Motivation in Collective Action

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Abstract

In some movements, participants are motivated by material considerations. In others, the motivation is ideological or psychological. Rivalry of material rewards introduces congestion externalities to the participants' coordination problem: changes that increase the likelihood of success also increase participation, so that each participant's share of future rewards is smaller. We show that movements whose members are materially rather than psychologically motivated to undertake collective action are less affected by repression, are less able to exploit an increase but better able to withstand a decrease in the supply of potential recruits, and are less able to turn early failures into future successes. A government attempting to control a movement should focus on raising participation costs when motivations are psychological and on destroying rewards when motivations are material. Moreover, we show rewards in the form of future government office rents share features of both material and psychological rewards.

Keywords: Regime Change, Rebellion, Repression, Recruitment, Material Incentives, Psychological Incentives

At least since Gurr's (1970) classic book, political scientists have been interested in the motivations that drive people to join social movements, protests, and rebellions. There is considerable heterogeneity in such motivations. In some movements, most members are motivated primarily by material considerations. In other movements, the primary motivation is ideological or psychological. For example, Weinstein (2007) and Humphrey and Weinstein (2008) find that rebel fighters in Sierra Leone were motivated by opportunities for looting, drug sales, and other material gains. In contrast, Wood (2003) finds that rebels in El Salvador were motivated by psychological rewards, ranging from vengeance to the opportunity to be "part of the making of history" (pp. 18–19). (See Blattman and Miguel (2010, p. 32–35) for a review of the literature.)

The question of motivations for collective action is foundational for a behavioral understanding of participation in collective action and has been studied extensively. But motivations also matter for the outcomes of collective action. To the extent that scholars have examined these consequences of motivations, the debate has largely focused on how different motivations affect group cohesion and discipline (Ellis 1999; Weinstein 2007). In this paper, we broaden the discussion, providing new theoretical insights into the consequences of variation in motivations for the outcomes of political collective action.

In our conceptualization, material and psychological rewards share an important feature and also differ in an important respect. The feature they share in common is that both are contingent on the success of the movement. The point of divergence is that material rewards are rival goods, while psychological rewards are non-rival.

We ask three key questions. First, all else equal, is repression more effective against movements whose members are motivated by material gains or against movements whose members are motivated by psychological or ideological concerns? Second, all else equal, which kind of movement is better able to exploit an increase in the supply of potential recruits or cope with a decrease in that supply? Third, all else equal, which type of movement is more likely to succeed following an early failure that leaves behind a residual core of highly committed members?

We show that movements whose members are materially rather than psychologically motivated are less affected by repression, are less able to exploit an increase but better able to withstand a decrease in the pool of potential recruits, and are less able to turn early failures into future successes.

The Nature of Material and Psychological Motivations As indicated above, we treat both psychological and material rewards as contingent on the success of the movement. But we treat material rewards as rival goods and psychological rewards as non-rival. Why these similarities and distinctions?

Material benefits take various forms, including direct payments, protection, opportunities for looting, and promises of future economic spoils. Some of these, such as looting while fighting, may be enjoyed by participants even during the course of a failing campaign. But all are larger in a successful movement and many, such as rents associated with taking over the economy, are only available following success. The economic spoils available at the end of a successful rebellion are finite. They must be divided among the participants in the victorious movement. As such, it is natural to think that they are rival—the larger the movement, the less each individual participant expects to receive.

The nature of psychological benefits has been the subject of long debate. Early work emphasized purely expressive motives (Davies 1962; Geschwender 1967; Gurr 1970). But later studies showed that even psychologically motivated individuals account for the likelihood of success and the costs of participation when deciding whether to mobilize (Tilly 1978, 2008; McAdam 1999; Tarrow 2011).[1] In particular, movements with no prospect of success are unlikely to be sustainable because the costs of participation exceed the psychological benefits. Later studies confirmed this insight and developed a success-contingent conception of psychological and ideological rewards. In particular, Wood's (2003) notion of pleasure-in-agency captures psychological rewards associated with participating in a successful movement. Wood defines pleasure-in-agency as, "the positive effect associated with self-determination, autonomy, self-esteem, efficacy, and pride that come from the successful assertion of intention" (p. 235). Based on extensive fieldwork and building on the historical and sociological literature, Wood found that agents motivated by psychological rewards not only account for the likelihood of success, but also act strategically: pleasure-in-agency is, "a frequency-based motivation: it depends on the likelihood of success, which in turn increases with the number participating (Schelling 1978; Hardin 1982)" (p. 235–6). Such findings suggest that whether psychological rewards derive from ideology, the satisfaction of "being part of the making of history," justice, honor, or vengeance, the net benefit is positive only if the movement succeeds (Petersen 2001;

[1] As Washington wrote to the Continental Congress in the 1770s, "The honor of making a brave defense does not seem to be a sufficient stimulus, when the success is very doubtful and the falling into the Enemy's hands probable" (Middlekauff 2005, p. 342).

Wood 2003; Morris and Shadmehr 2017; Pearlman 2018; Aytaç and Stokes 2019). By contrast with material rewards, the satisfaction from implementing an ideological vision, achieving justice, or being part of history is not diminished for being shared. As such, it is natural to think of psychological rewards as non-rival—as more people join the movement, the likelihood of success increases, with no associated diminution in individual rewards conditional on success.

This distinction creates a fundamental strategic difference for movements whose members are motivated materially versus psychologically. When rewards are material, as the movement becomes larger, success is more likely, but the rewards to each individual conditional on success are smaller. When rewards are psychological, as the movement becomes larger, success is more likely and rewards don't change. As such, the setting with psychological rewards features only strategic complements. But the setting with material rewards also has a force for strategic substitutes.

The Theoretical Framework Our model consists of a continuum of agents who simultaneously decide whether to participate in a costly rebellion. The rebellion succeeds if the measure of rebels exceeds a threshold that captures the regime's strength. In the setting with psychological rewards, if the rebellion succeeds, each participant receives a given reward regardless of the number of participants. In contrast, in the setting with material rewards, if the rebellion succeeds, a given reward is divided equally among the participants. We normalize the total size of available rewards, so that if all agents participate and the rebellion succeeds, each agent's payoff is the same in both settings. The regime's strength is uncertain and agents receive conditionally independent noisy private signals about it. Upon receiving their private information, players simultaneously decide whether to rebel, the success or failure of the movement is determined, and the payoffs are received.

In the complete information benchmark in which regime strength is known, psychological and material reward settings have similar properties. Both feature multiple equilibria, one in which no one rebels and one in which everyone rebels. Critically, these equilibria are insensitive to variations in the environment, such as increased repression (Proposition 1).

By contrast, with strategic uncertainty, equilibrium behavior is responsive to changes in the environment in ways that differ between the settings that represent material versus psychological rewards. Increases in repression reduce direct incentives to rebel in both settings, but the strategic effect differs across them (Proposition 3). Realizing that higher participation costs mean that others are less likely to rebel, a citizen's incentives to rebel fall even further in response to repression in the setting with psychological rewards. But this strategic effect is weaker in the material rewards setting and indeed may even counteract the direct effect because a smaller rebellion size implies higher rewards for participants if the rebellion does succeed. Consequently, repression is less effective against movements whose members are materially motivated—the likelihood of success is less responsive to repression for such movements than for movements whose members are psychologically motivated.

Increasing the supply of potential recruits raises rebellion incentives both directly and through the strategic channel in both settings (Proposition 4). However, these effects are weaker for movements whose members are materially motivated because, again, a larger rebellion implies smaller rewards for each participant upon success. This implies that materially motivated movements are less damaged by a decrease in the supply of potential recruits, but also less able to exploit an increase in that supply.

