STRATEGY

Strategy deals with managerial decisions when other firms (or persons) change their behavior in response to your behavior. **The key question is: how will other firms react to my decisions?"** We would like to:

- Predict how other firms react to our decisions.
- Make decisions accounting for the reaction of others.

For example, in the last section we learned that demand for a firm's product depends on what competitors charge. But the competitor's price was held fixed. We did not consider that, if we change our price, competitors may match our price change. In order to really assess the value of, say, cutting prices, we need to consider what others will do in response to our actions.

You must always ask yourself: "suppose I do x, how will my competitors react?" To answer this question we will use game theory.

I Framework: Two Player Games, Simultaneous Move

We start with a game with one other player. The other player could be another firm or another person.

A Payoff matrix

Definition 29 *The* **payoff matrix** *lists all rewards that each player gets as a function of decisions made by all players.*

For example, consider a **price war** game. Firm A (you) and Firm B are the dominant firms in the industry. Because of antitrust laws, you are forbidden from colluding with B to raise prices or even from discussing prices.

- If you cut prices and Firm B does not, you can take the majority of the market share and make a large profit, say \$6 million, while Firm B loses \$1 million.
- If Firm B cuts prices and you do not, then Firm B gains market share and makes \$4 million. However, and Firm A still makes \$1 million, as it has some loyal customers that will not be attracted to firm B's low price.

- If both firms cut prices, profits are zero for both firms.
- If both firms do not cut prices, the market is split between the two firms, but the relatively high prices allows each firm to make a profit of $3 million.

		Firm B	
		Hold prices	Cut prices
Firm A	Hold prices	3,3	1,4
	Cut prices	6,-1	0,0

Table 15: Payoff matrix for the price war game.

- Notice that the first number is always the payoff of the firm listed on the left (rows),
- the second number is the payoff of the firm listed on top (columns).

The payoffs depend on the other players actions. If Firm A holds prices, the payoff to Firm A depends on what Firm B does.

B Dominant Strategy

If your best action is independent of what the other player does, then the decision is easy.

Definition 30 *A* **Dominant Strategy** *is an action that is at least as good as all other actions regardless what the opposing player chooses.*

For example in the price war game (table 15),

- Suppose firm B elects to hold prices. Firm A should cut prices and steal firm B's customers, since $6 million is greater than $3 million in the payoff matrix (first column of table 15).
- Suppose firm B cuts prices. Then firm A should hold prices, since $1 million is greater than $0 (second column of table 15).

Thus firm A does not have a dominant strategy: the optimal decision depends on what firm B does. Firm B does have a dominant strategy, however:

- If Firm A holds prices, Firm B should cut prices and steal business away from Firm A ($4 million is greater than $3 million, first row of table 15).

- If Firm A cuts prices, then Firm B does better by cutting prices (second row, $0>$-1), splitting the market and making no profit is better than losing $1 million.

So the dominant strategy of cutting prices generates the highest possible profits for firm B regardless of the actions of firm A. Firm A should then predict firm B will cut prices. However, we still do not have a prediction for what firm A will do.

C Nash Equilibrium

We would like to compute the optimal strategy when no dominant strategy exists for at least one of the players. The trick is to predict the behavior of the other firm. In the the price war game with payoff matrix given in table 15:

- Firm A should predict that firm B will cut prices. After all, cutting prices is the dominant strategy for firm B, regardless of what firm A does.
- Given that firm B will cut prices, Firm A should hold prices.
- The last step is to check if the prediction is accurate: if firm A holds prices then firm B will certainly do better by cutting prices, so our prediction that firm B will cut prices is correct.

Therefore the optimal strategy for firm A is to hold prices and the optimal strategy for firm B is to cut prices. Together, these strategies are a Nash equilibrium and what we expect to see.

Now let us show that in equilibrium firm A does not cut prices.

- Suppose firm A does cut prices.
- Then firm B responds by cutting prices.
- But then it is no longer optimal for firm A to cut prices: firm A does better by holding prices.

So cutting prices is not a Nash equilibrium strategy for Firm A, since they will regret their decision if Firm B acts in an optimal way.

