First Quiz: Solutions
Eco 603

Question 1 (10 points).

We discussed in class that most (2/3) variation in hours are on the extensive margin (people entering and exiting the labor market). In contrast, the RBC model assumes only variation on the intensive margin (those who are already working work more or less hours). This can be fixed by making labor indivisible.

A second reason is changes in tax rates. Changes in labor tax rates are effectively a shock to the labor market, adding variation.

Question 2 (10 points) .

We have:

$$\text{var}(x_t) = \text{var}(sy_t) = s\text{var}(y_t). \tag{1}$$

$$\frac{\text{var}(x_t)}{\text{var}(y_t)} = s. \tag{2}$$

Since the savings rate is less than one, investment spending is less variable than GDP, which does not match up with the data.

Question 3 (15 points) .

a. On the spending side, we have an increase in consumption of \$10 from the purchase of the toy (I am assuming a toy is non-durable, which is realistic for Chinese toys in my view!). Net exports falls by \$10 so we have -\$10 in investment spending. GDP is unchanged since the production was not US owned.

b. On the spending side we have a \$100 increase in consumption. GNP also rises by \$100. On the income side, we have an increase in labor income of \$100. GNP again rises by \$100.

c. On the income side, we have government income so we assign $\gamma \cdot 100$ to capital income and $(1 - \gamma) \cdot 100$ to labor income. GNP rises by \$100. On the spending side \$100 of the total value of the car was "produced" in the US. Durable goods consumption rises by \$100, add \$100 to investment spending.

d. We have an increase in durables consumption by \$50, so add \$50 to investment spending. However, the net change in inventory is $-\$50$, so x decreases by \$50. On net, we have no change in investment spending, consumption, or GNP. On the income side, Chris is just selling an asset for cash. There is no income change.

e. On the income side, we have an increase in \$6M of labor income and an increase of \$6M on capital income. On the spending side, the government has purchased a durable asset, so add \$12M to investment spending.

Question 4 (10 points).

We can estimate the capital income from durable good service flows via $I_D = R \cdot K_D$. That is, we know the stock of durables and estimate the capital income to be at the same interest rate as private capital income. However, there is no corresponding way to get labor income from durables. Hence, by assumption the capital share of durables is one, which raises the overall capital share which is an average.

Question 5 (10 points).

a. We have:

$$I = \gamma Y = \$1000 + \gamma \$400. \tag{3}$$

$$\gamma \cdot 4400 = 1000 + \gamma \cdot 400. \tag{4}$$

Capital income is unambiguous capital income plus γ fraction of ambiguous capital income. Solving yields:

$$\gamma = \frac{1}{4} \tag{5}$$

Notice the capital share is unaffected by the sales taxes, because the sales taxes is assumed to have the same capital share.

b. We have capital income of $\gamma \cdot Y = 0.25 \cdot 4400 = \$1100$. Labor income is $4400 - 1100 = 3300$. GNP is 4400.

Question 6 (45 points).

a. For the resource constraint, we set resources equal to allocations. We have:

$$e^{z_t} k_t^{\gamma} h_t^{1-\gamma} = c_t + (1+\eta) k_{t+1} - (1-\delta) k_t + (1 - (1-\mu)^{\epsilon}) e^{z_t} k_t^{\gamma} h_t^{1-\gamma}. \tag{6}$$

So we just need to add the resource cost of abatement.

b. Simplifying and eliminating μ using $1 - \mu = E/y$:

$$(1-\mu)^{\epsilon} e^{z_t} k_t^{\gamma} h_t^{1-\gamma} = c_t + (1+\eta) k_{t+1} - (1-\delta) k_t. \tag{7}$$

$$E^{\epsilon}e^{(1-\epsilon)z_t}k_t^{\gamma(1-\epsilon)}h_t^{(1-\gamma)(1-\epsilon)} = c_t + (1+\eta)\,k_{t+1} - (1-\delta)\,k_t. \tag{8}$$

I find it best to introduce a new variable for the capital share, $\theta = \gamma\,(1-\epsilon)$.

