Vanguards in Revolution\textsuperscript{1}

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\textsuperscript{1}We thank Scott Ashworth, Raphael Boeslavsky, Markus Brunnermeier, Ethan Bueno de Mesquita, Odilon Camara, Chris Cotton, Soroush Ghazi, Sharan Grewal, Mark Harrison, Amaney Jamal, Gilat Levy, Stephen Morris, Michael Peress, Kris Ramsay, Manuel Santos, Scott Tyson, and seminar participants at the University of Chicago Harris School, University of Miami, University of Calgary, Midwest Economic Theory Conference, MPSA, SPSA, and Princeton-Warwick Political Economy Conference for helpful comments.

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Abstract

Revolutionary vanguards, their radicalism and coercive actions, and their interactions with ordinary citizens and the state are common threads in narratives of revolutionary movements. But what are the defining features of revolutionary vanguards? The literature is replete with terms that allude to some notion of a revolutionary vanguard (e.g., revolutionary entrepreneurs, entrepreneurs of violence, early-risers), but the essence of these conceptions and their implications for revolutionary process remain obscure. We identify and differentiate the two main notions of vanguards, the Leninist and “early-riser” notions, and develop a formal framework that captures their distinguishing features, deriving their implications for the likelihood of revolution. We then use this framework to study three related and overlooked topics: (a) state strategies in mitigating the vanguard’s influence on citizens; (b) citizens’ preferences for the degree of vanguard radicalism; and (c) the vanguards’ use of coercion against citizens.

Keywords: Vanguard, Revolution, Radicalism, Coercion, Coordination, Learning, Option to Delay.

JEL Classification: D74, D82, H00.
1 Introduction

Who are revolutionary vanguards, what roles do they play in revolutionary movements, and what is the nature of their interactions with the state? Lenin, in *What Is to be Done?*, envisioned them as professional revolutionaries, skilled in resisting the state and organizing the masses. Others consider vanguards as “early-risers” who come to streets before others (Tarrow 2002), potentially creating a snow-ball effect that topples a regime (Kuran 1991; Lohmann 1994; Pearlman 2016). Complicating matters, the literature uses a plethora of names to allude to *some* notion of a revolutionary vanguard (e.g., revolutionary entrepreneurs, or entrepreneurs of violence). However, the essence of these notions and their implications for revolutionary process remain unstudied, hindering the understanding of the roles of vanguards in revolutions and the nature of their interactions with the state and citizens. Our paper develops a framework to capture the essence of the two main notions of vanguards and investigates their implications for regime change. We then use this framework to glean insights into three related and overlooked topics: (a) state strategies in mitigating a vanguard’s influence on citizens; (b) citizens’ preferences for the degree of vanguard radicalism; and (c) a vanguard’s use of coercion against citizens.

A key differentiating feature in various conceptions of a vanguard is whether a vanguard is distinguished from other citizens by its knowledge of the know-hows of “protest technology,” access to organizational resources (e.g., religious funds or networks), or its skills in initiating anti-regime activities such as protests or armed attacks. This is Lenin’s idea of a vanguard, which closely resembles the notions of political entrepreneurs and entrepreneurs of violence in the social movements literature (Della Porta 1995; McAdam et al. 2001; Tilly and Tarrow 2007) and revolutionary entrepreneurs in formal models of revolutions (Bueno de Mesquita 2010; DeNardo 1985; Shadmehr 2015). The vast social movements literature highlights that sustaining a movement requires skilled activists who engage in extensive planning and coordination that typically cannot be provided by spontaneous contentious actions (Tarrow 2002; Tilly 1996, 2004; see also Goldstone (2001) and Morris and Staggenborg (2004)).

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1A large literature traces the critical role of skilled activists in movements that may *seem* spontaneous. Morris’s (1984) classical study of the U.S. civil rights movement, and Khatib and Lust’s (2014) study of the role of activists in the Arab Spring are two examples. The Leninists were convinced of the necessity of skilled revolutionaries. For example, in his *History of the Russian Revolution*, Trotsky (1932) discards the “kingdom
in the alternative “early riser” notion of vanguards, the vanguards are not skilled revolutionaries; rather, they emerge organically in the protest process as those citizens who protest first, for example, based on their assessments of the risks and rewards of the protest. A key goal of this paper is to identify the differences in the outcomes of revolution that arise from the presence of established, skilled vanguards versus the emergence of endogenous, organic vanguards. We discuss, in turn, the Leninist and early-riser models of a vanguard, and the consequences of each model for revolutions, highlighting our methodological contributions in our review of the literature, and delineating our substantive contributions in the Conclusion.

The key element of the Leninist notion of a vanguard is that a skilled vanguard must lead for a revolution to succeed. To capture this feature, we assume that a representative professional vanguard first decides whether to revolt. Then, a representative citizen observes the vanguard’s action and chooses whether to provide support. The status quo prevails unless both the vanguard and citizen revolt, and a solo revolter is punished by the regime. The payoffs from successful revolution can be decomposed into a certain component that captures how the vanguard’s preferences differ from the citizen’s—a more radical vanguard derives a higher payoff from successful revolution than the citizen—and an uncertain component that both the vanguard and citizen share. To capture the fact that the vanguard does not have a monopoly over information, we assume that both the vanguard and citizen receive private signals about the uncertain component of revolution payoffs.

Because the vanguard must decide whether to act before the citizen, it cannot directly base its choice on the citizen’s signal. Instead, the vanguard must rely on the information of spontaneity,” arguing that “to the question, who led the February revolution? we can then answer definitely enough: Conscious and tempered workers educated for the most part by the party of Lenin” (p. 152).

2 We abstract from coordination among citizens, which has been well studied in the literature. Coordination arises in the Leninist model because a vanguard cares about coordinating its actions with a representative citizen. Moreover, that the game begins with the vanguard’s decision does not preclude earlier anti-regime activities by other citizens. Model parameters can capture the consequences of such unmodeled interactions. For example, when such actions reveal information about revolution payoffs, that information is included in the priors of agents; and if such preceding actions reduce the likelihood of punishment by the regime, they are captured by varying the expected payoffs. Our goal is to focus instead on the essential feature of the Leninist professional-revolutionary notion of vanguards that a skilled vanguard must lead for a revolution to succeed.

3 For example, Khomeini’s followers valued an Islamic Republic more than many citizens who supported them in the Iranian Revolution. Alternatively, a vanguard may hold more optimistic prior beliefs about the payoffs from successful revolution. For example, Lenin’s followers shared his belief that, following a Bolshevik Revolution in Russia, European workers would revolt and end WWI; Lenin reiterated this theory on many occasions, including his April Theses and The Tasks of the Proletariat (Lenin 1964, p. 67; Harding 1996).
contained in the citizen’s equilibrium choice of whether to provide support; and the vanguard knows that the revolution cannot succeed unless it initiates revolt. In contrast, the citizen bases his action on both his own signal, and the vanguard’s decision of whether to initiate revolt. The strength of the information contained in the vanguard’s choice to revolt rises when the vanguard is more reluctant to revolt—which, in turn, makes the citizen more willing to revolt. We show that as a result, in equilibrium, the vanguard revolts and the citizen provides support whenever their assessments of the merits of revolution are sufficiently high—given standard conditions, the equilibrium is unique and is in cutoff strategies. Moreover, in contrast to settings without an established revolutionary vanguard (which always features an equilibrium with no revolution), the probability of a successful revolution is always positive.

We then develop an early-riser model of vanguards. To do so, we discard the central premise of the Leninist model that a revolution fails without the leadership of an established revolutionary vanguard. In the early-riser model, two ex-ante identical representative citizens receive private signals about payoffs from successful revolution, and each citizen can lead—no group has a “technological advantage” in leading revolt. Each citizen has two chances to act. By initiating revolt at date 1, a citizen can endogenously assume the mantle of a leader, revealing that he has good news about the revolution. Alternatively, a citizen can wait to see what the other citizen did, and base date-2 decisions on this information.

The analysis of the early-riser model of vanguards is challenging, and constitutes our primary methodological contribution. Without an established vanguard, citizens in the early-riser model face conflicting signaling and free-riding incentives. Citizens have incentives both to lead, risking punishment to convey a positive signal about revolution payoffs and deliver a successful revolution; and to delay, in order to free ride on the information conveyed by the other’s actions, and avoid punishment for leading a failed revolt. We show that an established vanguard is always more likely to initiate revolt than is any citizen in the early-riser model.\footnote{Revolution may still be more likely in the early-riser setting because both citizens can initiate revolt.} This is because in the Leninist model, the revolution cannot succeed unless the vanguard acts. The flip side of the relative reluctance of an organic leader to revolt is that it raises the good news conveyed by his action. Thus, it is more attractive to support an organic leader than an established vanguard. This informational logic has a sharp implication: Revolutions that begin organically are more likely to succeed. A numerical analysis further suggests that
when leaders emerge organically the likelihood of successful revolution is higher unless the punishment for failed revolt is small.

We then show how our framework can provide foundations for understanding the sources of a vanguard’s ideological radicalism, and state actions to contain the threat of revolution. This analysis is most transparently posed in the Leninist model with an established vanguard. To analyze the strategic interactions between vanguards and the state, we consider the problem of a state seeking to minimize the probability of regime change. Obviously, a state would like to face a conservative vanguard that is very reluctant to overturn the status quo. So, too, a state would like to catch and harshly punish any vanguard after a failed revolt—as then a vanguard is very unlikely to initiate revolt. But what should a state do if it can neither anoint a puppet as a leader (e.g., a faux union leader), nor punish so harshly? The state’s problem is subtle. A more radical vanguard revolts after worse information about revolution payoffs. But then the citizen needs better signals about revolution payoffs to be willing to provide support. Thus, a more radical vanguard revolts more, but is less likely to have a following.

We show that a state may be better off with an extremely radical vanguard than with a moderate vanguard; and it may be better off not punishing leaders of a failed revolt than punishing moderately. This result provides a rationale for why Assad “funded and co-operated with al-Qaeda in a complex double game even as the terrorists fight Damascus” (Telegraph, 20 Jan 2014). So, too, it sheds light on why a regime might implement light punishments for leaders of a failed revolt. For example, in the early 1960s when Khomeini first openly criticized the Shah, the Pahlavi regime only briefly kept him under arrest, even as it responded harshly to protesters on the streets. When, after his release, Khomeini again denounced the Shah, he was only forced into exile (Milani 1994; Parsa 1989, 1994).

We next observe that regimes are not the only actors with a stake in determining a vanguard’s radicalism. Citizens also care, and their decisions of which potential vanguards to promote can effectively select vanguards with their desired radicalism. We show that the representative citizen prefers a more radical vanguard to offset the effects of harsher punishment for failed revolt, or when the citizen’s own status quo payoff is lower. Indeed, if the status quo deteriorates below a critical threshold, moderate citizens prefer very radical vanguards who always revolt even though this blocks information-flow from vanguards to
citizens about the merits of the post-revolution regime. This result can reconcile sudden surges of support for radical vanguards that sometimes occur; for example, in the years just preceding the Iranian Revolution, Khomeini’s support rose dramatically at the expense of more moderate opposition (Katouzian 1981; Mottahedeh 1985).

Having analyzed two selection sources of a vanguard’s radicalism, we explore how a vanguard’s radicalism may affect its interactions with citizens, focusing on a vanguard’s use of coercion against citizens. In settings such as civil wars or guerrilla movements, a vanguard can punish a citizen who does not follow its lead, for example burning villages that do not cooperate. A vanguard’s strategic considerations are subtle: with a more compliant citizen, a vanguard faces less risk of a failed revolt; but a more compliant citizen also supports change even when his private information indicates that the vanguard is “mistaken.” This reduces a vanguard’s ability to protect against outcomes that could be worse than the costs of revolting alone, for example, protecting against a successful revolution that “devours its own,” as happened to the Jacobins in the French Revolution and the Marxists in the Iranian Revolution (Abrahamian 1982, 1999; Brinton 1965). We show that, as a result, a vanguard never wants to coerce a citizen too harshly. The logic is that a vanguard wants a citizen to take whatever action the vanguard would given the citizen’s information. Thus, the more radical is the vanguard, the more coercion the vanguard wants to use. Paradoxically, a citizen can benefit from a vanguard’s use of coercion, as it makes the vanguard more willing to initiate revolt.

Extending this logic, we show that if, rather than having common beliefs about the informational structure, a vanguard has more confidence in its knowledge than does the citizen, then it employs harsher coercion. This result highlights that the source of extreme coercive measures by vanguards is not that they believe they know far more than other citizens, but rather that they believe that other citizens do not understand how much they know. This also suggests why ideological vanguards (e.g., Bolsheviks) or clandestine revolutionary organizations tend to use more coercive measures. Those with faith in a world-view tend to be overconfident about their knowledge; and clandestine revolutionary organizations with limited outside contacts tend to over-weight their comrades’ views as independent rather than correlated evaluations (Della Porta 2013; Levy and Razin 2015).

