (2005), unemployment benefits randomly expire at a constant Poisson rate; even the individual worker does not know his remaining eligibility. This creates two stationary search problems, pre- and post-expiration. That is, all unemployed workers with benefits will solve their search problem identically, whether they have been unemployed one week or 100 weeks, because conditional on still having benefits at that point, the probability of losing the benefits is identical. Similarly, those without benefits solve their search problem independent of their unemployment duration. This stationarity simplifies the analysis, but also results in no more than two wages being offered or demanded in equilibrium.

In Coles and Masters (2004), unemployment benefits last exactly T periods.³ When an unemployed worker encounters a firm, strategic bargaining ensues to determine the wage. However, unlike the wage-posting environment in our paper of Albrecht and Vroman (2005), firms are informed of a particular worker's unemployment duration before an offer is made. Thus, no offers are rejected in equilibrium (which is their crucial mechanism to side-step non-stationarity issues), and thus the

²Later work has augmented these explanations of wage dispersion to obtain a closer fit to the observed wage distribution. Postel-Vinay and Robin (2002) allows the current employer to make a counteroffer when workers receive an outside offer. In Burdett and Coles (2003), firms' offers consist of both a wage and a duration. In Christensen, *et al* (2005), workers can influence the job arrival rate through choice of search effort. Carrillo-Tudela (2009) allows the firm to observe the workers' employment status. Chéron and Langot (2010) makes unemployment benefits proportional to the workers prior wage. All of these include on-the-job search; the first also permits unobserved heterogeneity in firms and workers.

³These benefits are actually only received by new entrants to the work force, because there is no layoff risk in this model. Job destruction only occurs when a worker exits the labor force (randomly) and new unmatched workers are born at the same rate. This simplification is another important component in their solution to non-stationarity.

hazard rate of leaving unemployment is constant throughout the unemployment spell.

Our model offers a more appealing set of assumptions, as well as richer predictions for individual behaviors. For instance, reservation wages steadily decline while exit rates from unemployment steadily increase until benefit expiration, holding constant thereafter. Furthermore, our equilibrium behavior need not replicate theirs; only in certain equilibria does our comparative statics analysis agree with those in both papers. One contribution all three papers make is explaining an atom in the wage distribution at the lowest wage, which arises because that wage is targeted at the strictly positive measure of workers whose benefits have expired. This also arises in Carrillo-Tudela (2009), where the lowest wage targets all unemployed workers, which firms can distinguish from those engaged in on-the-job search.

The second contribution of our work is its analysis of policy decisions in general equilibrium, primarily through comparative statics. Indeed, the source of wage dispersion is of vital concern for policy choices. For instance, when wage dispersion arises from on-the-job search as in Burdett and Mortensen (1998), an increase in unemployment benefits will raise and compresses the distribution of equilibrium wages. In our model, however, a benefit increase can actually decrease and expand the distribution of wages. Larger benefits encourage workers to delay their acceptance of jobs; as a consequence, a higher concentration of unemployed workers are closer to their benefit expiration date. Firms can exploit this fact, successfully offering lower wages as increasingly desperate workers lower their reservation wage.

Additionally, our model allows us to examine the effect of changes in benefit eligibility duration. We show that extending benefits will expand the wage distribution upward (while leaving the lowest wage unchanged) and increases the unemployment rate. While the policy change allows workers to delay accepting jobs, the deadline also moves farther away. The net effect is that workers get higher wages in equilibrium.

Therefore, our paper predicts the potential consequences of extending unemployment benefits, such as the one enacted by the Obama administration in 2009. The president signed into law an extension that will provide for an additional fourteen weeks for every state and another six weeks (for a total of 20 weeks) in states with unemployment rates above 8.5%. We estimate that this extension (in steady state) will increase the unemployment rate by 4.3 percentage points, and increase the average accepted wage by 0.03%.

Additionally, our work provides theoretical basis for some common empirical ob-

servations. For instance, beginning with Katz and Meyer (1990), many researchers have found that the hazard rate of exiting unemployment steadily climbs during the unemployment spell, with a significant spike immediately before benefits expire. They take the partial equilibrium model of Mortensen (1977) as the justification for this effect; but while this does produce an increasing hazard rate, it does not explain the spike.

Our model, however, can. In equilibrium, a positive measure of firms will always offer the lowest wage, which is only acceptable to workers whose benefits have expired. Thus, as workers lose eligibility, their probability of finding an acceptable offer jumps by the size of this atom in the distribution. We examine this and other empirical findings in further detail in Section 5.

We proceed as follows: in Section 2, we present the environment and define equilibrium. In Section 3, we solve for equilibrium and characterize its basic features. Section 4 provides analysis of policy changes via comparative statics, which are then compared against empirical findings in Section 5. Finally, we conclude in Section 6.

2 Model

Consider a continuous time environment with a measure 1 of infinitely-lived workers who are ex-ante identical but only differ in their realized employment shocks. If employed at a given wage, they remain employed at the same wage until the job is dissolved, which occurs randomly at Poisson rate δ . Workers have no savings technology, so they live hand-to-mouth, consuming their income each instant. For simplicity, we assume linear utility. Let ρ denote the discount factor of time preference.

When workers become unemployed, they receive an unemployment benefit b for up to T units of time. They also receive an exogenous utility from leisure, x , that continues for the full duration of unemployment. Job offers arrive randomly at Poisson rate λ . If an offer arrives, the particular wage is drawn randomly from the distribution of wage offers, $F(w)$. The worker then accepts or rejects that offer, with no recall being allowed.

These decisions will be characterized in terms of a reservation wage, $R(t)$, where t is the length of time until a worker loses his unemployment benefit. Given the random luck in when an acceptable offer is received, an endogenous distribution of unemployed workers will emerge, represented by cumulative distribution $H(t)$. Let u

denote the fraction of the population currently unemployed.

The distribution of offered wages is also endogenously determined, from the choices of profit maximizing firms. Any worker who is successfully hired will produce p dollars of value each instant. Firms choose wages so as to balance the realized profit, $p - w$, with the probability that wage w is accepted, taking $R(t)$ and $H(t)$ as given. We exclude on-the-job search, since this is known to produce wage dispersion.

We assume throughout that $p > x$. If this were violated, it would be inefficient to have anyone work, since their marginal product is less than their value of leisure. Additionally, we require that $\frac{\delta + \rho}{\lambda} \geq e^{-\frac{T\rho}{2}} \geq \frac{\rho}{2\lambda}$. If this were violated, job offers arrive too infrequently relative to the time preference, making households anxious to accept those that do arrive. As a consequence, wage dispersion would never arise in equilibrium.

2.1 Bellman Equations

The worker's decisions are presented recursively, with $V_e(w)$ representing the discounted present utility of a worker entering the period employed at wage w . $V_u(t)$ is the same for an unemployed worker who has t remaining periods of benefits.

A worker employed at wage w consumes his wage each period until job separation randomly occurs. At that point, he begins a new spell of unemployment benefits and his utility changes by $V_u(T) - V_e(w)$.

$$\rho V_e(w) = w + \delta (V_u(T) - V_e(w)) \quad (1)$$

Next, consider an unemployed worker whose benefits have expired. His recursive problem is stationary moving forward, receiving leisure utility x each period until he receives an acceptable job offer.

$$\rho V_u(0) = x + \lambda \left(\max_{R(0)} \int_{R(0)}^{\infty} (V_e(w) - V_u(0)) dF(w) \right) \quad (2)$$

Finally, an unemployed worker with t periods of benefits remaining has a similar recursive problem with two difference. First, he also receives benefit b each period. In addition, his state variable (time of remaining benefits) deterministically falls each instant, causing his utility to change at rate $-V'_u(t)$.

$$\rho V_u(t) = b + x - V'_u(t) + \lambda \left(\max_{R(t)} \int_{R(t)}^{\infty} (V_e(w) - V_u(t)) dF(w) \right) \quad (3)$$

Equation 1 reveals that $V_e(w)$ is increasing in w . Thus, unemployed workers will solve their utility maximization with a reservation wage such that $R(t)$ will satisfy

$$V_e(R(t)) = V_u(t). \quad (4)$$

2.2 Steady State Conditions

Consider the flows of workers between states of employment and unemployment. $(1 - u)G(w)$ will denote the measure of workers currently employed at or below wage w , where $G(w)$ is the cumulative distribution among employed workers at each wage. Similarly, $uH(t)$ denotes the measure of unemployed workers with t or fewer periods of benefits remaining, where $H(t)$ is a cumulative distribution of unemployed workers. We require that the flows between these states balance so that G , H , and u remain constant over time. Let $h(t)$ and $g(w)$ denote the respective probability density functions.

To allow for the possibility of atoms (*i.e.* discontinuous upward jumps) in the cumulative distribution function, we use the following notation for a mass of agents at a particular wage:

$$\mu_G(w) = G(w) - \lim_{\epsilon \rightarrow 0} G(w - \epsilon). \quad (5)$$

The notation similarly applies for a mass of firms offering a given wage, $\mu_F(w)$.

The transition from employment to unemployment is simply stated as

$$uh(T) = \delta(1 - u). \quad (6)$$

That is, the $1 - u$ employed agents become unemployed at rate δ , and enter unemployment with full benefits.

Next consider those who still have t periods of remaining benefits. A flow of $\lambda(1 - F(R(t)))h(t)$ workers will receive acceptable wage offers each instant and enter employment; thus the change in the density of unemployed workers is:

$$h'(t) = \lambda(1 - F(R(t)))h(t). \quad (7)$$

A positive measure of workers might not receive an acceptable job offer within T periods; thus, we allow $H(0) > 0$. The reservation wage of those whose benefits have expired, $R(0)$, is the lowest of any worker; thus it is also the lower bound of the support of F . In other words, workers without benefits accept any wage offered in equilibrium, receiving jobs at rate $\lambda H(0)$. To maintain steady state, these must be replaced by the flow of agents whose benefits have just expired, $h(0)$. Thus,

$$h(0) = \lambda H(0). \quad (8)$$

Among the employed, jobs at any given wage are lost at rate $\delta(1-u)g(w)$. These must be replaced by someone who receives and accepts a job at that wage, which occurs at rate $\lambda u f(w)H(R^{-1}(w))$, where $R^{-1}(w)$ denotes the inverse of the reservation wage function, and $f(w) = F'(w)$. Thus, the transition equation becomes:

$$\delta(1-u)g(w) = \lambda u f(w)H(R^{-1}(w)). \quad (9)$$

If F has an atom at w , G would need an atom at the same w . The same equation would still be required, replacing $g(w)$ and $f(w)$ with their respective atoms, $\mu_G(w)$ and $\mu_F(w)$.

2.3 Profit Maximization

The steady-state profit of a firm offering wage w is defined as the realized profits, $p - w$, times the average number of workers employed at w per firm offering w , $\frac{g(w)}{f(w)}$. The definition similarly applies when an atom occurs at w , only using $\frac{\mu_G(w)}{\mu_F(w)}$. Using Equation 9, we can state profits as:

$$\pi = \frac{\lambda u}{\delta(1-u)}(p - w)H(R^{-1}(w)). \quad (10)$$

We require that steady state profits equate for any firm offering a wage in the support of F .

2.4 Equilibrium

A steady state search equilibrium consists of firm profit π , a reservation wage policy $R(t)$, a measure of unemployed agents u , and distributions of employed workers $G(w)$,

unemployed workers $H(t)$, and firms wage offers $F(w)$, such that:

1. $R(t)$ maximizes utility for the worker, given $F(w)$.
2. All wages in the support of F produce the same profit π , while all other wages produce no more than π .
3. G , H , and u satisfy steady state conditions in Equations 6 through 9.

We begin by deriving some general properties of equilibrium, then investigate a particular equilibrium of interest.

Lemma 1. *The following conditions must hold in equilibrium:*

1. *An atom can occur in G if and only if one occurs in F at the same wage.*
2. *No atoms occur in H except at $H(0)$.*

In the following section, we present four equilibria that arise from this model. Which occurs depends on a comparison of the net social value of employment, $p - x$, to a present value of expected unemployment benefits, b .

