When Can Citizen Communication Hinder Successful Revolution?\textsuperscript{1}

Mehdi Shadmehr\textsuperscript{2} \quad Dan Bernhardt\textsuperscript{3}

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\textsuperscript{2}Department of Economics, University of Calgary, 2500 University Dr., Calgary, AB, T2N 1N4, Canada. E-mail: mehdi.shadmehr@ucalgary.ca

\textsuperscript{3}Department of Economics, University of Warwick and University of Illinois, 1407 W. Gregory Dr., Urbana, IL 61801. E-mail: danber@illinois.edu
Abstract

We show that when citizens are uncertain about whether a successful revolution will turn out better than the status quo, communication between citizens reduces the likelihood of successful revolution when the status quo is sufficiently bad. A bad regime faces a tradeoff: communication helps citizens to coordinate, facilitating revolution; but it also facilitates the dissemination of any negative information about the alternative to the status quo, forestalling revolution. When the regime is sufficiently bad, this latter effect dominates. This result contrasts with the literature that assumes that each citizen knows that he wants to change the regime, but he is uncertain about whether enough citizens will revolt. In such settings, communication always raises the likelihood of successful revolution.

Keywords: Revolution, Regime Change, Censorship, Media Freedom, Communication, Social Media.

Word Count: 4900.
Conventional wisdom holds that better communication facilitates revolutions. The logic is simple: If each citizen knows that he wants to change the regime, but does not know whether enough people will protest for a revolution to succeed, then better communication creates the common knowledge that, indeed, enough people will protest, enabling citizens to coordinate and overthrow the regime. However, this argument rests on the premise that citizens know that they want to change the regime. In this paper, we show that if citizens are uncertain about whether a successful revolution would be an improvement on an existing regime, then better communication technologies, freedom of expression and assembly, or other means that enable citizens to share information with each other can reduce the likelihood of successful revolution. In particular, we show that a bad regime faces a tradeoff: communication helps citizens to coordinate, facilitating revolution; but it also facilitates the dissemination of any negative information about the alternative to the status quo, forestalling revolution. When the regime is sufficiently bad, this latter effect dominates.

History is replete with instances of successful revolutions that seemed promising, but turned out badly. The Arab Spring highlights the great uncertainties that accompany successful revolutions. In Tunisia, revolution led to a more democratic government, but Egypt became a military dictatorship, and Syria and Yemen remain in civil wars. A consequence is that citizens want to learn each others’ information, not just to figure out whether revolution would succeed, but also to assess whether a successful revolution would even be desirable. To maintain the status quo, regimes use propaganda and censorship to manipulate the information available to citizens. Studies of propaganda and censorship analyze a regime’s manipulation of media reports, which alters the composition of information available to citizens, and reduces the total information available.

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1This result, interpreted as perfect communication among all citizens, holds in almost all models of revolutions, e.g., Egorov et al. (2009), Bueno de Mesquita (2010), Chen et al. (2015), Chen and Suen (2015), Edmond (2013), and Casper and Tyson (2014).

2A successful revolution is one that leads to a regime change; this does not imply that the new regime is better than the old one.

3Survey data from the Arab Barometer suggest that, as of March-April 2013, less than 13% of the Egyptians considered the Arab Spring a victory. Data and survey details are available at: http://www.arabbarometer.org/instruments-and-data-files. Such uncertainties may discourage revolutions, e.g., based on interviews, Jamal (2012) found that many Jordanians did not support a regime change, fearing that a revolution may bring about an anti-American government, reducing American support of Jordan.

We analyze a different form of communication—communication between citizens—and whether and when a regime gains by allowing such communication.\(^5\) In this context, communication does not change the total amount of information among citizens; rather, communication allows citizens to share this information and to coordinate.\(^6\) Communication also eliminates a citizen’s incentive to account for the information in the equilibrium actions of others. By eliminating learning-in-equilibrium incentives, communication reduces the likelihood of successful revolution. By facilitating coordination, communication raises the likelihood. Finally, by pooling citizen information, communication increases the information that each citizen has, raising the likelihood of successful revolution when the status quo is better than the ex-ante expected value of successful revolution, but reducing it when the status quo is worse.

To establish the overall effect of communication is challenging due to these three conflicting forces. We use Feller’s inequality to establish asymptotic results. We show that when a regime is sufficiently good, it does not want citizens to communicate because the coordination and information-pooling effects of communication swamp the elimination of the learning-in-equilibrium effects, making successful revolution more likely. However, when the status quo is bad enough, the information-pooling effect dominates: when a regime is sufficiently bad, communication reduces the likelihood of successful revolution.

This raises the question: how bad does a regime have to be to gain from fostering communication? To address this question, we numerically investigate how these asymptotic results extend to intermediate values of the status quo. We find that a regime can reduce the probability of successful revolution by allowing communication between citizens whenever the costs for participating in a failed revolution are small enough and the status quo is at least slightly worse than the ex-ante expected payoff from revolution. More generally, when the punishment is greater, a regime must be relatively worse to benefit from fostering communication. This is because communication mitigates the effect of punishment on citizens’

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\(^5\)Social media (as opposed to mass media) may better facilitate communication among citizens. Battaglini (2015) models social media as a technology that lets socially-connected citizens share information.

\(^6\)Communication among citizens is hard to censor or manipulate. Effective censorship requires a regime to observe and censor a citizen’s signal before it reaches others. Selective censorship requires complex and costly techniques. Regimes can also employ propaganda by hiring people who pretend to hold positive views. Citizens recognize that information may be propaganda or censored, reducing its effect. See Gunitsky (2015) and MacKinnon (2012) for reviews of modern censorship and propaganda techniques in dictatorships. We show that even without censorship and propaganda, an unpopular regime can benefit from communication between citizens.
incentives to revolt.

We first establish the results in the two-agent game of Shadmehr and Bernhardt (2011), where we can transparently delineate all of the underlying strategic forces that exist with a finite number of agents. We then characterize when a regime benefits from allowing communication in a setting where a continuum of citizens receive signals about the common-value payoff from revolution, and revolution succeeds if and only if enough citizens revolt. The key is the common-value nature of uncertain payoffs: as we show in the section on alternative models, when signals only concern the likelihood of successful revolution and not its merits (e.g., Morris and Shin 1998), or a citizen only knows his private-value payoff from revolution but not the payoffs of others (e.g., Bueno de Mesquita 2010), then the likelihood of successful revolution is always higher when citizens can communicate. This point emphasizes the scope of our model. We focus on communication between citizens about whether regime change is worthwhile: communication not only enables citizens to coordinate, it also causes them to update about the merits of regime change. When citizens use communication channels solely to coordinate, communication clearly hurts a regime. In essence, our paper focuses on early stages at which people may still change their minds about the merits of revolution. Once people conclude that they want to overthrow the regime (e.g., after some protesters are treated brutally or killed by security forces), communication largely serves to improve coordination.\footnote{Most empirical papers either do not distinguish between these phases of revolutions or they focus on late conflict stages, where coordination concerns dominate (e.g., Acemoglu et al. 2014).}

It is useful to compare our analysis with that in Shadmehr and Bernhardt (2011). That paper highlights learning-in-equilibrium incentives, contrasting equilibrium strategies when, rather than receiving private signals, citizens receive one public signal with the same noise as a private signal. They show that due to learning-in-equilibrium incentives, citizens are more willing to revolt with private signals if punishment for participating in a failed revolt is small. That paper does not take up the more complex question of how communication affects the likelihood of successful revolution, and their comparisons do not shed light on the effects of communication between citizens. Our analysis highlights how information pooling, which is at the heart of communication, and its combination with coordination and learning-in-equilibrium incentives affect the likelihood of successful revolution. Learning-in-equilibrium incentives are not a driving force for our findings: analogous results obtain with myopic
citizens who ignore the information content of each others’ equilibrium strategies.

