Repression and Repertoires

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Abstract

We formalize Tilly's (1978, 2006, 2008) concept of repertoires of contentious actions, and analyze how the structure of state repression affects the variety of observed contentious actions. Introducing the notion of repertoire width, we show that when state repression accelerates with higher levels of anti-regime actions, opposition leaders tend to call for many different forms of contentious actions, thereby generating a wider repertoire. In contrast, when repression decelerates with higher contentious actions, opposition leaders tend to call for few different forms of contentious actions, thereby generating a narrower repertoire. This result applies to both Tilly's "bounded rationality mode," in which opposition leaders must choose from a predetermined set of contentious actions (rigid/strong repertoire); and to his "efficiency model," in which leader can freely innovate and design contentious actions (weak/no repertoire). Methodologically, citizen interactions is modeled as a multi-action global game, and the leader's problem is a simple mechanism without transfers.

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1 Introduction

Opposition leaders (both individuals and organizations) can call for a variety of anti-regime actions, e.g., wearing particular colors for solidarity,[1] strikes and demonstrations, and armed resistance—Tilly (2008) called them *Contentious Performances.* In his influential book, *From Mobilization to Revolution*, Tilly (1978) introduced the notion of *repertoires of contentious actions* as the set of anti-regime actions used in a time period in an area. He posed a broad question, which has since attracted enormous scholarship (Tilly 2008; McAdam et al. 2001; Tarrow 2011): how does the environment (political, cultural, economic) influence the nature of repertoires of contention? Two key elements of a movement environment are opposition leaders who organize and coordinate anti-regime actions and the state's repressive apparatus that makes those actions costly. In this paper, we focus on these two elements and ask: Should opposition leaders call for one or a combination of contentious actions? What are their tradeoffs? How does the structure of state repression influence these choices? We develop a framework to formalize the concept of repertoires of contention and address these questions.

We say that a repertoire is wider when it includes a larger variety of contentious performances, and narrower when that variety is smaller. We define repression structure as a regime's punitive scheme that determines the expected punishment for different contentious actions—e.g., in a fully indiscriminate repression scheme, all actions are punished equally. Although many factors (e.g., culture and historical contingencies) affect what contentious actions appear in a movement, repression structure and opposition leaders play key roles in determining those actions—repression structures affect participation incentives and leaders can coordinate participants. We aim to identify conditions under which opposition leaders tend to call for a larger or smaller number of different contentious performances, thereby creating wider or narrower repertoires. The key condition that we focus on is the structure of repression. Thus, we investigate how a regime's repression structure affects the variety of observed contentious performances in a movement (i.e., the width of the movement's repertoire).

Our key insight is that repression structure affects how many different kinds of contentious performances are called for by the opposition that aims to overturn the status quo. When the repression scheme accelerates with higher anti-regime actions, the opposition tends to call for many different forms of contentious action, creating a wide repertoire for the movement. In

[1] Examples include Iran (green in 2009), Ukraine (orange in 2004), and the Philippines (yellow in 1986).

contrast, when the repression scheme decelerates with higher anti-regime actions, the opposition tends to call for few different forms of contentious action, creating a narrow repertoire for the movement. Specifically, when repression is a convex function of anti-regime action levels, opposition leaders try to innovate and call for many different forms and levels of contentious performances. For example, Alexei Navalny, a Russian opposition leader, aims to enable citizens to contribute to anti-regime (anti-Putin) actions in 15 minutes increments: "it fits into my conception that everyone should devote 15 minutes a day to struggling against the regime—I make it easier for people to do this" (Voronkov 2012, p. 165).[2] In contrast, when repression is a concave function of anti-regime action levels, opposition leaders call for just one form of contentious performance. A special case of such concave schemes is when repression is indiscriminate, so that the regime represses all anti-regime actions equally. Then, the opposition naturally tends to call for the highest possible anti-regime action.

The large literature on the nature of contentious actions and repertoires has focused on when contentious actions are violent or peaceful, investigating various topics, including the effect of the state's repression, concession, and capacity (Tilly 1978, 2006; Lichbach 1987; Moore 1998; Goodwin 2001; Kalyvas 2006; Earl 2011), the target of grievances (Parsa 2000; Walker et al. 2008; Tarrow 2011), and the support of the general population (Chenoweth and Stephan 2011; Bueno de Mesquita 2013; Wang and Piazza 2016; Davenport et al. 2019). The focus on violence provides important insights, but does not explore the rich variety of contentious actions, or how state repression influences repertoires of contention beyond the dichotomous categories of violent versus non-violent. This paper aims to provide a framework to study this vast under-explored area of research.

In our base model, different contentious performances correspond to different actions, which are represented by positive real numbers. Higher actions are more effective in overturning the status quo, but they are also more costly. A repression structure corresponds to the costs of these actions. There is an opposition leader and a large number of citizens. First, the leader calls for a set of actions (contentious performances). Then, each citizen decides which action to take (i.e., in which contentious performances to participate). The opposition leader cannot compel citizens to participate in contentious performances, but she has control over what contentious performances citizens may participate in. The status quo is overturned (regime change) whenever the aggregate action exceeds a threshold, which captures the strength of the

[2]We thank Daniel Treisman for pointing out this example and providing this translation from Voronkov's book.

status quo. This threshold is uncertain, and each citizen has his own private assessment of the strength of the status quo. Thus, citizens face a coordination problem with incomplete information. Citizens who are more optimistic that the status quo will be overturned have more incentives to participate in higher actions. But a citizen's degree of optimism is endogenous, and depends on his views about the strength of the status quo and other citizens' behavior.

We consider both settings in which the leader is constrained to choose from a given set of contentious performances (Tilly's "bounded rationality model"), and settings in which the leader can readily innovate and call for any action (Tilly's "efficiency model"). Tilly (1978, 2006, 2008) observed that, in a given time period and geographical area, activists and leaders tend to use previous forms of contentious performances with typically minor changes. He contrasts this "bounded rationality model" with the "efficiency model" in which all potential contentious actions are examined, and those that best serve the cause are selected. In Tilly's bounded rationality model, the set of contentious actions from which the leaders choose is given, e.g., there are five forms of actions that the opposition can call for. In his efficiency model, this constraint is relaxed and the leaders can innovate and choose from a much wider set of contentious actions. Our insights are robust across both settings. For example, when repression accelerates with higher contentious actions, in the bounded rationality model, leaders call for many contentious performances from a given set; in the efficiency model, leaders innovate and call for even more actions.

To convey the intuition, we begin with two simplistic benchmarks. Recall that a citizen who is more optimistic about the chances of overturning the status quo is more willing to engage in higher, more costly actions. First, consider a simplistic setting with just one citizen whose optimism is known to the opposition leader. The leader can call for the highest contentious action that the citizen would be willing to participate in, given the citizen's optimism and the structure of state repression that determines the expected costs of different contentious actions. Next, consider a more complicated setting in which there are many citizens with private degrees of optimism. The leader now faces a far more complex problem. Suppose she could choose to call for at most two different contentious actions with one being higher and more effective than the other, but also most costly for participants. If she only calls for the high action, citizens who are sufficiently optimistic about the chances of overturning the status quo will participate, and other, more pessimistic citizens will take no action. By contrast, if the leader calls for both the high and the low actions, some of the more pessimistic citizens join the movement, participating in the low contentious action. But, now some of those citizens who would have participated in the high action may switch to engage in the low action because it is less costly for them. These are the citizens who would have chosen the high action over no action by a small margin. But having a choice between no action, low action, and high action, they will choose the low action. Thus, whether the leader will call for just the high action, or for both the low and high action will depend on the distribution of the citizens' optimism, the difference between the effectiveness of those actions, and on the structure of state repression.

Our model corresponds to a more realistic setting in which the leader can choose from a number of contentious actions and the citizens' degrees of optimism are endogenously determined through their coordination interactions. Because the citizens' degrees of optimism will be determined endogenously in the coordination interaction that follows the leader's decision, the leader must anticipate the strategic interactions that will follow her decision of which contentious actions to call for. Because there can be many contentious actions, the leader faces many such tradeoffs that we discussed above. Our results show that how the leaders balance these tradeoffs depends on the concavity and convexity of the regime's repression structure, as we described above.

