Extremism in Revolutionary Movements

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Abstract

A revolutionary entrepreneur strategically chooses the revolutionary agenda to maximize the likelihood of revolution. Citizens have different preferences and can contribute varying degrees of support. We show: (1) Extremists exert a disproportionate influence over the revolutionary agenda; (2) Depending on the structure of repression, more severe repression can moderate or radicalize the revolutionary agenda. Specifically, increases in the “minimum punishment” (marginal cost of revolutionary effort at its minimum) radicalize the revolutionary agenda. This presents the elite with a tradeoff between extreme but unlikely revolutions and moderate but likely ones. (3) Competition between revolutionary entrepreneurs can radicalize the revolutionary agenda.

Keywords: Revolution, Revolutionary Entrepreneurs, Extremism, Repression, Repression Backlash, Competition.

JEL Classification: D74.

1. Introduction

Revolutionary movements vary in terms of the extent of the change they demand: Some are more extreme than others. During the 1979 Iranian Revolution, people were demanding a radical change that would involve a new constitution and a major restructuring of political, social, and economic institutions of Iran. Thirty years later, when election officials announced Ahmadinezhad’s victory in the 2009 Iranian presidential election, protests broke out throughout the country. However, the protesters had a much more moderate demand: They wanted a fair election between Mousavi and Ahmadinezhad because they believed Mousavi would have won if it was not for election fraud. In France, the 1830 Revolution was much more moderate than the 1848 Revolution. While the 1848 Revolution ended monarchy and established the Second Republic, “the change of men was probably
the most revolutionary aspect of the [1830] revolution” (Pinkney 1973, p. 276). It caused “no fundamental shift in the seat of power and resulted in no mandate for such change” (p. 367).

What determines the extremism of the revolutionary agenda?1 The literature has largely focused on coordination and free-riding issues that arise in revolutions and protests, ignoring the endogenous choice of the revolutionary agenda. We develop a simple model in which the revolutionary agenda is determined by revolutionary entrepreneurs in their interactions with citizens and the state. Our main contribution is to identify three factors that influence the extremism of the revolutionary agenda: (1) Citizens with extreme preferences exert a disproportionate influence over the revolutionary agenda; (2) Depending on the structure of state repression, higher repression can moderate or radicalize the revolutionary agenda; (3) Competition between two revolutionary entrepreneurs can radicalize the revolutionary agenda.

In addition to endogenizing the revolutionary agenda, we depart from the theoretical literature by allowing citizens to exert a continuum of effort toward changing the status quo.2 This “dissent gradation,” when combined with citizens’ heterogeneous preferences, generates a tradeoff for the revolutionary entrepreneur who chooses the revolutionary agenda.3 A more radical agenda is supported by fewer citizens, but the extremists who do support it exert more effort towards its success. That is, the revolutionary entrepreneur’s choice of revolutionary agenda features a tradeoff between

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1 Revolutionary agenda refers to the policies and programs that the revolution seeks to implement. A revolutionary agenda is more extreme if it seeks a larger change from the status quo. Precisely, we define extremism as the distance between the status quo policy and the revolutionary agenda—using Euclidean metric in a one-dimensional policy space. This “ideological extremism” is distinct from “tactical extremism,” which refers to the use of violent tactics.

2 The literature typically models revolutionary activities as a costly binary choice in which citizens decide whether or not to participate in the revolution (Acemoglu & Robinson 2001, 2006; Angeletos et al. 2007; Boix & Svolik 2013; Bueno de Mesquita and Smith 2014; Bueno de Mesquita 2010, 2013; Buenrostro et al. 2007; Casper & Tyson 2014, 2015; Chamley 1999; Chen et al. 2014; Chen & Xu 2014, 2015; DeNardo 1985; Edmond 2013; Egorov et al. 2009; Ellis & Foder 2011; Epstein et al. 2012; Fearon 2011; Gehlbach 2013; Ginkel & Smith 1999; Hollyer et al. 2013; Leventoglu 2005; Loepel et al. 2014; Little et al. 2013; Lohmann 1994; Meirowitz & Tucker 2013; Persson & Tabellini 2009; Rubin 2014; Shadmeir 2014; Shadmeir & Bernhardt 2011, 2014, 2015; Tyson & Smith 2014). However, the empirical and qualitative literature emphasizes that dissent comes in different intensities, and dissidents exhibit varying degrees of support for a revolutionary movement. For example, in the El Salvadoran civil war, some individuals participated in strikes and demonstrations, while others risked severe punishment by smuggling food and water to the guerrillas through army posts. Some contributed their silence: “We used to help them [the guerrillas] by telling the military, ‘No, haven’t seen anyone’ ” (Wood 2003, p. 126). Others informed the rebels about the movement of government forces. Of course, there were also those who took up arms, but even they put in various degrees of effort. As a regional commander of the Revolutionary Army of the People has told Wood (2003, p. 124): “there were various levels of militia participation. Some might be mobilized for activities for two, three, or five days—they would participate and then return to their homes. Others would join us for two weeks or a month.” El Salvador is not an exception. Studying Lithuanian resistance to German and Soviet occupation, Petersen (2001) documents varying degrees of resistance from “lower-risk, one-shot actions such as graffiti writing, singing antiregime songs on a bus, or showing up for demonstrations” to armed fighters (p. 23-4). Summarizing the literature, Kalyvas (2006, 100) argues that “support is not dichotomous but can be conceptualized as a continuum.”

3 The notion of “revolutionary entrepreneur” resonates with the notions of “professional revolutionaries” (as popularized by the Lenin’s treatise What is to be Done?), “social movement brokers,” and “entrepreneurs of violence” in social movements literature (Della Porta 1995, p. 108, 195-201; Diani 2003; Tilly and Tarrow 2007, p. 29-31), and has been used in formal models (Bueno de Mesquita 2010; Roemer 1985).
the extensive and intensive margins of support.\footnote{The extensive margin of support refers to the (expected) number of supporters, and intensive margin of support refers to the intensity of their support. We use support, effort, and anti-regime activities interchangeably. We emphasize that radicalizing the revolutionary agenda also reduces the efforts of some of the remaining supporters, so that the revolutionary entrepreneur also faces a tradeoff just along the intensive margin of support.}

This framework provides new insights into the interactions between repression and extremism in revolutionary movements. We show that the structure of punishments for citizens’ revolutionary efforts, which we refer to as \textit{repression structure}, influences the extremism of the revolutionary agenda. Expected punishments for dissent increase with the level of anti-regime activities. For example, punishments for attending a peaceful protest are typically less than those for organizing a demonstration, which, in turn, are less than punishments meted out for armed struggle. But in many countries, a citizen who is arrested even for very low levels of anti-regime activities incurs significant costs. For example, she may remain in custody for weeks until her case is processed, or she may be denied important opportunities such as higher-education just because her name has become associated with dissenting activities—no matter how small. Sometimes, these punishments are very severe. For example, many protesters who were arrested during and after the 2009 Iranian presidential election were tortured and raped. As the 2013 report of the UK-based \textit{Freedom from Torture} organization documents, some of these protesters had “no personal history of political or other activism, or family profile of dissent, prior to the 2009 presidential election period” \cite[p. 13]{FreedomFromTorture}, and “were arrested and detained for attendance at demonstrations alone” \cite[p. 14]{FreedomFromTorture}. We introduce the notion of \textit{minimum punishment} to refer to the expected costs that a citizen incurs for committing a minimal level of anti-regime activities—we formulate minimum punishment as the expected marginal cost of revolutionary effort at the minimum level of revolutionary effort. As one expects, higher levels of repression lower the likelihood of a successful revolution. However, increases in the minimum punishment have an additional strategic consequence: Higher minimum punishments make the revolutionary agenda more extreme.\footnote{Not all increases in repression radicalize the revolutionary agenda. For example, suppose the expected marginal cost of revolutionary effort, $e$, is $\alpha + \beta e^n$, with $\beta, \eta > 0$, so that $\alpha$ is the minimum punishment. We show that increases in $\alpha$ radicalize the revolutionary agenda, but increases in $\eta$ moderate it, while increases in $\beta$ do not affect the revolutionary agenda.}

The intuition for why higher minimum punishments lead to radicalism is strategic. Because even very small levels of support are subject to the minimum punishment, citizens whose support is already at a minimal level cannot further lower their support level to reduce the adverse effects of increases in the minimum punishment on their payoffs. Therefore, increases in the minimum punishment cause the most moderate supporters of the revolution to withdraw their support. The revolutionary entrepreneur can respond in two ways: He can either moderate his revolutionary
agenda to win back the moderates and partially recover the loss in the extensive margin of support, or he can make it more extreme to elicit more support from the extreme citizens along the intensive margin of support. The moderates’ contributions are small anyway because they do not gain as much from a successful revolution. Thus, the revolutionary entrepreneur strategically radicalizes the agenda to generate higher levels of support from the extremists. That is, higher minimum punishments decrease the likelihood that revolutions succeed, but make successful ones more extreme. This resonates with Della Porta’s (2013) summary of the qualitative literature that “repression tends to discourage the moderates, pushing them to return to private life and leaving room in the protest arena for the more radical wings” (p. 67).

The effect of increases in the minimum punishment contrasts with increases in indiscriminate repression—defined as repressive measures that affect all citizens, including those who do not support the movement (Kalyvas 2006). Indiscriminate repression increases everyone’s grievances against the status quo and pushes even very moderate citizens (those whose ideal policies are close to the status quo) to join the revolution. In response, the revolutionary entrepreneur moderates the revolutionary agenda to account for the preferences of these moderate supporters in order to maximize the total support for the revolution. That is, indiscriminate repression raises the likelihood of revolution, but moderates the revolutionary agenda.

What determines the structure of repression? The regime’s choice depends on the relative costs of establishing and maintaining the repression apparatus. Thus, one could model the regime’s decision to structure repression (choosing how repression increases with revolutionary effort) by introducing the costs of repression to the state and even the possibility of concession as in Acemoglu and Robinson (2001, 2006) or Boix (2003). However, in our model, this approach would simply highlight the substitution effects between different repression structures. Accordingly, we instead focus on a new tradeoff that is absent in those models. When choosing the minimum punishment, the ruling class faces a tradeoff between more likely but less extreme revolutions and less likely but more extreme ones. This tradeoff overturns the conventional wisdom that if it was not for the direct costs of repression, the regime would always gain by increasing repression. We show that when the ruling class is sufficiently risk averse and the minimum punishment is low, small increases

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6Because our model assumes quadratic utilities, the extremists’ gains are higher than the moderates’. However, our results extend even when the utility function is the absolute value function, so that it’s not strictly concave. In the Extensions, we consider the case where a citizen can also support the regime.

7For example, suppose that expected costs of exerting revolutionary effort $e$ is $C(e) = \alpha e + \frac{\beta}{\eta+1} e^{\eta+1}$, so that the expected marginal cost of effort becomes the sum of a non-negative constant and a homogeneous function of degree $\eta > 0$: $\alpha + \beta e^\eta$. Then, the regime’s choice of repression structure corresponds to choosing parameters $\alpha$, $\beta$, and $\eta$, which hinges on the marginal rate of substitution between these parameters and their relative costs.
in the minimum punishment hurt the ruling class. They only gain from increasing the minimum punishment after it rises past a threshold. Therefore, regimes with no minimum punishment tend not to raise it unless the direct costs are so low that they can increase the minimum punishment past that threshold. This, in turns, implies that minimum punishment can change discontinuously with direct costs of repression.

Finally, we study the effect of competition between two revolutionary entrepreneurs, each maximizing his support by choosing a revolutionary agenda. One may think that competition should always moderate the revolutionary agenda, or that with two revolutionary entrepreneurs, there will always be two “revolutionary camps” with distinct support bases. We show that with quadratic preferences, both revolutionary entrepreneurs choose the same revolutionary agenda, which is more extreme than the one chosen by a revolutionary entrepreneur who does not face any competition.

In the Extensions, we explore how outcomes are affected when we enrich the model in different ways. We study the model with a general distribution of citizens’ ideal points, showing that the revolutionary agenda becomes more extreme as this distribution shifts toward the extremists—in the sense of MLRP. We investigate how results are affected when (i) the post-revolution policy can be different from the revolutionary agenda promised by the revolutionary entrepreneur prior to the success of the revolution, (ii) repression also depends on the revolutionary agenda, (iii) citizens can also support the regime, or (iv) the regime can make concessions by changing the status quo policy.

We next discuss the literature. Section 2 presents the model. In section 3, we characterize the equilibrium. Section 4 studies the effects of repression structure and the risk attitude of the ruling class. Section 5 analyzes competition between revolutionary entrepreneurs. Extensions are in section 6. A conclusion and an appendix follow.

1.1. Related Literature

Our model has surface similarities to DeNardo (1985) that studies protest leaders’ choices of demands and tactics to elicit state concessions. However, the tradeoff between the extensive and intensive margins of support does not arise in that model: citizens have a binary choice of whether or not to support a movement and protest success depends on the number of supporters. Moreover, the effects of the structure of repression and the competition between revolutionary entrepreneurs on the extremism of revolutionary agenda do not appear in his analysis. In Bueno de Mesquita (2008), the leader of a terrorist organization chooses the group’s ideology to attract potential recruits who must decide whether or not to join the organization. He shows that increases in the cost of joining the group and the potential formation of a more radical splinter group radicalize the position of the
original group. Moreover, because the original group can provide more nonideological benefits to its members, the splinter group does not choose the same position as the original group.