We next discuss the literature. Section 2 develops a Leninist model of vanguards, and
Section 3 analyzes an early-riser model of vanguards. Section 4 studies how a vanguard’s interactions with the state and citizens both influence a vanguard’s radicalism and are influenced by it. A conclusion and an appendix containing proofs follow.

**Literature.** Our main methodological contribution is to analyze the *endogenous emergence of vanguards*, which we are the first to study. This analysis is tangentially related to research on dynamic global games (Angeletos et al. 2007). The closest is Dasgupta’s (2007) analysis of delay in a threshold global game of investment. In Dasgupta’s model, players trade off higher potential returns for better information; there is no payoff uncertainty, the cost of delay is exogenous, and a player’s action does not influence outcomes. Thus, neither the tradeoff between signaling and free-riding, nor learning-in-equilibrium considerations, both of which are central in revolution settings, arise in that paper. Our second point of departure is our emphasis on the common value nature of revolution payoffs, which gives rise to *learning-in-equilibrium* incentives. The literature focuses on a leader who has more information than others and attempts to credibly convey this information to potential followers. However, as Shadmehr and Bernhardt (2011) note, uncertainty about common value payoffs is an essential feature of revolutions. An implication is that a vanguard, too, can benefit from the information of citizens. Thus, in our model, one of the vanguard’s key strategic considerations is learning the citizen’s information via the information content of their equilibrium behavior.\(^5\) These common value and learning-in-equilibrium features underlie our substantive results on (a) state strategies in mitigating a vanguard’s influence on citizens; (b) citizens’ preferences for the degree of vanguard radicalism; and (c) a vanguards’ use of coercion against citizens, where we analyze how a vanguard uses coercion to render a following without excessively blocking equilibrium learning that can protect against outcomes that are even worse than the status quo.

Bueno de Mesquita (2010) studies the signaling role of vanguards. A vanguard moves first and exerts costly efforts to foment violence, which is observed by citizens who then decide whether to revolt. The intensity of violence is a noisy public signal of anti-regime sentiments,

\(^5\)To capture these uncertain common value payoffs, we structure of our Leninist model of vanguards so that it is the sequential analogue of the simultaneous game analyzed in Shadmehr and Bernhardt (2011) with a more general payoff and information structure—our Lemma 2 generalizes their best response characterization. Proposition 1 shows that the sequential timing structure ensures existence of an equilibrium with revolution and restores equilibrium uniqueness (see Morris (2014) for a general discussion). There is no overlap between these two papers in content beyond this contrast in the existence and uniqueness of finite-cutoff equilibria.
and the vanguard’s effort reduces strategic risk, facilitating coordination by citizens—see also Loeper et al. (2014). Lipnowski and Sadler (2017) show that in a star network centered on a leader, the leader’s decision to protest convinces players of each others’ intentions to act, generating a unique peer-confirming equilibrium. Morris and Shadmehr (2017) study a leader’s optimal design of rewards from successful revolution. Bueno de Mesquita (2013) studies a leader’s the choice of regular vs. irregular tactics. DeNardo (1985) and Shadmehr (2015) study a leader’s choice of revolutionary agenda. More broadly, a leadership literature has focused on the “leader as communicator.” In Hermalin (1998), the leader is a team member with more information about the returns to effort, who tries to credibly signal this information to other members to induce them to work harder. Dewan and Myatt (2007) study the effect of a leader’s public signal on the outcome of a party conference where players must coordinate on two potential outcomes.

A few papers study leadership in the beauty contest framework of Morris and Shin (2002), where players care about both coordinating actions and targeting the state of the world, about which they have private information. In Landa and Tyson (2017), an informed but biased leader wants followers to take her desired action, and followers incur exogenous costs of deviating from a leader’s announced action directive. Bolton et al. (2013) extend the beauty contest framework to allow for bottom-up information flow via the leader’s observation of the aggregate actions of followers. We abstract from the role of leaders in enhancing coordination among followers, which has been the subject of many studies, and focus on the novel aspects, described above, that have received minimal attention.

2 A Leninist Model of Vanguards

A representative vanguard and a representative citizen (follower) sequentially decide whether or not to revolt. The vanguard moves first and the citizen moves second. Figure 1 shows the sequence of moves and payoffs. The payoff \( \theta \) from a successful revolution is uncertain. The vanguard receives private signal \( s_1 \) about \( \theta \) and the citizen receives private signal \( s_2 \).

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6The literature on protest and revolution has mainly focused on citizen coordination and a state’s manipulation of information (Barbera and Jackson 2016; Casper and Tyson 2014; Chen and Suen 2017; Edmond 2013; Egorov et al. 2009; Egorov and Sonin 2017; Shadmehr 2017; Shadmehr and Bernhardt 2015, 2017; Siegel 2009, 2013; Tyson and Smith 2018).
Figure 1: Vanguard-citizen Game. \( R \) indicates revolt and \( \neg R \) indicates no revolt.

The other expected payoffs are common knowledge, with \( h_i > l_i \). Thus, when only one agent revolts, the revolution fails, and the sole revoler incurs a punishment cost.\(^7\) The vanguard’s degree of radicalism is captured by \( z \): a vanguard is more radical than the citizen if \( z > 0 \), and it is more conservative if \( z < 0 \). The signals and \( \theta \) are jointly distributed with a strictly positive, continuously differentiable density \( f(s_1, s_2, \theta) \) on \( \mathbb{R}^3 \). We assume that \( s_1, s_2 \) and \( \theta \) are strictly affiliated,\(^8\) the expectation of the absolute value of \( \theta \) is finite, and we impose minimal structure on the tail properties of \( f(s_1, s_2, \theta) \):

**Assumption A1.** For every \( k \), for \( i, j = 1, 2 \), with \( j \neq i \),

\[
\begin{align*}
(a) & \quad \lim_{s_i \to \infty} E[\theta|s_j < k, s_i] = \infty, \quad \lim_{s_i \to -\infty} E[\theta|s_j > k, s_i] = -\infty \\
(b) & \quad \lim_{s_i \to \infty} Pr(s_j > k|s_i) = 1, \quad \lim_{s_i \to -\infty} Pr(s_j > k|s_i) = 0 \\
(c) & \quad \lim_{s_i \to -\infty} Pr(s_2 > k|s_1) E[\theta|s_2 > k, s_1] > -\infty.
\end{align*}
\]

A1 is mild. For example, it holds in an additive, normal noise signal setting where \( s_i = \theta + \nu_i \), \( i \in \{1, 2\} \), and \( \theta, \nu_1 \) and \( \nu_2 \) are independently distributed normal random variables. Parts (a) and (b) are self explanatory. For example, \( \lim_{s_i \to \infty} E[\theta|s_j < k, s_i] = \infty \) in part (a) ensures that when a citizen’s signal is very good, he predicts that the state is also very good, even given beliefs that the other citizen’s signal is below some fixed threshold \( k \). Part (c) only requires that the left hand side not be unboundedly negative.

**Strategies.** A pure strategy for the vanguard is a function \( \rho_1 \) mapping its private signal \( s_1 \) into an action choice, \( a_1 \in \{-R, R\} \), where \( -R \) indicates no revolt and \( R \) indicates revolt. A

\(^7\)Because only the net expected payoffs of revolt or not enter action choices, if revolt has expected costs, the payoffs in Figure 1 capture them via normalization of \( w_i, h_i, \) and \( l_i \).

\(^8\)\( s_1, s_2 \) and \( \theta \) are strictly affiliated if, for all \( y, y' \in \mathbb{R}^3 \), with \( y \neq y' \), \( f(\min\{y, y'\})f(\max\{y, y'\}) > f(y)f(y') \), where min and max are defined component-wise (de Castro 2010; Milgrom & Weber 1982).
pure strategy for the citizen is a function $\rho_2$ mapping his private signal $s_2$ and the vanguard’s action $a_1$ into an action choice, $a_2 \in \{-R, R\}$. That is, $\rho_2 : \mathbb{R} \times \{-R, R\} \rightarrow \{-R, R\}$. The equilibrium concept is Perfect Bayesian Equilibrium.

**Equilibrium.** When the vanguard does not revolt, the citizen has a dominant strategy to do the same to avoid the punishment costs of $h_2 - l_2 > 0$. Thus, our model delivers an essential feature of the Leninist notion of professional revolutionary vanguards that the vanguard must lead for a revolution to succeed. Lemma A.1 in the Appendix shows that when the vanguard revolts, the citizen’s best response takes a cutoff form in which he revolts whenever his signal $s_2$ exceeds a threshold $k_2$ that depends on the vanguard’s strategy. Lemma A.2 in the Appendix shows that if the citizen’s strategy takes a cutoff form, then so does the vanguard’s. That is, the vanguard revolts whenever $s_1 \geq k_1$ for some $k_1$. These lemmas imply that in any equilibrium in which the vanguard sometimes revolts, both agents adopt cutoff strategies. Lemma 1 identifies the strategic forces that shape a citizen’s incentive to revolt.

**Lemma 1.** As the vanguard becomes more willing to revolt, the citizen revolts less. That is, the citizen’s best response always features strategic substitutes: $\frac{\partial k_2(k_1)}{\partial k_1} < 0$.

A vanguard who is more willing to revolt does so following worse signals about the revolution payoff—its cutoff $k_1$ is lower. This lowers a citizen’s forecast of the payoff from successful revolution—$E[\theta|s_1 \geq k_1, s_2]$ falls—reducing his incentive to revolt.

Unlike the citizen, the vanguard must decide whether to revolt without knowing whether it will be joined; and if the vanguard is the sole challenger to the regime, it expects to be punished. Thus, a vanguard faces a type of cost that the citizen does not—the miscoordination cost $\mu_1 \equiv h_1 - l_1 > 0$ that it pays when it revolts, but the citizen does not. With a more compliant citizen (someone who is more likely to follow the vanguard), the vanguard is less likely to incur the costs of miscoordination, and hence has more incentive to revolt. This provides a force for strategic complements in the vanguard’s calculations. However, as a citizen grows more compliant, he also follows the vanguard after receiving worse signals. This

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9While the citizen faces miscoordination costs, he never incurs them because he sees the vanguard’s action before deciding whether to revolt. Our results extend directly if the revolution can fail even when both the vanguard and citizen revolt: this amounts to a renormalization of payoffs. Also, our results extend qualitatively if there is a small probability $p$ that a revolution succeeds even if only one party revolts. When $p$ is positive and the vanguard does not revolt, the citizen will revolt whenever his signal is so high that it offsets the low probability of success and the negative news conveyed by the vanguard’s decision not to revolt.
reduces the vanguard’s expectation of the payoff from successful revolution—
\[ E[\theta|s_2 \geq k_2, s_1] \]
falls—reducing its incentive to revolt. That is, excessive compliance by a citizen deprives the vanguard of effectively aggregating the citizen’s information—information that can protect against a regime change that results in outcomes that are even worse than the status quo. This constitutes a force for strategic substitutes in the vanguard’s calculations.

Lemma 2 shows that with a barely-compliant citizen who rarely follows the vanguard, the force for strategic complements dominates: as a citizen becomes more likely to follow, the vanguard revolts more. However, as the citizen grows more compliant, the force for strategic substitutes rises relative to that for strategic complements. In fact, there is a unique threshold on a citizen’s level of compliance after which the force for strategic substitutes dominates: thereafter, as the citizen grows more compliant, the vanguard revolts less.

**Lemma 2.** There is a critical level \( k^* \) of the citizen’s cutoff such that if \( k_2 > k^* \), a vanguard’s best response features strategic complements; but if \( k_2 < k^* \), it features strategic substitutes.

If a vanguard’s best response featured global strategic complements, equilibrium would necessarily be unique—\( k_1(k_2) \) would be strictly increasing, and \( k_2(k_1) \) is strictly decreasing. However, because a vanguard’s best response exhibits strategic substitutes when \( k_2 \) is low, multiple equilibria might exist. When \( w_2 \) is sufficiently large, the crossing can only occur on the strategic complement part of the vanguard’s best response, so equilibrium is unique. This reflects that a higher \( w_2 \) makes a citizen more reluctant to revolt, raising his best response—for each \( k_1 \), the best response cutoff \( k_2(k_1; w_2) \) is higher. When \( w_2 \) rises past a threshold, the crossing occurs at \( k_2 > k^* \), i.e., from a strategic perspective, the high risk of punishment dominates the vanguard’s information aggregation concerns. To prove uniqueness more generally, we impose Assumption A2. Assumption A2 states that the conditional expectation of \( \theta \) is more sensitive to changes in a signal \( s_i = x \) than to changes in the cutoff \( x \leq s_i \).