The formal literature focuses on a general notion of repression as a state action that "raises the contender's cost of collective action" (Tilly 1978, p. 100). Beyond the distinction between discriminate versus indiscriminate repression (Kalyvas 2006; Rozenas 2018), little attention has been paid to the structure of repression. Instead, the literature studies topics such as repression backfire (Siegel 2011; Shadmehr and Boleslavsky 2019),[3] how regimes combine repression with other instruments of social control (Guriev and Treisman 2015; Shadmehr 2017; Tyson and Smith 2018; Di Lonardo et al. 2020), or the principal-agent problem between civilian authorities and armed forces responsible for repressing dissidents (Acemoglu et al. 2010; Svolik 2013; Tyson 2018).

A formal analysis of repertoires of contention or the effect of repression structure requires a characterization of multi-action coordination games, departing from the binary-action models of protest used in the literature (Chwe 2000; Bueno de Mesquita 2010; Boix and Svolik 2013; Edmond 2013; Casper and Tyson 2014; Chen and Suen 2016; Tyson and Smith 2018; Shadmehr 2019; Barbera and Jackson 2020). We use Morris and Shadmehr's (2019) characterization of continuous action global games to analyze the multi-action global game that arise among

[3]The large empirical literature on repression backfire includes Wood (2003), Francisco (2004), Lawrence (2017), and Aytaç and Stokes (2019).

citizens in our model—see also Frankel et al. (2003) and Guimaraes and Morris (2007). They study how transformative leaders can optimally manipulate pleasure in agency rewards (Wood 2003) to inspire regime change. By contrast, our focus is on "task-oriented" leaders who cannot change the people's psychological rewards from participation. However, they "plot a strategy suitable to resources and circumstances" (Goldstone 2001, p. 157) by carefully choosing the contentious performances of the movement to suit coordination and information frictions and the repression structure faced by citizens. At an abstract level, our model is related to mechanism design without contingent transfers, and in particular, to the delegation problem (Holmström 1977; Epstein and O'Halloran 1994; Bendor et al. 2001; Alonso and Matouschek 2008). Specifically, the leader's problem is *as if* the leader offers a representative agent a message space and a decision rule, mapping those messages to actions, so that when the agent sends a message, the leader is committed to take the action prescribed by the decision rule. The messages in our model will be the citizens' reported degree of optimism, and the actions correspond to the leader's chosen contentious performances. The conflict of interest arises because the leader always wants the highest action, while pessimistic citizens do not.

Section 2 presents the model. Our main analysis is in Section 3, where we also interprets the results in the context of repression and repertoires. Proof are in the Appendix.

2 Model

There is an opposition leader and a continuum of citizens, indexed by $i \in [0,1]$, who must decide how much effort $e \in [0, e_{max}]$ to contribute to the revolution. The revolution succeeds if and only if the total contribution $\int e_i di$ exceeds the regime's strength $\theta \in \mathbb{R}$. If the revolution fails, a citizen who contributes e pays a cost of $C(e)$, reflecting the regime's repression. A citizen who contributes e receives a benefit $B(e)$. The opposition leader wants to maximize the likelihood of successful revolution. She must decide whether or not to restrict the positive contributions of citizens to a subset of $N \in \mathbb{N}$ values, choosing $\{e_1, \cdots, e_N\}$, with $e_1 > \cdots > e_N > 0$. Citizens can always choose $e_{N+1} \equiv 0$.

The regime's strength θ is uncertain, and the citizens and opposition leader have a common improper prior that θ is distributed uniformly on $\mathbb{R}$. In addition, each citizen $i \in [0,1]$ receives a noisy private signal x_i about θ: $x_i = \theta + \nu_i$, where θ and ν_is are independent and $\nu_i \sim F$.

The timing of the game is as follows. First, the leader chooses $\{e_1, \cdots, e_N\}$. Next, citizens observe the leader's choice, receive their private signals, and then simultaneously decide how much to contribute. The success or failure of the revolution is determined and payoffs are received.We maintain the following Assumption throughout our analysis.

Assumption 1 *Costs $C(e)$ and benefits $B(e)$ of a contribution e have these properties:*

1. $C(0) = 0$, $C'(0) \geq 0$, *and* $C'(e) > 0$ *for* $e > 0$.

2. $B(0) = 0$, $B'(0) \geq 0$, *and* $B''(e) < 0 < B'(e)$ *for* $e > 0$.

3. $B''(e) < C''(e)$, $\lim_{e \to 0^+} \frac{C'(0)}{B'(0)} = 0$, *and* $C(e) > B(e)$ *for some* e.

4. $\frac{B(e)}{C(e)}$ *and* $\frac{B'(e)}{C'(e)}$ *are strictly decreasing.*

Remark 1. Assumption 1 implies there is a unique $e > 0$ at which $C(e) = B(e)$ and a unique $e > 0$ at which $C'(e) = B'(e)$. Let $\tilde{e}$ be the unique, strictly positive solution to $C(e) = B(e)$, and let $\bar{e}$ be the unique, strictly positive solution to $C'(e) = B'(e)$.[4] We assume that $\tilde{e} < e_{max}$.

Remark 2. If $C(e)$ is convex ($C''(e) \geq 0$), then part 4 of Assumption 1 is satisfied.

Interpretation. Different effort levels correspond to different contentious actions. For example, taking up arms corresponds to e_1, participating in a demonstration to e_2, and wearing a wristband to e_3, with $e_1 > e_2 > e_3 > e_4 = 0$. That the leader can restrict effort levels corresponds to the opposition leadership (organization) choosing different tactics. It is implicit in our setting that citizens will choose only among the tactics chosen by the opposition. An alternative approach is to consider contentious actions in a multidimensional space, with different forms of contentious actions corresponding to different dimensions. In the above example, we could have a 3-dimensional space, $(d_1, d_2, d_3) \in \{0, 1\}^3$, with $a_i^j \in \{0, 1\}$, $j \in \{1, 2, 3\}$, corresponding to whether citizen i participates in contentious action d_j. We then must posit a technology that specifies how these multidimensional actions translate into a probability of regime change. One possibility is that different kinds of actions add up with different weights, so that the regime collapses whenever $\sum_{j=1}^{N} w_j \left(\int a_i^j di\right) > \theta$. But these weights are the same as the unit of measurement in our current setting $\int e_i di > \theta$ with $e \in [0, e_{max}]$ because they determine the relative contributions of different forms of contentious actions to the overall likelihood of regime change. Once different contentious actions are mapped into $\mathbb{R}_+$, one can investigate

[4] That $\bar{e}$ is unique does not require part 4 of Assumption 1 that $B'(e)/C'(e)$ is strictly decreasing. One can show that $B''(e) < C''(e) \Rightarrow (B'(e)/C'(e))' < 0$ as long as $B'(e)/C'(e) \geq 1$, implying that $\bar{e}$ is unique.

how these actions map into the degrees of punishment, and determine the shape of the regime's punishment scheme, e.g., whether it is convex or concave.[5]

3 Analysis

A citizen j's strategy $s_j(x_j) : \mathbb{R} \to \{e_1, \cdots, e_N, e_{N+1}\}$ is a mapping from his signal x_j to an effort level $e_i \in \{e_1, \cdots, e_N, e_{N+1}\}$, where we recall that e_{N+1} is set at 0. We focus on the natural class of decreasing equilibria, so that if $x_i > x_i'$, then $s_j(x_i) \leq s_j(x_i')$.

Let p be a citizen's belief about the likelihood that the regime survives. Then, the citizen's problem is:

$$e^*(p) = \underset{e \in \{e_1, \cdots, e_N, 0\}}{\arg\max} \; B(e) - p\, C(e). \tag{1}$$

Definition 1 *Define p_i and m as follows:*

- $p_i = \frac{B(e_i) - B(e_{i+1})}{C(e_i) - C(e_{i+1})}$, *for* $i \in \{1, \cdots, N\}$, *with* $p_0 = 0$.
- $m = \max\{i \text{ s.t. } p_i \leq 1\}$, *if such an i exists, and otherwise set* $m = 0$.

The following Lemma characterizes the structure of a citizen's optimal effort.