Rebellions often unfold over time. A movement that fails at first may resurface again when another opportunity arises (McAdam, Tarrow, Tilly 2001; Tarrow 2011). Early failures often generate a core of committed members motivated by a sense of injustice, vengeance, or camaraderie. This pattern has been observed in a variety of places, including El Salvador (Wood 2003), Morocco (Lawrence 2017), Iran (Shadmehr 2017), Hong Kong (Bursztyn et al. 2019), and Syria (Pearlman 2020). The presence of a committed core increases the likelihood of success in both material and psychological rewards settings because the subsequent rebellion is sure to have the active support of the committed group. However, this effect is weaker with material rewards because uncommitted citizens recognize that material rewards have a tighter upper bound because the size of this subsequent rebellion is at least as large as the size of the committed core with whom the rewards will be shared. Consequently, conditional on a failure that gives rise to a committed core, movements whose participants are psychologically motivated are more likely to succeed (Proposition 5).

We end by analyzing how our results extend to movements in which material rewards are imperfectly divisible, such as with rents from holding government office. We show that this setting is a middle ground—the effects of higher repression, new potential recruits, and early failures are in between the effects found in the settings with psychological rewards and perfectly divisible material rewards (Proposition 6 and Figure 3). When the size of the rebel movement is smaller than the total number of government offices, a larger rebellion does not change the rewards of success, and each participant gets a political office. This feature shares the non-rival aspect of the psychological rewards setting. But when the rebellion size exceeds the number of government offices, an increase in participation reduces the chances that each rebel obtains an office should the rebellion succeed because there will not be enough offices to go around. This feature shares the rival aspect of the setting with perfectly divisible material rewards. It follows that when incentives are in the form of rents from future government offices, the rebel movement's response to variations in the environment falls in between its response when there are psychological rewards and its response when there are divisible material rewards.

1 Model and Analysis

There is a continuum of citizens of size $a > 0$, indexed by $i \in [0, a]$. Citizens simultaneously decide whether to join a rebel movement. The rebellion succeeds if and only if the size of the rebel movement, m, exceeds the state of the world, θ, which captures the strength of the status quo.

The payoff of a citizen who does not rebel is normalized to 0. If a citizen rebels, he pays a cost of $c \in (0, 1)$. If the rebellion succeeds, a citizen who participated in the rebellion receives a payoff u^j, $j = p, s$, where u^p is the reward in the setting with *psychological* rewards, and u^m is the reward in the setting with *material* rewards. Psychological rewards are normalized to 1, and material rewards are normalized to $\frac{a}{m}$, so that if the rebellion succeeds, the total available rewards in both settings is a. Figure 1 represents the payoffs.

psychological

	outcome: $m > \theta$	outcome: $m \leq \theta$
rebel	$1 - c$	$-c$
not rebel	0	0

material

	outcome: $m > \theta$	outcome: $m \leq \theta$
rebel	$\frac{a}{m} - c$	$-c$
not rebel	0	0

Figure 1: **Psychological versus Material Rewards.** The size of the population is a, the size of rebel movement is $m \leq a$, the cost of participation is c, and the regime's strength is θ. The left panel captures movements with psychological rewards: net rewards from participation do not depend on how many participate. The right panel captures movements with material rewards: net rewards from participation fall with more participation because participants must share the spoils.

The state of world is uncertain, and citizens share a common prior that θ is distributed on $\mathbb{R}$ according to an improper uniform distribution, so that there is no prior common knowledge.

Each citizen i receives a noisy private signals $x_i = \theta + \sigma\epsilon_i$, where $\epsilon_i \sim iid\ F$ and independent of θ. We assume that F is smooth and $F(x) \in (0, 1)$ for all $x \in \mathbb{R}$.

1.1 Complete Information Benchmark

We begin with the complete information benchmark in which the regime's strength θ is known.

Proposition 1 *The setting with psychological rewards and the setting with material rewards both have the same pure strategy equilibria:*

- *If $\theta \geq a$, there is a unique equilibrium in which no one rebels, the regime survives, and each citizen receives* 0.
- *If $\theta < 0$, there is a unique equilibrium in which everyone rebels, the regime collapses, and everyone receives* 1.
- *In between, both equilibria co-exist.*

Proposition 1 implies that the settings with psychological and material rewards generate the same outcomes. However, this complete information setting is misleading. Of course, it abstracts from information frictions that exist in the real. Moreover, it has two problematic properties: (1) there are multiple equilibria, which makes empirical prediction difficult; and (2) equilibrium outcomes are insensitive to parameters of the model like the costs of rebellion.

1.2 Equilibrium

We now analyze our main, incomplete information model. The left panel in Figure 1 is the quintessential regime change game (Morris and Shin 1998, 2003; Angeletos et al. 2007).[2] In it, equilibrium is characterized by two thresholds (x^p, θ^p), so that citizens with signal $x_i < x^p$ rebel, and the regime collapses if and only if $\theta < \theta^p$. These threshold are determined by the indifference (optimality) and belief consistency conditions:

$$aPr(x_i < x^p|\theta^p) = \theta^p \text{ (belief consistency)} \quad \text{and} \quad Pr(\theta < \theta^p|x_i = x^p) = c \text{ (indifference)} \qquad (1)$$

[2]Papers featuring variations of this game include Boix and Svolik (2013), Edmond (2013), Casper and Tyson (2014), Loeper et al. (2014), Chen et al. (2016), Rundlett and Svolik (2016), Shadmehr and Bernhardt (2017), Tyson and Smith (2018), and Shadmehr (2019b).

Because each citizen rebels whenever her signal of the regime's strength is below a threshold, for any given regimes strength θ, the aggregate size the rebellion is $aPr(x_i < x^p|\theta)$. Naturally, the size of the rebellion is decreasing in the regime's strength, implying that the regime collapses bellow a threshold of regime strength and survives above it. Thus, that critical threshold (which we call θ^p) is exactly the size of the rebellion at that critical threshold.

How do we find this critical rebellion size? Because a citizen rebels whenever her belief about the likelihood of success is larger than the cost of rebelling, to find the size of the rebellion at the critical threshold we need to know the distribution of these beliefs at that critical threshold. As Shadmehr (2019a) discusses in detail, when there is no common knowledge,[3] the distribution of these beliefs about the likelihood of success at the critical threshold is uniformly distributed on $[0, 1]$ among citizens. Thus, the size of the rebellion is the population size a times probability that a random citizen's belief is above the rebellion cost: $a(1 - c)$. That is,

$$\theta^p = a(1 - c). \tag{2}$$

The nature of strategic interactions is a pure coordination problem. The game is a standard global game of regime change, where the actions of citizens are always strategic complements: when one citizen believes that others are more likely to rebel, her incentives to join the rebellion increase because the rewards remain the same, but the likelihood of success increases.