We can characterize these equilibria based on how offered wages are targeted to potential employees. If a firm offers wage $R(t)$, they are effectively targeting the $uH(t)$ unemployed workers with t or fewer periods of benefit remaining.

For instance, when unemployment benefits are very generous, firms only target those whose benefits have expired. In this equilibrium, all firms offer the wage $R(0)$, and workers will refuse that wage until their benefits have expired. We refer to this as the *late degenerate equilibrium*, since workers only accept offers late in their unemployment spell.

On the other extreme, when benefits are small, all firms offer the wage $R(T)$. This wage is acceptable to any unemployed worker, even by those early in their unemployment spell. Thus we refer to this as the *early degenerate equilibrium*.

With moderate benefits, firms offer wages $R(t)$ that are individually targeted for workers across a spectrum of possible ts . Firms are indifferent among these wage offers because a lower wage is only acceptable to a smaller portion of the population, and in equilibrium, the two effects cancel to maintain constant expected profits within the support of F .

The distinction between these two remaining equilibria is which wages actually appear in the support of F . In one equilibrium, the support includes all wages from $R(0)$ to $R(Y)$, where $Y \in (0, T)$. As a consequence, workers who have been unemployed less than $T - Y$ periods reject all the equilibrium wage offers, and acceptances only occur late in the unemployment spell. Hence we refer to this as a *late dispersed equilibrium*.

In the other equilibrium, the support is from $R(Z)$ to $R(T)$, where $Z \in (0, T)$. Thus, workers are willing to accept at least some of the offered wages immediately on being unemployed. Moreover, those unemployed for more than $T - Z$ periods accept any of the equilibrium wage offers. We refer to these as *early dispersed equilibria*.

3 Equilibrium Solution

Our approach to finding the equilibrium solution is essentially to reformulate the equilibrium requirements into a second-order differential equation. In particular, we use the first equilibrium requirement to translate the Bellman equations into a relationship between $R(t)$, $R'(t)$, $R''(t)$, and $F(R(t))$. Meanwhile, the second and third equilibrium requirements are used to translate the steady state conditions into a distinct relationship between these same functions. Together, these allow us to substitute for $F(R(t))$ and solve the remaining differential equation for $R(t)$.

This system imposes four boundary conditions. Of course, in the solution to a second-order differential equation, two constants of integration can be set to match two boundary conditions. A third boundary condition simply determines the equilibrium unemployment rate. The fourth boundary condition determines which reservation wages $R(t)$ are actually offered by firms in equilibrium. In a late dispersed equilibrium, for instance, this condition pins down Y ; in an early dispersed equilibrium, it pins down Z .

In both dispersed equilibria, we cannot obtain a closed form solution. Rather, we are left with an equation which implicitly solves for Y or Z , and must express the equilibrium in terms of Y or Z . Both degenerate equilibria have a closed form solution, and are equivalent to setting $Y = 0$ or $Z = T$, respectively.

3.1 Late Equilibria

To characterize the equilibrium solution where offers are only accepted late in the unemployment spell, we introduce the following equation:

$$\phi(Y) \equiv \frac{p - x - b}{b} e^{\frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y\rho}{2}}\right)} + \frac{\rho + \delta (1 - e^{(Y-T)\rho})}{\rho} e^{-\frac{Y\rho}{2}} - \frac{\delta + \rho}{\lambda}. \quad (11)$$

Intuitively, $b \cdot \phi(Y)$ weighs the net social value of working, $p - x$, versus a particular discounted value of unemployment benefits b (which reflects the probabilities of getting an acceptable offer over the unemployment spell).

We are interested in the Y^* for which $\phi(Y^*) = 0$. If such a Y^* lies between 0 and T , what follows will constitute a late dispersed equilibrium. If $\phi(Y) < 0$ for all $Y \in [0, T]$, then setting $Y^* = 0$ in what follows will constitute a late degenerate equilibrium. If $\phi(Y) > 0$ for all $Y \in [0, T]$, then a late equilibrium does not exist (and, as we shall see later, an early degenerate equilibrium does). The following proposition establishes that, for any parameter values, the outcome among these three options is unique.

Lemma 2. *Assume $e^{-\frac{T\rho}{2}} \geq \frac{\rho}{2\lambda}$.*

1. *If there exists a solution $Y^* \in [0, T]$ such that $\phi(Y^*) = 0$, then it is the only such solution.*
2. *If $\phi(0) < 0$, then $\phi(Y) < 0$ for all $Y \in (0, T]$.*
3. *If $\phi(T) > 0$, then $\phi(Y) > 0$ for all $Y \in [0, T]$.*
4. *If $\phi(0) \geq 0$ and $\phi(T) \leq 0$, then there exists a unique $Y^* \in [0, T]$ such that $\phi(Y^*) = 0$.*

The assumption ensures that households are sufficiently patient (relative to the offer arrival rate) to await at least some future job offers. Without this, late equilibria would typically not exist.

Lemma 2 effectively partitions the parameter space among those that produce a late degenerate equilibrium, those that produce a late dispersed equilibrium, and those that produce neither. The first occurs when $\phi(0) \leq 0$, which is equivalent to

saying:

$$p - x \leq b \left(\frac{\delta + \rho}{\lambda} - \frac{\delta}{\rho} (1 - e^{-T\rho}) \right). \quad (12)$$

This is simply a comparison of the net social value of employment to an appropriately discounted value of the expected flow of unemployment benefits. If benefits are sufficiently generous, households find it optimal to enjoy the full stream of benefits, accepting jobs only after benefits expire. Moreover, firms find it optimal to only offer the wage that is only accepted by those without benefits.

On the other hand, neither late equilibrium occurs when $\phi(T) > 0$, which is equivalent to:

$$p - x > b \left(\frac{\delta + \rho}{\lambda} + e^{\frac{2\lambda}{\rho}} \left(1 - e^{-\frac{T\rho}{2}} \right) - e^{-\frac{T\rho}{2}} \right). \quad (13)$$

Again, the right-hand side can be thought of as the discounted expected value of benefits, but here it anticipates some workers receiving acceptable wages in the middle of their benefit eligibility (and thus truncating the flow of benefits). If this discounted value is too low, a late dispersed equilibrium cannot be sustained. In fact, in the next subsection, we show that this results in an early degenerate equilibrium.

We now present the equilibrium solution, whether for the late degenerate ($Y^* = 0$) or late dispersed ($Y^* \in (0, T]$) equilibrium. Qualitatively, equilibrium has the following features.

Reservation Wages: Newly unemployed workers will reject all wage offers initially, only accepting some of the offered wages once Y^* periods of benefits remain. As a worker nears the expiration of unemployment benefits, he becomes gradually less choosy about job offers. After expiration, he accepts any job offer in the support of F .

Offered Wages: The wage distribution contains two atoms (or for the degenerate case, one). One always occurs at the lowest wage, w_ℓ . This is equal to the reservation wage of those whose unemployment benefits have expired; they are the only ones who would accept such a wage offer, and there is a positive measure of them. Another can occur at the highest offered wage, w_h , which is accepted by all workers with less than S periods of benefits remaining. An atomless distribution also spans from w_h down to a wage w_ℓ .

Also, this atomless distribution has increasing density in w , so higher wages are more likely than low wages. This is typically the case in search models with endogenous wages and homogeneous productivity; the difference here is that the atom at w_ℓ skews the distribution towards lower wages.

We begin by characterizing the reservation wages.

$$R(t) = \begin{cases} p - \frac{(p-b-x)(\delta+\rho)\left(e^{\frac{Y^*\rho}{2}} - \lambda(1-e^{(Y^*-t)\rho})\right)}{\delta\lambda e^{(Y^*-T)\rho} + (\delta+\rho)\left(\rho e^{\frac{Y^*\rho}{2}} - \lambda\right)} & t \in (Y^*, T] \\ p - \frac{(\delta+\rho)b}{\lambda} e^{-\frac{2\lambda}{\rho}\left(1-e^{-\frac{t\rho}{2}}\right)} & t \in [0, Y^*]. \end{cases} \quad (14)$$

In equilibrium, the reservation wages for $t > Y^*$ are not offered by any firm; even so, these represent the wage at which they would be willing to enter employment rather than continue on unemployment benefits, taking F as given. Note that $R(t)$ is continuous everywhere, including at $t = Y^*$.

Next, we consider the distribution of offered wages, which can be expressed as follows:

$$F(w) = \begin{cases} 0 & w < R(0) \\ \frac{\rho}{2\lambda} \left(1 - \ln\left(\frac{\lambda(p-w)}{(\delta+\rho)b}\right)\right) & w \in [R(0), R(Y^*)) \\ 1 & w \geq R(Y^*). \end{cases} \quad (15)$$

It is also convenient to express this distribution in terms t , which is to say, what fraction of offers are at or below the reservation wage of a person with t periods of benefits remaining. This is done by substituting $w = R(t)$ into $F(w)$.

$$F(R(t)) = \begin{cases} 1 & t \in [Y^*, T] \\ \frac{\rho}{2\lambda} + 1 - e^{-\frac{t\rho}{2}} & t \in [0, Y^*]. \end{cases} \quad (16)$$

Since we have assumed $e^{-\frac{T\rho}{2}} \geq \frac{\rho}{2\lambda}$, we know that $\frac{\rho}{2\lambda} < 1$ and thus F is a well defined c.d.f. Also notice that there is an atom at $t = 0$ and $t = Y^*$, or equivalently, at the highest and lowest wages.

With these R and F , the distribution of the unemployed becomes:

$$H(t) = \begin{cases} \frac{e^{\frac{2\lambda}{\rho} \left(e^{-\frac{Y^*\rho}{2}} - e^{-\frac{t\rho}{2}} \right)}}{1 + \lambda(T - Y^*)e^{-\frac{Y^*\rho}{2}}} & t \leq Y^* \\ \frac{1 + \lambda(t - Y^*)e^{-\frac{Y^*\rho}{2}}}{1 + \lambda(T - Y^*)e^{-\frac{Y^*\rho}{2}}} & t > Y^*. \end{cases} \quad (17)$$

This distribution is continuous for all $t \in [0, T]$, including at Y^* , and leaves an atom at $H(0)$.

The distribution of the employed is:

$$G(w) = \begin{cases} 0 & w < R(0) \\ \frac{\rho(\delta + \rho)b}{2(p - w)\lambda^2} e^{\frac{Y^*\rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^*\rho}{2}} \right)} & w \in [R(0), R(Y^*)) \\ 1 & w \geq R(Y^*). \end{cases} \quad (18)$$

Atoms occur in G just as in F .

Steady state profits will be

$$\pi = \frac{(\delta + \rho)b}{\lambda} e^{\frac{Y^*\rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^*\rho}{2}} \right)}, \quad (19)$$

and the unemployment rate will be

$$u = 1 - \frac{\lambda}{\lambda(1 + \delta(T - Y^*)) + \delta e^{\frac{Y^*\rho}{2}}}. \quad (20)$$

Proposition 1. *Assume $e^{-\frac{T\rho}{2}} \geq \frac{\rho}{2\lambda}$, and that either $\phi(Y^*) = 0$, or $\phi(0) < 0$ and $Y^* = 0$. Then Equations 14 through 20 constitute an equilibrium.*

This is verified in the Appendix.

Using this solution, we compute typical labor statistics of interest. Of course, one of the most fundamental, the unemployment rate, was already reported in Equation 20. Next, consider descriptive statistics about the distribution of wages among current employees.

$$w_{\max} = p - \frac{(\delta + \rho)b}{\lambda} e^{-\frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^* \rho}{2}}\right)} \quad (21)$$

$$w_{\text{mean}} = p - \frac{(\delta + \rho)b}{\lambda} e^{\frac{Y^* \rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^* \rho}{2}}\right)} \quad (22)$$

$$w_{\text{median}} = p - \frac{\rho(\delta + \rho)b}{\lambda^2} e^{\frac{Y^* \rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^* \rho}{2}}\right)} \quad (23)$$

$$w_{\min} = p - \frac{(\delta + \rho)b}{\lambda} \quad (24)$$

One can quickly show that in a dispersed equilibrium, $g'(w)$ is strictly increasing for $w \in (w_\ell, w_h)$, which would normally imply a distribution of wages with a greater concentration on late wages; however, one must also consider the atoms at w_ℓ and w_h . A quick comparison reveals that $w_{\text{mean}} > w_{\text{median}}$ if and only if $\rho > \lambda$. Thus, when λ is smaller than ρ , the atom at w_ℓ is large enough to counteract the increasing density elsewhere and skew the distribution towards lower wages.

