Demagogues and the Fragility of Democracy

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Abstract

Our paper investigates the long-run susceptibility of Democracy to demagogues, studying the tension between far-sighted, magnanimous representatives who guard the long-run interests of voters, and office-seeking demagogues who cater to voters' short-run desires. We model the political decision process as a capital/social capital investment problem in which parties propose how to allocate existing resources between current consumption and investment. Voters with CRRA preferences base political choices on a comparison of the current period utility derived from policies and a stochastic valence shock. With log utility, we establish that the benevolent party's investment converges to zero when the depreciation rate goes to one, regardless of the valence advantage, even though, absent political competition from the demagogue, the economy would grow forever. When the coefficient of relative risk aversion exceeds one, we establish that, regardless of the current capital stock there is always a positive probability of entering a death spiral in which once capital falls below a critical level, it continues inevitably downward toward zero.

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1 Introduction

“The republican principle,” wrote Hamilton in Federalist No. 71, “does not require an unqualified complaisance to every sudden breeze of passion, or to every transient impulse which the people may receive from the arts of men, who flatter their prejudices to betray their interests.”

Quite the opposite, Hamilton argued: “When occasions present themselves, in which the interests of the people are at variance with their inclinations, it is the duty of the persons whom they have appointed to be the guardians of those interests, to withstand the temporary delusion.... conduct of this kind has saved the people from very fatal consequences of their own mistakes, and procured ...their gratitude to the men who had courage and magnanimity enough to serve them at the peril of their displeasure.” Still, should such magnanimous representatives cause too much displeasure, they would lose the election to those who will implement those “transient impulses” and pursue those “temporary delusions” full-force, paying “obsequious court to the people; commencing demagogues, and ending tyrants.”

Our paper studies the tension highlighted by Hamilton, between far-sighted, magnanimous representatives who guard the long-run interests of voters, and office-seeking demagogues who cater to voters’ short-run desires. In particular, we are interested in the long-run outcomes of democracy in a country populated by a short-sighted majority. Demagogues and short-sighted voters have long been considered inter-related vices of republican governments. For example, “Madison’s [belief] about democracy was based on [one] about human beings: man, by nature, preferred to follow his passion rather than his reason; he invariably chose short-term over long-term interests" (Wood (1969)). Researchers find this observation so self-evident that they define demagogues by this characteristic. For example, according to Guiso et al. (2018) parties led by demagogues “champion short-term policies while hiding their long-term costs by using anti-elite rhetoric to manipulate beliefs.”[1] What is not well-understood is how demagogues distort the behavior of far-sighted parties, and how a democratic country emerges from the long-run confrontation between selfish demagogues and socially benevolent, but pragmatic parties.

We analyze the dynamic political competition between two parties, a far-sighted, benevolent

[1]Guiso et al. (2018) use the term populists. Historically, populists were referred as demagogues but now these terms are often used interchangeably.

party that seeks to maximize voter welfare, and a demagogue, how is purely office-motivated and only cares about winning. The two parties and a representative short-sighted median voter interact repeatedly over time in an infinite horizon setting. To capture tensions between short- and long-term considerations, we model the political decision process as an investment problem in which parties propose how to allocate existing resources between current consumption and investment in capital that facilitates greater future consumption. The investment technology exhibits constant returns to scale, and capital depreciates at a constant rate, so absent sufficient investment, the capital stock and hence future consumption fall.

Each period, the benevolent party and demagogue propose investments (and hence consumption) for that period. The voter, who has CRRA preferences assesses proposed investment policies solely on the basis of the utility that he would derive in the current period. In addition to their policies, parties are distinguished by their valence, which captures other electoral concerns. The net valence is uncertain and its support is such that even if both parties choose the same policy, the demagogue, who may typically have a low valence reflecting the opprobrium with which demagogues are generally viewed, still has at least a slim chance of winning the election. After parties propose investment policies, valences are realized, and the median voter picks his preferred candidate. The winner's policy is implemented, period payoffs are realized, and the game proceeds to the next period with a capital stock consisting of the depreciated capital plus any new investment.

Absent a demagogue, the benevolent party would act like a social planner. Investments would more than compensate for capital depreciation, and capital stocks would grow without bound over time. Left on its own, the benevolent party would internalize voters' utility from future consumption, winning the race between the production and destruction of capital. Its investment policy would reflect the primitives of the economy in natural ways, for example, by reducing investments as the depreciation rate rises, because less capital needs to be replaced.

Demagogues, even ones who are likely to have low valences, change all that. Demagogues propose all consumption and no investment to appease short-sighted voters in order to maximize their chances in the election. Now, the benevolent party faces a dilemma. It can ignore the demagogue in its policy choices, but it does so at its own peril by losing electoral support. Alternatively, it can appropriate the demagogue's political agenda, but "trying to beat a populist insurgency by becoming one turns out to be a fool's errand ... [as it] has a huge .... economics cost."[2] In our model, the benevolent party must weigh the following two considerations. (1) The party can propose greater investment and lower current consumption that would benefit the voter in the long run *if* the benevolent party wins—but if the demagogue wins the election, there won't be any investment at all, and the voter views policies with less current consumption more negatively. (2) The party can increase its chances of winning and thus the chance of implementing a policy that is better than what the demagogue would offer by shifting its platform toward increased current consumption in exchange for reduced investment.

The policy choices of demagogue Huey Long and the response by President Roosevelt illustrate the benevolent party's dilemma. In the midst of the Great Depression, Long proposed a high progressive tax, and distributing the revenue to every American family, 5,000 dollars each, supposedly enough for a home, a car, and a radio. In addition, each family would be guaranteed an annual income of 2,500 dollars, topped off by shorter working hours, better veterans' benefits, education subsidies, and pensions. FDR's assessment was that "[Long] thinks he will have a hundred voted in the Democratic convention. Then he will set up as an independent with Southern and mid-western Progressives.... Thus, he hopes to defeat the Democratic party and put in a reactionary Republican. That would bring the country to such a state by 1940 that Long thinks he would be made dictator. *Thus, it is an ominous situation.*" FDR responded by proposing a Second New Deal that included a Wealth Tax Act designed.... "to save our system" from the "crackpot ideas" of Long and other demagogues. FDR recognized that his policy proposal was bad for the economy, but "I am fighting Communism, Huey Longism, Coughlinism, Townsendism," indicating his belief that the consequences of losing were worse.

We first consider voters who have logarithmic utility. We establish that the benevolent party's equilibrium probability of winning an election does not depend on the level of the capital stock. In turn, this lets us derive the benevolent party's value function, and then solve for the equilibrium. We establish that the benevolent party proposes to save a constant share of capital, $\lambda(\rho)$, where ρ is the fraction of capital that does not depreciate (i.e., $1-\rho$ is the depreciation rate).

One expects that political competition from the demagogue would induce the benevolent

[2] NY times Op Ed, https://www.nytimes.com/2018/07/16/opinion/to-defeat-far-right-nationalists-dont-try-to-imitate-them.html

party to adjust its proposed investment downward to increase its probability of winning and thus being able to implement its policy. What is surprising is the extent to which the benevolent party may mimic even a very unpopular demagogue who has little chance of winning. Indeed, we find that the benevolent party's investment policy $\lambda(\rho)$ converges to zero with ρ, *regardless* of the demagogue's expected valence disadvantage. In contrast, absent competition from the demagogue, the benevolent party's investment is highest rather than lowest when ρ is small, and it is always bounded away from zero. The benevolent party may be very likely to win the election, but by almost co-opting the demagogue's policy, an economic meltdown occurs as the capital stock converges to zero in the long run. We refer to this phenomenon as a "death spiral."

We identify necessary and sufficient conditions for death spirals to occur with probability one. We prove that there exist values of ρ such that a death spiral occurs even though there are feasible investment policies such that after accounting for competition from the demagogue, the economy would grow forever with probability one. With normally-distributed valences, absent a valence advantage for the responsible party, death spirals emerge unless ρ is quite high.

Even when ρ is large enough that death spirals are avoided, the demagogue's welfare impact can be large. For example, with a normal distribution of valence that gives the benevolent party a two standard deviation advantage, $\lambda(\rho)$ increases in ρ and the gap with the no-competition benchmark (which decreases in ρ) remains almost 50% even at $\rho = 1$, so capital does not depreciate. Indeed at $\rho = 1$, the welfare loss, measured by the associated certainty equivalent consumption level, is almost 70%. Even when the demagogue has a completely implausible valence disadvantage of seven standard deviations, i.e., the demagogue is about 1/1000 times less likely to win the election than winning the Powerball lottery if both choose the same platform, there remains a welfare shortfall of about 2% when $\rho = 0.4$ and 1% when $\rho = 0.6$.

Our initial analysis presumes that the demagogue only cares about winning in the current period, but the results extend when the demagogue is forward looking. We prove that the unique linear Markov equilibrium corresponds to the equilibrium with the myopic demagogue, as does the unique equilibrium of the limiting finite horizon economy.

The infinite horizon economy can also support equilibria in which both the demagogue and benevolent party propose increased investment policies such that the demagogue is more likely to win, using threats to revert to the 'myopic equilibrium'. However, these equilibria involve implausible levels of cooperation. Moreover, there are surprising limits to the feasible extent of cooperation. In particular, given a minimal equilibrium refinement that the benevolent party want its policy implemented rather than the demagogue's, we prove that no matter how high the discount factor is, such trigger strategies can never support equilibrium outcomes that are close to the first best that obtains in the demagogue's absence.

We then consider CRRA utilities with degrees of relative risk aversion that exceed one, consistent with estimates from macroeconomic models. Risk aversions that exceed one amplify the adverse consequences of a low capital stock—when consumption becomes small, utility goes to $-\infty$ at a faster rate. As a consequence, the benevolent party's optimal investment ceases to be linear in capital, and the probability of winning depends on capital stocks. While one can no longer determine the functional form of the value function, we identify a lower bound on the marginal benefit of raising investment. This bound lets us characterize the benevolent party's investment behavior and the economy's long-run fate. Using this bound, we establish that there exists some threshold $\bar{k}$ such that if capital stocks ever fall below $\bar{k}$, then the economy enters a self-enforcing death spiral that leads to zero capital stock and consumption in the long run. Thus, we get the paradoxical result that when the benevolent party is more concerned about low consumption levels (its coefficient of risk aversion exceeds one), it mimics the demagogue's non-investment policy even more closely.

The intuition for this result is that when the capital stock and hence the period consumption are very low, the short-term marginal effects of higher investments are very negative. This creates a strong electoral pressure for the benevolent party to propose lower investments lest it lose to the even worse no-investment platform of the demagogue. If capital stocks ever fall too low, the benevolent party proposes less investment than is needed to compensate for capital depreciation in the next period even when the benevolent party wins; this vicious cycle of low capital stocks and low investment lead the economy into oblivion.

Moreover, regardless of the existing capital stock, there is always a positive probability that the capital stock decreases—the demagogue always has a chance of winning the election. In turn, this implies that an economy is always just a few bad draws away from dropping capital below $\bar{k}$ and hence from an economic meltdown. Our analysis emphasizes the shocks on the real economy generated by election outcomes. These shocks take the form of a victory by the demagogue that reduces capital.[3] This analysis also implies that, consistent with casual observation, democracies in developing economies with lower capital stocks are more susceptible to meltdowns, because fewer and smaller shocks are sufficient to drop capital below the critical level $\bar{k}$ at which meltdowns become inevitable.

This vicious downward cycle is not a typical poverty trap in which people with low income cannot produce beyond subsistence levels to invest in productivity, thereby perpetuating economic misery. A benevolent party, free from the electoral pressure of demagogues, would propose more than enough investment to compensate for capital depreciation, leading to greater long-run consumption and voter utility. It is the presence of demagogues that drives this vicious cycle. We call this the populist trap.

There is a second dimension to this populist trap. The demagogue maximizes his probability of winning the current election by overspending to maximum extent possible, thereby reducing future capital. But, reducing future capital then amplifies the utility difference between the demagogue's zero-investment policy and any fixed investment policy. Thus, paradoxically, by failing in the sense of reducing capital stocks and destroying social capital associated with established institutions and property rights, the demagogue can increase his chance of winning.

We establish an important converse to these results, characterizing how the presence of demagogues alters investment when capital is high. One might conjecture that because the value of capital is reduced by competition from the demagogue that it always induces the benevolent party to lower its investment. This conjecture is false. Not facing such dire electoral concerns as when the capital stock is low, the benevolent party proposes even more investment than he would absent the demagogue in order to insure against the possibility that the demagogue may win, which would cause capital to drop. Thus, we uncover the paradoxical result that when capital is low, the benevolent party over-spends; but when capital is high, it over-invests, choosing an austerity policy when capital is already high. For example, this result can reconcile why the far-right EKRE party in Estonia[4] "promised to slash taxes." In contrast, "the two main parties support continued austerity policies, which have left Estonia with the lowest debt level of any

[3] Of course, adding other standard macroeconomic shocks such as real business cycle shocks or wars can also drive capital below the critical threshold, $\bar{k}$.

[4] https://www.bbc.com/news/world-europe-47430993 "Estonia general election: Opposition party beats Centre rivals," March 3, 2019.

Eurozone country but have caused anger in rural communities who feel left behind."

We conclude by providing a more uplifting counterpoint to our general theme of despair. While with CRRA preferences, there is always a positive probability of a death spiral no matter how high capital stocks are, we also establish that *if* a democracy is lucky enough at the outset to grow its capital stock to a sufficient level, then there is always a positive probability that despite the demagogue's presence, the capital stock never falls below that high level.

2 Related Literature

Bisin et al. (2015) develop a three-period model with voters who use hyperbolic discounting. Voters understand their self-control issues, and can use illiquid assets to prevent overspending in the second period. Bisin et al. (2015) show how two office-motivated candidates can undo this commitment device, by using excessive debt accumulation, hurting voters. In contrast, in our model voters are unaware of the consequences of current investment on future consumption. If we had two office-motivated candidates as in Bisin et al. (2015), no investments would be made. The questions we address are whether a benevolent party with a stochastic valence advantage will implement policies that are close to the socially-desirable policies, and how deviations from the optimal policy add up over time. In other words, we investigate the extent to which Madison's notion of a political leader can be effective in the presence of a demagogue.

Guiso et al. (2018) define a party as populist if it champions short-term policies while hiding their long-term cost. Using data from European countries between 2004 and 2016 they show that hard economic times lead to increased support for populist policies. They also show that establishment parties' policies become more populist in nature. Our theoretical model assumes that demagogues can hide the long-term consequences of economic policies from voters, precisely as in Guiso et al. (2018). Consistent with their empirical findings, we show that established parties increasingly mimic populists when the threat of populism rises. Moreover, as long as risk aversion coefficients exceed one, we show that consistent with Guiso et al. (2018), following hard times, established parties become more populist in response to the increased attraction of populist parties.