Definition 31 *A* **Nash Equilibrium** *is a set of strategies such that each player accurately believes that he/she is acting optimally given the strategy of the other players.*

The Nash Equilibrium anticipates what other players do. By assuming the other player will act optimally and accurately predict what you are doing, you can predict what the other player will do.

Firm B knows that Firm A will not match B's price cut, since Firm A does better by ignoring Firm B and just making money from its loyal customers. Therefore, Firm B can make inroads into Firm A's business by offering goods at low prices. Firm A is then forced into a relatively bad outcome.

Let's try another example.

		Firm B	
		Hold prices	Cut prices
Firm A	Hold prices	3,3	1,4
	Cut prices	5,1	0,0

Table 16: Price war game, version 2.

We can use an easy procedure (**"the circle method"**) to check for all Nash equilibria. First, for each choice of firm A (each row), circle the best response of firm B (if a tie exists, circle both).

		Firm B	
		Hold prices	Cut prices
Firm A	Hold prices	3,3	1,4
	Cut prices	5,1	0,0

Table 17: Price war game, version 2. Best responses of firm B to firm A's choices are circled.

Now for each choice of firm B (each column), circle the best response of firm A.

		Firm B	
		Hold prices	Cut prices
Firm A	Hold prices	3,3	1,4
	Cut prices	5,1	0,0

Table 18: Price war game, version 2. Best responses of both firms to each other's choices are circled.

The Nash equilibria are then the cells where both numbers are circled. In these cells, each player's action is a best response to the other players action. Here, two Nash equilibria exist. The previous Nash equilibrium continues to hold:

- If firm B predicts firm A holds,
- the firm B does better by cutting,
- in which case firm A should hold. Since the prediction that A would hold is correct, we have a Nash equilibrium. The Nash equilibrium is firm A holds and firm B cuts.

But another Nash equilibrium exists:

- Suppose firm B predicts firm A cuts.
- Then firm B does at better by holding,
- in which case Firm A should cut. Since the prediction is correct, we have a Nash equilibrium.

Therefore the set of strategies "firm A cuts and firm B holds" is also a Nash Equilibrium. Often more than one Nash equilibrium exists, in which case the behavior gets harder to predict.

Consider the "coordination" game of which side of the road to drive on.

		Car B	
		B's Left Side	B's Right Side
Car A	A's Left Side	10,10	-5,-5
	A's Right Side	-5,-5	10,10

Table 19: Typical coordination game.

Circle the best responses to get the Nash equilibria:

		Car B	
		B's Left Side	B's Right Side
Car A	A's Left Side	(10),(10)	-5,-5
	A's Right Side	-5,-5	(10),(10)

Table 20: Typical coordination game.

If both drivers drive on the opposite sides of the road (from their own perspective), it is a disaster for both players. But it does not matter if both players drive on their right or left. Therefore, two Nash equilibria exist, and we should be able to observe that in some countries people drive on the left and in other countries on the right.

II Some Simple Games

A Anti-Coordination Games: Avoiding Competition

Suppose firms need to decide which segment of the markets to target. Harley-Davidson can build cruiser or sport motorcycles, and the same for Buell (another motorcycle manufacturer). If both firms invest in the same business, competition will be fierce and each company will price close to marginal cost, which is a problem especially given the fixed costs of setting up the factory. However, if each focuses on a different market segment, both can charge more and increase profits.

		Buell	
		Sport Bikes	Cruisers
Harley	Sport Bikes	0,0	2,5
	Cruisers	5,2	0,0

Table 21: Anti-coordination game.

Two Nash equilibria exist:

		Buell	
		Sport Bikes	Cruisers
Harley	Sport Bikes	0,0	(2),(5)
	Cruisers	(5),(2)	0,0

Table 22: Anti-coordination game.

In one Nash equilibrium, Harley specializing in cruisers and Buell sport bikes, and the opposite in the second equilibrium. Notice that whoever gets the cruiser business is better off since the cruiser business makes profits equal to 5. This can make the outcome somewhat uneasy: both firms, deciding simultaneously, may choose cruisers and then hope the other firm chooses sport bikes. Or if the game is not perfectly simultaneous, both firms may rush into the cruiser business in the hopes that competitors will be deterred from doing so.

