ECO 301
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HW I Solutions

1. (i) $f(x) = -x^4 + 108x$

$$f'(x) = -4x^3 + 108$$

(ii) $f(x) = \frac{2x^2 + 5}{x - 6}$

$$f'(x) = \frac{4x(x-6) - (2x^2 + 5)1}{(x-6)^2}$$

$$= \frac{4x^2 - 24x - 2x^2 - 5}{(x-6)^2}$$

$$= \frac{2x^2 - 24x - 5}{(x-6)^2}$$

(iii) $f(x) = \left(3x^3 - 2x\right)\left(\frac{4}{x+3}\right)$

$$f'(x) = \left(9x^2 - 2\right)\left(\frac{4}{x+3}\right) + (3x^3 - 2x)\left(\frac{0(x+3) - 4.1}{(x+3)^2}\right) = \frac{4\left(9x^2 - 2\right)}{x+3} + \frac{-4(3x^3 - 2x)}{(x+3)^2}$$

$$= \frac{\left(36x^2 - 8\right)}{x+3} + \frac{(-12x^3 + 8x)}{(x+3)^2}$$

(iv) $f(x) = \ln\left(\frac{2x+5}{x^2}\right)$

$$f'(x) = \frac{\frac{2x^2 - (2x+5)2x}{x^4}}{\frac{2x+5}{x^2}} = \frac{\frac{2x^2 - (4x^2 + 10x)}{x^4}}{\frac{2x+5}{x^2}} = \frac{\frac{-2x^2 - 10x}{x^4}}{\frac{2x+5}{x^2}} = \frac{-2x^2 - 10x}{x^4}\frac{x^2}{2x+5} =$$

$$= \frac{-2x^2 - 10x}{x^2(2x+5)} = \frac{-2x(x+5)}{x^2(2x+5)}$$

$$= \frac{-2(x+5)}{x(2x+5)}$$

v) $f(x) = (4x^4 + 3x^2 - x + 2)^3$

$$f'(x) = 3(4x^4 + 3x^2 - x + 2)^2(16x^3 + 6x - 1)$$

(vi) $f(x) = -(2x-1)^2$

$f'(x) = -2(2x-1).2 = -4(2x-1)$

(vii)

$f(x) = \ln(x) + \ln(5-2x)$

$f'(x) = 1/x + (-2)/(5-2x)$

(viii) $f(x) = x^{\frac{1}{2}}(3x - x^2) \Rightarrow f'(x) = 1/2\, x^{-\frac{1}{2}}(3x - x^2) + x^{\frac{1}{2}}(3-2x)$

2. Solve the following equations in one unknown *x*:

(i) $6x - 5 = x^2$

$0 = x^2 - 6x + 5 = (x-5)(x-1) \Rightarrow x = 5$ or x=1

(ii) $\frac{x-2}{4} = \frac{2x}{5}$

$5(x-2) = 4.2x \Rightarrow 5x - 10 = 8x \Rightarrow 10 = -3x \Rightarrow x = -10/3$

(iii) $2x^3 + 8x^2 - x - 4 = 0$

$2x^2(x+4) - (x+4) = 0 \Rightarrow (2x^2-1)(x+4) = 0 \Rightarrow 2x^2 - 1 = 0$ or $x + 4 = 0$

$\Rightarrow 2x^2 = 1$ or x=-4

$\Rightarrow x^2 = 1/2$ or x=-4

$\Rightarrow$ x=-$(1/2)^{1/2}$ or x=$(1/2)^{1/2}$ or x=-4

(iv) $\frac{x(x-1)}{2x^2 - 3x + 1} = \frac{1}{4}$

$4x(x-1) = 2x^2 - 3x + 1 \Rightarrow 4x^2 - 4x = 2x^2 - 3x + 1 \Rightarrow 4x^2 - 4x - (2x^2 - 3x + 1) = 0$

$\Rightarrow 2x^2 - x - 1 = 0$

$\Rightarrow (2x+1)(x-1) = 0$

$\Rightarrow 2x + 1 = 0$ or x-1=0

$\Rightarrow x = -1/2$ or x=1

But, x=1 cannot be a solution since it makes the denominator of the term on the left hand side of the equality 0.