Acemoglu et al. (2013) develop a two-period model of populism in which a lobby that favors the wealthy can try to bribe politicians to choose policies that favor the wealthy. In their model, populists are not susceptible to bribes and signal this by choosing extreme left-wing policies. In contrast, in our model the defining feature of demagogues is that, to maximize the probability of winning election, they champion short-term policies that appeal to short-sighted voters, consistent with Guiso et al. (2018).

To be able to distinguish between demagogues and regular politicians, one needs a model in which their preferences and hence policies differ in meaningful ways. Importantly, we show how the political process itself generates endogenous shocks to the economy. Fundamental to these shocks is that there is policy divergence, so the election outcome matters, introducing dynamic stochastic economic distortions. We are not aware of other papers that share these features. There is a limited literature that embeds a probabilistic voting model à la Persson and Tabellini (2000) in a dynamic macroeconomic model (e.g., Cukierman and Meltzer (1989), Persson and Svensson (1989), Alesina and Tabellini (1990), Song et al. (2012), Battaglini (2014)). These models feature policy convergence: the political process generates an ex-ante distortion when parties choose platforms, but with policy convergence it does not matter who wins. In contrast, in our framework it greatly matters who wins, and the election itself generates uncertainty and additional dynamic economic distortions.

Battaglini and Coate (2008) adopts a different approach. They consider the effect of legislative bargaining on government debt and public good provision in a setting where debt constrains public good investment and pork provision, and each district is represented by a legislator. A first mover advantage in bargaining means that the proposer's identity matters for which district receives the most pork, but public good investment and debt levels are unaffected. Hence, unlike our model, but like the literature cited above, the political process itself does not generate intertemporal uncertainty.

By adopting these more tractable formulations of political competition to glean other insights, this literature implicitly assumes that it doesn't matter who wins an election; in our model, it does. Indeed, in our model the election result may determine whether the economy may grow forever, or whether it spirals down into disaster. Of course, it has been well-documented that election results matter for future economic outcomes; see e.g., Kelly et al. (2016) who analyze the impact of electoral outcomes on (forward-looking) financial markets.

Demagogues, derived from the Greek (Demos + Agogos) are rabble leaders, who appeal to the people solely to win power for themselves. The term populist is today often used interchangeably, but contains elements that we do not model. For example, according to Müller (2017), populists claim to represent the true people and true citizens against some elite who controls the levers of government at the expense of the true people. As a consequence, populists believe that is legitimate to move away from pluralistic democracy, because they and only they, are the legitimate representatives of the people. In contrast, in our model, majority rule is always preserved. If, instead, the demagogue were able to change the rules underlying fair political competition, making it harder for a benevolent party to regain power, it would strengthen our results.[5] In particular, because the benevolent party's costs of losing would increase, it would have even stronger incentives to mimic populist policies.

In our model, increasing current consumption comes at the expense of decreasing future consumption. One may argue that citizens should be able to come to understand this link. In reality, this link is less clear. In particular, populist governments can borrow for longer periods of time without discernible impacts on consumption. For example, Venezuela has borrowed from Russia on claims to its future oil streams. In practice, it can take decades before the links to adverse consequences become observable. As an abstraction, we capture this by assuming that citizens are short-sighted. In a historical companion paper (Bernhardt et al., 2019), we document the concerns of founding fathers of American Democracy regarding this short-sightedness. We believe that modeling how citizens come to learn that short-sighted excessive spending beyond revenues causes severe damage, and the consequences of that learning are important. In turn, it may matter whether that learning occurs in a country like Venezuela where the rules of the game were bent by the United Socialist party, or in a country like Greece where they were not.

3 Model

Two parties $P = b, d$ choose capital investment in each period, $t = 0, 1, 2, \ldots$. There is a consumption good and a capital good. If k_{t-1} is the amount of capital in period $t-1$, then

[5] One can also interpret a demagogue's efforts to weaken democratic institutions and norms in order to achieve short-term objectives, as a form of reduced investment in the social capital of democracy and property rights, with adverse consequences for future outcomes.

$f(k_{t-1}) = \phi k_{t-1}$ is the amount of the consumption good provided in period t. The consumption good can be invested and effectively turned into the capital good: if $i_t \geq 0$ is the investment, and $0 < 1 - \rho < 1$ is the depreciation rate of capital, then the capital stock at time t becomes $k_t = \rho k_{t-1} + i_t$. A party's policy in period t is a proposed investment level i_t.

The median voter's utility in period t is $u(c_t) + \upsilon_P$, where $c_t = f(k_{t-1}) - i_t$, and υ_P is a valence shock that measures the utility the voter derives if party P is in power. We focus on constant relative risk aversion utility preferences over consumption, i.e., $u(c) = c^{1-s}/(1-s)$ for $s > 1$, and $u(c) = \log(c)$ for relative risk aversion $s = 1$.[6] We interpret $\upsilon_{P,t}$, $P = b, d$ as measuring other, non-economic policy aspects that are out of the control of the party. Without loss of generality we assume that $\upsilon_{b,t} = 0$ and write υ_t instead of $\upsilon_{d,t}$. The median voter is myopic, and bases electoral decision solely on period utility. This captures the idea that voters are not sophisticated and do not understand the long-term impacts of economic policy (Guiso et al., 2018).

In contrast, parties are sophisticated and forward looking. Parties discount future payoffs by a discount factor β, where $0 < \beta < 1$. Party d only cares about winning; it receives a period payoff of 1 if it wins, and 0, otherwise. Party b only cares about policy, and its period utility from consumption corresponds to that of the median voter. This framework nests a setting in which multiple benevolent parties compete: Each would offer the same economic policy, and when party d loses, the benevolent party with the highest valence is elected.

We maintain the following assumptions.

Assumption 1 *υ_t is distributed i.i.d. with a cdf G and a pdf g. Moreover, $g(x) > 0$ for all x in the support of G, $\lim_{x\to-\infty} xG(x) = 0$, and $0 < G(0) < 1$.*

The *iid* assumption is largely made to simplify the exposition. Clearly, $\lim_{x\to-\infty} xG(x) = 0$ becomes superfluous when there is a lower bound on the support. The assumption that $0 < G(0) < 1$ implies that if party b perfectly mimics party d's platform, then each party has a strictly positive probability of winning.

Assumption 2 *$\beta\rho^{1-s} < 1$ and $\beta\phi > 1$.*

[6] The case where $s < 1$ does not capture the feature that consumers become increasingly desperate when capital becomes very small. As a result, when $s < 1$ there is minimal pressure for the benevolent party to mimic the demagogue when capital is low; but the reverse is true when capital is high.

The first inequality ensures that b's discounted expected utility is finite. The second inequality means that the project's return exceeds the discount factor, so that party b wants to invest.

The game proceeds as follows. At the beginning of each period t, the parties simultaneously propose policies $i_{P,t} \in [0, f(k_{t-1})]$, $P = b, d$. Then the valence shock υ_t is realized and the median voter elects his preferred party. The party that wins the election implements its announced policy, period payoffs are realized, and the game moves to the next period $t + 1$.

We consider subgame perfect equilibria of the game that satisfy an equilibrium refinement:

Assumption 3 *In any equilibrium, at every time t party b weakly prefers its own policy to that of the demagogue, i.e., party b is at least as well off if his policy wins in the current period rather than d's policy, keeping strategies in periods $k > t$ unchanged.*

The role of the refinement is to exclude a trivial and implausible equilibrium in which the demagogue proposes party b's most preferred investment level, while party b promises to invest everything. The median voter would then get unboundedly negative utility from party b's proposed policy, and hence would choose the demagogue regardless of the valence realization. This is an equilibrium because the demagogue only cares about winning, and the benevolent party only cares about policy, and not about who implements that policy. By proposing to invest everything, party b can effectively impose an arbitrarily high penalty on voters out of equilibrium, guaranteeing that d is elected. As a result, party b does not want to be elected, because it prefers its opponent's policy. While this is an equilibrium of our stylized model, it does not describe electoral competition in practice. In particular, it is reasonable to assume that candidates want to get elected, which is what Assumption 3 imposes.

4 No Political Competition from Demagogues

As a benchmark, we first suppose that the benevolent party faces no competition from the demagogue. Thus, party b is free to propose policies that are unconstrained by electoral concerns.

Let $\bar{k} = k_{-1} > 0$ be the initial level of capital. An unconstrained benevolent party b solves

$$\max_{i_t} \sum_{t=0}^{\infty} \beta^t u(\phi k_{t-1} - i_t) \text{ s.t. } k_t = \rho k_{t-1} + i_t,\ 0 \leq i_t \leq \phi k_{t-1}. \tag{1}$$

A challenge with analyzing this dynamic optimization problem is that the flow payoff, $u(c)$, is unbounded. Thus, the usual results that ensure differentiability of the value function as well as the contraction mapping results that ensure uniqueness do not apply. The problem also does not map into the approaches used when one has constant returns to scale (Stokey et al. (1989), Ch. 4.3). One approach is to exploit the monotonicity of the Bellman operator, finding an upper bound for the value function, and showing that it converges to its fixed point (Stokey et al. (1989), Theorem 4.14). Our simpler approach first establishes the scalability of the value function. This enables us to quickly arrive at the functional form of the value function. To verify scalability, we cast the problem in a standard format with k_t as the state variable:

$$V(\overline{k}) = \max_{\{k_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u((\rho+\phi)k_{t-1} - k_t) \text{ s.t. } \rho k_{t-1} \leq k_t \leq (\rho+\phi)k_{t-1}, \text{ with } k_{-1} = \overline{k}. \tag{2}$$

Let $\{\overline{k}_t\}_{t=0}^{\infty}$ be an optimal sequence. Now, suppose we multiply the initial capital by $\alpha > 0$, so that the initial capital stock is $\hat{k}_{-1} = \alpha\overline{k}$, and consider the sequence $\{\hat{k}_t\}_{t=0}^{\infty}$, where $\hat{k}_t = \alpha\overline{k}_t$. This new sequence, $\{\hat{k}_t\}_{t=0}^{\infty}$, satisfies all the constraints because $\rho\overline{k}_{t-1} \leq \overline{k}_t \leq (\rho+\phi)\overline{k}_{t-1}$ if and only if $\rho\hat{k}_{t-1} \leq \hat{k}_t \leq (\rho+\phi)\hat{k}_{t-1}$. Moreover, because $V(\hat{k}_{-1})$ reflects optimization given $\hat{k}_{-1}$ rather than $\overline{k}_{-1}$,

$$V(\hat{k}_{-1}) = V(\alpha\overline{k}) \geq \sum_{t=0}^{\infty} \beta^t u((\rho+\phi)\overline{k}_{t-1} - \overline{k}_t) = \begin{cases} \alpha^{1-s}V(\overline{k}) & \text{if } s > 1, \\ \frac{\log(\alpha)}{1-\beta} + V(\overline{k}) & \text{if } s = 1; \end{cases} \tag{3}$$

Because α and $\overline{k}$ are both arbitrary, we can use $\frac{1}{\alpha}$ instead of α, and $\alpha\overline{k}$ instead of $\overline{k}$ to get

$$V(\overline{k}) \geq \begin{cases} \frac{1}{\alpha^{1-s}}V(\alpha\overline{k}) & \text{if } s > 1, \\ \frac{\log(1/\alpha)}{1-\beta} + V(\alpha\overline{k}) & \text{if } s = 1; \end{cases} \tag{4}$$

Now, suppose the inequality in (3) was strict. Then, substituting the right-hand side of (3) into (4), yields

$$V(\alpha\overline{k}) > \alpha^{1-s}V(k) \geq \alpha^{1-s}\frac{1}{\alpha^{1-s}}V(\alpha\overline{k}) = V(\alpha\overline{k}),$$

a contradiction. Thus, we must have

$$V(\alpha\overline{k}) = \begin{cases} \alpha^{1-s}V(\overline{k}) & \text{if } s > 1, \\ \frac{\log(\alpha)}{1-\beta} + V(\overline{k}) & \text{if } s = 1. \end{cases}$$

Substituting $\overline{k} = 1$ and $\alpha = k$ yields

$$V(k) = \begin{cases} k^{1-s}V(1) & \text{if } s > 1, \\ \frac{\log(k)}{1-\beta} + V(1) & \text{if } s = 1; \end{cases}$$

where we recognize that $V(1)$ depends on s. It remains to show that $V(1)$ is finite. First, consider a lower bound, which is obtained when the benevolent party never invests, so that $k_{t-1} = \rho^t$, implying a consumption of $c_t = \phi k_{t-1} = \rho^t \phi$. When $s > 1$,

$$\sum_{t=0}^{\infty} \beta^t \frac{(\phi\rho^t)^{1-s}}{1-s} = \frac{\phi^{1-s}}{1-s} \sum_{t=0}^{\infty} (\beta\rho^{1-s})^t = \frac{\phi^{1-s}}{(1-s)(1-\beta\rho^{1-s})} > -\infty.$$

When $s = 1$,

$$\sum_{t=0}^{\infty} \beta^t \log(\phi\rho^t) = \sum_{t=0}^{\infty} \beta^t (\log(\phi) + t\log(\rho)) = \frac{1}{1-\beta}\log(\phi) + \frac{\beta}{(1-\beta)^2}\log(\rho) > -\infty.$$

Next, consider an upper bound obtained by letting the benevolent party invest all the output, i.e., $k_{t-1} = (\rho + \phi)^t$, but supposing that such saving is 'costless' so that consumption equals the sum of principle capital plus output, and hence $c_t = (\rho + \phi)k_{t-1} = (\rho + \phi)^{t+1}$. When $s > 1$,

$$\sum_{t=0}^{\infty} \beta^t \frac{((\phi+\rho)^{1+t})^{1-s}}{1-s} = \frac{(\rho+\phi)^{1-s}}{1-s} \sum_{t=0}^{\infty} (\beta(\phi+\rho)^{1-s})^t = \frac{(\rho+\phi)^{1-s}}{1-s} \frac{1}{1-\beta(\phi+\rho)^{1-s}} < \infty,$$

by Assumption 2, because $\beta\phi > 1$ implies $\beta(\phi + \rho)^{1-s} < 1$. The case where $s = 1$ is similar. Thus, $V(1)$ and the value functions are well defined.

