# Simple Decision Rules in Small Groups: Collegial Rule v. Rotational Rule

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**Abstract**

I analyze the decision by ex-ante identical group members with private preferences who must choose between two simple power-sharing schemes: collegial rule and rotational rule. Under collegial rule, members simultaneously express their preferred decisions, and the final decision takes the form of a simple compromise: the average of expressed decisions. Under rotational rule, one member is given the full authority to make decisions for a period of time, but this role (potentially) rotates among members. I identify the trade off between preference aggregation and information aggregation, and its interaction with group size and the extent of preference alignment among members.

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*JEL Classification:* D7.

## 1. Introduction

"We run Google as a triumvirate," wrote Larry Page in their 2004 Founder IPO Letter. "The three of us [Larry Page, Sergey Brin (co-founder), and Eric Schmidt (CEO)] run the company collaboratively.... For important decisions...differences are resolved through discussion and analysis and by reaching consensus." Google's collegial way of decision making is typical of many spinoffs and startups, especially in "emergent entrepreneurial teams" where members participate in "joint action of discovery" (Harper 2008). Under collegial rule, members of a group make decisions together by building consensus and making compromises, so that when there is a conflict, no member's preferred decision is implemented. One form of collegial rule is when each member expresses a preferred

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decision and the final decision takes the form of a simple compromise: the average of the expressed preferred decisions. A simple alternative to collegial rule is rotational rule, in which each member becomes the decision maker for a period of time and implements his preferred decisions, but this role (potentially) rotates among members. For example, in the Brazilian military regime (1964-1985) in which the power was concentrated in the executive branch, the presidency rotated among top Army generals.[1] This paper analyzes the trade offs between rotational and collegial rules of decision making in groups of equally powerful members.

Decision making in many real world settings resemble those of collegial or rotational rules. For example, in the Roman Republic, a group of annually elected *consuls* shared the highest authority in the state. Their authority included the right to exercise civil and criminal jurisdiction, summon the senate, supervise certain religious matters, and command the armed forces (Abbott 1901, p. 175-81). As a method of sharing authority between the *consuls*, consular power "alternated from month to month [between *consuls*] in the active exercise of that power over the city. When they were in joint command of an army in Italy they commonly alternated day by day. The possession of *fasces* [a collection of wooden rods that symbolized authority] passed from one to the other to indicate the change" (Abbott 1901, p. 155, see also p. 176).[2]

[1]The list of the Army generals who became president in the Brazilian military regime: Catello Branco (1964-1967), Artur da Costa e Silva (1967-1969), Emilio Medici (1969-1974), Ernesto Geisel (1974-1979), and Joao Figueiredo (1979-1985). Typically, the Army has the most personnel among the branches of armed forces, and it plays the dominant role in capturing the government, fighting domestic guerrilla movements, and containing strikes and demonstrations in coups. Therefore, "it is customary that the presidency during military regimes be taken by a top level General [of the army]" (Fontana 1987, p. 46). Following the 1964 coup, the military regime, through a series of Institutional Acts, concentrated the decision making power in the executive branch headed by the president. For example, the president could amend the constitution with minimal support from the (rubber-stamp) National Congress, had the authority to decree a recess of the legislature, and most starkly, the president had the authority "to suspend the political rights of any citizen for the period of (10) ten years and to end federal, state, and municipal elective terms of office " (Guerchon 1971, p. 267). For the details of the power arrangement and the decision making processes within the Brazilian military regime (1964-1985), see Bacchus (1990), Schneider (1971), Skidmore (1988), and Stepan (1988). Similar power arrangements existed in the Argentinian military regime (1976-1983), but the power of the president was more limited and the Military Junta, consisting of the commanders-in-chief of the three branches of the military, was the "supreme organ of the nation" with comparable decision making power to the president (Fontana 1987, p. 27). But even in Chile—which is associated with the personal dictatorship of Pinochet—before the coup, "the members of the Junta had informally agreed that the office [of president] rotate among them, with one-year terms" (Barros 2002, p. 52).

[2]In a democratic political setting, consider negotiating parties in a constitutional convention choosing between presidential and parliamentary systems. In a presidential system, in each period, the party who wins the election has significantly more decision-making power. However, the incumbent may lose the next election, in which case this power rotates between the parties. In contrast, in a parliamentary system, "where no party typically commands a majority of seats" in the parliament (Diermeier and Merlo 2004, p. 795), the decision making power is distributed more evenly among the parties, and they must compromise in each period. Clearly, the mean rule is too simplistic to capture the details of preference aggregation in parliamentary governments, and one may think that variations of more complex Baron-Ferejohn style bargaining models must have had great success in capturing the essence parliamentary politics. However, almost all such models imply the proposer/*formateur* advantage, while there is a strong empirical regularity (Gamson's Law) that "the proportion of cabinet ministries received by each government party...tends strongly to equal the proportion of legislative seats contributed by that party to the government seat total." This "portfolio allocation paradox" (Warwick and Druckman 2001, 2006) imply that "the profession's canonical theory of bargaining in legislatures is contradicted by one of the profession's strongest and most robust empirical laws" (Laver et al. 2011,

In a relationship setting, consider a couple who regularly go on dinner dates. They live in a lively city with a broad range of restaurants. They can take turns deciding which restaurant to go to or compromise each time they must decide.

Collegial rule aggregates preferences for every decision in a simple way that captures members' compromise, while rotational rule does not aggregate preferences at all. What complicates decision making is that members are often uncertain about each others' preferences when making decisions. For example, even though officers may be well-informed about each other's preferences at the time of a coup, they will be less informed about their preferences about new issues arising in the complex process of governance. This uncertainty is inconsequential for decision making under rotational rule in which the officer in charge implements his ideal decisions. However, under collegial rule, this uncertainty leads officers to act strategically: they express preferred decisions that are more extreme than their true ideal decisions in order to manipulate the process (Proposition 3).The magnitude of distortions in the members' expressed preferred decisions increases with the group size and decreases with the extent to which their preferences are aligned—as captured by the correlation between preference shocks (Proposition 4). As a result, when players' preferences are not too aligned, members forgo preference aggregation (collegial rule) in favor of rotational rule if and only if the group size exceeds a threshold. Alternatively, members prefer collegial rule to rotational rule if and only if their preferences are sufficiently aligned (Theorem 1).

Moreover, in most settings, members have private information and care about aggregating their information. For example, in a coup setting, junta officers may have private information about the merits of each decision due to their different background and advisors. When members have private information, collegial rule aggregates their information more effectively than rotational rule. Under collegial rule, the members' private information influences the final decision via their expressed preferred decisions. In contrast, under rotational rule, the sole decision maker has to decide based on his own information. To capture these information aggregation aspects, I endow members with noisy private signals about a common value payoff component. Interestingly, under collegial rule, members' equilibrium strategies fully aggregate their private information, as if a central decision maker collects all the members' signals and updates according to the Bayes' rule (Proposition 6).

p. 287-8). That is, "we still do not fully understand...how the preferences of cabinet members are aggregated to produce government policy" (Goodhart 2013, p. 205).

As another example, consider the chair of an academic department in a university. The position of the chair typically rotates among the tenured faculty. Of course, some faculty consider being the chair a costly service and do not care much about implementing their preferred policies. To the extent that these considerations are negligible, the power-sharing scheme in academic departments resembles the rotational rule.