In a complementary paper, Barbera and Jackson (2016) analyze the effects of different degrees of citizen communication in a private value coordination game featuring a binary state and binary signals. They find that communication among small groups (e.g., each citizen communicates with one other citizen) can reduce the likelihood of regime change, but consistent with our results, if enough citizens communicate, the likelihood of successful revolution always rises—see also Chen and Xu (2016).

More generally, our analysis provides a counterpoint to a common theme in the literature on revolutions and protests that public signals raise the ability of citizens to coordinate and overthrow a regime. In Egorov et al. (2009), Edmond (2013), and Shadmehr and Bernhardt (2015, Appendix), the public signal is a negative media report. In Egorov et al., a negative report “not only makes individual citizens aware of the dictator’s incompetence, but it also makes the dictator’s incompetence common knowledge, which is critical for a successful revolution. Revolutions involve a coordination problem: a citizen takes part...only if he knows that others will join” (p. 647). In Bueno de Mesquita (2010), the signal is the publicly-observed level of a vanguard’s violence—extensive violence signals high societal anti-regime sentiments. In Hollyer et al. (2015), the government’s economic performance acts as a public signal. Despite their differences, this literature takes a global games approach (Morris and Shin 1998, 2003) to modeling revolutions, in which citizens only have coordination concerns (e.g., about how many others will protest). In these settings, if citizens share information, so that their private signals become public, the likelihood of successful revolution always rises. Moreover, unlike Morris and Shin (2002) and Angeletos and Pavan (2007) that study the value of public information and efficient coordination, in our model, information sharing always improves welfare and leads to optimal coordination.

8Yanagizawa-Drott’s (2014, Online Appendix) model of the role of radio in genocides falls into this category: radio sends a noisy public signal about the level of a state’s sponsorship of genocide, and the payoff from participation in genocide rises in the measure of participants and the level of state sponsorship.

9Recent formal literature on revolution studies various topics such as the role of leaders and tactics (Bueno de Mesquita 2010, 2013; Loeper et al. 2014; Morris and Shadmehr 2016; Shadmehr and Bernhardt 2014; Wantchékon and García-Ponce 2014), the effect of globalization (Shadmehr 2017), elections (Egorov and Sonin 2015; Rozenas 2013), networks (Siegel 2009, 2013), and contagion (Chen and Suen 2016), interactions between pro- and anti-regime citizens (Tyson and Smith 2017), determinants of ideological extremism (DeNardo 1985; Shadmehr 2015), and middle class activism (Chen and Suen 2015).
Model and Analysis

We adopt the framework of Shadmehr and Bernhardt (2011) for most of the analysis; we later establish analogous results in a game featuring a continuum of citizens. Figure 1 presents the game. Two citizens simultaneously decide whether to revolt. A revolution succeeds if and only if both citizens revolt, in which case citizens receive the successful revolution payoff $R$. Otherwise, the status quo prevails, and citizens receive the status quo payoff $s$. If only one citizen revolts, the sole revolter pays an expected punishment cost of $\mu > 0$. The value of successful revolution $R$ is uncertain, and citizens receive noisy private signals, $x_i = R + \nu_i$, $i \in \{1, 2\}$, where $R$ and $\nu_i$'s are independent with $R \sim N(0, \sigma)$ and $\nu_i \sim N(0, \sigma_\nu)$. When citizens cannot communicate, after receiving their signals, they simultaneously decide whether to revolt. Due to the common value structure of payoffs, citizens have a shared interest in coordinating on the outcome with the higher expected payoff. As a result, when citizens can communicate they want to truthfully reveal their information.\(^{10}\) Thus, when citizens can communicate, after receiving their signals, they first share their information, and then simultaneously decide whether to revolt.

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Figure 1: Citizen Payoffs.

When citizens can communicate, it is as if they receive two public signals $p_i = R + \epsilon_i$, $R \sim N(0, \sigma^2)$, $\epsilon_i \sim N(0, \sigma^2_p)$, $i \in \{1, 2\}$, where $R$ and $\epsilon_i$'s are independent. We allow the noise in the private and public signals to differ to capture the possibility that communication improves knowledge by reducing signal error; or the possibility that communication may reduce knowledge by facilitating exaggeration or rumor, which serve to increase signal error.\(^{10}\) Given their joint information, citizens always want to coordinate: They want to revolt when $E[R|x_1, x_2] > s$, and they do not want to revolt when $E[R|x_1, x_2] \leq s$. Thus, a completely fully cheap talk equilibrium exists in which citizens truthfully report their signals, and then revolt if and only if $E[R|x_1, x_2] > s$: in this posited equilibrium, if citizen $j$ truthfully reveals his signal $x_j$, then citizen $i$ wants to reveal his signal $x_i$, and then revolt if and only if $E[R|x_1, x_2] > s$.\(^{10}\)
To analyze the effects of communication between citizens, we must contrast the likelihood of successful revolution when citizens receive two private signals versus two public signals.

In contrast to the literature, which focuses only on coordination concerns, citizens do not know how a successful revolution will turn out. Moreover, each citizen has relevant private information about the merits of a successful revolution, information that can help them determine whether revolution would be worthwhile. Our model structure captures the fact that when successful revolution represents a good outcome for one activist, it is usually a good outcome for others; and when a revolution turns sour for one citizen, it often ends badly for others.

**Analysis.** When citizens communicate, they both observe the same signals, and they can perfectly coordinate actions in any pure strategy equilibrium. Still, many pure strategy equilibria exist. Following any (public) signals $p_1$ and $p_2$, if a citizen believes that the other does not revolt, he does not revolt either; but if he believes that the other revolts, then he revolts if and only if $E[R|p_1, p_2] > s$. Therefore, in the equilibrium that features most successful revolutions, both citizens revolt (and the revolution succeeds) if and only if $E[R|p_1, p_2] = \frac{\sigma^2}{\sigma^2 + \sigma_p^2} (p_1 + p_2) > s$, i.e., the expected revolution payoff given their joint information is better than the status quo. We focus on this equilibrium because it is the worst case scenario for allowing communication from the ruler’s perspective—it bounds from below any benefits to the ruler from allowing communication—and it is the best equilibrium from a citizen welfare perspective (see, e.g., Shadmehr and Bernhardt (2011)).