(v) $\frac{1}{2x+1} = \frac{x}{x^2}$

$\Rightarrow \frac{1}{2x+1} = \frac{1}{x} \Rightarrow x = 2x + 1 \Rightarrow x = -1$

(vi)

$x^{1/3} = 1/2 \Rightarrow x = (1/2)^3 = 1/8$

(vii)

$$\frac{1}{x} = \frac{2}{5-x} \Rightarrow 5 - x = 2x \Rightarrow 5 = 3x \Rightarrow x = 3/5$$

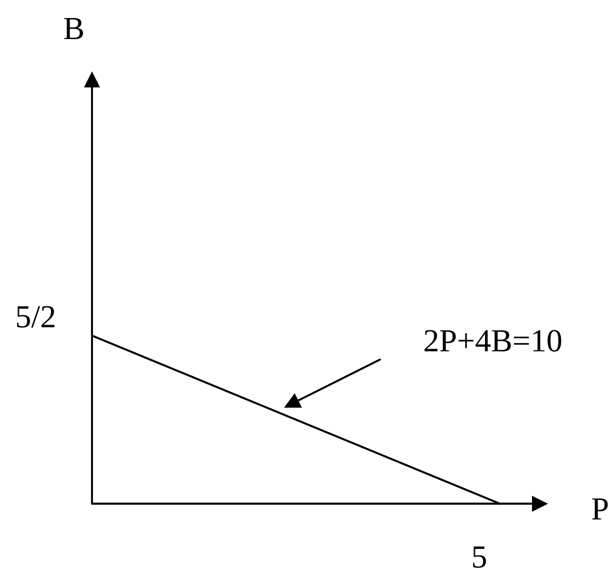
3. A consumer has to choose between consuming slices of pizza (P) and burritos (B). He has $10 to spend per day. His preferences over these two goods are summarized by the following utility function:

$$U(P, B) = 1.5 \ln P + \ln B$$

Price of a slice of pizza is $2 and a burrito is $4.

(i) What is the budget constraint for this consumer? Draw its graph. Make sure that the axes are labeled, and the graph is drawn according to a scale.

The budget constraint in this question is $2P + 4B = 10$



(ii) Find the optimal consumption bundle (P^*, B^*) for this consumer. Please write down the consumer's problem and each step of the substitution method explicitly while solving the problem. (You do not need to check the second order condition.)

Consumer's problem: $\max_{(P,B)>0}\{1.5\ln P+\ln B\}$ subject to 2P+4B=10

Step 1: B=(10-2P)/4 (from the budget constraint)
Step 2: Substitute in the utility function:

$$\max_{(P)>0}\left\{1.5\ln P+\ln\left(\frac{10-2P}{4}\right)\right\}$$

FOC: $1.5\frac{1}{P}+\frac{\frac{-2}{4}}{\frac{10-2P}{4}}=0$

$$\Rightarrow \frac{1.5}{P}-\frac{2}{4}\frac{4}{10-2P}=0\Rightarrow\frac{1.5}{P}-\frac{2}{10-2P}=0$$

$$\Rightarrow \frac{1.5}{P}=\frac{2}{10-2P}\Rightarrow 1.5(10-2P)=2P\Rightarrow 15-3P=2P\Rightarrow 15=5P\Rightarrow P^*=3$$

Hence, B*=(10-2P*)/4=(10-2.3) /4=1.

(iii) What would be the optimal choices of burritos and pizzas if his utility function was $U(P,B)=\ln P+1.5\ln B$ and the prices of the two goods stayed the same?