We now show that this functional form of the value function implies that a constant fraction of each period's capital is invested, and the remaining constant fraction is consumed. That is, the optimal investment in period t is λk_t, for some $\lambda > 0$ that does not depend on k. To see this, note that we showed that if $\overline{k}_t$ is the optimal sequence of capital stocks beginning from $\overline{k}_{-1}$, then $\hat{k}_t = \alpha\overline{k}_t$ is the optimal sequence of capital stocks beginning from $\alpha\overline{k}_{-1}$. Thus, if $\overline{i}_t = \overline{k}_t - \phi\overline{k}_{t-1}$ is the optimal investment sequence beginning from $\overline{k}_{-1}$, then $\hat{i}_t = \hat{k}_t - \phi\hat{k}_{t-1} = \alpha\overline{i}_t$ is the optimal investment sequence beginning from $\hat{k}_{-1} = \alpha\overline{k}_{-1}$.

In sum, the scalability of the initial stock of capital in our dynamic optimization problem generates a simple closed-form value function, which is differentiable and concave. Thus, we can characterize the solution either by using the Euler equations, or by posing the problem in its recursive form, and then directly optimizing using the functional form of the value function. Proposition 1 characterizes the solution.

Proposition 1 *Suppose the benevolent party faces no electoral competition. Then it solves the recursive optimization problem*

$$V(k) = \max_{i \in [0, \phi k]} u(\phi k - i) + \beta V(\rho k + i). \tag{5}$$

The value function takes the form

$$V(k) = \begin{cases} v\, k^{1-s} & \text{if } s > 1, \\ v + \frac{\log(k)}{1-\beta} & \text{if } s = 1; \end{cases},$$

where v does not depend on k. If k is the current capital, then investment is $i(k) = \lambda k$, where

$$\lambda = (\beta(\rho + \phi))^{1/s} - \rho > 1 - \rho \tag{6}$$

is increasing in ϕ and β, but decreasing in ρ.

Absent political competition from the demagogue, the benevolent party always grows capital: $\lambda > 1-\rho$. To see this, note that $\beta\phi > 1$ implies that $(\beta(\rho+\phi))^{1/s} > 1$. That the optimal level of investment increases in ϕ and β simply reflects that the value of investment is higher if the future is discounted by less or if the savings technology is more productive. The comparative statics with respect to ρ are more subtle, reflecting that greater depreciation raises the value of investment due to the concavity of payoffs in consumption, but reduces the value of investment because what is saved does not last for as long. When $s \geq 1$, increasing ρ, i.e., decreasing capital depreciation, reduces investment—the benevolent party can achieve the same level of capital with reduced investment when capital depreciates more slowly.

5 The Strategic Problem

5.1 Log Utility with Myopic Populist

We now analyze how electoral competition from a demagogue affects the policy choices of a benevolent party and long-run electoral outcomes. We first consider a myopic demagogue who only cares about winning the current period. We then show that this analysis extends if we focus on linear Markov Perfect Equilibria, or if we consider Subgame Perfect Equilibria of the game with finitely-many periods in the limit where the number of periods is arbitrarily large.

Electoral competition introduces uncertainty, which is captured by the sequence of wins and losses of the benevolent party b. Let $\Omega = \{w, \ell\}^{\mathbb{N}}$ be the sequence representing these wins and losses, and let $W_t \colon \Omega \to \{0, 1\}$ be the random variable that is 1 if party b wins in period t, and is

0, otherwise. The history at time t is given by the sequence of wins and losses up to time $t-1$. Let $\mathcal{F}_t$ be the filtration of Ω that corresponds to knowing that history. Then the capital stock and the investments $k_t, i_t \colon \Omega \to \mathbb{R}$ must be $\mathcal{F}_t$-measurable.

The probability distribution on Ω (i.e., the distribution over wins and losses) depends on k_t and i_t. To derive this distribution, let $\omega \in \Omega$. A myopic demagogue, who only cares about winning in the current period, has a dominant strategy not to invest. Let $i_t(\omega)$ be party b's period t investment policy given ω. Thus, the median voter in state ω at time t is indifferent between the parties at a net-valence level ϵ_t at which $\log(\phi k_{t-1}(\omega) - i_t(\omega)) = \log(\phi k_{t-1}(\omega)) + \epsilon_t$. Let D_t be the difference in utility that the median voter would derive from the two policy proposals:

$$D_t(\omega) = \log(\phi k_{t-1}(\omega) - i_t(\omega)) - \log(\phi k_{t-1}(\omega)) = \log\left(\frac{\phi k_{t-1}(\omega) - i_t(\omega)}{\phi k_{t-1}(\omega)}\right). \tag{7}$$

Then the probability that b wins at time t in state ω is $G(D_t(\omega))$. Letting $E[\cdot; i, k]$ be the resulting expectation over Ω, party b's optimization problem is

$$\max_{i_t} E\left[\sum_{t=0}^{\infty} \beta^t \Big(W_t(\omega)\log(\phi k_{t-1}(\omega) - i_t(\omega)) + (1 - W_t(\omega))\log(\phi k_{t-1}(\omega))\Big); \{i_t(\omega), k_t(\omega)\}_{t\in\mathbb{N}}\right] \tag{8}$$
$$\text{s.t. } \; k_t(\omega) = \rho k_{t-1}(\omega) + i_t(\omega), \; 0 \le i_t(\omega) \le \phi k_{t-1}(\omega), \text{ where } k_t, i_t \text{ are } \mathcal{F}_t \text{ measurable.}$$

A key observation is that, with log utility, the winning probabilities remain unchanged after scaling: $D_t = \log\left(\frac{\phi k_{t-1} - i_t}{\phi k_{t-1}}\right) = \log\left(\frac{\phi \alpha k_{t-1} - \alpha i_t}{\phi \alpha k_{t-1}}\right)$. This implies that, just as in the case where there is no political competition, the benevolent party's value function is scalable in k. In turn, this implies that its optimal investment is linear in capital stock.

Lemma 1 *The investment platform problem of party b can be written recursively, with a value function that takes the form $V(k) = v + \log(k)/(1-\beta)$.*

Using Lemma 1, we write party b's optimization as

$$\max_i G(D)\left(\log(\phi k - i) + \frac{\beta}{1-\beta}\log(\rho k + i)\right) + (1 - G(D))\left(\log(\phi k) + \frac{\beta}{1-\beta}\log(\rho k)\right), \tag{9}$$

where $D = \log((\phi k - i)/\phi k)$. Differentiating party b's objective with respect to i yields

$$G(D)\left(-\frac{1}{\phi k - i} + \frac{\beta}{(1-\beta)(\rho k + i)}\right) - \frac{g(D)}{\phi k - i}\left(\log\left(\frac{\phi k - i}{\phi k}\right) + \frac{\beta}{1-\beta}\log\left(\frac{\rho k + i}{\rho k}\right)\right).$$

Letting $i = \lambda k$, $D = \log\left(\frac{\phi k - i}{\phi k}\right)$ simplifies to $D = \log\left(\frac{\phi-\lambda}{\phi}\right)$, yielding first-order condition[7]

$$G(D)\left(-\frac{1}{\phi-\lambda} + \frac{\beta}{(1-\beta)(\rho+\lambda)}\right) = g(D)\frac{1}{\phi-\lambda}\left(\log\left(\frac{\phi-\lambda}{\phi}\right) + \frac{\beta}{1-\beta}\left(\log\left(\frac{\rho+\lambda}{\rho}\right)\right)\right). \tag{10}$$

The left-hand side of (10) is the expected marginal effect of increasing savings today on the discounted stream of future consumption. The right-hand side of (10) is the marginal cost of increasing savings that raise the chance that party d wins. It is the marginal reduction in the probability of winning due to increased savings, $g(D)/(\phi-\lambda)$, times the difference in the payoffs from winning and investing λk, versus losing to party d which will not invest.

To show how competition from the demagogue affects investment, it is useful to multiply both sides of (10) by $(\phi - \lambda)(1-\beta)(\rho+\lambda)$ to obtain

$$G(D)\left(\beta\phi - (1-\beta)\rho - \lambda\right) = g(D)(\rho+\lambda)\left((1-\beta)\log\left(\frac{\phi-\lambda}{\phi}\right) + \beta\log\left(\frac{\rho+\lambda}{\rho}\right)\right). \tag{11}$$

When unconstrained, the benevolent party chooses $\lambda = \beta\phi - (1-\beta)\rho$, which sets the left-hand side of (11) to zero. However, the right-hand side of (11) is strictly positive, when evaluated at that unconstrained λ. In other words, λ must be reduced. Posed differently, party b seeks to maximizes social welfare just as when it did not face electoral competition from the demagogue. However, now, b can only implement its policy if elected, so it invests less to reduce the probability that the demagogue controls the government. Proposition 2 uses (11) to characterize how the benevolent party adjusts its savings in response to competition from the demagogue.

Proposition 2 *Equilibrium investment is strictly reduced by political competition from the demagogue. Let $\lambda(\rho)$ be the equilibrium level of investment given the depreciation rate $1-\rho$. Then*

1. $\lim_{\rho\to 0}\lambda(\rho) = 0$*, implying that the winning probability $G(D)$ converges to $G(0)$.*

2. *The share of capital reinvested, $\lambda(\rho)$, converges to zero at the rate $-G(0)\phi/(g(0)\log(\rho))$, and hence $\lambda(\rho)$ is strictly increasing in ρ for small ρ. That is,*

$$\lim_{\rho\to 0}\lambda(\rho)\left(-\frac{g(0)}{G(0)}\frac{\log(\rho)}{\phi}\right) = 1.$$

[7] Observe that party b's objective function at the lower boundary of investment, $i = 0$, and the upper boundary of investment, $i = \phi k$, are the same. Moreover, the derivative of the objective at the lower boundary, $i = 0$, is positive if and only if $\beta(\rho + \phi) > \rho$, which holds by Assumption 2. Thus, we have an interior maximum.

By how much does political competition drive down party b's optimal investment? Proposition 2 provides a first step to answering this question: it provides an explicit analytical characterization of $\lambda(\rho)$ when the depreciation rate is high, $1 - \rho \approx 1$. One might expect that if party b has a very large ex-ante valence advantage, so that it is likely to win election, then it would only marginally reduce its investment relative to when it faces no political competition from the demagogue. However, the winning probability $G(D)$ is endogenous, and there can be a large marginal benefit of mimicking the demagogue's no-investment policy by lowering λ.

In fact, when the depreciation rate approaches 1, the endogenous winning probability goes to $G(0)$. Even when party b has a large ex-ante valence advantage, but $G(0)$ is less than 1, the effect on the benevolent party's investment becomes large when ρ is small. In particular, as ρ goes to zero, so does λ. Thus, the difference between the investment levels chosen by party b with and without political competition from the demagogue becomes maximal: even a remote threat of a demagogue winning can have large effects on the behavior of established parties.

To see why λ must go to zero as ρ goes to zero, observe that $\log((\rho + \lambda)/\rho)$ in equation (11) would go to infinity if λ were bounded away from zero. That is, the marginal electoral cost would become arbitrarily large, making it optimal to lower λ. This implies that even though absent competition, investment would be $\beta\phi > 1$, the socially-minded party b becomes so concerned about electoral competition that it drives investment down to zero. To see the rate of convergence of $\lambda(\rho)$ as ρ goes to zero, note that the right-hand side of equation (11) goes to

$$g(0)(\rho + \lambda(\rho))\beta \log\left(\frac{\rho + \lambda(\rho)}{\rho}\right) = \beta g(0)\Big((\rho + \lambda(\rho)) \log(\rho + \lambda(\rho)) - (\rho + \lambda(\rho)) \log(\rho)\Big).$$

Using $\lim_{x \to 0} x \log(x) = 0$, this simplifies to $-\beta g(0)(\rho + \lambda(\rho)) \log(\rho)$; and repeating this step, it simplifies further to $-\beta g(0)\lambda(\rho) \log(\rho)$. The left-hand side of equation (11) converges to $\beta G(0)\phi > 0$. In order for the right-hand side of equation (11) to converge to this value, $\lambda(\rho)$ must converge to zero at the rate $-G(0)\phi/(g(0) \log(\rho))$. The key term $\log\left(\frac{\rho+\lambda(\rho)}{\rho}\right)$ captures the future costs of losing the election to the demagogue. When the depreciation rate is almost 1, almost all capital disappears if the demagogue wins, generating huge future welfare losses. This effect becomes dominant, determining the rate at which party b reduces investment to avoid this outcome.

The presence of a demagogue alters equilibrium investment in other fundamental ways. Ab-

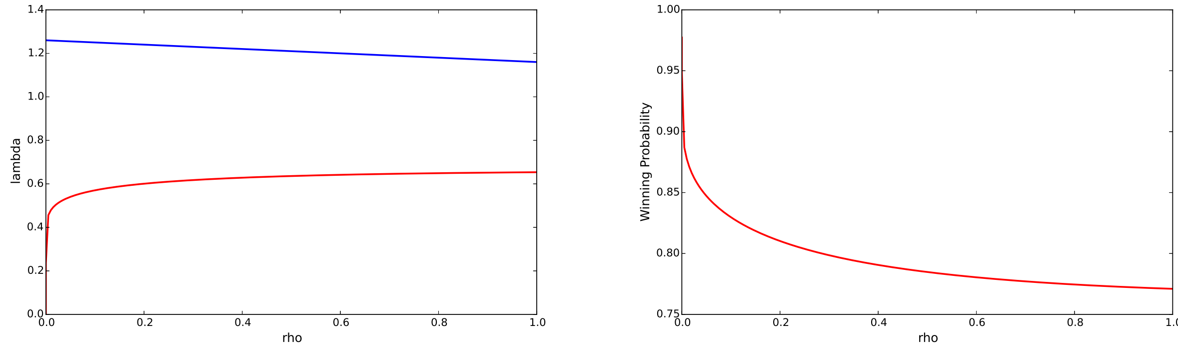

Figure 1: Investment (red), Social Optimal investment (blue), and b's winning probability for $\mu = -1$, $\sigma = 0.5$, $\phi = 1.4$, $\beta = 0.9$.