Thus, with communication, citizens revolt if and only if $p_1 + p_2 > k_p \equiv \frac{2\sigma^2 + \sigma_p^2}{\sigma^2} s$, and the probability of successful revolution is:

$$P_{pub}(k_p(s)) = \int_{-\infty}^{\infty} \int_{k_p(s)-p_2}^{\infty} h(p_1, p_2) dp_1 dp_2,$$

with $k_p(s) = \frac{2\sigma^2 + \sigma_p^2}{\sigma^2} s$, \hspace{1cm} (1)

where $h(p_1, p_2)$ is the joint pdf of $p_1$ and $p_2$.

Without communication, Shadmehr and Bernhardt (2011) show that a unique stable finite-cutoff equilibrium exists as long as the status quo payoff $s$ is not too high (or when $\mu = 0$). In this equilibrium, a citizen revolts if and only if his signal exceeds a threshold $\tilde{k}(s)$
given by the smallest root of the symmetric net expected payoff $\Delta_1(k; s)$,\footnote{Given player $i$’s signal $x_i$ and given that player $j \neq i$ adopts a cutoff strategy with cutoff $k_j$, player $i$’s net expected payoff from revolting rather than not revolting, denoted by $\Delta(x_i, k_j)$, is:

$$\Delta(x_i, k_j) = Pr(x_j > k_j | x_i = k_j) \left( E[R | x_j > k_j, x_i = k_j] - s \right) - Pr(x_j \leq k_j | x_i = k_j) \mu.$$}

$$\Delta_1(k; s) = Pr(x_j > k | x_i = k) \left( E[R | x_j > k, x_i = k] - s \right) - Pr(x_j \leq k | x_i = k) \mu$$

$$= \left[ 1 - \Phi(fk) \right] \left( bk + c \frac{\phi(fk)}{1 - \Phi(fk)} - s \right) - \Phi(fk) \mu, \quad (2)$$

where $a = \sqrt{\frac{\sigma_i^4 + 2\sigma_i^2\sigma_i^2}{\sigma_i^4 + \sigma_i^2}}$, $b = \frac{\sigma_i^2}{\sigma_i^4 + \sigma_i^2}$, $c = \frac{\sigma_i^2}{\sigma_i^4 + 2\sigma_i^2} a$, and $f = \frac{1-b}{a}$ are constants. When $\mu > 0$ and the status quo payoff $s$ is sufficiently high, citizens never revolt in equilibrium, in which case we set $\bar{k}(s) = \infty$. The probability of successful revolution in the stable finite-cutoff equilibrium is the probability that both citizens get signals that exceed $\bar{k}(s)$:

$$P_{pri}(\bar{k}(s)) = \int_{\bar{k}(s)}^{\infty} \int_{\bar{k}(s)}^{\infty} g(x_1, x_2) dx_1 dx_2, \quad (3)$$

where $g(x_1, x_2)$ is the joint pdf of $x_1$ and $x_2$.

As Figure 2 illustrates, $P_{pub}(k_p(s))$ and $P_{pri}(\bar{k}(s))$ are not easily comparable—the areas of integration differ and the location of $\bar{k}(s)$ relative to $k_p(s)$ varies in a complicated manner, reflecting the relative strengths of the different conflicting forces that we now discuss.

**Learning-in-equilibrium incentives.** Because citizens are uncertain about the value of successful revolution, they want to learn each others’ information, not just to learn if revolution would succeed, but also to assess whether a successful revolution would be desirable. This means that a citizen may revolt even when his information alone suggests that revolution is undesirable: each citizen recognizes that his assessment is imperfect, and that others may have information indicating that a successful revolution would be a great outcome. By revolting, he opens the possibility of a successful revolution by allowing the other citizen’s information to determine the revolution outcome. This creates a “learning-in-equilibrium” incentive to revolt. When citizens communicate and share information, this learning-in-equilibrium incentive vanishes, reducing the probability of successful revolution.

**Coordination incentives.** Communication lets citizens coordinate actions, directly raising the likelihood of successful revolution and indirectly raising the likelihood by reducing the
probability of participating in a failed revolt. By alleviating concerns about miscoordination and punishment, communication raises the probability of successful revolution.

**Information pooling incentives.** Communication *increases* the information available to each citizen, so that citizens make more accurate assessments of the merits of revolution. Lemma 1 shows that whether this information-pooling raises or lowers the likelihood of successful revolution depends on the status quo: the extra information raises the likelihood of successful revolution with a good status quo; but reduces it with a bad status quo. Thus, paradoxically, bad regimes, but not good ones, gain from information pooling.

**Lemma 1** *In the equilibrium with communication, greater noise in (public) signals raises the likelihood of successful revolution when the status quo is worse than the expected revolution payoff* \( E[R] = 0 \), *but reduces the likelihood of successful revolution when the status quo is better:*

\[
\frac{dP_{pub}(s)}{d\sigma^2_p} = \begin{cases} 
> 0 & ; s < 0 \\
= 0 & ; s = 0 \\
< 0 & ; s > 0.
\end{cases}
\]

To see the effects of information-pooling, suppose there is no punishment for participating in a failed revolt, and citizens are myopic, ignoring the information content of each others’
equilibrium actions. Now, citizens do not revolt whenever $E[R|p_1, p_2] = \frac{\sigma^2 + \sigma_p^2}{\sigma^2+\sigma_p^2} \frac{p_1+p_2}{2} < s$. When signals are less noisy ($\sigma_p$ is smaller), a citizen puts more weight on his signals and less on his prior. When the status quo is worse than the ex-ante expected revolution payoff, what can prevent a citizen from revolting is receiving sufficiently accurate signals that flip his view into believing that the revolution payoff is even worse than the bad status quo. There is also a conflicting effect: when the signal noise falls, the ex-ante chance that the (average) signal is less than the status quo falls, which tends to raise the ex-ante likelihood that citizens revolt. Lemma 1 shows that the first effect dominates: more precise information reduces the ex-ante likelihood that a citizen revolts. Still, this does not imply that the likelihood of successful revolution also falls because, without communication, citizens cannot perfectly coordinate actions, so that sometimes only one citizen revolts. Thus, when the status quo is bad, there is a tradeoff: sharing information reduces a citizen’s incentive to revolt, but it enables citizens to better coordinate, so that if they revolt, revolution succeeds.

Now suppose citizens are fully rational and internalize the information content of each other’s equilibrium strategy. To glean ideas about the strength of the “learning-in-equilibrium” incentive to revolt, we study how equilibrium strategies behave in the limit, when the status quo is very good or very bad. To ease exposition, we write $\lim_{x \to \infty} f(x) \approx g(x)$ if and only if $\lim_{x \to \infty} [f(x) - g(x)] = 0$, with $\lim_{x \to -\infty} f(x) \approx g(x)$ defined analogously.

Result: (a) For any given punishment cost $\mu \geq 0$, $\lim_{s \to -\infty} \bar{k}(s) \approx \frac{s}{b} = \frac{\sigma^2 + \sigma_p^2}{\sigma^2} s$. (b) If $\mu = 0$, then $\lim_{s \to \infty} \bar{k}(s) \approx \frac{s}{b+cf} = \frac{2\sigma^2 + \sigma_p^2}{2\sigma^2} s$.