B Coordination Games

Firms sometimes play location coordination games. For example, firms that sell complementary goods, such as clothing and shoe stores, have incentives to locate together.

C Mixed Coordination and Anti-Coordination

Let's try a tougher one of store location.

		Target			
		Uptown	Center City	East Side	West Side
Macy's	Uptown	30,40	50,95	55,95	55,120
	Center City	115,40	100,100	130,85	120,95
	East Side	125,45	95,65	60,40	115,120
	West Side	105,50	75,75	95,95	35,55

Table 23: Location game. Coordinate or not?

The Nash equilibrium is found via:

		Target			
		Uptown	Center City	East Side	West Side
Macy's	Uptown	30,40	50,95	55,95	55,120
	Center City	115,40	100,100	130,85	120,95
	East Side	125,45	95,65	60,40	115,120
	West Side	105,50	75,75	95,95	35,55

Table 24: Location game. Coordinate or not?

In table 24, East Side and Center City are relatively affluent areas. Therefore, Macy's generally does better in these locations (all of the red circles are in rows 2 or 3). Target does better in less affluent places (for example, two of Target's circles are on the West Side). However, if Target locates near Macy's, then Target could draw more customers by establishing the area as a place to shop (for example, when Macy's locates at Center City, target does best by also locating at Center City). Thus incentives exist to both coordinate and anti-coordinate.

The Nash equilibrium is for both stores to locate at Center City (coordinate) and compete. Restate the Nash equilibrium:

- Suppose Target forecasts that Macy's locates at Center City.
- From the second row of table 23, the best response of Target is to locate at Center City (100 beats all other payoffs).
- Check the forecast: if Target locates at Center City, Macy's best response is to locate at Center City (100 beats all alternatives in the second column). The forecast is correct and so we have a Nash equilibrium.

Notice that the outcome could be better for both firms: Target could get $120 on the West Side, but only if Macy's locates Uptown or East Side. However, Macy's will locate in Center City if Target locates on the West Side. Macy's does best by locating at Center City and Target locates on the East Side. However, if Macy's locates at Center City, Target will follow.

Advantages of coordinating or not coordinating.

1. Coordinate by locating near competitors to **draw shoppers** to the area.
2. Coordinate by locating near competitors if **demand in the area is better** than other areas (even with a competitor in the better area versus no competitor in the worse area).
3. Coordinate if differentiating your product and **being wrong** about customer preferences **yields more losses** than the gains from sharing the market, but producing an identical product.
4. Coordinate only if **incumbent firms will not react** forcefully to your entry.

D Prisoner's Dilemmas

1 Example

Suppose a symmetric version of the price war game:

	Firm B	
Firm A	Hold prices	Cut prices
Hold prices	3,3	0,5
Cut prices	5,0	1,1

Table 25: Prisoner's dilemma game.

	Firm B	
Firm A	Hold prices	Cut prices
Hold prices	3,3	0,(5)
Cut prices	(5),0	(1),(1)

Table 26: Prisoner's dilemma game.

So here a unique Nash Equilibrium exists where both firms cut prices (a price war ensues). Notice a peculiar feature of this game: the outcome is relatively poor for both firms.

Definition 32 *The* **Social Benefit** *is the sum of player payoffs, and the social benefit of the game is the sum of player payoffs when both players play their Nash equilibrium strategies.*

The social benefit is defined with respect to players only and not third parties like consumers. So the upper left corner has a social benefit (SB) to the two firms (obviously not to the consumers) of $SB = 3 + 3 = 6$. Similarly:

	Firm B	
Firm A	Hold prices	Cut prices
Hold prices	$SB = 6$	$SB = 5$
Cut prices	$SB = 5$	$SB = 2$

Table 27: Social benefit of the prisoner's dilemma game.