**Assumption A2.** For every \( x \) and \( y \),
\[
\frac{\partial E[\theta|s_1 = x, s_2 \geq y]}{\partial x} \frac{\partial E[\theta|s_1 \geq x, s_2 = y]}{\partial y} > \frac{\partial E[\theta|s_1 \geq x, s_2 \geq y]}{\partial y} \frac{\partial E[\theta|s_1 \geq x, s_2 = y]}{\partial x}.
\]

\( ^{10} \)This assumption regulates the slope of best responses, ensuring that the slope of the strategic substitutes segment of the vanguard’s best response is more negative than the slope of the citizen’s best response. To see the intuition, suppose \( l_1 = h_1 \), so that the vanguard’s best response features global strategic substitutes. Then a change of variable transforms the game into a game of global strategic complements (Vives 2001, p. 34). Assumption A2 rules out multiple equilibria by limiting the degree of strategic complementarities.
Lemma A.3 in the Appendix establishes that \( A_2 \) holds with the classical additive, normal noise signal structure. We maintain Assumptions \( A_1 \) and \( A_2 \) throughout the paper.

This leaves the possibility that, regardless of how promising its signal is, a vanguard never revolts. Then, a citizen’s beliefs cannot be determined via Bayes rule on an off-equilibrium path where the vanguard revolts. We impose a minimal plausibility condition on a citizen’s off-equilibrium beliefs: if the vanguard revolts, then \( E[\theta|a_1 = R, s_2] \) exceeds \( w_2 \) for all sufficiently high values of the citizen’s signal \( s_2 \).\(^{11}\) With this condition, an equilibrium in which there is never revolution does not exist.

**Proposition 1.** There is a unique equilibrium. Equilibrium strategies take cutoff forms, and revolution occurs with positive probability.

This result contrasts with what happens when there is no vanguard—i.e., when citizens move simultaneously. Shadmehr and Bernhardt (2011) show that without a vanguard, there is always an equilibrium in which citizens never revolt. Indeed, this no-revolt equilibrium is the sole equilibrium if miscoordination costs \( h_i - l_i \) are high. Without a vanguard, coordination breaks down because if each citizen believes that the other does not revolt, then s/he does not revolt to avoid paying \( h_i - l_i \). By being the first to challenge a regime and risking punishment, a vanguard helps coordination by ensuring that if the citizen revolts, he will not be alone. One can extend this result to show that even in a simultaneous move setting, asymmetric divisions of miscoordination costs facilitate coordination.\(^{12}\)

### 3 An Early-Riser Model of Vanguards

The preceding analysis developed a Leninist model of vanguards. This section develops an early-riser model of vanguards, in which it is not the heterogeneous attributes of a group of citizens (e.g., their skills and experience) that determines who initiates a protest, but rather their information. In contrast to the Leninist notion where the leaders of revolution are established professional revolutionaries, in the early-rise notion of vanguards, potential leaders

\(^{11}\)If, following the vanguard’s decision to revolt, the citizen believes that the vanguard’s signal \( s_1 \) cannot be unboundedly negative, then Assumption \( A_1(a) \) implies this minimal plausibility condition.

\(^{12}\)This logic is distinct from coordination-enhancing channels in which vanguards send public signals that reduce strategic risks and convey information about anti-regime sentiments (Bueno de Mesquita 2010; Lohmann 1994). See Morris (2014) for the link between timing frictions and equilibrium uniqueness.
of a revolution emerge organically based on their assessments of the risks and rewards of revolution. We now analyze this endogenous emergence of vanguards and show how revolution outcomes differ in Leninist and early-riser models of vanguards.

To study the early-riser notion of vanguards, we discard the key feature of the Leninist model that a revolution fails without the leadership of an established professional revolutionary vanguard. To highlight the contrast, we suppose instead that no group has a “technological advantage” over the other in initiating revolt. Thus, in our two-period model, two ex-ante identical citizens receive private signals about the payoffs from successful revolution. Both citizens can act at each date. At date 1, citizens simultaneously decide whether to revolt. If both revolt, the revolution succeeds, and each citizen receives $\theta$. Otherwise, citizens see each other’s date 1 actions, and those who did not revolt at date 1 have a second chance to act at date 2. The revolution succeeds whenever both citizens revolt by the end of date 2, when payoffs are realized. We maintain the payoff structure that (a) the status quo payoff is $h$; (b) if one citizen revolts, but the other does not, the sole revolter receives $l < h$, and the other citizen receives $w$; and (c) if both revolt, the revolution succeeds, and each citizen receives $\theta$. We impose symmetry on the signal structure: $f(s_1, s_2, \theta) = f(s_2, s_1, \theta)$ for all $s_1, s_2,$ and $\theta$.

When a citizen acts at date 2, he can base his decision on the information revealed by the other citizen’s date-one choice of whether or not to act. But by initiating revolt at date 1, a citizen can endogenously assume the mantle of a leader, revealing to the other citizen that he has information indicating that revolution is worthwhile. Thus, citizens have incentives both to lead at date 1, in order to signal good news about revolution payoffs and deliver a successful revolution; and to defer at date 1, in order to free ride on the information conveyed by the other citizen’s actions, thereby reducing the risk of being punished for leading a failed revolt.

We focus on symmetric equilibria in which citizens set cutoff $\alpha$ for revolt at date 1; cutoff $\beta$ for revolt at date 2 if the other citizen revolted at date 1; and cutoff $\gamma$ at date 2 if no one revolted at date 1. To ease presentation, we also assume $w \geq h$.

**Proposition 2.** A symmetric equilibrium with finite cutoffs $\alpha$ and $\beta$ always exists. In equilibrium, $\alpha > \beta$: a citizen is strictly more willing to revolt when the other citizen takes on the mantle of leadership. Further, when $\gamma$ is finite (i.e., when a failure of a leader to emerge does not preclude revolution), then $\beta < \gamma < \alpha$. 

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To understand the tradeoffs, first suppose that if no one revolts at date 1, then no one revolts at date 2. The benefit of revolt at date 1 is to induce the other citizen to support the revolution (at date 2) when he has signal $s \in [\beta, \alpha)$. The cost of revolt at date 1 is that it risks punishment when the other citizen’s signal is below $\beta$. Conversely, deferring revolt forgoes successful revolution outcomes when the other citizen has signal $s \in [\beta, \alpha)$, but does not risk punishment for leading a failed revolt when the other citizen has signal $s < \beta$. In equilibrium, a citizen with threshold signal $\alpha$ must be indifferent between initiating revolt to signal his information and deferring to free-ride. This means that

$$Pr(\beta \leq s_2 < \alpha|s_1 = \alpha) \left( E[\theta|s_1 = \alpha, \beta \leq s_2 < \alpha] - h \right) = Pr(s_2 < \beta|s_1 = \alpha) \ (h - l).$$

Equilibria in which revolt is initiated at date 2 can only exist under extreme parametric conditions. To see this, consider the tradeoffs in any hypothetical equilibrium in which revolt is sometimes initiated at date 2: if no one revolts at date 1, then a citizen revolts at date 2 when his signal exceeds $\gamma < \alpha$. Consider citizen 1 with signal $s_1 = \alpha$ at date 1. If citizen 2 revolts, then revolting and not revolting at date 1 does not make a difference because citizen 1 will revolt at date 2 as $s_1 = \alpha > \beta$. However, if citizen 2 does not revolt, then citizen 1 faces a tradeoff. Revolting (versus not revolting) induces citizen 2 to revolt for signals $s_2 \in [\beta, \gamma)$, but risks extra punishment when $s_2 \in [\beta, \gamma)$.

Equilibrium requires that

$$Pr(s_2 \in [\beta, \gamma]|\alpha)(E[\theta|s_1 = \alpha, s_2 \in [\beta, \gamma]) - h) = Pr(s_2 \in [\beta, \gamma]|\alpha)(l - h).$$

The left hand side is the incremental gain from revolting (vs. not revolting) for the marginal citizen with signal $\alpha$, and the right hand is the corresponding incremental cost. The proof of Proposition 2 reveals that necessary (demanding) conditions for the existence of an equilibrium cutoff $\gamma < \alpha$ are:

$$E[\theta|s_1 = \alpha, s_2 \in [\beta, \gamma)] = l, \text{ and } E[\theta|s_1 = \gamma, s_2 \in [\gamma, \alpha)] > E[\theta|s_1 \geq \alpha, s_2 = \beta] = w.$$

\footnote{Indeed, we have not been able to find such equilibria; our characterization is a necessary description if such equilibria exist.}

\footnote{If citizen 1 (with signal $s_1 = \alpha$) revolts at date one, he is punished when $s_2 < \beta$; and if he does not revolt, then he will revolt at date 2 (because $\gamma < s_1 = \alpha$) and is punished when $s_2 < \gamma$. Thus, the change in the expected punishment from revolting versus not revolting is $[Pr(s_2 < \beta|s_1 = \alpha) - Pr(s_2 < \gamma|s_1 = \alpha)] \ (h - l) = Pr(\beta \leq s_2 < \gamma|s_1 = \alpha) \ (l - h).$}
The first equality is the indifference condition for cutoff \( \alpha \)—equation (1) above. The last equality is the indifference condition for \( \beta \). The inequality is implied by the indifference condition for \( \gamma \)—see equation (18) in the Appendix—which also implies that \( \gamma > \beta \).

Having analyzed existence of equilibrium and the tradeoffs inherent in the equilibrium for both Leninist and early-riser models of vanguards, we compare the extent of revolt in these models. To make the models comparable, we consider the Leninist model of a vanguard with symmetric payoffs and symmetric signal structures.

**Proposition 3.** An established vanguard is more likely to initiate revolt than is an organic vanguard, but its revolt is less likely to be followed: \( \beta < k_2 < k_1 < \alpha \). That is, in the Leninist model of vanguards, a revolution is more likely to be initiated than in the early-riser model of vanguards; but revolutions that begin organically are more likely to succeed.

This proposition holds regardless of whether or not citizens initiate revolution at date 2. To see the intuition, note that “waiting” is very costly for an established vanguard: if the vanguard does not act, then revolution will not happen. When, instead, vanguards emerge organically, the opportunity cost of “not revolting” is lower because when one citizen does not revolt at date 1, a revolution can still happen: the other citizen may have initiated revolt (when its signal is high enough), which would then allow the first citizen to follow. As a result, with an endogenous, early-riser notion of a vanguard, a citizen is more reluctant to initiate revolt than a Leninist established vanguard would be. In turn, because citizens set a higher cutoff for initiating revolt when vanguards emerge organically, more good news is revealed when one of them revolts, causing citizens to set a lower cutoff for supporting a revolt.

Proposition 3 says that an established vanguard is more likely to revolt than an organic one. This result is driven by incentives to free-ride on the other citizen, to avoid the risk of being punished. One might conjecture that these free-riding incentives must reduce the probability of successful revolution below its level with an established vanguard, impairing citizen welfare. This reasoning is flawed—a Leninist established vanguard-follower structure does not even necessarily make revolution more likely. In particular, when the leadership mantle is endogenous, either citizen can lead a revolution, whereas in the Leninist model, no matter how promising the follower believes revolution payoffs to be, the vanguard must act first for a revolution to succeed. Moreover, conditional on a revolution being initiated,
Figure 2: Probability of regime change. Left: The solid (dashed) curve is the probability of regime change in the early-riser (Leninist) model of vanguards. Right: The solid curve minus the dashed curve. Parameters: $\sigma^2 = 1$, $\sigma^2_{\nu} = 0.85$, $l = -2$, and $h \in [-1.95, 1.95]$.

A revolution is more likely to succeed when leaders emerge endogenously, as an endogenous follower is always more willing to revolt.

This leads us to numerically investigate the probability of successful revolution in the two models in settings with an additive, normal noise signal structure when $w = h$. As Figure 2 illustrates, this analysis suggests that when (a) the expected status quo payoff is better than the ex-ante expected revolution payoff, and (b) the punishment from leading a failed revolt is not too small ($h > E[\theta] = 0$ and $h >> l$), then the probability of successful revolution is higher in the early-riser model (where either citizen’s information can induce him to lead) than in the Leninist model of established vanguard. More generally, when leading a failed revolution has sufficient costs, revolution is most likely to succeed with organic leaders.

However, this ordering on the likelihood of successful revolution is reversed when (a) leading a failed revolt has modest costs ($\mu = h - l$ is small), and (b) the status quo is bad ($h < 0$) so that the likelihood of revolution is high. In such settings, the likelihood of successful revolution can be higher in the Leninist model of established professional vanguard. The intuition is that when the punishment $\mu$ for leading a failed revolt is small, the fact that the established vanguard is less likely to be followed than an endogenous leader matters less. With one opportunity to act, a vanguard must revolt to let its follower’s information be pivotal in determining outcomes. When the punishment for failed revolt is small, the value of revolting to allow the other citizen’s information determine what happens swamps
the cost (in terms of $\mu$ and possible ex post regret). This incentive to revolt when $\mu$ is small exists with an established vanguard, but it is absent in the endogenous setting. Qualitatively, however, once the punishment $\mu$ for leading a failed revolt takes on more than modest values, or the status quo is “good enough,” this incentive is swamped, and the probability of successful revolution is higher in the early-riser endogenous model of vanguard.

**Welfare.** We now show that when $w = h$, so that the payoff of a citizen who does not revolt is unaffected by the other citizen’s action, citizens are too reluctant to revolt from a welfare perspective. As the logic is identical, we prove the result in the endogenous vanguard setting.