This result implies that the “learning-in-equilibrium” force becomes vanishingly small when the status quo is very bad. Intuitively, when citizens almost always revolt, equilibrium actions contain almost no information about revolution payoffs, so that a citizen revolts if and only if $E[R|x_j > \bar{k}(s), x_i] \approx E[R|x_i] > s$, implying $\bar{k}(s) \approx s/b$. In contrast, when the status quo is very good, and punishment is not a concern ($\mu = 0$), the “learning-in-equilibrium” force raises the incentive to revolt, reducing the equilibrium cutoff from $s/b$ to $s/(b+cf)$. That is, a citizen revolts if and only if his signal exceeds the $\bar{k}(s)$ that solves $E[R|x_j > \bar{k}(s), x_i = \bar{k}(s)] - s = b\bar{k}(s) + c 1 - \Phi(\frac{\phi(\bar{k}(s))}{1-\Phi(\bar{k}(s))}) - s = 0$. When $s$ is very large, a citizen almost never revolts, so $\lim_{s \to \infty} \bar{k}(s) = \infty$. In the proof of Proposition 1, we show that $\lim_{x \to \infty} \phi(x) \approx x$. Thus, $\lim_{s \to \infty} b\bar{k}(s) + c 1 - \Phi(\frac{\phi(\bar{k}(s))}{1-\Phi(\bar{k}(s))}) \approx b\bar{k}(s) + c\bar{k}(s) = (b+cf)\bar{k}(s)$, implying that $\bar{k}(s) \approx \frac{s}{b+cf}$.

To establish the net effect of these conflicting forces (information pooling, learning-in-
equilibrium, and coordination) is difficult. Proposition 1 derives the net effect asymptotically:

**Proposition 1** (a) When the status quo is very bad,

\[
\lim_{s \to -\infty} P_{pri}(s) - P_{pub}(s) = \begin{cases} 
0^- & \text{if } \sigma_p > \sqrt{2} \sigma_\nu \\
0^+ & \text{if } \sigma_p < \sqrt{2} \sigma_\nu.
\end{cases}
\]

(b) When the status quo is very good and \(\mu = 0\) so that \(\bar{k}(s)\) always exists,

\[
\lim_{s \to +\infty} P_{pri}(s) - P_{pub}(s) = \begin{cases} 
0^+ & \text{if } \sigma_p > \sigma_\nu \\
0^- & \text{if } \sigma_p \leq \sigma_\nu.
\end{cases}
\]

The proposition says that when the status quo is very bad, successful revolution is more likely with private signals than public signals as long as the public signals are not too much noisier than the private signals. The intuition is that when the status quo is very bad, citizens almost always revolt, so the learning-in-equilibrium effect is very small, and the risk of punishment for participating in a failed revolt almost vanishes. Hence, results hinge on the information-pooling effect. More information helps shift beliefs by more, possibly inducing a citizen to switch from revolting to not by convincing him that revolution payoffs are far worse than his prior led him to believe; and there is more information in two public signals than one private signal as long as they are not too much noisier, i.e., as long as \(\sigma_p^2 < 2\sigma_\nu^2\).

In contrast, when the status quo is really good and \(\mu = 0\), there is more successful revolution with private signals than public signals, as long as the private signals are not too much noisier than the public signals. Now signals must be really informative about a good revolution outcome to induce a citizen to try to overturn the good status quo, so more information makes revolution more likely. What complicates the calculation of how much noisier private signals have to be to overturn the result is the learning-in-equilibrium effect, which causes citizens to set lower cutoffs with private signals than with public signals in order to let the other citizen’s information influence the outcome. Ceteris paribus, this serves to raise the likelihood of successful revolution with private signals. After accounting for this effect, we use Feller’s (1968) inequality for the tails of normal distributions to establish that there is more successful revolution with public signals than private ones as long as public signals are at least as informative as private signals. Proposition 1 immediately implies:
Figure 3: $P_{\text{pri}}$ and $P_{\text{pub}}$ are the ex-ante probabilities of successful revolution when citizens do and do not communicate, respectively. The blue curve is the difference in these probabilities as a function of the status quo payoff $s$ when the punishments for participating in a failed revolt $\mu$ is zero. The blue curve is the corresponding probability difference when $\mu = 0.1$. When $\mu > 0$, the likelihood of successful revolution without communication drops to zero once the status quo payoff exceeds a threshold. $s^*$ depicts that threshold. Parameters: $\sigma = \sigma_{\nu} = \sigma_{p} = 1$.

Corollary 1 If communication does not alter signal noise, i.e., $\sigma_{\nu} = \sigma_{p}$, then:

(a) When the status quo is sufficiently bad, communication reduces the likelihood of successful revolution, i.e., $P_{\text{pri}}(s) > P_{\text{pub}}(s)$ for sufficiently negative $s$.

(b) When the status quo is sufficiently good, communication increases the likelihood of successful revolution, i.e., $P_{\text{pri}}(s) < P_{\text{pub}}(s)$ for sufficiently positive $s$ (even when $\mu = 0$).

These asymptotic results raise the questions: how bad does a regime have to be to gain from facilitating communication among citizens? And, how are answers affected when participants in a failed revolution expect to be punished by the regime? For intermediate values of the status quo $s$, it is difficult to analytically characterize the relative likelihoods of successful revolution. This leads us to establish numerically how the results extend. Figure 3 presents typical relationships between the status quo $s$ and the likelihoods of successful revolution with communication and without when $\mu$ is small.\textsuperscript{12} The figure shows that for

\textsuperscript{12}When $\mu$ is high, then for intermediate values of the status quo, coordination concerns dominate, and, as in standard settings, there is more revolution when citizens communicate.
a standard parameterization (all random variables \( \sim N(0, 1) \)), the probability of successful revolt is higher \textit{without} communication whenever the status quo is (at least) modestly worse than the ex-ante expected payoff from successful revolution. If, instead, the expected payoff from successful revolution is less than the status quo payoff, successful revolution is more likely with communication. Thus, a regime can reduce the probability of successful revolution by allowing communication between citizens whenever the costs for participating in a failed revolution are small enough and the status quo is at least slightly worse than the ex-ante expected payoff from revolution.

\textbf{Continuum of Agents.} One may wonder whether results depend on the number of citizens being small. One conjectures that the number of citizens should not alter our finding that a very bad regime benefits from permitting communication because the “information-pooling effect” drives this result. To show this, we modify the model by assuming a continuum of citizens and adopt the standard regime change technology that a revolution succeeds if and only if the measure of revolting citizens exceeds a threshold (i.e., the regime’s strength).

\begin{center}
\begin{tabular}{cc|cc}
\multicolumn{1}{c|}{outcome} & \multicolumn{1}{c}{\text{revolt}} & \multicolumn{1}{c}{\text{no revolt}} \\
\cline{2-4}
\text{citizen } i & \text{\(m > t\)} & \text{\(m \leq t\)} & \\
\hline
\text{\(R\)} & \text{\(s - \mu\)} & \\
\text{s} & \text{s} & \\
\end{tabular}
\end{center}

Figure 4: A continuum of citizens, indexed by \(i \in [0, 1]\), simultaneously decide whether to revolt. Revolution succeeds when the measure \(m\) of citizens who revolt exceeds the regime’s strength \(t \in (0, 1)\). A citizen who revolts receives a common revolution payoff \(R\) when a revolution succeeds, and \(s - \mu\) when the revolution fails. A citizen who does not revolt receives the status quo payoff \(s\). Citizens are uncertain about \(R\), and receive private noisy signals \(x_i = R + \nu_i, i \in [0, 1]\), where \(R\) and \(\nu_i\’s\) are independent with \(R \sim N(0, \sigma)\) and \(\nu_i \sim N(0, \sigma_\nu)\).