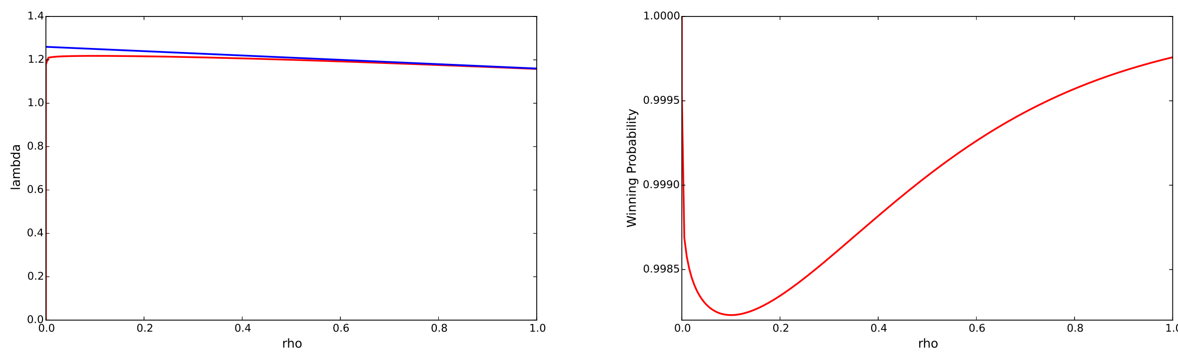

Figure 2: Investment (red), Social Optimal investment (blue), and b's winning probability for $\mu = -3.5$, $\sigma = 0.5$, $\phi = 1.4$, $\beta = 0.9$.

sent the demagogue, optimal investment decreases linearly with ρ (Proposition 1). In contrast, Proposition 2 establishes the surprising result that if ρ is small, then λ *strictly increases* in ρ—electoral competition *reverses* how party b behaves when depreciation is reduced. In fact, as Figures 1 and 2 illustrate, depending on parameters, $\lambda(\rho)$ can be globally increasing in ρ, or it can decrease in ρ once ρ is sufficiently close to one. This latter case occurs when the distribution of valence G is shifted sufficiently far in favor of party b, so that b is very likely to win. As a result, doing the 'right' thing begins to dominate electoral concerns when ρ is large.

The figures consider settings where party b has a large normally-distributed valence advantage. For the parameters of Figure 1, were b to perfectly mimic party d by offering $\lambda = 0$ then

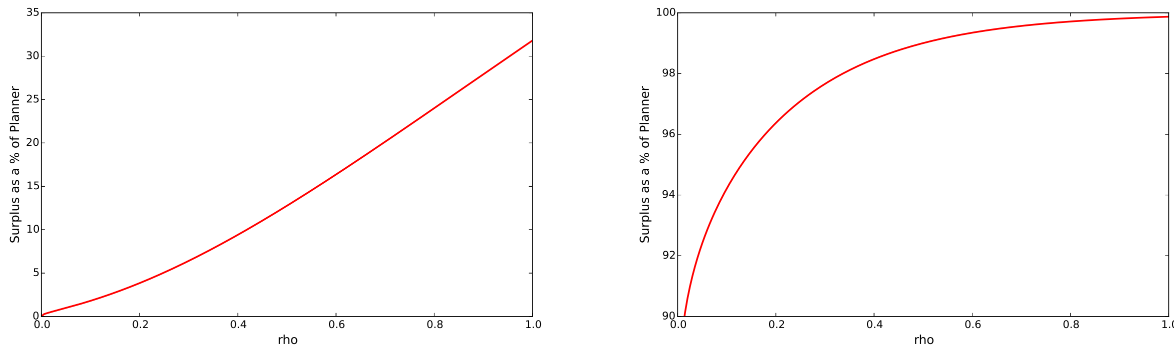

Figure 3: Welfare shortfall due to political competition, $\mu = -1.5$ left panel, and $\mu = -3.5$ right panel; $\sigma = 0.5$, $\phi = 1.4$, $\beta = 0.9$.

b would win with 97.7% probability. Yet, λ increases monotonically in ρ and even when ρ is high, it invests far less than when there is no political competition. One measure of the tradeoff that the benevolent party faces is that its equilibrium probability of winning falls below 0.8 for $\rho > 0.4$. That is, to achieve the equilibrium benefits of saving, b reduces its ex-ante electoral advantage by over 20 percent. At the same time, from the perspective of an outsider, it looks as if b is pandering to voters, reducing investment by approximately 50% from the social optimum. Thus, pandering to short-sighted voters has large welfare costs.

Figure 2 shows how investment rates vary with ρ when we increase the mean valence disadvantage of the demagogue from $\mu = -1$ to $\mu = -3.5$. At these values of μ and σ, the probability that candidate b loses if it perfectly mimics d is 1.28×10^{-12}, about 1,000 times less likely than winning the Powerball lottery. When ρ is small, λ rises with ρ, as established in Proposition 2. However, for larger values of ρ, considerations of the marginal benefits in (11) start to dominate the marginal costs, causing party b to behave closer to how it would in the demagogue's absence. In particular, once ρ rises sufficiently, b's choice of λ begins to decline with ρ. The non-monotonicity of λ is mirrored by the non-monotonicity of the winning probability: once λ decreases with ρ, party b's winning probability rises. Notably, even with such a large valence advantage, party b takes a 1/500 gamble of losing when $\rho = 0.1$, highlighting the costs of doing the right policy when voters are short-sighted.[8]

[8]The technical reason that the policy function does not converge uniformly as $\mu \to -\infty$ is that utility is unbounded from below.

The large differences in optimal investments with and without competition from a demagogue that arise even when b has a big valence advantage, translate into large welfare losses. To compute this welfare loss, we calculate the level of constant consumption streams, c_N (no competition) and c_C (competition) that would yield the same ex-ante expected utility. Figure 3 presents the ratio c_C/c_N in percent. In the left panel, party b has a three standard-deviation valence advantage over the demagogue. Nevertheless, the demagogue's presence results in a consumption equivalent that is always less than one third of that in a world without the demagogue. In the right-panel, party b's valence advantage is seven standard deviations. Nonetheless, even this extremely remote possibility that the demagogue wins meaningfully lowers welfare.[9]

Proposition 2 implies that if ρ is small enough, then party b lowers λ below the capital replacement rate of $1-\rho$. Because party d never re-invests, it follows that the capital stock then converges to zero. We say that the capital stock exhibits a death spiral if it converges to zero.

Corollary 1 *There exists a $\bar{\rho} > 0$ such that if $\rho \leq \bar{\rho}$, then the capital stock k_t exhibits a death spiral, converging to zero as $t \to \infty$.*

To prove Corollary 1 it suffices to establish that λ is close to zero when ρ is small. However, death spirals are not limited to such extreme cases. In fact, death spirals can occur even when ρ is large, so that party b's investment rate exceeds the replacement rate of $1-\rho$. Indeed, we now show that the probability of a death spiral may go to one, even when the expected investment rate of $G(D)(\rho+\lambda(\rho)) + (1-G(D))\rho$ (accounting for the probability that the demagogue wins) strictly exceeds the replacement rate $1-\rho$.

As a prelude, we provide necessary and sufficient conditions for a death spiral.

Lemma 2 *Let $\bar{k} > 0$ be the initial capital stock.*

1. *If $(\rho+\lambda)^{G(D)}\rho^{1-G(D)} < 1$, then for every $\varepsilon, \delta > 0$ there exists $\bar{t}$ such that the probability that $k_t < \delta\bar{k}$ exceeds $1-\varepsilon$ for all $t \geq \bar{t}$. That is, the probability of a death spiral goes to one.*
2. *If $(\rho+\lambda)^{G(D)}\rho^{1-G(D)} > 1$, then for every $\varepsilon > 0$ there exists $\bar{t}$ such that the probability that $k_t < \bar{k}$ is less than ε for all $t \geq \bar{t}$. That is, the probability of a death spiral goes to zero.*

[9] These computations do not account for welfare benefits from being able to elect a high-valence demagogue.

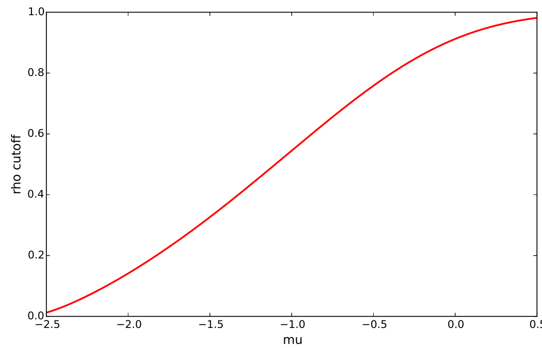

Figure 4: Cutoff value of ρ below which a death spiral occurs, $\sigma = 0.5, \phi = 1.4, \beta = 0.9$

Figure 4 plots the cutoff level of ρ below which death spirals are inevitable for different values of μ. Observe that if no party has a valence advantage, i.e., if $\mu = 0$, then death spirals occur unless ρ is close to one. Such a scenario could reflect a change in how the public perceives demagogues. If previous opprobrium by society toward demagogues declines, for example due to changes in political discourse or to changing attitudes about what is politically correct, then party b's valence advantage will decline. This contributes to death spirals in two ways. First, party d is more likely to win. But, what matters more, as our earlier discussion explains and Figures 1 and 2 illustrate, is that party b begins to adopt policies that are closer to the demagogue's.

When party b's valence advantage rises, i.e., as μ falls, death spirals only emerge when ρ is smaller. Nonetheless, for intermediate levels of ρ, death spirals still arise when party b's valence advantage is large. To provide insights into when death spirals obtain, we identify the value of λ that maximizes long-run capital growth, solving

$$\Gamma(\rho) = \max_{\lambda \in [0,\phi]} (\rho + \lambda)^{G(D)} \rho^{1-G(D)}, \tag{12}$$

where we recall that $D = \log((\phi - \lambda)/\phi)$. When this value of λ is chosen, Lemma 2 implies that no death spiral occurs when $\Gamma(\rho) > 1$, but they occur with probability 1 when $\Gamma(\rho) < 1$. By the Envelope Theorem, $\Gamma(\rho)$ strictly increases in ρ. Moreover, $\lim_{\rho \to 0} \Gamma(\rho) < 1 < \lim_{\rho \to 1} \Gamma(\rho)$. Thus, there exists a $\hat{\rho} > 0$ such that a death spiral can be prevented by choosing the λ that maximizes (12) if $\rho > \hat{\rho}$. When $\rho < \hat{\rho}$ a death spiral is inevitable. But party b does not choose its investment policy λ with the sole objective of avoiding a death spiral. Rather, it maximizes the median voter's discounted utility. The policies differ. To see this, note that the first-order condition of problem (12) is

$$G(D)(\phi - \lambda) - g(D)(\rho + \lambda)\log\left(\frac{\rho + \lambda}{\rho}\right) = 0. \tag{13}$$

In contrast, party b's first-order condition for optimization, equation (11), can be written as

$$\beta\left(G(D)(\phi - \lambda) - g(D)(\rho + \lambda)\log\left(\frac{\rho + \lambda}{\rho}\right)\right) + (1 - \beta)(\rho + \lambda)\left(G(D) - g(D)D\right) = 0. \tag{14}$$

When $\beta = 1$, equation (14) reduces to equation (13). This observation implies that when β is close to one, party b acts as if its primary objective is to avoid death spirals. More generally, party b's first-order condition for optimization is a convex combination of the first-order condition of the problem (12) with weight β and a weight $(1 - \beta)$ on $(\rho + \lambda)(G(D) - g(D)D)$, which measures the distortion in party b's objective away from long-run survival. Substituting the value of λ that solves (13) into the second term of (14) reveals that this term is strictly positive. That is, party b will choose a λ that is smaller than what maximizes long-run survival. Thus,

Corollary 2 *There are capital depreciation rates at which if party b invested "appropriately", then capital would grow arbitrarily large with probability one. However, party b, instead, invests sufficiently less that a death spiral occurs.*

5.2 Log Utility with Far-sighted Demagogues

To this point, we have focused on a demagogue who only cares about the present, and therefore gives citizens exactly what they want at the present, in order to maximize the probability of winning election. If the demagogue is, instead, far sighted and also cares about winning in the future, his interests may no longer be aligned with short-sighted citizens. For example, if high consumption today would lower the demagogue's chances of future re-election, then the demagogue may propose strictly positive investment, trading off between current and future probabilities of election, and no longer giving voters exactly what they want. Were this so, dynamic political competition would reduce the potential harm generated by the demagogue.[10]

[10]Obviously, we assume that, if elected, the demagogue cannot change the rules of the game while in office. As Levitsky and Ziblatt (2018) discuss, demagogues sometimes damage the "guardrails of democracy," by changing the basic rules and norms of democratic government to their advantage.

We now show that dynamic political competition may not discipline the demagogue. We first consider two natural equilibrium selection criteria and show that the equilibrium with a short-sighted demagogue also obtains when demagogues are far-sighted. In particular, the same equilibrium outcomes emerge with a far-sighted demagogue in (1) the unique linear Markov perfect equilibrium, and (2) the unique equilibrium to the stationary limit of a finite-horizon economy. The key as to why a far-sighted demagogue implements short-sighted policies that maximize his probability of winning today is that the demagogue's competitiveness in future elections does not depend on the level of the capital stock.

We then show that the demagogue's behavior can be improved if we allow for non-Markov (e.g., trigger) strategies. The intuition is that the benevolent party is happy to trade off a reduced probability of winning in return for a more far-sighted policy choice by the demagogue. In particular, b is prepared to increase investment toward the optimal level in the absence of electoral competition if the demagogue, in turn, moderates its policy by increasing its proposed investment. In this equilibrium, the increased investment by the benevolent party makes the demagogue more likely to win than in the equilibrium in which the demagogue never invests, while b gains from the better policies that are adopted. As long as the discount factor β is sufficiently close to one, this equilibrium can be supported by the threat of reverting to the 'myopic' equilibrium in which the demagogue does not invest. One might expect that if the discount rate goes to one so that parties care a lot about the long term, then like in repeated games, one should be able to get close to the Pareto efficient outcome by using harsh threats in the event of deviation. Surprisingly, we show that this is not so. Instead, implemented equilibrium policies remain bounded away from the first-best—threats can help, but only to a limited extent.

We begin with a formal statement of the result that equilibrium outcomes with a far-sighted demagogue correspond to the unique equilibrium with a myopic demagogue for natural equilibrium selection criteria.

Proposition 3 *In the unique linear MPE, behavior coincides with equilibrium outcomes when the benevolent party is farsighted and the demagogue is myopic.*[11] *Similarly, a far-sighted demagogue never invests in any period of a truncated finite-horizon model. Thus, the limit of the*

[11]This result holds if we allow for linear strategies in which the coefficients depend on time, so that $i_t^P(k) = \lambda_t^P k$, where λ_t^P depends on t, but not on the history of play.

finite-horizon model as the number of periods goes to infinity coincides with the equilibrium of the infinite-horizon model with a myopic demagogue.