In most other models of conflict, extremism is interpreted as violent actions or terrorism. Bueno de Mesquita (2005) studies tactical extremism in a terrorist organization consisting of a moderate faction and an extremist faction. The extremists derive a higher utility from defeating the state, and prefer to allocate more resources to violence. The state offers to each faction non-binding concessions which require the faction to disarm. In the equilibrium with concession, moderates accept concessions but extremists don’t, and the state honors its concessions because it needs the moderates’ help to fight extremists. In turn, the withdrawal of moderates from the terrorist organization removes their constraint on the extremists, and hence violence increases following an agreement between moderates and the state. In Wintrobe (2006), extremism is interpreted as high risk actions that have high potential rewards. Thus, if the “output function” is convex, groups prefer extremism to moderate actions. In Appelbaum (2008), extremism can improve a party’s bargaining power by providing a credible threat to destroy the contested prize. The large psychological literature that studies the effects of emotions on extremism belongs to this category, e.g., Midlarsky (2011). In our model, the level of extremism is the distance between the status quo and the revolutionary agenda, and is distinct from the degree of violence or “rebel tactics” studied in the literature (Bueno de Mesquita 2013; Della Porta 1995, 2013).

Besley and Persson (2011) study violence (defined as the occurrence of repression or civil war) in a setting with a government and an opposition. Both players can make costly investments in armed forces to increase the likelihood of taking over the state apparatus in the future. Civil wars refer to situations in which both the government and the opposition invest in armed forces, while repression is defined as situations in which only the government makes such investments. Their focus is on the effects of wages, natural resources, and public spendings on the likelihood of violence, and hence they abstract from citizen heterogeneity, dissent gradation, repression structure, and the endogenous choice of the revolutionary agenda which are the focus of our paper.

There is a large literature on ethnic conflict that involves heterogeneous players, but focuses on inter-group strategic interactions through a contest function. The few exceptions with within-group heterogeneity are related to our paper. For example, Sambanis and Shayo (2013) model how social, ethnic, and national identities influence ethnic conflicts. They distinguish between “core” and “non-core” group members, showing that inter-ethnic conflict between the core members may draw non-core members into conflict because, e.g., ethnic fightings are assumed to increase ethnic identity and to decrease national status. Esteban and Ray (2008, 2011) develop a model of ethnic
conflict in which individuals from two ethnic groups decide how much time to spend on activism and how much money to contribute to compensate the activists’ loss of income. Individuals are distinguished by their income (resources) and the intensity of their animosity against the other group. They show increases in within-group inequality cause the total group activism to rise because the opportunity costs of both activism by the poor and financial contributions by the rich fall. In contrast, a more disperse distribution of group animosity can reduce the group activism.\(^8\) Because of the distinct natures of revolutionary movements and ethnic conflicts, the endogenous choice of revolutionary agenda, the tradeoff between extensive and intensive margins of support, and the interactions between the structure of state repression and extremism do not appear in these papers.

A more distant literature studies the causes of extremism in elections. Glaeser et al. (2005) and Virag (2008) show that parties choose extreme platforms because their supporters are more likely to observe party mandates and voting is costly. In Boleslavsky and Cotton (2015), extremism arises because informative campaigns cause quality differentiation between the candidates, reducing incentives for policy moderation. In Bernhardt and Camara (2013), the incumbent can be more extreme when the pool of challengers becomes more moderate because the policy-motivated incumbent is less concerned about losing to a moderate challenger.

The tradeoff between intensive and extensive margins is also a feature of some spatial electoral models that seek to explain observed deviations from Downsian policy convergence by “base-mobilization” or “resource mobilization” (Aldrich 1983; Coate 2004; Moon 2004; Schofield and Miller 2007; Schofield and Sened 2005; see Peress (2013) for a review). However, these frameworks give rise to very different forms of predicted behavior. For example, in my revolution setting, extreme citizens have more incentive to exert effort to support the revolution. In contrast, in voting settings featuring “abstention due to alienation” (Adams and Merill 2003; Peress 2011), extreme voters are less likely than moderates to support a candidate, and it is this feature that drives divergent positions. In particular, when extreme voters are sensitive to candidate positions while moderates are not, candidates (who care about policy and vote shares) may move toward extremes to increase the likelihood of turnout among partisan voters by reducing abstention due to alienation. Moreover, these papers assume abstention due to alienation or indifference as behavioral features of the

\(^8\) For the most part, radicalism in Esteban and Ray (2011) refers to the degree of inter-ethnic animosity. The degree of radicalism is the intensity of individuals’ satisfaction from “group success” and “the individual perception of such nonmaterial, group-defined rewards” (p. 498). However, they also use radicalism when referring to the level of activism. The likelihood a group succeeds depends on the group’s level of activism through a contest function. Although individuals can contribute from a continuum of activism levels, in equilibrium, they either contribute the maximum level of activism or do not contribute at all, depending on whether the (endogenous) compensation rate for activism exceeds their income. However, individuals provide a varying degree of financial resources, depending on their levels animosity, income, activism, etc., that are used only to compensate activists.
voters.\(^9\) In addition, in these democratic settings, two candidates compete for vote shares in the absence of a status quo policy. In contrast, in my revolution setting, a revolutionary entrepreneur seeks popular support to topple an existing state, where the state’s power rests on its military forces, resources such as oil, and sometimes a narrow elite. Moreover, beyond the disproportionate effect of extreme citizens captured in Proposition 1, the very different structures of revolution vs. voting mean that none of my results have analogues in the voting literature. In particular, the effects of repression structure and the competition between revolutionary entrepreneurs on the extremism of the revolutionary agenda (Propositions 2 and 4, remarks 1 and 2), and the novel tradeoff that the elite face when choosing the minimum punishment (Proposition 3 and Corollary 1) obviously have no counterparts in voting settings.

2. Model

There are \(N\) citizens and a revolutionary entrepreneur.\(^10\) Each citizen is distinguished by his ideal policy in the unidimensional policy space \([0, 1]\). If policy \(p\) is implemented, the payoff of a citizen with ideal policy \(x\) is \(u(x, p) = -(x - p)^2\). The revolutionary entrepreneur is “office motivated”; she is interested in overthrowing the regime in order to capture a rent associated with the state’s leadership position. To overthrow the regime, the revolutionary entrepreneur needs the citizens’ support, and hence she must advocate a revolutionary agenda, \(p \in [0, 1]\), as an alternative to the regime’s status quo policy, \(s\), which we assume is 0. If the revolution succeeds, the revolutionary agenda \(p\) is implemented. Otherwise, the status quo prevails. Each citizen can provide a continuum of effort \(e_i \in [0, 1]\) to support the revolution at the expected marginal cost \(\alpha + \beta g(e)\), where \(\alpha, g'(\cdot) > 0\) and \(g(0) = 0\).\(^11\) The likelihood that the regime is overthrown is \(\Sigma_{i=1}^{N} e_i / N\), where \(N\) is the total number of citizens. Citizens’ ideal points are their private information and are uniformly distributed in the

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\(^9\)For example, in Moon (2004), the candidate \(i\)’s resource level \(r_i, i \in \{L, R\}\), is assumed to enter additively in voters’ payoff from voting for \(i\); and \(r_i\) is assumed to depend on the party activist’s ideal policy \(\theta_i\) and the candidates’ positions \(x_L\) and \(x_R\): \(r_i = \alpha + \omega_i - |x_i - \theta_i| + \{|x_j - \theta_i| - |x_i - \theta_i|\} - |x_L - 0.5|\), where \(\alpha\) and \(\omega_i\) are known constants, and 0.5 is the median voter’s position. See also Peress (2011).

\(^10\)The size of \(N\) depends on how we interpret a citizen in the model. For example, a citizen can be an influential individual, group, or organization whose support has non-negligible effects on the likelihood of success. Moreover, because even in large scale revolutions, only a very small fraction of the population participates, we can interpret citizens as the subgroup of the population that may potentially be involved in political activities.

\(^11\)Having both \(\alpha > 0\) and \(\beta > 0\) is important for the citizens’ equilibrium behavior to be consistent with real world observations. If \(\alpha\) were zero, then even a citizen whose preference is very close to the status quo would join the revolution when the revolutionary agenda is close to the status quo policy. If \(\beta\) were zero, then dissent gradation would not arise in equilibrium. Moreover, as we discussed in the introduction, we do not model the costs of repression to the regime because they do not provide new insights in our model. However, our Corollary 1 highlights how small changes in the costs of repression can lead to discontinuous changes in the state’s choice of minimum punishment due to the new tradeoff that arises in our model between radical but unlikely revolutions and moderate but likely ones.
policy space, $x_i \sim U[0,1]$.

**Timing.** First, citizens’ ideal points are realized and they are privately informed. Next, the revolutionary entrepreneur chooses the revolutionary agenda. Then, citizens simultaneously decide how much effort to devote to the revolution. If the revolution succeeds, the revolutionary agenda is implemented, otherwise the status quo prevails. Finally, payoffs are realized and the game ends.

**Extensions.** In the Extensions, we explore how outcomes are affected when we enrich the model in different ways. We study the model with more general distributions of citizens’ ideal points, showing that the revolutionary agenda becomes more extreme as this distribution shifts toward the extremists—in the sense of MLRP. We also study extensions in which (i) the post-revolution policy is uncertain, (ii) repression also depends on the revolutionary agenda, (iii) citizens can also support the regime, or (iv) the regime can make concessions by changing the status quo policy.

### 2.1. Discussion of the Model

**The Revolution Technology.** $RT(e_1, ..., e_N)$, is a mapping from the citizens’ effort choices into a probability the revolution succeeds. We assume $RT(e_1, ..., e_N) = \Sigma_{i=1}^N e_i / N$, so that the likelihood of success depends on the sum total of citizen efforts, and the marginal effect of a citizen’s effort is independent of the others’ efforts: $\frac{\partial^2 RT(e_1, ..., e_N)}{\partial e_j \partial e_i} = 0$. If this cross partial derivative were positive, citizens’ equilibrium actions would exhibit strategic complements (as in coordination games), which could result in multiple equilibria. If it were negative, citizens’ equilibrium actions would exhibit strategic substitutes as in private provision of public goods (Eichberger and Kelsey 2002), leading to free-riding considerations in individual decisions. In games with finite number of players and incomplete information, characterizing equilibria in the presence of coordination considerations is cumbersome. Papers either analyze two-player games or assume a continuum of players, so that a player’s action does not influence the outcome. Such assumptions are appropriate in settings where the revolutionary agenda is exogenous and the coordination problem is the most salient feature, but not for studying the endogenous choice of the revolutionary agenda, which requires that a player’s action influence the outcome. Thus, we select the revolution technology in a way to abstract from coordination and free-riding issues that have been the focus of the literature (Bueno de Mesquita 2010; Lohmann 1994). This assumption is based on two broad observations: (1) the restrictions on assemblies, civic groups, and public forums in dictatorships limit the ability of citizens to discuss their preferences, and (2) the punishment for dissenting views make citizens misrepresent their private preferences (Kuran 1991, 1995).

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12 Private knowledge of preferences is a standard assumption in formal models of revolution (e.g., Bueno de Mesquita 2010; Lohmann 1994). This assumption is based on two broad observations: (1) the restrictions on assemblies, civic groups, and public forums in dictatorships limit the ability of citizens to discuss their preferences, and (2) the punishment for dissenting views make citizens misrepresent their private preferences (Kuran 1991, 1995).

13 In addition, qualitative and empirical research suggests that individuals derive personal pleasure from supporting a revolutionary cause they believe in (e.g., Wood 2003), or that not joining the movement can be even more dangerous (e.g., Kalyvas and Kocher 2007).
2010, 2014; Casper & Tyson 2014, 2015; Chen et al. 2014; Chen & Xu 2014, 2015; Loeper et al. 2014; Lohmann 1994; Shadmehr & Bernhardt 2011, 2014; Tyson & Smith 2014), but complicate the model in other dimensions by allowing a revolutionary entrepreneur to endogenously choose the revolutionary agenda and by allowing citizens to take a continuum of actions.\footnote{In global games settings that are used in the literature (Carlsson and van Damme 1993; Morris and Shin 2003), the extension from a binary action to multiple or a continuum of actions is difficult. In particular, the uniqueness result that is central to the application of global games will not be robust to how the noise in the players’ private information vanishes (Frankel et al. 2003)—see Guimaraes and Morris (2007) for a special global game with a continuum of actions.}

**Revolutionary Entrepreneurs** are the leaders of a revolutionary movement who set the revolutionary agenda. To focus on the behavior of citizens, we abstract from some real world features of revolution leaders. Specifically, like Bueno de Mesquita (2008, 2010) and Roemer (1985), we assume a revolutionary entrepreneur who only seeks to maximize the probability of overturning the status quo. In the real world, a revolutionary entrepreneur may also be motivated by ideology. For example, suppose his preferences are given by $-(y - p)^2$, so that he chooses the revolutionary agenda $p$ to maximize $-E \left[ \sum_{i=1}^{N} e(x_i, p)/N \right] (y - p)^2$. This generates a force that tends to push the equilibrium revolutionary agenda toward the revolutionary entrepreneur’s ideal policy $y$. In particular, if $y = 1$, so that the revolutionary entrepreneur’s ideal policy is at the opposite end of the policy spectrum from the status quo, he chooses a more extreme revolutionary agenda than an office-motivated revolutionary entrepreneur. This additional force does not provide any new insight to our model, and hence we abstract from it to simplify exposition and focus on the new aspects of the model.

### 3. Equilibrium

We solve for the Subgame Perfect Nash Equilibrium using backwards induction. A citizen’s decision of how much to support the revolution depends on his preferences and the revolutionary agenda. For a given agenda, citizens whose ideal points are too close to the status quo contribute no effort toward the revolution. Although some of them may prefer the revolutionary agenda to the status quo, the revolution is not sufficiently attractive to them to justify the costs of even a small effort contribution. The size of this group depends on the minimum punishment level $\alpha$: if the minimum punishment is sufficiently large, then no citizen, no matter how aggrieved by the regime, provides any support for the revolution and the revolution fails. We focus on the case where revolution is possible. In particular, given a revolutionary agenda $p$, the marginal benefit of revolutionary effort for a citizen with ideal policy $x$ is $u(x, p) - u(x, s) = p(2x - p)$, which is less than $1/N$. The marginal cost of revolutionary effort is $\alpha + \beta g(e) \geq \alpha$. Therefore, if $\alpha \geq 1/N$, the probability that revolution succeeds is 0 no matter what the revolutionary agenda is.
In contrast, citizens whose preferences are far from the status quo have strong grievances and could exert full effort $e = 1$. To simplify the exposition, we also focus on the case in which no citizen contributes the maximum effort for any revolutionary agenda. In particular, if $1/N < \alpha + \beta g(1)$, then the marginal cost of the maximum revolutionary effort ($e = 1$) exceeds its marginal benefit.