Next, we are interested in the expected duration of unemployment. Ideally, this would be the expected time that a newly unemployed worker will wait before receiving an acceptable job offer. While this can be computed, the resulting expression is not particularly enlightening or analytically tractable for comparative statics. Instead, we adopt a simple measure:

$$H(0) = \frac{e^{-\frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^* \rho}{2}}\right)}}{1 + \lambda(T - Y^*)e^{-\frac{Y^* \rho}{2}}}. \quad (25)$$

This reports the fraction of unemployed workers who are no longer eligible for unemployment benefits at a given moment of time; an increase in this measure would typically indicate that expected duration has risen, since more people reach expiration.

This measure is also useful because it is used in computing the cost of providing unemployment benefits in any given moment. The total number of workers still covered by unemployment benefits at any given moment is $u(1 - H(0))$; if multiplied

by b , this gives the cost of unemployment benefits for one unit of time:

$$\text{Cost} = \frac{\delta \left(e^{\frac{Y^* \rho}{2}} \left(1 - e^{-\frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^* \rho}{2}} \right)} \right) + (T - Y^*) \lambda \right)}{\lambda (1 + \delta(T - Y^*)) + \delta e^{\frac{Y^* \rho}{2}}} b. \quad (26)$$

A frequently studied statistic is the hazard rate of unemployment exit, which is the rate at which workers find acceptable jobs. For workers who have t periods until benefit expiration, this is $\lambda(1 - F(R(t))) = \lambda e^{-\frac{t\rho}{2}} - \frac{\rho}{2}$. At $t = 0$, the hazard rate discretely jumps to λ , due to the atom at $R(0)$. The average hazard rate over all unemployed workers is $\frac{\lambda}{(T - Y^*)\lambda + e^{\frac{Y^* \rho}{2}}}$.

It is also possible to calculate expected utility, averaged over all workers, though it results in a lengthy expression. This is useful, however, in considering welfare impact of policy changes. In the degenerate equilibrium, expected utility is:

$$EU = \frac{x}{\rho} + \frac{\delta \lambda (T\rho - 1 + e^{-T\rho})}{\rho^2(\delta + \lambda + T\delta\lambda)} b. \quad (27)$$

3.2 Early Equilibria

The early equilibria have a similar development to the late equilibria. Qualitatively, the main difference is that newly unemployed workers are immediately targeted by some fraction of firms who offer $R(T)$. This is the only wage in the degenerate equilibrium; in the continuous equilibrium, the support is connected from $R(Z^*)$ to $R(T)$, where $Z^* > 0$. For those who have been unemployed longer than Z^* periods, they will accept any wage in the support since these are strictly above their reservation wage. Ex-post, firms will have left these workers with surplus; however, specifically targeting them would lower the probability of acceptance enough that firm profit would fall.

As in the late equilibrium, Z is found implicitly, as the solution to the following equation:

$$\psi(Z) \equiv \frac{\lambda(\delta + \rho)(p - x - b)}{\left((\delta + \rho)e^{\frac{T\rho}{2}} - \lambda e^{\frac{Z\rho}{2}} \right) b} e^{\frac{T\rho}{2} + \frac{2\lambda}{\rho} \left(1 - e^{\frac{1}{2}(Z-T)\rho} \right)} - (\delta + \rho)e^{-Z(\lambda + \rho)}. \quad (28)$$

If we find a $Z^* \in [0, T]$ for which $\psi(Z^*) = 0$, what follows will constitute an early

dispersed equilibrium. On the other hand, a late degenerate equilibrium exists when $\psi(T) > 0$, which is equivalent to the following condition:

$$p - x > b \left(1 + \frac{\delta + \rho - \lambda}{\lambda} e^{-T(\lambda + \rho)} \right). \quad (29)$$

Unlike the early equilibria, however, if a late dispersed equilibrium exists, a late degenerate equilibrium also exists.

Lemma 3. *Assume $\delta + \rho > \lambda$.*

1. *If there exists a solution $Z^* \in [0, T]$ such that $\psi(Z^*) = 0$, then it is the only such solution.*
2. *If $\psi(0) > 0$, then $\psi(Z) > 0$ for all $Z \in (0, T]$.*
3. *If $\psi(T) < 0$, then $\psi(Z) < 0$ for all $Z \in [0, T]$.*
4. *A solution $Z^* \in [0, T]$ such that $\psi(Z^*) = 0$ exists if and only if $\psi(0) \leq 0$ and $\psi(T) \geq 0$.*

The assumption $\delta + \rho > \lambda$ is sufficient to ensure that job offers do not arrive too frequently, so that it is optimal for workers to accept some jobs even as they begin unemployment. Note that, with this assumption, $\psi(0) > 0$ is equivalent to $\phi(T) > 0$, expressed in Equation 13.

We now characterize the reservation wages in the late equilibrium:

$$R(t) = \begin{cases} p - \frac{(\delta + \rho)b}{\lambda} e^{-Z^*(\lambda + \rho) - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(t - Z^*)\rho}{2}} \right)} & t \in (Z^*, T] \\ p - \frac{(\delta + \rho)b}{\lambda(\lambda + \rho)} (\lambda e^{-t(\lambda + \rho)} + \rho e^{-Z^*(\lambda + \rho)}) & t \in [0, Z^*]. \end{cases} \quad (30)$$

The distribution of offered wages is expressed as follows:

$$F(w) = \begin{cases} 0 & w < R(Z^*) \\ \frac{\rho}{2\lambda} \left(1 - (\lambda + \rho)Z^* - \ln \left(\frac{\lambda(p - w)}{(\delta + \rho)b} \right) \right) & w \in [R(Z^*), R(T)] \\ 1 & w \geq R(T). \end{cases} \quad (31)$$

or, expressed in terms of t :

$$F(R(t)) = \begin{cases} 1 & t = T \\ \frac{\rho}{2\lambda} + 1 - e^{\frac{(Z^*-t)\rho}{2}} & t \in [Z^*, T) \\ 0 & t \in [0, Z^*). \end{cases} \quad (32)$$

Since $\frac{\rho}{2\lambda} < 1$, F is non-decreasing from 0 to 1. Also notice that there is an atom at $t = Z^*$ and $t = T$, or equivalently, at the highest and lowest wages.

With these R and F , the distribution of the unemployed becomes:

$$H(t) = \begin{cases} e^{(t-Z^*)\lambda - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^*-T)\rho}{2}}\right)} & t \leq Z^* \\ e^{\frac{2\lambda}{\rho} \left(e^{\frac{(Z^*-T)\rho}{2}} - e^{\frac{(Z^*-t)\rho}{2}}\right)} & t > Z^*. \end{cases} \quad (33)$$

This distribution is continuous for all $t \in [0, T]$, including at Z^* , and leaves an atom at $H(0)$.

The distribution of the employed is:

$$G(w) = \begin{cases} 0 & w < R(Z^*) \\ \frac{\rho(\delta+\rho)b}{2(p-w)\lambda^2} e^{\frac{(T-Z^*)\rho}{2} - (\lambda+\rho)Z^* - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^*-T)\rho}{2}}\right)} & w \in [R(Z^*), R(T)) \\ 1 & w \geq R(T). \end{cases} \quad (34)$$

Atoms occur in G just as in F .

Steady state profits will be

$$\pi = \frac{(\delta + \rho)b}{\lambda} e^{\frac{(T-Z^*)\rho}{2} - (\lambda+\rho)Z^* - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^*-T)\rho}{2}}\right)}, \quad (35)$$

and the unemployment rate will be

$$u = \frac{\delta}{\delta + \lambda e^{\frac{(Z^*-T)\rho}{2}}}. \quad (36)$$

Proposition 2. *Assume that either $\psi(Z^*) = 0$, or $\psi(T) \geq 0$ and $Z^* = T$. Then Equations 30 through 36 constitute an equilibrium.*

This is verified in the Appendix.

We again compute key labor statistics. First, consider descriptive statistics about the distribution of wages among current employees.

$$w_{\max} = p - \frac{(\delta + \rho)b}{\lambda} e^{-Z^*(\lambda + \rho) - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^* - T)\rho}{2}}\right)} \quad (37)$$

$$w_{\text{mean}} = p - \frac{(\delta + \rho)b}{\lambda} e^{\frac{(T - Z^*)\rho}{2} - Z^*(\lambda + \rho) - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^* - T)\rho}{2}}\right)} \quad (38)$$

$$w_{\text{median}} = p - \frac{\rho(\delta + \rho)b}{\lambda^2} e^{\frac{(T - Z^*)\rho}{2} - Z^*(\lambda + \rho) - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^* - T)\rho}{2}}\right)} \quad (39)$$

$$w_{\min} = p - \frac{(\delta + \rho)b}{\lambda} e^{-Z^*(\lambda + \rho)} \quad (40)$$

As in the late equilibria, $w_{\text{mean}} > w_{\text{median}}$ if and only if $\rho > \lambda$.

Next, we report the fraction of unemployed workers who are ineligible for benefits, which is offered as a measure of unemployment duration.

$$H(0) = e^{-\lambda Z^* - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^* - T)\rho}{2}}\right)} \quad (41)$$

Also, the cost of providing these unemployment benefits is:

$$\text{Cost} = \frac{\delta \left(1 - e^{-\lambda Z^* - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^* - T)\rho}{2}}\right)}\right)}{\delta + \lambda e^{\frac{(Z^* - T)\rho}{2}}} b. \quad (42)$$

The hazard rate of unemployment exit for workers who have t periods until benefit expiration is $\lambda(1 - F(R(t))) = \lambda e^{\frac{(Z^* - t)\rho}{2}} - \frac{\rho}{2}$, and at $t = Z^*$, the hazard rate discretely jumps to λ . The average hazard rate is $\lambda e^{\frac{(Z^* - T)\rho}{2}}$.

3.3 Uniqueness of Equilibrium

We will not always have uniqueness of equilibria in this model. However, by combining the results of Lemmas 2 and 3, we can narrow down the possible equilibria which can simultaneously exist. Note also that when $Y^* = T$ and $Z^* = 0$, the two dispersed equilibria coincide, with a full support from $R(0)$ to $R(T)$.

Lemma 4. *Assume $e^{-\frac{T\rho}{2}} \geq \frac{\rho}{2\lambda}$ and $\delta + \rho > \lambda$. Then exactly one of the following will occur:*

- Only an early degenerate equilibrium exists (whenever $\phi(T) > 0$).
- Only a late equilibrium (dispersed or degenerate, but not both) exists (whenever $\psi(T) < 0$).
- An early degenerate equilibrium and one early dispersed equilibrium exists, as well as one late equilibrium (dispersed or degenerate, but not both) (whenever $\phi(T) \leq 0 \leq \psi(T)$).

Note that both assumptions are satisfied for an open set of parameter values. Also, only in the last case is it possible to have multiple equilibria. In particular, whenever an early dispersed equilibrium exists, an early degenerate equilibrium and a late equilibrium will also exist.

4 Comparative Statics

We now consider three policy experiments and determine their impact on the economy through comparative statics. In doing so, of course, we are comparing steady state outcomes, as the dynamic transition is not modeled. Also, our equilibria are implicitly solved in terms of Y^* and Z^* , respectively; nevertheless, we can use implicit differentiation to determine how these respond to changes in T , b , or x .

Indeed, in most cases, the signs are unambiguous. Assuming $e^{-\frac{T\rho}{2}} \geq \frac{\rho}{2\lambda}$ (as we did in Proposition 1), then $\frac{\partial Y^*}{\partial T} > 0$, $\frac{\partial Y^*}{\partial b} < 0$, and $\frac{\partial Y^*}{\partial x} < 0$. Assuming $\delta + \rho \geq \lambda$ (as we did in Proposition 2), then $\frac{\partial Z^*}{\partial b} > 0$, and $\frac{\partial Z^*}{\partial x} > 0$. Only $\frac{\partial Z^*}{\partial T}$ could take either sign; it is negative if and only if $\delta + \frac{\rho}{2} > \lambda e^{\frac{(Z^*-T)\rho}{2}}$.

