Economics 685
Managerial Economics
Notes

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INTRODUCTION

I what is Managerial Economics?

Definition 1 **Managerial Economics** *is the application of economic theory to decisions made by managers and firms.*

Definition 2 **Economics** *is the study of the allocation of scarce resources*

Economics is the study of the allocation of scarce resources. Because all decisions are essentially about the allocation of scarce resources, economics is in fact the study of decision making and problem solving in general. So we will simply apply economic rules for decision making to problems faced by managers.

As an example, consider a marketing executive for Tang (General Foods) considering both TV and "guerilla" advertising. The dollars in the marketing budget are scarce resources that must be allocated to one or the other type of advertising. Managerial economics provides a systematic, scientific method for making the where-to-advertise decision. Similarly, should Disney add a theme park in France? Capital is the scarce resource. The capital used to build the theme park could instead be used to build a theme park somewhere else, or used to invest in another business, perhaps video production.

II Managerial Economics' advice for decision making

Managerial economics provides a scientific method for making business decisions.

A The Steps

1. Formulate the problem: list the objective and the possible decisions.

2. Gather data on the objective and the decisions.

3. Using statistics, estimate the relationship between the objective and the decisions, known as the objective function.

4. Using calculus or game theory, choose the decision that maximizes the objective function.

B Example

Consider our Tang marketing executive again.

Step 1. The objective is presumably to maximizes sales (if the advertising budget is fixed and the goal is to get as much out of each marketing dollar as possible) or profits (if the executive for example wants to justify a larger budget as positively influencing profits). Let us suppose the former.

It is reasonable to conjecture that diminishing returns exist: as more and more advertisements are made, each additional dollar gives less and less additional sales. The first ad is fresh and reaches many previously unaware consumers. The 100th ad generates little additional sales, everyone is familiar with the product. Here is an objective function that has diminishing returns:

$$S = a + b_1 A_{tv} - c_1 A_{tv}^2 + b_2 A_g - c_2 A_g^2 \tag{1}$$

Here A is advertising expenses, S is sales, and a, b, and c are unknown parameters (A **parameter** is a variable which the manager cannot change).

Step 2. Our executive has data on past sales and advertising expenditures. We also of course need to know the cost of a TV ad and the cost of a guerrilla ad.

In step 3, we use fancy statistics to find the unknowns a, b and c. We get (sales are in millions of dollars):

$$S = 22 + 5A_g - 1.5A_g^2 + 24A_{tv} - 0.5A_{tv}^2 \tag{2}$$

We also have our budget numbers. We have \$0.5 million to spend, a block of TV ads costs \$5 million, while a set of guerilla ads costs \$1 million. So TV ads are more expensive, but reach a wider audience.

$$\$0.5 = 5A_{tv} + 1 \cdot A_g \tag{3}$$

In step 4, we use calculus to find the values of A_{tv} and A_g that maximize S, given our scarce resources (budget dollars). We would find that: $A_{tv} = 0.086$ ad blocks or about \$430,000 and $A_g = \$0.07$ ad blocks or about $\$70,000$.[1] Although TV ads are more expensive, the extra expense is more than offset by the larger audience.

C Alternative Methods of Making Decisions

Believe it or not, the vast majority of you, despite this class, will not make decisions this way. Instead, most of you will guess-timate or bluff your way through the problem. Let us be generous and call guessing relying on "experience" or "expertise."

One can actually do a pretty good job relying on experience, if you have it. With enough practice, a squirrel can figure out how many acorns to save without knowledge of internal rates of return. However, the scientific method requires no practice (other than data collection), and gives a better decision.

Both decision methods require common sense to work. The data used above may have been collected in a record year, giving numbers that are unlikely to be repeated. The executive may have data on TV advertising in one state and falsely suppose all states work the same.

III Decisions Studied in Managerial Economics

Although managerial economic principles can be applied to any decision (and in fact are applied elsewhere in for example your finance class), managerial economics typically focuses on three types of decisions.

1. How much to produce? How many calculators should the Rondo Corp. produce per day?

2. What inputs to use? How many workers and how many machine tools should Rondo employ?

[1] By the way, this executive should be arguing for a larger budget: spending \$2 million on guerilla marketing and \$128 million on TV ads (total budget of \$130 million) would maximize profits.