**Assumption 1.** $\alpha < 1/N < \alpha + \beta g(1)$. That is, repression structure is such that (i) revolution is possible, and (ii) no citizen exerts the maximum revolutionary effort.\(^{15}\)

The following lemma characterizes a citizen’s equilibrium level of revolutionary effort.

**Lemma 1.** Let $h(\cdot) = g^{-1}(\cdot)$. Given a revolutionary agenda $p$, the equilibrium revolutionary effort of a citizen with type $x$ is:

$$e^*(x, p) = \begin{cases} 0 & \text{if } x \leq x_L \\ h\left(\frac{\Delta(x, p) - \alpha N}{\beta N}\right) & \text{if } x \geq x_L, \end{cases}$$

where $x_L \equiv \frac{v^2 + \alpha N}{2p}$ and $\Delta(x, p) \equiv u(x, p) - u(x, s) = p(2x - p)$ is the change in the citizen’s payoff from a successful revolution.

Citizens whose preferences are too close to the status quo (citizens with $x_i \leq x_L$) do not provide any support for the revolution even though some of them prefer the revolution. The other citizens’ efforts are increasing in their types: those with more grievances gain more if the revolution succeeds, and hence support the revolution more. Next, we analyze the revolutionary entrepreneur’s decision.

**Revolutionary Entrepreneur’s Decision.** The revolutionary entrepreneur chooses a revolutionary agenda $p$ that maximizes the expected likelihood of successful revolution $E\left[\sum_{i=1}^{N} e^*_i / N\right] = \sum_{i=1}^{N} E[e^*_i]/N = E[e^*_i]$. When the regime imposes a minimum level of punishment, a revolutionary agenda close to the status quo does not garner any support, because the expected gains from a successful revolution are too small relative to the costs of providing any support. Lemma 1 reflects this intuition: if $\alpha > 0$ and $p$ is too small, $x_L$ exceeds 1, implying that no citizen participates. Thus, to generate support, the revolutionary entrepreneur has to distance himself sufficiently from the status quo by choosing a revolutionary agenda $p > p \equiv 1 - \sqrt{1 - \alpha N}$. Otherwise, no one participates. This\(^{15}\)Part (ii) also captures the observation that it is implausible that a non-negligible fraction of the population exerts maximum feasible effort. No matter how much effort an individual devotes, one can imagine even greater sacrifices that could increase the chance of revolution success. For example, an individual willing to sacrifice all of his wealth could risk his life as well; an individual willing to risk his life could also be willing to endanger family.

\(^{15}\)
Figure 1: $x_L$ as a function of the revolutionary agenda $p$. Although the extensive level of support is maximized at $\sqrt{\alpha N}$ where $x_L$ is at its minimum, the revolutionary entrepreneur chooses a more extreme revolutionary agenda $p^*$ to generate more support along the intensive margin from the more extreme citizens.

result resonates with the finding in Farhi’s (1990) comparative study: for a revolution to succeed, opposition ideology must be sufficiently different from the state’s—see also DeNardo (1985, p. 163).

We begin with the simple case where $\beta = 0$ so that the revolutionary effort is, in effect, a binary choice: a citizen either does not support the revolution or contributes the maximum effort $e = 1$. Thus, the revolutionary entrepreneur chooses the revolutionary agenda that maximizes the expected number of supporters (the extensive margin as captured by $Pr[x_i \geq x_L(p)]$), but not the magnitude of their efforts. This likelihood is non-monotone in $p$—see Figure 1. In fact, it has a unique maximum at $\sqrt{\alpha N}$. As the revolutionary agenda exceeds $p$, the most extreme citizens start to join the revolution. As the agenda becomes more extreme, more citizens join, so that at $p = \sqrt{\alpha N}$, all citizens who are more extreme than $\sqrt{\alpha N}$ support the revolution. Once the revolutionary agenda exceeds this threshold, the most moderate supporters begin to withdraw their support. Thus, without dissent gradation, in equilibrium, the revolutionary entrepreneur would choose $p = \sqrt{\alpha N}$. Proposition 1 characterizes the equilibrium revolutionary agenda in our model, which features dissent gradation.

**Proposition 1.** In equilibrium, the revolutionary entrepreneur chooses a more extreme revolutionary agenda $p^*$ than he would if citizen support for revolution was a binary choice: $p^* > \sqrt{\alpha N}$. In particular, if the marginal cost of revolutionary effort is $\alpha + \beta e^n$, $\eta > 0$, then there is a unique equilibrium in which:

$$p^* = \frac{1 + \sqrt{1 + \eta(2 + \eta)\alpha N}}{2 + \eta} > \sqrt{\alpha N}.$$  \hspace{1cm} (1)
Proposition 1 shows that dissent gradation radicalizes the revolutionary agenda.\footnote{The first part of Proposition 1 does not state that the equilibrium is unique. We can show that if \( g(e) \) is weakly convex, the equilibrium is unique. More generally, if there are multiple equilibria, the results of the paper hold in any of these equilibria.} The revolutionary entrepreneur chooses a more extreme revolutionary agenda to elicit more support from extreme citizens at the cost of losing the support of the most moderate supporters of the revolution. To illustrate, suppose \( g(e) = e^\eta \), \( \eta > 0 \), so that the marginal cost of revolutionary effort becomes \( \alpha + \beta e^\eta \).\footnote{When \( \eta < 1 \), the marginal cost is strictly concave, and when \( \eta > 1 \), it is strictly convex. When \( \eta = 1 \), the expected marginal cost of revolutionary effort is \( \alpha + \beta e \). Suppose that (1) the likelihood a dissident is caught is proportional to her effort level: \( a \times e \); and (2) the punishment imposed on apprehended dissidents is a linear function of their revolutionary effort: \( \alpha + \frac{a}{2} e \). Then, normalizing \( a \) to 1, the expected cost of revolutionary effort \( e \) is \( \alpha e + \beta e^2 / 2 \). Alternatively, suppose the state punishes the apprehended dissidents at the two levels of high or low. The low punishment imposes a cost \( l \) on a dissident and the high punishment imposes a cost of \( l + \delta \). The likelihood that the state imposes the high punishment is proportional to a dissident’s effort: \( b \times e \). Then, the expected cost of revolutionary effort \( e \) is \( e((1 - be)l + be(l + \delta)) = le + b\delta e^2 \), which can be written as \( \alpha e + \beta e^2 / 2 \).} Focusing on the relevant range of the revolutionary agenda, \( p > p_2 \), the expected probability of success for an agenda \( p \) is:

\[
E[e^*(x, p)] = \Pr[x \geq x_L(p)] \left[ \left( \frac{\Delta(x, p) - \alpha N}{\beta N} \right)^{1/\eta} \right] x \geq x_L(p) \quad (2)
\]

\[
= \frac{\eta}{1 + \eta} \frac{p(2 - p) - \alpha N}{2p} \left( \frac{p(2 - p) - \alpha N}{\beta N} \right)^{1/\eta} . \quad (3)
\]

The first term in equation (2) is the likelihood that a citizen supports a revolutionary agenda \( p \), capturing the extensive margin of support. The second term indicates the intensity of a citizen’s support, capturing the intensive margin of support. As \( p \) exceeds \( \sqrt{\alpha N} \), \( x_L(p) \) increases, hurting the revolutionary entrepreneur along the extensive margin. However, the second term rises, benefiting him along the intensive margin of support. That is, with dissent gradation, the revolutionary entrepreneur also takes into account how the revolutionary agenda affects the intensity of the citizens’ support as captured by this second term. Figure 2 illustrates this tradeoff when \( \eta = 1 \).

Remarkably, the likelihood of successful revolution in equilibrium \( E[e^*(x, p^*)] \) can be expressed in terms of the equilibrium revolutionary agenda \( p^* \) and the equilibrium effort level of the most extreme citizen \( e^*(1, p^*) \). In the Appendix (equation (15)), we show that with a change of variables, the first order condition of the revolutionary entrepreneur’s optimization problem simplifies to:

\[
E[e^*(x, p^*)] = e^*(1, p^*)(1 - p^*) . \quad (4)
\]

Figure 3 illustrates this result for the special case of \( g(e) = e \). This characterization only hinges on the monotonicity of the marginal cost, but not its shape, and is a key step that has allowed us to prove most of our results without imposing a specific functional form for the marginal cost.
Figure 2: Effort level as a function of citizens’ ideal points $x_i$ for two different revolutionary agendas, for a linear marginal cost of revolutionary effort: $\alpha + \beta e$. The dashed line is citizens’ efforts when $p = \sqrt{\alpha N}$, and the solid line is their efforts given the equilibrium revolutionary agenda $p^*$. The area under the solid line and above the dashed line captures the gains along the intensive margin of support. The revolutionary entrepreneur gains by setting a revolutionary agenda that is more extreme than what maximizes the extensive margin of support $\sqrt{\alpha N}$.

Figure 3: In equilibrium, the likelihood of successful revolution $E[e^*(x, p^*)]$ (red area) can be expressed in terms of the equilibrium revolutionary agenda $p^*$ and the equilibrium effort level of the most extreme citizen $e^*(1, p^*)$: the red area is the same as the blue area $E[e^*(x, p^*)] = e^*(1, p^*)(1 - p^*)$. 
4. Revolution and the Structure of Repression

Increases in the minimum punishment $\alpha$ affect revolutionary participation along both the intensive and extensive margins: all contributing citizens reduce support, and some citizens switch from contributing positive effort to contributing nothing, i.e., $x_L$ increases (see Lemma 1). This clearly reduces the equilibrium likelihood that the revolution succeeds. Here, we investigate the effects of increases in repression on the extremism of the revolutionary agenda. Increases in the minimum punishment could trigger two potential responses from the revolutionary entrepreneur: (1) he could moderate the revolutionary agenda, bring back the moderates who had dropped out, and raise the efforts of the moderates who had reduced their efforts, thereby partly recovering his support along both the extensive and intensive margins of support; or (2) he could radicalize the revolutionary agenda, lose even more efforts from the moderates, but compensate for that loss along the intensive margin from the more extreme supporters. Proposition 2 shows that he opts for the latter.

Proposition 2. The structure of repression matters:

- Suppose $g(e)$ is weakly convex. Then, increases in the minimum punishment radicalize the revolutionary agenda: $\frac{dp^*}{d\alpha} > 0$.

- Suppose $g(e) = e^\eta$, so that the marginal cost of revolutionary effort is $\alpha + \beta e^\eta$. Then, increases in $\alpha$, $\beta$, or $\eta$ have contrasting effects on the revolutionary agenda. While increases in $\beta$ do not change the revolutionary agenda, increases in $\alpha$ radicalize the agenda, and increases in $\eta$ moderate the revolutionary agenda: $\frac{dp^*}{d\eta} < 0 = \frac{dp^*}{d\beta} < \frac{dp^*}{d\alpha}$.

In the proof, we use equation (4) to show that $\frac{dp^*}{d\alpha} > 0$ if and only if $\frac{de^*(x,p^*)}{dx} \bigg|_{x=1}$, which captures the rates of change with $\alpha$ in the left and right hand sides of the equilibrium condition (4). The convexity of $g(e)$ implies the concavity of $e^*(x,p)$ in $x$ (for a given $p$), which, in turn, provides an upper bound for $\frac{de^*(x,p^*)}{dx} \bigg|_{x=1}$:

$$\frac{de^*(x,p)}{dx} \bigg|_{x=1} \leq \frac{e^*(1,p) - e^*(x_L(p),p)}{1 - x_L(p)} = \frac{e^*(1,p)}{1 - x_L(p)}.$$ 

Thus, a sufficient condition for $dp^*/d\alpha > 0$ is that $x_L(p^*) < p^*$, which is true from Figure 1 and Proposition 1. Of course, as the second part of the proposition shows, the convexity of $g(e)$ is not necessary. In particular, the radicalizing effect of higher minimum punishment $\alpha$ holds whenever $g(e) = e^\eta$, $\eta > 0$, even when $\eta < 1$ so that $g(e)$ is strictly concave.

Moreover, Proposition 2 highlights that not all kinds of increases in repression radicalize the revolutionary agenda. In particular, when repression structure is such that the expected marginal
cost of revolutionary effort is $\alpha + \beta e^\eta$, $\eta > 0$, increases in $\beta$ do not affect the revolutionary agenda, and increases in $\eta$ cause the revolutionary entrepreneur to moderate the revolutionary agenda.

By moderating the revolutionary agenda, the entrepreneur would recover some support from moderates, but the moderates brought back into the fold would not contribute much anyway. At the same time, moderating the agenda triggers a significant drop in the revolutionary efforts of the extremists. Thus, the entrepreneur responds to increases in $\alpha$ by setting a more extreme revolutionary agenda. This result resonates with Della Porta’s (2013) finding in her review of the empirical and qualitative literature that “repression tends to discourage the moderates, pushing them to return to private life and leaving room in the protest arena for the more radical wings” (p. 67).

Increases in the minimum punishment $\alpha$ affect the citizen efforts in two ways. First, because punishment is more severe, moderate citizens withdraw their support; second, the revolutionary entrepreneur radicalizes the revolutionary agenda, further shrinking the revolutionary base but increasing the efforts of the extremist supporters. Together, these confounding effects may raise the question of whether the probability of successful revolution could increase as the minimum punishment becomes more severe. However, by the Envelope Theorem, the likelihood of revolution must fall as the minimum punishment increases. However, the revolutionary effort of some extreme citizen can increase in response to the radicalization of the revolutionary agenda in response to increases in the minimum punishment.