$$E^{\epsilon}e^{\frac{\theta}{\gamma}z_t}k_t^{\theta}h_t^{(1-\theta-\epsilon)} = c_t + (1+\eta)\,k_{t+1} - (1-\delta)\,k_t. \tag{9}$$

c. The value function is:

$$v\,(k,z,m) = \max_{k',h,E}\left\{u\left[E^{\epsilon}e^{\frac{\theta}{\gamma}z_t}k_t^{\theta}h_t^{(1-\theta-\epsilon)} - (1+\eta)\,k_{t+1} + (1-\delta)\,k_t\right] - \psi h \right.$$
$$\left. -D\,(E+(1-\phi)\,m) + \beta\mathrm{E}\left[v\,(k',\rho z+\epsilon',E+(1-\phi)\,m)\right]\right\}. \tag{10}$$

The stock of pollution is a state like the stock of capital. Emissions is under control of the planner through abatement.

d. Let $s = [k, z, m]$ be a vector of states. We have:

$$(1+\eta)\,u_c\,(c) = \beta\mathrm{E}\,[v_k\,(s')]\,. \tag{11}$$

$$u_c\,(c)\,(1-\theta-\epsilon)\,\frac{y}{h} = \psi. \tag{12}$$

$$u_c\,(c)\,\epsilon\frac{y}{E} = D_e\,(m') + \beta\mathrm{E}\,[v_m\,(s')]\,. \tag{13}$$

For the envelopes:

$$v_k\,(s) = u_c\,(c)\left(\theta\frac{y}{k} + 1 - \delta\right). \tag{14}$$

$$v_m\,(s) = \{D_m\,(m') + \beta\mathrm{E}\,[v_m\,(s')]\}\,(1-\phi)\,. \tag{15}$$

e. For $\phi = 1$, $m = E$ and m drops out as a state. Equations (12) and (13) can be combined so that:

$$\frac{\psi\epsilon}{1-\theta-\epsilon}\frac{h}{E} = D_e\,(E)\,. \tag{16}$$

Now since h is procyclical, E must be as well since damages are convex. The idea is that you want to emit more when it is really productive to do so, in booms.

f. First note that from the law of motion for pollution, we have that:

$$m = \frac{1}{\phi} E. \tag{17}$$

We have:

$$(1+\eta)\, u_c(c) = \beta v_k(s). \tag{18}$$

$$u_c(c)\,(1-\theta-\epsilon)\,\frac{y}{h} = \psi. \tag{19}$$

$$u_c(c)\,\epsilon \frac{y}{E} = D_e\left(\frac{1}{\phi} m\right) + \beta v_m(s). \tag{20}$$

$$v_k(s) = u_c(c)\left(\theta \frac{y}{k} + 1 - \delta\right). \tag{21}$$

$$v_m(s) = \left\{ D_m\left(\frac{1}{\phi} E\right) + \beta v_m(s) \right\}(1-\phi). \tag{22}$$

Solve the envelope for v_m to get:

$$v_m(s) = \frac{\beta(1-\phi)}{1-\beta(1-\phi)} D_m\left(\frac{1}{\phi} E\right). \tag{23}$$

Substitute the envelopes into the first order conditions and use the beta trick to get:

$$\rho = \theta \frac{y}{k} - \delta. \tag{24}$$

$$u_c(c)\,(1-\theta-\epsilon)\,\frac{y}{h} = \psi. \tag{25}$$

$$u_c(c)\,\epsilon \frac{y}{E} = \frac{1}{1-\beta(1-\theta)} D_e\left(\frac{1}{\phi} m\right). \tag{26}$$

Equations (17), (24)-(26), and the resource constraint are 5 equations for unknowns c,

E, m, k, and h.

g. We have:

$$\text{cost} \ = \left(1-(1-\mu)^{\epsilon}\right) y_t, \tag{27}$$

$$\frac{\text{cost}}{y} = 1 - \left(\frac{E}{y}\right)^{\epsilon}. \tag{28}$$

$$\epsilon \log\left(\frac{E}{y}\right) = \log\left(1 - \frac{\text{cost}}{y}\right). \tag{29}$$

$$\epsilon = \frac{\log\left(1 - \frac{\text{cost}}{y}\right)}{\log\left(\frac{E}{y}\right)} \tag{30}$$

We have calibrated ϵ to match the fraction of income spent on environmental compliance. This assumes optimal government regulation, which is a bit of a stretch.

h. Multiplying equation (24) by k gives:

$$I = (\rho + \delta)\, k = \theta y \tag{31}$$

$$\frac{I}{y} = \theta. \tag{32}$$

The capital share is $\theta = \gamma\,(1-\epsilon)$.