3. What price to charge? Should Rondo use "cost-plus" pricing or some other method?

Notice that these decisions are very high-level decisions, typically made by CEOs and senior management, not newly minted MBAs (unless they are entrepreneurs or work in a small firm). This is one reason why the most common major among CEOs is economics. Nonetheless, as our Tang example demonstrates, even junior marketing executives can use these methods. Further, some of you no doubt aspire to be CEOs, and so will eventually need this information.

IV Theory of the Firm

A Objective of the firm

What is the objective or goal that a manager has in mind when making decisions? At the most general level, managerial economists suppose that managers try to maximize the value of a firm.

Definition 3 *The* **value** *of a firm is the present value of the firm's cash flows*

In a rough sense, cash flows are profits, which are revenues less costs. Thus:

$$pv = \sum_{t=1}^{n} \frac{\pi_t}{(1+i)^t} \tag{4}$$

$$\pi_t = TR_t - TC_t \tag{5}$$

Here pv is the present value of the firm, n is the planning horizon, i is the appropriate interest rate (or the rate of return that could be earned if the profits were invested elsewhere). Here π is profits and TR_t and TC_t are total revenues and total costs.

B Economic Profits

Note that profits here are economic profits.

Definition 4 Economic Profits *are profits after taking into account capital and labor provided by owners.*

Profits as normally recorded are known as accounting profits. Economic profits are lower, they subtract from total revenues opportunity costs. That is, they subtract the value of time and capital that could be spent elsewhere.

Example. Suppose the market rate of return is 10%. A shoe store owner is considering buying an additional factory for $240,000. The owner values the additional time required to supervise the new plant at $40,000 per year (or suppose he could hire someone at this price, but prefers to do it himself). He figures the accounting profit is $60,000 per year. To get the economic profit:

$$\text{Economic Profit } = \text{accounting profit } - \text{ return on capital used} - \text{ owner's labor} \tag{6}$$

So here the economic profit is $\$60,000 - \$24,000 - \$40,000 = -\$4,000$. Although the additional factory would make money, the owner would do better by investing the $240,000 in the market. Thus the optimal decision would be not to buy the factory. Clearly, accounting profits are good for maintaining records, but economic profits are needed to make decisions.

Finally, in the long run, economic profits are generally driven to zero. If economic profits are negative, firms will drop out of the market and prices will rise, increasing profits. Conversely, if economic profits are positive firms will enter the market and competition will drive prices down. Thus unless the industry has barriers which prevent firms from entering and exiting (eg. monopolies) or their are regulations on price setting (eg. electricity prices), economic profits tend to zero.

C Other objectives

1 Objectives of firms and managers

We claim that firms maximize profits (although individual managers may not). Firms that do not tend to go out of business. CEOs who do not maximize profits tend to be replaced, since shareholders clearly desire profits to be maximized.

Many other objectives, such as maximizing market share, are just intermediate goals toward the final goal of maximizing profits (remember, by setting a price of zero one could always maximize market share so it is doubtful if any firms, despite their claims, really do this). Similarly, satisficing, or meeting profit goals, are approximations to maximizing profits done when it is not clear what maximum profits are.

It is important to realize that individual managers or CEOs may have other objectives (as our Tang example indicates), unless motivated by stock options or other incentives.

2 Profit maximization, ethics, and welfare

Maximizing profits improves the welfare of society in two ways:

1. The firm provides products and services that consumers want (as evidenced by their willingness to pay for the products, and thus add to firm profits).

2. The firm stays in business, thus providing income to workers and stockholders.

Famous theorem in economics: in general, maximizing profits maximizes the welfare of society (this is not usually what is heard in the news). For example, consider "price gouging" of gasoline. By charging high prices when supply is low, gas retailers insure that the consumers who most need the gas (as evidenced by their willingness to pay high prices) get it. Further, the retailer generates income in the form of wages and profits for stockholders. Charging a low price means that whoever gets the small supply of gas is whoever gets to the station first, for example. Second example is electricity in California. During the recent shortage, retail prices were fixed. Thus, hospitals and swimming pools paid the same price for electricity, and rolling blackouts determined who got electricity. By raising prices, hospitals would have a chance to out bid swimming pool owners for the needed electricity, raising welfare. Thus, in most cases, the most ethical practice is to maximize profits.