**Remark 1.** Suppose $g(e) = e^\eta$, so that the marginal cost of revolutionary effort is $\alpha + \beta e^\eta$. When the minimum punishment marginally increases, the most extreme citizen raises his revolutionary effort if and only if $\eta > 2$ and the minimum punishment $\alpha$ is sufficiently small: $\frac{de^*, \eta > 2, g^*(\alpha, \alpha)}{d\alpha} > 0$ if and only if $\alpha N < \frac{\eta - 2}{4\eta}$.

**Minimum Punishment v. Indiscriminate Punishment.** Indiscriminate repression in the literature refers to the application of repression to all citizens, including those who do not support the revolutionary movement. For example, indiscriminate violence in Kalyvas (2006) means that the regime or the rebels use violence against individuals regardless of whether or not they cooperate with them or defect to the other side. In sharp contrast to raising the minimum punishment, higher indiscriminate punishments lead to a more moderate revolutionary agenda. To understand why, note that such indiscriminate punishments impose a cost on all citizens, reducing their status quo payoff from $u(x, s)$ to $u(x, s) - z$ for some $z > 0$. If citizens believe that indiscriminate punishment ends when the revolution succeeds, the gain of a citizen with ideal policy $x$ from revolution increases

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18 The proof is in the Appendix.
from $\Delta(x, p)$ to $\Delta(x, p) + z$. Following the logic of Lemma 1, the equilibrium revolutionary effort of a citizen with ideal point $x$ becomes:\(^{19}\)

$$e = h \left( \frac{\Delta(x, p) + z - \alpha N}{\beta N} \right). \quad (5)$$

This shows that increases in $\alpha$ and $z$ have opposite effects:

**Remark 2.** Indiscriminate repression raises the likelihood of revolution, but makes the revolutionary agenda more moderate.

Increases in indiscriminate repression $z$ reduce the status quo payoff of all citizens independent of whether or not they support a revolutionary agenda $p$. This has two effects: (1) those who already support the revolution put in more effort, and (2) some citizens who did not support the revolution join in. This triggers a strategic response from the revolutionary entrepreneur to moderate the revolutionary agenda to elicit more effort from the newcomers who are the most moderate supporters of the revolution.

**Minimum Punishment and Repression Backlash.** The literature has identified a confounding relationship between violent dissent and cruel treatment of dissidents. Some researchers argue that violent dissent triggers cruel repression, including torture (Poe and Tate 1994, Conrad and Moore 2010). Others argue that cruel acts of repression generate feelings of injustice, thereby radicalizing dissidents and increasing their use of violent methods (Della Porta 1995, 2013; Earl 2011; Wood 2003).\(^{20}\) Our result suggests a complementary alternative mechanism through which cruel treatments can lead to radicalization. To the extent that these cruelties are indiscriminate with respect to the intensity of anti-regime activities, they have similar effects to setting higher minimum punishments. As we discussed in Proposition 2, higher minimum punishments radicalize the revolutionary agenda through the revolutionary entrepreneur’s strategic response.

### 4.1. The Regime’s Tradeoff

Minimum punishments are indiscriminate with respect to the level of anti-regime activities: when a citizen is caught supporting the revolution, he bears the minimum punishment regardless of the level of his revolutionary efforts. From a normative perspective, this may be less acceptable than punishments that directly depend on the level of (illegal) revolutionary actions. This section investigates the regime’s choice of the level of minimum punishment. One could model the state’s

\(^{19}\)That is, the interior solution of the first order condition changes from (11) to (5).

\(^{20}\)A similar mechanism is also present in the literature on repression backlash (Francisco 2004; Rasler 1996; Shadmehr 2014, Appendix A; Siegel 2011).
choice of repression structure by introducing costs of repression to the state and even the possibility of concession as in Acemoglu and Robinson (2001, 2006), Boix (2003), or Moore (2000). For example, assuming the functional form $\alpha + \beta e^\eta$, one could investigate how the state structures repression by choosing $\alpha$, $\beta$, and $\eta$, given the exogenous costs of raising these repression parameters. However, in our model, this approach would simply highlight the substitution effects between increasing these repression parameters. We focus on a new tradeoff that does not arise in those models.

Increasing the minimum punishment reduces the likelihood of revolutions. One may, therefore, think that absent considerations of the direct costs of repression, the ruling class always prefers a higher minimum punishment. This view is incorrect. In fact, the ruling class may prefer a lower minimum punishment to a higher one. Because a higher minimum punishment radicalizes the revolutionary agendas, the ruling class faces a tradeoff: Increasing the minimum punishment reduces the likelihood of revolutions, but makes successful ones more extreme and hence more costly for the elite. This tradeoff reflects the political risk of revolution: If a revolution succeeds, the policy changes from the status quo to the revolutionary agenda. This suggests that the regime’s choice of repression may depend on the attitude of the ruling class toward policy uncertainty.\textsuperscript{21}

To capture this risk attitude, we assume the payoffs of the ruling class for a policy $p \in [0, 1]$ are given by $-p^m$, where $m \geq 1$ measures the degree of risk aversion for the ruling class whose ideal policy is the status quo $s = 0$: the coefficient of relative risk aversion is $m - 1$.\textsuperscript{22} We emphasize that the ruling class need not be similar to a typical member of the population, and hence they may have a different attitude toward risk from the regular members of the population. Moreover, the attitude of the ruling class toward the policy uncertainty induced by the revolutionary threat can reflect the sensitivity of their industries to political risk. For example, Boutchkova et al. (2012) show that different industries have different degrees of sensitivity to policy uncertainties in contract enforcement, labor-related issues, and trade policies (see also Kelly et al. (2014) and Strausz (2011)). Let $U_r(\alpha)$ be the expected payoff of the ruling class from choosing a minimum punishment $\alpha$:

$$U_r(\alpha) = -[p^*(\alpha)]^m E[e^*_i(p^*(\alpha), \alpha)].$$  \hspace{1cm} (6)

Raising the minimum punishment, $\alpha$, has two opposing effects. It makes the revolutionary agenda more extreme, harming the ruling class: $[p^*(\alpha)]^m$ rises. However, it also reduces the revolutionary efforts of the citizens, reducing the likelihood of successful revolution: $E[e^*_i(p^*(\alpha), \alpha)]$ falls. Propo-

\textsuperscript{21}The ruling class may also have institutional/organizational constraints on how far it can raise or reduce the minimum punishment. For example, a regime that relies on militia or does not have an independent judiciary may not be able to prevent its agents from severely torturing citizens who commit a minimal level of anti-regime activities.

\textsuperscript{22}This form of policy preferences is common in the literature (Boleslavsky and Cotton 2015).
Proposition 3. Suppose $g(e)$ is weakly convex. If the ruling class is sufficiently risk-averse, then marginal increases in the minimum punishment hurt the ruling class. That is, there exists $m^*(\alpha)$ such that $\frac{dU_r(\alpha)}{d\alpha} < 0$ if and only if $m > m^*(\alpha)$.

Suppose the marginal cost of revolutionary effort is $\alpha + \beta e^\eta$. Then, there exists $\alpha_m > 0$ such that $\frac{dU_r(\alpha)}{d\alpha} < 0$ if and only if $\alpha < \alpha_m$ and $m > m^*(0) = \frac{2 + \eta}{\eta^2}$.

The first part of the proposition states that at any given level of minimum punishment $\alpha$, slight increases in the minimum punishment reduce the payoff of the ruling class whenever $m$ exceeds a threshold $m^*(\alpha)$ that depends on $\alpha$. However, it does not characterize how this threshold $m^*(\alpha)$ varies with $\alpha$. As equation (22) in the proof of Proposition 3 shows, the derivative of $m^*(\alpha)$ depends on how fast $p^*(\alpha)$ increases with $\alpha$ in a complicated way. The second part of the proposition, in effect, shows that $m^*(\alpha)$ is strictly increasing in $\alpha$ by showing that $\frac{d^2p^*(\alpha)}{d\alpha^2} \leq 0$, using explicit calculations. This, in turn, implies that if $m \leq m^*(0)$, then raising the minimum punishment always benefits the ruling class—absent cost consideration. However, when $m > m^*(0)$, there exists a unique $\alpha_m > 0$ that solves $m^*(\alpha_m) = m$. Combined with the first part, this implies that if $\alpha < \alpha_m$, then $m^*(\alpha) < m$, and hence marginal increases in $\alpha$ hurt the ruling class.

Figure 4 illustrates the expected payoff of the ruling class when the marginal cost of revolutionary effort is $\alpha + \beta e$, i.e., $\eta = 1$. When the ruling class is sufficiently risk-averse and the minimum punishment is low, marginal increases in the minimum punishment lower the expected utility of the ruling class. Proposition 3 has two related implications. When the ruling class is sufficiently risk-averse, the minimum punishment is either zero or is bounded away from zero. That is, there exists $\hat{\alpha}$ such that the regime never chooses a minimum punishment $\alpha \in (0, \hat{\alpha})$. This, in turn, implies that repression structures, and hence the extremism of revolutionary movements, can be sensitive to small variations in the environment:

Corollary 1. Suppose the marginal cost of revolutionary effort is $\alpha + \beta e^\eta$. The level of minimum punishment $\alpha$ can change discontinuously with changes in repression costs.

To see this, suppose the costs of imposing a minimum punishment level are small, so that the regime chooses a positive minimum punishment, e.g., $\alpha_1 > \hat{\alpha} > 0$. Initially, as these costs increase, the

\[23\text{In particular, } \hat{\alpha} \text{ is the unique } \alpha > 0 \text{ such that } U_r(0) = U_r(\hat{\alpha}). \text{ See Figure 4.}\]
regime continuously decreases the minimum punishment to a lower level $\alpha_2 \in (\hat{\alpha}, \alpha_1)$. However, once these costs pass a threshold, the regime suddenly drops the minimum punishment from $\hat{\alpha}$ to 0.

5. Competition Among Revolutionary Entrepreneurs

One may think that competition between revolutionary entrepreneurs should always moderate the revolutionary agenda, or that with two revolutionary entrepreneurs, there will always be two “revolutionary camps” with distinct support bases. To study these conjectures, we analyze an extension of the model with two revolutionary entrepreneurs, each maximizing his support by choosing a revolutionary agenda. We assume that if a citizen is indifferent between supporting one revolutionary entrepreneur or the other, he supports them equally or randomly chooses one to support.\(^{24}\) Let $e_j^i$ be citizen $j$’s level of support for revolutionary entrepreneur $i$, so that citizen $j$’s total revolutionary effort is $e_j = e_j^1 + e_j^2$. As before, the likelihood that the regime collapses is $\Sigma_{j=1}^N e_j/N$, otherwise, the status quo prevails. If the regime does collapse, the likelihood that the revolutionary entrepreneur $i$ takes over the state and implements his revolutionary agenda $p_i$ is his supporters’ share of the total revolutionary effort $\frac{\Sigma_{j=1}^N e_j^i}{\Sigma_{j=1}^N e_j}$.\(^{25}\) If a citizen provides some support, he does so for the revolutionary

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\(^{24}\)Note that the idea of “abstention due to indifference” in the voting literature does not have an analogue in the competition of two revolutionary entrepreneurs.

\(^{25}\)This ratio formulation of contest function (Tullock 1980) is common in the formal models of contentious politics (Besley and Persson 2011; Svolik 2013).
agenda that is closer to his ideal. Thus, the revolutionary entrepreneur i’s problem is:

$$\max_{p_i} E[e^*(x, p_i, p_j)],$$

where $p_j$ is the other revolutionary entrepreneur’s revolutionary agenda, and $e^*(x, p_i, p_j)$ is the support for revolutionary entrepreneur $i$ by a citizen with ideal point $x$.

First, we recognize that no revolutionary entrepreneur chooses a revolutionary agenda so close to the status quo that there is still room to gain along both extensive and intensive margins of support, i.e., $p_i > \sqrt{\alpha N}$ in equilibrium. As Figure 1 illustrates, $x_L$ is minimized at $\sqrt{\alpha N}$, and hence, if $p_i < \sqrt{\alpha N}$, then $i$ can gain on both extensive and intensive margins of support by choosing a more extreme revolutionary agenda. Next, suppose the revolutionary entrepreneur 2’s agenda is more extreme: $\sqrt{\alpha N} \leq p_1 < p_2$. Then, $x_L = (p_1^2 + \alpha N)/(2p_1) < (p_1 + p_2)/2$, and hence the revolutionary entrepreneur 1’s problem becomes:

$$\max_{p_1 \in [\sqrt{\alpha N}, p_2]} \int_{x_L(p_1)}^{p_1 + p_2} h \left( \frac{\Delta(x, p_1) - \alpha N}{\beta N} \right) dx.$$ 

When the expected marginal cost of revolutionary effort is $\alpha + \beta e^\eta$, with $\beta, \eta > 0$, we can explicitly calculate the above integral:

$$E[e^*(x, p_1, p_2)] = \frac{\eta}{1 + \eta} \frac{1}{2(\beta N)^{1/\eta}} \frac{p_1 p_2 - \alpha}{p_1}^{1+\frac{1}{\eta}},$$

which is increasing in $p_1 \in [\sqrt{\alpha N}, p_2)$. The intuition is most clear along the extensive margin of support. By raising the revolutionary agenda from $p_1$ to $p_1 + dp_1$, revolutionary entrepreneur 1 loses his most moderate supporters: $x_L$ rises to $x_L + dx_L$. However, some of the other revolutionary entrepreneur’s supporters switch to support him: $(p_1 + p_2)/2$ rises to $(p_1 + p_2)/2 + dp_1/2$. Thus, the total change along the extensive margin becomes:

$$\left( \frac{1}{2} - \frac{dx_L(p_1)}{dp_1} \right) dp_1 = \left( \frac{1}{2} - \frac{1}{2} \frac{\alpha N}{2p_1^2} \right) dp_1 = \frac{\alpha N}{2p_1^2} dp_1 > 0.$$ 

Because more extreme citizens put in more effort to support a revolutionary agenda, the change in the intensive margin of support is also positive, further strengthening the revolutionary entrepreneur 1’s incentive to radicalize his agenda.