Figure 4 presents the game. With a continuum of types, when citizens can communicate, they learn the revolution payoff \(R\). Therefore, in the equilibrium that features the most revolution, all citizens revolt and the revolution succeeds whenever \(R > s\). When citizens cannot communicate, they must rely on their own private signals alone. Focusing on finite-cutoff strategies, the equilibrium is characterized by a pair \((x^*, R^*)\), such that a citizen revolts if and only if \(x_i > x^*\), and the revolution succeeds if and only if \(R > R^*\). In equilibrium, a citizen who receives the threshold signal \((x_i = x^*)\) must be indifferent between revolting and not.
This citizen assigns probability $Pr(R > R^* | x_i = x^*)$ to the revolution succeeding, in which case his net expected payoff from revolting versus not revolting is $E[R | R > R^*, x_i = x^*] - s$. With the remaining probability $Pr(R \leq R^* | x_i = x^*)$ the revolution fails, and the citizen’s expected net payoff from revolting is $-\mu$. Thus, in equilibrium:

$$E[R | R > R^*, x_i = x^*] - s = \frac{Pr(R \leq R^* | x_i = x^*)}{Pr(R > R^* | x_i = x^*)} \mu.$$  \hspace{1cm} (4)

Citizen beliefs must be consistent with equilibrium strategies: the fraction of citizens who revolt, $Pr(x_i > x^* | R)$, must exceed $t$ if and only if $R > R^*$. Because $Pr(x_i > x^* | R)$ rises in $R$,

$$Pr(x_i > x^* | R = R^*) = t.$$  \hspace{1cm} (5)

**Proposition 2** In the continuum citizen game, communication reduces the likelihood of successful revolution when $s$ is sufficiently low.

With common values, even with a continuum of types, all three effects of communication—coordination, learning-in-equilibrium, and information-pooling—are present. Once more, information pooling underlies the result.

**Alternative Regime Change Models**

To highlight the contrast with the literature, we show that communication always raises the probability of successful revolution in settings where citizens receive signals about private-value rather than common-value payoffs (Bueno de Mesquita 2010), and in the canonical global game of regime change (Morris and Shin 1998), where citizens receive signals about the strength of a regime. In both settings, information-pooling effects vanish, and communication only serves to foster coordination.

**Private-Value Payoffs (Bueno de Mesquita 2010).** With a common-value payoff structure, a citizen cares about the information of others not only to assess the likelihood that a revolution succeeds, but also to figure out whether successful revolution would be better than the status quo. In contrast, with a private-value payoff structure, public signals only facilitate coordination, and do not convey more information about whether successful revolution is a good outcome. To highlight the consequences, we show that in the two-player version of Bueno de Mesquita’s (2010) private-value game of revolution, citizens always revolt more
and the revolution is always more likely to succeed when they can communicate. Figure 5 presents the game.

<table>
<thead>
<tr>
<th>Citizen 1</th>
<th>Citizen 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Revolt</strong></td>
<td><strong>No Revolt</strong></td>
</tr>
<tr>
<td>$x_1, x_2$</td>
<td>$s - \mu, s$</td>
</tr>
<tr>
<td>$s, s - \mu$</td>
<td>$s, s$</td>
</tr>
</tbody>
</table>

Figure 5: Citizens know the expected status quo payoff $s$, but are uncertain about each others’ revolution payoffs: citizen $i$ knows $x_i$, but not $x_j$, $i \neq j \in \{1, 2\}$. Revolting alone is costly, $\mu > 0$, and citizens have private noisy signals about the revolution payoff: $x_i = R + \nu_i$, $i \in \{1, 2\}$, where $R$ and $\nu_i$’s are independent with $R \sim N(0, \sigma)$ and $\nu_i \sim N(0, \sigma_\nu)$, $i \in \{1, 2\}$.

When citizens can communicate then in the equilibrium with the most revolution, each citizen revolts if and only if $\min\{x_1, x_2\} > s$. In contrast, when their signals remain private, each citizen revolts if and only if his private signal exceeds $\bar{k}$:

$$Pr(x_j > \bar{k}|x_i = \bar{k})(\bar{k} - s) = Pr(x_j \leq \bar{k}|x_i = \bar{k}) \mu,$$

implying that $\bar{k} > s$, i.e. citizen $i$ revolts if and only if $x_i > \bar{k} > s$. For example, when $s < x_1 < \bar{k} < x_2$, both citizens revolt and the revolution succeeds if they can communicate; but if they cannot communicate so that their information remains private, only citizen 2 revolts, and the revolution fails. Thus, in a revolution game with private-values, there is always more revolution attempts and more successful revolution when citizens can communicate.\(^{13}\)

\(^{13}\)In Chen and Suen (2015), citizens are uncertain about whether a replacement regime would be drawn from a bad distribution or a good distribution, and they differ in the private benefits of revolt. Without communication, poorer citizens update more pessimistically about the likelihood a new regime would be drawn from a good distribution, resulting in an interval equilibrium, in which the middle class revolts, but the rich and poor do not. Communication, however, would resolve uncertainty about which type of distribution a new regime would be drawn from, retrieving a monotone equilibrium in which all save the wealthy are willing to revolt whenever it is attractive given that a new regime would be drawn from a good distribution. That is, communication resolves all uncertainty, resulting in a larger monotone equilibrium.
Canonical Global Game of Regime Change (Morris and Shin 1998). It is also enlightening to contrast our model with a model of revolution in which citizens know that they want to overthrow the regime, but are uncertain about the regime’s strength, and hence have trouble coordinating. Figure 6 presents the game.

<table>
<thead>
<tr>
<th>citizen</th>
<th>revolt</th>
<th>no revolt</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m &gt; \theta$</td>
<td>$R - c$</td>
<td>$s$</td>
</tr>
<tr>
<td>$m \leq \theta$</td>
<td>$-c$</td>
<td>$s$</td>
</tr>
</tbody>
</table>

Figure 6: A continuum of citizens, indexed by $i \in [0,1]$, simultaneously decide whether to revolt. Revolution succeeds if the measure $m$ of citizens who revolt exceeds the regime’s strength $\theta$. The status quo payoff is $s$, the successful revolution payoff is $R$, and revolting costs $c$, with $R - c > s$. Citizens are uncertain about the regime’s strength $\theta$. Citizens receive private noisy signals about $\theta$: $x_i = \theta + \nu_i$, $i \in [0,1]$, where $\theta$ and $\nu_i$’s are independent with $\theta \sim N(0, \sigma)$ and $\nu_i \sim N(0, \sigma_{\nu})$.