So the Nash Equilibrium is the worst possible outcome in terms of social benefit for the two firms! In fact, if firms could only collude on prices, they would surely agree to both hold prices high. **The prisoners dilemma is the basis for antitrust rules prohibiting price collusion and is perhaps the most important game in business. By prohibiting collusion or discussion of prices, the incentive to cut prices remains, benefiting consumers.**

2 Cartels, Collusion, and Cheating

Of course, in reality, prisoner's dilemmas are not always fully non-cooperative (meaning each player moves simultaneously without knowledge of the other's move). Some degree of information sharing about prices is always possible, and firms can sometimes agree to form a cartel and not cut prices. Further, firms sometimes Collude (agree to play strategies with the highest social benefit).

However, the incentive to cheat on a cartel or collusion agreement is very strong. If you know the other firm is sticking to the agreement and keeping prices high, you have a strong incentive to cut prices. Cartels such as OPEC see frequent cheating for exactly this reason.

3 Price Matching Offers

Why do Firms offer to match prices of other firms and do these offers benefit consumers? I was negotiating with a car dealer a few years ago. We had reached a sticky point in the negotiations and I said that I could get the car for less elsewhere. He responded all of the dealers in the area had agreed not to sell for less than this price (a bold admission of illegal price fixing!). I then said I was holding an offer from someone else for less. He said immediately to bring the offer in and he would match it (a price match clause, common among retailers). Let us see how the payoffs change.

		Firm B	
		Hold and match prices	Cut prices
Firm A	Hold and match prices	3,3	1,1
	Cut prices	1,1	1,1

Table 28: Price matching game.

Now if one firm cuts prices, the other firm automatically matches and the gain from cheating is erased. Two Nash Equilibria exist, one in which the cartel continues to hold and the other which has both firms cutting prices.

		Firm B	
		Hold and match prices	Cut prices
Firm A	Hold and match prices	(3), (3)	(1), 1
	Cut prices	1,(1)	(1), (1)

Table 29: Price matching game.

However, the Nash equilibrium where both players hold and match is much more likely to be observed, since both players know they can play this strategy and reach the highest payoff. We will therefore add one rule:

If multiple Nash equilibria exist and players have a dominant strategy, then we predict the dominant strategy equilibrium outcome.

So here both players have a dominant strategy to hold and match, so we predict both players hold and match.

A price matching agreement does not suffer from cheating, like a cartel does. Thus price-matching agreements are known to be instruments of price fixing, rather than benefiting consumers.

E Example with 3 Players

A takeover specialist wishes to acquire a firm that has 3 major shareholders (Jack, April, and Alec). The stock price is currently trading at \$48. The specialist makes the following offer: if one shareholder sells, she receives \$65. If two shareholders sell, they each receive \$50. If all three sell, they each receive \$45. The takeover succeeds if 2 or more shareholders sell. If the takeover succeeds, then the specialist will dilute the shares so that the value of any shares not sold is \$40. If the takeover does not succeed, the payoff to those not selling is \$48, which is the original stock price. The payoffs are therefore:

Alec Sells

	April	
Jack	Sells	Holds out
Sells	45,45,45	50,40,50
Holds Out	40,50,50	48,48,65

Alec Holds Out

	April	
Jack	Sells	Holds Out
Sells	50,50,40	65,48,48
Holds Out	48,65,48	48,48,48

Table 30: Takeover game.

Pick any two player's strategies, and circle the best response for the third player. For example, if Alec picks Sell and Jack picks Sell, then comparing the two boxes comprising the first row of the top table, we see that April should choose Sell because $45 > 40$ (use the second number in both boxes). If everyone else is selling, then the takeover will succeed, so April must sell to prevent her shares from being diluted.

Analogously, suppose Jack and April both hold out. Comparing the bottom right corner box in the top and bottom table, we see that Alec should sell since $65 > 48$ (last number in both boxes). In this case, Alec knows he is the only seller, and can therefore get the big payoff from the takeover specialist.

For the rest:

Alec Sells

		April	
		Sells	Holds Out
Jack	Sells	(45),(45),(45)	(50),40,(50)
	Holds Out	40,(50),(50)	48,48,(65)

Alec Holds Out

		April	
		Sells	Holds Out
Jack	Sells	(50),(50),40	(65),48,48
	Holds Out	48,(65),48	48,48,48

Table 31: Takeover game solution.