This result implies that, in any equilibrium, the revolutionary entrepreneurs must choose the same revolutionary agenda, i.e., $p_1 = p_2 = p^{**}$. This, in turn, implies that the total efforts of citizens with ideal points to the left and right of $p^{**}$ must be equal, otherwise, a revolutionary entrepreneur can gain from slight deviations from $p^{**}$. However, this condition does not guarantee that $p^{**}$ is an equilibrium in our setting: because effort schedules vary with revolutionary agendas,
when a revolutionary entrepreneur radicalizes, the extreme citizens raise their support for him (see Figure 5). In the proof, we show that such gains along the intensive margin never dominate the losses along the extensive margin. Therefore, the revolutionary agenda $p^{**}$ is an equilibrium if and only if the total efforts of citizens with ideal points to the left and right of $p^{**}$ are equal.

Using equation (7), direct calculations reveal that (i) there is a unique $p^{**}$, and (ii) this $p^{**}$ is more extreme than the equilibrium revolutionary agenda $p^*$ when there is only one revolutionary entrepreneur—equation (1). In particular,

$$p^{**} = \frac{1 + \sqrt{1 + \left(\frac{2}{1 + \eta} - 1\right)\alpha N}}{1 + 2\frac{\eta}{1 + \eta}} > p^* = \frac{1 + \sqrt{1 + \eta(2 + \eta)\alpha N}}{2 + \eta}.$$

Proposition 4 states that these findings generalize to convex expected marginal costs of revolutionary effort, $\alpha + \beta g(e)$.

**Proposition 4.** Suppose $g(e)$ is weakly convex. When there are two revolutionary entrepreneurs who each maximizes his support, in the unique equilibrium, they both choose the same revolutionary agenda $p^{**}$. Moreover, the revolutionary agenda becomes more extreme when there are two revolutionary entrepreneurs rather than one: $p^{**} > p^*$.

As we showed above, when the expected marginal cost of revolutionary effort takes the functional form of $\alpha + \beta \ e^\eta$, these results are valid even when $\eta \in (0, 1)$, so that $g(e)$ is concave. We suspect that a weaker sufficient condition is that $\frac{e^* (x=1, p)}{e^* (x=p,p)}$ be weakly decreasing in $p$, which is satisfied when
Figure 6: The solid (dashed) curve is the equilibrium revolutionary agenda with two (one) revolutionary entrepreneurs as a function of the minimum punishment \( \alpha \in (0, 1/N) \) for a linear expected marginal cost of revolutionary effort: \( \alpha + \beta e \). Competition between revolutionary entrepreneurs radicalizes the revolutionary agenda.

\[ g(e) = e^\eta, \ \eta > 0. \] However, we could only prove the results when \( g(e) \) is weakly convex, i.e., when the third derivative of the cost function is non-negative: \( C'''(e) \geq 0 \).

Figure 6 shows that \( p^{**}(\alpha) > p^*(\alpha) \) for all \( \alpha N \in [0, 1) \). The radicalizing effect of competition between revolutionary entrepreneurs stems from the disproportionate influence of extreme citizens and the presence of the state’s status quo policy. If both revolutionary entrepreneurs choose the agenda \( p^* \), the total efforts of the citizens who are more extreme than \( p^* \) exceeds the efforts of the more moderate supporters. As a result, a revolutionary entrepreneur radicalizes his agenda to attract all the citizens who are more extreme than \( p^* \).

We highlight that our analysis of the competition between revolutionary entrepreneurs partly depends on the assumption that the status quo policy is sufficiently extreme that both revolutionary entrepreneurs choose their agendas on the same side of the policy space. Clearly, if the status quo policy is close to the middle of the policy space (i.e., if \( s = \frac{1}{2} \)), then revolutionary entrepreneurs choose their agendas on the opposite sides of the status quo, in which case they do not compete for the citizens’ support. Then, the likelihood that the regime is overthrown increases, but a revolutionary entrepreneur does not radicalize his revolutionary agenda due to the presence of another revolutionary entrepreneur. However, in many dictatorships, the status quo policy is extreme relative to the population. Moreover, when the status quo is sufficiently close to one extreme, both revolutionary entrepreneur will choose the same side of the policy space, in which case their competition leads to the radicalization of the revolutionary agenda.
6. Extensions

**Distribution of Citizen Preferences.** In the text, we focused on the case in which the distribution of citizen types is uniform, i.e., \( x_i \sim U[0,1] \). Clearly, the assumption of uniform distribution is restrictive: with non-uniform distributions, the revolutionary entrepreneur assigns different weights to different ideal policies when he calculates the citizens’ expected total effort. However, such generalizations do not provide any deeper insight in our paper. For example, consider a case with a large group of citizens concentrated around a moderate policy and a small group of extremists. The tradeoff between the extensive and intensive margins of support exists in this setting. However, the loss in the extensive margin of support may be so large to offset the gains along the intensive margin, in which case the revolutionary entrepreneur does not cater to extremists. Instead, we focus on more interesting comparative statics. Suppose \( x_i \sim iid f(x_i; \theta) \), where \( \theta \in \mathbb{R} \) parameterizes a family of distributions \( \{f(x_i, \theta)\} \). From (2),

\[
E[e^*_i; \theta] = \int_{x_L(p)}^1 h \left( \frac{\Delta(x_i, p) - \alpha}{\beta} N \right) f(x_i; \theta) dx_i.
\]

The revolutionary entrepreneur chooses a revolutionary agenda that maximizes \( E[e^*_i; \theta] \):

\[
p^*(\theta) = \arg \max_p \int_0^1 1_{\{x_i \geq x_L\}} h \left( \frac{\Delta(x_i, p) - \alpha}{\beta} N \right) f(x_i; \theta) dx_i,
\]

where \( 1_{\{A\}} \) is the indicator function that is 1 if \( A \) is true and is 0 otherwise. Then, the literature on monotone comparative statics (Ashworth and Bueno de Mesquita 2006; Athey 2002) implies: If the family of distributions \( \{f(x_i, \theta)\} \) has increasing likelihood ratio property, then \( p^*(\theta) \) is increasing in \( \theta \). That is, when the distribution of citizens’ ideals shifts to the right (in the sense of increasing likelihood ratio), so that there are more extremists, the revolutionary entrepreneur makes his revolutionary agenda more extreme.

**Uncertain Revolutionary Agenda.** In the text, we assumed that if the revolution succeeds, the revolutionary entrepreneur’s revolutionary agenda \( p \) will be implemented, or at least, citizens believe so. However, given the great uncertainties involved in a revolution, the post-revolution policy that is eventually implemented may not be exactly the proposed revolutionary agenda. Thus, we consider a variation of the model in which if the revolutionary agenda is \( p \) and the revolution succeeds, the post-revolution policy will be a random variable \( \hat{p} \) distributed according to a pdf \( f(\hat{p}) \) with a mean \( p \) and a variance \( v(p) \). Then, the expected revolution payoff of a citizen with ideal point \( x \) becomes \( E[-(x - \hat{p})^2|p] \), where the expectation is over \( \hat{p} \). Using \( E[\hat{p}^2|p] = v + (E[\hat{p}|p])^2 = v(p) + p^2 \), we have \( E[-(x - \hat{p})^2|p] = -(x - p)^2 - v(p) \). Therefore, given a revolutionary agenda \( p \), the expected payoff
of citizen $i$ with ideal point $x_i$ from contributing revolutionary effort $e_i$ becomes:\footnote{See (10) in the proof of Lemma 1.}

$$u(x_i, s) + \frac{\sum_{j=1}^{N} e_j}{N} [\Delta(x_i, p) - v(p)] - C(e_i).$$

That is, the uncertainty in the post-revolution policy reduces the citizens’ marginal gain from supporting the revolution, and this reduction increases with the magnitude of the uncertainty as captured by the variance $v(p)$. When $v(p)$ is constant for $p \geq \sqrt{\alpha N}$, the effect of this uncertainty is the same as the effect of raising the expected minimum punishment from $\alpha$ to $\alpha + v/N$.

**When Repression Also Depends on the Revolutionary Agenda.** In the text, we assumed that the costs of revolutionary efforts do not depend on the revolutionary agenda. In cases in which the regime is at its maximum capacity for repression, this specification is plausible. In other cases, a natural question is whether our results still hold if repression rises as the revolutionary agenda becomes more extreme. This may create an incentive for the revolutionary entrepreneur to moderate the agenda. However, as far as these costs do not rise too sharply with $p$, the strategic forces that we identify dominate and the qualitative nature of our results do not change.

To illustrate, we consider the case in which the marginal cost of revolutionary effort is $\frac{\partial C(e, p)}{\partial e} = A(p) + \alpha + \beta g(e)$, where $A(0) = 0$, $A'(p) > 0$, $A''(p) \geq 0$, and $N = 1$ to simplify exposition. Mirroring the calculations of Lemma 1, for a given revolutionary agenda $p$, differentiating a citizen’s payoff with respect to the revolutionary effort $e$ yields $\Delta(x, p) - A(p) - \alpha - \beta g(e)$. Therefore, the ideal policy of the most moderate supporter is $x_L(p) = \frac{p^2 + \alpha}{2p} + \frac{A(p)}{2p}$, which exceeds the original $\frac{p^2 + \alpha}{2p}$.

In the original model, we needed to have $\alpha < 1$ for $\min_{p \in [0,1]} x_L(p) < 1$, so that the revolution had a chance of success. In the modified model this condition becomes tighter.\footnote{As in the text, we also assume repression is high enough that $e^*(x, p) < 1$ for $x, p \in [0,1]$.} For this additive separable specification, net from the tighter upper bound on $\alpha$, minimal assumptions on $A(p)$ deliver our main results when there is a single revolutionary entrepreneur.

**Remark 3.** Suppose $g(e)$ is weakly convex, $A(0) = 0$, $A'(p) > 0$, and $A''(p) \geq 0$. Then, (i) $x_L(p)$ has a unique minimum at a policy $p_b \in (0,1)$, and no other extremum, (ii) $p^* > p_b$, and (iii) $\frac{dp^*}{d\alpha} > 0$.

For example, suppose $h(x) = x$ and $A(p) = p^2$. Then, $p_b = \sqrt{\alpha/2}$, $x_L(p_b) = \sqrt{2\alpha}$, and hence we assume $\alpha < 1/2$. Moreover, $e^*(1, p) = \frac{p^2(2-p) - \alpha - p^2}{3p^2}$ and $E[e^*(x, p)] = \frac{\beta}{2p} \int_0^{e^*(1, p)} z dz = \frac{\beta}{2p} [e^*(1, p)]^2$. Thus, from equation (30), $1 - 2p^* - p^*(2-p^*) - \alpha(p^*)^2 = 0$, implying $p^* = 1 + \sqrt{1 + 4\alpha \frac{1}{2}}$. In contrast, if $A(p) = 0$, then revolutionary agenda would raise to $\frac{1 + \sqrt{1 + 4\alpha}}{2}$. Thus, although $A(p) > 0$ moderates the agenda, the revolutionary entrepreneur chooses a more extreme agenda than what he would
choose with binary efforts \((p^* > p_b)\) for \(\alpha < 1/2\), and \(p^*\) still rises with the minimum punishment \(\alpha\).

Because higher minimum punishments \(\alpha\) radicalize the equilibrium revolutionary agenda, the results of Proposition 3 that are based on the elite’s tradeoff between likely but moderate and unlikely but radical revolutions remain. Finally, consider our results on the competition between revolutionary entrepreneurs. To convey intuition, recall that we earlier showed that the moderate revolutionary entrepreneur always gains along the extensive margin if he moves closer to the radical revolutionary entrepreneur (p. 21). This result does not hold when \(A(p_1) - p_1 A'(p_1)\) is sufficiently negative:

\[
\left(\frac{1}{2} - \frac{dx_L(p_1)}{dp_1}\right) dp_1 = \left(\frac{1}{2} - \frac{1}{2} + \frac{\alpha + A(p_1) - p_1 A'(p_1)}{2\beta N}\right) dp_1.
\]

For example, it holds when \(A(p)\) is linear, so that \(A(p_1) - p_1 A'(p_1) = 0\), but not when \(A(p) = p^m\) and \(m\) is large, so that repression, \(A(p)\), rises sharply with the revolutionary agenda. Mirroring the steps of the proof of Proposition 4, one can show that this intuition extends: when \(A(p)\) rises slowly (e.g., it is linear), still \(p_1 = p_2\) is any pure strategy equilibrium; however, when it rises sufficiently fast, two revolutionary entrepreneurs may choose different revolutionary agendas in equilibrium.

**Supporting the Regime.** In the text, we assumed that the citizens can only support the revolution, but not the regime. Here, we allow a citizen \(i\) to support the regime by exerting effort \(r_i \in [0, 1]\). The marginal cost of exerting effort \(r\) is: \(\alpha + \beta g(r)\). The likelihood that revolution succeeds is \(\max\{0, \Sigma_{i=1}^N (e_i - r_i)/N\}\), so that supporting the regime reduces the likelihood of successful revolution. Clearly, a citizen either supports the regime or the revolution. Moreover, citizens that are sufficiently indifferent between the status quo and the revolutionary agenda do not exert costly effort to support the regime or the revolution. We have already established that, for a given revolutionary agenda \(p\), only citizens whose ideal policies are larger than a threshold \(x_L(p) = \frac{p^2 + \alpha}{2p}\) support the revolution.