In contrast, the game in right panel of Figure 1 is not a pure coordination game. In this game, when a citizen believes that others are more likely to rebel, her incentives to protest may fall because, although success is more likely, the limited rewards from that success will be shared among a larger group, so that each participant will expect to receive less reward conditional on success. That is, the game does not feature global strategic complements due to congestion externalities. In particular, for a given level of regime strength, θ, the net payoff from revolting versus not revolting is:

$$\mathbf{1}_{\{\theta<m\}} \cdot \frac{a}{m} - c.$$

This net payoff is non-monotone in the size of the rebellion m. It jumps up from 0 to $\frac{a}{m} - c$ at $m = \theta$ (the threshold at which regime change succeeds), but then falls smoothly to $1 - c$ as more people join the movement. Therefore, the best response to a monotone strategy is not monotone in general, and monotone equilibria may not exist. When a citizen receives a lower

[3]For example, when citizens share a prior that θ is distributed accordingly to an improper uniform distribution on $\mathbb{R}$, or when the noise in private signals is vanishingly small.

signal, she updates that the regime is weaker and the size of the rebellion larger. This updating increases her assessment of the chances of success, but reduces her assessment of the reward conditional on that success (the citizen updates $\mathbf{1}_{\{\theta<m\}}$ upward, but $\frac{a}{m}$ downward). Despite this non-monotonicity, Proposition 2 shows that our assumptions are enough to deliver the existence of monotone equilibria.

We look for symmetric monotone equilibria in which a citizen rebels if and only if her signal is below a finite threshold $x_i < x^m$. As before, this monotone strategy implies that the regime collapses if and only if $\theta < \theta^m$, where

$$aPr(x_i < x^m|\theta^m) = \theta^m \quad \text{(belief consistency)}. \tag{3}$$

A citizen with signal x_i rebels if and only if her expected payoff from rebellion exceeds its costs. What complicates this expectation relative to the psychological rewards setting is that the expected rewards do not boil down to the likelihood of success, because higher chances of success also imply a larger rebellion size, which in turn, implies a smaller reward for each participant.

$$E\left[\mathbf{1}_{\{\theta<\theta^m\}} \cdot \frac{a}{m}\bigg|x_i\right] = E\left[\mathbf{1}_{\{\theta<\theta^m\}} \cdot \frac{a}{aPr(x_j < x^m)}\bigg|x_i\right] = \int_{\theta=-\infty}^{\theta^m} \frac{pdf(\theta|x_i)}{Pr(x_j < x^m|\theta)} d\theta > c. \tag{4}$$

If a symmetric monotone equilibrium exists, the marginal citizen who receives the threshold signal $x_i = x^m$ must be indifferent between rebelling or not. Moreover, as Shadmehr (2019b, Lemma 3) shows, when there is no prior common knowledge, the marginal citizen believes that the size of the rebellion is uniformly distributed on $[0, 1]$. But because a rebel only receives the rewards if the rebellion succeeds, the maximum reward size is $\frac{1}{Pr(x_j<x^m|\theta=\theta^m)}$. Thus, the marginal citizen's expected payoff from rebellion and the indifference condition become

$$\int_{Pr(x_j<x^m|\theta^m)}^{1} \frac{du}{u} = -\log(Pr(x_j < x^m|\theta^m)) = c.$$

Combining this with (3) yields $-\log(\theta^m/a) = c$, so that the unique equilibrium regime change threshold is

$$\theta^m = ae^{-c}. \tag{5}$$

Proposition 2 summarizes these results.

Proposition 2 *The setting with psychological rewards has a unique equilibrium in which the rebellion succeeds if and only if the strength of the regime is below a threshold $\theta^p = a(1-c)$. The setting with material rewards has a unique equilibrium in which the rebellion succeeds if and only if the strength of the regime is below a threshold $\theta^m = ae^{-c}$.*

Proposition 2 implies that $\theta^m > \theta^p$. Because total rewards in the material setting are divided among rebel participants, and some citizens always choose not to rebel in equilibrium due to information frictions, the equilibrium incentives are stronger in the material rewards setting ($1 < a/m$ for $m \in (0, a)$).

2 Repression and Recruitment

We now compare the effect of increases in the direct costs of rebellion (e.g., state repression) and the effect of a change in the size of the pool of potential recruits.

We capture the idea of increased repression with an increase in the cost of rebellion, c. Of course, repression may raise both a citizen's direct costs of rebelling and a citizen's direct benefit from rebelling due to a sense of injustice or a desire for vengeance (Siegel 2011; Lawrence 2017; Aytaç and Stokes 2019; Shadmehr and Boleslavsky 2019). In our model, the cost c is, in fact, the ratio of the costs and benefits of rebellion. Representing increased repression with an increase c means that even though both the numerator and the denominator may rise, we assume the direct cost-benefit ratio is increasing. This is consistent with the standard view of higher repression as reducing political opportunities (Tilly 1978, 2006; McAdam 1999; Davenport 2007; Tarrow 2011).

Proposition 2 immediately implies:

Proposition 3 *The equilibrium regime change threshold is less responsive to repression in the material rewards setting than in the psychological rewards setting:* $\frac{\partial \theta^p}{\partial c} < \frac{\partial \theta^m}{\partial c} < 0$.

Proposition 3 shows how the likelihood of success responds to variation in repression, modeled as the costs of rebellion. (Notice, this result could also be interpreted in terms of, say, the economic opportunity costs of mobilization.) The direct effect of higher rebellion costs is to reduce incentives to protest in both settings. But there is also a strategic effect: a citizen recognizes that higher costs mean that others have less incentives to rebel, and adjust her behavior accordingly. In the psychological rewards setting, this further reduces the incentives to rebel because the likelihood of success is lower. This strategic effect in weaker in the material rewards setting and may off-set parts of the direct effect (if actions are strategic substitutes at equilibrium). The reason is that even though the likelihood of success is lower, the size of the rebellion is also smaller, so that if the rebellion succeeds each participant receives a larger reward. Due to this strategic effect, the likelihood of success in the material reward is less sensitive to increases in the direct costs of rebellion. Thus, repression is less effective against groups whose members are materially motivated.

In contrast to repression, which tends to reduce the chances of success, access to new recruits tends to increase the chances of success. How does having access to a larger pool of potential recruits affect the likelihood of success in settings with psychological and material rewards? Suppose that a population of size $b \geq 0$ is added to the original population of size $a > 0$. To ensure that a best response to a monotone strategy is monotone, we strengthen our assumption that $c \in (0, 1)$ to the following assumption:

Assumption 1 $c \in \left(0, \frac{a}{a+b}\right)$.

Label the new equilibrium regime change thresholds θ_r^j, $j \in \{p, m\}$.

Proposition 4 *With the new pool of potential recruits, the equilibrium regime change thresholds are* $\theta_r^p = (b + a)(1 - c)$ *and* $\theta_r^m = (a + b)e^{-c\frac{a+b}{a}}$. *Increasing the pool of potential recruits always raises the likelihood of regime change, but the marginal effect is larger in the psychological than in the material rewards setting:* $\theta_r^p - \theta^p > \theta_r^m - \theta^m > 0$.

New potential recruits raise the potential size of the rebellion. In the psychological rewards setting, the direct and strategic effects increase incentives to rebel because a larger rebellion increases the likelihood of success. However, in the material rewards setting, there are also congestion externalities that pull in the opposite direction. A larger rebellion means that the rewards of success will be divided among a larger group. Therefore, the marginal effect of access to a larger pool of recruits is smaller in the material rewards setting. Put differently, materially inspired movements will be less affected by a decrease in the pool of potential recruits.

Increasing rebellion costs and decreasing the size of the recruitment pool work through similar channels. But there is a subtle difference. With changing costs, the congestion externalities associated with material rewards on appear in the strategic effect. But with a changing recruitment pool, the congestion externalities appear in both the direct and strategic effects. Nonetheless, overall, the same basic logic that makes settings with material rewards less sensitive to repression also makes them less sensitive to the size of the pool of new recruits.