Lemma 1 *We have: $p_i < p_{i+1}$, for $i \in \{1, \cdots, N-1\}$. Moreover,*

$$e^*(p) = \begin{cases} e_1 & ; p \in [0, p_1) \\ e_2 & ; p \in [p_1, p_2) \\ \vdots & ; \quad \vdots \\ e_{m+1} & ; p \in [p_m, 1]. \end{cases} \tag{2}$$

To provide some intuition, suppose $N = 3$, so that citizens must choose from $\{e_1, e_2, e_3, e_4\}$, with $e_{max} \geq e_1 > e_2 > e_3 \geq e_4 \equiv 0$. Let $e_i \succ_p e_j$ mean that a citizen with belief p strictly prefers action e_i over e_j. Then, by comparing the payoffs we have: $e_i \succ_p e_{i+1} \Leftrightarrow p < p_i$, for $i = 1, 2, 3$. The proof shows that $0 < p_1 < p_2 < p_3$. That is, citizens who are more optimistic about regime change exert more effort. Moreover, there is no gap in the effort levels: if some citizens exert e_1 and some citizens exert e_3, there will be some who exert e_2. Thus, if $p_3 < 1$, then: citizens with beliefs $p \in [0, p_1)$ take action e_1, those with $p \in [p_1, p_2)$ take action e_2, those

[5] Davenport et al. (2019) show that people's perception of the intensity of contentious actions can be measured in a uni-dimensional space.

with beliefs $p \in [p_2, p_3)$ take action e_3, and the most pessimistic citizens with $p \in [p_3, 1]$ take the lowest action $e_4 = 0$. Of course, it may be the case that, for example, $0 < p_1 < 1 < p_2 < p_3$, in which case all citizens either take action e_1 or e_2.

Because $e^*(p)$ is decreasing we can apply Proposition 1 of Morris and Shadmehr (2019) to characterize the equilibrium of the citizens' coordination game. In particular, there exists an essentially unique monotone equilibrium in which the revolution succeeds if and only if $\theta < \theta^*$, where

$$\theta^* = \int_{p=0}^{1} e^*(p) dp. \tag{3}$$

Combining this with Lemma 1, we have:

Proposition 1 *Fix the set of citizen choices to $\{e_1, \cdots, e_N, e_{N+1}\}$, with e_{N+1} set to 0. There is a unique equilibrium, in which there is regime change if and only if $\theta < \theta^*_m$. Moreover,*

$$\theta^*_m = \boldsymbol{1}_{\{m \geq 1\}} \sum_{i=1}^{m} (p_i - p_{i-1})\ e_i + (1 - p_m)\ e_{m+1}, \text{ with } p_0 = 0, \tag{4}$$

where $\boldsymbol{1}_{\{m \geq 1\}}$ is an indicator function that is 1 if $m \geq 1$, and 0 otherwise.

Our main result characterizes how the equilibrium regime change threshold changes when we remove one of the citizens' effort choices. We will see that a consecutive application of this result will allow us to characterize the relationship between the repression structure $C(e)$ and the repertoire width (i.e., the number of different contentious actions that appear in equilibrium).

Theorem 1 *Suppose $m \geq 2$. Let θ^*_{m-1} be the equilibrium regime change threshold that is obtained without e_i, with $1 < i \leq m$. If C is strictly concave, then $\theta^*_{m-1} > \theta^*_m$. If C is linear, then $\theta^*_{m-1} = \theta^*_m$. If C is strictly convex, then $\theta^*_{m-1} < \theta^*_m$.*

Theorem 1 takes m as given, and states what happens when we remove one of $\{e_2, \cdots, e_m\}$ from the set of feasible actions. The consecutive application of Theorem 1 implies that when $C(e)$ is strictly concave, the equilibrium regime change threshold increases by reducing the set of feasible efforts from $\{e_1, \cdots, e_m, e_{m+1}, 0\}$ to, at most, two positive effort levels $\{e_1, e_{m+1}, 0\}$. To see this, begin with $\{e_1, \cdots, e_N, e_{N+1}\}$, and note that only $\{e_1, \cdots, e_m, e_{m+1}\}$ are taken in equilibrium. Now, consecutively, remove e_2, e_3, and so on until only $\{e_1, e_{m+1}\}$ are taken in equilibrium, and note that in each step the new m for that step remains weakly larger than 2. Thus, when $e_{m+1} = 0$, the equilibrium regime change threshold increases by removing all positive effort levels except the largest. An example of this is when $e_1 > \cdots > e_N \geq \tilde{e}$, so that $e_m = e_N$ and $e_{m+1} = 0$. Then, the consecutive application of Theorem 1 implies that when $C(e)$ is strictly concave, restricting efforts to just e_1 raises the equilibrium regime change threshold. When $e_{m+1} > 0$, then $0 < p_1 \leq 1 < p_{m+1}$, and hence $m = 1$ and we cannot apply the Theorem because a key for the application of Theorem 1 is that $m \geq 2$. To see this, suppose some feasible effort levels are below $\tilde{e}$, e.g., $e_1 > e_2 > \tilde{e} > e_3 > 0$. Now, $0 < p_1 < p_2 < 1 < p_3$ and $m = 2$. According to the Theorem, we can remove e_2. But then $m = 1$, preventing the application of the Theorem. More broadly, as long as all feasible effort levels are (weakly) above $\tilde{e}$, we can freely apply the theorem in both adding and removing effort levels.

Moreover, because even the most pessimistic citizens (with $p = 1$) are willing to put in the effort level $\tilde{e}$, it follows that if the leader wants to restrict efforts to a single level, her optimal choice $\hat{e}$ must be larger than $\tilde{e}$ (see equation (6)). A corollary of these arguments is that, when $C(e)$ is strictly concave and all feasible efforts are above $\tilde{e}$, restricting efforts to just one effort level benefits the leader. Analogous arguments hold for the case of linear, and strictly convex costs. The following Corollary summarizes these results, and provides an independent proof for the case of a continuum of efforts $e \in [\tilde{e}, e_{max}]$.

Corollary 1 *Let θ_1^{**} be the highest equilibrium regime change threshold that is obtained from having only one (strictly) positive action: $\theta_1^{**} = \max_{e_1 \geq 0} \theta_1^*$. Let $\tilde{\theta}_\infty^{**}$ be the equilibrium regime change threshold that is obtained when all actions $e \in [\tilde{e}, e_{max}]$ are feasible.*

- *If $C(e)$ is strictly concave, then $\theta_1^{**} > \tilde{\theta}_\infty^{**}$.*
- *If $C(e)$ is linear, then $\theta_1^{**} = \tilde{\theta}_\infty^{**}$.*
- *If $C(e)$ is strictly convex, then $\theta_1^{**} < \tilde{\theta}_\infty^{**}$.*

To understand the intuition, consider equation (4), and remove effort level e_i. Now, some people who would have put in effort e_i raise their effort to e_{i-1}, but others reduce their effort to e_{i+1}. Let p' be the type of the marginal citizen who is indifferent between exerting effort e_{i-1} and e_{i+1}. Lemma 3 in the Appendix shows that $p_{i-1} < p' < p_i$. Now, if $1 < i \leq m$ and $p_i \leq 1$ (which follows from the requirement that $m \geq 2$), we have

$$\theta_{m-1}^* - \theta_m^* = (p' - p_{i-1})\,(e_{i-1} - e_i) - (p_i - p')\,(e_i - e_{i+1}). \qquad (5)$$

The first term captures the gains from removing effort level e_i, and the second term captures the losses. To communicate the intuition, suppose effort levels are equidistant from each other ($e_{i-1} - e_i = e_i - e_{i+1}$). Then, because types are uniformly distributed, the question becomes whether the majority of the types in (p_{i-1}, p_i) increase or decrease their effort when we remove the middle effort level. In particular, $\theta^*_{m-1} > \theta^*_m$ if and only if $p' > (p_{i-1} + p_i)/2$, i.e., whenever the middle type is smaller than the indifferent type. But these three thresholds are linked. Letting $\Delta B_{i-1} = B(e_{i-1}) - B(e_i)$ and $\Delta C_{i-1} = C(e_{i-1}) - C(e_i)$, we have:

$$p_{i-1} = \frac{\Delta B_{i-1}}{\Delta C_{i-1}}, \; p_i = \frac{\Delta B_i}{\Delta C_i}, \; p' = \frac{\Delta B_{i-1} + \Delta B_i}{\Delta C_{i-1} + \Delta C_i}.$$

Thus,

$$\theta^*_{m-1} > \theta^*_m \;\Leftrightarrow\; p' > \frac{p_{i-1} + p_i}{2} \;\Leftrightarrow\; \frac{\Delta B_{i-1} + \Delta B_i}{\Delta C_{i-1} + \Delta C_i} > \frac{1}{2}\left(\frac{\Delta B_{i-1}}{\Delta C_{i-1}} + \frac{\Delta B_i}{\Delta C_i}\right) \;\Leftrightarrow\; \Delta C_i > \Delta C_{i-1},$$

where the last step uses the fact that $p_{i-1} < p_i$. This simple algebra reveals the critical role of convexity and concavity of $C(\cdot)$ in determining the leader's optimal decision. Our proof in the Appendix generalizes this logic.