By symmetry, when citizens can support the regime, only those whose ideal policies are less than the threshold \(x_r(p) = p - x_L(p) = \frac{p^2 - \alpha}{2p}\) support the regime. That is, the citizens with ideal policies \(x_i \in [x_r(p), x_L(p)] = [\frac{p}{2} - \frac{\alpha}{2p}, \frac{p}{2} + \frac{\alpha}{2p}]\) are sufficiently indifferent between the status quo and the revolution alternative that do not put in any costly effort. Using analogous calculations that determine \(e^*(x, p)\), given a revolutionary agenda \(p\), a citizen with ideal point \(x \in [0, x_r(p)]\) exerts an effort level \(r^* = h\left(\frac{-p(2x - p) - \alpha}{\beta N}\right)\) to support the regime. The revolutionary entrepreneur chooses a revolutionary agenda to maximize the expected likelihood of revolution

\[
E[e^*(x, p)] - E[r^*(x, p)] = -\int_0^{x_r(p)} h\left(\frac{-p(2x - p) - \alpha}{\beta N}\right) dx + \int_{x_L(p)}^1 h\left(\frac{p(2x - p) - \alpha}{\beta N}\right) dx
\]

For simplicity, suppose \(h(x) = x\), which allows for a closed form solution. Substituting for \(x_r(p) = \frac{p^2 - \alpha}{2p}\) and \(x_L(p) = \frac{p^2 + \alpha}{2p}\) and integrating yields \(E[e^*(x, p)] - E[r^*(x, p)] = (p^2 + (1 + \alpha N)p - \alpha N)/\beta N\).
The revolutionary entrepreneur chooses the revolutionary agenda \( \hat{p} \) that maximizes this expression, and hence:
\[
\sqrt{\alpha N} < \hat{p} = \frac{1 + \alpha N}{2} < p^* = \frac{1 + \sqrt{1 + 3\alpha N}}{3}.
\]
Indeed, if some citizens can support the regime, the revolutionary entrepreneur moderates the revolutionary agenda to reduce their support for the regime. However, as in the original model, increases in the minimum punishment \( \alpha \) radicalize the revolutionary agenda: \( \hat{p} \) is increasing in \( \alpha \).

**General Status Quo and Concession.** In the text, we assumed that the status quo policy is 0. Changing the status quo from 0 to \( s \) does not change the nature of our results. With one revolutionary entrepreneur, the revolutionary entrepreneur’s problem becomes:
\[
\int_{sN+(p-s)^2}^{1} h \left( \frac{(2(x-s)-(p-s))(p-s)-\alpha N}{\beta N} \right) dx.
\]
To simplify the exposition, suppose \( g(e) = e \), so that the marginal cost of revolutionary effort is \( \alpha + \beta e \). Direct calculations reveal that:
\[
p^* = 1 + 2s + \frac{\sqrt{(1-s)^2 + 3\alpha N}}{3}. \tag{8}
\]
Changing the location of the status quo policy is a simple rescaling. When \( s = 0 \), from equation (1),
\[
p^* = \frac{1 + \sqrt{1 + 3\alpha N}}{3},
\]
and hence when \( s > 0 \), we must have \( p^* - s = \frac{1 - s + \sqrt{(1-s)^2 + 3\alpha N}}{3} \), which simplifies to equation (8). As before, increases in the minimum punishment \( \alpha \) radicalize the revolutionary agenda.

When there are two revolutionary entrepreneurs, however, \( s > 0 \) can undermine the competition between two revolutionary entrepreneurs because they may choose their revolutionary agendas on the opposite sides of the status quo, in which case they do not compete for the citizens’ support. Then, the likelihood that the regime is overthrown increases, but a revolutionary entrepreneur does not radicalize his revolutionary agenda due to the presence of another revolutionary entrepreneur.

We model state concessions by allowing the state to commit to a status quo policy \( s > 0 \) to mitigate the likelihood of revolution. Similar to increases in the minimum punishment, policy concessions (increases in \( s \in [0, 1/2] \)) radicalize the revolutionary agenda, but reduce the revolutionary entrepreneur’s ability to garner support. In particular, given a repression structure, the state’s problem is:
\[
\min_s E[e^*(s, p^*(s))] (p^*(s))^m + (1 - E[e^*(s, p^*(s))]) s^m.
\]
Increases in \( s \) raise \( p^*(s) \), but reduce \( E[e^*(s, p^*(s))] \). Therefore, when choosing how much to concede, the state faces the same tradeoffs that arise in the choice of the minimum punishment.
7. Conclusion

What determines the extremism of revolutionary movements? The literature on protests and revolutions offers little guidance. To investigate the sources of extremism of revolutions, we develop a simple model in which the revolutionary agenda is determined by revolutionary entrepreneurs in their interactions with citizens and the state. At its core, the paper identifies three determinants of extremism in revolutionary movements: The disproportionate influence of extreme citizens, the structure of state repression, and competition between revolutionary entrepreneurs.

People’s preferences determine their grievances, and hence their potential for anti-regime activities. Heterogeneous citizenry and dissent gradation present the revolutionary entrepreneurs with a tradeoff between the extensive and intensive margins of support: set a moderate agenda and cultivate the low-intensity support of an extensive number of citizens, or set an extreme agenda and cultivate the intense support of a narrower subset of citizens. We show that the structure of repression matters: Higher minimum punishments dampen the likelihood of successful revolutions, but radicalize the revolutionary agenda. This contrasts with indiscriminate repression that punishes all supporters and non-supporters alike: unlike higher minimum punishments, more indiscriminate repression increases the likelihood of successful revolution, but moderates the revolutionary agenda. We show how the attitude of the ruling class toward (political) risk influences their choice of repression structure: slight increases in the minimum punishment hurt the ruling class when they are sufficiently risk-averse and the minimum punishment is low. This implies that low levels of minimum punishment have inertia, but they can jump in response to small decreases in repression costs. Finally, we show that competition between two revolutionary entrepreneurs can radicalize the revolutionary agenda.

To underscore the empirical import of our theoretical findings, we return to our opening examples of the Iranian and French revolutions. Although revolutions are complex processes, our simple model can shed light on why the levels of extremism in these revolutions differed. In France, the state was more tolerant of minor dissent (had a lower minimum punishment) under the late Bourbon Monarchy in the years preceding the 1830 Revolution than under the late Orleans Monarchy preceding the 1848 Revolution. As early as 1835, the (Orleans) July Monarchy “was associated with...political reaction and vigorous repression of all opposition” (Fortescue 2005, p. 27). Even minimal dissenting activities were punished as political gatherings were illegal, and associations, strikes, and protests were harshly repressed (Pilbeam 1991, Ch. 8; Pinkney 1973). Consistent with our theoretical result, the 1848

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28The state could not raise repression so much to completely deter dissent due to its limited resources (Price 1982).
Revolution that ended monarchy and established the Second Republic was far more extreme than the 1830 Revolution that replaced the (restored) House of Bourbon with the House of Orleans. Turning to Iran, the competition between the socialist and Islamic revolutionaries contributed to the extremism of the 1979 Iranian Revolution. Islamic revolutionary entrepreneurs were anxious about losing their supporters to the socialists (especially, among the youth). In response, ideologues such as Shariati, Motahhari, and even Khomeini adapted many radical elements of socialist platform such as anti-imperialism, social justice, and redistribution to win the poor and radical youth (Boroujerdi 1996; Dabashi 1993).

On the empirical front, current datasets on repression (Cingranelli et al. 2013; Gibney et al. 2012; Freedom House) do not capture how punishments vary with the level of anti-regime activities, and current datasets on protests and revolutions (Banks and Wilson 2013; Chenoweth and Lewis 2013; Minorities at Risk Project 2009) do not measure the extremism of their agendas. By showing the strategic interactions between the structure of repression and the extremism of revolutionary agendas, we hope that this paper informs future data collections and empirical projects.

The paper focused on the effects of repression, competition, and dissent gradation on the extremism of revolutionary agenda, abstracting from other factors such as coordination among citizens and fragmentation of the opposition. In many settings, opposition groups interact to form and break coalitions (Christia 2012; Pearlman and Cunningham 2012). Analyzing their strategic interactions is a fruitful area for future research that sheds light on both pre-revolution struggles and post-revolution consolidation processes.

Appendix: Proofs

Proof of Lemma 1: Given a revolutionary agenda \( p \), the expected payoff of citizen \( i \), with ideal policy \( x_i \), who contributes effort \( e_i \) is:

\[
\frac{\sum_{j=1}^{N} e_j}{N} u(x_i, p) + \left(1 - \frac{\sum_{j=1}^{N} e_j}{N}\right) u(x_i, s) - C(e_i),
\]

(9)

where \( C(e_i) \) is the expected cost of contributing effort \( e_i \). With probability \( \frac{\sum_{j=1}^{N} e_j}{N} \) the revolution succeeds and \( i \) receives \( u(x_i, p) \). With the remaining probability, the status quo prevails and \( i \) receives the status quo payoff \( u(x_i, s) \). Let \( \Delta(x_i, p) \equiv u(x_i, p) - u(x_i, s) \) be the change in citizen \( i \)'s payoff from a successful revolution, so that \( \Delta(x_i, p) = p(2x_i - p) \leq 1 \). Rewrite (9) as:

\[
u(x_i, s) + \frac{\sum_{j=1}^{N} e_j}{N} \Delta(x_i, p) - C(e_i).
\]

(10)
Citizen $i$’s marginal benefit from exerting effort is $\Delta(x_i, p)/N$, and his marginal cost is $\alpha + \beta g(e_i)$. Thus, the interior solution of the first order condition is:

$$e^*_i(x_i, p) = g^{-1} \left( \frac{\Delta(x_i, p) - \alpha N}{\beta N} \right) = h \left( \frac{\Delta(x_i, p) - \alpha N}{\beta N} \right). \quad (11)$$

Provided this value is feasible, i.e., $e^*_i \in [0, 1]$, it is the citizen’s optimal effort choice. Otherwise, the citizen’s effort contribution is either zero or one. Because $g'(\cdot) > 0$, we have $h'(\cdot) > 0$. Moreover, $\frac{\Delta(x_i, p) - \alpha N}{\beta N} = \frac{p(2x_i - p) - \alpha N}{\beta N}$ is strictly increasing in $x_i$. Let $x_L$ be the unique solution to $p(2x_i - p) - \alpha N = 0$, so that $x_L = \frac{p^2 + \alpha N}{2p}$. Thus, $h \left( \frac{\Delta(x_i, p) - \alpha N}{\beta N} \right) > 0$ if and only if $x_i > x_L$. Similarly, let $x_H$ be the unique solution to $1 = h \left( \frac{\Delta(x_i, p) - \alpha N}{\beta N} \right)$, so that $x_H \equiv \frac{p^2 + (\alpha + \beta g(1))N}{2p}$. Thus, $h \left( \frac{\Delta(x_i, p) - \alpha N}{\beta N} \right) < 1$ if and only if $x_i < x_H$. Therefore,

$$e^*(x_i, p) = \begin{cases} 
0 & \text{if } x_i \leq x_L \\
h \left( \frac{\Delta(x_i, p) - \alpha N}{\beta N} \right) & \text{if } x_L \leq x_i \leq x_H \\
1 & \text{if } x_H \leq x_i,
\end{cases}$$

where $x_L \equiv \frac{p^2 + \alpha N}{2p}$ and $x_H \equiv \frac{p^2 + (\alpha + \beta g(1))N}{2p}$.

Moreover, $x_H(p)$ is minimized at $p = \sqrt{(\alpha + \beta g(1))N}$ at which $x_H = \sqrt{(\alpha + \beta g(1))N}$. From Assumption 1, $x_H > 1$, and hence irrelevant for the analysis. □

**Proof of Proposition 1:** The revolutionary entrepreneur chooses a $p$ to maximize the likelihood of successful revolution:

$$E[e^*(x, p)] = Pr(x \geq x_L) E \left[ h \left( \frac{\Delta(x, p) - \alpha N}{\beta N} \right) \bigg| x \geq x_L \right]. \quad (12)$$

The revolutionary entrepreneur chooses a $p$ for which $x_L < 1$, otherwise, the likelihood of success is 0. From Lemma 1, $x_L(p) = \frac{p^2 + \alpha N}{2p}$, and hence, for $p \in [0, 1]$, $x_L < 1$ if and only if $p > p_L \equiv 1 - \sqrt{1 - \alpha N}$. Thus, for the relevant range of $p \geq p_L$, equation (12) can be written as:

$$E[e^*(x, p)] = \int_{x_L(p)}^{1} h \left( \frac{\Delta(x, p) - \alpha N}{\beta N} \right) dx. \quad (13)$$

Differentiating with respect to $p$ yields:

$$\frac{dE[e^*(x, p)]}{dp} = -\frac{dx_L(p)}{dp} h \left( \frac{\Delta(x_L(p), p) - \alpha N}{\beta N} \right) + \int_{x_L(p)}^{1} \frac{d}{dp} h \left( \frac{\Delta(x, p) - \alpha N}{\beta N} \right) dx$$

$$= \frac{1}{\beta N} \int_{x_L(p)}^{1} \frac{d\Delta(x, p)}{dp} h' \left( \frac{\Delta(x, p) - \alpha N}{\beta N} \right) dx$$

$$= \frac{2}{\beta N} \int_{x_L(p)}^{1} (x - p) h' \left( \frac{\Delta(x, p) - \alpha N}{\beta N} \right) dx,$$
where the second equality follows from \( h\left(\frac{\Delta(x_L(p),p) - \alpha N}{\beta N}\right) = h(0) = 0 \), and the third follows from \( \frac{d\Delta(x,p)}{dp} = 2(x-p) \). Because \((x-1) \leq 0 < h'(\cdot), \frac{dE^*[x,(p)]}{dp}\right|_{p=1} < 0 \). Moreover, for \( p \in [\beta, \sqrt{\alpha N}] \), we have \( p \leq x_L(p) \), and hence \( \frac{dE^*[x,(p)]}{dp} > 0 \). In particular, when \( p = \sqrt{\alpha N} \), then \( x_L(p) = \sqrt{\alpha N} \), and hence \( \frac{dE^*[x,(p)]}{dp}\right|_{p=\sqrt{\alpha N}} > 0 \). Thus, the equilibrium revolutionary agenda \( p^* \in (\sqrt{\alpha N}, 1) \). The result for \( p^* \) in the special case of \( g(e) = e^\eta, \eta > 0 \), is derived from direct calculations, using equation (13). \( \square \)

**Proof of Proposition 2:** In the special case of \( g(e) = e^\eta \), with \( \eta > 0 \), the results are derived from the explicit equation of \( p^* \) in Proposition 1. Here, we prove \( \frac{dp^*}{da} > 0 \) for the general \( g(e) \).