3 Failure and a Committed Core

Our analysis thus far has focused on the differential effect of repression and the size of the recruitment pool on movements with material versus psychological rewards. We showed that materially inspired movements are more resilient to both repressive actions that raise participation costs and to reductions in the size of recruitment pool. We now ask which type of movement is more resilient to an initial failure?

Many movements do not succeed or fail in a single episode. A movement that fails initially may resurface later when another political opportunity arises (McAdam, Tarrow, Tilly 2001; Tarrow 2011).

To study the effect of failure on the likelihood of success, we extend the model to two periods, and normalize the population size to 1. The stage game in the first period is identical to the previous setting. If the regime survives, citizen play the same stage game again with one difference: we assume a fraction $1 - a \in (0, 1)$ of citizens will surely rebel in the second period. This captures the idea that an early episode of rebellion generates a variety of direct incentives for rebelling among a committed core. These incentives include a desire to avenge state repression as Wood (2003) and Lawrence (2017) observed in El Salvador and Morocco, respectively. They also include the camaraderie and friendships formed among rebels as Bursztyn et al. (2019) observed in Hong Kong, or Pearlman (2020) observed in Syria. (See also Diani and McAdam, 2003). Thus, there are two differences between periods 1 and 2: in the second period, a fraction $1 - a$ of citizens are committed to the rebellion and citizens have the additional, common knowledge that the regime survived the first period. To ease exposition, we focus on Normal distributions of noise so that $F = N(0, 1)$.

In period 2, each citizen has three pieces of information: her signal from the first period, her signal from the second period, and the fact that the regime has survived. Because conditional expectations of normally distributed variables are linear, a citizen's private information in period 2 is the average of her private signals in periods 1 and 2. Let x_2 be that average. We refer to this average signal as a citizen's private signal in period 2.

As before, we focus on symmetric monotone equilibria. In period 1, a citizen rebels if and only if her private signal is below a threshold x_1^*. In period 2, a fraction $1 - a$ of citizens will be committed and rebel, and a citizen from the remaining group rebels if and only if her private signal x_2 is below a threshold x_2^*. As in the static setting, there is no equilibrium in which a citizen always revolts: $x_t^* < \infty$. If the regime survives the first period, it means that $\theta > 0$. Thus, there is always an equilibrium in which $x_2^* = -\infty$, and only the fraction $1 - a$ of (committed) citizens rebel. We focus on the interesting case of finite cutoff equilibria, so that $x_2^* \in \mathbb{R}$.

Because a single citizen's action does not change the outcome (each individual is too small to make a non-negligible difference), in the first period the equilibrium behavior of citizens in the same as in the static game. Let θ_t^j, $j \in \{p, m\}$, $t = 1, 2$, be the period t equilibrium regime change threshold in the settings with psychological ($j = p$) and material ($j = m$) rewards, respectively. Let x_t^p and x_t^m be the corresponding equilibrium citizen cutoffs. From our earlier analysis, $\theta_1^p = 1 - c$ and $\theta_1^m = e^{-c}$. In the second period, in the setting with material rewards, any pair of cutoffs (θ_2^m, x_2^m) that satisfy the following belief consistency and indifference conditions constitute an equilibrium:

$$\theta_2^m = (1-a) + aPr(x_i < x_2^m | \theta_2^m) \tag{6}$$

$$c = E\left[\frac{\mathbf{1}_{\{\theta < \theta_2^m\}}}{(1-a) + aPr(x_j < x_2^m)} \middle| x_i = x_2^m, \theta > \theta_1^m\right] \tag{7}$$

In the second period, in the setting with psychological rewards, any (θ_2^p, x_2^p) that satisfy the following conditions constitute an equilibrium:

$$\theta_2^p = (1-a) + aPr(x_i < x_2^p | \theta_2^p) \tag{8}$$

$$c = Pr(\theta < \theta_2^p | x_i = x_2^p, \theta > \theta_1^p) \tag{9}$$

These equilibrium conditions reflect the two differences between period 2 and period 1. The information content of the regime's survival is reflected in conditioning on $\theta > \theta_1^j$, $j \in \{p, m\}$, in the indifference conditions. The emergence of a committed rebel core is reflected by $1 - a$ in the belief consistency conditions.

It is bad news for the rebels that the regime survived the first period because it mean the regime is stronger that the typical citizens had thought in period 1. This may prevent rebellion in period 2 all together (finite-cutoff equilibria may not exist). But if citizens' private information is sufficiently precise, they effectively discard the relatively imprecise information that $\theta > \theta_1^j$, $j \in \{p, m\}$: compared to their precise private information, this public information receives little weight in their Bayesian updating. We focus on this limit when the noise in private signals is vanishingly small ($\sigma \to 0$), to abstract from the informational link across periods, which has been studied in the literature (Angeletos et al. 2007). Thus, the key difference between period becomes the presence of a committed rebel core.

Proposition 5 *Suppose the noise in private signals becomes vanishingly small ($\sigma \to 0$), and we focus on the largest equilibrium. Conditional on failure in the first period, the chances of success is higher in the psychological rewards setting than in the material rewards setting.*

The logic is similar to our earlier analysis. A failure in the first attempt creates a group of committed rebels. This increases the likelihood of success in both settings, but the effect is weaker in the material rewards setting because, conditional on success, the group that will share the rewards is surely larger than the size of the committed members. Combining Propositions 2 and 5 implies that movements with psychological rewards are less likely to succeed in the first period, but conditional on a failure in the first period, they are more likely to succeed than movements with material rewards. This result resonates with the finding in Shadmehr and Bernhardt (2019) that it is more difficult for a movement to begin organically (without a revolutionary vanguard); but movements that begin organically are more likely to succeed.

4 Between Purely Material and Psychological Rewards

We have compared the two extreme kinds of rewards, purely material or purely psychological. But as Kennedy (1999) aptly puts in his study of "the rumbles of discontent" during the Great Depression, people "can subsist on solely spiritual nourishment little longer than they can live on bread alone" (p. 218). We now analyze movements in which incentives to rebel are a combination of material and psychological rewards. We generalize our payoffs in Figure 1 by adding a parameter $\bar{m} \in [0, 1]$ that generates our purely material rewards setting in one extreme ($\overline{m} = 0$) and our purely psychological rewards setting in the other extreme ($\overline{m} = 1$). Figure 2 shows the payoffs—as in the previous section, we normalize the population size to 1.

Interestingly, this middle ground can also be thought of as representing a movement where material rewards are imperfectly divisible, such as promises to hold future government office. Suppose there are a total of $\overline{m}$ offices available. If the rebellion is small, $m < \overline{m}$, and succeeds, each participant in the rebellion gets an office. But there are too many offices for the rebels to fill all of them. So some offices must be left in the hands of their previous holders. (Think of a small rebel group not fully purging the bureaucracy after taking control of the state.) However, if the rebellion is large, $m > \overline{m}$, there are not enough offices to go around and the congestion externality returns.

	outcome	
	$m \geq \theta$	$m < \theta$
rebel	$\frac{1}{m}(\mathbf{1}_{\{m \leq \overline{m}\}} + \mathbf{1}_{\{m \geq \overline{m}\}} \cdot \frac{\overline{m}}{m}) - c$	$-c$
not rebel	0	0

Figure 2: Payoffs capture rents from future government offices. Official posts are available for at most $\overline{m} \in [0,1]$ fraction of the population, and if the fraction of rebels exceeds this threshold, limited offices are assigned randomly to participants.