PRODUCTION THEORY

Production Theory helps managers decide what inputs to use. The book presents the material largely from a manufacturing prospective, for example how many machines and how many laborers to use. But the material could equally apply to, say, a financial planning firm deciding how many assistants should be assigned to each financial planner.

I The Production Function

Definition 5 *The* **Production Function** *is a graph, table, or equation showing the maximum output rate that can be achieved by any specified set of inputs*

For example, consider the production function in the book, for Thompson Machine Co. Thompson has five machines in a shop that produces machine parts. Let's suppose the number of machines input is fixed in the short run. Let Q be hundreds of parts produced per year, and let L be the number of full time workers, again per year. Then:

$$Q = 30L + 20L^2 - L^3 \tag{7}$$

Here we have the information in table form.

Full time laborers (L)	Parts Produced (Q)
0	0
1	49
2	132
3	243
6	684
9	1161
12	1512
14	1596
15	1575

What can we do with the production function?

1. Set inputs to produce a specified quantity of outputs. If the manager expects 700 orders this year, he knows to hire 7 workers (from the table, 6 produces only 684).
2. Hire workers to meet objective of maximizing production. See below.
3. With information on wages, hire workers to meet objective of maximizing profits. See below.

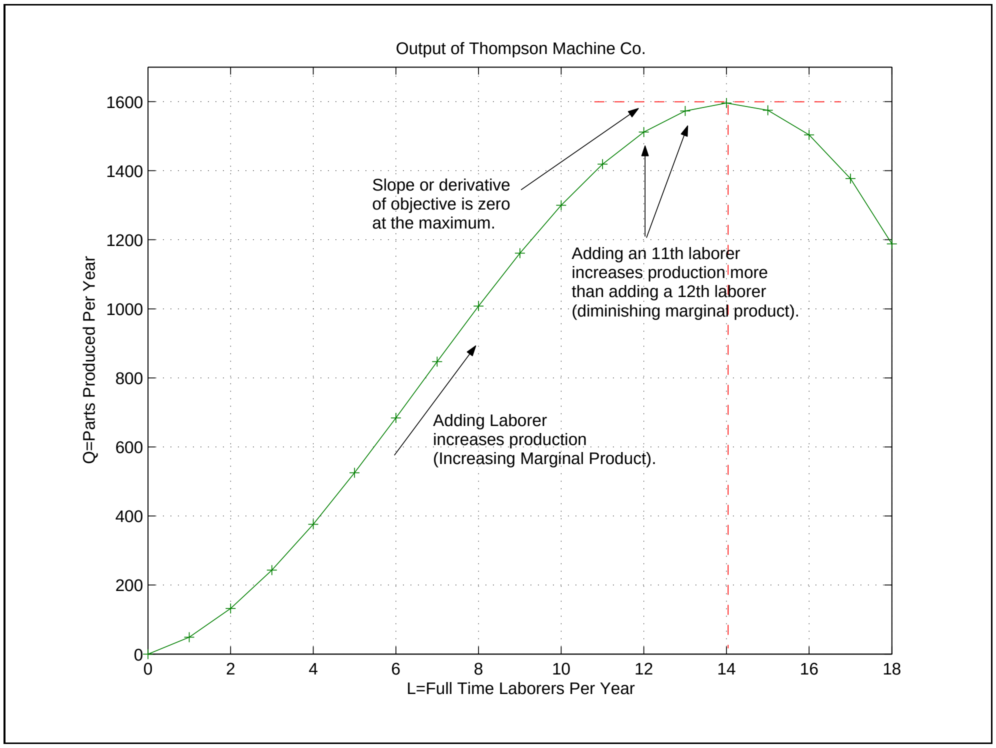
Where does the production function come from? We estimate a production function statistically, see below.

What are the properties of the production function? It is helpful to understand the production function by studying its properties.

1. Zero inputs implies zero output. Product cannot be produced without inputs.
2. **Definition 6 Law of diminishing marginal returns:** *if all other inputs are held constant, then the additional output from increasing one input eventually falls.*

   Hold the number of machines constant at five. Then going from four to five workers is no problem: each can use one machine. Adding one more worker has a lower marginal return: he can only assist one of the machine operators. Adding (in this case) the 15th worker results in zero additional output: he can only stand and watch. Adding still more workers decreases output: these workers simply get in the way.
3. Positive marginal product. Up to a point, adding additional workers increases output.

The graph below illustrates these concepts.



