Question 1

Consider the growth model with exogenous growth in productivity $(A_t = (1 + \theta) A_{t-1})$ from homework 1. Add the continuous productivity shocks as in class.

 a. Write the value function for the social planning problem.

 b. Calculate the first order condition(s). Interpret.

 c. Calculate the relevant envelope equation(s). Interpret.

 d. Calculate the comparative statics $\frac{\partial k'}{\partial k}$ and $\frac{\partial k'}{\partial z}$ (hint: use the implicit function theorem). Under what conditions is k' an increasing function of k and z?

 e. Write the equations which determine the certainty equivalence steady state.

 f. Solve the steady state equations given the functional forms used in the calibration.

 g. Consider the calibration of the discount factor β. Under what conditions is the calibrated value of β in the model with exogenous productivity growth larger or smaller than the discount factor found in the model from class? Explain this result (hint: refer to homework 1).

Question 2

Assign each of the following NIPA categories to a particular variable or set of variables so that the model is consistent with the data. If assigning to more than one category, give the proportion assigned to each category.

 a. Sales taxes.

 b. Corporate Profits.

 c. Durable goods consumption.

 d. Change in Inventories.

 e. Income from sole proprietorships.

Question 3

For this problem, you will need to calibrate part of the model as shown in class. Start by obtaining NIPA data on unambiguous capital income, ambiguous capital income, GNP, depreciation, private capital, inventories, government capital, consumer durables capital, gross investment, net exports, change in inventories, government investment and depreciation, and durable goods consumption. Construct the land values using the FED data as real

estate values less replacement costs for all 3 sectors reported. The FED data is available at: www.federalreserve.gov/releases/z1.

Piketty in a recent book argues empirically that the capital share of income has been increasing over time.

a. Construct the capital share considering only unambiguous private capital income. Has the capital share been increasing over time?

b. Construct the capital share considering ambiguous and unambiguous private capital income. Has the capital share been increasing over time?

c. Construct the capital share including government and durable capital income. Has the capital share been increasing over time?

d. Construct r and $g = \frac{Y_{t+1}}{Y_t}$. Piketty also argues that $r > g$. Is this supported in the US data?

Question 4

Suppose Ralph imports a BMW motorcycle for $10,000. BMW pays a 10% tariff to the US government, then sells the remaining dollars for Euros in the foreign exchange market to two German buyers. One buyer of the dollars buys a $4,000 Apple computer. The other buyer invests the remaining dollars in US treasury bills.

a. Calculate consumption, investment spending, and GNP according to the NIPA.

b. Calculate consumption, investment spending, and GNP according to the RBC model.

Question 5

Our model assumes households can choose any $h \in [0,1]$. However, many workers choose only to work or not work. Once the decision to work is made, many workers work a fixed number of hours (say 40 per week).

Consider an alternative model, which has all assumptions of the RBC model with the continuous distribution of the productivity shocks, except the following.

- Suppose a household can only work a fixed number of hours, or not at all. Therefore, all variation in hours comes from variation in the number of people employed. Changes in h associated with people entering and exiting the workforce is called the extensive margin, whereas changes in h associated with people working more hours is called the intensive margin. Let x denote then the *fraction of employed workers*. Assume (without loss of generality) $h = h_0 \in (0,1)$ for the employed and $h = 0$ for those not working. In the actual data, two thirds of hours variation is caused by variation along the extensive margin.

- Suppose individual utility is separable in consumption and leisure: $U = (1 - \alpha) \log [c] + \alpha (l)^{1+\sigma}$.

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The problem of the social planner is then to determine the fraction of workers to employ, in addition to the usual decision(s).

a. Intuitively, how will the social planner allocate consumption between households that work and those that do not? Derive mathematically using part (b) if you are not sure.

b. Write the resource constraint with all variables in per-capita terms. Write the value function for the social planner who maximizes the utility of all households.

c. Find the first order condition(s) and envelope equation(s).

d. Consider now the intertemporal elasticity of substitution for leisure. Roughly speaking, a change in the return to labor over time may not induce much extra working/leisure if households prefer relatively smooth leisure over time. That is, if households prefer to work 8 hours each day rather than 16 hours one day, and none the next.

- Let $1 + r = e^z F_k + 1 - \delta$ denote the gross return to capital. Let $w = e^z F_h$ denote the return to leisure. Use the first order conditions to derive an equation relating the substitution of leisure across time:

$$\frac{l'}{l} = G\left[\frac{(1+r)\,w}{w'}\right] \tag{1}$$

. Do so for both divisible and indivisible labor cases.

- Derive the intertemporal elasticity of leisure,

$$e = \frac{\partial \frac{l'}{l}}{\partial \frac{(1+r)w}{w'}} \frac{\frac{(1+r)w}{w'}}{\frac{l'}{l}} \tag{2}$$

for both divisible and indivisible labor cases.

- Can the divisible labor model replicate both the micro studies indicating a strong preference for smoothing leisure over time and the macro results of high variation in hours?

- Can the indivisible labor model replicate both the micro studies indicating a strong preference for smoothing leisure over time and the macro results of high variation in hours?

e. Assume that due to consumption smoothing c does not change with z. Is x an increasing or decreasing function of z? Intuitively, is x pro- or counter-cyclical?

f. Graph the first order condition for x.

g. Again, assuming consumption does not change with productivity shocks, show on the graph that total hours worked varies more when variation is along the extensive margin, then when variation is along the intensive margin.

h. Explain how to calibrate the preference parameter for leisure (α). What data is needed?