It remains to analyze what happens when some feasible effort levels are strictly below $\tilde{e}$. The key difference now is that the consecutive application of Theorem 1 brings m to 1 because some of the remaining effort levels will have $p_i > 1$. But then, equation (5) reveals that this reduces the losses. Thus, if before (e.g., with concave costs), removing effort levels would benefit the leader, now, with the losses being even lower, removing effort levels benefits the leader even further. The flip side is that convexity is no longer sufficient for removing effort levels to reduce the equilibrium regime change threshold. Proposition 2 focuses on a natural extension of Theorem 1 for the case of concave costs.

Proposition 2 *Suppose $C'' \leq 0$. If the set of feasible efforts is $\{e_1, \cdots, e_N, 0\}$, with $e_{max} \geq e_1 > \cdots > e_N > 0$, then the (weakly) highest equilibrium regime change threshold is obtained when only the highest effort e_1 is allowed. Moreover, let θ^{**}_∞ be the equilibrium regime change threshold when all actions $e \in [0, e_{max}]$ are feasible. Then, $\theta^{**}_1 > \theta^{**}_\infty$.*

Proposition 2 characterizes the leader's optimal choice when $C(e)$ is concave. Our earlier discussion reveals the difficulties of providing an analogous characterization when $C(e)$ can be strictly convex; still, we can derive sharp qualitative implications. Suppose $C(e)$ is strictly convex, and there are some $e_i < \tilde{e}$. If the leader decides to keep $\max\{e_i < \tilde{e}\}$, she (weakly) benefits from removing all other $e_i < \tilde{e}$, because even the most pessimistic type $p = 1$ chooses $\max\{e_i < \tilde{e}\}$ over 0—clearly, this does not hinge on convexity or concavity of the costs $C(e)$. However, perhaps the leader is better off by removing $\max\{e_i < \tilde{e}\}$, and keeping the second largest of all $e_i < \tilde{e}$. He should still remove all lower effort levels, because even the most pessimistic type $p = 1$ chooses this second largest $e_i < \tilde{e}$ over 0. Thus, when $C(e)$ is strictly convex, the leader keeps all $e_i \geq \tilde{e}$ and at most one $e_i < \tilde{e}$. In the Appendix, we provide an example with power function that contrasts θ_1^{**}, $\tilde{\theta}_\infty^{**}$, and θ_∞^{**}.

Remark 3 (Indiscriminate Repression). Suppose the leader could only restrict efforts to one level. Then, the leader's problem becomes:

$$\hat{e} \equiv \underset{e \in [0, e_{max}]}{\arg\max} \ \min\left\{\frac{B(e)}{C(e)}, 1\right\} \cdot e = \underset{e \in [0, e_{max}]}{\arg\max} \begin{cases} e & ; e \leq \tilde{e} \\ \\ \frac{B(e)}{C(e)}\ e & ; e \geq \tilde{e}, \end{cases} \tag{6}$$

where we recall that $\tilde{e}$ is the unique, strictly positive effort level at which $B(e) = C(e)$. If $C'' \leq 0$, then $\hat{e} = e_{max}$. The reason is that when C is concave, $C(e)/e$ is weakly decreasing, and hence $\frac{B(e)}{C(e)}\ e$ is strictly increasing. Particularly, indiscriminate repression, in which the regime punishes all levels of anti-regime actions equally, corresponds to a concave cost function: $C(e) = c > 0$ for $e > 0$ and $C(0) = 0$. Although this function is discontinuous at 0, it can be approximated by a continuous concave function.

Before we summarize the implications of these results in the context of repression and repertoires, we highlight the applicability of our results to the natural case where the repression scheme is a left-continuous step function, so that punishment jumps at certain contentious actions. Then, any continuous function approximation is neither convex nor concave. However, note that in equilibrium the opposition leader will only choose from the set of actions corresponding to the jumps, because all the actions between one jump and another have the same cost, and higher actions are better. This means that our results apply if the repression scheme, restricted to the jump points, is either convex or concave. For example, consider the following repression scheme, where $a = e_{max}/3$ and $\alpha \geq 1$:

$$C(e) = \begin{cases} 0 & ; (0, a] \\ c & ; (a, 2a] \\ \alpha c & ; (2a, 3a]. \end{cases}$$

Then, in equilibrium the leader only chooses from $e_1 = 3a$, $e_2 = 2a$, and $e_3 = a$, with the corresponding costs of $C(e_1) = \alpha c$, $C(e_2) = c$, and $C(e_3) = 0$. Then, if $\alpha < 2$ ($\alpha > 2$), our results for the case of continuous and strictly concave (convex) cost function carry over.

3.1 Implications: Repression Structure & Repertoires of Contention

We now summarize the implications of our analysis for the relationship between repression structure and the width of repertoires of contention. In line with Tilly (1978, 2008), we consider two environments. In one environment, the set of contentious performances are fixed, and opposition leader have to choose to call for a contentious action from this exogenously given set of possibilities. This corresponds to the "bounded rationality model" in Tilly (1978) and strong or rigid repertoires in Tilly (2008). In the other environment, opposition leaders can freely innovate and come up with novel contentious performances. This corresponds to the "efficiency model" in Tilly (1978) and weak or no repertoires in Tilly (2008). We will see that our general insights apply to both these environment.

Rigid/Strong Repertoires. In some settings, the leader can innovate very little over existing contentious performances. The reasons for such rigidities can be lack of skills to come up with a different feasible form of contentious performance, personality traits that value tradition, or environmental constraints, such as strong norms of protest, which cause deviations to be perceived as mob actions. In Tilly's (2008) terminology, the repertoire is rigid or strong. To capture strong repertoires, suppose available contentious performances are fixed, and the leader must decide which contentious performances to call for. For example, the leader can call for a sit-in, a demonstration, a strike, or any combination of these, but nothing else. Should she call for one or a combination of them, and which one or ones? Each of these contentious performances maps into an action level, e.g., going on a strike corresponds to e_1, participating in a demonstration corresponds to e_2, and participating in a sit-in corresponds to e_3, with $e_4 = 0 < e_3 < e_2 < e_1$. When repertoires are strong, the leader has to choose from an exogenously given contentious repertoire $\{e_1, \cdots, e_N\}$. Our results show that what the leader does depends on whether the regime's repression scheme $C(e)$ is concave or convex. When it is strictly concave, the leader severely limits the variety of contentious actions, only calling for the highest effort level e_1, thereby creating a narrow repertoire of contentious performances for the movement. When it is strictly convex, she does the opposite, calling for all $e_i \geq \tilde{e}$ and sometimes even an additional contentious action among $\{e_i < \tilde{e}\}$, thereby creating a wide repertoire of contentious performances for the movement.

Weak/No Repertoires. The opposite extreme of the above setting is when the leader is not constrained by previous repertoires and can design and call for any contentious performance as long as its required effort is less than some given level e_{max}. In Tilly's (2008) terminology, there is no repertoire, or the repertoire is weak. This means that the leader can design any $\{e_1, \cdots, e_N\}$ for any $N \in \mathbb{N}$. What would she do? Our results show that when $C(e)$ is strictly concave, again the leader severely limits the variety of contentious performances, only calling for the action that maximizes θ_1^{**}, that is, $\hat{e} = \arg\max_{e\in[0,e_{max}]} \max\{\frac{B(e)}{C(e)}, 1\}\ e$. The only difference from the case of strong repertoires is that then the leader had to pick the maximum action from the existing repertoire, but now he picks exactly what maximizes θ_1^{**}, namely $\hat{e}$. In contrast, when $C(e)$ is strictly convex, the leader designs many contentious performances, at least covering $[\tilde{e}, e_{max}]$, and perhaps even including a subset of $(0, \tilde{e})$. In sum, as before, repression schemes that accelerate punishment with anti-regime actions tend to create wide repertoires, while repression schemes that decelerate punishment with anti-regime actions tend to create narrow repertoires.

4 Conclusion

We formalized the concept of repertoires of contention, and developed a model in which a set of contentious actions arise from the interactions between opposition leaders and citizens, facing a repression structure as well as coordination and information frictions. We analyzed the trade-offs involved in the opposition's choice of contentious actions, and analyzed how the structure of repression can lead to a wider or narrower repertoire. Our paper provides a framework to study repertoires, contentious actions and repression structures beyond the binary choices of violent versus not-violent and discriminate versus indiscriminate.