II Application 1: Maximizing Production

Let us now back up and do a step-by-step application. Suppose you a manager assigned to a particular shop with five machines. A shortage of machine parts exists. The orders are to crank out as many parts as possible.

1. Formulate the problem. The objective is clearly to maximize output of parts. The decision is in fact how many workers to hire.

2. Gather data on the objective and the decisions.

3. Using statistics, estimate the objective function. The objective function here is the production function (which relates choices, how many workers to hire, to the objective, how much is produced. We cannot yet do these steps. Therefore, let us instead assume we have completed these steps, and have thus come up with the production function $Q = 30L + 20L^2 - L^3$.

4. Using calculus, determine the optimal decision.

So mathematically, the problem is:

$$\max_{L} 30L + 20L^2 - L^3 \tag{8}$$

By looking at the graph, we can see that output is maximized when adding an additional worker adds zero output (the marginal product is zero).

Definition 7 *The* **Marginal Product** *is the additional output from an additional unit of an input.*

If we view units as "full time workers" then by following the graph we see that the maximum occurs at about 14 workers (1596 parts).

Alternatively, if through overtime or part time, we can hire part of a worker, we could use calculus. Notice that when output is maximized, the slope of the tangent line (the derivative) is zero.

Thus:

$$\text{marginal product of labor } = \frac{\partial Q}{\partial L} = 0 \tag{9}$$

The derivative is:

$$\frac{\partial Q}{\partial L} = 30 + 40L - 3L^2 = 0 \tag{10}$$

$$-3L^2 + 40L + 30 = 0 \tag{11}$$

The solution is given by the quadratic formula (page 62):

$$L = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-40 \pm \sqrt{1600 - 4\,(-3)\,30}}{2 \cdot (-3)} = 14.05 \tag{12}$$

Notice I have discarded the negative solution. So in fact (if possible) the manager should pay some overtime so that the equivalent of 14.05 full time workers are employed to maximize production.

I submit to you that coming up with 14 workers would be difficult to come up with relying solely on expertise.

III Application 2: maximizing productivity

Let us suppose now that there is no shortage, but the manager is paid a bonus based on the productivity of the workers.

Definition 8 *The* **Productivity** *or* **average product** *of an input is the output divided by the number of inputs.*

So we have:

$$\text{average product of labor } = \frac{Q}{L} \tag{13}$$

Follow the steps:

1. The objective is to maximize the average product of labor. The choice is once again how many full time workers to hire.
2. The objective function is $Q/L = 30 + 20L - L^2$.
3. We do not need to estimate this function, we have it already.
4. The last step is to maximize.

So mathematically:

$$\max_L 30 + 20L - L^2 \tag{14}$$

We can create another table or use calculus again. Set the slope or derivative equal to zero:

$$\frac{\partial 30 + 20L - L^2}{\partial L} = 20 - 2L = 0 \quad \rightarrow L = 10 \tag{15}$$

So hiring 10 workers maximizes productivity, with $Q/L = 130$. On average, the workers produce at most 130 units each.

IV Application 3: Maximizing Profits

A Example

Suppose now the manager is paid a bonus based on the profits of the company. To maximize profits we need to know how output affects total revenues and how inputs affect total costs. Suppose the parts can be sold for $500 each and workers earn a salary of $45,000 per year.

Total revenues are thus:

$$TR = \$500 \cdot Q = \$500 \cdot \left(30L + 20L^2 - L^3\right) \tag{16}$$

Total costs are:

$$TC = \$45,000L \tag{17}$$

Follow our steps again.

1. The objective is to maximize profits, the choice is the number of full time employees.

2. The only data we need are the cost of labor ($45,000) and the price of the parts ($500). Profits are:

$$\pi = TR - TC = \$500 \cdot \left(30L + 20L^2 - L^3\right) - \$45,000L \tag{18}$$

3. We again do not need to do any estimation beyond the production function.

4. Maximize.

We have:

$$\max_L \$500 \cdot \left(30L + 20L^2 - L^3\right) - \$45,000L \tag{19}$$

$$15,000 + 20,000L - 1,500 \cdot L^2 - 45,000 = 0 \tag{20}$$

Divide by 1000:

$$20L - 1.5 \cdot L^2 - 30 = 0 \tag{21}$$

$$L = \frac{-20 \pm \sqrt{20^2 - 4 \cdot (-1.5) \cdot (-30)}}{2 \cdot (-1.5)} = 11.6 \tag{22}$$

Either answer works, I have chosen the larger.