So the Nash equilibrium is that everyone sells and receives a price of \$45, which is less than the original value of the stock (\$48)! The game is a prisoner's dilemma in that everyone would be better off everyone held out. But the specialist cleverly designed the payoffs to give each shareholder a strong incentive to cheat on any agreement not to sell (if one or two cheat, the payoffs are \$65 and \$50). Further, if two are selling, than the third must sell to avoid dilution.[3]

III Mixed Strategies

When a player has an interest in disguising their actions, it is often optimal to choose a random or mixed strategy.

Definition 33 *A* Mixed Strategy Nash Equilibrium *is a Nash Equilibrium where the strategy of at least one player assigns a random probability to choosing each strategy.*

Definition 34 *A* Pure Strategy Nash Equilibrium *is a Nash Equilibrium where the strategy of each player is chosen with probability one.*

We have previously only considered pure strategies, but will now look at mixed strategies.

[3]This example combines some actual cases. Decreasing the sale price if more shareholders sell has been used in the takeover of Federated Department Stores in 1998. Dilution or otherwise penalizing hold outs is a common strategy. See for example, the Facebook movie, or the treatment of hold out creditors by Argentina.

A Using Mixed Strategies to Prevent Opponents From Reacting

Sellers sometimes offer sales at random times. The classic example is the "Blue Light Special" where at random times a sale is announced over the retailer's PA system. The seller wants to get rid of excess inventory. But if the seller announces a sale on a specific date, then all customers would shop on that date only, including those who would have bought the product at the high price. What would be ideal is if non price sensitive customers did not change their behavior in response to when the sale was.

		Customer	
		Shop today	Shop tomorrow
Firm	Sale today	4,12	8,6
	Sale Tomorrow	12,8	6,10

Table 32: Blue light special game.

The customer (lets say this customer would buy at full price) is better off shopping on the day when the sale is. The firm is better off if this customer comes on a day when there is no sale. The firm wants to anti-coordinate and the shopper wants to coordinate.

No Pure Strategy Nash Equilibrium exists in this game. Suppose the firm predicts customers will shop today. Then the firm chooses to have the sale tomorrow. But then after announcing the sale, customers will shop tomorrow, so customers shopping today is not a Nash Equilibrium. Similarly, if the firm predicts shoppers will shop tomorrow and announces a sale for today, then customers react by shopping today and the firm regrets the decision to have a sale today.

But an equilibrium exists in which consumers and firms assign probabilities to strategies (the Blue Light Special). Suppose the firm offers a sale today with probability q. Then the customer who shops today gets 12 with probability q and 8 with probability $1-q$ for a total of $12q+8(1-q)$. The payoff for shopping tomorrow is $6q+10(1-q)$. **The seller must equalize the two average payoffs.** If the average payoffs are different, the shopper will always shop on one day, but then the firm will want to have the sale on the other day, and the prediction that the seller randomizes is not correct. So the equilibrium breaks down if the the average payoffs are not equal.

So we have:

$$12q+8(1-q)=6q+10(1-q) \quad (155)$$

$$4q = 1 \quad \rightarrow \; q = \frac{1}{4} \tag{156}$$

So having a sale one quarter of the time today, and 3/4 of the time tomorrow is the best the firm can do. Notice that consumer overall prefers to shop today (adding the columns, $12 + 8 > 6 + 10$). Therefore, the seller must have a higher probability of a sale tomorrow. If the probabilities of a sale were equal on the two days, the shopper would definitely shop today and the equilibrium would break down. Only by making a sale less likely today can the seller make the buyer indifferent.

Similarly, the consumer does not want to give away to the firm which day they will be shopping, so they also randomize. Let p be the probability that a consumer shops today. The payoff to the firm of a sale today is then $4p + 8(1-p)$ and the payoff of a sale tomorrow is $12p + 6(1-p)$. Equalizing these two gives:

$$4p + 8(1-p) = 12p + 6(1-p) \tag{157}$$

$$1 - p = 4p \quad \rightarrow \; p = \frac{1}{5} \tag{158}$$

So the shopper sets p to equalize the average payoffs to the seller. If the payoffs to seller are not equal, the seller always holds the sale on one day, but then the shopper does not randomize, and the equilibrium breaks down. So the mixed strategy Nash Equilibrium is to shop today with probability 1/5 and to have a sale today with probability 1/4.

