COST THEORY

Cost theory is similar to production theory, they are often used together. However, the question is usually how much to produce, as opposed to which inputs to use. That is, assume that we use production theory to choose the optimal ratio of inputs (eg. 2 fewer engineers than technicians), how much should we produce in order to minimize costs and/or maximize profits? We can also learn a lot about what kinds of costs matter for decisions made by managers, and what kinds of costs do not.

I What costs matter?

A Opportunity Costs

Remember from Section (IV) of the Introduction, that in addition to accounting profit, managers must consider the cost of inputs supplied by the owners (owners capital and labor).

Definition 13 Explicit Costs: *Accounting Costs, or costs that would appear as costs in an accounting statement.*

Definition 14 Implicit Costs: *Other costs, such as the cost of the owners capital and labor, and/or the cost of alternative uses of each input.*

Definition 15 Opportunity Costs: *The value of all inputs to a firm's production in their most valuable alternative use.*

Recall the example from Section (IV), where the decision was whether or not to buy the shoe factory. the implicit costs were the $24,000 per year the money could be earning elsewhere, and the owners time cost of $40,000, which exceeded the accounting profit of $60,000.

Another example, suppose we run a financial planning firm with one planner, making $60,000. Each account takes 10% of her time, and she already has 9 clients. Our valued sales force gets us two new accounts: one is a restaurant owner with $2 million to invest (assume a 1% management fee), the second is a doctor with $1 million to invest. What are the opportunity costs of managing the restaurant owner's account? The costs are of course the explicit cost of $6,000 in salary (10% time), but also the opportunity cost of $10,000 that could be earned managing another account. Similarly, the opportunity cost of the doctor's account is $20,000 and $6,000. The choice is obvious, manage the restaurant owner's account, but just measuring the accounting costs of $6,000 in either case tells us nothing about which account we should manage. (For fun, calculate the economic profit for each account). Digression about financial planning and keeping an eye on the valued customers. Do you consider the opportunity cost of your time when you respond to questions from a low value account (often the low value accounts are the most time consuming)?

B Fixed costs, variable costs, and sunk costs

The short run is a time period such that some inputs cannot be changed. We will define the short run as a period of time in which capital and much professional or salaried labor cannot be changed. We also assume the short run is long enough so that production workers or non-professional labor inputs can be changed. We call inputs that can be changed in the short run variable inputs, and inputs that cannot be changed fixed inputs.

Definition 16 Total Variable Cost *The total cost of all inputs that change with the amount produced (all variable inputs).*

Definition 17 Total fixed costs *The total cost of all inputs that do not vary with the amount produced (all fixed inputs).*

Consider the Thompson machine company. The cost of the 5 machines used to make machine parts was fixed in the short run, and therefore a fixed cost, while the number of workers could change and varied with the amount produced. These were variable costs. A fixed cost cannot be changed and thus cannot vary with the amount produced.

Definition 18 Sunk costs *Are costs that have been incurred and cannot be reversed*

Any costs incurred in the past, or indeed any fixed cost for which payment must be made regardless of the decision is irrelevant for any managerial decision. Whether you pumped $2 or \$2 million trying to break into a new market last year is irrelevant, the only question is whether an additional dollar investment will make sufficient return. The principle of sunk costs is equivalent to the saying "don't throw good money after bad" in poker.

Sometimes a decision can be made to recover part of a fixed cost. Perhaps one could sell a factory and recover part of the fixed costs. Then only the difference is sunk. For example, if we can sell a building for which we paid \$500,000 for \$300,000, then only \$200,000 is sunk. Conversely, once we pay a signing bonus, the bonus is sunk (and should not affect our decision about firing the employee).

Sunk costs are perhaps one of the most psychologically difficult things to ignore. Last night I watched the world series of poker. In one instance, the odds of drawing a flush (and almost for sure winning the hand) was 1 in 5. The pot was huge: say about \$200,000. So the player should call any bet less than or equal to \$40,000. Yet the commentator advised that the player would call regardless of the bet, because he already had so much money in the pot (sunk costs).