Using the calculated implicit derivatives, we can then determine the impact of any particular policy change on the various statistics previously reported. Of course, this does depend on which equilibrium is considered; we report the comparative signs of each comparative static in each equilibrium.

4.1 Policy Experiment: Increase in b

Suppose that benefits are made more generous, keeping the duration unchanged. As previously noted, $\frac{\partial Y^*}{\partial b} < 0$, which means that in a late dispersed equilibrium, unemployed workers will wait until *later* in their unemployment spell before accepting

any wage offer. This can be considered one measure of moral hazard, as the more generous benefit delays the exit from unemployment.

In an early dispersed equilibrium, workers accept some wage offers from the beginning of their unemployment spell, but only accept all offers once Z^* or fewer periods remain. Since $\frac{\partial Z^*}{\partial b} > 0$, a higher benefit causes workers to start accepting all offers *earlier* in their unemployment spell. Hence, this accelerates exit from unemployment.

Table 1 summarizes the comparative statics for the key statistics computed earlier. The same results apply to both degenerate and dispersed equilibria, with the exception that b has no impact on u or $H(0)$ in either degenerate equilibrium. This is because, even though the wage changes, when it is accepted does not change; thus the hazard rate of leaving unemployment is constant. Note that early and late equilibria move in opposite directions; and in the dispersed equilibrium in which the support of F runs from $R(0)$ to $R(T)$, all of the comparative statics have a derivative of zero.

Table 1: Impact of an increase in temporary benefits level, b

$\partial/\partial b$	early equilibria	late equilibria
w_{\max}	+	-
$w_{\text{mean}}, w_{\text{median}}$	+	-
w_{\min}	+	-
u	-	+
$H(0)$	-	+
π	-	+

The results in the early equilibria follow what most might intuitively expect. The larger benefit creates a higher opportunity cost of accepting an offer, so firms must counter by offering higher wages. Perhaps the only unexpected result is that the unemployment rate actually falls. This is because the increase in Z^* not only increases wages, but also compresses the wage offer distribution; thus workers receive acceptable offers earlier in their unemployment spell, and fewer of them exhaust their unemployment benefit eligibility.

Given this intuitive explanation, it is all the more striking that the late equilibrium is exactly opposite. The more generous benefit encourages the unemployed to demand higher reservation wages initially and wait longer before acceptable offers arrive. However, this blessing of increased patience becomes its own curse as benefit

expiration nears. As $t = 0$ approaches, the reservation wage falls at a faster rate than before, as workers are more anxious to secure jobs before benefits are lost. In addition, there are more workers near expiration than before. Both of these effects allow firms to offer lower wages than before and yet be more likely to fill the positions. This is particularly visible in the fact that more unemployment spells outlive the temporary benefit after the policy change.

The difference between the two effects comes from the fact that, in the late equilibrium, the reservation wage of those whose benefits have expired plays a pivotal role. In the early equilibrium, though, it is non-binding. Thus, those who are running out of time drag down all wages in the former, but not in the latter.

The cost of providing these benefits, $u(1 - H(0))b$, has competing effects in the first and second terms. However, in late equilibria, benefit costs unambiguously rise. In early equilibria, the net effect could go either direction, depending on parameter values.

Finally, it is worth noting that as b increases, a dispersed equilibrium (early or late) will move closer to becoming a degenerate equilibrium. For large enough b , all early equilibria will cease to exist.

4.2 Policy Experiment: Increase in x

To this point, we have considered x as an exogenous benefit from leisure, but suppose government policy can directly augment x . For instance, one could introduce a supplemental unemployment benefit that lasts the entire duration of the unemployment spell.

As with b , an increase in x causes Y^* to fall, meaning workers will wait longer into the unemployment spell before accepting any offered wages. Also, an increase in x causes Z^* to rise, so unemployed workers accept all wages earlier in their unemployment spell. Table 2 summarizes the comparative statics for the key statistics. Again, early and late equilibria move in opposite directions of each other. In almost all cases, the comparative statics are the same as those for b .

The intuition for these results mirrors that in the previous experiment. In an early equilibrium, the larger permanent benefit encourages workers to be more selective in accepting wages. Firms counter by offering higher wages in a more compressed wage distribution. The net effect is that workers re-enter employment faster, leaving fewer

Table 2: Impact of an increase in permanent benefits level, x

$\partial/\partial x$	early equilibria	late equilibria
w_{\max}	+	-
$w_{\text{mean}}, w_{\text{median}}$	+	-
w_{\min}	+	0
u	-	+
$H(0)$	-	+
π	-	+

people unemployed and fewer workers exhausting their limited benefits.

In a late equilibrium, the larger permanent benefit encourages delay in accepting offers as well. However, as workers near expiration of temporary benefits, they accept jobs more readily. This allows firms to reduce wage offers, again compressing the wage distribution. As a consequence, workers wait longer, but get less due to their hurried acceptance near benefit expiration.

The fact that, in the late equilibrium, the lowest wage w_{\min} does not change is particularly interesting. This wage is only accepted by those whose temporary benefits have expired. Even though their permanent unemployment benefits have increased, their average wage offer is lower as well. These precisely cancel to give them the same reservation wage after the increase in x . Effectively, the firms are expropriating the increase in benefit through the shift in the wage distribution.

The effect on the cost of benefits is ambiguous. In the late dispersed equilibrium, the cost of the limited benefits unambiguously falls, but the increased cost of permanent benefits can offset this. In the early dispersed equilibrium, even the cost of limited benefits is ambiguous.

In either degenerate equilibrium, u and $H(0)$ are unaffected by changes in x . Hence, the cost of providing temporary benefits b is unchanged, though the cost of augmenting x certainly has risen. Another important distinction is that in *both* the early and late degenerate equilibria, the single wage offered *increases*. In other words, the expropriation described above only works because the firms are reducing the average wage offer; once the distribution collapses to degeneracy, further increases in x cannot be expropriated or ignored by the firms.

Concerning transitions between equilibria, note that an increase in x has the same

effect as an increase in b . Dispersed equilibria move closer to becoming degenerate equilibria, and eventually early equilibria cease to exist.

4.3 Policy Experiment: Increase in T

Finally, our model allows us to ask what happens when temporary benefits are offered over a longer time span. As seen in Table 3, the answers can depend on parameter values.

Table 3: Impact of an increase in benefit duration, T

$\partial/\partial T$	early equilibria	late equilibria
w_{\max}	+	+
$w_{\text{mean}}, w_{\text{median}}$	$\delta + \rho < 2\lambda e^{\frac{(Z^*-T)\rho}{2}}$	+
w_{\min}	$2\delta + \rho < 2\lambda e^{\frac{(Z^*-T)\rho}{2}}$	0
u	+	$\delta \left(1 + \frac{\rho}{\lambda} e^{\frac{Y^*\rho}{2}}\right) e^{(Y^*-T)\rho} < \delta + \rho$
$H(0)$	-	-
π	$\delta + \rho > 2\lambda e^{\frac{(Z^*-T)\rho}{2}}$	-

Notes: Where a condition is listed, it means the comparative static is strictly positive if and only if this condition is satisfied.

In late dispersed equilibria, an increase in T results in higher (and more dispersed) wages. Since it is offered in this equilibrium, the reservation wage $R(0)$ plays a pivotal role, just as it did in the preceding experiments. Increases in b or x encouraged delay followed by a hasty exit; increases in T actually stretch out the exit, making workers less desperate and forcing firms to offer higher wages. Note that fewer unemployment spells outlive the benefits, since $H(0)$ falls after the policy change.

To be more particular, $\frac{\partial Y^*}{\partial T} > 0$, so workers begin accepting wages earlier after the change. Of course, this is relative to the date at which benefits expire, which has moved farther away. To have unemployed workers accept offers earlier relative to the beginning of their unemployment spell, we would need $\frac{\partial Y^*}{\partial T} > 1$, which may or may not hold, and is equivalent to:

$$\frac{\rho}{2\lambda} > e^{-\frac{Y^*\rho}{2}} \left(\frac{(1 - e^{(Y^*-T)\rho}) \delta + \rho}{(1 + e^{(Y^*-T)\rho}) \delta + \rho} \right).$$

To the extent that $\frac{\partial Y^*}{\partial T} < 1$, we might view this as one measure of moral hazard, since the longer unemployment benefit induces a longer wait before accepting offers.

Regardless of this moral hazard, the net effect is unambiguous in raising wages, and a larger fraction of the unemployed remain covered. The effect on unemployment rates can go either way, but typically we expect it to increase, since normally δ is significantly smaller than λ (*i.e.* the job offer rate is much larger than the job destruction rate).

The early dispersed equilibria proceed quite similarly. The curious result is that wages could actually fall, which is most likely to happen when Z^* is close to 0 (*i.e.* the support of F nearly spans from $R(0)$ to $R(T)$). However, for larger Z^* as well as the early degenerate equilibrium, wages will rise.

As T increases, Z^* falls if and only if $2\delta + \rho > 2\lambda e^{\frac{(Z^*-T)\rho}{2}}$. Moreover, $\frac{\partial Z^*}{\partial T} < -1$ would indicate that unemployed workers wait longer from the beginning of their unemployment spell to accept all offers. This occurs if and only if:

$$e^{\frac{1}{2}(Z^*+T)\rho}\lambda(2\delta + \lambda + 2\rho) > 2e^{Z^*\rho}\lambda^2 + e^{T\rho}(\delta + \rho)(\lambda + \rho).$$

The late degenerate equilibrium, however, is quite different from the others. An increase in T actually reduces the (only) wage offered. To explain this, note that $V_u(0) = \frac{x}{\rho}$ in equilibrium. In the degenerate equilibrium, firms offer a wage which makes these expired workers indifferent: $V_u(0) = \frac{R(0)+\delta V_u(T)}{\delta+\rho}$. But the longer benefit increases the present value of starting unemployment, $V_u(T)$. Hence the firms expropriate the unemployment benefit by offering a lower wage than before.

Concerning transitions between equilibria, as T increases, a dispersed equilibrium moves away from the degenerate equilibria.

5 Empirical Evidence

Many empirical studies of unemployment insurance have recognized the importance of limited benefits in worker behavior, including Katz and Meyer (1990), van Ours and Vodopivec (2006), Card, Chetty and Weber (2007), and Lalive (2008). In this section, we summarize these results and compare them to the predictions of our model.

To allow some quantitative comparisons, we begin by calibrating the model to

match a few stylized facts about the US labor market. We consider a period to equal one month. Also, marginal product of a worker is normalized to $p = 1$; this choice proportionately scales b and x , but has no impact on equilibrium outcomes other than to proportionately scale reservation wages. Other choices are made as follows:

- $T = 6$, matching the typical US benefit duration.
- $\rho = 0.0041$, matching a 5% annual interest rate.
- $\delta = 0.017$, the average monthly separation rate. With other parameters, implies a 6% unemployment rate.
- $\lambda = 0.27$ ensures that 20% of unemployed workers exhaust benefits so that $H(0) = 0.2$, consistent with Katz and Meyer (1990).
- $b = 0.397$, matching a 40% replacement of the average wage.
- $Y^* = T$, to replicate the fact that the hazard rate is positive and increasing throughout the entire unemployment spell.
- $x = 0.675$, to satisfy the equilibrium condition Equation 11.

These parameters rule out a late degenerate equilibrium; a unique dispersed price equilibrium occurs (exactly on the border between a late and an early equilibrium), and an early degenerate equilibrium is possible as well.

The hazard rate of exiting unemployment has been the focus of much research. Katz and Meyer (1990) were the first to study the impact of limited benefit duration on hazard rates. They found that hazard rates gradually rise as benefit exhaustion approaches, with a significant spike (78% increase) in the last week of benefits. Similar results have been found in a variety of studies, such as Addison and Portugal (2004).