Two directions for future research stand out. We took the structure of repression as given. Although repression structure may have some inertia, states adjust their repression strategies in anticipation of or in response to contentious actions (e.g., McAdam 1983). One could contemplate extending this framework by endogenizing the repression structure. Our paper provides one ingredient for that analysis, but leaves it to future research. A second direction is about the efficacy of different contentious actions. When interpreting our framework, we implicitly assume that leaders and citizens know the typical effect of different contentious actions. Therefore, there is little room for experimentation. However, the opposition may be uncertain about the efficacy of different contentious actions. For example, the effectiveness of sit-ins, strikes, or demonstrations relative to each other may not be uncertain. In such settings, opposition leaders may experiment to get a better assessment of the efficacy of different tactics. Integrating this uncertainty and experimentation into the framework is another fruitful direction for future research.

References

Acemoglu, Daron, Davide Ticchi, and Andrea Vindigni. 2010. "A Theory of Military Dictatorships." *American Economic Journal: Microeconomics* 2: 1-42.

Alonso, Ricardo, and Niko Matouschek. 2008. "Optimal Delegation." *Review of Economic Studies* 75: 259-93.

Aytaç, Erdem, and Susan Stokes. 2019. *Why Bother?: Rethinking Participation in Elections and Protests.* New York, NY: Cambridge University Press.

Barbera, Salvador, and Matthew Jackson. 2020. "A Model of Protests, Revolution, and Information." *Quarterly Journal of Political Science. Forthcoming.*

Bendor, Johnathan, Amihai Glazer, and Thomas Hammond. 2001. "Theories of Delegation." *Annual Review of Political Science* 4: 235-69.

Boix, Carles, and Milan Svolik. 2013. "The Foundations of Limited Authoritarian Government: Institutions and Power-Sharing in Dictatorships." *Journal of Politics* 75: 300-16.

Bueno de Mesquita, Ethan. 2010. "Regime Change and Revolutionary Entrepreneurs." *American Political Science Review* 104 (3): 446-66.

Bueno de Mesquita, Ethan. 2013. "Rebel Tactics." *Journal of Political Economy* 121: 323-57.

Casper, Brett, and Scott Tyson. 2014. "Popular Protest and Elite Coordination in a Coup d'etat." *Journal of Politics* 76: 548–64.

Chen, Heng, and Wing Suen. 2016. "Falling Dominoes: A Theory of Rare Events and Crisis Contagion." *American Economic Journal: Microeconomics* 8: 1-29.

Chenoweth, Erica, and Maria Stephan. 2011. *Why Civil Resistance Works: The Strategic Logic of Nonviolent Conflict.* New York: Columbia University Press.

Chwe, Michael Suk-Young. 2000. "Communication and Coordination in Social Networks." *Review of Economic Studies* 67: 1-16.

Davenport, Christian, and Dave Armstrong, and Thomas Zeitzoff. 2019. "Acceptable or Out of Bounds? Scaling Perceptions of Challenger and Government Tactics as well as Understanding Contentious Interaction." Mimeo.

Di Lonardo, Livio, Jessica Sun, and Scott Tyson. 2020. "Autocratic Stability in the Shadow of Foreign Threats." *American Political Science Review. Forthcoming.*

Earl, Jennifer. 2011. "Political Repression: Iron Fists, Velvet Gloves, and Diffuse Control."

Annual Review Sociology 37: 261-84.

Edmond, Chris. 2013. "Information Manipulation, Coordination and Regime Change." *Review of Economic Studies* 80: 1422-58.

Epstein, David, and Sharyn O'Halloran. 1994. "Administrative Procedures, Information, and Agency Discretion." *American Journal of Political Science* 38: 697-722.

Francisco, Ronald. 2004. "After the Massacre: Mobilization in the Wake of Harsh Repression." *Mobilization* 9: 107-26.

Frankel, David, Ady Pauzner and Stephen Morris. 2003. "Equilibrium Selection in Global Games with Strategic Complementarities." *Journal of Economic Theory* 108: 1-44.

Goldstone, Jack A. 2001. "Toward a Forth Generation of Revolutionary Theory." *Annual Review of Political Science* 4: 139-87.

Goodwin, Jeff. 2001. *No Other Way Out: States and Revolutionary Movements, 1945–1991.* New York, NY: Cambridge University Press.

Guimaraes, Bernardo, and Stephen Morris. 2007. "Risk and Wealth in a Model of Self-fulfilling Currency Attack." *Journal of Monetary Economics* 54: 2205-30.

Guriev, Sergei, and Daniel Treisman. 2015. "How Modern Dictators Survive: An Informational Theory of the New Authoritarianism." Mimeo.

Holmström, Bengt. 1977. "On Incentives and Control in Organizations." Ph.D. Thesis, Stanford University.

Kalyvas, Stathis N. 2006. *The Logic of Violence in Civil War.* NY: Cambridge University Press.

Lawrence, Adria. 2017. "Repression and Activism among the Arab Spring's First Movers: Evidence from Morocco's February 20th Movement." *British Journal of Political Science* 47: 699-718.

Lichbach, Mark Irving. 1987. "Deterrence or Escalation? The Puzzle of Aggregate Studies of Repression and Dissent." *Journal of Conflict Resolution* 31: 266-97.

McAdam, Doug. 1983. "Tactical Innovation and the Pace of Insurgency." *American Sociological Review*, 48: 735-54.

McAdam, Doug, Sydney Tarrow, and Charles Tilly. 2001. *Dynamics of Contention.* New York: Cambridge University Press.

Moore, Will. 1998. "Repression and Dissent: Substitution, Context, and Timing." *American*

Journal of Political Science 42: 851-73.

Morris, Stephen, and Mehdi Shadmehr. 2019. "Inspiring Regime Change." Mimeo.

Parsa, Misagh. 2000. *Sates, Ideologies, & Social Revolutions: A Comparative Study of Iran, Nicaragua, and the Philippines.* Cambridge, UK: Cambridge University Press.

Rozenas, Arturas. 2018. "A Theory of Demographically Targeted Repression." Mimeo.

Shadmehr, Mehdi. 2019. Investment in the Shadow of Conflict: Globalization, Capital Control, and State Repression. *American Political Science Review* 113: 997-1011.

Shadmehr, Mehdi, and Raphael Boleslavsky. 2019. International Pressure, State Repression and the Spread of Protest. *Journal of Politics. Forthcoming.*

Siegel, David. 2011. "When Does Repression Work? Collective Action in Social Networks." *Journal of Politics* 73: 993-1010.

Svolik, Milan. 2013. "Contracting on Violence: Moral Hazard in Authoritarian Repression and Military Intervention in Politics." *Journal of Conflict Resolution* 57: 765-94.

Tarrow, Sydney. 2011. *Power in Movement: Social Movements and Contentious Politics, 3rd Ed.* New York: Cambridge Univ. Press.

Tilly, Charles. 1978. *From Mobilization to Revolution.* Addison-Wesley Publishing Company.

Tilly, Charles. 2006. *Regimes and Repertoires.* Chicago: University of Chicago Press.

Tilly, Charles. 2008. *Contentious Performances.* New York: Cambridge University Press.

Tyson, Scott. 2018. "The Agency Problem Underlying the Use of Repression." *Journal of Politics* 80: 1297-1310.

Tyson, Scott, and Alastair Smith. 2018. "Dual-Layered Coordination and Political Instability." *Journal of Politics* 80: 44-58.

Voronkov, Konstantin. 2012. *Aleksey Navalny: Groza zhulikov I vorov.* Moscoe: Eksmo.

Walker, Edward, Andrew Martin, and John McCarthy. 2008. "Confronting the State, the Corporation, and the Academy: The Influence of Institutional Targets on Social Movement Repertoires." *American Journal of Sociology* 114:35-76.

Wang, Dan, and Alessandro Piazza. 2016. "The Use of Disruptive Tactics in Protest as a Trade-Off: The Role of Social Movement Claims." *Social Forces* 94: 1675-1710.

Wood, Elisabeth J. 2003. *Insurgent Collective Action and Civil War in El Salvador.* New York: Cambridge University Press.