Information aggregation considerations tend to make collegial rule more attractive. When both information and preference aggregation considerations are present, with quadratic preferences, the difference between a member's payoff from rotational rule versus collegial rule is a quadratic function of the group size (equation 18). As a result, as the group size increases, members' preference for rotational rule versus collegial rule can change, at most twice. In particular, in two-member groups, members prefer collegial rule unless they have extreme conflict of interest, i.e., their preferences are sufficiently negatively correlated (Proposition 7). In large groups, as long as members' private signals are not too informative (so that additional signals are sufficiently valuable), information aggregation advantage dominates when group size is sufficiently large, so that members prefer collegial rule to rotational rule (Theorem 2).

Next, I discuss the nature of collegial rule. Then, I present the basic model, followed by the analyses of rotational rule, collegial rule, and their comparison. In the following section, I extend the model by endowing the members with noisy private signals about a common value payoff component, and analyze the effects of information aggregation considerations. A conclusion follows.

## 2. The Nature of Collegial Rule

By collegial rule I refer to the (more or less informal) decision making processes by which a group of friends (whether in an entrepreneurial or casual setting) or family members come to a final decision. Whatever the exact underlying decision making process is, I argue that the final decision in such groups has two interrelated essential features: (1) The intensity of each member's preferences are accounted for in the final decision. That is, the final decision is sensitive to the expressed preferences of *every* member, so that the final decision changes if a member expresses a different preferred decision. (2) When there is a conflict, no member's preferred decision is implemented, so that (almost always) *every* member has to compromise. These features are not present in decision rules that only depend on the ranking of expressed preferences. For example, one might be tempted to model the collegial way of decision making by positing that the players use a median rule so that the final decision becomes the median of the players' expressed preferred decisions—with an even number of players, pick any of the two expressed preferred decisions that are in the middle, i.e., use the extended median rule.[3] However, the median rule is insensitive to the members' expressed preferences—except

[3]With the median rule, players report their preferences truthfully (Moulin 1980; Sprumont 1995), so the strategic manipulation distortions that arise with the mean rule are absent. However, the median rule is *not* generally superior to the mean rule even with no information aggregation consideration. In particular, the mean rule dominates the median rule when the members' preferences are sufficiently aligned—in the following model, when $\rho$ is sufficiently high.

the median's. Moreover, the preferred decision of the median member is implemented, while other members have to compromise—sometimes a lot. This is inconsistent with the nature of decision making in groups that we consider under collegial rule.[4]

To illustrate in a family setting, consider a mother, a father and a son who must choose where to dine tonight. The father loves hamburgers and beer, mildly likes Italian food, and hates Chinese food, while the mother and son are in the mood for Chinese food tonight, have mild interest in pasta, and don't want hamburgers tonight. With the median rule, the family will go to a Chinese restaurant and the father must submit to the majority rule. However, in many families, that will not happen; instead, the family goes to an Italian restaurant. Alternatively, the family may decide to give the final decision to a member and rotate this power among members, so that, for example, tonight the mother decides where to go, next time the son decides, and the time after that the father decides, and so on.[5]

One may also be tempted to model the collegial way of decision making with some underlying bargaining game, e.g., a random member makes a proposal that will be implemented if passed according to some voting rule such as unanimity or majority rule. However, analyzing such multilateral bargaining games with private preferences is extremely difficult. Moreover, it is not even clear whether such bargaining games capture the nature of collegial decision making process. Therefore, I study the simplest rule that has the two essential features of collegial way of decision making discussed above: the mean rule in which each member expresses his preferred decision and the final decision is the average of all expressed preferred decisions. The mean rule is sensitive to all members' expressed decisions and has the important feature that every member has to compromise. Moreover, the mean rule is the optimal rule with truth-telling, i.e., it minimizes $\Sigma_{i=1}^{N}(\nu_i - p)^2$ where $\nu_i$ is members $i$'s private preference and $p$ is the final decision. Moreover, the mean rule leads to optimal information aggregation when players have private information about a common value payoff component: the final decision is made as if a central decision maker collects all private information and updates according to Bayes' rule (Proposition 6).

[4]One may think that because the median of expressed preferred decisions is the Condorcet winner, the median rule has the additional advantage of being the outcome of some majority voting in a Downsian model. However, Downsian models are inherently models of electoral competition which require multiple parties with no policy preferences competing for votes, a setting that does not fit decision making processes in small groups.

[5]In some of the examples, one may think that there is little uncertainty about the members' preferences, e.g., family members know each others' taste for food. However, a person's preference for a particular type of food in a particular time depends on many variables such as the recent history of the food that person has consumed, his/her mood at the time, commercials that s/he has recently watched on T.V., etc. Such uncertainties become more salient in the more complex settings such as governing a country under a military regime. For example, the particular time at which the military should retreat to barracks and hand over the government to civilians, what kinds of freedom should be granted to citizens under particular circumstances, etc.

**Average, Weighted Average, and Optimal Mechanism.** The purpose of this paper is to analyze preference and information aggregation considerations when a group must choose between two simple mechanisms (rotational and collegial) that reasonably represent the underlying decision making process in some real world settings. However, it is useful to place these mechanisms in the context of other possible mechanisms. To do so, consider a single-period game in which a member's preferences consist only of a preference shock $\nu_i$. Using the revelation principle, the problem of finding the optimal preference aggregation mechanism $p(\nu_1, ..., \nu_N)$ can be formulated as:

$$\min_{p(\cdot)} \quad E[\Sigma_{i=1}^N (\nu_i - p(\nu_i, \nu_{-i}))^2]$$
$$\nu_i \in \arg\min_{\nu_i'} E_{\nu_{-i}}[(\nu_i - p(\nu_i', \nu_{-i}))^2 | \nu_i] \quad \text{(Incentive Compatibility).}$$

The incentive compatibility constraint yields

$$E_{\nu_{-i}} \left[ -2(\nu_i - p(\nu_i', \nu_{-i})) \; \frac{dp(\nu_i', \nu_{-i})}{d\nu_i'} \middle| \nu_i \right]_{\nu_i' = \nu_i} = 0. \tag{1}$$

The collegial rule (mean rule) belongs to the class of weighted average mechanisms in which $p(\nu_1, ..., \nu_N) = \sum_{i=1}^N \omega_i \nu_i$, with $\omega_i > 0$ for all $i$. For this class of mechanisms, $\frac{dp(\nu_i', \nu_{-i})}{d\nu_i'} = \omega_i$, and hence the incentive compatibility constraint (equation (1)) implies $E_{\nu_{-i}}[\nu_i - p(\nu_i, \nu_{-i}) | \nu_i] = 0$, i.e., $\nu_i = E_{\nu_{-i}}[p(\nu_i, \nu_{-i}) | \nu_i] = \omega_i \nu_i + \sum_{j \neq i} \omega_j E_{\nu_j}[\nu_j | \nu_i] = \omega_i \nu_i + \rho \nu_i \sum_{j \neq i} \omega_j$, where the last equality follows from linear conditional expectation (Assumption 1 in the analysis below). Thus, $1 = \omega_i + \rho \ \sum_{j \neq i} \omega_j$, for all $i$. Hence, $\omega_i = \omega_j$ for all $i$ and $j$, and hence $\omega_i = \frac{1}{1+\rho(N-1)}$. Thus,

$$p(\nu_1, ..., \nu_N) = \frac{1}{1 + \rho(N-1)} \ \sum_{i=1}^N \nu_i.$$

That is, all the mechanisms within the class of weighted average mechanisms with positive weights yield the same final decision, and hence are equivalent from the players' perspective. Of course, different weights result in different strategic behaviors, but these behaviors are such that the final decision $p(\nu_1, ..., \nu_N)$ remains the same.