It is important to understand what the Nash Equilibrium is:

- The shopper predicts the firm will have a sale today **with probability** 1/4. The shopper does not predict an outcome. The shopper only predicts a strategy (a probability).
- The firm predicts the shopper will shop today with probability 1/5. The firm also does not know the outcome, but only predicts a probability.
- Given the predictions, the shopper does at least as well shopping today with probability 1/5 as shopping today with any other probability. So the shopper plays this strategy and the prediction (of a probability, not an outcome) is correct. The same reasoning holds for the firm.

Other examples of games of this type include Rock, paper, scissors; baseball (fastball or curve); penalty kicks (left or right side); and tennis (serve inside or outside). Studies have found that humans are not great randomizers. Two common pitfalls:

1. Players **switch too often**. For example, studies of tennis players show they switch too often between serving inside and outside. A smart opponent can therefore gain an advantage by guessing the serve will go to the opposite side as the previous serve.

2. Players **try too hard to ensure all possible strategies occur**. In the TV show "Numbers" a serial killer appears to be killing in random neighborhoods. However, the police are able to catch the killer by predicting he will strike in a previously untouched neighborhood, rather than striking a second time in another neighborhood.

B Using Mixed Strategies to Avoid Coordination

Consider two firms, Macy's and TJ Maxx. Macy's is a high priced retailer that carries a variety of clothes in all sizes (and thus higher inventory costs and higher prices) and TJ Maxx is a retailer who charges low prices, but carries whatever surplus clothes manufacturers happen to have, usually in odd sizes. Customers can shop at Macy's, putting up with high prices but knowing they will find what they want, or they can go to TJ Maxx, paying lower prices if TJ Maxx happens to have what they want. Further, the more customers go to TJ Maxx, the more likely they are to be out of something. So the payoffs are:

		Jack	
		Shop TJ Maxx	Shop Macy's
April	Shop TJ Maxx	4,4	10,7
	Shop Macy's	7,10	5,5

Table 33: Anti-coordination game.

Here two Nash Equilibria exist, corresponding to the customers shopping at different stores.

- If Jack predicts April will shop at Macy's, then Jack knows TJ Maxx likely has Jack's size and will have lower prices. But then April will shop at Macy's and we have a Nash equilibrium.

- The game is symmetric (Jack and April are identical in terms of payoffs), so April shops at TJ Maxx and Jack shops at Macy's is also an equilibrium.

Now compute a mixed strategy.

- Suppose Jack predicts April will shop at TJ Maxx with probability q.
- Then Jack's payoff is $4q + 10(1-q)$ if Jack shops at TJ Maxx and $7q + 5(1-q)$ if Jack shops at Macy's. Payoffs are equal if $q = \frac{5}{8}$.
- The game is symmetric, so a mixed strategy exists in which each customer shops at the discount retailer with probability 5/8.

Notice that the mixed strategy is not very efficient. With probability $\frac{5}{8} \cdot \frac{5}{8} = \frac{25}{64}$ the customers "bump into each other" at the discount store, which does not lead to a high payoff for either player. The social benefit of either pure strategy Nash equilibria is $7 + 10 = 17$. The social benefit of the mixed strategy is the sum of the social benefits times the probability of end up in each box:

$$\frac{5}{8} \cdot \frac{5}{8} \cdot (4+4) + \frac{5}{8} \cdot \frac{3}{8} \cdot (10+7) + \frac{3}{8} \cdot \frac{5}{8} \cdot (7+10) + \frac{3}{8} \cdot \frac{3}{8} \cdot (5+5) = 13.5, \tag{159}$$

which is less than the social benefit of the pure strategy Nash. Nonetheless, the mixed Nash may be more realistic, because both players prefer the discount store, so both players have an incentive to pick TJ Maxx.