Consider the game without communication. We focus on cutoff strategies in which a player $i$ revolts if and only if his private signal $x_i$ is below a threshold ($x_i < x^*$). Because the game is standard, we just present the result: equilibria are characterized by a pair $(\theta^*, x^*)$ such that a citizen revolts if and only if his private signal exceeds $x^*$, and the revolution succeeds if and only if $\theta < \theta^*$. Any solution to the following system of equations constitutes an equilibrium $(\theta^*, x^*)$:

$$Pr(\theta < \theta^*|x_i = x^*) = \frac{c + s}{R} \quad \text{and} \quad Pr(x_i < x^*|\theta^*) = \theta^*.$$  

With an additive normal signal structure, there are generically three or one equilibrium, and in all equilibria $\theta^* < 1$. In contrast, when citizens can communicate, $\theta$ becomes known, so that whenever $\theta < 1$, in the equilibrium with the most revolution, all citizens revolt and revolution succeeds. Unlike with private values, all players face the same payoff structure. However, because payoffs are known, citizens are only concerned about coordination. Communication allows them to coordinate, and avoid incurring punishment for participating in a failed revolt. Thus, in the revolution game with uncertainty about a regime’s strength, there are always more revolution attempts and more successful revolution if citizens can communicate.
Conclusion

Conventional wisdom has been that better communication among citizens must facilitate revolution. The logic relies on communication facilitating coordination when there is no uncertainty about the merits of revolution. We show that when one incorporates the fact that citizens are typically uncertain about whether a successful revolution would improve on an existing regime, then communication raises the likelihood of successful revolution when a regime is sufficiently good, but not when it is sufficiently bad. In fact, regimes sometimes do open up communication among citizens in early stages of unrest. For example, Syria unblocked access to YouTube and Facebook in 2011 amid the Tunisian and Egyptian Revolutions (The New York Times, 2/10/2011); and the Shah relaxed censorship in the midst of the Iranian Revolution (Milani 1994, p. 117).

Historians of the Iranian Revolution argue that the people who overthrew the Pahlavi regime accepted Khomeini’s leadership, not because they shared his vision of an Islamic state, but because Khomeini meant different things to different groups. In his 1970 treatise on an Islamic State, Khomeini was clear that it would involve the control of the state by the clergy. However, his book reached a limited audience because the regime suppressed its dissemination. One wonders whether Khomeini and his staunch followers would have commanded the formidable coalition of non-religious forces that brought the Shah down had the Shah earlier allowed people to communicate more freely.

Appendix: Proofs

Proof of Lemma 1: Differentiating $P_{pub}(s)$ from equation 1 yields the result:

$$\frac{dP_{pub}(s)}{d\sigma_p^2} = -\frac {dk_p(s)}{d\sigma_p^2} \int_{-\infty}^{\infty} h(k_p - x, x)dx = \frac{-s}{\sigma^2} \int_{-\infty}^{\infty} h(k_p - x, x)dx$$

$$= -\frac{s}{\sigma^2} \sqrt{\frac{1}{2(2\sigma^2 + \sigma_p^2)}} \phi\left(\frac{k_p(s)}{\sqrt{2(2\sigma^2 + \sigma_p^2)}}\right),$$

where $\phi(\cdot)$ is the $N(0, 1)$ pdf, and the second equality follows from direct integration. □

\footnote{Abrahamian (1982) argues, “to the petty bourgeoisie, he was not only the sworn enemy of dictatorship but also the guardian of private property. To the intelligentsia, he appeared, despite his clerical garb, to be a militant nationalist. To the urban worker, he was a man of the people, eager to enforce social justice” (p. 531; see also Milani (1994)).}
Proof of Proposition 1: First, we show preliminary results used to prove parts (a) and (b). From our discussion of equation 2, recall that \( \Delta_1(\tilde{k}(s)) = 0 \). Thus, focusing on finite-cutoff strategies, we have \( \lim_{s \to \pm \infty} \tilde{k}(s) = \pm \infty \). Differentiating \( P_{pri}(s) \) from equation 3 yields:

\[
\frac{dP_{pri}(\tilde{k}(s))}{ds} = -\frac{\partial \tilde{k}(s)}{\partial s} \left( \int_{k(s)}^{\infty} g(\tilde{k}(s), x_2)dx_2 + \int_{k(s)}^{\infty} g(x_1, \tilde{k}(s))dx_1 \right)
\]

\[
= -2 \frac{\partial \tilde{k}(s)}{\partial s} \int_{k(s)}^{\infty} g(x_1, \tilde{k}(s))dx_1 \quad \text{(by symmetry)}
\]

\[
= -2 \frac{\partial \tilde{k}(s)}{\partial s} \ Pdf(\tilde{k}) \int_{k(s)}^{\infty} g(x_1|x_2 = \tilde{k}(s))dx_1
\]

\[
= -\frac{2}{\sqrt{\sigma^2 + \sigma^2_{\nu}} \phi(\frac{\tilde{k}(s)}{\sqrt{\sigma^2 + \sigma^2_{\nu}}}) \left[ 1 - \Phi(f\tilde{k}(s)) \right] \frac{\partial \tilde{k}(s)}{\partial s},
\]

(6)

where \( \phi(\cdot) \) is the pdf and \( \Phi(\cdot) \) is the cdf of the standard normal distribution, and \( PdF(x_2) = \frac{1}{\sqrt{\sigma^2 + \sigma^2_{\nu}}} \phi(\frac{x_2}{\sqrt{\sigma^2 + \sigma^2_{\nu}}}) \) because \( x_2 = R + \nu_2 \). The last equality follows from using \( x_1|x_2 \sim N(bx_2, a) \), so that \( \int_{k(s)}^{\infty} g(x_1|x_2 = \tilde{k}(s))dx_1 = 1 - \Phi(\frac{\tilde{k}(s)-bk(s)}{a}) = 1 - \Phi(f\tilde{k}(s)) \).

Similarly, differentiating \( P_{pub}(s) \) from equation 1 yields:

\[
\frac{dP_{pub}(s)}{ds} = -\frac{\partial k_p(s)}{ds} \int_{-\infty}^{\infty} h(k_p - x, x)dx = -\frac{2\sigma^2 + \sigma^2_{p}}{\sigma^2} \int_{-\infty}^{\infty} h(k_p - x, x)dx
\]

\[
= -\frac{2\sigma^2 + \sigma^2_{p}}{\sigma^2} \frac{1}{\sqrt{2(2\sigma^2 + \sigma^2_{p})}} \phi \left( \frac{k_p(s)}{\sqrt{2(2\sigma^2 + \sigma^2_{p})}} \right)
\]

\[
= -\sqrt{2\sigma^2 + \sigma^2_{p}} \phi \left( \frac{k_p(s)}{\sqrt{2(2\sigma^2 + \sigma^2_{p})}} \right)
\]

(7)

where the second equality is the result of direct integration, which gives \( \frac{1}{2\sqrt{\pi}\sqrt{2\sigma^2 + \sigma^2_{p}}} \Exp\{ -\frac{(k_p(s))^2}{4(2\sigma^2 + \sigma^2_{p})} \} \).