Online Appendix: Examples and Proofs

Example

Suppose $B(e) = e^m$, $0 < m < 1$, and $C(e) = c^l$, $l > m$, and set $e_{max} = 2 > \tilde{e} = 1$. Then, $C'(\bar{e}) = B'(\bar{e})$ implies $(\bar{e})^{l-m} = m/l$. Moreover, using integration by parts, equation (3) can be written as:

$$\int_{p=0}^{1} e^*(p)dp = \bar{e} + \int_{e=\bar{e}}^{e_{max}} \frac{B'(e)}{C'(e)}de.$$

Thus,

$$\begin{aligned}
\theta_\infty^{**} &= \bar{e} + \int_{e=\bar{e}}^{e_{max}} \frac{m}{l}\ e^{m-l}de = \bar{e} + \left[\frac{m}{l}\frac{(e)^{m-l+1}}{m-l+1}\right]_{e=\bar{e}}^{e=e_{max}} \\
&= \bar{e} + \frac{m}{l}\frac{(e_{max})^{m+1-l}}{m+1-l} - \frac{m}{l}\frac{(\bar{e})^{m+1-l}}{m+1-l} \\
&= (\bar{e})^{m+1-l}\ \frac{m}{l} + \frac{m}{l}\frac{(e_{max})^{m+1-l}}{m+1-l} - \frac{m}{l}\frac{(\bar{e})^{m+1-l}}{m+1-l} \quad \text{(substituting for } \bar{e}\text{)} \\
&= (e_{max})^{m+1-l}\ \frac{m}{l}\frac{1}{m+1-l} + (\bar{e})^{m+1-l}\ \frac{m}{l}\left(1 - \frac{1}{m+1-l}\right).
\end{aligned} \tag{7}$$

If the opposition leader restricts efforts to a single effort level e, from (6), we have

$$\hat{e} = \arg\max_{e\in[0,e_{max}]} \begin{cases} e & ; e \le \tilde{e} \\ \\ e^{m-l+1} & ; e \ge \tilde{e} \end{cases} = \begin{cases} \tilde{e} & ; m+1 < l \\ [\tilde{e}, e_{max}] & ; m+1 = l \\ e_{max} & ; m+1 > l. \end{cases}$$

Thus, recalling that $\tilde{e} = 1$,

$$\theta_1^{**} = \begin{cases} \tilde{e} & ; m+1 \le l \\ \\ (e_{max})^{m+1-l} & ; m+1 \ge l. \end{cases} \tag{8}$$

First, consider the comparison between θ_1^{**} and $\tilde{\theta}_\infty^{**}$ from Corollary 1. Mirroring the calculations leading to equation (7),

$$\begin{aligned}
\tilde{\theta}_\infty^{**} &= \tilde{e} + \int_{e=\tilde{e}}^{e_{max}} \frac{m}{l}\ e^{m-l}de = \tilde{e} + \left[\frac{m}{l}\frac{(e)^{m-l+1}}{m-l+1}\right]_{e=\tilde{e}}^{e=e_{max}} \\
&= \tilde{e} + \frac{m}{l}\frac{(e_{max})^{m+1-l}}{m+1-l} - \frac{m}{l}\frac{(\tilde{e})^{m+1-l}}{m+1-l}.
\end{aligned}$$

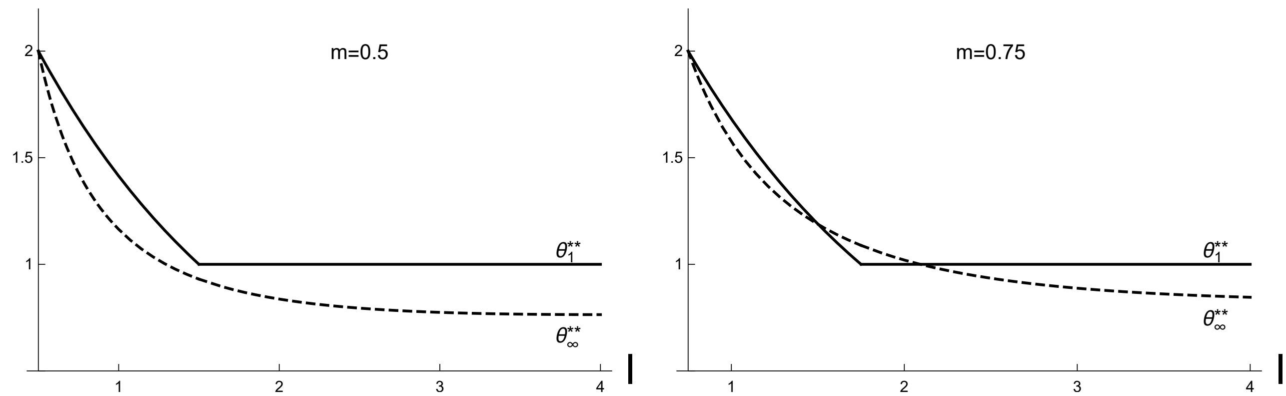

Figure 1: θ_1^{**} and θ_∞^{**} as a function of l for two values of m. When $l \leq 1$, so that $C(e)$ is concave, $\theta_1^{**} > \theta_\infty^{**}$. When $l > 1$, so that $C(e)$ is strictly convex, we can have $\theta_1^{**} < \theta_\infty^{**}$. Parameters: $B(e) = e^m,\ 0 < m < 1,\ C(e) = c^l,\ l > m$, and $e_{max} = 2 > \tilde{e} = 1$.

Thus, recognizing that $\tilde{e} = 1$,

$$\theta_1^{**} - \tilde{\theta}_\infty^{**} = \begin{cases} -\int_{e=\tilde{e}}^{e_{max}} \frac{m}{l}\ e^{m-l} de & ; m+1 < l \\ (e_{max})^{m+1-l}\ \left(1 - \frac{m}{l}\frac{1}{m+1-l}\right) - \left(1 - \frac{m}{l}\frac{1}{m+1-l}\right) & ; m+1 > l. \\ = \left[(e_{max})^{m+1-l} - 1\right]\ \frac{(l-m)}{l(m+1-l)}\ (1-l), & \end{cases}$$

as prescribed by Corollary 1.

Now, consider the comparison of θ_1^{**} and θ_∞^{**}. If $m + 1 > l$, then from equations (7) and (8),

$$\begin{aligned} \theta_1^{**} - \theta_\infty^{**} &= (e_{max})^{m+1-l}\ \left(1 - \frac{m}{l}\frac{1}{m+1-l}\right) - (\bar{e})^{m+1-l}\ \frac{m}{l}\ \left(1 - \frac{1}{m+1-l}\right) \\ &= \frac{l-m}{l(m+1-l)}\ \left[(e_{max})^{m+1-l}\ (1-l) + (\bar{e})^{m+1-l}\ m\right] \end{aligned}$$

If $C(e)$ is concave, so that $l \leq 1$, then $\theta_1^{**} - \theta_\infty^{**} > 0$ as prescribed by Proposition 2. This result also reflects that even when $l > 1$, when efforts are not restricted to be greater than $\tilde{e}$, convexity is not sufficient to deliver $\theta_1^{**} < \theta_\infty^{**}$.

If $m + 1 < l$, then from equations (7) and (8),

$$\theta_1^{**} - \theta_\infty^{**} = [\tilde{e} - \bar{e}] - \left[(e_{max})^{m+1-l} - (\bar{e})^{m+1-l}\right]\ \frac{m}{l}\frac{1}{m+1-l}.$$

Figure 1 illustrates.

Proofs

First, we a prove a lemma that states the key implications of Assumption 1 that we will use.

Lemma 2 *The following results hold under Assumption 1:*

1. *There is a unique* $e^*(p) = \arg\max_{e \in [0, e_{max}]} B(e) - pC(e)$. *Further,* $e^*(p)$ *is continuous and decreasing in* p.

2. *If* $C(e)$ *is strictly convex, then* $\frac{B'(e)}{C'(e)} > \left(e\frac{B(e)}{C(e)}\right)'$. *If* $C(e)$ *is linear, then* $\frac{B'(e)}{C'(e)} = \left(e\frac{B(e)}{C(e)}\right)'$. *If* $C(e)$ *is strictly concave, then* $\frac{B'(e)}{C'(e)} < \left(e\frac{B(e)}{C(e)}\right)'$.

3. *For any* $e_1 > e_2 > e_3 \geq 0$,
$$\frac{C(e_1) - C(e_2)}{B(e_1) - B(e_2)} < \frac{C'(e_2)}{B'(e_2)} < \frac{C(e_2) - C(e_3)}{B(e_2) - B(e_3)}.$$

Proof of Lemma 2: *Proof of Part 1.* Let $\Omega(e) = B(e) - pC(e)$, so that $\Omega'(e) = B'(e) - pC'(e)$ and $\Omega''(e) = B''(e) - pC''(e)$. If $C''(e) \geq 0$, then $\Omega''(e) < 0$. If $C''(e) < 0$, then $\Omega''(e) = B''(e) - pC''(e) \leq B''(e) - C''(e) < 0$ by our assumption that $B''(e) < C''(e)$. Thus, because $\Omega''(e) < 0$, $e^*(p)$ is unique. Moreover $\frac{\partial^2 \Omega(e;p)}{\partial p \partial e} = -C'(e) < 0$, and hence $e^*(p)$ is decreasing in p. Continuity is by the Maximum Theorem.