The proposition uses the standard notion of a Markov perfect equilibrium, where the state in each period is the initial capital stock. Thus, a Markov strategy for party $P \in \{b, d\}$ is a mapping from that capital stock to an investment, $i_P(k) : [0, \infty) \to [0, \phi k]$. A linear Markov strategy for party P takes the form $i_P(k) = \lambda_P k$, for some $\lambda_P \in [0, \phi]$. A linear Markov perfect equilibrium is a linear strategy profile, (λ_b, λ_d), of mutual best responses in every subgame, where one must verify that the best response to a rival's linear strategy is also linear.

To prove the result for the finite-horizon model, we proceed by backward induction. In the terminal period, obviously neither party invests, so election outcomes do not hinge on the capital stock. Continuing inductively, the non-dependence of future electoral outcomes on the capital stock ensure that the demagogue does not want to save, as saving reduces the current probability of winning without affecting future probabilities of election. Thus, the demagogue never invests, just as in the myopic setting.

It is immediate that allowing for non-Markov strategies, the demagogue can be induced to use better policies whenever β is sufficiently large. In particular, let b choose the unconstrained optimum, λ^*, and select λ_d so that d's winning probability is marginally larger. Clearly, this is welfare improving from b's perspective. Further, for all β sufficiently close to 1, d does not want to deviate given the threat to revert to the myopic equilibrium after a deviation. What is surprising is that even for β close to one, such trigger strategies cannot support the first best.

Proposition 4 *Suppose both parties use trigger strategies, where a deviation results in reversion to the unique linear Markov equilibrium. Then the investment levels are bounded away from the first-best levels for all β.*

At first blush, these trigger strategies in which the benevolent party strategically induces the demagogue to moderate looks like a situation in which a responsible party attempts to tame the demagogue, for example by including them in a governing coalition. As an illustration, in 2000, the Austrian center-right people's party (ÖVP) invited the Freedom Party (FPÖ) to join the government in the hope that the burden of governing would moderate the FPÖ. However, this is not a situation that our model describes. Instead, we have two separate parties with two separate policies, and any coordination has to occur over time. In fact, rather than taming the demagogue, the responsible party chooses to help the demagogue to win, in the hope that the demagogue will behave more responsibly. This seems like a disastrous campaign strategy, and it is hard to understand how intertemporal coordination of this non-Markovian type would work. Further, the case where β is even remotely close to 1 badly approximates reality, because (i) an election winner holds office for a non-trivial period, and (ii) many offices feature term limits on individuals.

6 The Inevitability of Death Spirals

The previous section established that when voters have log preferences, the equilibrium probability with which the benevolent party b wins does not vary with the capital stock. As a result, party b's optimally proposes a constant investment strategy. Lemma 2 shows that this means that the occurrence of death spirals depends on model parameters, but not on the capital stock. We now show that this is no longer true if the level of risk aversion exceeds 1, which is consistent with most macroeconomic calibrations of risk aversion of consumer preferences. In fact, when voters have CRRA utility with relative risk aversion $s > 1$, then regardless of the other parameters describing the economy, given any initial capital stock k, the probability of a death spiral is strictly positive, bounded away from zero. Moreover, death spirals become inevitable once capital levels fall too low.

Characterization of equilibrium behavior becomes far harder when $s > 1$, because the optimal investment as a share of capital is no longer constant. This reflects that for a given proposed investment rate policy λ, the utility that voters derive from the two policies now hinges on k. In particular, $u((\phi - \lambda)k) - u(\phi k) = k^{1-s} \frac{(\phi-\lambda)^{1-s} - \phi^{1-s}}{1-s}$ increases in k for $s > 1$. This means that, ceteris paribus, party b is more likely is to win the election when the capital stock is larger.

The value function (and its derivative if it exists) that recursively describes party b's expected payoffs now depend in potentially complicated ways on the properties of the valence density. In particular, there is no reason for $G(u((\phi - \lambda)k) - u(\phi k))$ to be well-behaved. As a result, the standard arguments used to prove concavity or differentiability of the value function cannot be employed. This complicates characterizing the optimal investment rate policy $\lambda(k) = \frac{i(k)}{k}$.

A key step in our analysis is to identify a lower bound on the derivative of party b's value function—if that derivative exists. This lower bound is the derivative of the differentiable value function associated with zero investment in every period, where with zero investment by party b, neither investment rates nor the probabilities of winning vary with the capital stock. This lower bound delivers a lower bound on the marginal value of a positive investment rate. Let $V_N(k)$ denote the benevolent party's value function if investment is zero in every period:

$$V_N(k) = \sum_{t=0}^{\infty} \beta^t (\phi \rho^t k)^{1-s}/(1-s) = \frac{(\phi k)^{1-s}}{1-s} \sum_{t=0}^{\infty} (\beta \rho^{1-s})^t = \frac{(\phi k)^{1-s}}{1-s} \frac{1}{1-\beta\rho^{1-s}},$$

where from Assumption 2, $\beta\rho^{1-s} < 1$. Thus, $V_N(k)$ is differentiable with respect to k and

$$V_N'(k) = \frac{(1-s)V_N(k)}{K}. \tag{15}$$

We establish the following bound.

Lemma 3 *Let $V(k)$ be the value function for party b. Then $V_N(k)$ is differentiable, and*

$$\liminf_{k' \to k} \frac{V(k) - V(k')}{k - k'} \geq V_N'(k).$$

Lemma 3 enables us to characterize party b's investment decisions. Party b's optimization problem can be written recursively as:

$$V(k) = \max_{\lambda \in [0,\phi]} G(D)\Big(u((\phi-\lambda)k) + \beta V((\rho+\lambda)k)\Big) + (1-G(D))\Big(u(\phi k) + \beta V(\rho k)\Big), \tag{16}$$

where

$$D = u((\phi-\lambda)k) - u(\phi k) = k^{1-s}\, \frac{(\phi-\lambda)^{1-s} - \phi^{1-s}}{1-s}.$$

The marginal cost of raising investment (i.e., of raising λ) is

$$MC = \frac{g(D)k^{1-s}}{(\phi-\lambda)^s}\Big(u((\phi-\lambda)k) + \beta V((\rho+\lambda)k) - u(\phi k) - \beta V(\rho k)\Big).$$

From Lemma 3, the marginal benefit of raising investment (i.e., of raising λ) satisfies:

$$MB \geq G(D)k^{1-s}\Big(-ku'((\phi-\lambda)k) + k\beta V_N'((\rho+\lambda)k)\Big) = G(D)\left(-\frac{1}{(\phi-\lambda)^s} + \beta\frac{\phi^{1-s}}{1-\beta\rho^{1-s}}\right).$$

Because for any $k > 0$, $MC(\lambda = 0) = 0$, while $MB(\lambda = 0) = G(0)\frac{\beta(\phi+\rho^{1-s})-1}{\phi^s(1-\beta\rho^{1-s})} > 0$, party b always benefits from marginally raising his investment above zero at any $k > 0$. Moreover,

$\lim_{k\to 0} \lambda(k) = 0$. That is, party b can always get a higher payoff than it would get from choosing $\lambda = 0$. In turn, if the support of valence is bounded so that $G(D) = 0$ for sufficiently negative D, this implies that $\lim_{k\to 0} \lambda(k) = 0$. To see this, observe that if $\lim_{k\to 0} \lambda(k) > 0$, then $\lim_{k\to 0} D = -\infty$, and hence $\lim_{k\to 0} G(D) = 0$. That is, for sufficiently small capital stocks, party b always loses. If b always loses, its payoff equals what it would get from choosing $\lambda = 0$. But we just showed that party b can always get a higher payoff than that from choosing $\lambda = 0$. Therefore, $\lim_{k\to 0} \lambda(k) = 0$. By the same logic, $G(D(k)) \in (0, G(0))$. This proves Proposition 5 below:

Proposition 5 *Party b's optimal investment and probability of winning are strictly positive:* $\lambda(k) > 0$ *and* $0 < G(D(k)) < G(0)$ *for all* $k > 0$. *Moreover, if the support of G is finite,*[12] *then* $\lim_{k\to 0} \lambda(k) = 0$.

Proposition 5 has two implications. Because $\lim_{k\to 0} \lambda(k) = 0$, when k is sufficiently small, $\lambda(k) < 1 - \rho$. It follows that if capital ever falls sufficiently low, then it falls in every period thereafter: even when party b wins, investment fails to compensate for depreciation. This reduction in capital, in turn, reduces party b's future investment, creating a death spiral for the capital stock. Moreover, even when the initial capital stock is large enough that b's investment exceeds the rate of depreciation, $\lambda(k) > 1 - \rho$, the demagogue may win. If the demagogue wins enough times, capital depreciation can bring capital stock below the critical threshold that begins a death spiral. Let $\bar{k}$ be that capital threshold, and suppose the initial capital stock is $k > \bar{k}$. If the demagogue wins n consecutive times, then the capital stock falls from k to $\rho^n k$. Thus, a death spiral necessarily occurs if $n > n^*$, where $n^* = \min\{n \in \mathbb{N} \ s.t. \ \rho^n k < \bar{k}\} = \left\lceil \frac{\log(\bar{k}/k)}{\log(\rho)} \right\rceil$. The probability that the demagogue wins n^* consecutive times is at least $(1 - G(0))^{n^*}$. Thus,

Proposition 6 *There exists a capital level* $\bar{k}$ *such that if* $k \leq \bar{k}$ *then a death spiral occurs with probability 1. Given any capital stock* $k > \bar{k}$, *the probability of dropping below* $\bar{k}$ *and entering a death spiral exceeds* $(1 - G(0))^{n^*} > 0$, *where* $n^* = \lceil (\log(\bar{k}/k))/\log(\rho) \rceil$.

It is natural to intepret k as including the social capital associated with the institutional norms of democracy. This proposition implies that younger democracies that have lower social capital are more vulnerable to demagogues, as the requisite n^* for a death spiral is smaller.

[12] We still maintain assumption 1. Thus, $0 < G(0) < 1$.

7 Investment in Good Times

We next characterize party b's investment policy choices when capital stocks are large. When capital stocks are small, b invests so little that a death spiral results. In contrast, we now show that when capital stock are high, b over-invests in the sense that political competition from the demagogue causes it to save even more than it would absent electoral concerns.

This result is perhaps surprising, because political competition always reduces the level of the value function for any given capital stock. One might therefore think that political competition *must* reduce derivative of the value function with respect to capital. This turns out not to be so. The key step is to show that for large k, solutions of Problem (16) are close to solutions of the recursive optimization problem:

$$V_b(k) = \max_{\lambda\in[0,\phi]} G(0)\Big(u((\phi-\lambda)k) + \beta V_b((\rho+\lambda)k)\Big) + (1-G(0))\Big(u(\phi k) + \beta V_b(\rho k)\Big), \tag{17}$$

in which party b's probability of winning is set at its highest level $G(0)$—-and does not depend on its action. Replacing $G(0)$ by 1 in (17) yields the benevolent party's problem when there is no demagogue, whose recursive form is given in (5). Thus, by the same argument as in Proposition 1, the value function takes the form $V_b(k) = \frac{v_b}{1-s} k^{1-s}$, and the investment share λ is independent of k. Letting λ^* be that optimal λ, the first-order condition of (17) yields:

$$\lambda^* = \frac{(\beta v_b)^{1/s}\phi - \rho}{1 + (\beta v_b)^{1/s}}. \tag{18}$$

Lemma 4 shows that for large k, solutions of problem (16) are close to this λ^*.

Lemma 4 *Let λ^* be the solution of problem* (17) *given by* (18). *Let $\lambda(k)$ solve problem* (16). *Then for every $\varepsilon > 0$ there exists $\hat{k}$ such that $|\lambda(k) - \lambda^*| < \varepsilon$, for all $k \geq \hat{k}$.*

In view of the lemma, it suffices to compare the benevolent party's problem absent a demagogue to (17). Proposition 1 shows that the value function for the party's problem absent a demagogue is $V_P(k) = k^{1-s}v_P/(1-s)$, where $v_P/(1-s) = V_P(1)$. Because ex-ante utility is maximized when there is no political competition, $V_P(1) > V_b(1)$. Because $1-s < 0$, it follows that $v_P < v_b$. In general, if the value function takes the form $k^{1-s}v/(1-s)$ then the investment share is

$$\lambda = \frac{(\beta v)^{1/s}\phi - \rho}{1 + (\beta v)^{1/s}}. \tag{19}$$

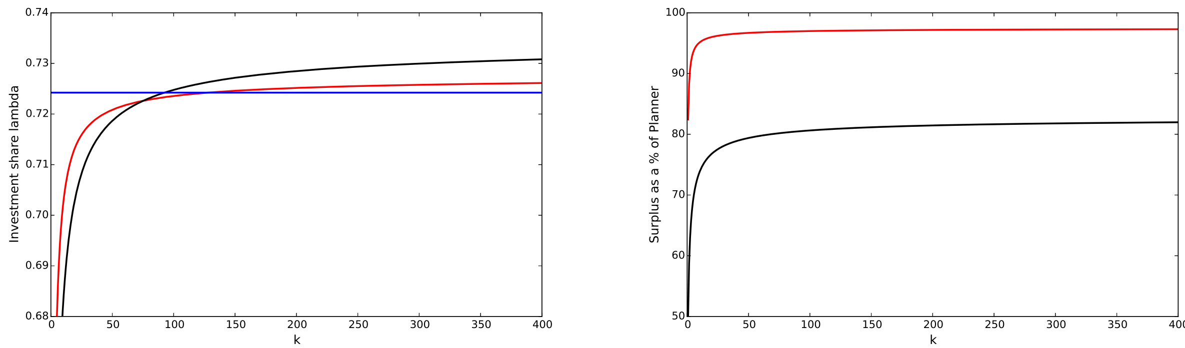

Figure 5: Investments and Welfare for $\mu = -1$ (red), $\mu = -0.5$ (black), and Social Optimal investment (blue), and investment for $\sigma = 0.5$, $\phi = 1.4$, $\beta = 0.9$, $\rho = 0.9$, and $s = 1.5$

Note that λ as defined in (19) is strictly increasing in v. Because $v_P < v_b$ it follows that the investment share from problem (17), λ_b strictly exceeds the share, λ_P, absent competition. Hence, Lemma 4 implies that $\lambda(k) > \lambda_P$ for large k. The same argument implies that that the solution of problem (17) decreases in $G(0)$. In other words, if the valence distribution becomes more favorable to the demagogue, then b responds by increasing investment when the capital stock is large. Proposition 7 summarizes.