Consumer's problem: $\max_{(P,B)>0}\{\ln P+1.5\ln B\}$ subject to 2P+4B=10

Step 1: B=(10-2P)/4 (from the budget constraint)
Step 2: Substitute in the utility function:

$$\max_{(P)>0}\left\{\ln P+1.5\ln\left(\frac{10-2P}{4}\right)\right\}$$

FOC: $\frac{1}{P}+1.5\frac{\frac{-2}{4}}{\frac{10-2P}{4}}=0$

$$\Rightarrow \frac{1}{P}-1.5\frac{2}{4}\frac{4}{10-2P}=0\Rightarrow\frac{1}{P}-\frac{3}{10-2P}=0$$

$$\Rightarrow \frac{1}{P}=\frac{3}{10-2P}\Rightarrow (10-2P)=3P\Rightarrow 10=5P\Rightarrow P^*=2$$

Hence, B*=(10-2P*)/4=(10-2.2) /4=3/2.

The intuition behind your answer:

In (iii), the individual consumes more pizzas and less burritos compared to her consumption levels (ii). This results from the fact that, in (iii), her preferences are biased towards consuming more burritos rather than pizzas as opposed to her preferences in (ii).

4. An individual has to decide how much leisure to have and how much to consume in one day. A day can be devoted either to work (*L*) or leisure, so let's denote fraction of one day devoted to leisure as (1-*L*). His preferences over leisure and consumption are represented by

$$U(C,1-L)=C(1-L).$$

One unit of consumption good is *p* dollars and the individual makes *w* dollars per unit of time he works.

(i) Write down the budget constraint for this individual.

pC=wL

(ii) Find the optimal amount of work and consumption for this individual. Please write down the consumer's problem and each step of the substitution method explicitly while solving the problem.

Consumer's problem: $\max_{(C,L)>0}\{C(1-L)\}$ subject to Cp=wL

Step 1: C=$\frac{w}{p}L$ (from the budget constraint)

Step 2: Substitute in the utility function:

$$\max_{(L)>0}\left\{\frac{w}{p}L(1-L)\right\}$$

FOC: $\frac{w}{p}(1-L)+\frac{w}{p}L(-1)=0$

$$\Rightarrow \frac{w}{p}-\frac{w}{p}L-\frac{w}{p}L=0$$

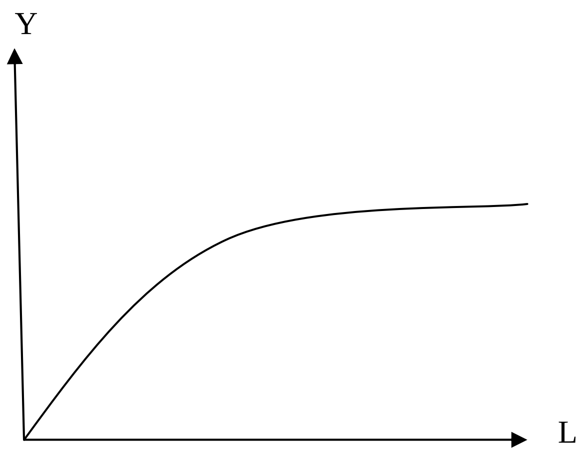
$$\Rightarrow \frac{w}{p}-2\frac{w}{p}L=0$$

$$\Rightarrow \frac{w}{p}=2\frac{w}{p}L \Rightarrow L^*=1/2$$

Hence, C*=w.L*/p=w/(2p).

5. (i) A production function shows the relationship between quantity of inputs used and the quantity of output that can be produced by using those inputs. In this question, we have $Y = \left(L^D\right)^{2/3}$.