General optimal decision mechanisms, while appealing for their efficiency properties, involve significantly more sophistication. With both preference and information aggregation, an optimal mechanism has not been identified. However, a few recent papers address optimal preference aggregation mechanisms. Carrasco and Fuchs (2009a) consider a static setting with *two* players, quadratic preferences, and log-concave distribution of a bounded set of types (preferred decisions). They show that an optimal decision can be implemented by a mechanism of "dividing and discarding": Divide the type interval evenly into left and right; if players' expressed types are on the opposite sides, then take the decision corresponding to the middle of the interval; if players' expressed types are on the left (right) half, discard the right (left) half, and repeat this process on the left (right) half.[6] In an *infinitely* repeated version, Carrasco and Fuchs (2009b) show that, with a symmetric distribution of types with bounded support and sufficiently patient players, the optimal mechanism puts time-varying weights on the players' expressed types: A player who claims to have an extreme type today gets a higher weight today, but lower weights in the future. Jackson and Sonnenschein (2007) establish related results using a "budgeting mechanism" in which each player gets a quota of how many times s/he can announce to be of each type. In both papers, characterizations require (very) patient decision makers and strong technical assumptions (e.g., symmetric distributions of types), and, even with this structure, the optimal mechanisms are *extremely* complex.

Given the complexities of optimal mechanisms, some papers have focused on studying simple collective decision rules. Grüner and Kiel (2004) investigate preference aggregation under median and mean rules in a group in which members have "interdependent preferences." Each member $i$ has a quadratic utility with ideal point $\theta_i^* = (1-\alpha)\theta_i + \frac{\alpha}{N-1}\sum_{j\neq i}\theta_j$, where $\theta_i$ is $i$'s private information, $N$ is the group size, and $\alpha$ is the extent of interdependency. Importantly, $\theta_i$'s are independently distributed. Their main result is that when $\alpha$ is sufficiently small, the group welfare is higher under the median rule, while the opposite obtains when $\alpha$ is sufficiently large. In the same environment and building on the delegation literature (Holmstrom 1977; Alonso and Matouschek 2008; Callander and Krehbiel 2012), Rosar (2012) investigates the effect of imposing restrictions on the set of expressed preferred decisions (reports). He shows that if the distribution of types have monotone hazard rate and when the reporting space is binary, then under the *optimal* (binary) reporting space, average mechanism yields a higher welfare than the median one. Moreover, he relaxes the restriction to binary reporting spaces when either types are distributed uniformly or the group size is sufficiently large. In contrast to these papers, I compare the collegial (average) rule with the rotational rule, and investigate *both* preference and information aggregation considerations. Moreover, in this paper, the members' private preferences are correlated, which generates richer strategic interactions than Grüner and Kiel's "interdependent" structure of preferences—see footnote 8.

## 3. Model

$N \geq 2$ ex-ante identical players must decide how to allocate the right to make a decision in each of the following $N$ periods. They must choose between two simple power-sharing schemes: collegial

[6] Carrasco and Fuchs (2009a) further assume a symmetric distribution of types, but assert that this assumption is not necessary for their results. They also show that for utility functions that satisfy a monotone hazard property, if types are distributed uniformly, then "dividing and discarding" mechanism is optimal among monotonic mechanisms.

rule and rotational rule. Under collegial rule, no player has the exclusive right to make the decision. In each period, each player announces a preferred decision, and the final decision is the average of the expressed preferred decisions. Under rotational rule, in each period, one player is given the exclusive right to make the decision, and this right potentially rotates among the players. In the first period, each player is selected with equal probability $1/N$. Once a player becomes the decision maker, he will stay in office in the next period with probability $\gamma \in [0, 1]$. If he is removed from office, which happens with probability $1 - \gamma$, then other players (who have not been in office before) are selected with equal probability.[7]

Player $i$'s preference in period $t$ is quadratic with an ideal point that consists of a period-specific common value $\theta_t$ and a period-specific private value $\nu_{it}$, reflecting that players have both common and conflicting interests. Thus, player $i$'s ideal decision in period $t$ is $\theta_t + \nu_{it}$, where $(\theta_t, \nu_{1t}, ..., \nu_{Nt}) \sim iid\ f$ for $t \in \{1, ..., N\}$, where $\theta_t$ and $(\nu_{1t}, ..., \nu_{Nt})$ are independent, and $E[\nu_{it}]$ is normalized to 0. Moreover, $(\nu_{1t}, ..., \nu_{Nt})$ is distributed with $Var(\nu_{it}) = \sigma^2$ and $E[\nu_{it}\nu_{jt}] = \sigma^2\rho$ for $j \neq i$, with $\rho < 1$, where the positive-definitness of the variance-covariance matrix implies $-\frac{1}{N-1} < \rho$.[8] Letting $p_t$ be the decision made in period $t$, player $i$'s period $t$ payoff is $u_{it} = -((\theta_t + \nu_{it}) - p_t)^2$. Players discount future payoffs by $\beta \in [0, 1]$, so that period $k$ payoffs are discounted by $\beta^{k-1}$.

When deciding between collegial rule and rotational rule, players do not know the values of $\theta_t$ or $\nu_{it}$. At the beginning of period $t$, $\theta_t$ and $\nu_{it}$s are realized and players are informed of $\theta_t$ and their private preference shocks. The equilibrium concept is Perfect Bayesian.

Two points are worth emphasizing: (1) Because preferences are independent over time, the game is static in essence. Therefore, one could analyze the trade offs in a one period game. Then, under rotational rule, each player is selected with probability $1/N$ to become the decision maker. However, we set up the model in a multiple period settings because (a) it better fits the real world settings, in which there are multiple periods and decision makers rotate, and (b) this multiple-period setup does not significantly complicate the model: terms such as $\sum_t \beta^t/N$ appear in the analysis, but

[7] In many settings, the person in charge may be able to change the rules of the game to stay longer in power, and $\gamma$ captures the extent to which this can happen.

[8] $\rho$ captures the level of preference alignment between the members, i.e., lower $\rho$ is interpreted as more conflict of interest. One may be tempted to set $\rho = 0$, and model the players' bliss points as $\alpha\theta_t + (1-\alpha)\nu_{it}$, where $\alpha$ captures the intensity of the players' conflict of interests. However, the strategic considerations that arise in the setting with $\rho$ are absent in the setting with $\alpha$. For example, mirroring the calculations leading to equation (5), one can show that in the setting with $\alpha$, player $i$ with preferred decision $\alpha\theta_t + (1-\alpha)\nu_{it}$, expresses a referred decision $p_{it} = \alpha\theta_t + N(1-\alpha)\nu_{it}$. That is, she exaggerates the private component $\nu_{it}$ by a magnitude of $N$. But this strategic behavior arises in the current setup $(\theta_t + \nu_{it})$ in the special case of $\rho = 0$. Therefore, the setting with $\alpha$ does not fully capture the considerations that arise when preference shocks are correlated. Finally, the results of generalizing the ideal points from $\theta_t + \nu_{it}$ to $\alpha\theta_t + (1-\alpha)\nu_{it}$ are easy to see: Due to linear conditional expectations (assumptions 1 and 2), throughout the paper, $\theta_t$ is replaced by $\alpha\theta_t$ and $\nu_{it}$ with $(1-\alpha)\nu_{it}$.

they are easy to derive and interpret. (2) The common value component $\theta$ plays no role in the parts of the paper with only preference aggregation, but becomes essential when I model information aggregation aspects by endowing player with private signals about $\theta$. I kept $\theta$ from the beginning to simplify comparing those results from only preference aggregation and those with both preference and information aggregations.