Another example: the Iraq war. We have sunk billions, but that should not enter our decision about whether or not to stay.

Another example: Consider restaurants in a high rent district (say an airport). Should they take the rent into account when setting prices? No.

II Short run costs

We use short run costs primarily to compute how much to produce while minimizing costs or maximizing profits. We use long run costs to answer questions like should the firm expand, contract, merge, etc.

Definition 19 Average Costs: *Costs divided by output.*

Definition 20 Marginal Costs: *The cost of one additional unit of an input.*

Here is the notation:

Type of Cost	Total Cost equals	Variable Costs	Plus Fixed Costs
Total	$TC =$	TVC	$+TFC$
Average	$ATC = \frac{TC}{Q} =$	$AVC = \frac{TVC}{Q}$	$+AFC = \frac{TFC}{Q}$
Marginal	$MC = \frac{\partial TC}{\partial Q}$		

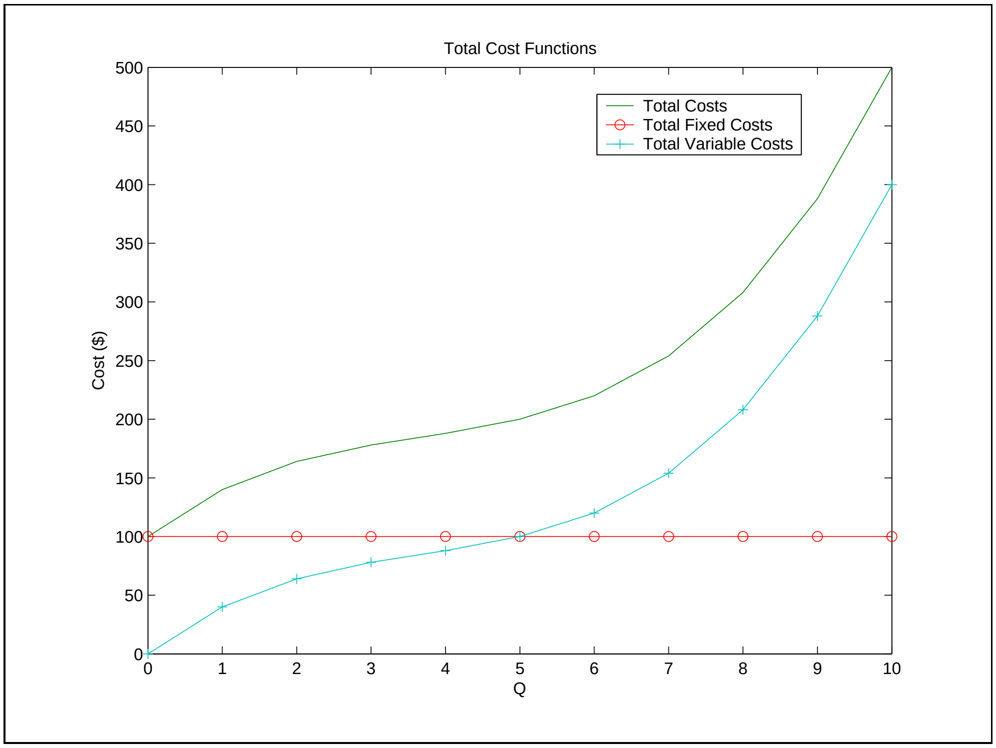
Properties of cost functions in the short run:

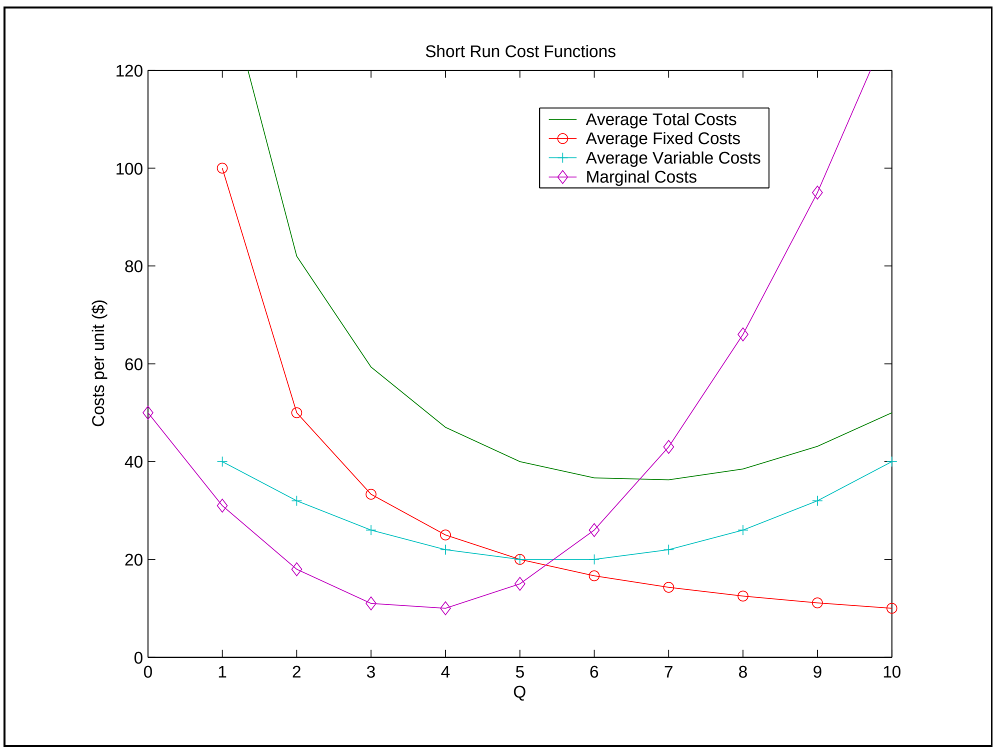
1. Total costs of course increase with Q, the quantity produced.

2. Average Costs decline with Q, but eventually rise. The fixed costs are spread over many more units of production at high Q, reducing average costs. All of the extra workers required for producing additional units when the factory is near capacity starts to increase average costs eventually.

3. Marginal costs usually decline then increase, but must eventually increase. At first, producing one additional unit is cheaper than the last unit. Suppose the firm goes from one to two workers. The workers can now specialize increasing efficiency. However, eventually diminishing returns sets in and the workers just get in each other's way. Then a very large number of additional workers might be needed to produce an additional unit.

Here is a graph of the cost curves.


Total Cost Functions
Total Costs
Total Fixed Costs
Total Variable Costs
Cost ($)
Q
0
50
100
150
200
250
300
350
400
450
500
0
1
2
3
4
5
6
7
8
9
10


Short Run Cost Functions
Average Total Costs
Average Fixed Costs
Average Variable Costs
Marginal Costs
Costs per unit ($)
Q
0
20
40
60
80
100
120
0
1
2
3
4
5
6
7
8
9
10

III Examples of using Short run cost curves.

A Profit maximization

Let us suppose that you are a hypothetical manager of a group of sugar cane farms. Using data from your horticulturalists have estimated the short run cost function to be (we will see how to do this estimation below):

$$TC = 60 + \frac{Q^2}{20} \tag{65}$$

Some costs are fixed and in the short run, sunk: regardless of Q, you must pay \$60 in fixed costs, so this will not enter your decision. Suppose the futures price of sugar cane is \$3 per basket, so you can sell any reasonable amount of sugar cane at this price.

Maximize profits:

$$\max \pi = TR - TC = 3Q - 60 - \frac{Q^2}{20} \tag{66}$$

Take the derivative to get the slope and set the slope equal to zero:

$$3 - \frac{Q}{10} = 0 \text{ or } MR = MC \tag{67}$$

Notice that the fixed costs have dropped out. The math agrees: fixed costs do not matter for our decision.