Payoffs are the same as the psychological rewards setting when $\overline{m} = 1$ and are the same as the material rewards setting when $\overline{m} = 0$, where we recognize that $\overline{m}$ in the denominator is canceled with the one in the numerator. Proposition 6 characterizes the unique equilibrium. Figure 3 illustrates the result.

Proposition 6 *Let θ^* be the equilibrium regime change threshold in the setting with material rewards in the form of future government office rents. Then,*

$$\theta^* = \begin{cases} e^{-c} & ; \overline{m} \leq e^{-c} \\ \\ \overline{m}\ (1 - c - \log(\overline{m})) & ; \overline{m} \geq e^{-c}. \end{cases}$$

Moreover, $\theta^m > \theta^*(\overline{m}) > \theta^p$ for $\overline{m} \in (0,1)$, with $\lim_{\overline{m} \to 0} \theta^* = \theta^m$ and $\lim_{\overline{m} \to 1} \theta^* = \theta^p$.

Proposition 6 and Figure 3 show that when material rewards are in the form of future government offices, effects of repression, the size of the pool of potential recruit, and early failure are in between those effects in the settings with material and psychological rewards analyzed earlier. The key is the discreteness of rewards. While in the pure material rewards setting, the total rewards are divisible, the value of future government offices are not as easily divisible. Therefore, if the number of participants is smaller than the number of offices, an increase in participation in the rebellion does not diminish the rewards an individual enjoys should they success. For example, if 1000 offices are available, whether 600 or 800 citizens rebel, there are enough offices for each to get one. This feature shares the non-rival aspect of the psychological rewards setting. However, if the number of participants exceed 1000, further increases in the

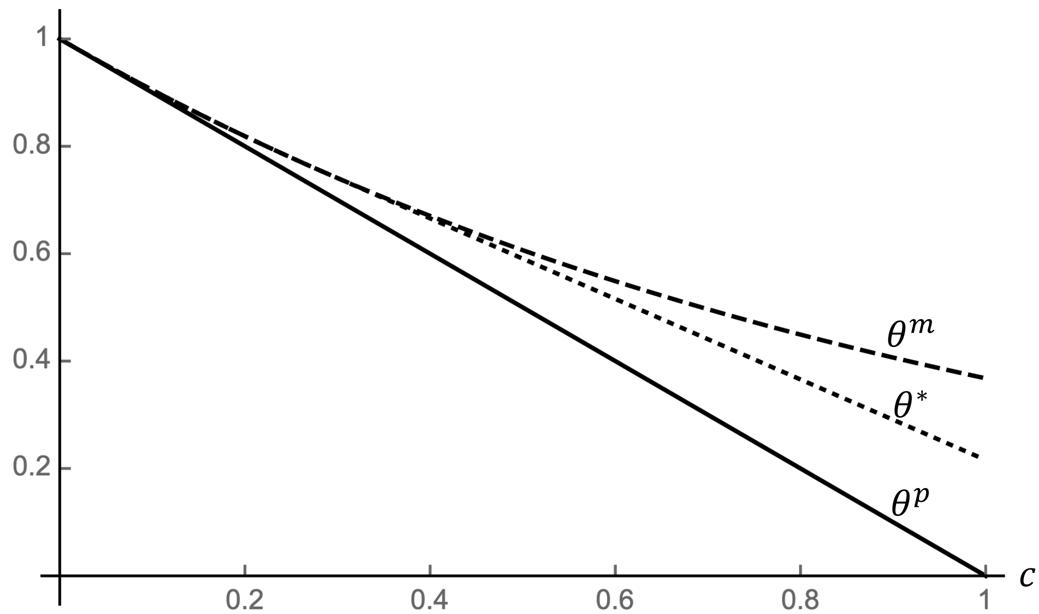

Figure 3: The equilibrium regime change threshold for the settings with pure psychological rewards (solid line, θ^p), pure material rewards (dashed curve, θ^m), and rewards in the form of government offices (dotted curve, θ^*). Parameters: $\overline{m} = 0.75$.

number of participants reduces the chances that each rebel receives an office upon success because there will not be enough government offices to go around. This feature shares the rival aspect of the material rewards setting.

The real world, of course, is not so clear cut. More offices can be created and responsibilities may be shared. However, the insight that government offices tend to be more discrete than, for example, cash, diamonds, or land remains true. As such, in settings where such offices are the main reward of victory, the effect of changes to the environment on the rebel movement fall between the effects in settings with pure (continuous) material rewards and settings with psychological rewards.

5 Conclusion

We explored how different motivations for participation affect how collective action movements respond to changes in the environment. While movements with material rewards are less affected by repression and decreases in the pool of potential recruits, movements with psychological rewards can better exploit an increase in the supply of potential recruits and are more resilient to early failures. A key ingredient of the underlying logic is the rivalry of material rewards versus the non-rivalry of psychological rewards—the other ingredient is strategic uncertainty as Propositions 1 and 2 reveal. Because material rewards must be shared upon success, changes that increase participation (e.g., new pool of potential recruits or the emergence of a committed group) have less influence on movements with material motives: such changes increase the likelihood of success, but by increasing participation they also reduce the rewards of success in movements with material motives. Conversely, changes like higher repression or a diminution in the pool of recruits are less damaging to movements with material motives: they reduce the likelihood of success, but this effect is partly counteracted with material motives because lower participation means that the rewards of success will be shared among a smaller group. Moreover, we showed that when rewards are in the form of future rents from government offices, the nature of incentives and the effects of changes to the environment fall in between the extremes of purely psychological and purely material incentives.

These insights have policy implications. Policy makers should be more concerned about the size of the potential recruitment pool when confronting movements with psychological and ideological motives rather than material motives. They should also be more cautious about the long-run efficacy of early victories when facing a group whose members are psychologically or ideologically motivated as they are more resilient to such early failures than are movements using primarily material motives. By contrast, policy makers should recognize the movements motivated by material reward are more resilient to increased costs of participation—whether the result of repression, law enforcement, or economic opportunity costs. However, it is often easier to reduce material rewards than to discredit ideologies or change individual's sense of injustice or vengeance. Combining these observations suggests the optimal policy that aims to reduce the chances of success are qualitatively different across movements whose members differentially motivated. When dealing with movements whose members are motivated by psychological or ideological rewards, the focus should be on the raising the costs of participation. In contrast, in dealing with movements whose members are materially motivated, the focus should be on reducing the material rewards that participants hope to gain should the movement succeed.

References

Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan. 2007. "Dynamic Global

Games of Regime Change: Learning, Multiplicity and Timing of Attacks." *Econometrica* 75: 711–756.

Aytaç, Erdem, and Susan Stokes. 2019. *Why Bother?: Rethinking Participation in Elections and Protests.* New York, NY: Cambridge University Press.

Blattman, Christopher, and Edward Miguel. 2010. "Civil War." *Journal of Economic Literature* 48: 3–57.

Boix, Carles, and Milan Svolik. 2013. "The Foundations of Limited Authoritarian Government: Institutions and Power-Sharing in Dictatorships." *Journal of Politics* 75: 300–16.

Bursztyn, Leonardo, Davide Cantoni, David Y. Yang, Noam Yuchtman, and Y. Jane Zhang. 2019. "Persistent Political Engagement: Social Interactions and the Dynamics of Protest Movements." Mimeo.

Casper, Brett, and Scott Tyson. 2014. "Popular Protest and Elite Coordination in a Coup d'etat." *Journal of Politics* 76: 548–64.

Chen, Heng, Yang K. Lu, and Wing Suen. 2016. "The Power of Whispers: A Theory of Rumor, Communication and Revolution." *International Economic Review* 57: 89–116.