However, the result is significantly weaker in Card, Chetty and Weber (2007). They find that the spike mainly occurs because people stop registering in the unemployment system once they lose benefits, not because they become more likely to find an acceptable job. After distinguishing between these two outcomes, they still find a spike in the hazard rate of exiting unemployment to employment, but it is only a 12 to 14% jump. Using more detailed administrative data, van Ours and Vodopivec (2006) also distinguish between stopping registering and finding a job using Slovenian data; there the spike is nearly as large as in Katz and Meyer (1990).

Katz and Meyer (1990) cites the partial equilibrium model of Mortensen (1977), which takes the wage offer distribution as given, as the theoretical basis for their empirical findings. This model does predict a steady increase in the hazard rate up until benefit exhaustion, but does not explain why there would be a sudden jump at the time of benefit expiration. However, our model can offer an explanation. In the late equilibrium, a positive measure of firms always offer the lowest wage, which is acceptable only to workers whose benefits have expired. Thus, the hazard rate jumps from $\lambda - \frac{\rho}{2}$ to λ ; for our calibration, that produces a 1% jump.

Another key question is what effect policy changes have on reservation wages. This question is particularly important in light of Shimer and Werning (2007); they conclude that a policy change is welfare-improving if and only if a risk-averse worker's after-tax reservation wage increases. This analysis takes place in a partial equilibrium model, including in a case in which benefits have limited duration.

Lalive (2007) studies a change in Austrian unemployment insurance which allowed certain age groups and states to receive significantly longer benefits. While this resulted in longer unemployment spells, they report no significant change in the realized wages on returning to employment. Our model predicts an increase in wages, but the increase is miniscule; for our calibration, a one-month extension only results in a 0.01% increase in the average accepted wage.

Blau and Robins (1986) examines not only offered wages, but also infers the reservation wage of workers. They find that a 10% increase in the UI replacement rate (the ratio of benefit to average wage) causes reservation wages to increase by 2.1% for men and by 1.5% for women. Even so, the more generous benefits do not create a statistically significant change in offered wages, though the point estimates are positive.

Our model predicts that an increase in benefit b will cause individual reservation wages and average accepted wages to fall (and hence the replacement rate to rise). In our calibration, a 10% increase in b will cause the average wage to fall by 0.3%, with similar effect on the maximum, median, and minimum wages. Of course, these are general equilibrium predictions. To the extent that Blau and Robins (1986) rely on individual variation that is unobservable to the firm (as opposed to variation between state programs which creates distinct labor markets), these would not be an adequate test of our model.

The calibration of our model also allows us to numerically evaluate the welfare

effect of the various policy changes. Here, we define welfare as the average expected utility across all workers, plus firm profits, minus the cost of the UI program (so the program is effectively financed by lump-sum taxation). In this environment, a one-month extension of benefits is actually welfare enhancing, causing total welfare to increase by 0.14%. Increasing the size of benefits by 10%, however, causes welfare to fall by 0.14%, while an increase in x by the same amount results in a 0.06% fall in welfare.

One final empirical question was raised by Hornstein, *et al* (2009). They evaluate a number of wage dispersion models for their ability to create the degree of dispersion observed in the data. They propose the ratio of mean-to-minimum prices as a measure of dispersion. In our model, that ratio would be:

$$Mm \equiv 1 + \frac{b(\delta + \rho)}{p\lambda - b(\delta + \rho)} \left(1 - e^{\frac{\tau\rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{\tau\rho}{2}} \right)} \right) = 1.025. \quad (43)$$

This is approximately the same degree of dispersion obtained by other models examined in Hornstein, *et al* (2009), which is well below the 1.68 mean-to-minimum ratio they measure in the data. Allowing a higher ρ or δ , or a lower λ , will increase Mm ; but to reach 1.68 would require implausible values. It should be noted that when the model is solved with mild risk aversion (log preferences), the resulting wage distribution and Mm look nearly identical to a 12-fold increase in ρ . Thus, greater risk aversion seems to be the most promising route for achieving a better fit. Unfortunately, the problem ceases to be analytically tractable for most parameter values of CRRA preferences.

6 Conclusion

Our paper contributes to two strands of literature. First, we show how a finite limit on the duration of unemployment benefits can generate equilibrium wage dispersion among homogeneous workers and firms. This provides a tractable environment where we can isolate the incentive effects of unemployment benefits without resorting to on-the-job search or heterogeneity among agents. We provide sufficient conditions for its existence and uniqueness of dispersed wage equilibria.

Second, our general equilibrium framework, which is calibrated to replicate the

key labor market statistics in the US, enables us to quantitatively evaluate the effects of extended benefits on the unemployment rate, distribution of wages, unemployment duration, and welfare in the economy. We find that a 10% increase in b will cause the average wage to fall by 0.3% and welfare to fall by 0.14%. On the other hand, a one-month extension results in a 0.01% increase in the average accepted wage, 1.4% increase in the unemployment rate, and a 0.14% increase in welfare. Our theory is also successful in generating and explaining the spike in the exit rate from unemployment just before the benefit expiration rate, which is documented by many empirical studies.

Despite its success in generating dispersed wages in equilibrium, our model fails to account for the extent of dispersion quantitatively. Hornstein, *et al* (2009) find that the mean-to-minimum ratio of wages is 1.67 in the data, whereas our model can only generate a ratio of 1.02 with reasonable degrees of risk aversion or discounting.

Our model can easily be extended in several useful ways to capture relevant institutional details of unemployment insurance. First, unemployment benefits are typically financed via employment taxes, and these may introduce additional distortions. Aside from financing, we may also consider budget-neutral policy changes, where increases in the size of benefit are covered by reduction in the duration of benefit eligibility.

Other important (and more difficult) extensions to the model would be to endogenize search effort or the job offer arrival rate, allowing workers or firms to increase the number of draws via costly search. This could alter our comparative statics results. For example, if search effort is endogenous, one might expect that an extension of benefits would reduce search per unit of time, which would increase unemployment duration, increase the rate of unemployment, and reduce post-unemployment wages as workers would be sampling less from the wage distribution. Similarly, if a tax on employers, which increases the cost of providing employment, causes them to extend fewer wage offers than they otherwise would, then the job offer arrival rate would be lower, causing an increase in both the duration and the length of unemployment. We leave these important extensions for future work, as they would make the model analytically intractable.

A Proofs

A.1 Lemma 1

1. *An atom can occur in G if and only if one occurs in F at the same wage.*

If $\mu_G(w) > 0$ while $\mu_F(w) = 0$, a firm offering wage w would garner an infinite number of workers and thus would have profits far above firms offering other wages. If reversed, a firm offering wage w would garner an infinitesimal number of workers and earn zero profits, strictly less than other firms. Either would violate equal steady state profits.

2. *No atoms occur in H except at $H(0)$.*

First, note that a steady flow (but not an atom) enters unemployment and thus state $t = T$ each instant. From that point (as benefit time declines), note that the steady state conditions dictate h' in terms of h . While there can be discontinuous changes in h' (*e.g.* the flow out of unemployment can increase suddenly if there is a time t where people accept a reservation wage $R(t)$ and there is an atom $\mu_F(R(t)) > 0$.), this will not result in discontinuous changes in h . This is purely a consequence of the Poisson process of job offer arrivals, which means that even if many are willing to accept a wage offered by a positive measure of firms, only a steady stream of them will encounter such a wage.

For the same reason, there will be a positive measure of unemployed workers who do not encounter an acceptable wage in T periods. Hence, there will be an atom at $H(0)$.

3. *$R(0)$ is always in the support of F .*

Suppose that the lowest wage offered in the support was $w = R(s)$ for some $s > 0$. Note that the reservation wages $R(t)$ are still well defined for $t < s$, even though they are strictly lower than any offered wage. We pose the question whether a single firm could earn higher expected profits by deviating, offering a wage $R(t)$ just below $R(s)$. Here, we compare expected profits $\Pi(t) \equiv (p - R(t))\lambda uH(t)$, where the latter indicates the expected flow of workers who would accept such an offer.

From Eq. 7 and 8, we have $h'(t) = \lambda h(t)$ and $h(0) = \lambda H(0)$, respectively. The solution to this differential equation on $t \in [0, s)$ is $H(t) = H(0)e^{t\lambda}$.

Similarly, the Bellman equation in Eq. 2 is $\rho V_u(0) = x + \lambda (V_{avg} - V_u(0))$, where V_{avg} is the average utility from accepted wages:

$$V_{avg} = \int_{R(s)}^{\infty} V_e(w) dF(w).$$

Meanwhile, the Bellman equation in Eq. 3 becomes $\rho V_u(t) = b + x - V'_u(t) + \lambda (V_{avg} - V_u(t))$. Again, this differential equation has the solution:

$$V_u(t) = \frac{b(1 - e^{-t(\lambda+\rho)}) + x + \lambda V_{avg}}{\lambda + \rho}.$$

Reservation wages are defined by $V_u(t) = \frac{R(t) + \delta V_u(T)}{\delta + \rho}$, so:

$$R(t) = \left(\frac{\delta + \rho}{\lambda + \rho} \right) (b(1 - e^{-t(\lambda+\rho)}) + x + \lambda V_{avg}) - \delta V_u(T).$$

We can now substitute these into expected profits. Assuming this is an equilibrium outcome, it must be that $\Pi'(s) = 0$; otherwise, profits could be increased by choosing a higher or lower s . This is equivalent to requiring

$$p + \delta V_u(T) = \frac{\delta + \rho}{\lambda + \rho} \left(x + \lambda V_{avg} + b \left(1 + \frac{\rho}{\lambda} e^{-s(\lambda+\rho)} \right) \right).$$

However, if we take the second derivative evaluated at $t = s$, we get:

$$\Pi''(s) = u\lambda^3 e^{s\lambda} \left(p + \delta V_u(T) - \frac{\delta + \rho}{\lambda + \rho} \left(x + \lambda V_{avg} + b \left(1 - \frac{\rho^2}{\lambda^2} e^{-s(\lambda+\rho)} \right) \right) \right) H(0).$$

If we substitute for $p + \delta V_u(T)$ using the $\Pi'(s) = 0$ condition, this simplifies to $\Pi''(s) = u\lambda\rho(\delta + \rho)be^{-s\lambda}H(0) > 0$. Thus, even if $\Pi'(s) = 0$, this is in fact a minimum rather than a maximum, and expected profits can be increased by offering $R(t)$ with $t < s$.

A.2 Lemma 2

Proof. Assume that there is a $Y^* \in [0, T]$ such that $\phi(Y^*) = 0$. The first derivative of ϕ is:

$$\phi'(Y) = \frac{1}{2}e^{-\frac{Y\rho}{2}} \left(\frac{2(p-b-x)\lambda}{b} e^{-\frac{2\lambda}{\rho}(-1+e^{-\frac{Y\rho}{2}})} - \delta - e^{(Y-T)\rho}\delta - \rho \right). \quad (44)$$

If evaluated at Y^* , we may use $\phi(Y^*) = 0$ to substitute for $e^{-\frac{2\lambda}{\rho}(-1+e^{-\frac{Y^*\rho}{2}})}$, obtaining:

$$\phi'(Y^*) = \frac{e^{-Y^*\rho} (\delta (1 - e^{(Y^*-T)\rho}) + \rho) (\rho e^{\frac{Y^*\rho}{2}} - 2\lambda)}{2\rho}. \quad (45)$$

Since we only consider $Y^* \in [0, T]$, $1 - e^{(Y^*-T)\rho} > 0$ and $e^{\frac{Y^*\rho}{2}} < e^{\frac{T\rho}{2}} < \frac{2\lambda}{\rho}$, where the last inequality is assumed throughout the model. Thus, ϕ is declining wherever $\phi = 0$. Since ϕ is continuous, there is at most one Y^* such that $\phi(Y^*) = 0$.