## 4. Rotational Rule

Under rotational rule, in each period the decision maker chooses his ideal decision and receives the maximum payoff—normalized to zero. By symmetry, a player's expected payoff is the same in each period he is not the decision maker. In particular, player $i$'s expected payoff in period $t$ in which he is not in office is $-E[(p_t - (\theta_t + \nu_{it}))^2] = -E[(\nu_{jt} - \nu_{it})^2]$, with $j \neq i$:

$$E[(\nu_{jt} - \nu_{it})^2] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\nu_{jt} - \nu_{it})^2 \ f(\theta, \nu_{jt}, \nu_{it}) d\theta d\nu_{jt} d\nu_{it} = 2\ (1-\rho)\sigma^2.$$

By symmetry, ex-ante, each player is in office with equal probability, and hence the expected number of periods in which a player is in office is the same for each player. In addition, a player is equally likely to be in office in each of the $N$ periods, and hence his expected payoff is[9]

$$-\frac{(N-1)\ \Sigma_{t=0}^{N-1}\beta^t}{N}\ E[(\nu_{jt} - \nu_{it})^2],\ j \neq i,$$

where $(N-1)\ \Sigma_{t=0}^{N-1}\beta^t/N$ is the expected discount factor for the period in which a player is not the decision maker:

$$\frac{(\Sigma_{t=0}^{N-1}\beta^t - 1) + (\Sigma_{t=0}^{N-1}\beta^t - \beta) + ... + (\Sigma_{t=0}^{N-1}\beta^t - \beta^{N-1})}{N} = \frac{(N-1)\ \Sigma_{t=0}^{N-1}\beta^t}{N}.$$

Proposition 1 combines these results.

**Proposition 1.** *Under rotational rule, a player's expected payoff is*

$$U_r^P(N,\rho) = -2(N-1)\ \frac{\Sigma_{t=0}^{N-1}\beta^t}{N}\ (1-\rho)\sigma^2.$$

$(N-1)\ \Sigma_{t=0}^{N-1}\beta^t/N$ is simply the expected number of periods, discounted by $\beta$, in which a player is not the decision maker. Without discounting, i.e., when $\beta = 1$, this expression simplifies to $N-1$. At the other extreme, when players do not care about future payoffs at all, i.e., when $\beta = 0$, this expression simplifies to $\frac{N-1}{N}$, which is the probability that a player is not the decision maker in the

[9]Observe that because players' expectations are from an ex ante perspective, $\gamma$ does not enter the calculations.

first period, i.e., the only period that matters to the players. The term $2(1-\rho)\sigma^2$ captures the alignment of preferences. At the extreme case of perfect correlation, i.e., when $\rho = 1$ or $\sigma^2 = 0$, preferences are identical, i.e., $\nu_{it} = \nu_{jt}$ for all $i$ and $j$, and hence all players receive the maximum payoff zero no matter who becomes the decision maker. As $\rho$ decreases or $\sigma^2$ increases, preferences become less aligned, intensifying the utility loss for a player in each period that he is not in power.

## 5. Collegial Rule

Under collegial rule, no player has an exclusive right to make a decision. In each period, players express their preferred decision, and the final decision is the average of the expressed decisions. Thus, if players report their preferred decisions truthfully, the absolute value of player $i$'s payoff is

$$\Sigma_{t=1}^{N}\beta^{t-1}\ E\left[\left(\theta_t + \nu_{it} - \frac{\Sigma_{k=1}^{N} p_{kt}}{N}\right)^2\right] = E\left[\left(\theta_t + \nu_{it} - \frac{\Sigma_{k=1}^{N}\theta_t + \nu_{kt}}{N}\right)^2\right]\ \Sigma_{t=1}^{N}\beta^{t-1}.$$

Proposition 2 states the players' expected payoffs when they report truthfully.

**Proposition 2.** *If players reported their preferred decisions truthfully, a player's expected payoff under collegial rule would be*

$$-(N-1)\ \frac{\Sigma_{t=1}^{N}\beta^{t-1}}{N}\ (1-\rho)\sigma^2.$$

**Proof:** In the Appendix.

Propositions 1 and 2 imply that the loss under rotational rule is exactly twice as much as what it would be under collegial rule if players reported their preferences truthfully. This reflects the quadratic utility functions. To see this, consider only two players and let $d = |\nu_1 - \nu_2|$. Then, $|\nu_1 - \frac{\nu_1+\nu_2}{2}| = |\nu_2 - \frac{\nu_1+\nu_2}{2}| = |\frac{\nu_1-\nu_2}{2}| = \frac{d}{2}$, moreover, $d^2 = 2((\frac{d}{2})^2 + (\frac{d}{2})^2)$. The advantage of quadratic preferences is that they allow us to express results in terms of variance and correlation coefficient $\rho$. Our qualitative results hinge only on symmetry and convexity of the loss function.

Proposition 2 assumes that players report their preferred decisions truthfully, or alternatively, that players' preferred decisions are public knowledge so that they cannot be misreported. However, in many settings, a player's period preference is private knowledge. Self-interested players who want to maximize their payoffs do not simply reveal their ideal decisions, they act strategically: they express preferred decisions that might differ from their true ideal decisions. Next, I analyze the behavior of players under collegial rule when they strategically express preferred decisions, knowing that the final decision will be the average of expressed preferred decisions. I focus on symmetric pure-strategy equilibria.

Let $p_i(\theta, \nu_i)$ be player $i$'s expressed preferred decision in the last period, where I suppress the time index. Given $\theta$, $\nu_i$, and other players' strategies $p_j(\theta, \nu_j)$, player $i$ expresses a preferred decision $p_i^*$ that solves

$$\min_{p_i} E\left[\left(\theta + \nu_i - \frac{p_i + \Sigma_{j\neq i} p_j(\theta, \nu_j)}{N}\right)^2 \middle| \theta, \nu_i\right]. \tag{2}$$

Thus,

$$\begin{aligned} p_i(\theta, \nu_i) &= E[N(\theta + \nu_i) - \Sigma_{j\neq i} p_j(\theta, \nu_j)|\theta, \nu_i] = N(\theta + \nu_i) - E[\Sigma_{j\neq i} p_j(\theta, \nu_j)|\theta, \nu_i] \\ &= N(\theta + \nu_i) - \Sigma_{j\neq i} E[p_j(\theta, \nu_j)|\theta, \nu_i] \\ &= N(\theta + \nu_i) - (N-1)E_{\nu^j}[p(\theta, \nu_j)|\theta, \nu_i],\ j \neq i, \end{aligned} \tag{3}$$

where I drop the index of $p_j(\theta, \nu_j)$ because I focus on symmetric equilibria. First, I analyze the polar cases of independent preference shocks ($\rho = 0$) and completely aligned preference shocks ($\rho = 1$). When players' preferences are independent, equation (3) simplifies to:

$$p(\theta, \nu_i) = N(\theta + \nu_i) - (N-1)E_{\nu^j}[p(\theta, \nu_j)|\theta],\ \text{independent preference shocks.} \tag{4}$$