We establish in the early riser model where revolution is not initiated at date 2 that from a given citizen’s perspective, the other citizen does not revolt enough, i.e., his cutoffs $\alpha$ and $\beta$ for revolution are too high. First, consider the choice of $\beta$: suppose that citizen 1 has revolted, and citizen 2 must decide whether to follow. Citizen 2 compares the payoff $w$ received when he does not revolt, with his expected payoff from revolt. However, citizen 1 would have citizen 2 weigh the payoff $l$ that citizen 1 receives when citizen 2 does not revolt against the expected payoff from revolt. Because $l < w$, citizen 1 would prefer citizen 2 lower his cutoff below $\beta$. Now, consider the choice of $\alpha$. Citizen 1’s payoffs are only affected by a marginal reduction in $\alpha$ set by citizen 2 when citizen 1 receives a signal $s_1 \in [\beta, \alpha)$: with the reduction, citizen 1 would receive $E[\theta|s_2 = \alpha, \beta \leq s_1 < \alpha] \approx h$ rather than $h$. However, citizen 2’s indifference condition for initiating revolt given signal $\alpha$ implies $E[\theta|s_2 = \alpha, \beta \leq s_1 < \alpha] - h > 0$ because citizen 2 worries that $s_1 < \beta$, and he gets punished. It follows that citizens are too reluctant from a social welfare perspective to initiate revolt.

In particular, the socially optimal symmetric cutoffs $\alpha_s$ and $\beta_s$ solve:

$$Pr(\beta_s \leq s_2 < \alpha_s | s_1 = \alpha_s) \left( E[\theta|s_1 = \alpha_s, \beta_s \leq s_2 < \alpha_s] - h \right) = Pr(s_2 < \beta_s | s_1 = \alpha_s) \frac{h - l}{2},$$

and

$$E[\theta|s_1 \geq \alpha_s, s_2 = \beta_s] = \frac{h + l}{2}.$$ 

In contrast, equilibrium indifference conditions demand that the term $\frac{h - l}{2}$ be $(h - l)$ and the term $\frac{h + l}{2}$ be $h$. Thus, citizens would gain from an ex-ante perspective if they could commit to reducing their cutoffs below equilibrium levels.

We next underscore that variations in primitive parameters can have perverse welfare effects due to the impact on information aggregation from equilibrium actions. Consider the
effects of harshening the consequences for leading a failed revolution. Obviously, the direct
effect of reducing $l$ is to reduce citizen welfare. Moreover, the first order indirect strategic
effect reduces welfare further: reducing $l$ makes citizens more reluctant to initiate revolt, i.e.,
they raise $\alpha$. But, the second order indirect strategic effect raises citizen welfare: increases
in $\alpha$ cause citizens who see that a revolt was initiated to update more positively about
revolution payoffs, causing them to reduce the cutoff $\beta$ for providing support. A numerical
investigation shows that the welfare-increasing effect of the reduction in $\beta$ can sometimes
dominate, so that as $l$ falls, citizen welfare can rise. Similar conflicting welfare forces can
arise in the Leninist model with an established vanguard.

4 State, Citizens, and Vanguard Radicalism

We have developed a framework that captures the essence of the Leninist and early-riser
notions of vanguards. We now use these frameworks to study the sources of a vanguard’s
ideological radicalism, and state actions that aim to curb the vanguard’s influence and con-
tain the threat of revolution. To ease exposition, we pose the analysis in the Leninist model
with an established vanguard. We begin with two critical, albeit simple, observations. A
more radical vanguard is more eager to revolt, which means that it sets a lower cutoff $k_1$
for revolting. But, this means that it revolts after receiving worse signals about successful
revolution payoffs $\theta$. In turn, this reduces the citizen’s incentive to revolt by lowering his
estimate of the successful revolution payoffs, $E[\theta|s_1 \geq k_1, s_2]$.\(^{15}\)

**Proposition 4.** The more radical is the revolutionary vanguard, the more likely it is to
revolt, and the less likely is the citizen to provide support:

$$\frac{\partial k_1(z)}{\partial z} < 0 < \frac{\partial k_2(z)}{\partial z},$$

where $k_1(z)$ and $k_2(z)$ are the endogenous equilibrium cutoffs.

\(^{15}\)One can show that even if $z = 0$, the vanguard may appear to be more radical than the citizen, setting
a lower cutoff for revolt. That is, even though the vanguard alone risks punishment by the regime, it may
still be more willing than a citizen to act given the same signal about revolution payoffs. This is because
the vanguard must also weigh the possible gains of acting in order to let the citizen’s information determine
whether the revolution succeeds. The vanguard is more willing to act than the citizen whenever the citizen’s
payoff from not supporting the vanguard is high (so that the citizen is not too willing to act), but not too
high (so that the vanguard does not face an excessively high risk of miscoordination, and hence punishment).
**State and Vanguard Radicalism.** Proposition 4 highlights key tradeoffs for a regime that aims to prevent regime change. One might think that greater rewards to a citizen for defying a revolutionary vanguard, and harsher punishments for failed revolt are always complementary tools for the state, and that increasing either always has value to the regime. Indeed, if a regime can punish failed revolters extremely harshly at minimal expense, or radically raise the reward $w_2$ to a citizen for defying a vanguard, or “deradicalize” a vanguard by decreasing $z$ sufficiently, then it can reduce the probability of successful revolt almost to zero. However, when such actions are not possible, it can be optimal for the state to go in the opposite direction. The state does not care about the probability of revolt, per se, but rather about the probability that a revolt succeeds, and successful revolt requires that both the vanguard and citizen act. As a result, to reduce the probability of successful revolt, rather than anoint a modestly conservative citizen as a puppet vanguard of the opposition (e.g., a union vanguard), the regime may do better to radicalize the vanguard. A vanguard with a larger $z$ is more eager to revolt, delegitimizing the vanguard in the eyes of citizens.

Increasing $z$ has conflicting effects: the direct, non-strategic effect is to increase the vanguard’s incentive to revolt; but the indirect, strategic effect is to decrease the citizen’s incentive to follow the vanguard. This strategic effect further feeds back into the revolutionary vanguard’s strategic considerations, which can mitigate (amplify) the direct effect when the vanguard’s best response features strategic complements (substitutes). When the strategic effect dominates, the likelihood of successful revolution falls as the vanguard become more radical—see Figure 3. Let $P$ be the probability of successful revolution, i.e., $P = Pr(s_1 \geq k_1, s_2 \geq k_2)$. Then,

$$\frac{dP}{dz} = \left( \frac{\partial P}{\partial k_1} + \frac{\partial P}{\partial k_2} \frac{\partial k_2}{\partial k_1} \right) \frac{dk_1}{dz}.$$}

The first term in the parenthesis captures the direct, non-strategic effect of having a more radical vanguard, and is positive. The second term captures the strategic effect of having a more radical vanguard and is negative: $\frac{\partial k_2}{\partial k_1} < 0$, reflecting that the follower’s best response always features strategic substitutes. As Figure 3 illustrates in an additive normal noise setting, $P(z)$ is single-peaked in $z$, and the state wants to avoid that maximum. When the vanguard is sufficiently conservative, the non-strategic effect dominates. From that point, as the vanguard’s level of radicalism begins to rise, so does the probability of successful revolt.
However, once the vanguard’s level of radicalism $z$ rises far enough, a threshold is reached after which the strategic effect dominates and the likelihood of successful revolution falls as the vanguard becomes ever more radical. In sum, the state would like a sufficiently conservative vanguard that is unlikely to revolt; failing that, the state would like a sufficiently extreme vanguard that is unlikely to win the following required for a revolution to succeed. The same logic underlies why a state may be better off refraining from punishing the leaders of a failed revolt to reduce their following. These results provide a theoretical lens to interpret seemingly paradoxical interactions of some authoritarian regimes with their opponents, e.g., Assad’s double game of supporting al-Qaeda, and the Shah’s mild punishment of Khomeini.

![Figure 3: Probability of successful revolution as a function of the revolutionary vanguard’s radicalism $z$. $s_i = \theta + \epsilon_i$, with $\theta, \epsilon_i \sim iidN(0, 1)$, $h_1 = 0.1$, $l_1 = 0$, $w_2 = 1.2$.](image)

As Figure 3 shows, when a citizen’s disloyalty payoff $w_2$ is high, the probability of a successful revolution may be highest with a slightly conservative vanguard, whose payoff from successful revolt is less than the citizen’s. Figure 4 numerically illustrates how with normally distributed uncertainty, when the state raises the citizen’s disloyalty payoff $w_2$, the level of the vanguard’s radicalism that maximizes the likelihood of successful revolution falls. Paradoxically, the vanguard’s increased willingness to revolt can make the citizen sufficiently less willing to follow that it reduces the likelihood that the regime is overthrown. Thus, a regime must be wary about combining the twin tools of more generously rewarding a citizen who turns on the vanguard, and of also punishing the vanguard after a failed revolt somewhat
more harshly. Doing so can backfire and increase the probability of successful revolt.

**Citizen Preferences for Vanguard Radicalism.** The preceding analysis suggests that one source of vanguard radicalism can be a state that tries to promote radical vanguards in order to reduce their appeal to citizens. But citizens also care about the radicalism of vanguards. In early stages of the formation of vanguard organizations, decisions by citizens of which potential vanguards to support can effectively determine the radicalism of the future established vanguards that will decide whether and when to initiate revolt. Proposition 5 characterizes how the representative citizen’s preferred level of radicalism in a vanguard hinge on the various payoffs to agents from successful and unsuccessful revolution:

**Proposition 5.** Fixing the other parameters, there exists a \( \bar{h} < w_2 \) such that if \( h_2 > \bar{h} \) then the citizen’s preferred level of radicalism in a vanguard is given by

\[
   z^* = \frac{Pr(s_2 < k_2|s_1 = k_1) (w_2 - l_1) - (h_2 - h_1)}{Pr(s_2 \geq k_2|s_1 = k_1)},
\]

where \( k_1 \) and \( k_2 \) are the endogenous equilibrium cutoffs. If, instead \( h_2 < \bar{h} \) then the citizen always wants the vanguard to revolt, i.e., \( z^* = \infty \). That is, the citizen’s preferred level of vanguard radicalism features a threshold effect.

The proposition shows that small changes in the citizen’s status quo payoff can dramatically change the citizen’s preferred vanguard. As long as the citizen’s status quo payoff \( h_2 \)
is not too low, he prefers a vanguard that only revolts whenever its signal indicates that revolution payoffs are high, i.e., $z^*$ is finite. When a vanguard revolts selectively, its action reveals information about the value of successful revolution, but sometimes it prevents revolution when the citizen wants the revolution to succeed. However, once a citizen’s status quo payoff falls below a critical threshold, he suddenly switches to preferring an extremely radical vanguard that always revolts, i.e., $z^* = \infty$. The intuition for the result is that the value of such information is bounded, and when the citizen’s status quo is sufficiently bad, having the option to change the regime dominates the value of the vanguard’s information.

This threshold effect sheds light on abrupt surges of support for vanguards with extreme interests in revolution. For example, in the late Pahlavi Regime, many politically-active Iranians supported the Liberation Movement of Iran (LMI) and the Nationalists who organized protests, but were far less eager than Khomeini’s faction to mount a revolution. However, in the years just preceding the Iranian Revolution, support for Khomeini grew so rapidly that even Bazargan, a founding member of the LMI, was marginalized for being too conservative in advocating revolution (Chehabi 1990; Katouzian 1981; Mottahedeh 1985). Our model suggests that the Shah’s increased authoritarian approach following the oil boom of the mid-1970s\textsuperscript{16} together with the subsequent economic crisis reduced status quo payoffs below the threshold that justified following a “cautious vanguard” that would initiate revolt selectively, causing many Iranians to switch support to Khomeini.

To understand how a citizen’s preferred level of radicalism in a vanguard varies with other characteristics, recognize that choosing $z$ amounts to choosing the equilibrium level of the vanguard’s cutoff $k_1$. From the citizen’s perspective, the optimal cutoff $k_1$ only depends on his payoffs $w_2$ and $h_2$. However, the vanguard’s payoffs $h_1$ and $l_1$ influence its willingness to revolt, and hence the level of radicalism that the citizen seeks in a vanguard. When the punishment for failed revolt rises, i.e., when $l_1$ is lower, the vanguard is more reluctant to revolt. To reduce $k_1$ back to the citizen’s preferred level, the citizen wants to increase $z$, i.e., $z^*$ is decreasing in $l_1$.\textsuperscript{17} The effect of raising the citizen’s disloyalty payoff $w_2$ is more complicated because the direct effect of raising $w_2$ is to induce the citizen to revolt less, i.e.,

\textsuperscript{16}In 1974, the secretary general of Amnesty International stated that “no country in the world has a worse record in human rights than Iran” (Bill 1988, p. 187).

\textsuperscript{17}Similarly, one can show that $z^*$ increases in $h_1$. 