Next, we use Feller’s inequality for the tail of a normal distribution (Feller 1968, p. 175):

\[
\left( \frac{1}{x} - \frac{1}{x^3} \right) \phi(x) < 1 - \Phi(x) < \frac{1}{x} \phi(x), \text{ for } x > 0.
\]

Hence, for \( x > 1 \), we have \( x < \frac{\phi(x)}{1-\Phi(x)} < \frac{x^3}{x^3-1} \), and hence \( 0 < \frac{\phi(x)}{1-\Phi(x)} - x < \frac{x}{x^3-1} \). Letting \( o(x) \equiv \frac{\phi(x)}{1-\Phi(x)} - x \), this implies that \( \lim_{x \to \infty} o(x) = 0 \), and:

\[
\lim_{x \to \infty} x \times o(x) \leq 1.
\]

(8)
Moreover, recall that when \( \mu = 0 \), \( E[R|x_i > \bar{k}, x_i = \bar{k}] - s = b\bar{k}(s) + c \frac{\phi(\bar{f}(s))}{1 - \Phi(\bar{f}(s))} - s = b\bar{k}(s) + c \left( \frac{\phi(\bar{f}(s))}{1 - \Phi(\bar{f}(s))} - f\bar{k}(s) + f\bar{k}(s) \right) = s = (b + cf)\bar{k}(s) + c \phi(f\bar{k}(s)) - s = 0, \) and hence:

\[
\bar{k}(s) = \frac{s - c \phi(f\bar{k}(s))}{b + cf}.
\] (9)

Finally, from our Result in the text (p. 9), recall that:

\[
\lim_{s \to -\infty} \frac{\partial \bar{k}(s)}{\partial s} = \frac{1}{b} \quad \text{and} \quad \lim_{s \to \infty} \frac{\partial \bar{k}(s)}{\partial s} = \frac{1}{b + cf} \quad \text{(when} \ \mu = 0). \quad \text{(10)}
\]

**Proof of part (a):** We investigate the derivative of \( P_{pri}(s) - P_{pub}(s) \) with respect to \( s \) when \( s \) is very negative. From equations 6 and 7, \( \lim_{s \to -\infty} \frac{dP_{pri}(k(s))}{ds} - \frac{dP_{pub}(k(s))}{ds} = \)

\[
\frac{\sqrt{2\sigma^2 + \sigma_p^2}}{2\sqrt{2\sigma^2}} \times \phi \left( \frac{k_p}{\sqrt{2(2\sigma^2 + \sigma_p^2)}} \right) \times \left( 1 - \frac{1}{\sqrt{2\sigma^2 + \sigma_p^2}} \frac{2}{\sqrt{2\sigma^2 + \sigma_p^2}^2} \phi \left( \frac{\bar{k}(s)}{\sqrt{2\sigma^2 + \sigma_p^2}} \right) \right) \cdot \left( [1 - \Phi(f\bar{k}(s))] \frac{\partial \bar{k}(s)}{\partial s} \right)^{-1}.
\]

Moreover, \( \lim_{s \to -\infty} [1 - \Phi(f\bar{k}(s))] = 1 \), and from 10, \( \lim_{s \to -\infty} \frac{\partial \bar{k}(s)}{\partial s} = \frac{1}{b} > 0. \) Further,

\[
\lim_{s \to -\infty} \frac{\phi \left( \frac{\bar{k}(s)}{\sqrt{2\sigma^2 + \sigma_p^2}} \right)}{\phi \left( \frac{k_p(s)}{\sqrt{2(2\sigma^2 + \sigma_p^2)}} \right)} = \lim_{s \to -\infty} \text{Exp} \left\{ -\left( \frac{1}{\sigma^2 + \sigma_p^2} \right) \left[ \frac{\bar{k}(s)}{\sigma^2 + \sigma_p^2} \right]^2 + \frac{1}{2(2\sigma^2 + \sigma_p^2)} \left[ \frac{k_p(s)}{\sigma^2 + \sigma_p^2} \right]^2 \right\} \]

\[
= \lim_{s \to -\infty} \text{Exp} \left\{ -\left( \frac{1}{\sigma^2 + \sigma_p^2} \right) \left( \frac{s}{\bar{s}} \right)^2 \frac{1}{2} + \frac{1}{2(2\sigma^2 + \sigma_p^2)} \left( \frac{2\sigma^2 + \sigma_p^2}{\sigma^2 + \sigma_p^2} \right)^2 \left( \frac{s}{\bar{s}} \right)^2 \right\} \quad \text{if} \ \sigma_p^2 \neq \sigma_\nu^2
\]

\[
= \lim_{s \to -\infty} \text{Exp} \left\{ \frac{\sigma_p^2 - 2\sigma_\nu^2}{4\sigma^4} \frac{s^2}{\bar{s}^2} \right\}.
\]

Algebra shows that \(-\left( \frac{1}{\sigma^2 + \sigma_p^2} \right) \left( \frac{1}{\bar{s}} \right)^2 + \frac{1}{2(2\sigma^2 + \sigma_p^2)} \left( \frac{2\sigma^2 + \sigma_p^2}{\sigma^2 + \sigma_p^2} \right)^2 = \frac{\sigma_p^2 - 2\sigma_\nu^2}{2\sigma^4}.\) Thus,

\[
\lim_{s \to -\infty} \left( \frac{dP_{pri}(s)}{ds} - \frac{dP_{pub}(s)}{ds} \right) = \begin{cases} 
0^- & ; \sigma_p > \sqrt{2} \sigma_\nu \\
0^+ & ; \sigma_p < \sqrt{2} \sigma_\nu
\end{cases}
\]

Because \( \lim_{s \to -\infty} P_{pri}(s) = P_{pub}(s) = 1 \), this implies:

\[
\lim_{s \to -\infty} P_{pri}(s) - P_{pub}(s) = \begin{cases} 
0^- & ; \sigma_p > \sqrt{2} \sigma_\nu \\
0^+ & ; \sigma_p < \sqrt{2} \sigma_\nu
\end{cases}
\]
Proof of part (b):

\[
\lim_{s \to \infty} \frac{P_{pri}(s)}{P_{pub}(s)} = \lim_{s \to \infty} \frac{\int_{k(s)}^{\infty} \int_{k(s)}^{\infty} g(x_1, x_2)dx_1dx_2}{\int_{-\infty}^{\infty} \int_{k_p(s)-p_2}^{\infty} h(p_1, p_2)dp_1dp_2}
\]

(Next, use L'Hôpital's rule)

\[
= \lim_{s \to \infty} \frac{-\frac{2}{\sqrt{\sigma^2 + \sigma_p^2}} \phi \left( \frac{k(s)}{\sqrt{\sigma^2 + \sigma_p^2}} \right) \left[ 1 - \Phi(f \tilde{k}(s)) \right]}{-\frac{\sqrt{2\sigma^2 + \sigma_p^2}}{\sqrt{2\sigma^2}} \phi \left( \frac{k_p(s)}{\sqrt{2(2\sigma^2 + \sigma_p^2)}} \right)}
\]

(from equations 6 and 7)

\[
= \lim_{s \to \infty} \frac{2}{\sqrt{2\sigma^2 + \sigma_p^2}} \frac{1 - \Phi(f \tilde{k}(s))}{b + cf} \frac{\phi \left( \frac{k(s)}{\sqrt{\sigma^2 + \sigma_p^2}} \right)}{\phi \left( \frac{k_p(s)}{\sqrt{2(2\sigma^2 + \sigma_p^2)}} \right)}
\]