So to maximize profits, we should hire 11.6 workers. Again, I don't know of a way to obtain this without using math.

B Marginal Revenue Product

To help with the intuition, note that:

$$\pi = TR - TC \quad \rightarrow \quad \frac{\partial \pi}{\partial \text{input}} = \frac{\partial TR}{\partial \text{input}} - \frac{\partial TC}{\partial \text{input}} = 0 \tag{23}$$

$$\frac{\partial TR}{\partial \text{input}} = \frac{\partial TC}{\partial \text{input}} \tag{24}$$

We call the first term the marginal revenue product (MRP).

Definition 9 **Marginal Revenue Product** *is the amount of additional revenue from an additional unit of an input.*

The second term is the marginal expenditure (ME).

Definition 10 *The* **Marginal Expenditure** *is the amount of additional costs from an additional unit of input.*

So we hire additional inputs until marginal revenue product equals marginal expenditure. Ignoring the fancy jargon, we see that we should hire a worker if the worker produces more revenue than the cost of hiring that person. Such a worker makes money for the firm.

Finally, notice that:

$$MRP = \frac{\partial TR}{\partial \text{input}} = \frac{\partial TR}{\partial Q}\frac{\partial Q}{\partial \text{input}} = MR \cdot MP \tag{25}$$

Thus the additional revenue from an additional unit of input is equal to the marginal revenue times the marginal product.

V Multiple Inputs

A Marginal Product and Price Ratios

We now suppose there is more than one input to choose from (in principle, everything is the same). Let L and K denote the two types of inputs, for example workers and machines (capital), or workers and managers. The price of capital is P_K and the price of labor is P_L. Let us suppose again our desire is to maximize profits.

$$\max_{K,L} TR(L,K) - P_K K - P_L L \tag{26}$$

Identical to the one input case, we are looking for a maximum, we find the place where the slope (derivative) is zero.

$$\frac{\partial TR}{\partial K} - P_k = 0 \tag{27}$$

$$\frac{\partial TR}{\partial L} - P_L = 0 \tag{28}$$

Notice that we have two equations for the two unknowns, the amount of labor and capital.

$$MRP_K = \frac{\partial TR}{\partial K} = P_k = ME_K \tag{29}$$

$$MRP_L = \frac{\partial TR}{\partial L} = P_L = ME_L \tag{30}$$

Notice that we have the exact same result as before: the marginal revenue product equals the marginal expenditure. Hire additional workers until adding one more worker costs more in salary then the revenue that worker brings in. Buy one more machine until the cost of the machine is higher than the value of the product produced.

Now divide the two equations:

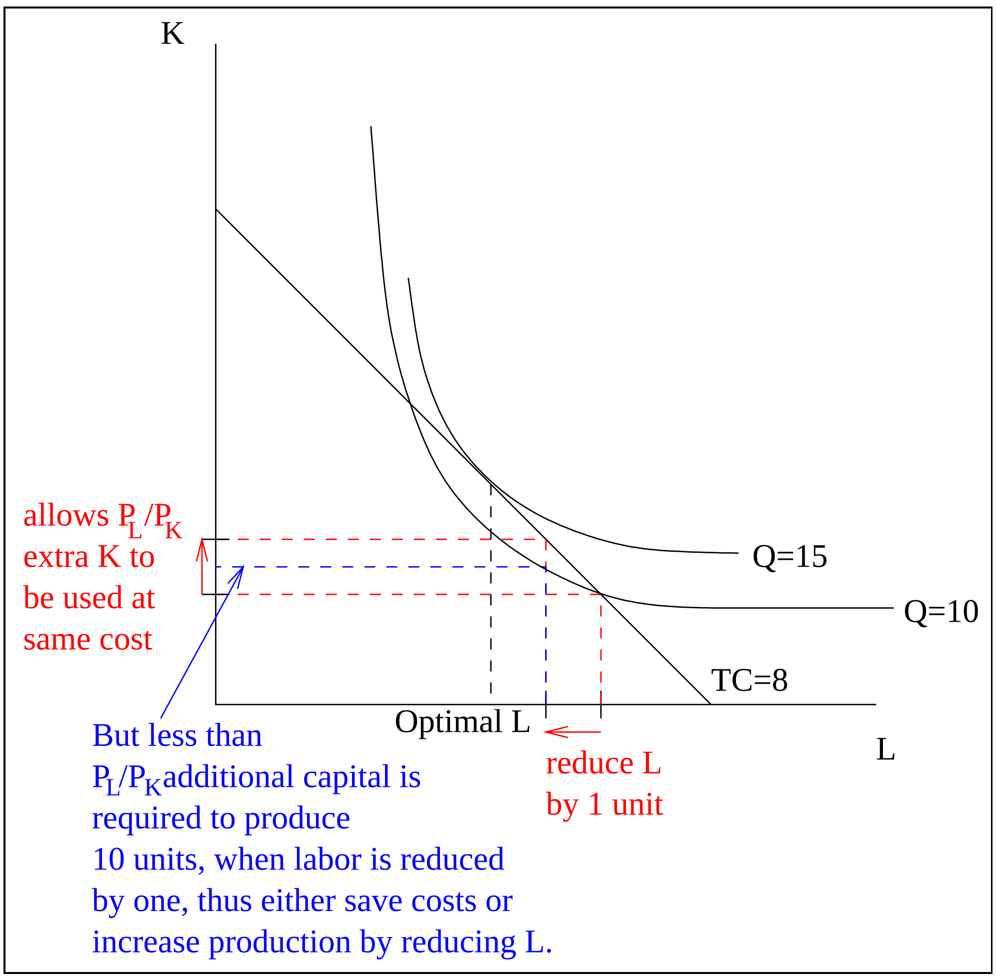
$$\frac{MRP_K}{MRP_L} = \frac{MP_k \cdot MR}{MP_L \cdot MR} = \frac{P_k}{P_L} \tag{31}$$

Cancelling out the MR, we see that:

$$\frac{MP_K}{MP_L} = \frac{\frac{\partial f(K,L)}{\partial K}}{\frac{\partial f(K,L)}{\partial L}} = \frac{P_k}{P_L} \tag{32}$$

So after all this math, we see that the ratio of marginal products must equal the price ratio. This combination of inputs minimizes the costs of production and maximizes profits.

We can give a graphical intuition of this result.



Suppose the manager has two inputs K and L and a total budget of \$8 million. Draw all of the possible ways to spend the \$8 million. For example, we could spend \$8 million on labor, which at price P_L results in $L = 8/P_L$ units of labor. Reduce the number of laborers by one. Then we have P_L extra dollars to spend on K. Thus we can buy $P_K K = P_L$ or $K = P_L/P_K$ units of capital. So the slope is $-P_L/P_K$. This line is called the isocost line.

Now draw all of the input combinations that produce 10 units. The curve is convex: because of diminishing returns, when a lot of labor is used and very little capital, it takes a lot of labor to keep production at 10 units when we reduce the capital used by one unit.

Now consider a point where the price ratio is not equal to the ratio of the marginal products, like the red point. At that point, $P_L/P_K > MP_L/MP_K$. Reduce L by one unit. That means we can buy P_L/P_K additional K. But we need less than this amount of K to continue producing 10 units, since the marginal product of labor is small relative to the marginal product of capital at this point. So we can either spend all of the money on K, increasing production to 15 units, or we can reduce costs and continue to produce 10 units. Thus the optimum is reached when the ratio of marginal products equals the price ratio.

B Example: Multiple Inputs

Suppose an engineering analysis firm uses engineers and technicians to do their consulting. Engineers are paid \$4,000 per month and Technicians \$2,000. The production function was found to be:

$$Q = 20E - E^2 + 12T - 0.5 \cdot T^2 \tag{33}$$

The firm charges \$1,000 to do an engineering analysis.

1. How many engineers and technicians should be hired if the manager is given a maximum wage bill of \$28,000?

We can skip many of the steps here, we know that profit and production maximization and cost minimization all say the marginal product and price ratios must be equal. We have:

$$\frac{MP_E}{MP_T} = \frac{\frac{\partial Q(T,E)}{\partial E}}{\frac{\partial Q(T,E)}{\partial T}} = \frac{P_E}{P_T} \tag{34}$$

Or:

$$\frac{20-2E}{12-T}=\frac{\$4000}{\$2000}=2 \tag{35}$$

$$20-2E=2\,(12-T) \tag{36}$$

$$-2E=4-2T \tag{37}$$

$$E=T-2 \tag{38}$$

The firm should always have two fewer engineers than technicians.