The revolutionary entrepreneur maximizes the probability of successful revolution. Focusing on the relevant range of \( p \geq p^* > 0 \), from (13), the revolutionary entrepreneur’s optimization problem is

\[
\max_p E[e^*(x,p)] = \max_p \int_{x_L(p)}^{1} h\left(\frac{\Delta(x,p) - \alpha N}{\beta N}\right) \, dx.
\]

Let \( z \equiv \frac{\Delta(x,p) - \alpha N}{\beta N} = \frac{2px - p^2 - \alpha N}{\beta N} \), so that:

\[
E[e^*(x,p)] = \frac{\beta N}{2p} \int_0^{\frac{\Delta(x,p) - \alpha N}{\beta N}} h(z) \, dz.
\]

Differentiating with respect to \( p \) yields:

\[
\frac{\partial E[e^*(x,p)]}{\partial p} = \frac{\beta N}{2p^2} \left( h\left(\frac{\Delta(1,p) - \alpha N}{\beta N}\right) \frac{d\Delta(1,p)}{dp} \frac{1}{\beta N} - \int_0^{\frac{\Delta(1,p) - \alpha N}{\beta N}} h(z) \, dz\right) = \frac{1}{p} \left( h\left(\frac{\Delta(1,p) - \alpha N}{\beta N}\right) (1-p) - \frac{\beta N}{2p} \int_0^{\frac{\Delta(1,p) - \alpha N}{\beta N}} h(z) \, dz\right) = e^*(1,p)(1-p) - E[e^*(x,p)]
\]

(15)

Because \( p^* \) is an interior maximum, to prove \( p^* \) is strictly increasing in \( \alpha \), we must show that \( \frac{\partial^2 E[e^*(x,p)]}{\partial \alpha \partial p}\right|_{p=p^*} > 0 \). From equation (15),

\[
\frac{\partial^2 E[e^*(x,p)]}{\partial \alpha \partial p} = \frac{1}{p} \left[ \frac{\partial e^*(1,p;\alpha)}{\partial \alpha} (1-p) - \frac{\partial E[e^*(x,p;\alpha)]}{\partial \alpha} \right],
\]

(16)

where we have made explicit the dependency of \( e^*(x,p;\alpha) \) and \( E[e^*(x,p;\alpha)] \) on \( \alpha \). Moreover,

\[
\frac{\partial e^*(1,p;\alpha)}{\partial \alpha} (1-p) = \frac{1-p}{\beta} h'\left(\frac{\Delta(1,p) - \alpha N}{\beta N}\right) = -(1-p) \frac{N}{2p} \frac{d}{dx} h'\left(\frac{\Delta(1,p) - \alpha N}{\beta N}\right) |_{x=1}.
\]

(17)

Further, from equation (14),

\[
\frac{\partial E[e^*(x,p;\alpha)]}{\partial \alpha} = -\frac{N}{2p} h\left(\frac{\Delta(1,p) - \alpha N}{\beta N}\right) = -\frac{N}{2p} e^*(1,p).
\]

(18)
Substituting from equations (17) and (18) into (16) yields
\[
\frac{\partial^2 E[e^*(x, p)]}{\partial \alpha \partial p} = \frac{1}{p} \left[ -(1-p) \frac{N}{2p} \frac{de^*(x, p)}{dx} \right]_{x=1} + \frac{N}{2p} e^*(1, p) = \frac{N}{2p^2} \left[ -(1-p) \frac{de^*(x, p)}{dx} \right]_{x=1} + e^*(1, p).
\]

Thus,
\[
\frac{\partial^2 E[e^*(x, p)]}{\partial \alpha \partial p} > 0 \text{ if and only if } e^*(1, p) > (1-p) \frac{de^*(x, p)}{dx} \bigg|_{x=1}.
\]

Moreover, the convexity of \(g(\cdot)\) implies the concavity of its inverse \(h(\cdot)\), and hence
\[
\frac{de^*(x, p)}{dx} \bigg|_{x=1} \leq \frac{e^*(1, p) - e^*(x_L(p), p)}{1 - x_L(p)} = \frac{e^*(1, p)}{1 - x_L(p)}
\]
where we recall that \(h\left(\frac{p(2x_L(p)-p) - \alpha N}{\beta N}\right) = e^*(x_L(p), p) = 0\) and \(h\left(\frac{p(2-p) - \alpha N}{\beta N}\right) = e^*(1, p)\). Thus, from (19) and (20), a sufficient condition for \(\frac{\partial^2 E[e^*(x, p)]}{\partial \alpha \partial p} \bigg|_{p=p^*} > 0\) is that \(1 > \frac{1-p^*}{1-x_L(p^*)}\), where we recall that \(p^*\) is the equilibrium revolutionary agenda. Finally, \(1 > \frac{1-p^*}{1-x_L(p^*)}\) if and only if \(x_L(p^*) < p^*\), which is true from Proposition 1 and our earlier observation that \(x_L(p) < p\) for \(p \in (\sqrt{\alpha N}, 1)\).

**Proof of Remark 1:** From Lemma 1 and Proposition 1,
\[
\frac{de^*(x=1, p^*(\alpha), \alpha)}{d\alpha} = \frac{\partial e^*(x=1, p^*(\alpha), \alpha)}{\partial p^*} \frac{\partial p^*}{\partial \alpha} + \frac{\partial e^*(x=1, p^*(\alpha), \alpha)}{\partial \alpha} = \frac{1}{\beta N} h'\left(\frac{p^*(2-p^*) - \alpha N}{\beta N}\right) \left(2(1-p^*) \frac{\partial p^*}{\partial \alpha} - N\right).
\]

Thus, \(\frac{de^*(x=1, p^*(\alpha), \alpha)}{d\alpha} > 0\) if and only if \(\frac{\partial p^*}{\partial \alpha} > \frac{N}{2(1-p^*)}\), recognizing that \(p^* < 1\).

Differentiating equation (1) with respect to \(\alpha\) yields:
\[
\frac{\partial p^*(\alpha)}{\partial \alpha} = \frac{N\eta}{2\sqrt{1+\eta(2+\eta)\alpha N}} = \frac{N\eta}{2} \frac{1}{(2+\eta)p^*-1},
\]
where we have used \(\sqrt{1+\eta(2+\eta)\alpha N} = (2+\eta)p^*-1 > 0\). Thus, \(\frac{de^*(x=1, p^*(\alpha), \alpha)}{d\alpha} > 0\) if and only if \(\frac{N\eta}{2(2+\eta)p^*-1} > \frac{N}{2(1-p^*)}\), i.e., \(\frac{1}{2} > p^*, \text{ i.e.,} \frac{1}{2} > \frac{\sqrt{1+\eta(2+\eta)\alpha N}}{2+\eta} > \frac{1}{\sqrt{1+\eta(2+\eta)\alpha N}}\), which simplifies to \(\alpha N < \frac{\eta - 2}{4\eta}\).

**Proof of Proposition 3:** From equation (6),
\[
\frac{dU_*}{d\alpha} = \left[ -\frac{d[p^*(\alpha)]^m}{d\alpha} E\left[ e^*(p^*(\alpha), \alpha) \right] + [p^*(\alpha)]^m \frac{dE[e^*(p^*(\alpha), \alpha)]}{d\alpha} \right] = \left[ -[p^*(\alpha)]^{m-1} \left[ \frac{m}{p^*(\alpha)} \frac{dE^*}{d\alpha} \frac{E^*}{p^*(\alpha)} e^*(p^*(\alpha), \alpha) \right] \right],
\]
where we have used the fact that \(\frac{\partial E[e^*(p, \alpha)]}{\partial p} \bigg|_{p=p^*} = 0\). Thus, \(\frac{dU_*}{d\alpha} < 0\) if and only if \(m > m^*(\alpha)\), where:
\[
m^*(\alpha) \equiv -\frac{\partial E^*[e^*(p^*(\alpha), \alpha)]}{\partial \alpha} \left( \frac{dp^*(\alpha)/d\alpha}{p^*(\alpha)} \right)^{-1}.
\]

32
From equations (15) and (18), at \( p = p^* \), we have:

\[
\frac{\partial E[e^*(p^*(\alpha), \alpha)]/\partial \alpha}{E[e^*(p^*(\alpha), \alpha)]]} = -\frac{N}{2 p^*} e^*(1, p^*) e^*(1, p^*)(1 - p^*) = -\frac{N}{2 p^*(1 - p^*)},
\]

(22)

Substituting from equation (22) into (21) yields:

\[ m^*(\alpha) = \frac{N}{2} \left( 1 - p^* \right) \left( \frac{d p^*(\alpha)}{d \alpha} \right)^{-1}. \]

(23)

Differentiating with respect to \( \alpha \) yields:

\[
\frac{d m^*(\alpha)}{d \alpha} = \frac{N}{2} \left( 1 - p^* \right) \left( \frac{d p^*(\alpha)}{d \alpha} \right)^{-2} \left( \frac{d p^*(\alpha)}{d \alpha} - (1 - p^*) \frac{d^2 p^*(\alpha)}{d \alpha^2} \right).
\]

(24)

Because \( \frac{d p^*(\alpha)}{d \alpha} > 0 \) (by Proposition 2), if \( \frac{d^2 p^*(\alpha)}{d \alpha^2} \leq 0 \), then \( \frac{d m^*(\alpha)}{d \alpha} > 0 \).

Next, we prove the second part. When \( g(e) = e^n \), the revolutionary entrepreneur’s objective function is given by equation (3). Differentiating with respect to \( p \) yields:

\[
\frac{d E[e^*(x, p)]}{d p} = \frac{(-\alpha N + 2p - p^2)^{1/n} (\eta \alpha N + 2p - (2 + \eta)p^2)}{2(1 + \eta)p^2}.
\]

The first order condition for the maximum yields \( p^* = \frac{1}{2} \sqrt{\frac{1 + (2 + \eta)\alpha N}{2 + \eta}} \). Hence,

\[
\frac{d^2 p^*(\alpha)}{d \alpha^2} = -\left( \frac{\eta N}{2} \right) \frac{(2 + \eta)}{(1 + \alpha N(2 + \eta))^3} < 0 < \frac{\eta N}{2 \sqrt{1 + \alpha N(2 + \eta)}}
\]

and hence \( \frac{d m^*(\alpha)}{d \alpha} > 0 \). Moreover, substituting these results in equation (23) yields \( m^*(\alpha) = \frac{2 + \eta}{\eta} \frac{\sqrt{1 + (2 + \eta)\alpha N}}{1 + \eta - \sqrt{1 + (2 + \eta)\alpha N}} \), so that \( m^*(\alpha = 0) = \frac{2 + \eta}{\eta} \). Thus, if \( m \leq m^*(0) \), then \( \frac{d u_i(\alpha)}{d \alpha} \geq 0 \). However, if \( m > m^*(0) \), then \( \frac{d u_i(\alpha)}{d \alpha} < 0 \) if and only if \( \alpha < \alpha_m \equiv m^{-1}(m) \). \( \square \)

**Proof of Proposition 4:** Given the revolutionary agendas \( p_1 \) and \( p_2 \), the expected payoff of citizen \( i \), with ideal policy \( x_i \), who contributes \( e_i \) and \( e_i^2 \) is:

\[
\left( 1 - \frac{\sum_{j=1}^{N} e_j}{N} \right) u(x_i, s) + \frac{\sum_{j=1}^{N} e_j}{N} \left( \frac{\sum_{j=1}^{N} e_j}{N} \right) u(x_i, p_1) + \frac{\sum_{j=1}^{N} e_j^2}{N} u(x_i, p_2) - C(e_i)
\]

\[
\left( 1 - \frac{\sum_{j=1}^{N} e_j}{N} \right) u(x_i, s) + \frac{\sum_{j=1}^{N} e_j}{N} u(x_i, p_1) + \frac{\sum_{j=1}^{N} e_j^2}{N} u(x_i, p_2) - C(e_i).
\]

Clearly, if a citizen provides some support, he does so for the revolutionary agenda that is closer to his ideal. Moreover, we established in the text that there is no equilibrium in which \( p_i < \sqrt{\alpha N} \), and hence, in any equilibrium, \( x_L(p_i) \leq p_i, i \in \{1, 2\} \). Given revolutionary agendas \( p_i \) and \( p_j \), let \( e^*(x, p_i, p_j) \) be the optimal level of support for revolutionary entrepreneur \( i \) by a citizen with ideal
The left hand side is increasing. Thus, if the right hand side is decreasing, then \( \Omega' \), where:
\[
e^\ast (x_i, p_1, p_2) = \begin{cases} 
0 & \text{if } x_i \leq x_L(p_1) \\
h \left( \frac{\Delta(x_i, p_1) - \alpha N}{\beta N} \right) & \text{if } x_L(p_1) < x_i < \frac{p_1 + p_2}{2} \\
0 & \text{if } x_i < \frac{p_1 + p_2}{2} \\
\end{cases}
\]
and \( e^\ast (x_i, p_1, p_2) = e^\ast (x_i, p_2, p_1) = \frac{1}{2} h \left( \frac{p_1 p_2 - \alpha N}{\beta N} \right) \) whenever \( x_i = \frac{p_1 + p_2}{2} \).