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to increase $k_2$; and the impact of raising $k_2$ on $k_1$ depends on whether the vanguard’s best response exhibits strategic complements ($k_1$ rises) or strategic substitutes ($k_1$ falls).$^{18}$

**Vanguard Radicalism and Coercion.** A vanguard cannot typically choose its followers. However, in settings such as civil wars or guerrilla movements, a vanguard can punish those who do not follow its lead. For example, guerrillas contemplating an attack on a government military post near a village can subsequently punish the villagers if they do not cooperate (Kalyvas 2006; Wood 2003) by setting houses on fire or kidnapping them. Ahlquist and Levi (2011) highlight in their review of the leadership literature that vanguards can often use coercion to make others follow their lead, and yet “no model so far encapsulates noninformational tools available to leaders, such as coercion” (p. 14).

In the Leninist model (where coercion can be relevant), the vanguard’s use of coercion amounts to reducing the citizen’s payoff from $w_2$ to $w_2 - c$ when the vanguard revolts, but the citizen does not. How much coercion $c$ should a vanguard employ? We show that a vanguard does better to allow for some “dissent” in order to make more effective use of a citizen’s information. If the vanguard punishes a citizen severely whenever he does not provide support, the citizen revolts even when his information suggests that the outcome of successful revolution would be far worse than the status quo. This hurts the vanguard. Given this logic, one may conjecture that as the precision of a vanguard’s information rises and it grows more confident in its information, the vanguard feels less need to rely on the citizen’s information, and hence finds harsher coercive measures optimal. This reasoning is wrong: the citizen already accounts for the quality of a vanguard’s information in his strategic calculations. As a result, the vanguard’s optimal choice of coercion is unrelated to the quality of the citizen’s and vanguard’s information:

**Proposition 6.** If, prior to observing $s_1$, a vanguard can choose how much to punish the citizen when he does not provide support, then it chooses $c = \max\{w_2 - l_1 + z, 0\}$.

In most settings, the vanguard is more radical than the citizen and suffers more following a failed revolt. In such circumstances, $c = w_2 - l_1 + z > 0$, and the vanguard uses

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$^{18}$Formally, one can show the following result. Suppose $h_2 = h_1$, $w_2 > l_1$, and the vanguard’s best response features strategic substitutes. Then, $z^*$ is increasing in $w_2$.

$^{19}$If $w_2 + z < l_1$, a vanguard would like to compensate the citizen—if it could. This situation may arise in a civil war in which the government uses violence indiscriminately in a region with guerrilla activities.
more coercion when it is more radical, or when it risks a harsher punishment for initiating a failed revolt. To see the intuition, recall that the citizen joins the vanguard whenever $E[\theta | s_1 \geq k_1, s_2] \geq w_2$. By having the citizen internalize its payoff when the citizen does not support the revolution, the vanguard induces the citizen to make the decision that is optimal from the vanguard’s perspective based solely on the citizen’s information.

The result that a vanguard wants to exactly align the citizen’s preferences with its own contrasts with what a citizen wants in the vanguard. Most obviously, once the citizen’s status quo payoffs are low enough, the citizen wants an extremely radical vanguard that always revolts. This allows the citizen to avoid low status quo payoffs and to choose between the disloyalty and revolution payoffs, albeit at the cost of learning less about those revolution payoffs.

Paradoxically, the vanguard’s use of coercion can benefit the citizen by raising the vanguard’s confidence that the citizen will support it, making the vanguard more willing to initiate revolution. To highlight most transparently that a citizen can gain from coercion, suppose that $h_2 = h_1 = h$ and $z = 0$ so that with optimal coercion, the vanguard and citizen’s payoffs are the same. Then, when $E[\theta] >> h$ and $w_2$ is sufficiently large, not only the vanguard, but also the citizen, benefit from the vanguard’s coercion. When $w_2$ is very large, absent coercion, the vanguard almost never revolts, so the citizen and vanguard almost always receive $h$, and the citizen’s expected payoff is only marginally above $h$. With coercion, the vanguard revolts whenever its signal is high enough that it expects to gain over the status quo, and since the citizen’s payoff with coercion equals the vanguard’s, the citizen must also benefit.

The result that the quality of the vanguard’s information does not influence its use of coercion hinges on the assumption that the vanguard and citizen share common beliefs about the quality of each others’ signals. If, instead, the vanguard has more faith in its information than does the citizen, i.e., if the vanguard thinks that the citizen believes that the vanguard’s information is less precise than the vanguard believes its own information to be, then the vanguard wants to use more coercion to adjust for the citizen’s lesser incentive to revolt.\(^\text{20}\)

**Proposition 7.** Suppose $w_2 - l_1 + z > 0$, $E[\theta | s_1 \geq k_1, s_2]$ is strictly decreasing in the variance of $\theta | s_1$, and it is common knowledge that the vanguard and citizen disagree about the quality

\(^{20}\)In Proposition 7, the assumption that $E[\theta | s_1 \geq k_1, s_2]$ is strictly decreasing in the variance of $\theta | s_1$ is satisfied by the additive normal noise signal structure—see equation (9) in the Appendix.

However, in such scenarios, guerrillas likely cannot protect citizens who do not cooperate.
of the vanguard’s signal: the vanguard believes that \( \text{var}(\theta|s_1) = v_1 \), and the citizen believes that \( \text{var}(\theta|s_1) = v_2 \). Then, the vanguard’s choice of coercion, \( c(v_1, v_2) \), increases in \( v_2 - v_1 \), with \( c(v, v) = w_2 - l_1 + z \).

When the citizen believes that the vanguard’s information is less precise, then from the vanguard’s perspective, the citizen underweights the positive information about the revolution payoffs conveyed by the vanguard’s decision to revolt. To correct for this, the vanguard reduces the citizen’s payoff when he fails to support the vanguard. Many vanguards have strong ideological convictions that cause them to be overconfident in their knowledge, others live underground with limited contacts where “correlation neglect” (Della Porta 2013; Levy and Razin 2015) may cause them to put excessive weight on confirmations received from like-minded comrades. Proposition 7 shows that when these disparities are more severe, a vanguard uses harsher coercion to elicit the desired behavior from citizens.

5 Conclusion

Revolutionary vanguards, their radicalism and coercive actions, and their interactions with citizens and the state are common threads in narratives of revolutionary movements. But what are the defining features of revolutionary vanguards? We identify two main notions of vanguards and develop frameworks that capture their essence and explore their implications:

- **Conceptualization:** A key differentiating feature in various conceptions of vanguards is whether a vanguard is distinguished from other citizens by its protest expertise, or its skills in initiating anti-regime activities. This is Lenin’s idea of vanguards, which closely resembles the notion of revolutionary entrepreneurs in the literature. In contrast, in the alternative “early riser” notion of vanguards, the vanguard is not comprised of skilled revolutionaries; rather, the vanguard emerges organically in the protest process. That is, the vanguard is comprised of those citizens who have the most optimistic assessments of the risks and rewards of protest, and hence are the first to act.

- **Implications of Different Vanguard Notions:** We identify the differences in the outcomes of revolution that arise from these two vanguard notions. With the Leninist notion of a vanguard, a revolution is more likely to be initiated but less likely to win
support from citizens than with the early-riser notion of a vanguard. That is, revolutions that begin organically are more likely to succeed. A numerical analysis suggests that when the vanguard emerges organically, the likelihood of successful revolution is higher unless the punishment for failed revolt is small.

We then show how these frameworks can be used to study: (a) state strategies in mitigating a vanguard’s influence on citizens; (b) the sources of a vanguard radicalism; and (c) a vanguards’ use of coercion against citizens.

- **State Strategies and Vanguard Radicalism.** A more radical vanguard revolts more, but is less likely to generate a following. Paradoxically, a state that aims to prevent regime change may prefer radical to moderate vanguards (e.g., Assad’s double-game with ISIS), or set mild punishments for leaders of a failed revolt (e.g., the Shah’s punishment of Khomeini in the 1960s and 1970s).

- **Preferences over Vanguard Radicalism.** If the status quo deteriorates too low, moderate citizens prefer very radical vanguards who always revolt even though this blocks the flow of information from vanguards to citizens about the merits of a post-revolution regime. This result can reconcile the abrupt surges in support for radical vanguards that sometimes occur (e.g., in the years just before the Iranian Revolution when Khomeini’s support rose sharply at the expense of more moderate opposition).

- **Vanguard’s Use of Coercion.** In contrast, a vanguard never wants to block all information flow from citizens via their equilibrium actions. That is, a vanguard’s desire to use citizens’ information about the merits of revolution—to protect against truly bad outcomes—always moderates its use of punishment against citizens who do not cooperate. Still, if a vanguard is more radical or if it has more confidence in its knowledge than do citizens then it employs harsher coercion. This result can explain why ideological vanguards (e.g., Bolsheviks) or clandestinerevolutionary organizations who tend to be overconfident about their knowledge also tend to use more coercive measures.
6 Appendix

Lemma A.1. Suppose the vanguard sometimes revolts, i.e., that \( \rho_1(s_1) = R \) for some \( s_1 \). Then the citizen’s best response to \( \rho_1(\cdot) \) takes a cutoff form: There exists a finite cutoff \( k_2(\rho_1) \) such that the citizen revolts if and only if \( s_2 \geq k_2(\rho_1) \).

Proof of Lemma A.1: The citizen’s best response following \( a_1 \) and \( s_2 \) is to take action \( R \) if and only if \( E[\theta|\rho_1(s_1) = R, s_2] \geq w_2 \). The limit properties in Part (a) of A1 imply the existence of sufficiently good and bad signals, so that there exists a signal \( s_2 = k_2 \) such that \( E[\theta|\rho_1(s_1) = R, k_2] = w_2 \). Strict affiliation of signals implies that \( E[\theta|\rho_1(s_1) = R, s_2] > w_2, \forall s_2 > k_2 \), and \( E[\theta|\rho_1(s_1) = R, s_2] < w_2, \forall s_2 < k_2 \). \( \square \)

Lemma A.2. Suppose that \( \rho_2(s_2, R) = R \) if and only if \( s_2 \geq k_2 \). Then, given Assumption A1, there exists a \( k_1 \) such that the vanguard’s best response is to revolt if and only if \( s_1 \geq k_1 \).

Proof of Lemma A.2: Given the citizen’s cutoff \( k_2 \), the vanguard’s expected net payoff from revolt is \( \Delta(s_1; k_2) \equiv Pr(s_2 \geq k_2|s_1) (E[\theta|s_2 \geq k_2, s_1] + z) + Pr(s_2 < k_2|s_1)l_1 - h_1 \), which simplifies to

\[
\Delta(s_1; k_2) = Pr(s_2 \geq k_2|s_1) (E[\theta|s_2 \geq k_2, s_1] + z - l_1) + l_1 - h_1. \tag{3}
\]

From parts (a) and (b) of A1, \( \lim_{s_1 \to -\infty} \Delta(s_1; k_2) < 0 < \lim_{s_1 \to +\infty} \Delta(s_1; k_2) \). Moreover, both \( Pr(s_2 \geq k_2|s_1) \) and \( E[\theta|s_2 \geq k_2, s_1] \) rise with \( s_1 \) due to affiliation. Thus, from equation (3), if \( \Delta(s_1 = x; k_2) = 0 \), then \( \Delta(s_1; k_2) > 0, \forall s_1 > x \). Thus, for every \( k_2 \), there exists a unique \( s_1 = k_1 \) such that \( \Delta(k_1; k_2) = 0 \), with \( \Delta(s_1; k_2) > 0, \forall s_1 > k_1 \), and \( \Delta(s_1; k_2) < 0, \forall s_1 < k_1 \). Further,

\[
\frac{\partial \Delta(s_1; k_2)}{\partial s_1} \bigg|_{s_1=k_1} > 0. \tag{4}
\]

Proof of Lemma 1: Let \( k_2(k_1) \) be the citizen’s best response to the vanguard’s strategy with associated cutoff \( k_1 \). From Lemma A.1, \( E[\theta|s_1 \geq k_1, s_2 = k_2] - w_2 = 0 \). Thus,

\[
\frac{\partial k_2(k_1)}{\partial k_1} = -\left( \frac{\partial E[\theta|s_1 \geq k_1, s_2 = k_2]}{\partial s_2} \bigg|_{s_2=k_2} \right)^{-1} \frac{\partial E[\theta|s_1 \geq k_1, s_2 = k_2]}{\partial k_1}.
\]

By affiliation, both terms are positive, and hence \( \frac{\partial k_2(k_1)}{\partial k_1} < 0 \). \( \square \)