Davenport, Christian. 2007. "State Repression and Political Order." *Annual Review of Political Science* 10: 1–23.

Davies, James C. 1962. "Toward a Theory of Revolution." *American Sociological Review* 27: 5–19.

Diani, Mario, and Doug McAdam. 2003 *Social Movements and Networks: Relational Approach to Collective Action.* New York: Oxford University Press.

Edmond, Chris. 2013. "Information Manipulation, Coordination and Regime Change." *Review of Economic Studies* 80: 1422–58.

Ellis, Stephen. 1999. The Mask of Anarchy: The Destruction of Liberia and the Religious Dimension of an African Civil War. New York: New York University Press.

Geschwender, James. 1967. "Continuities in Theories of Status Inconsistency and Cognitive Dissonance." *Social Forces* 46: 165–7.

Gurr, Ted. 1970. *Why Men Rebel?* Princeton, NJ: Princeton University Press

Hardin, Russell. 1982. *Collective Action.* Baltimore, MD: Johns Hopkins University Press.

Humphreys, Macartan, and Jeremy Weinstein. 2008. "Who Fights? The Determinants of

Participation in Civil War." *American Journal of Political Science* 52(2): 436–55.

Lawrence, Adria. 2017. "Repression and Activism among the Arab Spring's First Movers: Evidence from Morocco's February 20th Movement." *British Journal of Political Science* 47: 699–718.

Kennedy, David M. 1999. Freedom from Fear: The American People in Depression and War, 1929-1945. New York: Oxford University Press.

Loeper, Antoine, Jakub Steiner, and Colin Stewart. 2014. "Influential Opinion Leaders." *Economic Journal* 124: 1147–67.

McAdam, Doug. 1999. *Political Process and the Development of Black Insurgency, 1930-1970. 2nd Ed.* Chicago: University of Chicago Press.

McAdam, Doug, Sidney Tarrow, and Charles Tilly. 2001. *Dynamics of Contention.* New York: Cambridge University Press.

Middlekauff, Robert. 2005. *The Glorious Cause: The American Revolution, 1763–1789.* New York, NY: Oxford University Press.

Morris, Stephen, and Mehdi Shadmehr. 2017. "Inspiring Regime Change." Mimeo.

Morris, Stephen and Hyun Song Shin. 1998. "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks." *American Economic Review* 88 (3): 587–597.

Morris, Stephen, and Hyun Song Shin. 2003. "Global Games: Theory and Application." In *Advances in Economics and Econometrics, Theory and Applications, Eighth World Congress, Volume I,* edited by M. Dewatripont, L. P. Hansen, and S. J. Turnovsky. New York: Cambridge University Press.

Pearlman, Wendy. 2018. "Moral Identity and Protest Cascades in Syria." *British Journal of Political Science* 48: 877–901.

Pearlman, Wendy. 2020. "Mobilizing From Scratch: Large-Scale Collective Action Without Preexisting Organization in the Syrian Uprising." *Comparative Political Studies. Forthcoming.*

Petersen, Roger D. 2001. *Resistance and Rebellion: Lessons from Eastern Europe.* New York: Cambridge University Press.

Rundlett, Ashlea, and Milan Svolik. 2016. "Deliver the Vote! Micromotives and Macrobehavior in Electoral Fraud." *American Political Science Review* 110: 180–97.

Schelling, Thomas C. 1978. *Micromotives and Macrobehavior.* New York: Norton.

Shadmehr, Mehdi. 2017. "Khomeini's Theory of Islamic State and the Making of the Iranian Revolution." Mimeo.

Shadmehr, Mehdi. 2019a. "Multiplicity and Uniqueness in Regime Change Games." *Journal of Politics* 81: 303-8.

Shadmehr, Mehdi. 2019b. Investment in the Shadow of Conflict: Globalization, Capital Control, and State Repression. *American Political Science Review* 113: 997–1011.

Shadmehr, Mehdi, and Dan Bernhardt. 2017. "When Can Citizen Communication Hinder Successful Revolution?" *Quarterly Journal of Political Science* 12: 301–23.

Shadmehr, Mehdi, and Dan Bernhardt. 2019. "Vanguards in Revolution." *Games and Economic Behavior* 115: 146–66.

Shadmehr, Mehdi, and Raphael Boleslavsky. 2019. International Pressure, State Repression and the Spread of Protest. *Journal of Politics. Forthcoming.*

Siegel, David. 2011. "When Does Repression Work? Collective Action in Social Networks." *Journal of Politics* 73: 993–1010.

Tyson, Scott, and Alastair Smith. 2018. "Dual-Layered Coordination and Political Instability: Repression, Cooptation, and the Role of Information." *Journal of Politics* 18: 44–58.

Tarrow, Sidney. 2011. *Power in Movement: Social Movements and Contentious Politics. 3rd Ed.* New York: Cambridge University Press.

Tilly, Charles. 1978. *From Mobilization to Revolution.* Reading, MA: Addison-Wesley.

Tilly, Charles. 2006. *Regimes and Repertoires.* Chicago: University of Chicago Press.

Tilly, Charles. 2008. *Contentious Performances.* New York: Cambridge University Press.

Weinstein, Jeremy. 2007. *Inside Rebellion: The Politics of Insurgent Violence.* New York, NY: Cambridge University Press.

Wood, Elisabeth J. 2003. *Insurgent Collective Action and Civil War in El Salvador.* New York, NY: Cambridge University Press.

6 Appendix: Proofs

Proof of Proposition 2: To complete the proof in the text, we finish the characterization of the equilibrium in the material rewards setting. Let $z(\hat{\theta}) = Pr(\theta < \hat{\theta}|x_i = x^m)$, with $z^m = Pr(\theta < \theta^m|x_i = x^m)$. Because there is no prior common knowledge about θ: $Pr(\theta < \theta^m|x_i = x^m) = 1 - Pr(x_i < x^m|\theta = \theta^m)$. Thus, $1 - z^m = \theta^m/a$. Moreover, the left hand side of the inequality (4) can be re-written in terms of z, so that the indifference condition becomes:

$$\int_{\theta=-\infty}^{\theta^m} \frac{pdf(\theta|x_i = x^m)}{Pr(x_j < x^m|\theta)} d\theta = \int_{z=0}^{z^m} \frac{dz}{1-z} = -\log(1-z)\Big|_{z=0}^{z^m} = -\log(1-z^m) = c. \quad (10)$$

Because $1 - z^m = \theta^m/a$, this shows $\theta^m = ae^{-c}$. It remains to show that the best response to a monotone strategy is also monotone. Let $\pi(\theta) = \mathbf{1}_{\{\theta<\theta^m\}} \cdot \frac{1}{Pr(x_j<x^m|\theta)} - c$. If $c \in (0,1)$, then $\pi(\theta)$ has a single-crossing property. Because (x, θ) satisfy the monotone likelihood ratio property, by Karlin's theorem (Shadmehr 2019b, Online Appendix), the single-crossing property holds under the integral transformation of equation (4), and the best response to a monotone strategy is also monotone. This means that the marginal citizen with signal $x_i = x^m$ must be indifferent between rebelling or not. □

Proof of Proposition 4: In the psychological rewards setting, the equilibrium conditions are:

$$(b + a)Pr(x_i < x_r^p|\theta_r^p) = \theta_r^p \quad \text{and} \quad Pr(\theta < \theta_r^p|x_i = x_r^p) = c.$$