Thus since the wage bill is $28,000:

$$\$28,000=\$4000E+\$2000T \tag{39}$$

$$14=2E+T \tag{40}$$

$$14=2\,(T-2)+T \tag{41}$$

$$18=3T \;\;\rightarrow\;\; T=6 \;\rightarrow\;\; E=4 \tag{42}$$

3. How many engineers and technicians should be hired to maximize profits?

We know already that we need two fewer engineers than technicians.

$$\max_{T,E}\ \$1,000\left(20E-E^2+12T-0.5\cdot T^2\right)-\$4,000E-\$2,000T \tag{43}$$

So set the slope equal to zero. With respect to E:

$$\$1,000\,(20-2E)=\$4,000 \tag{44}$$

$$20 - 2E = 4 \tag{45}$$

$$E = 8 \ \rightarrow T = 10 \tag{46}$$

So we have 8 engineers, two fewer than the 10 technicians.

4. How many engineers and technicians should be hired if the firm needs to perform an output of 166 engineering analysis?

We need

$$Q = 166 = 20E - E^2 + 12T - 0.5 \cdot T^2 \tag{47}$$

$$166 = 20\left(T - 2\right) - \left(T - 2\right)^2 + 12T - 0.5 \cdot T^2 \tag{48}$$

$$166 = 20T - 40 - T^2 + 4T - 4 + 12T - 0.5T^2 \tag{49}$$

$$166 = 36T - 44 - 1.5T^2 \tag{50}$$

$$0 = 1.5T^2 - 36T + 210 \tag{51}$$

$$T = \frac{36 + \sqrt{36^2 - 4 \cdot (210) \cdot 1.5}}{2 \cdot 1.5} = 42/3 = 14 \ \rightarrow \ E = 12 \tag{52}$$

VI Mergers and Spinoffs

A Returns to Scale

Here we think about the size of our production processes. Should the firm expand (say through a merger) or contract (say through a spin-off). Should the firm build a second factory or increase the size of the current factory? Should the firm close one plant and move operations to another? Should the firm outsource some production processes?

The production function tells us the answer.

Definition 11 *The production function exhibits* **Increasing (decreasing, constant) returns to scale** *if a doubling of all inputs more than (less than, exactly) doubles output*

Doubling all inputs can be thought of as building an identical factory or merging with an identical firm. If increasing returns to scale exists than the merger is beneficial, since the total costs do not change (the costs are the same as the costs of the two firms acting separately), but output is greater. Thus the cost per unit falls. Similarly, with decreasing returns to scale, if half the firm is spun-off, then costs are split in half, but output falls by less than half. Thus cost per unit falls.

Consider the above production function, and let us double all inputs:

$$Q' = 20\,(2E) - (2E)^2 + 12\,(2T) - 0.5\,(2T)^2 \tag{53}$$

$$= 40E - 4E^2 + 24T - 2T^2 \tag{54}$$

On the other hand, if we double output, we get:

$$2Q = 2 \cdot \left(20E - E^2 + 12T - 0.5\left(T^2\right)\right) \tag{55}$$

$$2Q = 40E - 2E^2 + 24T - T^2 \tag{56}$$

So $Q' < 2Q$, doubling all inputs resulted in less than double the output. This firm has decreasing returns. In the long run, I would recommend splitting this plant into two smaller units.

Why returns to scale? The primary reason for decreasing returns is, well, too much management. Coordination of a large enterprise requires many employees who do not directly contribute to the overall production. Information does not flow well, each individual worker has little impact on profits, and therefore little incentive to engage in profit maximizing. Conversely, increasing returns to scale can occur for many reasons.

1. Indivisibilities. It may be difficult to hire part time accountants, one full time accountant must be used. But then doubling the size might not require any additional accountants.

2. Engineering Reasons. It may be the case that doubling the size of the warehouse might not require double the steel, electricity, etc.

3. Specialization. Large firms can have employees be more efficient by specializing. A larger firm can hire an accountant, rather than have the head sales guy also do the accounting.