The second and third claims are simple extensions. If $\phi(0) < 0$ and ϕ is continuous, there cannot be a $\hat{Y} \in [0, T]$ such that $\phi(\hat{Y}) \geq 0$ unless there exists a $Y^* \in [0, \hat{Y}]$ such that $\phi(Y^*) = 0$ and $\phi'(Y^*) > 0$, which contradicts. Similarly, if $\phi(T) > 0$, there cannot be a $\hat{Y} \in [0, T]$ such that $\phi(\hat{Y}) \leq 0$ unless there exists a $Y^* \in [\hat{Y}, T]$ such that $\phi(Y^*) = 0$ and $\phi'(Y^*) > 0$, which contradicts.

The fourth claim follows from the continuity of ϕ . □

A.3 Proposition 1

Proof. We begin by verifying that the steady state equations hold. To do so, we need the p.d.f.s of the respective distributions, which are found by taking a first derivative:

$$h(t) = \begin{cases} \frac{\lambda e^{\frac{2\lambda}{\rho} \left(e^{-\frac{Y^* \rho}{2}} - e^{-\frac{t \rho}{2}} \right) + \frac{(Y^* - t) \rho}{2}}}{e^{\frac{Y^* \rho}{2}} + \lambda(T - Y^*)} & t \leq Y^* \\ \frac{\lambda}{e^{\frac{Y^* \rho}{2}} + \lambda(T - Y^*)} & t > Y^* \end{cases} \quad (46)$$

$$g(w) = \begin{cases} \frac{\rho(\delta + \rho)b}{2(p-w)^2 \lambda^2} e^{\frac{Y^* \rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^* \rho}{2}} \right)} & w \in (w_\ell, w_h) \\ 0 & w \in [0, w_\ell) \cup (w_h, \infty) \end{cases} \quad (47)$$

$$f(w) = \begin{cases} \frac{\rho}{2\lambda(p-w)} & w \in (w_\ell, w_h) \\ 0 & w \in [0, w_\ell) \cup (w_h, \infty) \end{cases} \quad (48)$$

where $w_\ell = R(0)$ and $w_h = R(Y^*)$.

Consider Eq. 6, governing those entering unemployment. Substituting the proposed solution, we get:

$$\left(\frac{\lambda(1 + \delta(T - Y^*)) + \delta e^{\frac{Y^* \rho}{2}} - \lambda}{\lambda(1 + \delta(T - Y^*)) + \delta e^{\frac{Y^* \rho}{2}}} \right) \frac{\lambda}{e^{\frac{Y^* \rho}{2}} + \lambda(T - Y^*)} = \frac{\delta \lambda}{\lambda(1 + \delta(T - Y^*)) + \delta e^{\frac{Y^* \rho}{2}}}$$

which holds. Next, consider Eq. 7, governing the unemployed still receiving benefits. Substituting $F(R(t))$ and h into this equation, we get:

$$\lambda \left(e^{-\frac{t \rho}{2}} - \frac{\rho}{2\lambda} \right) \frac{\lambda e^{\frac{2\lambda}{\rho} \left(e^{-\frac{Y^* \rho}{2}} - e^{-\frac{t \rho}{2}} \right) + \frac{(Y^* - t) \rho}{2}}}{e^{\frac{Y^* \rho}{2}} + \lambda(T - Y^*)} = \frac{\lambda \left(2\lambda - \rho e^{\frac{t \rho}{2}} \right) e^{\frac{2\lambda}{\rho} \left(e^{-\frac{Y^* \rho}{2}} - e^{-\frac{t \rho}{2}} \right) + \frac{(Y^* - 2t) \rho}{2}}}{2 \left(e^{\frac{Y^* \rho}{2}} + \lambda(T - Y^*) \right)}.$$

This simplifies to: $2\lambda \left(e^{-\frac{t \rho}{2}} - \frac{\rho}{2\lambda} \right) = \left(2\lambda - \rho e^{\frac{t \rho}{2}} \right) e^{-\frac{t \rho}{2}}$, which holds for all t .

Similarly, Eq. 8, governing the flow of workers whose benefits have expired, clearly holds after substitution:

$$\lambda e^{-\frac{2\lambda}{\rho} \left(1 - e^{-\frac{T \rho}{2}} \right)} = \lambda e^{-\frac{2\lambda}{\rho} \left(1 - e^{-\frac{T \rho}{2}} \right)}.$$

Turning to the transitions for employed individuals, note that:

$$\frac{g(w)}{f(w)} = \frac{(\delta + \rho)b}{(p - w)\lambda} e^{\frac{Y^*\rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^*\rho}{2}}\right)} \text{ for } w \in [w_\ell, w_h]. \quad (49)$$

Similarly,

$$\frac{\mu_G(w_\ell)}{\mu_F(w_\ell)} = e^{\frac{Y^*\rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^*\rho}{2}}\right)} \text{ and } \frac{\mu_G(w_h)}{\mu_F(w_h)} = e^{\frac{Y^*\rho}{2}}.$$

We also need to compute $H(R^{-1}(w))$, which after algebraic manipulation, becomes:

$$H(R^{-1}(w)) = \frac{b(\delta + \rho)}{(p - w)\lambda \left(e^{\frac{Y^*\rho}{2}} + (T - Y^*)\lambda\right)} e^{\frac{Y^*\rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^*\rho}{2}}\right)}.$$

We then verify Eq. 9 which governs transition from unemployment to employment.

With substitution, we get:

$$\delta \frac{(\delta + \rho)b}{(p - w)\lambda} e^{\frac{Y^*\rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^*\rho}{2}}\right)} = \frac{\delta \left(e^{\frac{Y^*\rho}{2}} + (T - Y^*)\lambda\right) b(\delta + \rho)}{(p - w)\lambda \left(e^{\frac{Y^*\rho}{2}} + (T - Y^*)\lambda\right)} e^{\frac{Y^*\rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^*\rho}{2}}\right)},$$

which also holds for all w .

We check the same equation for both atoms in the distribution. At w_h , this is:

$$\delta e^{\frac{Y^*\rho}{2}} = \lambda \frac{\delta \left(e^{\frac{Y^*\rho}{2}} + (T - Y^*)\lambda\right)}{\lambda} \frac{e^{\frac{Y^*\rho}{2}}}{e^{\frac{Y^*\rho}{2}} + (T - Y^*)\lambda},$$

and at w_ℓ ,

$$\delta e^{\frac{Y^*\rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^*\rho}{2}}\right)} = \lambda \frac{\delta \left(e^{\frac{Y^*\rho}{2}} + (T - Y^*)\lambda\right)}{\lambda} \frac{e^{\frac{Y^*\rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^*\rho}{2}}\right)}}{e^{\frac{Y^*\rho}{2}} + (T - Y^*)\lambda}.$$

Since these both hold, the proposed system indeed maintains all steady state conditions.

Next, consider the equal profit condition. Since steady state profits are $(p - w) \frac{g(w)}{f(w)}$, substitution of Eq. 49 will give the proposed equilibrium profit from Eq. 19, which

does not depend on w . Indeed, those offering wage w_h or w_ℓ earn the same profit:

$$\frac{b(\delta + \rho)}{\lambda} e^{-\frac{2\lambda}{\rho} \left(1 - e^{-\frac{Y^* \rho}{2}}\right)} e^{\frac{Y^* \rho}{2}} = \pi.$$

Moreover, all wages outside the support will result in no greater profit. Consider if one of the infinitesimal firms deviated, offering a wage outside the support but not changing the distribution of wage offer. Offering a wage below w_ℓ would result in zero profit, since all unemployed workers have a reservation wage at or above w_ℓ , given F .

On the other hand, if the firm offers a wage $w = R(t) > R(Y^*) = w_h$, they would be able to successfully attract additional workers (whose benefits are between Y^* and t periods until expiration), but this also reduces realized profit on those who do accept. We will show that the net of these two effects will result in lower profit.

Recall that unemployed workers with $t > Y^*$ periods of benefits remaining require $R(t) = p - \frac{(p-b-x)(\delta+\rho)}{\delta\lambda e^{(Y^*-T)\rho} + (\delta+\rho) \left(\rho e^{\frac{Y^* \rho}{2}} - \lambda\right)} \left(e^{\frac{Y^* \rho}{2}} - \lambda (1 - e^{(Y^*-t)\rho})\right)$ as a reservation wage. The fraction of unemployed workers willing to accept such an offer is $H(t) = \frac{1+\lambda(t-Y^*)e^{-\frac{Y^* \rho}{2}}}{1+\lambda(T-Y^*)e^{-\frac{Y^* \rho}{2}}}$.

Thus, the expected profit from offering a wage targeted for t is $(p - R(t))\lambda uH(t)$, which becomes:

$$\Pi(t) \equiv \frac{\delta\lambda(p-b-x) \left(e^{\frac{Y^* \rho}{2}} + \lambda(t-Y^*)\right) (\delta+\rho) \left(e^{\frac{Y^* \rho}{2}} \rho - (1 - e^{(Y^*-t)\rho}) \lambda\right)}{\left(\lambda + \delta \left(e^{\frac{Y^* \rho}{2}} + (T-Y^*)\lambda\right)\right) \left(e^{(Y^*-T)\rho} \delta\lambda - (\delta+\rho) \left(\lambda - e^{\frac{Y^* \rho}{2}} \rho\right)\right)}.$$

At $t = Y^*$, $\Pi(Y^*)$ provides the same expected profit that is experienced by offering a wage in the support of F . (Note that expected profit of offering a wage is not the same average profit, but both are constant across the support in a steady state equilibrium).

If $\phi(Y^*) = 0$ (*i.e.* this is a dispersed equilibrium), then the first derivative of Π evaluated at $t = Y^*$ is:

$$\Pi'(Y^*) = \frac{\delta\lambda^2(\delta+\rho)(p-b-x) \left(\lambda - \rho e^{\frac{Y^* \rho}{2}} + \rho e^{\frac{Y^* \rho}{2}} - \lambda\right)}{\left(\lambda + \delta \left(e^{\frac{Y^* \rho}{2}} + (T-Y^*)\lambda\right)\right) \left(e^{(Y^*-T)\rho} \delta\lambda - (\delta+\rho) \left(\lambda - e^{\frac{Y^* \rho}{2}} \rho\right)\right)} = 0.$$

The second derivative of Π evaluated at $T = Y^*$ yields:

$$\Pi''(Y^*) = \frac{-\delta\rho\lambda^2(\delta + \rho)(p - b - x) \left(2\lambda - \rho e^{\frac{Y^*\rho}{2}}\right)}{\left(\lambda + \delta \left(e^{\frac{Y^*\rho}{2}} + (T - Y^*)\lambda\right)\right) \left(e^{(Y^*-T)\rho}\delta\lambda - (\delta + \rho) \left(\lambda - e^{\frac{Y^*\rho}{2}}\rho\right)\right)} < 0.$$

Thus, as a firm attempts to target t near but greater than Y^* , his expected profits will fall.

On the other hand, if $\phi(Y^*) < 0$ (so $Y^* = 0$), then

$$\Pi'(0) = \frac{\delta\lambda}{\delta + \lambda(1 + T\delta)} \left(p - x - b \left(\delta + \rho - \frac{\delta\lambda}{\rho} (1 - e^{-T\rho}) \right) \right).$$

In fact, comparison to Eq. 12 reveals that $\Pi'(0) < 0$ if and only if $\phi(Y^*) < 0$.

Even so, as t increases, $\Pi'(t)$ increases and could eventually change sign. Indeed, $\Pi'(t)$ could also become positive. If it does, we only need to verify that $\Pi(T) < \Pi(Y^*)$, since all values in between would then be strictly lower than $\Pi(Y^*)$. This comparison simplifies to the assumption for Proposition 1.

$$(1 - e^{(Y^*-T)\rho}) \left(1 + (T - Y^*)\lambda e^{-\frac{Y^*\rho}{2}}\right) > \rho(T - Y^*) \quad (50)$$

Finally, we construct the value functions for unemployed workers under this equilibrium, and show that they indeed solve the Bellman equations.

$$V_u(T) = \frac{b + x}{\rho} - \frac{\lambda(p - b - x)e^{(Y^*-T)\rho}}{(\delta + \rho) \left(-\lambda + \rho e^{\frac{Y^*\rho}{2}}\right) + \delta\lambda e^{(Y^*-T)\rho}} \quad (51)$$

$$V_u(t) = \frac{R(t) + \delta V_u(T)}{\delta + \rho} \text{ for } t \in [0, T] \quad (52)$$

First, recall that from Eq. 1, $V_e(w) = \frac{w + \delta V_u(T)}{\delta + \rho}$ and that in equilibrium, we need $V_e(R(t)) = V_u(t)$. This is the construction of both these equations. We must now show that these also agree with the literal definition of the Bellman equations from Eq. 3. As in the proof of Proposition 1, we do this by showing that the solution satisfies a system of differential equations equivalent to the Bellman equations.