Solving for Q we see that $Q = 30$. Producing the 30th basket gives us \$3, just enough to cover our costs of producing *the 30th basket*, $\frac{Q}{10} = \$3$. However, the 31st basket requires more labor than previous baskets due to diminishing returns. Producing any more is not profitable. Management costs, taxes, rent on the farm, and other fixed costs are irrelevant. The firm is losing money here ($TR < TC$). But that is irrelevant. We have already paid the fixed costs, so we might as well lose as little as possible.

B Break Even Analysis

An important consideration when deciding whether to continue operations in a particular market, expand into a market, or start a new business is a break even analysis. We can do a break even analysis very easily with our cost functions.

In a break even analysis, the question is how much profit is required to exactly pay off all fixed costs. Alternatively, how much revenue is required to pay off the average variable costs and the fixed costs:

$$\pi = 0 = TR - TC \tag{68}$$

$$0 = P \cdot Q - TFC - TVC \tag{69}$$

$$0 = P \cdot Q - TFC - AVC \cdot Q \tag{70}$$

$$Q = \frac{TFC}{P - AVC} \tag{71}$$

Here I have assumed linear total costs, so that average variable cost is constant. One could assume (more realistically) that total costs are quadratic, and then solve for Q using the quadratic formula.

C Minimizing Average Costs

The book pays attention to minimizing cost per unit, or average costs. Consider a consulting firm with the total cost function:

$$TC = 10Q - 6Q^2 + Q^3 \tag{72}$$

The firm has no fixed costs. Average costs are:

$$ATC = \frac{TC}{Q} = 10 - 6Q + Q^2 \tag{73}$$

So here is the problem:

$$\min_{Q} ATC = 10 - 6Q + Q^2 \tag{74}$$

Find the slope and set equal to zero:

$$-6 + 2Q = 0 \quad \rightarrow \quad Q = 3 \tag{75}$$

When two units are produced:

$$AVC = 10 - 6Q + Q^2 = 10 - 6 \cdot 3 + 9 = 1 \tag{76}$$

$$MC = 10 - 12Q + 3Q^2 = 10 - 12 \cdot 3 + 3 \cdot 9 = 1 \tag{77}$$

So $AVC = MC$ when AVC is minimized.

