Review, First Quiz, Solutions
Managerial Economics: Eco 685
Quiz Date: February 4, 2005

Do not read solutions until you have worked through the problems!

Question 1

This problem is identical to the one in the notes, except different numbers.

a. According to the formula sheet, the marginal product of an input is the derivative of the production function with respect to the input:

$$MP_s = \frac{\partial Q}{\partial s} = 30 + 0 - \tfrac{1}{2} \cdot 2s - 0 = 30 - s \tag{1}$$

$$MP_u = \frac{\partial Q}{\partial u} = 0 + 20 - 0 - \frac{1}{4} \cdot 2u = 20 - \tfrac{1}{2}u \tag{2}$$

$$\frac{MP_s}{MP_u} = \frac{30 - s}{20 - \frac{1}{2}u} \tag{3}$$

The ratio of prices is $\frac{P_s}{P_u} = \frac{10}{5} = 2$.

b. Solving using $\frac{MP_s}{MP_u} = \frac{P_s}{P_u}$ results in:

$$\frac{30 - s}{20 - \frac{1}{2}u} = 2 \tag{4}$$

$$30 - s = 2\left(20 - \tfrac{1}{2}u\right) \tag{5}$$

$$30 - s = 40 - u \tag{6}$$

$$u = s + 10 \tag{7}$$

So we hire 10 more unskilled than skilled workers.

c. The firm has too many skilled workers, for 3 skilled workers we need 13 unskilled workers.

d. The budget is:

$$60 = 10s + 5u \tag{8}$$

Since we need 10 more unskilled:

$$60 = 10s + 5\,(s + 10) \tag{9}$$

$$60 = 10s + 5s + 50 \tag{10}$$

$$10 = 15s \quad \rightarrow \quad s = \frac{2}{3} \tag{11}$$

We should hire 2/3 skilled workers and 10 and 2/3 unskilled.

Question 2

a. The fixed costs are the Field Rental, T-shirt set up, salaries, and chalk for a total of $2,135.

b. The Field rental cost of $1,600 is sunk.

c. Total costs are total fixed costs ($2,135) plus total variable costs. The t-shirts are a variable cost equal to $\$6Q$, where Q is the number of players. The party is a variable cost equal to $\$6 \cdot \frac{1}{2}Q$. On average we need to spend $3 per player on the party. Thus total costs are:

$$TC = TFC + TVC = \$2,135 + \$9Q \tag{12}$$

d. Average variable costs equal total costs divided by Q or $9.

e. Using the formula:

$$Q_{\text{break even}} = \frac{\$2,135}{\$25 - \$9} = 133.44 \tag{13}$$

f. We needed 133.44 players to break even. Actual demand was 135 players, so we should make a small profit.

$$\pi = TR - TC = \$25Q - \$2,135 - \$9Q = \$25 \cdot 135 - \$2,135 - \$9 \cdot 135 = \$25 \tag{14}$$

The profit is $25. Needless to say, if the opportunity cost of my time was larger, we might decide not to have the league.

Question 3

The left hand side says that if we decrease s by one, then we require $\frac{MP_s}{MP_u}$ units of u to keep production constant. Similarly, if we decrease s by one, we can spend the savings on u and hire $\frac{P_s}{P_u}$ units of u. So reducing s by one and adding $\frac{MP_s}{MP_u}$ units of u is a losing strategy. Production is unchanged, but costs rise. We can hire $\frac{P_s}{P_u}$ without increasing costs, but we need to hire more because the left hand side is greater.

Let's try the reverse. Notice that:

$$\frac{MP_u}{MP_s} < \frac{P_u}{P_s} \tag{15}$$

Now if we reduce u by one unit and hire $\frac{MRP_u}{MRP_s}$ units of s, then we keep production constant, but costs fall: we can spend $\frac{P_u}{P_s}$ and costs will be constant, but we spent less than that since the left hand side is smaller. So we can increase profits by reducing unskilled labor by one unit and hiring $\frac{MRP_u}{MRP_s}$ skilled workers.

Question 4

a. Total fixed costs are \$16.68. Average total costs are TC/Q or:

$$\frac{TC}{Q} = \frac{16.68}{Q} + 0.125 + 0.00439Q \tag{16}$$

Marginal costs are:

$$\frac{\partial TC}{\partial Q} = 0 + 0.125 + 2 \cdot 0.00439Q = 0.125 + 0.00878Q \tag{17}$$

b. Marginal revenue here is the price which equals \$0.13. So setting marginal revenue equal to marginal cost gives:

$$\$0.13 = 0.125 + 0.00878Q \quad \rightarrow \quad Q = 0.57 \tag{18}$$

It costs \$0.13 to make the $Q = 0.57$ kilowatt hour, so the company breaks even. After that, it costs more than \$0.13 to make another kilowatt hour, so the firm should not produce any more than $Q = 0.57$.