Homework 3

Complete question 7c from homework 3.

Question 1 (REQUIRES TUESDAY'S NOTES).

Give two reasons why fiscal shocks explain more business cycle variation than monetary shocks in the Cooley-Hansen cash good/credit good model. Give one critique of the Cooley Hansen model as applied to this question.

Question 2 (REQUIRES TUESDAY'S NOTES).

In the Cooley-Hansen cash good/credit good framework, which is more distorting, a consumption tax or the inflation tax. Explain.

Question 3 (REQUIRES TUESDAY'S NOTES).

Which of the following increases the welfare gain from stabilizing business cycles?

    a. A decrease in the coefficient of relative risk aversion.

    b. An increase in the variance of the productivity shock.

    c. An increase in the variance of tax shocks.

Longer Questions

Question 4.

Consider the Cooley-Hansen Cash good - Credit good model with constant money growth rate μ consumption tax, τ, which I summarize below.

To mimick the actual economy, let us assume that when a cash good is purchased, the consumption tax is paid in cash at the time of purchase. Further, the helicopter drop consists of only seniorage, S, not consumption tax revenue. Therefore, the CIA constraint is:

$$(1 + \tau)\, c_1 \leq \frac{m}{P} + S. \tag{1}$$

Money growth is $\bar{M}_{t+1} = (1 + \mu)\, \bar{M}_t$.

No government spending or bonds exist, and the government rebates all consumption tax revenue back to the *worker*. So the government constraint is:

$$TR = S + \tau C. \tag{2}$$

Lastly, remember when you buy a good on credit (e.g. with a credit card), you finance the entire purchase including tax. Therefore, cash is not required to pay the consumption tax on credit consumption.

a. Normalize the problem letting $\hat{p} = p/\bar{M}'$ and derive the household budget constraint.

b. Write the value function.

c. Derive the equilibrium first order condition(s), envelope equation(s). List all equations required for equilibrium.

d. Does the inflation tax distort the ratio of cash to credit goods $\frac{C_1}{C_2}$? Does the consumption tax distort the $\frac{C_1}{C_2}$ ratio? Show mathematically and explain intuitively.

e. Derive the effective tax rate on cash goods consumption as a function of the money growth rate and consumption tax rate. That is, what fraction multiplied by C_1 equals government revenue from both sources? Explain your result.

f. Calculate the consumption tax revenue and inflation tax revenue. Let $\frac{\mu}{1+\mu} = \tau$ so that both tax rates are identical. Derive a condition for α as a function of τ and the parameters for which the inflation tax raises more revenue. Is the condition satisfied for standard parameter values?

g. Derive the steady state equations. Solve for C_1 and C_2 as a function of the paramters, tax rate, and money growth rate.

h. Does the Friedman rule hold? Explain.

Question 7.

Consider the following specification of the OG model with money (taken from Benhabib and Day (1982)):

$$U\left(C_{1,t}\right) = (1-d)\,C_{1,t} \tag{3}$$

$$V\left(C_{2,t+1}\right) = aC_{2,t+1} - \frac{b}{2}\left(C_{2,t+1}\right)^2 \tag{4}$$

$$C_{1,t} = w_1 - \frac{M_t}{P_t}, \quad C_{2,t+1} = w_2 + \frac{M_t}{E\left[P_{t+1}\right]} \tag{5}$$

$$E\left[P_{t+1}\right] = P_{t-1}, \quad M_t = \bar{M} \text{ in equilibrium} \tag{6}$$

So we have the adaptive model and are looking at the dynamics under learning. Note that $C_{2,t}$ is bounded above by $\frac{a}{b}$ and let $d < 1$.

a. Find the first order condition for equilibrium money balances. Solve for the difference equation governing real money balances as a function of previous real money balances.

b. Find the conditions under which money has positive value in equilibrium.

c. Solve for the steady state(s).

For the rest of the questions, consider the special case of $a = b = 1$ and $w_2 = 0$.

d. Find conditions on d for stability of the monetary steady state.

e. Draw the phase diagram and show on the graph how the dynamics change as d increases from zero to one (you may want to draw the dynamics on different graphs for different values of d). For what values of d will the economy display endogenous competitive business cycles?