(from equation 10)

\[
= \lim_{s \to \infty} \frac{\phi(f \tilde{k}(s))}{\phi \left( \frac{k(s)}{\sqrt{\sigma^2 + \sigma_p^2}} \right)} \frac{\phi \left( \frac{k(s)}{\sqrt{\sigma^2 + \sigma_p^2}} \right)}{\phi \left( \frac{k_p(s)}{\sqrt{2(2\sigma^2 + \sigma_p^2)}} \right)}
\]

(by definition of \(o(x)\))

\[
= \lim_{s \to \infty} \frac{2}{\sqrt{2\sigma^2 + \sigma_p^2} \sqrt{2\sigma^2}} \frac{1}{(b + cf)\sqrt{2\pi}} \frac{\text{Exp} \left\{ \frac{\sigma_p^2 - \sigma_p^2}{4\sigma^2} s^2 \right\}}{f \tilde{k} + o(f \tilde{k})}
\]

(if \(\sigma_p \neq \sigma_v\), (12)

where the last equality follows from the following calculations:

\[
\lim_{k \to \infty} \phi(f \tilde{k}(s)) \frac{\phi \left( \frac{k(s)}{\sqrt{\sigma^2 + \sigma_p^2}} \right)}{\phi \left( \frac{k_p(s)}{\sqrt{2(2\sigma^2 + \sigma_p^2)}} \right)}
\]

\[
= \lim_{s \to \infty} \text{Exp} \left\{ -\left( \frac{1}{\sigma^2 + \sigma_v^2} + f^2 \right) \frac{[k(s)]^2}{2} + \frac{1}{2(2\sigma^2 + \sigma_p^2)} \frac{[k_p(s)]^2}{2} \right\}
\]

\[
= \lim_{s \to \infty} \text{Exp} \left\{ -\left( \frac{1}{\sigma^2 + \sigma_v^2} + f^2 \right) \left( s - c o(f \tilde{k}(s)) \right)^2 \frac{1}{2} + \frac{1}{2(2\sigma^2 + \sigma_p^2)} \left( \frac{2\sigma^2 + \sigma_p^2}{\sigma^2} \right)^2 \frac{1}{2} s^2 \right\},
\]

where the last equality follows from substituting for \(k_p(s)\) and for \(\tilde{k}(s)\) from 9. Next, collect all the coefficients of \(s^2\) and simplify to get:

\[
\lim_{s \to \infty} \text{Exp} \left\{ \frac{\sigma_p^2 - \sigma_v^2}{4\sigma^4} s^2 - \left( \frac{1}{\sigma^2 + \sigma_v^2} + f^2 \right) \frac{2c o(f \tilde{k}(s))s + [c o(f \tilde{k}(s))]^2}{(b + cf)^2} \right\}
\]

(13)

Algebra yields

\[
\left( \frac{1}{\sigma^2 + \sigma_v^2} + f^2 \right) \left( \frac{1}{b + cf} \right)^2 + \frac{1}{2(2\sigma^2 + \sigma_p^2)} \left( \frac{2\sigma^2 + \sigma_p^2}{\sigma^2} \right)^2 = \frac{\sigma_p^2 - \sigma_v^2}{2\sigma^4}
\]

Thus, equation 12 implies:

If \(\sigma_p \neq \sigma_v\), then

\[
\lim_{s \to \infty} \frac{P_{pri}(s)}{P_{pub}(s)} = \begin{cases} 
\infty & ; \sigma_p > \sigma_v \\
0 & ; \sigma_p < \sigma_v.
\end{cases}
\]

(14)
If \( \sigma_p = \sigma_\nu \), then from 13,
\[
\lim_{s \to \infty} \phi(f \bar{k}(s)) \frac{\phi \left( \frac{k(s)}{\sqrt{\sigma^2 + \sigma_\nu^2}} \right)}{\phi \left( \frac{k_\nu(s)}{\sqrt{2(\sigma^2 + \sigma_\nu^2)}} \right)} = \lim_{s \to \infty} \text{Exp} \left\{ \left( \frac{1}{\sigma^2 + \sigma_\nu^2} + f^2 \right) \frac{c \phi(f \bar{k}(s)) \times s}{(b + cf)^2} \right\}
\]
\[
\leq \text{Exp} \left\{ \left( \frac{1}{\sigma^2 + \sigma_\nu^2} + f^2 \right) \frac{c}{(b + cf)^2} \frac{b + cf}{f} \right\},
\]
where we use equations 8 and 9 to show:
\[
\lim_{s \to \infty} \phi(f \bar{k}(s)) \times s \leq \frac{1}{f \bar{k}(s)} s \approx \frac{1}{f \bar{k}(s)} s = \frac{b + cf}{f}.
\]
This together with 11 implies:
\[
\text{If } \sigma_p = \sigma_\nu, \text{ then } \lim_{s \to \infty} \frac{P_{pri}(s)}{P_{pub}(s)} = 0. \tag{15}
\]
Moreover, \( \lim_{s \to \infty} P_{pri}(s) = \lim_{s \to \infty} P_{pub}(s) = 0 \). This together with 14 and 15 implies:
\[
\lim_{s \to \infty} P_{pri}(s) - P_{pub}(s) = \begin{cases} 0^+ & ; \sigma_p > \sigma_\nu \\ 0^- & ; \sigma_p \leq \sigma_\nu \end{cases}. \quad \Box
\]

**Proof of Proposition 2:** Recall that \( R|x_i \sim N(bx_i, v^2) \), where \( b = \frac{\sigma^2}{\sigma^2 + \sigma_\nu^2} \) and \( v^2 = \sigma_\nu^2 \). Thus, equation 5 become \( 1 - \Phi\left( \frac{x^* - R^*}{\sigma_\nu} \right) = t \), which implies:
\[
x^* = R^* + \sigma_\nu \Phi^{-1}(1 - t) \Rightarrow R^* - bx^* = (1 - b)R^* - b\sigma_\nu \Phi^{-1}(1 - t). \tag{16}
\]
Focusing on finite-cutoff strategies, observe that \( \lim_{s \to -\infty} R^* = \lim_{s \to -\infty} x^* = -\infty \). Thus, \( \lim_{s \to -\infty} E[R|R > R^*, x_i = x^*] \approx E[R|x_i = x^*] \approx bx^* \). With equation 16, this implies:
\[
\lim_{s \to -\infty} E[R|R > R^*, x_i = x^*] \approx bR^* + b\sigma_\nu \Phi^{-1}(1 - t). \tag{17}
\]
Moreover, from 16, we have \( \lim_{s \to -\infty} R^* - bx^* = -\infty \), and hence \( \lim_{s \to -\infty} \Phi\left( \frac{R^* - bx^*}{\sigma_\nu} \right) = 0 \). Substituting this and 17 into equation 4 yields \( \lim_{s \to -\infty} bR^* + b\sigma_\nu \Phi^{-1}(1 - t) - s = 0 \), implying \( \lim_{s \to -\infty} R^* \approx \frac{s}{b} - \sigma_\nu \Phi^{-1}(1 - t) < s \), where we recall that \( b < 1 \). \quad \Box

**References**