The graph of the production function is:



(ii) The marginal product of labor is the percentage change in output for one percent change in the labor input. You can equivalently define this as the contribution of each unit of labor to output. In this example, we have diminishing marginal product of labor, because the first derivative of Y is decreasing in L:

$$\text{MPL=dY/dL=}\frac{2}{3}\left(L^D\right)^{-1/3}.$$

(iii) Firm's maximization problem:

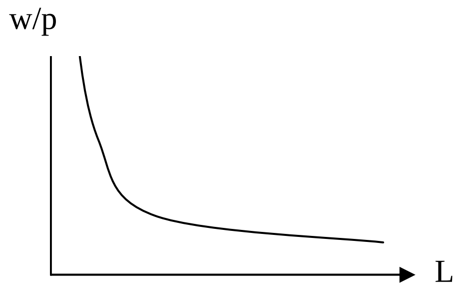
$$\max_{Y,L^D}\left\{pY - wL^D\right\} \text{ } subject \text{ } to \text{ } Y = \left(L^D\right)^{2/3}.$$

Substitution method:

$$\max_{L^D}\left\{p\left(L^D\right)^{2/3} - wL^D\right\}$$

$$FOC: p\frac{2}{3}\left(L^D\right)^{-1/3} - w = 0 \Rightarrow \frac{2}{3}\left(L^D\right)^{-1/3} = \frac{w}{p} \Rightarrow L^D = \left(\frac{3w}{2p}\right)^{-3}$$

The graph of the labor demand function:



6.

(i) In equilibrium, labor demanded and labor supplied have to be equal at the equilibrium (or "market clearing") real wage. So,

$$\frac{1}{2} = \left(\frac{3}{2}\frac{w}{p}\right)^{-3} \Rightarrow \left(\frac{w}{p}\right)^{*} = \left(\frac{1}{2}\frac{2}{3}\right)^{-1/3} = (1/3)^{-1/3} = 1.44$$

Plugging this back into either the labor supply or labor demand curve gives equilibrium employment in the economy:

$L^{*} = 1/2.$

(ii) When the government imposes the 25% tax on nominal wage income, the budget constraint of the individual becomes:

$$pC = 0.75wL^{S}$$

Therefore, the individual's maximization problem is:

$$\max_{C,L} C(1 - L^{S}) \ s.t. \ pC = 0.75wL^{S}$$

Solving for C from the budget constraint and substituting for it in the utility function gives:

$$\max_{L} \frac{0.75wL^{S}}{p}(1 - L^{S}) \ .$$

The FOC is:

$$\frac{0.75w}{p}(1 - L^{S}) - \frac{0.75wL^{S}}{p} = 0 \Rightarrow \frac{0.75w}{p} - \frac{0.75wL^{S}}{p} - \frac{0.75wL^{S}}{p} = 0$$

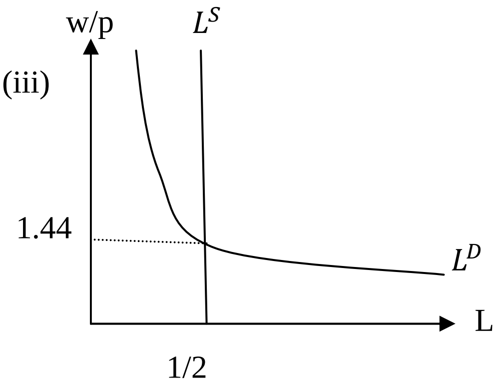
$$\Rightarrow \frac{0.75w}{p} - \frac{1.5wL^{S}}{p} = 0$$

$$\Rightarrow \frac{0.75w}{p} = \frac{1.5wL^{S}}{p} \Rightarrow L^{S} = 1/2$$

Note that the individual's labor supply didn't change as a result of the tax. This is due the functional form of the utility function.

Since the firm's problem hasn't changed, we still have the same labor demand demand function as in question 5. Therefore, the equilibrium employment and real wage will not change.

(iii)



As discussed in (ii) above, neither the equilibrium employment nor the real wage changed as a result of the tax. With the given preferences, the substitution effect and income cancel each other out, causing the labor supplied to be the same no matter what the real wage is.