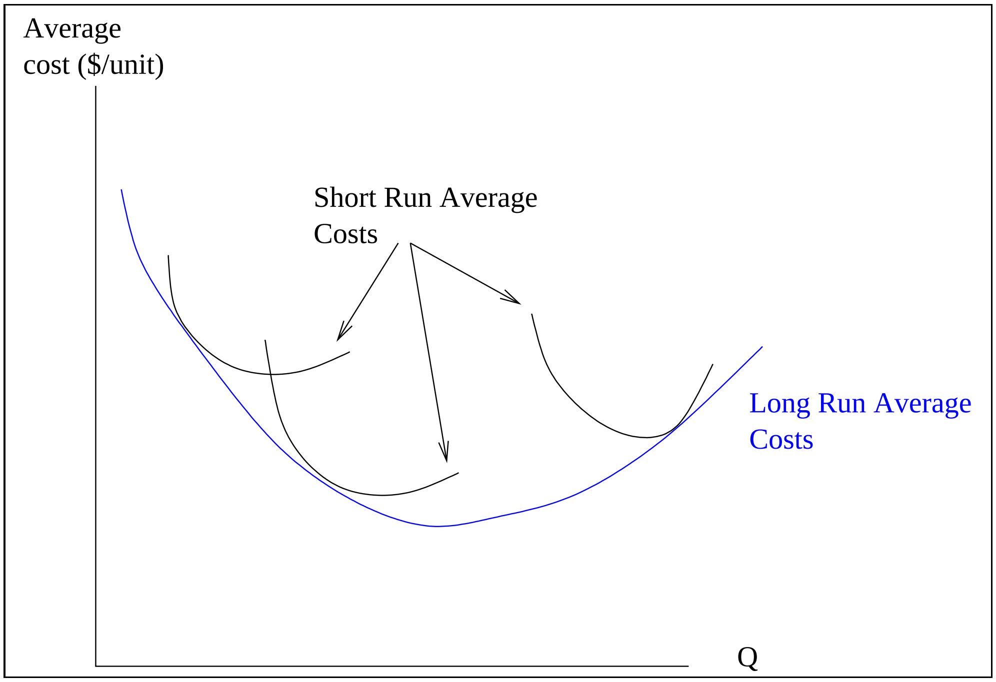
IV Long Run Costs

We use long run costs to decide scale issues, for example mergers. We assume the long run is long enough for all costs to be variable.

In the long run, we can build any size factory we wish, based on anticipated demand, profits, and other considerations. Once the plant is built, we move to the short run as described above. Therefore, it is important to forecast the anticipated demand. Too small a factory and marginal costs will be high as the factory is stretched to over produce. Conversely too large a factory results in large fixed costs (eg. air conditioning, or taxes) and low profitability.

Definition 21 Long Run Average Costs: *The minimum cost per unit of producing a given output level when any sized plant can be built*

Graphically:



Long run average costs may be increasing and then decreasing, but also may be strictly decreasing. Here are some LRAC curves for some industries.

1. Nursing Homes have decreasing LRAC. Nursing homes have many fixed management costs. Further, larger nursing homes are able to negotiate lower prices for many raw materials.

2. Cruise Ships: Huge cruise ships have lower average costs than small cruise ships, economizing on many services provided on the ships.

When the LRAC curve is decreasing, it is often in the interest of the industry to consolidate. A merger with another firm can increase the customer base but reduce the cost per unit, thus increasing profits.

One reason for increasing long run costs is coordination and information problems. In a large firm, many individuals do not meaningfully affect profits, and thus have the wrong incentives. Smaller operations may know their customers and production processes better.

In this case, spin-offs and divestments may be optimal. A compromise is franchising. Nationalize just the parts for which increasing returns works.

Another reason for increasing LRAC curves is regulation. In many countries large firms are taxed to a much greater extent. In addition, large firms are attractive targets for lawsuits (the ladder industry is small). Unions may drive up costs in large firms. In such industries, smaller may be better. Consider the taxi industry in Peru, dominated by one-employee firms. Why not large firms like in the US? Because of taxes and other regulations.

V Application: Banking mergers

Consider two banks. The first bank services $Q = 15$ customers and the second (smaller) bank services $Q = 5$ customers. The long run average cost function in the industry is:

$$LRAC = 700 - 40Q + Q^2 \tag{78}$$

Revenue is constant at \$300 dollars of loan revenue per customer.

Should these two firms merge? The size of the customer base (Q) which minimizes long run average costs is:

$$\min LRAC = 700 - 40Q + Q^2 \tag{79}$$

$$-40 + 2Q = 0 \quad \rightarrow \quad Q = 20 \tag{80}$$

Costs per unit fall until $Q = 20$. Thus these two firms can reduce costs by merging from two firms of size $Q = 15$ and $Q = 5$) into one firm with $Q = 20$.

Profit *per unit* in each case are:

$$\pi = 300 - \left(700 - 40Q + Q^2\right) = -400 + 40Q - Q^2 \tag{81}$$

$$\pi\left(Q = 15\right) = -400 + 40 \cdot 15 - 15^2 = -25 \tag{82}$$

$$\pi\left(Q = 5\right) = -400 + 40 \cdot 5 + 5^2 = -225 \tag{83}$$

$$\pi\left(Q = 20\right) = -400 + 40 \cdot 20 - 20^2 = 0 \tag{84}$$

Individually, the two banks lose money but together they break even.

VI Measuring Cost Functions

We use the same procedure as with production functions: Obtain data on total costs and quantity produced, and use Excel to fit the data. Both total cost and total quantity produced may appear to be easier to obtain than input data. However, one must remember that costs represent opportunity costs, which are not always straightforward.