Proof of Lemma 2: Let \( k_1(k_2) \) be the vanguard’s best response cutoff to the citizen’s cutoff strategy with the associated cutoff \( k_2 \). To ease exposition, sometimes we drop the argument
of \( k_1(k_2) \) and write \( k_1 \). By the Implicit Function Theorem,
\[
\frac{\partial k_1(k_2)}{\partial k_2} = - \left( \frac{\partial \Delta(s_1; k_2)}{\partial s_1} \mid_{s_1 = k_1} \right)^{-1} \frac{\partial \Delta(s_1 = k_1; k_2)}{\partial k_2},
\]
where (4) in Lemma A.2 ensures that \( \frac{\partial \Delta(s_1; k_2)}{\partial s_1} \mid_{s_1 = k_1} > 0 \). Rewrite equation (3) as
\[
\Delta(s_1; k_2) = \int_{k_2}^{\infty} (E[\theta|s_2, s_1] + z) g(s_2|s_1) ds_2 + G(k_2|s_1) l_1 - h_1,
\]
where \( g(\cdot|s_1) \) is the PDF of \( s_2 \) conditional on \( s_1 \), and \( G(\cdot|s_1) \) is the corresponding CDF. Let \( \delta(x, y) \equiv E[\theta|s_2 = x, s_1 = y] + z - l_1 \). Thus,
\[
\frac{\partial \Delta(k_1; k_2)}{\partial k_2} = g(k_2|s_1 = k_1) ( -E[\theta|s_2 = k_2, s_1 = k_1] - z + l_1 ) = -g(k_2|s_1 = k_1) \delta(k_2, k_1),
\]
and hence from equations (4), (5), and (6), \( \text{sign} \left( \frac{\partial k_1}{\partial k_2} \right) = \text{sign} \left( \delta(k_2, k_1) \right) \). Next, we sign \( \delta \), establishing its monotonicity properties:
\[
\frac{d\delta(k_2, k_1(k_2))}{dk_2} = \frac{dE[\theta|s_2 = k_2, s_1 = k_1(k_2)]}{dk_2} = \frac{\partial E[\theta|k_2, k_1(k_2)]}{\partial k_2} \frac{\partial k_1}{\partial k_2} + \frac{\partial E[\theta|k_2, k_1(k_2)]}{\partial k_1} \frac{\partial \Delta(s_1; k_2)}{\partial s_1} \mid_{s_1 = k_1(k_2)} \frac{\partial \Delta(k_1; k_2)}{\partial k_2},
\]
where the third equality follows from equation (5) and the fourth from equation (6). Both \( \frac{\partial E[\theta|k_2, k_1]}{\partial k_1} \) and \( \frac{\partial E[\theta|k_2, k_1]}{\partial k_2} \) are positive because \( s_1, s_2, \) and \( \theta \) are affiliated; and \( \frac{\partial \Delta(s_1; k_2)}{\partial s_1} \mid_{s_1 = k_1} > 0 \) from equation (4). Thus, \( \frac{d\delta(k_2, k_1(k_2))}{dk_2} > 0 \) for all \( \delta \geq 0 \), which implies that \( \delta(k_2, k_1(k_2)) \) has a single-crossing property as a function of \( k_2 \). Next, we show \( \delta \) changes sign from negative (strategic substitutes) to positive (strategic complements). From equation (3) and A1, \( \lim_{k_2 \to -\infty} k_1(k_2) < \infty \), and hence \( \lim_{k_2 \to -\infty} \delta(k_2, k_1(k_2)) = -\infty \). Finally, we show that \( \lim_{k_2 \to -\infty} k_1(k_2) > -\infty \). Suppose not, so that \( \lim_{k_2 \to -\infty} k_1(k_2) = -\infty \). Then,
\[
\lim_{k_2 \to -\infty} \Delta(k_1(k_2), k_2) = \lim_{k_2 \to -\infty} Pr(s_2 \geq k_2|s_1 = k_1(k_2)) E[\theta|s_2 \geq k_2, s_1 = k_1(k_2)] - (h_1 - l_1) \\
\leq \lim_{k_2 \to -\infty} Pr(s_2 \geq k_2|s_1 = k_1(k_2)) E[\theta|s_2 \geq k_2] - (h_1 - l_1) \\
\leq \lim_{k_2 \to -\infty} Pr(s_2 \geq k_2) E[\theta|s_2 \geq k_2] - (h_1 - l_1) < 0,
\]
where the first equality follows from the fact that if \( \lim_{k_2 \to \infty} k_1(k_2) = -\infty \), then \( \lim_{k_2 \to \infty} Pr(s_2 \geq k_2 | s_1 = k_1)(z - l_1) = 0 \). The last inequality follows from the following observation:

\[
\lim_{k_2 \to \infty} Pr(s_2 > k_2) E[\theta | s_2 > k_2] = \lim_{k_2 \to \infty} \int_{\theta = -\infty}^{\infty} \theta \left( \int_{s_2 = k_2}^{\infty} pdf(\theta, s_2) ds_2 \right) d\theta
\]

\[
= \lim_{k_2 \to \infty} \int_{\theta = -\infty}^{\infty} \theta \left( \int_{s_2 = k_2}^{\infty} pdf(s_2 | \theta) ds_2 \right) pdf(\theta) d\theta
\]

\[
= \int_{\theta = -\infty}^{\infty} \theta \left( \lim_{k_2 \to \infty} \int_{s_2 = k_2}^{\infty} pdf(s_2 | \theta) ds_2 \right) pdf(\theta) d\theta
\]

\[
= \int_{\theta = -\infty}^{\infty} \theta \times 0 d\theta = 0.
\]

We can move the limit inside since substituting \( |\theta| \) for \( \theta \) and 1 for \( \int_{s_2 = k_2}^{\infty} pdf(s_2 | \theta) ds_2 \) yields:

\[
\int_{\theta = -\infty}^{\infty} \theta \left( \int_{s_2 = k_2}^{\infty} pdf(s_2 | \theta) ds_2 \right) pdf(\theta) d\theta \leq \int_{\theta = -\infty}^{\infty} |\theta| pdf(\theta) d\theta = E[|\theta|] < \infty.
\]

Inequality (8) a contradiction because \( \Delta(k_1(k_2), k_2) = 0 \). Thus, \( \lim_{k_2 \to \infty} k_1(k_2) > -\infty \), and hence \( \lim_{k_2 \to \infty} \delta(k_2, k_1(k_2)) = \lim_{k_2 \to \infty} E[\theta | s_2 = k_2, s_1 = k_1(k_2)] = \infty \). □

**Lemma A.3.** Assumption A2 holds with an additive, normal noise signal structure, \( s_i = \theta + \nu_i \), where \( \theta \), \( \nu_1 \) and \( \nu_2 \) are independently normally distributed with \( \theta \sim N(0, \sigma_0^2) \) and \( \nu_i \sim N(0, \sigma_i^2) \).

**Proof of Lemma A.3:** For \( i \in \{1, 2\} \), let \( b_i = \sigma_i^2 / (\sigma_0^2 + \sigma_i^2) \), \( a_i = \sqrt{1 + b_i} \sigma_i^2 \), and \( \Sigma = \sigma_0^2 \sigma_1^2 + \sigma_0^2 \sigma_2^2 + \sigma_1^2 \sigma_2^2 \). Then,

\[
E[\theta | k_i, s_j \geq k_j] = b_i k_i + \frac{\sigma_0^2 \sigma_i^2 a_i}{\Sigma} \frac{\phi(x_i)}{1 - \Phi(x_i)},
\]

where \( x_i = (k_j - b_i k_i) / a_i \) and \( \phi(x) \) and \( \Phi(x) \) are pdf and cdf of standard normal distribution, respectively. Let \( A(x) \equiv \frac{\partial}{\partial x} \frac{\phi(x)}{1 - \Phi(x)} \). Moreover, \( A(x) \in (0, 1) \) (Sampford 1953). Thus, Assumption A2 holds if and only if

\[
\left( 1 - \frac{\sigma_0^2 \sigma_1^2}{\Sigma} A(x_1) \right) \left( 1 - \frac{\sigma_0^2 \sigma_2^2}{\Sigma} A(x_2) \right) b_2 b_1 > \left( \frac{\sigma_0^2 \sigma_2^2}{\Sigma} A(x_2) \right) \left( \frac{\sigma_0^2 \sigma_1^2}{\Sigma} A(x_1) \right).
\]

That is,

\[
b_1 b_2 \left( 1 - \frac{\sigma_0^2 \sigma_1^2}{\Sigma} A(x_1) - \frac{\sigma_0^2 \sigma_2^2}{\Sigma} A(x_2) \right) > (1 - b_1 b_2) \frac{\sigma_0^2 \sigma_1^2}{\Sigma} \frac{\sigma_0^2 \sigma_2^2}{\Sigma} A(x_1) A(x_2).
\]
Next, observe that
\[ b_1 b_2 = \frac{\sigma_1^2}{(\sigma_0^2 + \sigma_1^2)(\sigma_0^2 + \sigma_2^2)}, \quad \text{and hence} \quad 1 - b_1 b_2 = \frac{\Sigma}{(\sigma_0^2 + \sigma_1^2)(\sigma_0^2 + \sigma_2^2)}. \]  
(11)

Substituting (11) into (10) and rearranging yields
\[ \Sigma - \sigma_0^2 \sigma_1^2 A(x_1) - \sigma_0^2 \sigma_2^2 A(x_2) > \sigma_1^2 \sigma_2^2 A(x_1)A(x_2), \]
which is true because \( A(x) \in (0, 1) \), and hence
\[ \Sigma - \sigma_0^2 \sigma_1^2 A(x_1) - \sigma_0^2 \sigma_2^2 A(x_2) > \Sigma - \sigma_0^2 \sigma_1^2 - \sigma_0^2 \sigma_2^2 = \sigma_1^2 \sigma_2^2 > \sigma_1^2 \sigma_2^2 A(x_1)A(x_2). \]
\[ \square \]

**Proof of Proposition 1:** From the proof of Lemma 2 recall that \( \lim_{k_1 \to +\infty} k_2(k_1) = -\infty \), \( \lim_{k_1 \to -\infty} k_2(k_1) \) is finite, and \( \lim_{k_2 \to +\infty} k_1(k_2) = -\infty \). Thus, the continuity of \( k_1(k_2) \) and \( k_2(k_1) \) implies that they cross, at least once.

Next, consider the citizen. His best response satisfies \( E[\theta|k_1, s_2 \geq k_2] = l_1 - z \), and hence, for the vanguard’s best response,
\[ \frac{\partial k_1(k_2)}{\partial k_2} = -\left( \frac{\partial E[\theta|k_1, s_2 \geq k_2]}{\partial k_1} \right)^{-1} \frac{\partial E[\theta|k_1, s_2 \geq k_2]}{\partial k_2}. \]  
(12)

Next, consider the citizen. His best response satisfies \( E[\theta|s_1 \geq k_1, k_2] = w_2 \). Similar calculations for the citizen’s best response yields
\[ \frac{\partial k_2(k_1)}{\partial k_1} = -\frac{\partial E[\theta|s_1 \geq k_1, k_2]}{\partial k_1} \left( \frac{\partial E[\theta|s_1 \geq k_1, k_2]}{\partial k_2} \right)^{-1}. \]  
(13)

To prove uniqueness, it suffices to show that the vanguard’s best response curve in \((k_1, k_2)\)-space, for all relevant \( k_1 \)'s, always has a sharper negative slope that the citizen’s. That is, the inverse of (12) is a larger negative number that (13), i.e.,
\[ -\frac{\partial E[\theta|s_1 \geq k_1, k_2]}{\partial k_1} \left( \frac{\partial E[\theta|s_1 \geq k_1, k_2]}{\partial k_2} \right)^{-1} > -\frac{\partial E[\theta|k_1, s_2 \geq k_2]}{\partial k_1} \left( \frac{\partial E[\theta|k_1, s_2 \geq k_2]}{\partial k_2} \right)^{-1}. \]

Due to affiliation all terms are positive, and hence rearrangement yields
\[ \frac{\partial E[\theta|s_1 \geq k_1, k_2]}{\partial k_1} \left( \frac{\partial E[\theta|k_1, s_2 \geq k_2]}{\partial k_2} \right) < \frac{\partial E[\theta|k_1, s_2 \geq k_2]}{\partial k_1} \left( \frac{\partial E[\theta|s_1 \geq k_1, k_2]}{\partial k_2} \right), \]
which holds by Assumption A2. If \( h_1 > l_1 \), then the slope of the strategic substitute segment of the vanguard’s best response becomes even more negative. Thus, if a crossing
happens on the strategic substitutes segment of the vanguard’s best response, it is unique. Finally, clearly, crossing can happen only once on the strategic complements segment of the vanguard’s best response. □

**Proof of Proposition 2:** Citizen 1’s expected net payoff from revolting versus not revolting at date 1 when he receives signal $s_1$ and citizen 2 sets cutoffs $\alpha, \beta$ and $\gamma$ is:

$$Pr(s_2 \geq \min\{\alpha, \beta\}|s_1) \ E[\theta|s_1, s_2 \geq \min\{\alpha, \beta\}] + Pr(s_2 < \min\{\alpha, \beta\}|s_1)l$$

$$- Pr(s_2 \geq \alpha|s_1) \ \max\{w, E[\theta|s_2 \geq \alpha]\} - Pr(s_2 < \alpha|s_1) v,$$

(14)

where $v > l$ is the expected continuation payoff to citizen 1 if neither citizen initiates revolt at date 1, and we note that $v = h$ is always an equilibrium, supported by beliefs that there will be no revolt at date 2, if there is none at date 1.