Taking expectations with respect to $\nu_i$ on both sides yields

$$E_{\nu_i}[p(\theta, \nu_i)|\theta] = N\theta - (N-1)E_{\nu_j}[p(\theta, \nu_j)|\theta]. \tag{5}$$

By symmetry, $E_{\nu_i}[p(\theta, \nu_i)|\theta] = E_{\nu_j}[p(\theta, \nu_j)|\theta]$, which together with equation (5) yields $E_{\nu_i}[p(\theta, \nu_i)|\theta] = E_{\nu_j}[p(\theta, \nu_j)|\theta] = \theta$. Substituting this into equation (4), I can identify equilibrium strategies:

$$p(\theta, \nu_i) = \theta + N\nu_i,\ \text{for independent preference shocks.} \tag{6}$$

In contrast, when preferences are completely aligned, $E[p(\theta, \nu_j)|\theta, \nu_i] = p(\theta, \nu_i)$. Therefore, from equation (3),

$$p(\theta, \nu_i) = \theta + \nu_i,\ \text{for } \rho = 1. \tag{7}$$

That is, players express their true preferred decisions.

Now consider a general $\rho$. Because $\theta$ is known when players express their preferred decisions, it simply shifts players' strategies. Thus, by symmetry, we have $p(\theta, \nu_i) = \theta + q(\nu_i)$. Substituting this into equation (3) yields

$$q(\nu_i) = N\nu_i - (N-1)E[q(\nu_j)|\nu_i]. \tag{8}$$

To make the analysis tractable for general levels of correlation, I focus on linear strategies:

$$q(\nu_i) = A\ \nu_i,\ \text{for some } A \in \mathbb{R}. \tag{9}$$

From equation (8), $A\nu_i = N\nu_i - (N-1)A\ E[\nu_j|\nu_i]$, and hence

$$A(\nu_i + (N-1)E[\nu_j|\nu_i]) = N\nu_i, \text{ for all } \nu_i. \tag{10}$$

Equation (10) implies that for an equilibrium in linear strategies to exist, we must have $E[\nu_j|\nu_i] = \alpha\nu_i$, which together with $E[\nu_i\nu_j] = \rho\sigma^2$ implies $E[\nu_j|\nu_i] = \rho\nu_i$. Therefore, I make the following assumption in the rest of the paper.

**Assumption 1.** *The conditional expectations of preference shocks are linear, i.e.,* $E[\nu_j|\nu_i] = \rho\nu_i$.

Assumption 1 is satisfied by the exponential family of distributions, which includes such familiar distributions as Normal, Gamma, Beta, Chi Square, Bernoulli, Binomial, Poisson, and Dirichlet. Assumption 1 together with equation (10) yields $q(\nu_i) = \frac{N\nu_i}{1+(N-1)\rho}$, and hence,

$$p(\theta, \nu_i) = \theta + \frac{N}{1+(N-1)\rho}\ \nu_i, \text{ for } -\frac{1}{N-1} < \rho \leq 1. \tag{11}$$

Note that although equation (11) implies our results for $\rho = 0$ and $\rho = 1$ as special cases, our derivation of those results does not hinge on linear strategies or on linear conditional expectations over preference shocks when preference shocks are independent or when $\rho = 1$.

That a player's equilibrium strategy in the last period is unique together with the logic of backward induction implies that players use the same equilibrium strategy in every period. Proposition 3 formally states these results.

**Proposition 3.** *Under collegial rule, when players act strategically, their expressed preferred decisions are more extreme than their true preferred decisions. Formally, a player with ideal point* $\theta + \nu_i$ *expresses a preferred decision*

$$p(\theta, \nu_i) = \theta + A(N, \rho)\ \nu_i, \text{ with } A(N, \rho) = \frac{N}{1+(N-1)\rho} \geq 1.$$

Three aspects of the equilibrium strategies are worth emphasizing: (1) Players appear more extreme than they really are. When a player's preference shock is positive ($\nu_i > 0$), he exaggerates it and pretends to be even more to the right than he really is ($A\nu_i > \nu_i$). Similarly, when his preference shock is to the left ($\nu_i < 0$), he pretends to be more left than he really is ($A\nu_i < \nu_i$). (2) As players' preferences become more aligned, i.e., when $\rho$ increases, players have less incentive to distort their reports, and hence their expressed preferred decisions become closer to their true preferred decisions. (3) Players pretend to be more extreme in larger groups. Corollary 1 summarizes these findings.

**Corollary 1.** *As players' preferences become less aligned or the group size increases, players appear to be more extreme. Formally,*

$$\frac{\partial A(N,\rho)}{\partial \rho} < 0 < \frac{\partial A(N,\rho)}{\partial N}.$$

To calculate player payoffs, substitute equilibrium strategies from equation (11) into equation (2):

$$E\left[\left(\theta + \nu_i - \frac{\Sigma_{j=1}^N p(\theta,\nu_i)}{N}\right)^2\right] = \frac{E[((1+(N-1)\rho)\nu_i - \Sigma_{j=1}^N \nu_j)^2]}{(1+(N-1)\rho)^2}. \tag{12}$$

Proposition 4 combines these findings.

**Proposition 4.** *Under collegial rule, when players act strategically, a player's expected payoff is*

$$U_c^P(N,\rho) = -\frac{(N-1)\sigma^2 \ (1+(N-2)\rho-(N-1)\rho^2)}{(1+(N-1)\rho)^2} \ \Sigma_{t=0}^N \beta^t.$$

*Moreover, as players' preferences become less aligned or the group size increases, players become worse off. Formally,*[10]

$$\frac{\partial U_c^P(N,\rho)}{\partial N} < 0 < \frac{\partial U_c^P(N,\rho)}{\partial \rho}.$$

**Proof:** In the Appendix.

## 6. Collegial Rule v. Rotational Rule

Propositions 1, 2, and 4 allow us to compare collegial rule with rotational rule. If players acted myopically and reported truthfully, or if their preferences were completely aligned ($\rho = 1$), then they would prefer collegial rule to rotational rule. However, when players act strategically under collegial rule, equilibrium payoffs fall. Let $\Delta^P$ be the difference in expected payoffs between rotational and collegial rules, adjusted for discounting. Then,

$$\begin{aligned}\Delta^P(N,\rho) &= \frac{U_r^P(N,\rho) - U_c^P(N,\rho)}{\Sigma_{t=0}^{N-1}\beta^t} \\ &= \frac{(N-1)(1-\rho)\ [N(1-2\rho) - 2(1-\rho)]\ \sigma^2}{N(1+(N-1)\rho)}.\end{aligned} \tag{13}$$

When the players' conflicts of interest are small, i.e., when $\rho \geq \frac{1}{2}$, then the players' strategic manipulation (and hence the resulting welfare loss) is small, so they prefer collegial rule that aggregates their period preferences over rotational rule, i.e., $\Delta^P < 0$. However, when their conflicts of interest

[10] In fact, $\frac{\partial U_c^P(N,\rho)}{\partial N} = -\frac{(1-\rho)\sigma^2}{(1+(N-1)\rho)^2} \ \Sigma_{t=0}^N \beta^t < 0$ and $\frac{\partial U_c^P(N,\rho)}{\partial \rho} = \frac{N(N-1)\sigma^2}{(1+(N-1)\rho)^2} \ \Sigma_{t=0}^N \beta^t > 0$.