Some additional issues:

A Choice of Cost Function

One choice is whether to use a linear, quadratic, or cubic function:

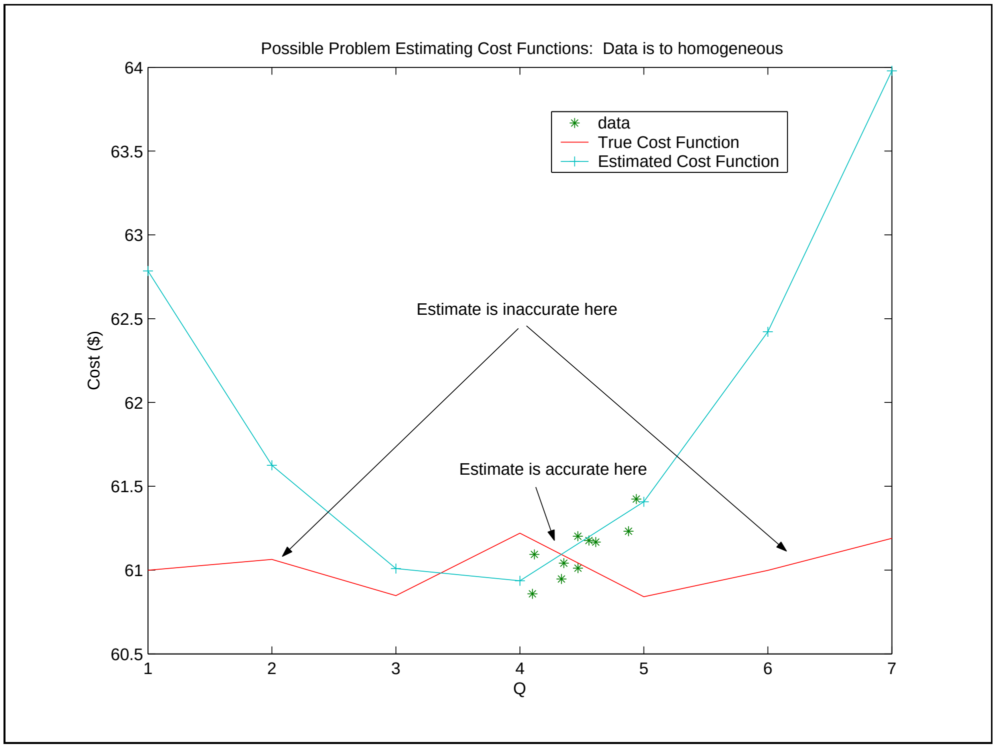
$$TC = a + bQ \tag{85}$$

$$TC = a + bQ + cQ^2 \tag{86}$$

$$TC = a + bQ + cQ^2 + dQ^3 \tag{87}$$

Under most circumstances, the linear cost function does a reasonable job over a narrow range of Q (for example in the short run), but the quadratic and cubic terms must matter theoretically, especially for a wider range of Q. A good strategy might therefore be to estimate the cubic or quadratic.

If the t-stats are low for the quadratic and cubic terms, then predictions are likely to be unreliable for Q that falls outside the data. This indicates using some caution before, for example, committing to large mergers. The following graph illustrates the problem.



B Data issues

Some problems with the data that often need correcting:

1. Definition of cost: as mentioned earlier, we use opportunity costs not accounting costs.

2. Price level changes: Historical data is likely to be inaccurate if the price of some inputs or outputs have changed dramatically.

3. What costs vary with output: Some costs have a very limited relationship with output. For example, the number of professionals required may vary in some limited way with output. A firm with $1 million in sales may have two accountants. The firm can obviously increase output to some degree without needing more accountants (so the cost would be fixed). But for larger Q additional accountants are needed (like a variable cost).

4. The cost data needs to match the output data. Often the cost of producing some output may be accounted for in some other period.

5. The firm's technology may change over time.

When estimating long run costs, it is usually preferable to use a cross section of firms across an industry. An individual firm is unlikely to have changed size significantly enough to generate data for a wide range of Q.