First, consider $V_u(t)$ for $t \in (Y^*, T]$. All wages in the support of F are below the

reservation wage $R(t)$, so the Bellman equation simply becomes:

$$\rho V_u(t) = b + x - V'_u(t) \implies \frac{R(t) + \delta V_u(T)}{\delta + \rho} = \frac{b + x}{\rho} - \frac{R'(t)}{\rho(\delta + \rho)}.$$

Note that, in this range of t , $R'(t) = \frac{e^{(Y^*-t)\rho}(p-b-x)\lambda\rho(\delta+\rho)}{\delta\lambda e^{(Y^*-T)\rho} + (\delta+\rho)\left(\rho e^{\frac{Y^*\rho}{2}} - \lambda\right)}$. Thus, the Bellman equation requires:

$$\begin{aligned} \frac{p}{\delta + \rho} - \frac{(p - b - x) \left(e^{\frac{Y^*\rho}{2}} - \lambda (1 - e^{(Y^*-t)\rho}) \right)}{\delta\lambda e^{(Y^*-T)\rho} + (\delta + \rho) \left(\rho e^{\frac{Y^*\rho}{2}} - \lambda \right)} + \frac{\delta V_u(T)}{\delta + \rho} \\ = \frac{b + x}{\rho} - \frac{e^{(Y^*-t)\rho}(p - b - x)\lambda}{\delta\lambda e^{(Y^*-T)\rho} + (\delta + \rho) \left(\rho e^{\frac{Y^*\rho}{2}} - \lambda \right)} \\ \implies \delta V_u(T) = \frac{\delta(b + x)}{\rho} - p + b + x + \frac{(\delta + \rho)(p - b - x) \left(e^{\frac{Y^*\rho}{2}} - \lambda \right)}{\delta\lambda e^{(Y^*-T)\rho} + (\delta + \rho) \left(\rho e^{\frac{Y^*\rho}{2}} - \lambda \right)} \\ \implies V_u(T) = \frac{b + x}{\rho} - \frac{(p - b - x)\lambda e^{(Y^*-T)\rho}}{\delta\lambda e^{(Y^*-T)\rho} + (\delta + \rho) \left(\rho e^{\frac{Y^*\rho}{2}} - \lambda \right)}. \end{aligned}$$

This agrees with our definition of $V_u(T)$, and hence the Bellman equation holds for all $t \in (Y^*, T]$.

Next, consider the Bellman equation at $t = Y^*$. Because there is an atom in F at w_h , this becomes:

$$\rho V_u(Y^*) = b + x - V'_u(Y^*) + \lambda (V_e(w_h) - V_u(Y^*)) \mu_F(w_h).$$

However, $w_h = R(Y^*)$ and $V_e(R(Y^*)) = V_u(Y^*)$, so the last term cancels. The resulting equation is identical to the one which held for $t \in (Y^*, T]$, and will still hold as $t \rightarrow Y^*$.

The only lingering concern is that for V_u to be continuous at Y^* , $V'_u(t)$ must be the same whether t approaches Y^* from below or from above. Since $V_u(t) = \frac{R(t) + \delta V_u(T)}{\delta + \rho}$ from the definition of the reservation wage, this is equivalent to stating that $R'(t)$ is

continuous at Y^* . The derivative for $t \geq Y^*$ is stated above; for $t \leq Y^*$ it is:

$$R'(t) = (\delta + \rho)be^{-\frac{t\rho}{2} - \frac{2\lambda}{\rho}\left(1 - e^{-\frac{t\rho}{2}}\right)}.$$

When these two derivatives are evaluated at Y^* and set equal, we obtain the equilibrium condition for Y^* , Equation 11. Thus, in a late dispersed equilibrium, this is automatically satisfied. If instead this is a late degenerate equilibrium, this is trivially satisfied, since $R'(t)$ is only defined for $t \geq Y^* = 0$.

We then move to the Bellman equation for $t \in (0, Y^*)$, which is:

$$\rho V_u(t) = b + x - V'_u(t) + \lambda \left(V_e(w_h)\mu_F(w_h) + \int_{R(t)}^{w_h} V_e(w)f(w)dw - (1 - F(R(t)))V_u(t) \right).$$

If we substitute for $V_e(w)$ with $\frac{w + \delta V_u(t)}{\delta + \rho}$ and then take the derivative with respect to t , we obtain the differential equation:

$$(\rho + \lambda(1 - F(R(t))))V'_u(t) = -V''_u(t) \implies \left(\frac{\rho}{2} + \lambda e^{-\frac{t\rho}{2}} \right) \frac{R'(t)}{\delta + \rho} = -\frac{R''(t)}{\delta + \rho}.$$

This equation holds for all $t \in [0, Y^*]$, since

$$R''(t) = -(\delta + \rho)be^{-\frac{t\rho}{2} - \frac{2\lambda}{\rho}\left(1 - e^{-\frac{t\rho}{2}}\right)} \left(\frac{\rho}{2} + \lambda e^{-\frac{t\rho}{2}} \right).$$

Thus, because this differential equation is satisfied, $V_u(t)$ has the right rate of change on $t \in (0, Y^*]$. Moreover, since V_u agrees with the Bellman equation definition at initial condition Y^* , it will also do so for $t \in (0, Y^*]$.

Finally, we consider the Bellman equation at $t = 0$. Because benefits are lost and all wages are accepted, here the definition changes to:

$$\rho V_u(0) = x + \lambda \left(V_e(w_h)\mu_F(w_h) + \int_{w_\ell}^{w_h} V_e(w)f(w)dw + V_e(w_\ell)\mu_F(w_\ell) - V_u(0) \right).$$

We must ensure that V_u is continuous at 0; as $t \rightarrow 0$ the definition is:

$$\begin{aligned} \rho V_u(0) &= b + x - V'_u(0) \\ &\quad + \lambda \left(V_e(w_h)\mu_F(w_h) + \int_{R(t)}^{w_h} V_e(w)f(w)dw - (1 - \mu_F(w_\ell))V_u(0) \right). \end{aligned}$$

However, bearing in mind that $V_e(w_\ell) = V_u(0)$, the first definition simplifies to:

$$\rho V_u(0) = x + \lambda \left(V_e(w_h) \mu_F(w_h) + \int_{w_\ell}^{w_h} V_e(w) f(w) dw - (1 - \mu_F(w_\ell)) V_u(0) \right).$$

Thus, V_u is continuous at $t = 0$ if and only if $V'_u(0) = b$. But note that $R'(0) = b(\delta + \rho)$, so this condition is satisfied. \square

A.4 Lemma 3

Proof. Assume that there is a $Z^* \in [0, T]$ such that $\psi(Z^*) = 0$. The first derivative of ψ evaluated at Z^* is:

$$\psi'(Z^*) = \frac{b(\delta + \rho)}{e^{(\lambda + \rho)Z^*}} \left(\rho + \lambda \left(1 - e^{\frac{(Z^* - T)\rho}{2}} \right) + \frac{\lambda \rho}{2 \left((\delta + \rho) e^{\frac{(T - Z^*)\rho}{2}} - \lambda \right)} \right). \quad (53)$$

Since $Z^* \in [0, T]$, $1 - e^{\frac{(Z^* - T)\rho}{2}} > 0$. Moreover, we have assumed that $\delta + \rho > \lambda$, so $(\delta + \rho) e^{\frac{(T - Z^*)\rho}{2}} > \lambda$. Thus, $\psi'(Z^*) > 0$; or in other words, ψ is increasing wherever it crosses zero. Since ψ is continuous, there is at most one Z^* such that $\psi(Z^*) = 0$.

The second and third claims are simple extensions. If $\psi(0) \geq 0$ and ψ is continuous, there cannot be a $\hat{Z} \in [0, T]$ such that $\psi(\hat{Z}) \leq 0$ unless there exists a $Z^* \in [0, \hat{Z}]$ such that $\psi(Z^*) = 0$. But $\psi'(Z^*) > 0$, which contradicts. Similarly, if $\psi(T) < 0$, there cannot be a $\hat{Z} \in [0, T]$ such that $\psi(\hat{Z}) \geq 0$ unless there exists a $Z^* \in [\hat{Z}, T]$ such that $\psi(Z^*) = 0$. But $\psi'(Z^*) > 0$, which contradicts.

For the fourth claim, the *if* follows from the continuity of ϕ , while the *only if* is an immediate consequence of the first through third claims. \square

A.5 Proposition 2

Proof. We begin by verifying that the steady state equations hold. To do so, we need the p.d.f.s of the respective distributions, which are found by taking a first derivative:

$$h(t) = \begin{cases} \lambda e^{(t-Z^*)\lambda - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^*-T)\rho}{2}}\right)} & t \leq Z^* \\ \lambda e^{\frac{(Z^*-t)\rho}{2} - \frac{2\lambda}{\rho} \left(e^{\frac{(Z^*-t)\rho}{2}} - e^{\frac{(Z^*-T)\rho}{2}}\right)} & t > Z^* \end{cases} \quad (54)$$

$$g(w) = \begin{cases} \frac{\rho(\delta+\rho)b}{2(p-w)^2\lambda^2} e^{\frac{(T-Z^*)\rho}{2} - (\lambda+\rho)Z^* - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^*-T)\rho}{2}}\right)} & w \in (w_\ell, w_h) \\ 0 & w \in [0, w_\ell) \cup (w_h, \infty) \end{cases} \quad (55)$$

$$f(w) = \begin{cases} \frac{\rho}{2\lambda(p-w)} & w \in (w_\ell, w_h) \\ 0 & w \in [0, w_\ell) \cup (w_h, \infty) \end{cases} \quad (56)$$

where $w_\ell = R(Z^*)$ and $w_h = R(T)$.

Consider Eq. 6, governing those entering unemployment. Substituting the proposed solution, we get:

$$\frac{\delta}{\delta + \lambda e^{\frac{(Z^*-T)\rho}{2}}} \lambda e^{\frac{(Z^*-T)\rho}{2}} = \delta \left(1 - \frac{\delta}{\delta + \lambda e^{\frac{(Z^*-T)\rho}{2}}}\right)$$

which holds. Next, consider Eq. 7, governing the unemployed still receiving benefits. When $t < Z^*$, $F(R(t)) = 0$ and this equation becomes:

$$\lambda^2 e^{(t-Z^*)\lambda - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^*-T)\rho}{2}}\right)} = \lambda \cdot \lambda e^{(t-Z^*)\lambda - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^*-T)\rho}{2}}\right)}$$

which obviously holds. When $t \geq Z^*$, we substitute for $F(R(t))$ and h into this equation to get:

$$\begin{aligned} & \left(\lambda e^{\frac{(Z^*-t)\rho}{2}} - \frac{\rho}{2} \right) \lambda e^{\frac{(Z^*-t)\rho}{2} - \frac{2\lambda}{\rho} \left(e^{\frac{(Z^*-t)\rho}{2}} - e^{\frac{(Z^*-T)\rho}{2}} \right)} = \\ & \lambda \left(e^{\frac{(Z^*-t)\rho}{2}} - \frac{\rho}{2\lambda} \right) \lambda e^{\frac{(Z^*-t)\rho}{2} - \frac{2\lambda}{\rho} \left(e^{\frac{(Z^*-t)\rho}{2}} - e^{\frac{(Z^*-T)\rho}{2}} \right)}, \end{aligned}$$

which holds for all t .