B A special production function and output elasticity

Definition 12 *The* **output elasticity** *is the percentage increase in output from a one percent increase in inputs.*

Consider the telephone industry in Canada. The production function was found to be:

$$Q = 0.70L^{0.70}K^{0.41} \tag{57}$$

A one percent increase in inputs gives:

$$Q' = 0.70\,(1.01L)^{0.70}\,(1.01K)^{0.41} \tag{58}$$

$$= 0.70\,(1.01)^{0.70}\,L^{0.70}\,(1.01)^{0.41}\,K^{0.41} \tag{59}$$

$$= (1.01)^{0.70}\,(1.01)^{0.41}\,0.70L^{0.70}K^{0.41} \tag{60}$$

$$= (1.01)^{0.70+0.41}\,Q \tag{61}$$

$$= 1.0111Q \tag{62}$$

Thus if inputs increase by 1%, outputs increase by 1.11%, so the output elasticity is 1.11. The above production function has the special feature that the output elasticity is constant. Further, for this production function the output elasticity is simply the sum of the coefficients: $0.71+0.40 = 1.11$. Does this function have increasing, decreasing, or constant returns to scale? How could a telephone monopoly increase or decrease it's size?

VII How Do We Find the Production Function?

We can do many great thing with the production function. However, I'm willing to bet none of you have come across one in your business career. How do we obtain the production function? Several ways exist, all of which involve gathering company data (or even competitors data) and then using statistics (regressions). What we need is data on inputs used and how much output was obtained.

A Obtain input-output data

The first step is to obtain some data. Here are several possibilities.

1. Time Series Data. Get historical data of the firm's inputs and outputs.

2. Cross Section Data. Get data of all plants (or factories) owned by the firm in a single time frame. Or get data on all of the firms in the industry.

3. Use technical information supplied by engineers.

4. Conduct a randomized study. Select a random factory or factories and change the inputs.

5. Benchmarking. Observe firms outside the industry that specialize in this type of production.

Each has advantages and disadvantages. Time series data is often easy to come by. The manager does not need to request data from other managers or look up data on other firms. However, things change over time. Suppose the time series data is something like:

Date	Full time laborers (L)	Parts Produced (Q)
April 3	6	684
April 4	7	681

One might look at this data and think that the MP becomes negative after 6 workers, that is, the firm should never hire more than six workers. But it is also possible that something odd happened on April 4 that did not happen on April 3. For example, it could have been someone's birthday on April 4, and the firm wasted a few hours giving out cake.

Cross sectional studies take more time to acquire the data. To a lesser degree, cross sectional data has the same problem. There might be something special about one factory that the manager does not know about.

Plant	Full time laborers (L)	Parts Produced (Q)
West Palm Beach	6	684
Miami	7	681

With this data, we would again conclude that hiring more than 6 workers is a mistake. But it could be that the Miami plant uses older equipment, and so the workers are not as productive.

The randomized study, as is done in medicine, does not suffer from these problems, but is the most expensive way to obtain data. Because the plant selected is random, differences between plants are random and tend to average out. But you have to make the plant use odd combinations of inputs, possibly resulting in low production, just to gather data.

B Choosing a production function

The second step is to choose a production function. Suppose we have two inputs, K and L again. We would like our function to have all of the properties listed above like diminishing marginal products. Here are two that have these properties.

$$Q = aLK + bL^2K + cLK^2 - dL^3K - eLK^3 \tag{63}$$

$$Q = aL^bK^c \tag{64}$$

C Regression

We now use the data to find values for a, b, c, d, and e. No production function is exactly like the above two. What the regression tells us is what values of a-e make the above production function 'closest' to the data (ie closest to the real world).

Excel has a simple function "linest" which will be sufficient. On the website is an example Excel file that demonstrates how to do a linear regression. Statistics can tell us many things besides the values a - e. We will focus on two other things the Excel file tells us.

1. T-stat. The T-stat tells us if K, L, K^2, etc. likely have any effect on Q. If the T-stat for d is small, then we may want to drop the term L^3K from the production function.

2. R^2. This tells us the percentage of the variation in output that is explained by variation in the inputs. If the number is close to one, the production function is doing a good job describing the real production process. If the number is close to zero, there are some aspects of production being missed. Perhaps another input.

We now have all the pieces in place. The steps are as follows:

1. Determine the objective and the decision.

2. Gather data on inputs and outputs.

3. Choose a production function and estimate the coefficients.

4. Use calculus to find the decision which maximizes the objective.

I have provided an example problem, with all steps complete.