Thus, from proposition 2, $\theta_r^p = (b + a)(1 - c)$, which is increasing in b. In the material rewards setting, the belief consistency condition is:

$$(b + a)Pr(x_i < x_r^m|\theta_r^m) = \theta_r^m.$$

A citizen with signal x_i rebels if and only if:

$$\int_{\theta=-\infty}^{\theta_r^m} \frac{a\ pdf(\theta|x_i)}{(b + a)Pr(x_j < x_r^m|\theta)} d\theta - c > 0.$$

Let $\pi_r(\theta) = \mathbf{1}_{\{\theta<\theta_r^m\}} \cdot \frac{a}{(a+b)Pr(x_j<x_r^m|\theta)} - c$. By Assumption 1, $c < \frac{a}{a+b}$. Hence, $\pi_r(\theta)$ has a single-crossing property, and hence, as in the proof of Proposition 2, the best response to a monotone strategy is monotone. Moreover, using the same change of variables as in the proof of Proposition 2, the indifference condition is:

$$c = \int_{\theta=-\infty}^{\theta_r^m} \frac{a\ pdf(\theta|x_i = x_r^m)}{(b + a)Pr(x_j < x_r^m|\theta)} d\theta = \frac{a}{a+b}\int_{z=0}^{z_r^m} \frac{dz}{1-z} = -\frac{a}{a+b}\log(1-z)\Big|_{z=0}^{z_r^m} = -\frac{a}{a+b}\log(1-z_r^m).$$

Moreover, $1 - z_r^m = 1 - Pr(\theta < \theta_r^m | x_r^m) = Pr(x_i < x_r^m | \theta_r^m) = \frac{\theta_r^m}{a+b}$. Thus,

$$\frac{a}{a+b}\log(1 - z_r^m) = \frac{a}{a+b}\log\left(\frac{\theta_r^m}{a+b}\right) = -c.$$

Thus,

$$\theta_r^m = (a+b)e^{-c\frac{a+b}{a}}.$$

$\frac{\partial \theta_r^m}{\partial b} > 0$ for $c < a/(a+b)$, which is true by Assumption 1. Showing $\theta_r^p - \theta^p > \theta_r^m - \theta^m$ is equivalent to showing $b(1-c) - (a+b)e^{-c\frac{a+b}{a}} - ae^{-c} > 0$. Letting $z = b/a > 0$, this inequality is equivalent to $(1-c) > \frac{(1+z)e^{-(1+z)c} - e^{-c}}{z}$, for $c < 1/(1+z)$. Now, $(1-c) > e^{-c}(1-c) = \lim_{z \to 0} \frac{(1+z)e^{-(1+z)c} - e^{-c}}{z} \geq \frac{(1+z)e^{-(1+z)c} - e^{-c}}{z}$, where the last inequality follows because $(1+z)e^{-(1+z)c}$ is concave in z for $c < 1/(1+z)$. $\square$

Proof of Proposition 5: First, we prove a lemma that characterizes finite cutoff equilibria.

Lemma 1 *Let θ_2^m be an equilibrium regime change threshold for the setting with material rewards, let θ_2^p be the corresponding threshold for the setting with psychological rewards, and recall that $\theta_1^m = e^{-c}$ and $\theta_1^p = 1 - c$. θ_2^m is an equilibrium regime change threshold of the material rewards setting if and only if is satisfies*

$$\theta_2^m = \left(1 - a + aF\left(\frac{\theta_2^m - \theta_1^m}{\sigma} + F^{-1}\left(\frac{\theta_2^m - (1-a)}{a}\right)\right)\right) e^{-ac\ F\left(\frac{\theta_2^m - \theta_1^m}{\sigma} + F^{-1}\left(\frac{\theta_2^m - (1-a)}{a}\right)\right)} \quad (11)$$

and θ_2^p is an equilibrium regime change threshold of the psychological rewards setting if and only if is satisfies

$$\theta_2^p = 1 - a + a(1-c)F\left(\frac{\theta_2^p - \theta_1^p}{\sigma} + F^{-1}\left(\frac{\theta_2^p - (1-a)}{a}\right)\right). \quad (12)$$

Proof of Lemma 1: We first consider the material rewards setting. From equation (7), the marginal citizen's net expected payoff from rebellion is:

$$E\left[\frac{\mathbf{1}_{\{\theta<\theta_2^m\}}}{1-a+aPr(x_j<x_2^m)}\middle| x_i=x_2^m,\theta>\theta_1^m\right]$$

$$= \int_{\theta_1^m}^{\theta_2^m}\frac{1}{1-a+aPr(x<x_2^m|\theta)}\frac{pdf(\theta|x_2^m)}{Pr(\theta>\theta_1^m|x_2^m)}d\theta$$

$$= \int_{\theta_1^m}^{\theta_2^m}\frac{1}{1-a+aF\left(\frac{x_2^m-\theta}{\sigma}\right)}\frac{f\left(\frac{x_2^m-\theta}{\sigma}\right)g(\theta)}{\int_{\theta_1^m}^{\infty}f\left(\frac{x_m-\theta}{\sigma}\right)g(\theta)d\theta}\,d\theta \quad \text{(let } g(\theta) \text{ be the prior pdf of } \theta\text{)}$$

$$= \frac{\int_{\theta_1^m}^{\theta_2^m}\frac{f\left(\frac{x_2^m-\theta}{\sigma}\right)}{1-a+aF\left(\frac{x_2^m-\theta}{\sigma}\right)}d\theta}{\int_{\theta_1^m}^{\infty}f\left(\frac{x_2^m-\theta}{\sigma}\right)d\theta} \quad (g(\theta)=1 \text{ for uniform})$$

$$= \frac{\frac{1}{a}\log\left(\frac{1-a+aF\left(\frac{x_2^m-\theta_1^m}{\sigma}\right)}{1-a+aF\left(\frac{x_2^m-\theta_2^m}{\sigma}\right)}\right)}{F\left(\frac{x_2^m-\theta_1^m}{\sigma}\right)}$$

$$= \frac{1}{a}\frac{\log\left(\frac{1-a+aF\left(\frac{x_2^m-\theta_1^m}{\sigma}\right)}{\theta_2^m}\right)}{F\left(\frac{x_2^m-\theta_1^m}{\sigma}\right)} \quad \text{(from equation (6)).} \tag{13}$$

Substituting from (13) into (7) yields:

$$\log\left(1-a+aF\left(\frac{x_2^m-\theta_1^m}{\sigma}\right)\right)-\log(\theta_2^m)=acF\left(\frac{x_2^m-\theta_1^m}{\sigma}\right). \tag{14}$$

Substituting x_2^m from (6) into (14) yields:

$$\theta_2^m=\left(1-a+aF\left(\frac{\theta_2^m-\theta_1^m}{\sigma}+F^{-1}\left(\frac{\theta_2^m-(1-a)}{a}\right)\right)\right)\ e^{-ac\ F\left(\frac{\theta_2^m-\theta_1^m}{\sigma}+F^{-1}\left(\frac{\theta_2^m-(1-a)}{a}\right)\right)}. \tag{15}$$

Similarly for the psychological rewards setting, from equation (9), the marginal citizen's net expected payoff from rebellion is:

$$E\left[\mathbf{1}_{\{\theta<\theta_2^p\}}\middle| x_i=x_2^p,\theta>\theta_1^p\right]=\frac{F\left(\frac{x_2^p-\theta_1^p}{\sigma}\right)-F\left(\frac{x_2^p-\theta_2^p}{\sigma}\right)}{F\left(\frac{x_2^p-\theta_1^p}{\sigma}\right)}. \tag{16}$$