Proof of Part 2.

$$\begin{aligned}
\frac{B'(e)}{C'(e)} - \left(e\frac{B(e)}{C(e)}\right)' &= \frac{B'(e)}{C'(e)} - \frac{B(e)}{C(e)} - e\left(\frac{B(e)}{C(e)}\right)' \\
&= \frac{B'(e)}{C'(e)} - \frac{B(e)}{C(e)} - e\,\frac{C'(e)}{C(e)}\left(\frac{B'(e)}{C'(e)} - \frac{B(e)}{C(e)}\right) \quad \text{(from equation (11))} \\
&= \left(1 - e\,\frac{C'(e)}{C(e)}\right)\left(\frac{B'(e)}{C'(e)} - \frac{B(e)}{C(e)}\right).
\end{aligned}$$

Because $\frac{B(e)}{C(e)}$ is strictly decreasing, we have $\left(\frac{B'(e)}{C'(e)} - \frac{B(e)}{C(e)}\right) < 0$. If $C(e)$ is strictly convex, $\left(1 - e\,\frac{C'(e)}{C(e)}\right) < 0$. If $C(e)$ is linear, $\left(1 - e\,\frac{C'(e)}{C(e)}\right) = 0$. If $C(e)$ is strictly concave, $\left(1 - e\,\frac{C'(e)}{C(e)}\right) > 0$.

Proof of Part 3. By Cauchy's Mean Value Theorem, there exists $e_{12} \in (e_2, e_1)$ and $e_{23} \in (e_3, e_2)$ such that:
$$\frac{B(e_1) - B(e_2)}{C(e_1) - C(e_2)} = \frac{B'(e_{12})}{C'(e_{12})} \text{ and } \frac{B'(e_{23})}{C'(e_{23})} = \frac{B(e_2) - B(e_3)}{C(e_2) - B(e_3)}. \tag{9}$$

From part 4 of Assumption 1, $B'(e)/C'(e)$ is strictly decreasing. Further, $e_{12} > e_2 > e_{23}$. Thus,

$$\frac{B'(e_{12})}{C'(e_{12})} < \frac{B'(e_2)}{C'(e_2)} < \frac{B'(e_{23})}{C'(e_{23})}. \tag{10}$$

Combining (9) and (10) yields the result:

$$\frac{B(e_1) - B(e_2)}{C(e_1) - C(e_2)} < \frac{B'(e_2)}{C'(e_2)} < \frac{B(e_2) - B(e_3)}{C(e_2) - B(e_3)}.$$

□

Proof of Remark 2: Observe that

$$\left(\frac{B(e)}{C(e)}\right)' = \frac{B'(e)C(e) - C'(e)B(e)}{[C(e)]^2} = \frac{C'(e)}{C(e)}\left(\frac{B'(e)}{C'(e)} - \frac{B(e)}{C(e)}\right). \tag{11}$$

Because $C(e)$ is convex and $C(0) = 0$, we have $C'(e) \geq \frac{C(e)}{e}$. Because $B(e)$ is strictly concave and $B(0) = 0$, we have $B'(e) < \frac{B(e)}{e}$. Therefore, $\frac{B'(e)}{C'(e)} < \frac{B(e)/e}{C(e)/e} = \frac{B(e)}{C(e)}$. Moreover,

$$\left(\frac{B'(e)}{C'(e)}\right)' = \frac{B''(e)C'(e) - C''(e)B'(e)}{[C'(e)]^2},$$

which is negative if $C''(e) \geq 0$. □

Proof of Lemma 1: From part 3 of Lemma 2, $p_i < p_{i+1}$ for $i \in \{1, \cdots, N-1\}$. The result follows immediately from the discussion following Lemma 1 in the text. □

Proof of Theorem 1: Without e_i, $1 < i < m$, citizens' choices are $\{e_1, \cdots, e_{i-1}, e_{i+1}, \cdots, e_m, e_{m+1}\}$, where $e_1 > \cdots > e_m > e_{m+1} \geq 0$. The case of $i = m$ is analogous. Let p' be the threshold belief that makes a citizen indifferent between taking actions e_{i-1} and e_{i+1}, so that

$$p' = \frac{B(e_{i-1}) - B(e_{i+1})}{C(e_{i-1}) - C(e_{i+1})},$$

and recall that

$$p_{i-1} = \frac{B(e_{i-1}) - B(e_i)}{C(e_{i-1}) - C(e_i)} \quad \text{and} \quad p_i = \frac{B(e_i) - B(e_{i+1})}{C(e_i) - C(e_{i+1})}. \tag{12}$$

Lemma 3 $p_{i-1} < p' < p_i$.

Proof of Lemma 3: Suppose $0 \leq x < y$, and write p' as $p' = \frac{B(y)-B(x)}{C(y)-C(x)}$. Then, p_{i-1} is obtained from p' by raising x, and p_i is obtained from p' from reducing y. Now, observe that

$$\frac{d}{dy}\frac{B(y) - B(x)}{C(y) - C(x)} < 0 \Leftrightarrow \frac{B'(y)}{C'(y)} < \frac{B(y) - B(x)}{C(y) - C(x)},$$

and

$$\frac{d}{dx}\frac{B(y)-B(x)}{C(y)-C(x)} < 0 \iff \frac{B(y)-B(x)}{C(y)-C(x)} < \frac{B'(x)}{C'(x)}.$$

If the above derivatives are negative, the result follows. Moreover, by Cauchy Mean Value Theorem, $\frac{B(y)-B(x)}{C(y)-C(x)} = \frac{B'(z)}{C'(z)}$ for some $z \in (x, y)$. Then, part 4 of Assumption 1 implies:

$$\frac{B'(y)}{C'(y)} < \frac{B(y)-B(x)}{C(y)-C(x)} < \frac{B'(x)}{C'(x)},$$

and hence the derivatives are negative. □

Note that because $1 < i \leq m$, we also have $0 < p_{i-1} < p' < p_i \leq 1$. Thus,

$$\theta^*_m - \theta^*_{m-1} = (p_i - p')\ (e_i - e_{i+1}) - (p' - p_{i-1})\ (e_{i-1} - e_i), \tag{13}$$

where the first term captures the losses of removing e_i, and the second term captures the corresponding gains.

Lemma 4 *There is an $A > 0$ such that*

$$(p' - p_{i-1})(e_{i-1} - e_i) = \frac{A}{C(e_{i-1}) - C(e_{i+1})}\ \frac{e_{i-1} - e_i}{C(e_{i-1}) - C(e_i)}, \tag{14}$$

and

$$(p_i - p')(e_i - e_{i+1}) = \frac{A}{C(e_{i-1}) - C(e_{i+1})}\ \frac{e_i - e_{i+1}}{C(e_i) - C(e_{i+1})}. \tag{15}$$

Proof of Lemma 4: Observe that:

$$\begin{aligned}
&(p' - p_{i-1})(e_{i-1} - e_i) \\
=\ & \left(\frac{B(e_{i-1}) - B(e_{i+1})}{C(e_{i-1}) - C(e_{i+1})} - \frac{B(e_{i-1}) - B(e_i)}{C(e_{i-1}) - C(e_i)}\right)(e_{i-1} - e_i) \\
=\ & \frac{[C(e_{i-1}) - C(e_i)]\ [B(e_{i-1}) - B(e_{i+1})] - [C(e_{i-1}) - C(e_{i+1})]\ [B(e_{i-1}) - B(e_i)]}{C(e_{i-1}) - C(e_{i+1})} \\
\times\ & \frac{e_{i-1} - e_i}{C(e_{i-1}) - C(e_i)},
\end{aligned} \tag{16}$$

and

$$\begin{aligned}
&(p_i - p')(e_i - e_{i+1}) \\
=\ & \left(\frac{B(e_i) - B(e_{i+1})}{C(e_i) - C(e_{i+1})} - \frac{B(e_{i-1}) - B(e_{i+1})}{C(e_{i-1}) - C(e_{i+1})}\right)(e_i - e_{i+1}) \\
=\ & \frac{[C(e_{i-1}) - C(e_{i+1})]\ [B(e_i) - B(e_{i+1})] - [C(e_i) - C(e_{i+1})]\ [B(e_{i-1}) - B(e_{i+1})]}{C(e_{i-1}) - C(e_{i+1})} \\
\times\ & \frac{e_i - e_{i+1}}{C(e_i) - C(e_{i+1})}.
\end{aligned} \tag{17}$$

Critically,

$$\begin{aligned}(C(e_{i-1})-C(e_i))\,(B(e_{i-1})-B(e_{i+1})) &- (C(e_{i-1})-C(e_{i+1}))\,(B(e_{i-1})-B(e_i))\\ &=\\ (C(e_{i-1})-C(e_{i+1}))\,(B(e_i)-B(e_{i+1})) &- (C(e_i)-C(e_{i+1}))\,(B(e_{i-1})-B(e_{i+1}))\end{aligned}$$

if and only if

$$\begin{aligned}(B(e_{i-1})-B(e_{i+1})) &\times (C(e_{i-1})-C(e_i)+C(e_i)-C(e_{i+1}))\\ &=\\ (B(e_i)-B(e_{i+1})+B(e_{i-1})-B(e_i)) &\times (C(e_{i-1})-C(e_{i+1})),\end{aligned}$$

which is true. Now, let

$$A=\frac{[C(e_{i-1})-C(e_{i+1})]\,[B(e_i)-B(e_{i+1})]-[C(e_i)-C(e_{i+1})]\,[B(e_{i-1})-B(e_{i+1})]}{C(e_{i-1})-C(e_{i+1})}.$$

Substituting A into equations (16) and (17) yields (14) and (15), respectively.