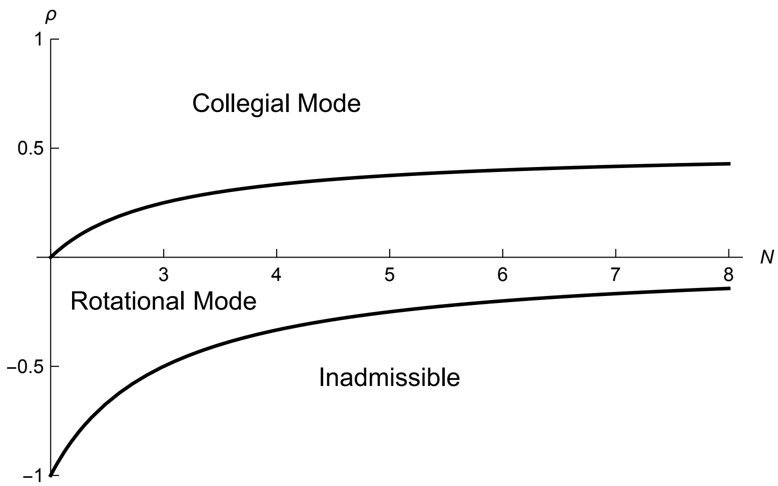


Figure 1: Collegial Rule v. Rotational Rule in $\rho$-$N$ space.

are higher, i.e., $\rho < \frac{1}{2}$, then $\Delta^P > 0$ if and only if $N > N^P(\rho) \equiv \frac{2-2\rho}{1-2\rho}$. As Corollary 1 shows, players misreport their preferences more severely in larger groups. The resulting welfare loss is so much that when group size is sufficiently large, it dominates the gains from preference aggregation under collegial rule. Moreover, as players' conflicts of interest decrease ($\rho$ rises), group size must be larger for players to prefer rotational rule to collegial rule: $\frac{\partial N^P(\rho)}{\partial \rho} > 0$. See Figure 1. Similarly, for a given group size, $\Delta^P < 0$ if and only if $\rho^P \equiv \frac{1}{2}\frac{N-2}{N-1} < \rho < 1$. Theorem 1 summarizes these findings.

**Theorem 1.** *When players act strategically and their preferences are not too aligned ($\rho < \frac{1}{2}$), then they prefer rotational rule to collegial rule if and only if group size is sufficiently large: $N > N^P(\rho) = \frac{2-2\rho}{1-2\rho}$. Alternatively, players prefer collegial rule to rotational rule if and only if their preferences are sufficiently aligned: $\rho^P(N) < \rho < 1$. However, if players' preferences are sufficiently aligned ($\rho \geq \frac{1}{2}$), or if they reported their preferred decisions truthfully, then they would always prefer collegial rule to rotational rule.*

Theorem 1 sheds light on why, typically, military juntas opt for rotational rule, while "emergent entrepreneurial teams" opt for collegial rule, at least in the beginning. Military juntas consist of aged military officers with conflicting interests (low $\rho$) who have made a coalition against a "common enemy". In contrast, many emergent entrepreneurial team are young friends who have a lot more in common in their vision and preferences (high $\rho$). As Theorem 1 shows, members prefer collegial rule to rotational rule if and only if their preferences are sufficiently congruent, i.e., $\rho$ is high.

Although the level of preference alignment $\rho$ is exogenous in this paper, one could speculate that as time passes, members specialize in different aspects of the company and develop different preferences with age, which decreases the congruency of their preferences—$\rho$ decreases. As a result, groups who had opted for collegial rule become more likely to switch to rotational rule over time.

## 7. Preference Aggregation, Information Aggregation, and Group Size

Previous sections compared preference aggregation aspects of collegial and rotational rules, abstracting from information aggregation considerations. But information aggregation is an important feature of group decision making in many settings. In such settings, if members can aggregate their private information, they can make more informed decisions that benefit them all. To integrate information aggregation aspects of group decision making, I extend the model by allowing the members to receive noisy private signals about the common value component of their payoffs, $\theta_t$. In each period $t$, a group member $i$ receives a private signal $s_{it}$ about $\theta_t$, where $(s_{1t}, ..., s_{Nt})$ are identically distributed, independent across periods, and independent from members' private payoffs $(\nu_{1t}, ..., \nu_{Nt})$, with $E[\theta_t^2] = \sigma_\theta^2$, $E[s_{it}^2] = \sigma_s^2$. As in section 4 (see the discussion prior to Assumption 1), I assume:

**Assumption 2.** *$E[\theta_t|s_{it}] = E[s_{jt}|s_{it}] = rs_{it}$, with $r \in (0, 1)$.*

For example, for normally distributed, additive noise structure $s_{it} = \theta_t + \epsilon_{it}$, where $\theta_t$ and $\epsilon_{it}$ are independent for $i, t \in \{1, ..., N\}$, with $\theta_t \sim N(0, \sigma_\theta^2)$ and $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$, we have $r = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\epsilon^2} \in (0, 1)$.

**Rotational Rule.** Under rotational rule, the player in office can make whatever decision he wants, but his ability to make the decision that maximizes his payoff is limited by his incomplete information about the state of the world $\theta_t$. Given quadratic preferences, if player $j$ is in office in period $t$, he chooses $p_t = E[\theta_t|s_{jt}] + \nu_{jt}$. Thus, his expected payoff is $E[(E[\theta_t|s_{jt}] - \theta_t)^2]$, while any other player $i$'s expected payoff is $E[(E[\theta_t|s_{jt}] - \theta_t)^2] + E[(\nu_{jt} - \nu_{it})^2]$. The second term, $E[(\nu_{jt} - \nu_{it})^2]$, is the same as the one that appears in the setting without information aggregation. Moreover,

$$E[(E[\theta_t|s_{jt}] - \theta_t)^2] = E[(rs_{jt} - \theta_t)^2] = r^2\sigma_s^2 - 2rE[s_{jt}\theta_t] + \sigma_\theta^2 = \sigma_\theta^2 - r^2\sigma_s^2,$$

where I use $E[s_{jt}\theta_t] = E[s_{jt}E[\theta_t|s_{jt}]] = rE[s_{jt}^2]$. Proposition 5 summarizes these findings.

**Proposition 5.** *Under rotational rule, a player's expected payoff is*

$$U_r = U_r^P(N, \rho) - U_r^I, \text{ with } U_r^I = \Sigma_{t=0}^{N-1} \beta^t \ (\sigma_\theta^2 - r^2\sigma_s^2).$$

With normal noise structure, $E[(E[\theta_t|s_{jt}] - \theta_t)^2] = \frac{\sigma_\theta^2 \sigma_\epsilon^2}{\sigma_\theta^2 + \sigma_\epsilon^2} = Var(\theta|s_i)$, where I have suppressed the time index in $s_{it}$. Thus,

$$U_r = U_r^P(N, \rho) - Var(\theta|s_i)\ \Sigma_{t=0}^{N-1} \beta^t.$$

Having uncertainty about the state of the world $\theta_t$ reduces players' expected payoffs. The more informative the players' signals are, i.e., the lower $Var(\theta|s_i)$ is, the more accurately the player in charge can forecast the common value $\theta$ based on his private signal, which in turn increases all players' expected payoff. Of course, if the decision maker had more signals about $\theta$, he could make better decisions. However, rotational rule hampers the players' abilities to aggregate their information. As I now show, this contrasts with collegial rule.