Thus, any (θ_2^p,x_2^p) that satisfies the following equations constitutes an equilibrium:

$$1-a+aF\left(\frac{x_2^p-\theta_2^p}{\sigma}\right)=\theta_2^p \quad \text{and} \quad \frac{F\left(\frac{x_2^p-\theta_1^p}{\sigma}\right)-F\left(\frac{x_2^p-\theta_2^p}{\sigma}\right)}{F\left(\frac{x_2^p-\theta_1^p}{\sigma}\right)}=c. \tag{17}$$

Substituting x_2^m from the belief consistency condition into the indifference condition yields:

$$\theta_2^p = 1 - a + a(1-c)F\left(\frac{\theta_2^p - \theta_1^p}{\sigma} + F^{-1}\left(\frac{\theta_2^p - (1-a)}{a}\right)\right). \tag{18}$$

□

Lemma 2

$$\lim_{\sigma\to 0}\max\{\theta_2^p(\sigma)\} = 1 - a + a(1-c) \quad \textit{and} \quad \lim_{\sigma\to 0}\max\{\theta_2^m(\sigma)\} = e^{-ac}.$$

Proof of Lemma 2: Suppose $1 - a < 1 - c$. The other case is simpler because there will be only two equilibria: the one we will focus on and the one with $x_2^m = -\infty$. We have:

$$\lim_{\sigma\to 0} F\left(\frac{\theta_2^m - \theta_1^m}{\sigma} + F^{-1}\left(\frac{\theta_2^m - (1-a)}{a}\right)\right) = \begin{cases} 1 & ;\theta_2^m > \theta_1^m = e^{-c} \\ 0 & ;\theta_2^m < \theta_1^m = e^{-c}. \end{cases}$$

Thus, when $\theta_2^m > \theta_1^m = e^{-c}$, the right hand side of (11) is about $e^{-ac} > e^{-c}$, so that the largest crossing of the 45 degree line is around e^{-ac}:

$$\lim_{\sigma\to 0}\max\{\theta_2^m(\sigma)\} = e^{-ac}.$$

Similarly,

$$\lim_{\sigma\to 0}\max\{\theta_2^p(\sigma)\} = 1 - a + a(1-c).$$

□

Lemma 2 reflects that when noise in private signals is small, citizens discard their public information. Thus, the informational channel is shut down in the limit. From Lemma 2 and Proposition 2,

$$\Delta^m = \lim_{\sigma\to 0}\max\{\theta_2^m(\sigma)\} - \theta_1^m = e^{-ac} - e^{-c} \quad \text{and} \quad \Delta^p = \lim_{\sigma\to 0}\max\{\theta_2^p(\sigma)\} - \theta_1^p = (1-a)c. \tag{19}$$

$\Delta^m < \Delta^p$ because e^{-x} is decreasing and convex. □

Proof of Proposition 6: The net payoff from rebelling versus not is:

$$\frac{1}{\overline{m}}\left(\mathbf{1}_{\{\theta<m,m\leq\overline{m}\}} + \mathbf{1}_{\{\theta<m,m\geq\overline{m}\}} \cdot \frac{\overline{m}}{m}\right) - c \tag{20}$$

This net payoff is non-monotone in the fraction of rebels m. It jumps up at $m = \theta$ (the threshold at which regime change succeeds), but then falls, weakly is some range and strictly in others, as more citizens join the movement.

As before, given a value of θ, the fraction of rebels is $m(\theta) = Pr(x < x^*|\theta)$, and $Pr(x_i < x^*|\theta^*) = \theta^*$. Moreover, $m(\theta) < \overline{m}$ if and only if $\theta > \bar{\theta}$, where $Pr(x_i < x^*|\bar{\theta}) = \overline{m}$. Then, the net expected payoff from rebellion versus not is:

$$\int_{\theta=-\infty}^{\infty} \frac{1}{\overline{m}} \left(\mathbf{1}_{\{\theta<\theta^*,\theta\geq\bar{\theta}\}} + \mathbf{1}_{\{\theta<\theta^*,\theta\leq\bar{\theta}\}} \cdot \frac{\overline{m}}{Pr(x_i < x^*|\theta)} \right) \, pdf(\theta|x_i) - c. \tag{21}$$

As before, if $c < \min\{1, 1/\overline{m}\} = 1$, we can invoke the Karlin's Theorem to conclude to that the best response to a monotone strategy is also monotone. The indifference condition is:

$$\int_{\theta=-\infty}^{\infty} \left(\mathbf{1}_{\{\theta<\theta^*,\theta\geq\bar{\theta}\}} + \mathbf{1}_{\{\theta<\theta^*,\theta\leq\bar{\theta}\}} \cdot \frac{\overline{m}}{Pr(x_i < x^*|\theta)} \right) \, pdf(\theta|x_i = x^*) = \overline{m}\ c. \tag{22}$$

First, suppose $\bar{\theta} > \theta^*$. Then,

$$\int_{\theta=-\infty}^{\infty} \mathbf{1}_{\{\theta<\theta^*\}} \cdot \frac{\overline{m}}{Pr(x_i < x^*|\theta)} \, pdf(\theta|x_i = x^*) = \overline{m}\ c. \tag{23}$$

Thus,

$$\theta^* < \bar{\theta} \;\Rightarrow\; \theta^* = e^{-c}, \tag{24}$$

where we recognize that $\bar{\theta}$ is endogenous and depends on x^*. However, recall that $Pr(x < x^*|\bar{\theta}) = \overline{m}$ and $Pr(x < x^*|\theta^*) = \theta^*$. Thus, $\theta^* < \bar{\theta}$ is equivalent to $\theta^* > \overline{m}$. Given (24), $\theta^* > \overline{m}$ is equivalent to: $-c > \log(\overline{m})$.

Next, suppose $\bar{\theta} < \theta^*$, i.e., $\theta^* < \overline{m}$. Then,

$$\begin{aligned} \overline{m}\ c &= \int_{\theta=-\infty}^{\bar{\theta}} \frac{\overline{m}}{Pr(x_i < x^*|\theta)} \, pdf(\theta|x_i = x^*) d\theta + \int_{\bar{\theta}}^{\theta^*} pdf(\theta|x_i = x^*) d\theta \\ &= -\overline{m}\ \log(1 - Pr(\theta < \bar{\theta}|x_i = x^*)) + Pr(\theta < \theta^*|x_i = x^*) - Pr(\theta < \bar{\theta}|x_i = x^*). \end{aligned} \tag{25}$$

Substituting for $Pr(x_i < x^*|\bar{\theta}) = \overline{m} = 1 - Pr(\theta < \bar{\theta}|x_i = x^*)$ and $Pr(\theta < \theta^*|x_i = x^*) = 1 - \theta^*$ yields $-\overline{m}\ \log(\overline{m}) + \overline{m} - \theta^* = \overline{m}\ c$, i.e.,

$$\theta^* = \overline{m}\ (1 - \log(\overline{m})) - \overline{m}\ c. \tag{26}$$

Thus, $\theta^* < \overline{m}$ if and only if $-c < \log(\overline{m})$.

Combining these results yield:

$$\theta^* = \begin{cases} e^{-c} & ; c \leq -\log(\overline{m}) \\ \overline{m}\ (1 - c - \log(\overline{m})) & ; c \geq -\log(\overline{m}) \end{cases} \tag{27}$$

We observe that

$$\left.\frac{d\theta^*(c)}{dc}\right|_{c=-\log(\overline{m})} = -\overline{m}.$$

□