At a symmetric equilibrium, optimization by citizen 1 requires that when $s_1 = \alpha$, he be indifferent between actions:

$$Pr(s_2 \geq \min\{\alpha, \beta(\alpha)\}|s_1 = \alpha)\ E[\theta|s_1 = \alpha, s_2 \geq \min\{\alpha, \beta(\alpha)\}] - l] + l$$

$$= Pr(s_2 \geq \alpha|s_1 = \alpha)\ \max\{w, E[\theta|s_2 \geq \alpha]\} - v] + v,$$

(15)

where best-responding when citizen 2 revolts for signals $s_2 \geq \alpha$ means that $\beta(\alpha)$ solves

$$E[\theta|s_2 \geq \alpha, s_1 = \beta(\alpha)] = w.$$  

(16)

The left-hand side of (15) is citizen 1’s expected payoff if he revolts at date 1 following signal $s_1 = \alpha$. With probability $Pr(s_2 \geq \min\{\alpha, \beta(\alpha)\}|s_1 = \alpha)$, citizen 2 revolts too—either at date 1 or date 2—and the revolution succeeds, yielding an expected payoff of $E[\theta|s_1 = \alpha, s_2 \geq \min\{\alpha, \beta(\alpha)\}]$ from citizen 1’s perspective. With the remaining probability, citizen 2 does not revolt, and citizen 1 gets $l$. The right-hand side captures citizen 1’s expected payoff if he does not revolt at date 1. With probability $Pr(s_2 \geq \alpha|s_1)$, citizen 2 initiates revolt at date 1. At date 2, citizen 1 revolt if his expected payoff from revolting $E[\theta|s_1 = \alpha, s_2 \geq \alpha]$ exceeds that of not revolting $w$. With the remaining probability, no one revolts at date 1, and both citizens receive $v$.

If $\beta(\alpha) \geq \alpha$, then equation (15) becomes

$$Pr(s_2 \geq \alpha|s_1 = \alpha)\ E[\theta|s_1 = \alpha, s_2 \geq \alpha] - l] + l$$

$$= Pr(s_2 \geq \alpha|s_1 = \alpha)\ \max\{w, E[\theta|s_2 \geq \alpha]\} - v] + v.$$
A contradiction immediately obtains since \( v > l \), and \( \max\{w, E[\theta|s_1 = \alpha, s_2 \geq \alpha]\} \geq E[\theta|s_1 = \alpha, s_2 \geq \alpha] \). Hence, \( \alpha > \beta(\alpha) \), and equation (15) becomes

\[
Pr(s_2 \geq \beta(\alpha)|s_1 = \alpha)[E[\theta|s_1 = \alpha, s_2 \geq \beta(\alpha)] - l] + l
= Pr(s_2 \geq \alpha|s_1 = \alpha)[\max\{w, E[\theta|s_1 = \alpha, s_2 \geq \alpha]\} - v] + v.
\]

Next use \( E[\theta|s_2 \geq \alpha, s_1 = \beta(\alpha)] = w \) together with \( \alpha > \beta(\alpha) \) to see that \( \max\{w, E[\theta|s_1 = \alpha, s_2 \geq \alpha]\} = E[\theta|s_1 = \alpha, s_2 \geq \alpha] \). Thus, equation (15) becomes

\[
Pr(s_2 \geq \beta(\alpha)|s_1 = \alpha)[E[\theta|s_1 = \alpha, s_2 \geq \beta(\alpha)] - l] + l
= Pr(s_2 \geq \alpha|s_1 = \alpha)[E[\theta|s_1 = \alpha, s_2 \geq \alpha] - v] + v, \quad \text{or equivalently,}
= Pr(s_2 \geq \alpha|s_1 = \alpha) \{E[\theta|s_1 = \alpha, s_2 \geq \beta(\alpha)] - l\} - Pr(s_2 < \alpha|s_1 = \alpha)(v - l)
= Pr(s_2 \geq \alpha|s_1 = \alpha) \{E[\theta|s_1 = \alpha, s_2 \geq \alpha] - l\}. \tag{17}
\]

To establish that there exist \((\alpha^*, \beta(\alpha^*))\) that satisfy equations (16) and (17), first observe that \( \beta(\alpha) \) is continuous and strictly decreasing in \( \alpha \), with \( \lim_{\alpha \to -\infty} \beta(\alpha) > -\infty \), and \( \lim_{\alpha \to +\infty} \beta(\alpha) = -\infty \). Thus, there exists a unique \( \alpha_\beta \) such that \( \beta(\alpha_\beta) = \alpha_\beta \). Further, so long as \( v > l \), at \( \alpha_\beta \), the left-hand side of (17) is less than the right-hand side; and, as \( \alpha \to \infty \), the left-hand side goes to infinity, while the right-hand side goes to zero. By continuity, there exists \((\alpha^*, \beta(\alpha^*))\) that satisfies equations (16) and (17).

When \( \gamma < \alpha \) is a posited cutoff for revolt in the second date given that no one revolted in the first date, equilibrium demands that

\[
Pr(s_2 < \gamma|s_1 = \gamma, s_2 < \alpha) l + Pr(s_2 \geq \gamma|s_1 = \gamma, s_2 < \alpha) E[\theta|s_1 = \gamma, \gamma \leq s_2 < \alpha]
= Pr(s_2 < \gamma|s_1 = \gamma, s_2 < \alpha) h + Pr(s_2 \geq \gamma|s_1 = \gamma, s_2 < \alpha) w.
\]

Substitute in for \( w = E[\theta|s_1 \geq \alpha, s_2 = \beta(\alpha)] \) and re-arrange:

\[
Pr(s_2 \geq \gamma|s_1 = \gamma, s_2 < \alpha) \{E[\theta|s_1 = \gamma, \gamma \leq s_2 < \alpha] - E[\theta|s_1 \geq \alpha, s_2 = \beta(\alpha)]\}
= Pr(s_2 < \gamma|s_1 = \gamma, s_2 < \alpha) (h - l) > 0.
\]

For the left-hand side to be positive, it must be that

\[
E[\theta|s_1 = \gamma, \gamma \leq s_2 < \alpha] > E[\theta|s_1 \geq \alpha, s_2 = \beta(\alpha)] . \tag{18}
\]
Since \( \alpha > \gamma \) is necessary for a revolt at date 2 when there is no revolt at date 1, the symmetry of \( f(s_1, s_2) \) then implies that for (18) to hold, we must have \( \alpha > \gamma > \beta(\alpha) \).

It remains to verify that at the equilibrium level of \( \alpha \)

\[
Pr(s_2 \geq \beta(\alpha)|s_1) \{E[\theta|s_1, s_2 \geq \beta(\alpha)] - l\} - Pr(s_2 < \alpha|s_1)(v - l)
- Pr(s_2 \geq \alpha|s_1) \{E[\theta|s_1, s_2 \geq \alpha] - l\}
\]

is positive for all \( s_1 > \alpha \), and negative for all \( s_1 < \alpha \). Equivalently, we must sign

\[
Pr(\beta(\alpha) \leq s_2 < \alpha|s_1) \{E[\theta|s_1, \beta(\alpha) \leq s_2 < \alpha]\} - Pr(\beta(\alpha) \leq s_2 < \alpha|s_1) v,
\]

which we rewrite as

\[
Pr(\beta(\alpha) \leq s_2 < \alpha|s_1) (E[\theta|s_1, \beta(\alpha) \leq s_2 < \alpha] - v) - Pr(s_2 < \beta(\alpha)|s_1) (v - l),
\]

which is equivalent to signing

\[
[E[\theta|s_1, \beta(\alpha) \leq s_2 < \alpha] - v] - \frac{Pr(s_2 < \beta(\alpha)|s_1)}{Pr(\beta(\alpha) \leq s_2 < \alpha|s_1)} (v - l).
\]

**Case I: Constant** \( v \). The bracketed term increases in \( s_1 \). The last term falls with \( s_1 \). To see this, let \( A \equiv Pr(s_2 < \alpha|s_1) \) and \( B \equiv Pr(s_2 < \beta|s_1) \), so that \( \frac{Pr(s_2 < \beta|s_1)}{Pr(\beta \leq s_2 < \alpha|s_1)} = \frac{B}{A-B} \).

Differentiating with respect to \( s_1 \) yields \( \frac{B'(A-B)-(A-B)'B}{(A-B)^2} = \frac{B'A-A'B}{(A-B)^2} \), which is negative if and only if \( \frac{B'}{A-B} < \frac{A'}{A} \). Thus, a sufficient condition for \( \frac{B}{A-B} \) to be decreasing in \( s_1 \) for all \( \alpha \) and \( \beta \) with \( \beta < \alpha \) is that \( \frac{\partial}{\partial x} \frac{\partial \ln[Pr(s_2 > x|s_1)]]}{\partial s_1} > 0 \), i.e., \( \frac{\partial}{\partial s_1} \frac{\partial \ln[Pr(s_2 > x|s_1)]]}{\partial x} = \frac{\partial}{\partial s_1} \frac{PDF(s_2=x|s_1)}{CDF(s_2=x|s_1)} > 0 \). But this follows from our assumption that \( s_1 \) and \( s_2 \) are affiliated, i.e., they have the monotone likelihood ratio property.

**Case II: non-constant** \( v \). Dropping dependence on \( \alpha \)

\[
v = \begin{cases} 
  h & ; s_1 < \gamma \\
  Pr(s_2 < \gamma|s_1, s_2 < \alpha) \cdot l + Pr(\gamma < s_2|s_1, s_2 < \alpha) \cdot E[\theta|s_1, \gamma < s_2 < \alpha] & ; s_1 > \gamma,
\end{cases}
\]

which one can rewrite as

\[
v = \begin{cases} 
  h & ; s_1 < \gamma \\
  \frac{Pr(s_2 < \gamma|s_1) \cdot l + Pr(\gamma < s_2 < \alpha|s_1) \cdot E[\theta|s_1, \gamma < s_2 < \alpha]}{Pr(s_2 < \alpha|s_1)} & ; s_1 > \gamma.
\end{cases}
\]

We have shown that a necessary condition for \( v \neq h \), is \( \beta < \gamma < \alpha \). Rewrite (19) as

\[
Pr(\beta \leq s_2 < \alpha|s_1) E[\theta|s_1, \beta \leq s_2 < \alpha] + Pr(s_2 < \beta|s_1) l - Pr(s_2 < \alpha|s_1) v,
\]

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and then substitute for $v$ into this expression to obtain

$$Pr(\beta \leq s_2 < \alpha|s_1) \ E[\theta|s_1, \beta \leq s_2 < \alpha] + Pr(s_2 < \beta|s_1) \ l$$

$$- \begin{cases} 
   Pr(s_2 < \alpha|s_1) \ h & ; s_1 < \gamma \\
   Pr(s_2 < \gamma|s_1) + Pr(\gamma < s_2 < \alpha|s_1) \ E[\theta|s_1, \gamma < s_2 < \alpha] & ; s_1 > \gamma,
\end{cases}$$

Then, if $s_1 > \gamma$, the above expression becomes

$$Pr(\beta \leq s_2 < \alpha|s_1) \ E[\theta|s_1, \beta \leq s_2 < \alpha] + Pr(s_2 < \beta|s_1) \ l$$

$$- \begin{cases} 
   Pr(s_2 < \gamma|s_1) \ l + Pr(\gamma < s_2 < \alpha|s_1) \ E[\theta|s_1, \gamma < s_2 < \alpha] & ; s_1 > \gamma,
\end{cases}$$

Hence,

$$\begin{cases} 
   Pr(\beta \leq s_2 < \alpha|s_1) \ (E[\theta|s_1, \beta \leq s_2 < \alpha] - h) - Pr(s_2 < \beta|s_1) \ (h - l), & ; s_1 < \gamma \\
   Pr(\beta \leq s_2 < \gamma|s_1) \ (E[\theta|s_1, \beta \leq s_2 < \gamma] - l) & ; s_1 > \gamma,
\end{cases}$$

We have established that the first line for $s_1 \in \mathbb{R}$ has a single crossing property. The second line, obviously has that too, and hence, if is has a solution, then it is unique. Further, payoffs are continuous at $s_1 = \gamma$, implying a unique solution in $s_1$. That a solution exists follows from the limiting properties of the expectations as $s_1 \to \pm \infty$. □

**Proof of Proposition 3:** Index cutoffs in the exogenous and endogenous settings by $ex$ and $en$, respectively, so that $k_1 = \alpha_{ex}$ and $k_2 = \beta_{ex}$. In both settings, the best response function following a revolt at date 1 is the same: $\beta_{ex}(\cdot) = \beta_{en}(\cdot)$. Thus, $\beta_{ex}(\alpha_{ex}) < \beta_{en}(\alpha_{en})$ if and only if $\alpha_{ex} > \alpha_{en}$. Moreover, in the exogenous setting,

$$Pr(s_2 \geq \beta_{ex}|s_1 = \alpha_{ex}) \ E[\theta|s_1 = \alpha_{ex}, s_2 \geq \beta_{ex}] + Pr(s_2 < \beta_{ex}|s_1 = \alpha_{ex}) \ l = h, \quad (21)$$

which has a unique solution by Proposition 1. In the endogenous setting,

$$Pr(s_2 \geq \beta_{en}|s_1 = \alpha_{en}) \ E[\theta|s_1 = \alpha_{en}, s_2 \geq \beta_{en}] + Pr(s_2 < \beta_{en}|s_1 = \alpha_{en}) \ l$$

$$= \begin{cases} 
   Pr(s_2 \geq \alpha_{en}|s_1 = \alpha_{en}) \ E[\theta|s_1 = \alpha_{en}, s_2 \geq \alpha_{en}] + Pr(s_2 < \alpha_{en}|s_1 = \alpha_{en}) \ v > h.
\end{cases}$$

