Exercise 3: Eco 603
Cash Good/Credit Good.
Due (questions 1-8 except 7d): Tuesday October 31

## Questions from Class Notes

### Question 1.

As noted in class, redo the money-in-utility model with endogenous labor supply. Show that the neutrality result is not robust.

### Question 2.

Give conditions on $\psi\left(c_t, \frac{M_t}{P_t}\right)$ for which steady state real balances are decreasing in the money growth rate.

## Cash Good/Credit Good

Here we consider an alternative cash-in-advance model. The model differs in two respects: the transfer is given to the shopper, and only some consumption goods are purchased with cash. Here is a quick summary of the model:

- Assume two consumption goods exist: $c_{1t}$ and $c_{2t}$. The production can be allocated to either good (think of them as red and blue shmoos).
- Consumption of good one requires cash in advance, the other good is purchased on credit.
- The transfer goes to the shopper and can be used to fund cash expenses.
- Utility is logarithmic with indivisible labor: $U = \alpha \log c_{1t} + (1-\alpha)\log c_{2t} - \psi h_t$.

The framework is described fully on page 195-98 of the Cooley textbook, with government spending and bonds equal to zero (except as described below). Also, Cooley uses Cobb-Douglas production, but this is not necessary for any of the questions below, except question 6.

### Question 3.

Normalize the problem using the Cooley Hansen method. Write the value function for the problem. You should get identical answers as in the book.

### Question 4.

a. Derive the first order conditions and envelope conditions for the model. Interpret each.

b. Define equilibrium.

c. Derive the set of equations which solve for all equilibrium variables.

**Question 5.**

Prove how each equilibrium decision variable is affected by the money growth rate.

**Question 6.**

a. Solve for the equations which determine the certainty equivalence steady state.

b. Is money neutral? Super-neutral?

c. Write the social planning problem and derive the first order conditions and envelope equations, and then the certainty equivalence steady state. Derive and explain the optimal money growth rate.

d. Solve for the steady state variables for the special case of Cobb-Douglas production.

**Question 7.**

These questions pertain to the consumption velocity. One immediate question is whether to use $\bar{M}$ or $\bar{M}'$ in the equation $MV = PC$. Following Cooley, we will let $V_c$ denote the velocity using $\bar{M}$ and $VEL$ as the velocity using $\bar{M}'$.

a. Which makes more sense as a measure of the consumption velocity in the data, $V_c$ or $VEL$?

b. Express the consumption velocity $V_c$ as a function of the money growth rate and the parameters only (not necessarily steady state).

c. Does the consumption velocity have the following properties of the data? Explain.

   (a) Velocity rises with inflation.

   (b) Velocity is pro-cyclical.

   (c) Velocity is less than one.

d. Derive equation (26) in the text, VEL $\equiv \frac{V_c}{e^{\mu t}}$ as a function of the nominal rate.

e. Solve for the certainty equivalence steady state velocity, $VEL$. Let $\nu \equiv \frac{c_1}{c_1+c_2}$ be the fraction of consumption transactions that are done with cash. Derive equation (27), $\alpha$ as a function of the steady state fraction of transactions which use cash.

**Question 8.**

Consider the following set of data:

- Ralph gets paid on February 28th. A direct deposit of \$1,500 is added to his checking account (assume previous balance was zero).

- Ralph goes to the ATM and withdraws \$200, which he puts in his wallet (assume previous balance was zero).
- On March 1, Ralph pays his rent with a \$900 check.
- In March, Ralph buys \$200 worth of gas using his credit card.
- On March 15, Ralph gets a hot stock tip and invests \$100, paid with a check.
- For the month of March, Ralph consumes \$100 worth of doppio espresso shots, all paid from cash in his wallet.

a. Calculate the consumption velocity using the cash good/credit good model above.

b. Calculate consumption velocity according to the CIA model in class.

c. What is the true consumption velocity? That is, how many times was the average dollar spent on consumption?

d. Does (a) correctly, over, or under estimate the consumption velocity? Explain why.

e. Does (b) correctly, over, or under estimate the consumption velocity? Explain why.

**Question 9.**

Now introduce an debt instrument $b_t$ that pays $1 + R_{t-1}$ (in cash) in period $t$, which can be purchased for \$1 in the previous period. Thus $b_{t+1}$ are bonds purchased in $t$ and redeemed in $t+1$. Here we can interpret $R$ as the nominal interest rate. Let both bond redemptions and new purchases occur prior to when the shopper leaves. Finally, suppose bonds have supply equal to zero each period.

a. Normalize $b_t$ consistent with above (so that $p$ is replaced by $\hat{p}$ and the problem is stationary).

b. Show:

$$R = \left(\beta E\left[e^{-\mu_{t+1}}\right]\right)^{-1} - 1 \tag{1}$$

Interpret the result.

c. For the certainty equivalence steady state, find the Fisher relation (the relationship between the nominal rate, the real rate, and inflation).

d. Calculate the optimal steady state nominal rate of interest. Explain your answer.

**Question 10.**

Show that $R_t > 0$ implies the CIA constraint binds (hint: show that if the CIA constraint does not bind, then $R_t = 0$ and then use $(A \Rightarrow B) \Rightarrow (\text{ not } B \Rightarrow \text{ not } A)$).

**Question 11.**

Prove the Ricardian equivalence proposition holds/does not hold in the model with bonds.

**Question 12.**

a. Derive the conventional deficit.

b. Derive the primary deficit.

c. Is a permanent primary deficit possible with seniorage?

d. Is a permanent conventional deficit possible with seniorage?

e. Is a permanent primary deficit possible without seniorage?

f. Is a permanent conventional deficit possible without seniorage?