**Collegial Rule.** Let $p_i(s_i, \nu_i)$ be player $i$'s expressed preferred decision in the last period, given his private signal $s_i$ and preference shock $\nu_i$. Given $s_i$, $\nu_i$, and other players' strategies $p_j(s_j, \nu_j)$, player $i$ expresses the preferred decision $p_i^*$ that solves

$$\min_{p_i} E\left[\left(\theta + \nu_i - \frac{p_i + \Sigma_{j\neq i} p_j(s_j, \nu_j)}{N}\right)^2 \middle| s_i, \nu_i\right]. \tag{14}$$

As before, I focus on symmetric, linear strategies so that

$$p(s_i, \nu_i) = A'\ \nu_i + B\ s_i,\ \text{for some } A', B \in \mathbb{R}.$$

Mirroring the steps that led to equation (11), one can show that Assumption 2 together with equation (14) yield

$$p(s_i, \nu_i) = \frac{N}{1 + (N-1)\rho}\ \nu_i + \frac{Nr}{1 + (N-1)r}\ s_i,\ \text{for } -\frac{1}{N-1} < \rho \leq 1. \tag{15}$$

Note that $A' = A \geq 1$, while $B = \frac{Nr}{1+(N-1)r}$, and hence

$$B < 1,\ 0 < \frac{\partial B(N)}{\partial N} = \frac{r(1-r)}{(1 + (N-1)r)^2},\ \text{and } \lim_{N\to\infty} B(N) = 1.$$

To calculate a player's welfare, substitute equilibrium strategies from equation (15) into the following expectation:

$$E\left[\left(\theta + \nu_i - \frac{\Sigma_j p(s_j, \nu_j)}{N}\right)^2\right] = E\left[\left(\nu_i - \frac{\Sigma_j A \nu_j}{N}\right)^2\right] + E\left[\left(\theta - \frac{\Sigma_j B s_j}{N}\right)^2\right],$$

where the first term captures preference aggregation (it also appears in Proposition 4), and the second term captures information aggregation. That a player's equilibrium strategy in the last period is unique together with the logic of backward induction implies that players employ the same equilibrium strategy in every period. Proposition 6 summarizes these findings.

**Proposition 6.** *Under collegial rule, when players act strategically, a player's expected payoff is*

$$U_c = U_c^P(N,\rho) - U_c^I, \text{ with } U_c^I = -\Sigma_{t=0}^{N-1}\beta^t \left(\sigma_\theta^2 - \frac{N}{1+(N-1)r} r^2\sigma_s^2\right).$$

With a normal noise structure, $\sigma_\theta^2 - \frac{N}{1+(N-1)r} r^2\sigma_s^2 = \frac{\sigma_\theta^2\sigma_\epsilon^2}{N\sigma_\theta^2+\sigma_\epsilon^2} = Var(\theta|s_1,...,s_N)$, and hence

$$U_c = U_c^P(N,\rho) - Var(\theta|s_1,...,s_N)\ \Sigma_{t=0}^{N-1}\beta^t.$$

That is, players' equilibrium strategies aggregate information as if a decision maker updates according to Bayes rule, given all the available information. This result is consistent with Hastie and Kameda (2005), who perform simulations to study the performance of a group consisting of member with identical preferences but private information about the best course of action. Simulations suggest that, among several collective decision rules, the average rule yields higher group performance as measured by how effectively a group decision aggregates the members' private information.

**Rotational Rule v. Collegial Rule.** From Propositions 5 and 6, the net gain from information aggregation under collegial rule versus rotational rule is $\Sigma_{t=0}^{N-1}\beta^t\ \Delta^I$, where

$$\Delta^I(N,r,\sigma_s^2) = \frac{(N-1)(1-r)}{1+(N-1)r} r^2\sigma_s^2 > 0. \tag{16}$$

For the additive, normal noise structure where $r = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\epsilon^2}$ and $\sigma_s^2 = \sigma_\theta^2 + \sigma_\epsilon^2$, we have $r\sigma_s^2 = \sigma_\theta^2$, and hence

$$\Delta^I(N,\sigma_\theta^2,\sigma_\epsilon^2) = \frac{\sigma_\epsilon^2\sigma_\theta^4}{(\sigma_\epsilon^2+\sigma_\theta^2)(\sigma_\epsilon^2+N\sigma_\theta^2)}.$$

As the group size increases, the information aggregation advantage of collegial rule increases because rotational rule does not take advantage of $N-1$ signals in each period.

$$\frac{\partial\Delta^I}{\partial N} = \frac{(1-r)r^2\sigma_s^2}{(1+(N-1)r)^2} > 0.$$

The total net gain from rotational rule versus collegial rule is composed of a preference aggregation term and an information aggregation term:[11]

$$\Delta(N) = \frac{U_r - U_c}{\Sigma_{t=0}^{N-1}\beta^t} = \Delta^P(N,\rho) - \Delta^I(N,r,\sigma_s). \tag{17}$$

**Two-Player Teams.** The case of two players has particular real world relevance. For example, data indicate that the majority of non-solo nascent entrepreneurs in the US are two-player teams

[11] In the analysis of group size $N$, I focus on a single period, and hence divide the net gain by the term $\sum_{t=0}^{N-1}\beta^t$. This is to isolate the comparative statics with respect to group size $N$, and not the number of periods. Clearly, because periods are ex ante identical, increasing the number of periods from, say, $T$ to $T+1$, adds to the net gain $\Delta(N)$ a factor of $\beta^T\Delta(N)$.

(Vereshchagina 2012). Substituting $N = 2$ in equation (17), using equations (13) and (16) yields

$$\Delta(N = 2) = -\frac{(1-\rho)\rho}{(1+\rho)}\,\sigma^2 - \frac{(1-r)r}{1+r}\,(r\sigma_s)^2,$$

which implies that players prefer collegial rule to rotational rule for two-member groups unless they have extreme conflicts of interest, i.e., their preference shocks are sufficiently negatively correlated. In particular, if $\rho \geq 0$, then $\Delta(N = 2) < 0$.

**Proposition 7.** *In two-member groups, players prefer collegial rule to rotational rule unless their preference shocks are sufficiently negatively correlated.*

**Large Groups.** To characterize the players' preference for collegial and rotational rule in larger groups, one needs to sign $\Delta(N)$ in equation (17). From equations (13) and (16), it can be shown that

$$\Delta(N) = A(aN^2 + bN + c), \tag{18}$$

and hence $\Delta(N)$ changes sign at most twice. For example, for some set of parameters, players prefer collegial rule in small and large groups, but prefer rotational rule in medium-size groups.