Finally, $A>0$ if and only if

$$A>0 \;\Leftrightarrow\; \frac{B(e_i)-B(e_{i+1})}{C(e_i)-C(e_{i+1})}>\frac{B(e_{i-1})-B(e_{i+1})}{C(e_{i-1})-C(e_{i+1})} \;\Leftrightarrow\; p_i>p',$$

which we showed to be true in Lemma 3. □

Now, substituting from equations (14) and (15) in Lemma 4 into (13), we have:

$$\theta_m^*-\theta_{m-1}^* \;=\; \frac{A}{C(e_{i-1})-C(e_{i+1})}\left(\frac{e_i-e_{i+1}}{C(e_i)-C(e_{i+1})}-\frac{e_{i-1}-e_i}{C(e_{i-1})-C(e_i)}\right). \tag{18}$$

Thus, from (18),

$$\theta_m^*-\theta_{m-1}^*\begin{cases}>0 & ;\text{if } C(e) \text{ is strictly convex}\\ =0 & ;\text{if } C(e) \text{ is linear}\\ <0 & ;\text{if } C(e) \text{ is strictly concave}\end{cases}$$

□

Proof of Corollary 1: From part 1 of Lemma 2, $e^*(p)$ is decreasing and continuous. Thus, from equations (3), we have:

$$\tilde{\theta}_\infty^{**}=\int_{p=0}^{1}e^*(p)dp=\tilde{e}+\int_{e=\tilde{e}}^{e_{max}}\frac{B'(e)}{C'(e)}\,de.$$

First, suppose $C(e)$ is strictly concave. From part 2 of Lemma 2, $\frac{B'(e)}{C'(e)} < \left(e\frac{B(e)}{C(e)}\right)'$, and hence:

$$\begin{aligned}
\tilde{\theta}_{\infty}^{**} &= \tilde{e} + \int_{e=\tilde{e}}^{e_{max}} \frac{B'(e)}{C'(e)}\, de \\
&< \tilde{e} + \int_{e=\tilde{e}}^{e_{max}} \left(e\, \frac{B(e)}{C(e)}\right)' de \\
&= \tilde{e}\left(1 - \frac{B(\tilde{e})}{C(\tilde{e})}\right) + e_{max}\, \frac{B(e_{max})}{C(e_{max})} \\
&= e_{max}\, \frac{B(e_{max})}{C(e_{max})} \\
&\leq \theta_1^{**} \qquad \text{(because the leader can choose } e_{max}\text{).}
\end{aligned}$$

Next, suppose $C(e)$ is linear, so that $C(e) = \alpha \cdot e$ for some $\alpha > 0$. From equation (6), $\theta_1^{**} = \frac{B(e_{max})}{\alpha}$. From part 2 of Lemma 2, $\frac{C'(e)}{B'(e)} = \left(e\frac{C(e)}{B(e)}\right)'$. Thus, mirroring the above calculations, we have $\tilde{\theta}_{\infty}^{**} = e_{max}\, \frac{B(e_{max})}{C(e_{max})} = \frac{B(e_{max})}{\alpha} = \theta_1^{**}$.

Finally, suppose $C(e)$ is strictly convex. From part 2 of Lemma 2, $\frac{B'(e)}{C'(e)} > \left(e\frac{B(e)}{C(e)}\right)'$, and hence:

$$\begin{aligned}
\tilde{\theta}_{\infty}^{**} &= \tilde{e} + \int_{e=\tilde{e}}^{e_{max}} \frac{B'(e)}{C'(e)} de \\
&\geq \tilde{e} + \int_{e=\tilde{e}}^{\hat{e}} \frac{B'(e)}{C'(e)} de \quad \text{(for any } \hat{e} \in (\tilde{e}, e_{max}]\text{)} \\
&> \tilde{e} + \int_{e=\tilde{e}}^{\hat{e}} \left(e\frac{B(e)}{C(e)}\right)' de \\
&= \tilde{e} + \hat{e}\, \frac{B(\hat{e})}{C(\hat{e})} - \tilde{e}\, \frac{B(\tilde{e})}{C(\tilde{e})} \\
&= \hat{e}\, \frac{B(\hat{e})}{C(\hat{e})}.
\end{aligned}$$

If $\theta_1^{**} = \tilde{e}$, then clearly $\tilde{\theta}_{\infty}^{**} > \theta_1^{**}$. Otherwise, from (6), $\theta_1^{**} = e\, \frac{B(e)}{C(e)}$ for some $e \in (\tilde{e}, e_{max}]$, in which case the above calculations show that $\tilde{\theta}_{\infty}^{**} > \theta_1^{**}$. □

Proof of Proposition 2: *Proof of Part 1.* We consider the three cases of $m = 0, 1$ and $m \geq 2$ separately. Suppose $m = 0$. Then, it must be that $\tilde{e} > e_1 > \cdots > e_N$. Because even type $p = 1$ is willing to take action e_1, if we remove all $\{e_2, \cdots, e_N\}$, all types will take the highest action e_1, which gives the highest equilibrium regime change threshold.

Suppose $m = 1$. Then, it must be that $e_1 \geq \tilde{e}$. Now, remove an e_i, $i \in \{2, \cdots, N\}$. If θ^* does not change (because no type change their decision), then continue removing effort levels.

If θ^* changes, it must be that e_2 has been removed. Then, if $p' \geq 1$, again we are done. If, instead, $p' < 1$, then as we discussed in the text, the gains remain $(p' - p_1)(e_1 - e_2)$, while the loss becomes $(1 - p')(e_2 - e_3) < (p_2 - p')(e_2 - e_3)$—because $p_2 > 1$. Now, θ^* is larger because, by Theorem 1, even if the losses were $(p_2 - p')(e_2 - e_3)$, θ^* would have been larger.

Finally, suppose $m \geq 2$. Then, by Theorem 1, the equilibrium regime change threshold increases by removing some effort levels $i > 1$ until either only the highest action e_1 remains, or $m = 1$, which is the case that we discussed above.

Proof of Part 2. From part 1 of Lemma 2, $e^*(p)$ is decreasing and continuous. Thus, from equations (3), we have:

$$\theta^{**}_\infty = \int_{p=0}^{1} e^*(p)dp = \overline{e} + \int_{e=\overline{e}}^{e_{max}} \frac{B'(e)}{C'(e)}\, de.$$

From part 2 of Lemma 2, $\frac{B'(e)}{C'(e)} \leq \left(e\frac{B(e)}{C(e)}\right)'$, and hence:

$$\begin{aligned}
\theta^{**}_\infty &= \overline{e} + \int_{e=\overline{e}}^{e_{max}} \frac{B'(e)}{C'(e)}\, de \\
&\leq \overline{e} + \int_{e=\overline{e}}^{e_{max}} \left(e\, \frac{B(e)}{C(e)}\right)'\, de \\
&= \overline{e}\left(1 - \frac{B(\overline{e})}{C(\overline{e})}\right) + e_{max}\, \frac{B(e_{max})}{C(e_{max})} \\
&< e_{max}\, \frac{B(e_{max})}{C(e_{max})} \quad \text{(because } \overline{e} < \tilde{e},\ \tfrac{B(e)}{C(e)} \text{ is strictly decreasing, and } \tfrac{B(\tilde{e})}{C(\tilde{e})} = 1) \\
&= \theta^{**}_1 \quad \text{(from Remark 3).}
\end{aligned}$$

□