The left-hand sides of (21) and (22) are equal when evaluated at the same $\alpha$, but the right-hand side of (22) exceeds that of (21) because $v \geq h$ and $E[\theta|s_1 = \alpha_{en}, s_2 \geq \alpha_{en}] > E[\theta|s_1 = \beta_{en}, s_2 \geq \alpha_{en}] = h$. When evaluated at $\alpha_{en}$, the left-hand side of (21) exceeds the right-hand
Moreover, from proposition 1, the left-hand side of (21) crosses $h$ from below at a unique point $\alpha_{ex}$, and hence $\alpha_{ex} < \alpha_{en}$. □

**Proof of Proposition 4:** From equation (3), $\frac{\partial \Delta(k_1;k_2,z)}{\partial z} = Pr(s_2 \geq k_2|s_1 = k_1) > 0$. Thus, $-\frac{\partial k_1(z)}{\partial z} = -(\frac{\partial \Delta(k_1;k_2,z)}{\partial k_1})^{-1} \frac{\partial \Delta(k_1;k_2,z)}{\partial z} < 0$ at any best response cutoff $k_1$, including the equilibrium cutoff. Moreover, since the citizen’s best response exhibits strategic substitutes (Lemma 1), his equilibrium cutoff decreases in $z$, $0 < \frac{\partial k_1(z)}{\partial z}$. □

**Proof of Proposition 5:** Let $E[U_2|k_1,k_2]$ be the citizen’s ex ante expected utility given equilibrium cutoffs $k_1$ and $k_2$. Then

$$\frac{dE[U_2|k_1,k_2]}{dz} = \left( \frac{\partial E[U_2|k_1,k_2]}{\partial k_1} + \frac{\partial E[U_2|k_1,k_2]}{\partial k_2} \right) \frac{dk_1}{dz} \quad (23)$$

where we use $\frac{\partial E[U_2|k_1,k_2]}{\partial k_2} = 0$ because $k_2$ is the citizen’s equilibrium cutoff. Moreover,

$$E[U_2|k_1,k_2] = Pr(s_1 < k_1)h_2 + Pr(s_1 \geq k_1, s_2 < k_2)w_2 + Pr(s_1 \geq k_1, s_2 \geq k_2) E(\theta|s_1 \geq k_1, s_2 \geq k_2)$$

$$= Pr(s_1 < k_1)h_2 + w_2 \int_{s_1=k_1}^{\infty} \int_{s_2=-\infty}^{k_2} f(s_1,s_2) ds_1 ds_2$$

$$+ \int_{s_1=k_1}^{\infty} \int_{s_2=k_2}^{\infty} E(\theta|s_1, s_2) f(s_1,s_2) ds_1 ds_2,$$

where $f(s_1,s_2)$ is the joint of $s_1$ and $s_2$. Let $h(\cdot)$ be the pdf of $s_1$, and recall that $g(\cdot|s_1)$ is the pdf of $s_2$ conditional on $s_1$. Hence, $\frac{\partial E[U_2|k_1,k_2]}{\partial k_1} = h(k_1) h_2 - w_2 h(k_1) \int_{-\infty}^{k_2} g(s_2|s_1=k_1) ds_2 - h(k_1) \int_{k_2}^{\infty} E(\theta|s_1 = k_1, s_2) g(s_2|s_1 = k_1) ds_2$ (24)

$$= h(k_1) \{h_2 - w_2 Pr(s_2 < k_2|s_1 = k_1) - Pr(s_2 \geq k_2|s_1 = k_1) E(\theta|s_2 \geq k_2, s_1 = k_1)\} \quad (25)$$

We proceed in a number of steps. First, substituting $\lim_{k_1 \to -\infty} k_2(k_1) = -\infty$ into (24) reveals that $\frac{\partial E[U_2|k_1,k_2]}{\partial k_1} < 0$ for $k_1$ sufficiently large. Thus, $\frac{dE[U_2|k_1,k_2]}{dz} > 0$ for sufficiently negative $z$, implying that $z^* > -\infty$. Second, since $E(\theta|s_1 \geq k_1, s_2 = k_2) = w_2$ and $\lim_{k_1 \to -\infty} k_2(k_1)$ is finite, $\lim_{k_1 \to -\infty} E(\theta|k_1, s_2 \geq k_2) = -\infty$ and $\lim_{k_1 \to -\infty} E(\theta|s_1 \geq k_1, k_2) = w_2$. Substituting these limits into (25) reveals that when $h_2 = w_2$, $\frac{\partial E[U_2|k_1,k_2]}{\partial k_1} > 0$ for sufficiently negative $k_1$. Therefore, when $h_2 = w_2$, $\frac{dE[U_2|k_1,k_2]}{dz} < 0$ for all sufficiently large $z$, i.e., $z^* < \infty$ when
$h_2 = w_2$. Third, $h_2$ does not affect $k_1$ or $k_2$. Thus, if $Pr(s_2 \geq k_2|s_1 = k_1)E[\theta|s_2 \geq k_2, s_1 = k_1]$ is bounded from below as we vary $z$, then, from (24), $\frac{\partial E[U_2|k_1,k_2]}{\partial k_1}$ is always negative for sufficiently negative $h_2$ (holding $w_2$ fixed). Therefore, $\frac{\partial E[U_2|k_1,k_2]}{\partial z}$ is always positive, and hence $z^* = \infty$. Moreover, assumption A1(c) implies $Pr(s_2 \geq k_2(k_1)|s_1 = k_1)E[\theta|s_2 \geq k_2(k_1), s_1 = k_1]$ is bounded from below for all $z$. To see this, note that $\lim_{k_1 \to -\infty} k_2(k_1) = k \in \mathbb{R}$, and hence $\lim_{k_1 \to -\infty} Pr(s_2 \geq k_2|s_1 = k_1)E[\theta|s_2 \geq k_2, s_1 = k_1] \approx \lim_{k_1 \to -\infty} Pr(s_2 \geq k|s_1 = k_1)E[\theta|s_2 \geq k, s_1 = k_1] > -\infty$ where the last inequality follows from A1(c). Fourth, from (24), $\frac{\partial E[U_2|k_1,k_2]}{\partial k_1}$ rises in $h_2$ because $h_2$ does not affect $k_1$ or $k_2$. Hence, $z^*$ falls with $h_2$. Thus, there exists $\tilde{h} \in \mathbb{R}$ such that $z^*$ is finite if and only if $h_2 > \tilde{h}$.

Next, we derive $z^*$ when it is finite. Because $k_1$ is the vanguard’s equilibrium cutoff,

$$Pr(s_2 \geq k_2|s_1 = k_1)E[\theta|s_2 \geq k_2, s_1 = k_1] = h_1 - l_1 Pr(s_2 < k_2|s_1 = k_1) - Pr(s_2 \geq k_2|s_1 = k_1)z.$$ 

Substituting for $E[\theta|s_2 \geq k_2, s_1 = k_1]$ from this equation into (24) yields $\frac{1}{h(k_1)} \frac{\partial E[U_2|k_1,k_2]}{\partial k_1} = h_2 - w_2 Pr(s_2 < k_2|s_1 = k_1) - h_1 + l_1 Pr(s_2 < k_2|s_1 = k_1) + Pr(s_2 \geq k_2|s_1 = k_1)z$

$$= (h_2 - h_1) - (w_2 - l_1) Pr(s_2 < k_2|s_1 = k_1) + Pr(s_2 \geq k_2|s_1 = k_1)z.$$ 

Combining equations (23) and (26) yields

$$\frac{dE[U_2]}{dz} = \frac{dk_1}{dz} h(k_1) [(h_2 - h_1) - Pr(s_2 < k_2|s_1 = k_1) (w_2 - l_1) + Pr(s_2 \geq k_2|s_1 = k_1) z].$$ 

Solving this first-order condition yields the result in the proposition. □

**Proof of Proposition 6:** Let $E[U_1|k_1,k_2]$ be the vanguard’s ex ante expected utility given cutoffs $k_1$ and $k_2$:

$$E[U_1|k_1,k_2] = Pr(s_1 \geq k_1, s_2 \geq k_2) (E[\theta|s_1 \geq k_1, s_2 \geq k_2] + z)$$ 

$$+ Pr(s_1 \geq k_1, s_2 < k_2) l_1 + Pr(s_1 < k_1) h_1$$ 

$$= Pr(s_1 \geq k_1) \int_{s_2=k_2}^{\infty} (E[\theta|s_1 \geq k_1, s_2] + z) g(s_2|s_1 \geq k_1) ds_2$$ 

$$+ Pr(s_1 \geq k_1) Pr(s_2 < k_2|s_1 \geq k_1) l_1 + Pr(s_1 < k_1) h_1.$$ 

(27)

In equilibrium, $k_1$ and $k_2$ are best responses, and hence $\frac{\partial E[U_1|k_1,k_2]}{\partial k_1} = 0$. Moreover, from Lemmas A.1 and A.2, only $k_2$ explicitly depends on $w_2$. Thus,

$$\frac{\partial E[U_1|k_1(k_2(w_2)),k_2(w_2)]}{\partial w_2} = \frac{\partial E[U_1]}{\partial k_2} \frac{\partial k_2}{\partial w_2} + \frac{\partial E[U_1]}{\partial k_1} \frac{\partial k_1}{\partial w_2}$$ 

$$= \frac{\partial E[U_1]}{\partial k_2} \frac{\partial k_2}{\partial w_2}.$$ 

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It is easy to see that $\frac{\partial k_2}{\partial w_2} > 0$, and hence the first order condition, $\frac{\partial E[U_1|k_1(k_2(w_2)),k_2(w_2)]}{\partial w_2} = 0$, holds if and only if $\frac{\partial E[U_1]}{\partial k_2} = 0$. From equation (27),

$$\frac{\partial E[U_1]}{\partial k_2} = -Pr(s_1 \geq k_1) (E[\theta|s_1 \geq k_1, k_2] + z) g(k_2|s_1 \geq k_1) + f(k_2|s_1 \geq k_1) Pr(s_1 \geq k_1) l_1$$

$$= Pr(s_1 \geq k_1) g(k_2|s_1 \geq k_1) (l_1 - z - E[\theta|s_1 \geq k_1, k_2]).$$

Thus, $\frac{\partial E[U_1]}{\partial k_2} = 0$ if and only if $E[\theta|s_1 \geq k_1(k_2(w_2)), k_2(w_2)] = l_1 - z$. Moreover, from Lemma A.1, $E[\theta|s_1 \geq k_1(k_2(w_2)), k_2(w_2)] = w_2$. Thus, the first order condition holds if and only if $w_2 = l_1 - z$. It is easy to see that this is a maximum. Thus, $\frac{\partial E[U_1]}{\partial k_2} = 0$ if and only if $E[\theta|s_1 \geq k_1(k_2(w_2)), k_2(w_2)] = l_1 - z$. Moreover, from Lemma A.1, $E[\theta|s_1 \geq k_1(k_2(w_2)), k_2(w_2)] = w_2$. Thus, the first order condition holds if and only if $w_2 = l_1 - z$. It is easy to see that this is a maximum. Thus, the vanguard wants to choose $c$ such that $w_2 - c = l_1 - z$, i.e., $c = w_2 - l_1 + z$. Clearly, when we restrict $c$ to be positive, $c = \max\{w_2 - l_1 + z, 0\}$. □

**Proof of Proposition 7:** Let $E_v$ be an agent’s expectation when the agent believes that $\text{var}(\theta|s_1) = v$. By assumption, $E_v[\theta|s_1 \geq k_1(k_2(w_2 - c)), k_2(w_2 - c)]$ is decreasing in $v$. The equilibrium level of $k_2$ is (uniquely) determined by $E_{v_2}[\theta|s_1 \geq k_1(k_2(w_2 - c)), s_2 = k_2(w_2 - c)] = w_2 - c$, where we have made explicit that $k_2$ depends on $w_2 - c$. From Proposition 6, the vanguard’s optimal choice is to pick a $c$ that yields a $k_2(w_2 - c)$ that satisfies $E_{v_1}[\theta|s_1 \geq k_1(k_2(w_2 - c)), s_2 = k_2(w_2 - c)] = l_1 - z$. Because these expectations are increasing in $s_2$, the vanguard’s optimal $c$ is increasing in $v_2 - v_1$. Clearly, if $v_1 = v_2$, then $w_2 - c = l_1 - z$. □

### 7 References