Next, we show that there is no equilibrium in which \( p_1 \neq p_2 \). Without loss of generality, suppose \( \sqrt{\alpha N} < p_1 < p_2 \). Given 2's revolutionary agenda \( p_2 \), the expected support for the revolutionary entrepreneur 1 who chooses a revolutionary agenda \( p_1 \) is:
\[
\int_{x_L(p_1)}^{\frac{p_1 + p_2}{2}} h \left( \frac{\Delta(x, p_1) - \alpha N}{\beta N} \right) \, dx = \frac{\beta N}{2p_1} \int_0^{\frac{p_1 p_2 - \alpha N}{\beta N}} h(z) \, dz,
\]
where we have used a change of variable \( z = \frac{\Delta(x, p_1) - \alpha N}{\beta N} \). Differentiating with respect to \( p_1 \) yields:
\[
= \frac{\beta N}{2p_1} \left\{ \frac{p_1 p_2 - \alpha N}{\beta N} h \left( \frac{p_1 p_2 - \alpha N}{\beta N} \right) - \int_0^{\frac{p_1 p_2 - \alpha N}{\beta N}} h(z) \, dz \right\}
> \frac{\beta N}{2p_1} \left\{ \frac{p_1 p_2 - \alpha N}{\beta N} h \left( \frac{p_1 p_2 - \alpha N}{\beta N} \right) - \int_0^{\frac{p_1 p_2 - \alpha N}{\beta N}} h(z) \, dz \right\} > 0,
\]
where the last inequality follows from the Mean Value Theorem for integration together with the fact that \( h \left( \frac{p_1 p_2 - \alpha N}{\beta N} \right) \) is increasing in \( p_1 \).

Next, suppose \( p_1 = p_2 = p \). For \( p \) to be an equilibrium, a necessary condition is that \( \Omega(p) = 0 \), where:
\[
\Omega(p) \equiv \int_0^{\frac{p^2 - \alpha N}{\beta N}} h(z) \, dz - \int_{\frac{p^2 - \alpha N}{\beta N}}^{\frac{(2-p)^2 - \alpha N}{\beta N}} h(z) \, dz - 2 \int_0^{\frac{p^2 - \alpha N}{\beta N}} h(z) \, dz - \int_0^{\frac{(2-p)^2 - \alpha N}{\beta N}} h(z) \, dz. \tag{25}
\]
\( \Omega(p = \sqrt{\alpha N}) < 0 < \Omega(p = 1) \), and hence \( \Omega(p) = 0 \) has, at least one solution. We show that this solution is unique. Because \( h(z) \) is increasing, from (25) at any \( p \) that satisfies \( \Omega(p) = 0 \), we must have \( \frac{p^2 - \alpha N}{\beta N} > \frac{(2-p)^2 - \alpha N}{\beta N} \), i.e., \( 2(p^2 - \alpha N) > p(2-p) - \alpha N \), i.e., \( 3p^2 - 2p - \alpha N > 0 \), implying that \( p^* > \frac{1 + \sqrt{1 + 4\alpha N}}{3} > \frac{2}{3} > \frac{1}{2} \). Moreover, differentiating \( \Omega(p) \) with respect to \( p \) yields:
\[
\Omega'(p) = \frac{2}{\beta N} \left\{ 2p h \left( \frac{p^2 - \alpha N}{\beta N} \right) - (1-p) h \left( \frac{(2-p)^2 - \alpha N}{\beta N} \right) \right\}.
\]
Thus, for \( p \in (\sqrt{\alpha N}, 1) \), \( \Omega'(p) = 0 \) if and only if \( \frac{2p}{1-p} = h \left( \frac{(2-p)^2 - \alpha N}{\beta N} \right) \). The left hand side is increasing. Thus, if the right hand side is decreasing, then \( \Omega'(p) = 0 \) has at
most one solution, which, in turn, implies that \( \Omega(p) = 0 \) has a unique solution at which \( \Omega'(p) > 0 \). To show that the right hand side is decreasing, differentiate \( \frac{e^*_{(x=1,p)}}{e^*_{(x=p,p)}} \) with respect to \( p \):

\[
\frac{d}{dp} \frac{e^*(1,p)}{e^*(p,p)} = 2(1-p) \left( \frac{p(2-2p-\alpha N)}{\beta N} \right) h \left( \frac{p^2-\alpha N}{\beta N} \right) - 2p h' \left( \frac{p^2-\alpha N}{\beta N} \right) h \left( \frac{p(2-2p-\alpha N)}{\beta N} \right) \frac{\beta N}{h \left( \frac{p^2-\alpha N}{\beta N} \right)^2}.
\]

Given that \( p > 1/2 \), if \( h(\cdot) \) is (weakly) concave, so that \( h'(\cdot) \) is decreasing, then this derivative is negative, and hence \( \frac{e^*_{(1,p)}}{e^*_{(p,p)}} \) is decreasing.

It remains to show that \( p^{**} \) is, indeed, an equilibrium. That is, when \( p_1 = p_2 = p^{**} \), no player deviates to another revolutionary agenda. We have already shown that if \( p_i < p_j = p^{**} \), then revolutionary entrepreneur \( i \) gains by raising \( p_i \). Thus, it suffices to show that if \( p_j = p^{**} < p_i \), then revolutionary entrepreneur \( i \) gains by reducing \( p_i \). When \( p_2 = p^{**} < p_1 \), revolutionary entrepreneur \( 1 \)'s expected payoff is:

\[
\int_{\frac{1}{1+p^{**}}}^{1} h \left( \frac{\Delta(x,p_1) - \alpha N}{\beta N} \right) dx = \frac{\beta N}{2p_1} \int_{\frac{p_1(2-2p_1-\alpha N)}{\beta N}}^{p_1(2-2p_1-\alpha N)} h(z) dz,
\]

where we have used a change of variable \( z = \frac{\Delta(x,p_1) - \alpha N}{\beta N} = \frac{p_1(2x-p_1-\alpha N)}{\beta N} \). Differentiating (26) with respect to \( p_1 \) yields:

\[
\frac{\beta N}{2p_1} \left[ \frac{2(1-p_1)}{\beta N} h \left( \frac{p_1(2-p_1) - \alpha N}{\beta N} \right) - \frac{p^{**}}{\beta N} h \left( \frac{p_1p^{**} - \alpha N}{\beta N} \right) \right] - \frac{\beta N}{2p_1} \int_{\frac{p_1(2-2p_1-\alpha N)}{\beta N}}^{p_1p^{**} - \alpha N} h(z) dz
\]

\[
= \frac{(1-p_1)}{p_1} e^*(1,p_1) - E[e^*(x,p_1)] + \frac{\beta N}{2p_1} \int_{0}^{\frac{p_1p^{**} - \alpha N}{\beta N}} h(z) dz - \frac{p^{**}}{2} h \left( \frac{p_1p^{**} - \alpha N}{\beta N} \right).
\]

Because \( \frac{\beta N}{2p_1} \int_{0}^{\frac{p_1p^{**} - \alpha N}{\beta N}} h(z) dz < \frac{p_1p^{**} - \alpha N}{\beta N} h \left( \frac{p_1p^{**} - \alpha N}{\beta N} \right) < E^* \beta N h \left( \frac{p_1p^{**} - \alpha N}{\beta N} \right) \), it suffices to show that \( (1-p_1) e^*(1,p_1) - E[e^*(x,p_1)] \leq 0 \). Recall that \( p^* \) is the equilibrium revolutionary agenda when there is only one revolutionary entrepreneur. From equation (15), recall that \( p^* \) satisfies the first order condition \( \frac{dE[e^*(x,p)]}{dp} = (1-p^*) e^*(1, p^*) - E[e^*(x,p^*)] = 0 \). Moreover, \( \frac{dE[e^*(x,p)]}{dp} < 0 \) for \( p \) sufficiently close to 1. Thus, the largest \( p \) that satisfies \( \frac{dE[e^*(x,p)]}{dp} = 0 \) is a maximum, and hence if \( p_1 \) is larger than all such \( p^* \)’s, then \( \frac{dE[e^*(x,p)]}{dp} \mid_{p=p_1} = (1-p_1) e^*(1,p_1) - E[e^*(x,p_1)] < 0 \).

Finally, we show that \( p^{**} > p^* \). Because \( (1-p^*) e^*(1,p^*) = E[e^*(x,p^*)] \), if \( h(\cdot) \) is linear, then \( 1-p^* = p^* - x_L(p^*) \); and hence, if \( h(\cdot) \) is concave, then \( 1-p^* > p^* - x_L(p^*) \). Thus, \( p^{**} > p^* \). Because \( (1-p^*) e^*(1,p^*) = E[e^*(x,p^*)] \), if \( h(\cdot) \) is linear, then \( 1-p^* = p^* - x_L(p^*) \); and hence, if \( h(\cdot) \) is concave, then \( 1-p^* > p^* - x_L(p^*) \). Thus, \( \int_{x_L(p^*)}^{p^*} h \left( \frac{\Delta(x,p^*) - \alpha N}{\beta N} \right) dx < (p^* - x_L(p^*)) h \left( \frac{\Delta(p^*,p^*) - \alpha N}{\beta N} \right) \leq (1-p^*) h \left( \frac{\Delta(p^*,p^*) - \alpha N}{\beta N} \right) < \int_{p^*}^{1} h \left( \frac{\Delta(x,p^*) - \alpha N}{\beta N} \right) dx \),

(27)
Moreover, $\partial$

Next, recall that the revolutionary entrepreneur's problem is:

From (25) and (27), $\Omega(p^*) < 0$, and hence $p^* < p^{**}$. □

**Proof of Remark 3:**

(i) Differentiating $x_L(p) = \frac{p^2 + \alpha}{2p} + \frac{A(p)}{2}$ with respect to $p$ yields:

$$
\frac{dx_L(p)}{dp} = \frac{p^2 - \alpha - A(p) + pA'(p)}{2p^2} \quad \text{and} \quad \frac{d^2x_L(p)}{dp^2} = \frac{1}{p} \left( 1 + \frac{A'(p)}{2} - 2 \frac{dx_L(p)}{dp} \right),
$$

and hence $\frac{dx_L(p)}{dp} = 0$ implies $\frac{d^2x_L(p)}{dp^2} > 0$. Thus, if $\frac{dx_L(p)}{dp} \bigg|_{p=p_b} = 0$, then $p_b$ is unique and is a global minimand. Next, we show $\frac{dx_L(p)}{dp} = 0$ has a solution in $(0,1)$. Clearly, $\frac{dx_L(p)}{dp} < 0$ for sufficiently small $p$. Thus, it suffices to show that $\frac{dx_L(p)}{dp} > 0$ for some $p \in (0,1)$. Suppose not, so that $\frac{dx_L(p)}{dp} < 0$ for all $p \in (0,1]$. Then, $\min_{p \in [0,1]} x_L(p) = x_L(1) = \frac{1 + \alpha + A(1)}{2} < 1$, implying that $0 < 1 - \alpha - A(1)$. Thus, $\frac{dx_L(p)}{dp} \bigg|_{p=1} = \frac{1 - \alpha - A(1) + A'(1)}{2} > A'(1) > 0$, which is a contradiction. Thus, there exists a $p \in (0,1]$ such that $\frac{dx_L(p)}{dp} > 0$.

(ii) Using $\Delta(x_L(p), p) - A(p) - \alpha = 0$ and $\Delta(x, p) = p(2x - p)$, rewrite $\frac{dx_L(p)}{dp}$ as:

$$
\frac{dx_L(p)}{dp} = -\frac{1}{2p} \frac{\partial[\Delta(x = x_L(p), p) - A(p)]}{\partial p}.
$$

From part (i), $\frac{dx_L(p)}{dp} \leq 0$ for $p \leq p_b$ with equality only at $p_b$. Thus,

$$
\frac{\partial[\Delta(x = x_L(p), p) - A(p)]}{\partial p} \geq 0 \text{ for } p \leq p_b \text{, with equality only at } p = p_b.
$$

Moreover, $\frac{\partial[\Delta(x, p) - A(p)]}{\partial p}$ is increasing in $x$. Thus,

$$
\frac{\partial[\Delta(x, p) - A(p)]}{\partial p} > 0 \text{ for } x > x_L(p) \text{ and } p \leq p_b.
$$

Next, recall that the revolutionary entrepreneur’s problem is:

$$
\max_p E[e^*(x, p)] = \max_p \int_{x_L(p)}^{1} h \left( \frac{\Delta(x, p) - A(p) - \alpha}{\beta} \right) dx.
$$

Mirroring the calculations in the proof of Proposition 1 yields:

$$
\frac{\partial E[e^*(x, p)]}{\partial p} = \frac{1}{\beta} \int_{x_L(p)}^{1} \frac{\partial[\Delta(x, p) - A(p)]}{\partial p} h' \left( \frac{\Delta(x, p) - A(p) - \alpha}{\beta} \right) dx.
$$

Because $h'(\cdot) > 0$, from (29), $\frac{\partial E[e^*(x, p)]}{\partial p} > 0$ at $p \leq p_b$.

(iii) Mirroring the derivation of equation (15),

$$
\frac{\partial E[e^*(x, p)]}{\partial p} = e^*(1, p)(1 - p - A'(p)/2) - E[e^*(x, p)],
$$

(30)

In the derivation of equation (15), $\frac{d\Delta(1,p)}{dp}$ becomes $\frac{d\Delta(1,p)}{dp} - A'(p) = 2[(1 - p) - A'(p)/2]$.  

36
which, in turn, yields:

$$\frac{\partial^2 E[e^*(x,p)]}{\partial \alpha \partial p} > 0 \text{ if and only if } e^*(1,p) > [1 - p - A'(p)/2] \left. \frac{de^*(x,p)}{dx} \right|_{x=1}.$$ 

The convexity of $g(\cdot)$ implies the concavity of $h(\cdot)$. Hence:

$$\left. \frac{de^*(x,p)}{dx} \right|_{x=1} \leq \frac{e^*(1,p) - e^*(x_L(p),p)}{1 - x_L(p)} = \frac{e^*(1,p)}{1 - x_L(p)},$$

and hence $\frac{\partial^2 E[e^*(x,p)]}{\partial \alpha \partial p} > 0$ if $x_L(p) < p + A'(p)/2$. Substituting for $x_L(p) = p^2 + p + A(p)$, this inequality becomes $p^2 - \alpha - A(p) + pA'(p) > 0$. From parts (i) and (ii), $\left. \frac{dx_L(p)}{dp} \right|_{p=p^*} > 0$, which together with equation (28) yields $(p^*)^2 - \alpha - A(p^*) + p^*A'(p^*) > 0$. □

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9. References