Proposition 7 *If k is sufficiently large then:*

1. *Party b invests more than would a social planner who always gets his policy implemented, i.e., $\lambda(k) > \lambda_P$.*

2. *If party b's valence decreases (i.e., $G(0)$ decreases), then $\lambda(k)$ increases.*

As k grows very large, the probability that the benevolent party b loses goes to $G(0)$, becoming insensitive to b's equilibrium investment. This leads party b to 'over-invest' to account for the fact that investments are only made when it wins. That is, b over invests when capital is high to insure against capital being depreciated too much due to a string of wins by the demagogue. In some sense, relative to a scenario with no political competition, the benevolent party pursues too much of an austerity policy in good times.

The left panel in Figure 5 compares the socially-optimal investment to b's investment choice.

As Proposition 7 indicates, competition causes b to increase investment past its unconstrained level whenever k is sufficiently large. The gap between these investment levels depends on $G(0)$.

We have shown that given any capital stock level k, no matter how high, the probability of a death spiral is always strictly positive. In particular, there exists a critical capital stock level $\bar{k}$ such that if k ever drops below $\bar{k}$, then it continues to decrease monotonically toward zero. We now establish a more positive counterpart to this result: If the capital stock k ever grows sufficiently high, the probability that electoral competition from the demagogue never drives capital below that high level is bounded strictly away from zero.

Proposition 8 *Conditional on the economy reaching capital stock level $\tilde{k} > \hat{k}$, the probability that the capital stock never falls below $\tilde{k}$ exceeds $\frac{G(0)^{j+1}}{1-G(0)^j+G(0)^{j+1}}$, where j is the smallest integer solving $(\beta(\rho+\phi))^{j/s}\rho \geq 1$.*

When $k > \hat{k}$, the benevolent party proposes to invest $\lambda(k) > \lambda_P = (\beta(\rho+\phi))^{1/s} - \rho > 1 - \rho$. Thus, if b wins j consecutive times, and then loses to the demagogue, capital remains above $\tilde{k}$: a strict lower bound on the capital stock following that scenario is

$$(\rho + \lambda_P)^j \rho \tilde{k} = (\beta(\rho+\phi))^{j/s})\rho\tilde{k} \geq \tilde{k}.$$

For the demagogue ever to drive the capital stock below $\tilde{k}$, it must win at the outset often enough that the ratio of wins by the benevolent party to wins by the demagogue falls below j. Thus, defining $G \equiv G(0)$, a strict upper bound on this probability is

$$\begin{aligned} & (1+G+G^2+\ldots+G^{j-1})(1-G) + (1+G+G^2+\ldots+G^{j-1})G^j(1-G)^2+\ldots \\ = \; & (1-G)(1+G+G^2+\ldots+G^{j-1})\sum_{i=0}^{\infty}(G^j(1-G))^i \\ = \; & (1-G^j)\sum_{i=0}^{\infty}(G^j(1-G))^i \\ = \; & \frac{1-G^j}{1-G^j(1-G)}. \end{aligned}$$

That is, the ratio falls below j if the demagogue wins any of the first j elections, which happens with probability $(1+G+G^2+\ldots+G^{j-1})(1-G)$, or if the demagogue does not win any of the first j elections, but wins twice before the benevolent party wins $2j$ elections, and so on. Thus, the probability that the capital stock does not fall below $\tilde{k}$ exceeds

$$1 - \frac{1 - G^j}{1 - G^j + G^{j+1}} = \frac{G^{j+1}}{1 - G^j + G^{j+1}} > 0,$$

establishing the proposition.

The probability that the capital stock remains high increases in the benevolent party's valence advantage, as captured by $G(0)$, and the investment level $\lambda(k)$. The lower bound λ_P on $\lambda(k)$, which, as Figure 5 illustrates, is tight when $G(0)$ is small, and, more importantly, is almost proportional to $\lambda(k)$ when k is large, also gives a rough lower bound on the probability that the capital stock falls below $\tilde{k}$. This lower bound on $\lambda(k)$ is increasing in the discount factor β and the productivity ϕ of capital, and decreasing in the rate of depreciation $1 - \rho$. For example, using the parameters from Figure 5, the lower bound on the probability that the economy never reaches a death spiral once the capital stock is sufficiently high is 81.7% for $\sigma = 0.5, \mu = -0.5$, and 97.7% when $\sigma = 0.5, \mu = -1$.

Summarizing the content of Propositions 6 and 8, if at the outset, an economy has the good fortune of electing enough benevolent leaders, those leaders may grow the capital by so much that there is a good chance of forestalling a retreat below the current strong level. Concretely, enlightened leadership by Washington, Adams, Madison, Lincoln, Roosevelt,... can build enough institutional capital to forestall the adverse effects of later occasionally drawing a demagogue. If, instead, the economy has the misfortune at the outset of drawing a few demagogues before enough institutional capital has been created, it may necessarily doom the economy forever.

8 Conclusion

Our paper investigates the long-run susceptibility of Democracy to demagogues, studying the tension between far-sighted, magnanimous representatives who guard the long-run interests of voters, and office-seeking demagogues who cater to voters' short-run desires, highlighted by Hamilton. We analyze these political dynamics in a model with three players: (i) Short sighted voters who vote based on current but not future payoffs; (ii) a far-sighted politician, who seeks to guard voters' long-run interests; and (iii) a demagogue who exploits voters' short-sightedness for the sole purpose of gaining power. We model the political decision process as a (social) capital investment problem in which politicians propose how to allocate existing resources between current consumption and investment. Voters base political choices on a comparison of the current period utility derived from policy proposals and a stochastic valence shock.

The demagogue's sole focus on winning leads him to cater to short-sighted voters by underinvesting. The far-sighted politician faces a fundamental tradeoff: He can choose a policy that would be better for voters in the long-run if implemented, but because only the winner gets to choose the policy, he must also be concerned about appealing to short-sighted voters.

One of the main insights of our analysis is that even if the demagogue's chance of winning is remote (because the demagogue's ex-ante expected valence is very negative), his presence can nevertheless have an outsized influence on the far-sighted politician's policy and long run outcomes. This influence can be so large that the economy enters a death spiral with capital declining relentlessly to zero. Indeed, with CRRA preferences and risk aversion levels exceeding one, death spirals always occur with positive probability regardless of the current capital stock or the demagogue's expected valence disadvantage. Young democracies with limited social capital, and democracies hit by capital-reducing economic shocks are especially vulnerable. Thus, our paper provides theoretical underpinnings for Hamilton's concerns about the fragility of democracy in the presence of short-sighted voters and demagogues.

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9 Appendix

Proof of Proposition 1. We showed in the text that the value function is differentiable in k. We proceed by using Euler equations.

$$\begin{aligned} V(k_t) &= \max_{i_t \in [0, \phi k_{t-1}]} u(\phi k_t - i_{t+1}) + \beta V(k_{t+1}) \\ & \quad s.t.\ k_{t+1} = \rho k_t + i_{t+1}. \end{aligned}$$

Use the change of variable from i to $c_t = \phi k_t - i_{t+1}$ to rewrite the problem as

$$\begin{aligned} V(k_t) &= \max_{c_t \in [0, \phi k_t]} u(c_t) + \beta V(k_{t+1}) \\ & \quad s.t.\ k_{t+1} = (\rho + \phi) k_t - c_t. \end{aligned}$$

The associated first-order condition yields

$$u'(c_t) = \beta V'(k_{t+1}). \tag{20}$$

Similarly, for the next period, $k_{t+2} = (\rho + \phi) k_{t+1} - c_{t+1}$,

$$u'(c_{t+1}) = \beta V'(k_{t+2}). \tag{21}$$

By the Envelope Theorem,

$$V'(k_t) = \beta(\rho + \phi) V'(k_{t+1}). \tag{22}$$

Thus, for the next period,

$$V'(k_{t+1}) = \beta(\rho + \phi) V'(k_{t+2}). \tag{23}$$

Thus, from equations (20)–(23),

$$\frac{u'(c_{t+1})}{u'(c_t)} = \frac{V'(k_{t+2})}{V'(k_{t+1})} = \frac{1}{\beta(\rho + \phi)}. \tag{24}$$

The budget constraint for the next period is

$$k_{t+2} = (\rho + \phi) k_{t+1} - c_{t+1} \quad \Leftrightarrow \quad k_{t+1} = \frac{c_{t+1}}{\rho + \phi} + \frac{k_{t+2}}{\rho + \phi}.$$

Next, recursively apply the budget constraint to get

$$\begin{aligned} c_t &= (\rho + \phi) k_t - k_{t+1} \\ &= (\rho + \phi) k_t - \frac{c_{t+1}}{\rho + \phi} - \frac{k_{t+2}}{\rho + \phi} \\ &= (\rho + \phi) k_t - \frac{c_{t+1}}{\rho + \phi} - \frac{c_{t+2}}{(\rho + \phi)^2} - \frac{k_{t+3}}{(\rho + \phi)^2} \\ &\vdots \\ &= (\rho + \phi) k_t - \frac{c_{t+1}}{\rho + \phi} - \frac{c_{t+2}}{(\rho + \phi)^2} - \frac{c_{t+3}}{(\rho + \phi)^3} - \cdots . \end{aligned} \tag{25}$$

Thus, assuming existence, if (24) yields $c_{t+1} \propto c_t$, then it follows that $c_t \propto k_t$.

When $u(c) = \frac{c^{1-s}}{1-s}$, for $s \neq 1$, and $u(c) = \log(c)$, for $s = 1$, then (24) yields

$$\left(\frac{c_t}{c_{t+1}}\right)^s = \frac{1}{\beta(\rho+\phi)} \quad \Leftrightarrow \quad \frac{c_{t+1}}{c_t} = [\beta(\rho+\phi)]^{1/s}. \tag{26}$$

Now, substituting from (26) into (25),

$$c_t = (\rho+\phi)k_t - \frac{[\beta(\rho+\phi)]^{1/s}}{\rho+\phi}c_t - \frac{([\beta(\rho+\phi)]^{1/s})^2}{(\rho+\phi)^2}c_t - \frac{([\beta(\rho+\phi)]^{1/s})^3}{(\rho+\phi)^3}c_t - \cdots.$$

Thus,

$$\begin{aligned}(\rho+\phi)k_t &= c_t + \left(\frac{[\beta(\rho+\phi)]^{1/s}}{\rho+\phi}\right)c_t + \left(\frac{[\beta(\rho+\phi)]^{1/s}}{\rho+\phi}\right)^2 c_t + \left(\frac{[\beta(\rho+\phi)]^{1/s}}{\rho+\phi}\right)^3 c_t + \cdots \\ &= \frac{c_t}{1 - \frac{[\beta(\rho+\phi)]^{1/s}}{\rho+\phi}}, \text{ assuming } \beta(\rho+\phi)^{1-s} < 1, \text{ which is true by Assumption 2.}\end{aligned}$$

Cross-multiplying, yields that consumption is a linear function of k_t.

$$c_t = \left(\rho + \phi - [\beta(\rho+\phi)]^{1/s}\right)k_t.$$

Thus, investment is given by

$$i_{t+1} = \phi k_t - c_t = \left(\phi - \rho - \phi + [\beta(\rho+\phi)]^{1/s}\right)k_t = \left([\beta(\rho+\phi)]^{1/s} - \rho\right)k_t.$$

For the FOC to hold, i.e., for investment to be positive, we need $(\beta(\rho+\phi))^{1/s} > (\beta\phi)^{1/s} > 1 > \rho$, which holds by Assumption 2. ■

Proof of Lemma 1. Party b's stream of discounted payoffs is bounded by the same argument that we presented in the text for the social planner.

In state ω, let $i_t(\omega)$ be an optimal investment given an initial capital stock $\bar{k}$, and let $\tilde{i}_t(\omega)$ be an optimal investment given $\alpha\bar{k}$, where $\alpha > 0$. Let $k(\omega)$ and $\tilde{k}(\omega)$ be the associated capital stocks, and $D_t(\omega)$ and $\tilde{D}_t(\omega)$ be the associated valence cutoffs.

Now, consider a possibly different investment $\hat{i}_t(\omega) = \alpha i_t(\omega)$. Note that the constraints of Problem (8) are satisfied by $\hat{i}_t(\omega)$ when the initial capital stock is $\alpha\bar{k}$. Moreover, the capital stock is $\hat{k}_t(\omega) = \alpha k_t(\omega)$. Thus, letting $\hat{D}_t(\omega)$ be the associated valence cutoff, $\hat{D}_t(\omega) = D_t(\omega)$. Thus, the expectation $E[\cdot]$ under k_t, i_t, starting at $\bar{k}$, is the same as the expectation $E[\cdot]$ under $\hat{k}_t, \hat{i}_t$, starting at $\alpha\bar{k}$.

The expected discounted payoff under $\{\tilde{i}_t(\omega), \tilde{k}_t(\omega)\}_{t=0}^{\infty}$ is higher than under $\{\hat{i}_t(\omega), \hat{k}_t(\omega)\}_{t=0}^{\infty}$ because $\{\tilde{i}_t(\omega), \tilde{k}_t(\omega)\}_{t=0}^{\infty}$ is an optimal solution (the inequality below). Factor α out of the objective (the equality below) to obtain:

$$
\begin{aligned}
&E\left[\sum_{t=0}^{\infty}\beta^t\Big(W_t(\omega)\log(\phi\tilde{k}_{t-1}(\omega)-\tilde{i}_t(\omega))+(1-W_t(\omega))\log(\phi\tilde{k}_{t-1}(\omega))\Big);\{\tilde{i}_t(\omega),\tilde{k}_t(\omega)\}_{t\in\mathbb{N}}\right]\\
&\geq E\left[\sum_{t=0}^{\infty}\beta^t\Big(W_t(\omega)\log(\phi\alpha k_{t-1}(\omega)-\alpha i_t(\omega))+(1-W_t(\omega))\log(\phi\alpha k_{t-1}(\omega))\Big);\{\alpha i_t(\omega),\alpha k_t(\omega)\}_{t\in\mathbb{N}}\right]\\
&=\frac{\log(\alpha)}{1-\beta}+E\left[\sum_{t=0}^{\infty}\beta^t\Big(W_t(\omega)\log(\phi k_{t-1}(\omega)-i_t(\omega))+(1-W_t(\omega))\log(\phi k_{t-1}(\omega))\Big);\{i_t(\omega),k_t(\omega)\}_{t\in\mathbb{N}}\right].
\end{aligned}
$$

Let $V(k)$ denote the expected discounted payoff to party b. Then this argument shows that

$$V(\alpha k)\geq\frac{\log(\alpha)}{1-\beta}+V(k). \tag{27}$$

Because k and α are arbitrary it follows that

$$V(k)=V\left(\frac{1}{\alpha}(\alpha k)\right)\geq\frac{\log\left(\frac{1}{\alpha}\right)}{1-\beta}+V(\alpha k). \tag{28}$$

Suppose that the inequality in (27) is strict. Combining (27) and (28) yields:

$$V(k)\geq\frac{\log\left(\frac{1}{\alpha}\right)}{1-\beta}+V(\alpha k)>\frac{\log\left(\frac{1}{\alpha}\right)}{1-\beta}+\frac{\log(\alpha)}{1-\beta}+V(k)=V(k),$$

a contradiction. Thus, $V(k)=\log(k)/(1-\beta)+V(1)$. ■

Proof of Proposition 2. Part 1. We first prove that $\lambda(\rho)$ is strictly less than the socially optimal investment. Equation (6) of Proposition 1 shows that, with log utility ($s = 1$), the social optimum is $\lambda = \beta\phi - (1-\beta)\rho$. Inserting this into (11) implies that the left-hand side is zero. We next show that the right-hand side of (11), which represents the marginal cost of increasing λ, is strictly positive.