Also, Eq. 8, governing the flow of workers whose benefits have expired, clearly

holds after substitution:

$$\lambda e^{-Z^* \lambda - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^* - T)\rho}{2}}\right)} = \lambda e^{-Z^* \lambda - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^* - T)\rho}{2}}\right)}.$$

Turning to the transitions for employed individuals, note that:

$$\frac{g(w)}{f(w)} = \frac{(\delta + \rho)b}{(p - w)\lambda} e^{\frac{(T - Z^*)\rho}{2} - (\lambda + \rho)Z^* - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^* - T)\rho}{2}}\right)} \text{ for } w \in [w_\ell, w_h]. \quad (57)$$

Similarly,

$$\frac{\mu_G(w_\ell)}{\mu_F(w_\ell)} = e^{\frac{(T - Z^*)\rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{(Z^* - T)\rho}{2}}\right)} \text{ and } \frac{\mu_G(w_h)}{\mu_F(w_h)} = e^{\frac{(T - Z^*)\rho}{2}}.$$

We also need to compute $H(R^{-1}(w))$, which after algebraic manipulation, becomes:

$$H(R^{-1}(w)) = \frac{b(\delta + \rho)}{(p - w)\lambda} e^{-Z^*(\lambda + \rho) - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(T - Z^*)\rho}{2}}\right)}.$$

We then verify Eq. 9 which governs transition from unemployment to employment. With substitution, we get:

$$\begin{aligned} \delta \left(1 - \frac{\delta}{\delta + \lambda e^{\frac{(Z^* - T)\rho}{2}}}\right) \frac{(\delta + \rho)b}{(p - w)\lambda} e^{\frac{(T - Z^*)\rho}{2} - (\lambda + \rho)Z^* - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^* - T)\rho}{2}}\right)} = \\ \lambda \left(\frac{\delta}{\delta + \lambda e^{\frac{(Z^* - T)\rho}{2}}}\right) \frac{b(\delta + \rho)}{(p - w)\lambda} e^{-Z^*(\lambda + \rho) - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(T - Z^*)\rho}{2}}\right)} \end{aligned}$$

which also holds for all w .

We check the same equation for both atoms in the distribution. At w_h , this is:

$$\delta \left(1 - \frac{\delta}{\delta + \lambda e^{\frac{(Z^* - T)\rho}{2}}}\right) e^{\frac{(T - Z^*)\rho}{2}} = \lambda \left(\frac{\delta}{\delta + \lambda e^{\frac{(Z^* - T)\rho}{2}}}\right),$$

and at w_ℓ ,

$$\delta \left(1 - \frac{\delta}{\delta + \lambda e^{\frac{(Z^* - T)\rho}{2}}}\right) e^{\frac{(T - Z^*)\rho}{2} - \frac{2\lambda}{\rho} \left(1 - e^{-\frac{(Z^* - T)\rho}{2}}\right)} = \lambda \left(\frac{\delta}{\delta + \lambda e^{\frac{(Z^* - T)\rho}{2}}}\right) e^{-\frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^* - T)\rho}{2}}\right)}.$$

Since these both hold, the proposed system indeed maintains all steady state condi-

tions.

Next, consider the equal profit condition. Since steady state profits are $(p-w)\frac{g(w)}{f(w)}$, substitution of Eq. 57 will give the proposed equilibrium profit from Eq. 35, which does not depend on w . Indeed, those offering wage w_h or w_ℓ earn the same profit.

Moreover, all wages outside the support will result in no greater profit. Consider if one of the infinitesimal firms deviated, offering a wage outside the support but not changing the distribution of wage offers. Offering a wage above w_h always strictly decreases profit; every unemployed worker is already willing to accept $w_h = R(T)$, so the higher offer results in the same probability of acceptance but less profit on acceptance.

On the other hand, if the firm offers a wage $w = R(t) < R(Z^*) = w_\ell$, they would receive fewer acceptances (from those whose benefits are between Z^* and t periods until expiration), but increase realized profit on those who do accept. We will show that the net of these two effects will result in lower profit.

The fraction of unemployed workers willing to accept an offer of $R(t)$ where $t < Z^*$ is $H(t)$. Thus, the expected profit from offering a wage targeted for t is $(p - R(t))\lambda uH(t)$, which becomes:

$$\Pi(t) \equiv \frac{(\delta + \rho)b}{\lambda(\lambda + \rho)} \frac{\lambda\delta (\lambda e^{-t(\lambda+\rho)} + \rho e^{-Z^*(\lambda+\rho)})}{\delta + \lambda e^{\frac{(Z^*-T)\rho}{2}}} e^{(t-Z^*)\lambda - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^*-T)\rho}{2}}\right)}.$$

At $t = Z^*$, $\Pi(Z^*)$ provides the same expected profit that is experienced by offering a wage in the support of F .

If $\psi(Z^*) = 0$ (*i.e.* this is a dispersed equilibrium), then the first derivative of Π is:

$$\Pi'(t) = -\frac{\delta\lambda\rho(\delta + \rho)b (e^{Z^*(\lambda+\rho)} - e^{t(\lambda+\rho)})}{(\lambda + \rho) \left(\delta + \lambda e^{\frac{(Z^*-T)\rho}{2}}\right)} e^{-(t+Z^*)\rho - 2\lambda Z^* - \frac{2\lambda}{\rho} \left(1 - e^{\frac{(Z^*-T)\rho}{2}}\right)}.$$

When evaluated at $t = Z^*$, $\Pi'(Z^*) = 0$. For $t < Z^*$, $\Pi'(t) < 0$. Thus, as a firm attempts to target t lower than Z^* , his expected profits will fall.

On the other hand, if $\psi(Z^*) > 0$ (so $Z^* = T$), then

$$\Pi'(t) = -\frac{\delta\lambda\rho(\delta + \rho)b (e^{T(\lambda+\rho)} - e^{t(\lambda+\rho)})}{(\lambda + \rho)(\delta + \lambda)} e^{-2T\lambda - T\rho - t\rho}.$$

Hence $\Pi'(t) < 0$ for all $t < T$.

Finally, we construct the value functions for unemployed workers under this equilibrium, and show that they indeed solve the Bellman equations.

$$V_u(T) = \frac{R(T)}{\rho} \quad (58)$$

$$V_u(t) = \frac{R(t) + \delta V_u(T)}{\delta + \rho} \text{ for } t \in [0, T]. \quad (59)$$

First, recall that from Eq. 1, $V_e(w) = \frac{w + \delta V_u(T)}{\delta + \rho}$ and that in equilibrium, we need $V_e(R(t)) = V_u(t)$. This is the construction of both these equations. We must now show that these also agree with the literal definition of the Bellman equations from Eq. 3.

One could do this by substituting for $R(t)$ and $F(w)$ and evaluating the integrals, though it results in large amounts of algebraic manipulation. A more elegant approach is to demonstrate that we satisfy a system of differential equations equivalent to the Bellman equations.

First, consider the Bellman Equation evaluated at T . Because $V_e(w_h) = V_u(T)$, this reduces to:

$$\rho V_u(T) = b + x - V'_u(T) \implies R(T) = b + x - \frac{R'(T)}{\delta + \rho}.$$

In an early dispersed equilibrium, substitution for $R(T)$ and $R'(T)$ shows that this is identical to our equilibrium condition on Z^* , Eq. 28. In an early degenerate equilibrium, however, one should substitute for $R(T)$ and $R'(T)$ using the definitions when $t \leq Z^*$. This yields:

$$b + x - \frac{b((\lambda - \delta)e^{-T(\lambda + \rho)} + (\delta + \rho)e^{-T(\lambda + \rho)})}{\lambda + \rho} = b + x - \frac{b(\delta + \rho)e^{-T(\lambda + \rho)}}{\delta + \rho},$$

which always holds.

Next, consider the Bellman equation $V_u(t)$ for $t \in (Z^*, T]$, which is:

$$\rho V_u(t) = b + x - V'_u(t) + \lambda \left(V_e(w_h) \mu_F(w_h) + \int_{R(t)}^{w_h} V_e(w) f(w) dw - (1 - F(R(t))) V_u(t) \right).$$

If we substitute for $V_e(w)$ with $\frac{w + \delta V_u(T)}{\delta + \rho}$ and then take the derivative with respect to

t , we obtain the differential equation:

$$(\rho + \lambda(1 - F(R(t)))) V'_u(t) = -V''_u(t) \implies \left(\frac{\rho}{2} + \lambda e^{\frac{(Z^* - t)\rho}{2}} \right) \frac{R'(t)}{\delta + \rho} = -\frac{R''(t)}{\delta + \rho}.$$

This equation holds for all $t \in (Z^*, T)$, since

$$R''(t) = -\left(\frac{\rho}{2} + \lambda e^{\frac{(Z^* - t)\rho}{2}} \right) R'(t).$$

Thus, the differential equation satisfied for interior $t \in (Z^*, T)$. Since the initial condition at $t = T$ is also satisfied, $V_u(t)$ will agree with the direct derivation of the Bellman equation throughout this range.

Next, we must verify that the Bellman equation is continuous at $t = Z^*$. Since $V_u(Z^*) = V_e(w_\ell)$, the definition of $V_u(t)$ converges to the same thing whether approached from above or below, so this naturally holds.

Next, consider $V_u(t)$ for $t \in (0, Z^*]$:

$$\rho V_u(t) = b + x - V'_u(t) + \lambda \left(V_e(w_h) \mu_F(w_h) + \int_{w_\ell}^{w_h} V_e(w) f(w) dw + V_e(w_\ell) \mu_F(w_\ell) - V_u(t) \right)$$

The analogous differential equation would be:

$$(\rho + \lambda) V'_u(t) = -V''_u(t) \implies (\rho + \lambda) \frac{R'(t)}{\delta + \rho} = -\frac{R''(t)}{\delta + \rho}.$$

This equation holds for all $t \in [0, Z^*]$, since $R''(t) = -(\rho + \lambda) R'(t)$ in this range. Thus, the differential equation satisfied for $t \in (0, Z^*)$.

Finally, we must inspect the Bellman equation at the final condition of $t = 0$. On reaching that state, benefits are lost, so the definition changes to:

$$\rho V_u(0) = x + \lambda \left(V_e(w_h) \mu_F(w_h) + \int_{w_\ell}^{w_h} V_e(w) f(w) dw + V_e(w_\ell) \mu_F(w_\ell) - V_u(0) \right).$$

We must ensure that V_u is continuous at 0; as $t \rightarrow 0$ the definition is:

$$\begin{aligned} \rho V_u(0) &= b + x - V'_u(0) \\ &\quad + \lambda \left(V_e(w_h) \mu_F(w_h) + \int_{R(t)}^{w_h} V_e(w) f(w) dw + V_e(w_\ell) \mu_F(w_\ell) - V_u(0) \right). \end{aligned}$$

Thus, V_u is continuous at $t = 0$ if and only if $V'_u(0) = b$. But note that $R'(0) = b(\delta + \rho)$, so this condition is satisfied. \square

A.6 Lemma 4

Proof. If $\phi(T) > 0$, then Lemma 2 establishes that $\phi(Y) > 0$ for all $Y \in [0, T]$. Thus, no late degenerate or dispersed equilibrium can occur. Moreover, since $\phi(T) > 0$, this is equivalent to $\psi(0) > 0$, and Lemma 3 establishes that $\psi(Z) > 0$ for all $Z \in [0, T]$. Thus there is no early dispersed equilibrium, but there is an early degenerate equilibrium.

If $\psi(T) < 0$, then there is no early degenerate equilibrium, and by Lemma 3, $\psi(Z) < 0$ for all $Z \in [0, T]$, so there is no early dispersed equilibrium. Also $\psi(0) < 0$ is equivalent to $\phi(T) < 0$. This allows for either a late dispersed or late degenerate equilibrium (but not both), depending on whether $\phi(0) > 0$ or not, respectively.

If $\psi(T) \geq 0$, an early degenerate equilibrium exists. If $\phi(T) \leq 0$, a late equilibrium exists (either degenerate or dispersed, but not both). Moreover, $\phi(0) \leq 0$. The continuity of ψ ensures that there exists a $Z^* \in (0, T]$ such that $\psi(Z^*) = 0$. If $Z^* = T$, then there is only a degenerate equilibrium; otherwise there is a distinct early dispersed equilibrium as well as an early degenerate equilibrium. \square

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