Collegial and rotational rules of decision making are plausible for small groups. However, it is worthwhile to analyze the asymptotic behavior of $\Delta(N)$ because it illustrates the trade off between preference and information aggregation considerations under collegial and rotational rules. Recall that when $\rho > \frac{1}{2}$, players prefer collegial rule to rotational rule even without information aggregation incentives. Moreover, the required restriction on possible range of correlation coefficient $-\frac{1}{N-1} < \rho$ implies that $0 < \rho$ when $N \to \infty$. Thus, I focus on $0 \leq \rho < \frac{1}{2}$. From equations (13) and (16),

$$\lim_{N\to\infty} \Delta^P(N) = \frac{(1-\rho)(1-2\rho)}{\rho}\,\sigma^2, \quad \lim_{N\to\infty} \Delta^I(N) = (1-r)r\sigma_s^2.$$

With additive, normal noise structure, $\lim_{N\to\infty} \Delta^I(N) = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\epsilon^2} = Var(\theta|s_i)$. Thus,

$$\lim_{N\to\infty} \Delta(N) > 0 \text{ if and only if } \frac{(1-\rho)(1-2\rho)}{\rho}\,\sigma^2 > Var(\theta|s_i).$$

Theorem 2 summarize these findings.

**Theorem 2.** *If players' private signals are not too informative, then players prefer collegial rule to rotational rule in sufficiently large groups.*

Theorem 1 showed that players prefer rotational rule to collegial rule in large groups unless they have very limited conflicts of interest, i.e., unless $\rho > \frac{1}{2}$. The underlying reason hinged on strategic behavior of players who try to manipulate the preference aggregation mechanism under collegial rule: In large groups, players lie so much in equilibrium (pretending to be far more extreme than they are) that the resulting welfare loss dominates the gains obtained by preference aggregation. But collegial rule also allows information aggregation. In large groups, players have many signals, and hence can forecast the state of the world almost accurately. Thus, the information aggregation advantage of collegial rule hinges on the accuracy of signals. Ironically, if players' signals are very accurate, i.e., $Var(\theta|s_i)$ is small, then the informational gain from integrating $N-1$ signals is marginal, i.e., $Var(\theta|s_i)-Var(\theta|s_1,...,s_N)$ is small, and hence the preference aggregation considerations dominates. However, as Theorem 2 shows, if players' signals are not too accurate so that aggregating more signals is sufficiently valuable, information aggregation considerations dominate so that players prefer collegial rule in large groups.

## 8. Conclusion

I contrast two rules of decision making, collegial and rotational, that map group members' preferred decisions into a collective final decision. These decision making rules are distinguished by their simplicity and their application in real world small groups. I highlight (1) how the distortions arising from strategic decision making can render collegial rule inferior to the rotational rule even though collegial rule seemingly aggregates preferences more, and (2) how effectively collegial rule aggregates players' private information (e.g., relative to rotational rule) *despite* the inefficiencies that arise in the preference aggregation process under collegial rule. The point of this paper is not that optimal mechanisms that better aggregate preferences and information do not exist, but rather to analyze the preference and information aggregation considerations that arise when choosing between two particularly simple mechanisms that naturally arise in many real world settings.

In this paper, information is aggregated through the players' equilibrium strategies. In some settings pre-play communication is prohibitively costly. In others, players may send cheap talk messages before announcing their preferred decisions. Unless the cheap talk equilibrium in the rotational rule is *fully* revealing (which is not due to the players' conflict of interest), the information aggregation advantage of the collegial rule remains. In a fully revealing cheap talk equilibrium, the decision maker in rotational rule would receive all the available signals and updates according to Bayes rule; but in the collegial rule, even absent cheap talk, equilibrium strategies aggregate information as if a decision maker receives all the available signals and updates according to Bayes rule. Still, to what extent the out-of-power players can communicate their information to the ruler (cheap talk in rotational rule) is an important question. In two-players cheap talk settings with known preferences, recent papers show that the decision maker's private information hampers communication by making his decision incensitive to the expert's messages (Chen 2009; Ishida and Shimizu 2014; Lai 2014; Moreno de Barreda 2013). The decision maker's private preference shocks and the presence of multiple agents with uncertain biases can make the decision maker even less sensitive to the expert's message (from the expert's perspective), further hindering communication.[12] Investigating the interactions between these effects is an interesting area of research.

## 9. Appendix

**Proof of Proposition 2:** The absolute value of player $i$'s payoff in period $t$ is

$$E\left[\left(\theta_t + \nu_{it} - \frac{\Sigma_{k=1}^N \theta_t + \nu_{kt}}{N}\right)^2\right] = E\left[\left(\frac{(N-1)\nu_{it} - \Sigma_{k\neq i}\nu_{kt}}{N}\right)^2\right]$$
$$= \frac{(N-1)^2 E[\nu_{it}^2] + \Sigma_{k\neq i}E[\nu_{kt}^2] - 2(N-1)\Sigma_{k\neq i}E[\nu_{it}\nu_{kt}] + 2\Sigma_{k\neq j\neq i}E[\nu_{jt}\nu_{kt}]}{N^2}$$
$$= \frac{(N-1)^2\sigma^2 + (N-1)\sigma^2 - 2(N-1)(N-1)\sigma^2\rho + 2\ C_{N-1}^2\ \sigma^2\rho}{N^2}$$
$$= \frac{(N-1)N(1-\rho)\sigma^2}{N^2} = \frac{N-1}{N}\ (1-\rho)\sigma^2,$$

where $C_{N-1}^2 = \frac{(N-1)!}{(N-3)!\ 2!}$ when $N \geq 3$, and 0, otherwise. Result follows. □

**Proof of Proposition 4:** The numerator in equation (12) is:

$$\begin{aligned} & E[((1+(N-1)\rho)\nu_i - \Sigma_{j=1}^N \nu_j)^2] = E[((N-1)\rho\nu_i - \Sigma_{j\neq i}\nu_j)^2] \\ = \quad & (N-1)^2\rho^2\sigma^2 + (N-1)\sigma^2 - 2(N-1)^2\rho^2\sigma^2 + (N-1)(N-2)\rho\sigma^2 \\ = \quad & (N-1)\sigma^2(1+(N-2)\rho - (N-1)\rho^2). \qquad (19) \end{aligned}$$

The results follow from substituting (19) into equation (12). □

**Proof of Proposition 6:** The preference aggregation term $U_c^P(N,\rho)$ has been calculated in Proposition 2. I calculate the information aggregation term $U_c^I$ in a player's expected payoff in a period:

$$E\left[\left(\theta - \frac{\Sigma_j B s_j}{N}\right)^2\right] = E[\theta^2] - 2\ \frac{B}{N}\ E[\theta\Sigma_j s_j] + \left(\frac{B}{N}\right)^2\ E[(\Sigma_j s_j)^2]$$
$$= \sigma_\theta^2 - 2\ \frac{B}{N}\ Nr\sigma_s^2 + \left(\frac{B}{N}\right)^2\ \left(N\ \sigma_s^2 + 2\ \frac{N(N-1)}{2}\ r\sigma_s^2\right)$$
$$= \sigma_\theta^2 - \frac{N}{1+(N-1)r}\ r^2\sigma_s^2. \quad \square$$

[12]There are also other forces. For example, because experts are risk averse and an expert is uncertain about the others' equilibrium messages, he may coarsen his message intervals to reduce the variance of outcome. Moreover, even when an expert believes that his ideal point is close to the decision maker's, he may not reveal his signal to avoid being mistaken with a biased type (Morris 2001).

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