(11) implies that $\lambda > 0$ in any social optimum. Thus, the dynamic payoff from investing $\lambda > 0$ exceeds the payoff from investing nothing. Thus the cost to party b of losing the election is given by $\log((\phi-\lambda)/\phi)+\beta\log((\rho+\lambda)/\rho) > 0$. Thus, marginal cost is strictly positive. As a consequence it is optimal to lower λ from the social optimum, i.e., to set $\lambda(\rho) < \beta\phi-(1-\beta)\rho$ for all $0 < \rho \leq 1$.

Part 2(a). We show that $\lim_{\rho\downarrow 0}\lambda(\rho) = 0$. Let $\rho \downarrow 0$. Suppose by way of contradiction that λ remains bounded away from zero. Then there exists a sequence ρ_n, $n \in \mathbb{N}$ such that associated levels, λ_n converge to $\bar{\lambda} > 0$. If $n \to \infty$ then the left-hand side of (11) converges to $G((\phi - \bar{\lambda})/\phi)\left(\beta\phi - \bar{\lambda}\right)$. We show that $\lim_{\rho\downarrow 0} \lambda(\rho) = 0$. Let $\rho \downarrow 0$. Suppose by way of contradiction that λ remains bounded away from zero. Then there exists a sequence ρ_n, $n \in \mathbb{N}$ such that associated levels, λ_n converge to $\bar{\lambda} > 0$. If $n \to \infty$ then the left-hand side of (11) converges to $G((\phi - \bar{\lambda})/\phi)\left(\beta\phi - \bar{\lambda}\right)$. In contrast, the right-hand side of (11) goes to ∞ because $\log((\rho_n + \lambda_n)/\rho_n) \to \infty$, while $\lim_{n\to\infty} g(D)(\rho + \lambda) = g((\phi - \bar{\lambda})/\phi)\bar{\lambda}$. Thus, the right-hand side of (11) exceeds the left-hand side for all sufficiently large n, a contradiction to the assumption that λ_n satisfies the first-order condition for ρ_n. Thus, $\lambda(\rho)$ must converge to zero as $\rho \downarrow 0$.

Parts 2(b). We show that $\lim_{\rho\downarrow 0} \lambda(\rho)/\rho = \infty$. The left-hand side of (11), i.e., the marginal benefit of saving converges to $G(0)\beta\phi$ as $\rho \downarrow 0$, which is strictly positive. Thus, the right-hand side of (11), i.e., the marginal cost, must also be non-zero in the limit. Again, note that D converges to 0 as $\rho \downarrow 0$. Next, because $\lim_{\rho\downarrow 0} \lambda(\rho) = 0$ it follows that $\log((\phi - \lambda(\rho))/\phi)$ goes to zero. Thus, $(\rho + \lambda(\rho)) \log((\rho + \lambda(\rho))/\rho)$ must be non-zero in the limit. Given that both ρ and $\lambda(\rho)$ go to zero, it follows that $\log((\rho + \lambda(\rho))/\rho) = \log(1 + \lambda(\rho)/\rho)$ goes to infinity. Thus, $\lambda(\rho)/\rho$ goes to infinity.

Next, we show that $\lim_{\rho\downarrow 0} \rho \log((\rho+\lambda(\rho))/\rho) = 0$. Suppose by way of contradiction that there exists a sequence ρ_n such that $\lim_{\rho\downarrow 0} \rho_n \log((\rho_n + \lambda(\rho_n))/\rho_n) = a \neq 0$. Then this and the fact that $\lambda(\rho)/\rho$ becomes unbounded implies that $\lambda(\rho_n) \log((\rho_n + \lambda(\rho_n))/\rho_n)$ goes to infinity, a contradiction because the left-hand side of (11) is bounded, as shown above. This can only be the case if $\lambda(\rho)/\rho$ goes to infinity if $\rho \downarrow 0$. Taking the limit on both sides of (11) as $\rho \downarrow 0$ therefore yields

$$G(0)\beta\phi = g(0)\beta \lim_{\rho\downarrow 0} \lambda(\rho) \log\left(\frac{\rho + \lambda(\rho)}{\rho}\right).$$

Thus,

$$\begin{aligned} G(0)\phi/g(0) &= \lim_{\rho\to 0} \lambda(\rho) \log\left(\frac{\rho + \lambda(\rho)}{\rho}\right) = \lim_{\rho\to 0} \lambda(\rho) \log\left(\frac{\lambda(\rho)}{\rho}\right) \\ &= \lim_{\rho\to 0} \lambda(\rho) \log(\lambda(\rho)) - \lim_{\rho\to 0} \lambda(\rho) \log(\rho) = -\lim_{\rho\to 0} \lambda(\rho) \log(\rho), \end{aligned}$$

where we have used the fact that $\lim_{x\to 0} x \log x = 0$. Thus,

$$\lim_{\rho\to 0} \lambda(\rho) \left(-\frac{g(0)}{G(0)} \frac{\log(\rho)}{\phi}\right) = 1.$$

That is, $\lambda(\rho)$ goes to zero at the indicated rate. ■

Proof of Lemma 2. Party b's investment as a share of capital, λ, is given by (11), and is independent of k. Thus, if b wins in a period n then $k_n = (\rho + \lambda)k_{n-1}$. In contrast, if b loses, then the capital stock becomes $k_n = \rho k_{n-1}$. Let u be the number of times that b wins and $t - u$ the number of times that d wins in t elections. Then the capital stock at the beginning of period t,

$$k_{t-1} = \bar{k}(\rho + \lambda)^u \rho^{t-u} = \bar{k}\left((\rho + \lambda)^{\frac{u}{t}} \rho^{1-\frac{u}{t}}\right)^t, \tag{29}$$

Let $p = G(D)$ be the probability that b wins. Note that D is independent of k. Let X_t, $t \in \mathbb{N}$ be the stochastic process that assumes the value 1 if b wins, and 0, otherwise. Let $a > 0$. Then the weak law of large numbers implies that $\lim_{t\to\infty} \text{Prob}\left(\left\{(1/t)\sum_{n=0}^{t-1}\left|X_n - p\right| > a\right\}\right) = 0$. Hence for every $\delta > 0$, there exists $\bar{t}$ such $u/t < p + a$ for all $t \geq \bar{t}$ and $1 - u/t > 1 - p - a$. Thus, (29) implies

$$k_{t-1} \leq \bar{k}\left((\rho + \lambda)^{p+a} \rho^{1-p-a}\right)^t. \tag{30}$$

Thus, if $(\rho + \lambda)^{p+a} \rho^{1-p-a} < 1$ then (30) implies that k_{t-1} converges to zero. The result follows because a was arbitrary. ■

Proof of Corrolary 2. Divide equation (14) by $\rho + \lambda$ to obtain:

$$\beta(G(D)\frac{\phi - \lambda}{\rho + \lambda} - g(D)\log(\frac{\rho + \lambda}{\rho})) + (1 - \beta)\Big(G(D) - g(D)\log(\frac{\phi - \lambda}{\phi})\Big) = 0.$$

We now show that the term with weight $1 - \beta$ is negative at the value that maximizes survival, i.e., when the term with weight β is zero. That is, at $G(D) = g(D)\log(\frac{\rho+\lambda}{\rho})\frac{\rho+\lambda}{\phi-\lambda}$, we show that

$$\log\left(\frac{\rho + \lambda}{\rho}\right)\frac{\rho + \lambda}{\phi - \lambda} > \log\left(\frac{\phi - \lambda}{\phi}\right).$$

Multiplying both sides by $\phi - \lambda$ yields

$$\log\left(\left(\frac{\rho + \lambda}{\rho}\right)^{\rho+\lambda}\right) > 0 > \log\left(\left(\frac{\phi - \lambda}{\phi}\right)^{\phi-\lambda}\right)$$

It follows that at the value of λ that maximizes survival, the marginal benefit of increasing λ is less than the marginal cost, i.e., party b will choose a smaller λ. Letting $x(\lambda)$ be the term multiplying β and $y(\lambda) < x(\lambda)$ be the term multiplying $1 - \beta$, we have $\frac{d\lambda}{d\beta} = -\frac{(x-y)}{\beta x' + (1-\beta)y'} > 0$ ■

Proof of Proposition 3. We first prove the result for Linear Markov Perfect Equilibria.

Let $V_P(k; \lambda_{-P})$ be the value function of party $P = b, d$, when the initial capital stock is k and the other party uses a linear strategy $k' \mapsto \lambda_{-P}\, k'$. Then, (λ_b, λ_d) characterize a linear MPE if and only if they solve the following optimization problems for all values of $k > 0$:

$$\begin{aligned} V_b(k; \lambda_d) = \max_{\lambda_b \in [0,\phi]} & G\left(\log\left(\frac{\phi - \lambda_b}{\phi - \lambda_d}\right)\right)(\log((\phi - \lambda_b)k) + \beta V_b((\rho - \lambda_b)k; \lambda_d)) \\ & + \left(1 - G\left(\log\left(\frac{\phi - \lambda_b}{\phi - \lambda_d}\right)\right)\right)(\log((\phi - \lambda_d)k) + \beta V_b((\rho - \lambda_d)k; \lambda_d)). \end{aligned}$$

$$V_d(k;\lambda_b) = \max_{\lambda_d\in[0,\phi]} G\left(\log\left(\frac{\phi-\lambda_b}{\phi-\lambda_d}\right)\right)(\beta V_d((\rho-\lambda_b)k;\lambda_b))+\left(1-G\left(\log\left(\frac{\phi-\lambda_b}{\phi-\lambda_d}\right)\right)\right)(1+\beta V_d((\rho-\lambda_d)k;\lambda_b))$$

Expanding the demagogue's value function reveals that it does not depend on the initial capital k:

$$v_d(\lambda_b) = V_d(k;\lambda_b) = \max_{\lambda_d\in[0,\phi]} \sum_{t=0}^{\infty} \beta^t\Big(1 - G\Big(\log\Big(\frac{\phi-\lambda_b}{\phi-\lambda_d}\Big)\Big)\Big). \tag{31}$$

Therefore, the demagogue's strategy is not to invest at all, i.e., to choose $\lambda_d = 0$. But from our analysis of the myopic case, we know that if the demagogue does not invest, investment strategies are linear. It remains to show that, if party b chooses a linear strategy, then the demagogue's best response is also linear. Using the scalability argument from before, we show that, in fact, if party b chooses a linear strategy, the the demagogue's best response is not to invest. Let $V_d(k)$ be party d's value function when the initial capital stock is k, and party b uses a linear strategy $\lambda_b k$.

$$V_d(\bar{k}) = \max_{\{i_t^R\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\Big(1 - G\Big(\log\Big(\frac{(\phi-\lambda_b)k_{t-1}}{\phi k_{t-1} - i_t^R}\Big)\Big)\Big). \tag{32}$$

Let $\tilde{i}^R$ be the demagogue's best response. Now, suppose the initial capital changes to $\alpha\bar{k}$, and conside the demagogue's strategy in which the demagogue chooses $\hat{i}_t^R = \alpha\tilde{i}^R$. We have

$$V_d(\alpha\bar{k}) \geq \sum_{t=0}^{\infty}\Big(1 - G\Big(\log\Big(\frac{(\phi-\lambda_b)k_{t-1}}{\phi k_{t-1} - \tilde{i}_t^R}\Big)\Big)\Big) = V_d(\bar{k}). \tag{33}$$

Now, use $\frac{1}{\alpha}$ instead of α, and $\alpha\bar{k}$ instead of $\bar{k}$ to get $V_d(\bar{k}) \geq V_d(\alpha\bar{k})$. Thus, $V_d(\bar{k}) = V_d(\alpha\bar{k})$, and hence $V_d(k) = V_d(1)$. That is, if party b's uses a linear strategy, then the demagogue's payoff does not depend on the capital stock. Hence, the future capital stock is irrelevant for the demagogue, and hence he does not invest. This proves the result for linear Markov perfect equilibria.

Next, we show that our results with a myopic demagogue also hold for finite horizon economies as the number of periods goes to infinity.

The game begins in period $t = 0$ with initial capital $k_{-1} = \bar{k}$, and ends in period $T \geq 1$. We proceed by backward induction. In period T both parties obviously have a dominant strategy not to invest. Thus,

$$i_T^R = i_T^L = 0, \text{ and } V_T(k_{T-1}) = \log(\phi k_{T-1}),$$

where V_T is party b's value function in period T. In period $T-1$, the demagogue does not invest because it hurts his current period payoff, and it does not change his future period payoffs. To analyze party b's optimization problem at $T-1$, let $V_{T-1}(k_{T-2})$ be party b's value function at time $T-1$, and define $\lambda_{T-1} = i^L_{T-1}/k_{T-2}$. Then

$$V_{T-1}(k_{T-2}) = \max_{\lambda_{T-1}\in[0,\phi]} G\Big(\log\Big(\frac{\phi-\lambda_{T-1}}{\phi}\Big)\Big)\Big(\log((\phi-\lambda_{T-1})k_{T-2}) + \beta V_T((\rho-\lambda_{T-1})k_{T-2})\Big)$$
$$+ \Big(1 - G\Big(\log\Big(\frac{\phi-\lambda_{T-1}}{\phi}\Big)\Big)\Big)\Big(\log(\phi k_{T-2}) + \beta V_T(\rho k_{T-2})\Big).$$

Using the fact that $V_T(k_{T-1}) = \log(\phi k_{T-1})$ yields

$$V_{T-1}(k_{T-2}) = (1+\beta)\log(k_{T-2}) + \max_{\lambda_{T-1}\in[0,\phi]} G\Big(\log\Big(\frac{\phi-\lambda_{T-1}}{\phi}\Big)\Big)(\log(\phi-\lambda_{T-1}) + \beta\log(\rho-\lambda_{T-1}))$$
$$+ \Big(1 - G\Big(\log\Big(\frac{\phi-\lambda_{T-1}}{\phi}\Big)\Big)\Big)(\log(\phi) + \beta\log(\rho)). \tag{34}$$

Thus, the optimal λ_{T-1} depends on the model parameters (ρ,ϕ,β), but not on capital. Let $\tilde{V}_{T-1}$ be the value of the maximization problem on the right-hand side of (34). Then

$$V_{T-1}(k_{T-2}) = (1+\beta)\log(k_{T-2}) + \tilde{V}_{T-1}(\beta,\phi,\rho). \tag{35}$$

Next, consider period $T-2$. Again, d chooses $i^R_{T-2} = 0$ because it maximizes his current period payoff (i.e., the winning probability), and his future payoffs do not depend on the capital stock, because party b's behavior, and hence d's winning probabilities do not depend on the capital stock. Now, consider b's optimization in period $T-2$. We repeat the above procedure.

$$\begin{aligned} V_{T-2}(k_{T-3}) = \max_{\lambda_{T-2}\in[0,\phi]} & G\Big(\log\Big(\frac{\phi-\lambda_{T-2}}{\phi}\Big)\Big)(\log((\phi-\lambda_{T-2})k_{T-3}) + \beta V_{T-1}((\rho+\lambda_{T-2})k_{T-3})) \\ & + \Big(1 - G\Big(\log\Big(\frac{\phi-\lambda_{T-2}}{\phi}\Big)\Big)\Big)(\log(\phi k_{T-3}) + \beta V_{T-1}(\rho k_{T-3}). \end{aligned} \tag{36}$$

We substitute (35) into (36), factoring out the term $\log(k_{T-3})$, and rearrange to obtain

$$\begin{aligned} V_{T-2}(k_{T-3}) = {} & (1+\beta+\beta^2)\log(k_{T-3}) + \beta\tilde{V}_{T-1} \\ & + \max_{\lambda_{T-2}\in[0,\phi]} G\Big(\log\Big(\frac{\phi-\lambda_{T-2}}{\phi}\Big)\Big)(\log(\phi-\lambda_{T-2}) + \beta(1+\beta)\log(\rho+\lambda_{T-2})) \\ & + \Big(1 - G\Big(\log\Big(\frac{\phi-\lambda_{T-2}}{\phi}\Big)\Big)\Big)(\log(\phi) + \beta(1+\beta)\log(\rho)). \end{aligned} \tag{37}$$

Thus, the optimal λ_{T-2} does not depend on capital. Define $\tilde{V}_{T-2}$ to be the value of the maximization problem on the right-hand side of (37). Then

$$V_{T-2}(k_{T-3}) = (1+\beta+\beta^2)\log(k_{T-3}) + \tilde{V}_{T-2}(\beta,\phi,\rho) + \beta\tilde{V}_{T-1}(\beta,\phi,\rho). \tag{38}$$

Continuing inductively implies

$$V_{T-n}(k_{T-(n+1)}) = (1+\beta+\cdots+\beta^n)\log(k_{T-(n+1)}) + \tilde{V}_{T-n} + \beta\tilde{V}_{T-(n-1)} + \cdots + \beta^{n-1}\tilde{V}_{T-1}, \tag{39}$$

where

$$\tilde{V}_{T-k} = \max_{\lambda_{T-k}\in[0,\phi]} G\Big(\log\Big(\frac{\phi-\lambda_{T-k}}{\phi}\Big)\Big)\Big(\log(\phi-\lambda_{T-k})+\beta\sum_{i=0}^{k-1}\beta^i\log(\rho+\lambda_{T-k})\Big) \\ +\Big(1-G\Big(\log\Big(\frac{\phi-\lambda_{T-k}}{\phi}\Big)\Big)\Big)\Big(\log(\phi)+\beta\sum_{i=0}^{k-1}\beta^i\log(\rho)\Big). \tag{40}$$

Choose $n = T$ to get

$$V_0(k_{-1};T) = \sum_{t=0}^{T}\beta^t\log(k_{-1}) + \sum_{t=0}^{T-1}\beta^t\tilde{V}_t. \tag{41}$$

Note that (40) implies that for given β, ϕ and ρ there exists $M > 0$ such that $|\tilde{V}_{T-k}| < M$ for all T and k. Thus, $v = \lim_{T\to\infty}\sum_{t=0}^{T-1}\beta^t\tilde{V}_t$ exists. Recall that $k_{-1} = \bar{k}$. Then

$$V(\bar{k}) = \lim_{T\to\infty} V_0(\bar{k};T) = \frac{\log(\bar{k})}{1-\beta} + v. \tag{42}$$

Thus, we get the same value function as when d is myopic, establishing the result. ■

Proof of Proposition 4. We construct trigger strategies, in which the two parties offer strategies $\lambda_b(k)$ and $\lambda_d(k)$, unless one of the parties deviates, in which case we revert to the unique linear Markov equilibrium.

Let $D = \log(\phi - \lambda_b(k)) - \log(\phi - \lambda_d(k))$. Recall that the demagogue's winning probability in that period is $1 - G(D)$. Let $\hat{\lambda}_b$, $\hat{\lambda}_d = 0$ be the strategies in the linear Markov equilibrium.

Suppose by way of contradiction that there exists a sequence of discount factors β_n and a sequence of strategies $\lambda_{b,n}(k)$ and $\lambda_{d,n}(k)$ that can be supported as a trigger-strategy equilibrium, which converges to the first-best strategies, i.e., $\lim_{n\to\infty}\lambda_{b,n}(k) = \lim_{n\to\infty}\lambda_{d,n}(k) = \lambda^*$. Then this implies that the demagogue's winning probability converges to $1 - G(0)$.

Let $\hat{\lambda}_{b,n}$, $\hat{\lambda}_{d,n} = 0$ be the linear Markov equilibrium strategies given discount factor β_n. Let $\hat{D}_n = \log(\phi - \hat{\lambda}_{b,n}) - \log(\phi)$. Given our characterization of the linear Markov equilibrium it is immediate that there exists $\kappa > 0$ such that $\hat{\lambda}_{b,n} \geq \kappa$ for all n. From assumption 1, $0 < G(0) < 1$ and the density $g(0) > 0$. Thus, $\limsup_n 1 - G(\hat{D}_n) < 1 - G(0)$. However, this means that the demagogue would strictly better off deviating to 0 for sufficiently large n, a contradiction. Therefore, the first-best cannot be obtained in the limit. ■

Proof of Lemma 3. Pick $k', k > 0$, and without loss of generality, suppose $k' < k$. Let $\alpha = k'/k \in (0,1)$. If $\lambda(k) = \lambda(k')$, then $\alpha^{1-s}V(k) \geq V(\alpha k)$. To see this, observe that if the winning probabilities remained unchanged, then $V(\alpha k) = V(k') = \alpha^{1-s}V(k)$. However, the winning probability is increasing in k, and hence $V(\alpha k) = V(k') \leq \alpha^{1-s} V(k)$. Then,

$$\liminf_{k' \to k} \frac{V(k) - V(k')}{k - k'} \geq \liminf_{\alpha \to 1} \frac{V(k) - \alpha^{1-s} V(k)}{(1-\alpha)k} = \lim_{\alpha \to 1} \frac{V(k)}{k} \frac{1 - \alpha^{1-s}}{1 - \alpha} = \frac{(1-s)V(k)}{k}$$
$$\geq \frac{(1-s)V_N(k)}{k} = V_N'(k),$$

where the last equality follows from eqaution (15). ■

Proof of Lemma 4. We first prove a lemma.

Lemma 5 *Suppose that the coefficient of relative risk aversion* $s > 1$. *Then there exists* $\bar{\lambda} < \phi$ *and* $\bar{k} > 0$ *such that* $\lambda(k) < \bar{\lambda}$ *for all* $k \geq \bar{k}$.

Proof of Lemma 5. Suppose by way of contradiction that there exists a sequence $k_n \to \infty$ such that $\lim_{n\to\infty} \lambda(k_n) = \phi$. The net benefit of investment contingent on winning the election is

$$u((\phi - \lambda)k_n) - u(\phi k_n) + \beta \left(V((\rho + \lambda)k_n) - V(\rho k_n)\right) < u((\phi - \lambda)k_n) - u(\phi k_n) - \beta V_N(\rho k_n), \quad (43)$$

where the inequality follows because utility is negative for $s > 1$ and V_N is a lower bound on the continuation utility. Note that

$$\begin{aligned} &\lim_{n\to\infty} u((\phi - \lambda)k_n) - u(\phi k_n) - \beta V_N(\rho k_n) \\ &\quad = \lim_{n\to\infty} \frac{1}{(1-s)k_n^{s-1}} \left((\phi - \lambda(k_n))^{1-s} - \phi^{1-s} - \frac{\beta \phi^{1-s}}{1 - \beta \rho^{1-s}} \right) < 0, \end{aligned} \quad (44)$$

because $\lim_{n\to\infty} \lambda(k_n) = \phi$ implies $\lim_{n\to\infty} (\phi - \lambda(k_n))^{1-s} = -\infty$, and the other terms are bounded. This implies that the net benefit from investment given by (43) is strictly negative. By choosing $\lambda = 0$ candidate b can guarantee a higher payoff, contradicting the optimality of $\lambda(k_n)$. ■

Now, we turn to the proof of Lemma 4. Let $W_b(k) = k^{s-1} V_b(k)$. Then $W(k)$ solves

$$\begin{aligned} W_b(k) &= \max\nolimits_{\lambda \in [0,\phi]} G(0)\Big(k^{s-1} u((\phi - \lambda)k) + \beta W_b((\rho + \lambda)k)\Big) + (1 - G(0))\Big(k^{s-1} u(\phi k) + \beta W_b(\rho k)\Big) \\ &= \max\nolimits_{\lambda \in [0,\phi]} G(0) \left((\phi - \lambda)^{1-s} + \beta W_b((\rho + \lambda)k) \right) + (1 - G(0)) \left(\phi^{1-s} + \beta W_b(\rho k) \right). \end{aligned} \quad (45)$$

It immediately follows that $W_b(k) = k^{s-1} V_b(k) = v_b/(1-s)$, is independent of k. Further, the objective of (45) is strictly concave in λ.

Similarly, we define $W(k) = k^{s-1} V(k)$, where $V(k)$ is the value function for problem (16). Let $\lambda(k)$ be be the associated optimal investment share. Then $W(k)$ and $\lambda(k)$ solve

$$W(k) = \max_{\lambda \in [0,\phi]} G(D)\Big(k^{s-1} u((\phi - \lambda)k) + \beta W((\rho + \lambda)k)\Big) + (1 - G(D))\Big(k^{s-1} u(\phi k) + \beta W(\rho k)\Big). \quad (46)$$

Next, assume by way of contradiction that $\lambda(k)$ does not converge to λ^* as $k \to \infty$. Then there exists a sequence $k_n \to \infty$ with $\lim_{n\to\infty} \lambda(k_n) \neq \lambda^*$. Without loss of generality, suppose that $\lambda(k_n) < \bar{\lambda} < \lambda^*$, for some $\bar{\lambda}$ (the analysis when $\bar{\lambda} > \lambda^*$ is analogous). Recall that λ^* solves problem (45) and that the objective is strictly concave in λ, and independent of k. From the strict concavity, choosing $\lambda(k_n) < \bar{\lambda} < \lambda^*$, reduces payoffs by an amount that is bounded away from zero: there exists a $\delta > 0$ such that

$$\begin{aligned} &G(0)\left((\phi - \lambda(k_n))^{1-s} + \beta W_b((\rho + \lambda(k_n))k_n)\right) + (1 - G(0))\left(\phi^{1-s} + \beta W_b(\rho k_n)\right) + \delta \\ &\quad < G(0)\left((\phi - \lambda^*)^{1-s} + \beta W_b((\rho + \lambda^*)k_n)\right) + (1 - G(0))\left(\phi^{1-s} + \beta W_b(\rho k_n)\right). \end{aligned} \tag{47}$$

Recall that the social planner's value function takes the form $V_S(k) = v_s/(1-s)k^{1-s}$. Similarly, the value function if no investment takes place takes the form $V_N(k) = v_n/(1-s)k^{1-s}$. The value function of (16) is bounded from above and below by these two value functions. Therefore,

$$\frac{v_N}{1-s} = k^{s-1}V_N(k) \leq W(k) \leq k^{s-1}V_S(k) = \frac{v_S}{1-s}. \tag{48}$$

Thus, $W(k)$ is bounded.

Let $\varepsilon = (1 + /(1-\beta)|v_s/(1-s)|) < \delta/2$. Then Lemma 5 and the argument in the text imply that there exists a $k_1 > 0$ such that $|G(D_k) - G(0)| < \varepsilon$ for all $k \geq k_1$.

Further, because $\beta < 1$, there exists a $T \in \mathbb{N}$ such that $\left|\beta^T V_N(k_1)\right| < \varepsilon$. Let $k_2 = k_1/\rho^T$. This, and (48) imply that if we start in period $t = 0$ with initial capital stock $k \geq k_2$ then the utility impact of actions in periods $\tau > T$ is less than ε. In addition, in periods $\tau \leq T$ capital $k \geq k_1$. Thus, the fact that $|G(D_k) - G(0)| < \varepsilon$ and the boundedness of W established in (48) imply that changing the probabilities from $G(0)$ to $G(D_k)$ impacts utility by less than $\varepsilon|v_s/(1-s)|$ in each period, and hence by less than $\varepsilon\beta^T/(1-\beta)|v_s/(1-s)| < \varepsilon/(1-\beta)|v_s/(1-s)|$ in the first $T+1$ periods. Given the choice of ε it follows that $|W(k) - W_b(k)| < \delta$ for all $k \geq k_2$.

Finally, equation (47) implies that if we replace λ^* by $\lambda(k_n)$ then utility decreases by more than δ. Further, we have shown that replacing W_b by W changes payoffs by less than $\delta/2$. Thus,

$$\begin{aligned} &G(0)\left((\phi - \lambda(k_n))^{1-s} + \beta W((\rho + \lambda(k_n))k_n)\right) + (1 - G(0))\left(\phi^{1-s} + \beta W(\rho k_n)\right) \\ &\quad < G(0)\left((\phi - \lambda^*)^{1-s} + \beta W((\rho + \lambda^*)k_n)\right) + (1 - G(0))\left(\phi^{1-s} + \beta W(\rho k_n)\right), \end{aligned} \tag{49}$$

which contradicts that $\lambda(k_n)$ is the optimal investment share given k_n. This contradiction proves that $\lim_{k\to\infty} \lambda(k_n) = \lambda^*$. ■