By the way, this game was featured in the movie about John Nash: "A Beautiful Mind." However, the movie inexplicably states that the Nash Equilibrium is for all consumers to shop at the high priced retailer. Nash proved in his dissertation that every game has at least one mixed or pure strategy equilibrium.

IV Dynamic Sequential Games

Two versions of games played over time exist: games as above but repeated, and sequential games in which one player moves before the other. We will look at only the sequential games.

A Extensive Form

It is easiest to represent sequential games in extensive form. In the extensive form, player strategies are represented as branches on a tree.

Consider an anti-coordination game where one player now moves first. Apple, the first mover, can either expand capacity in the tablet market or not expand capacity. Then Samsung has the same choice. As in the anti-coordination game, if both firms expand in the tablet market, the competition will be fierce and profits are low for both firms. If one firm expands and the other firm does not, profits will be high for the firm that expands. The firm that does not enter makes more money than the case where both firms expand into the tablet market, but less than if the firm entered and the other firm did not.

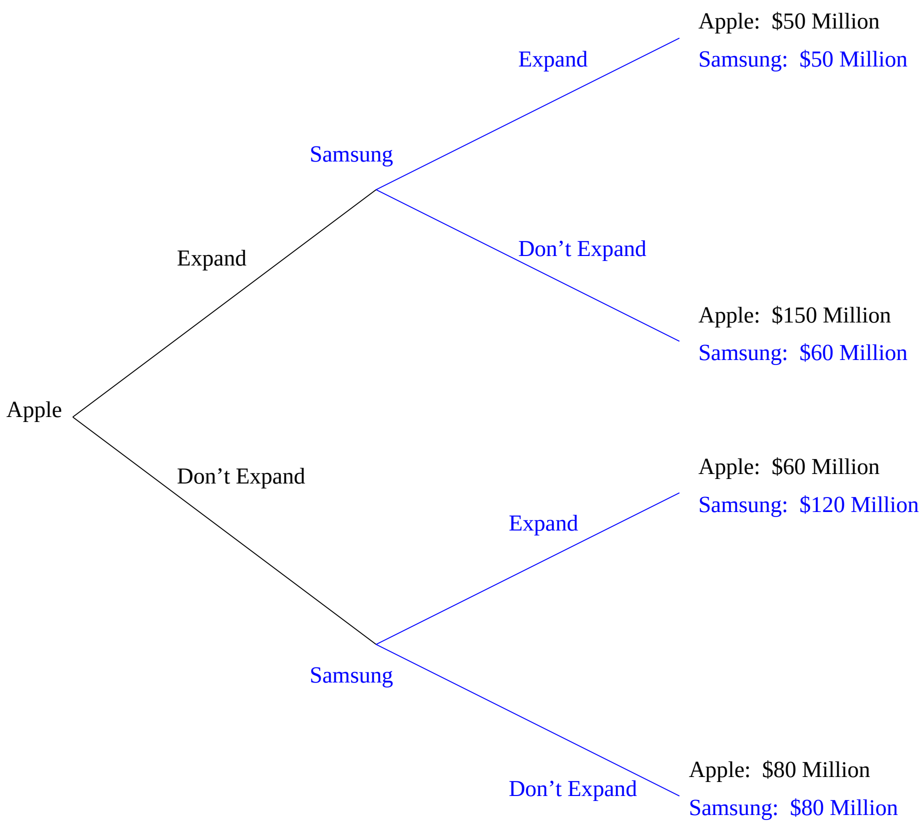

Figure 9: Anti-coordination game in sequential form.

To solve this problem, work backwards. If Apple expands, Samsung does better by ceding Apple the market and focusing resources elsewhere. If Apple does not expand, Samsung does better by expanding into the tablet business. Knowing this, Apple can either expand, knowing Samsung will not expand, and make $150 Million, or not expand, knowing Samsung will expand, and make $60 million. Thus Apple will expand. The equilibrium, which we will call Sub-Game Perfect, is for Apple to Expand and Samsung not to Expand. Apple makes $150 Million and Samsung makes $60 Million.

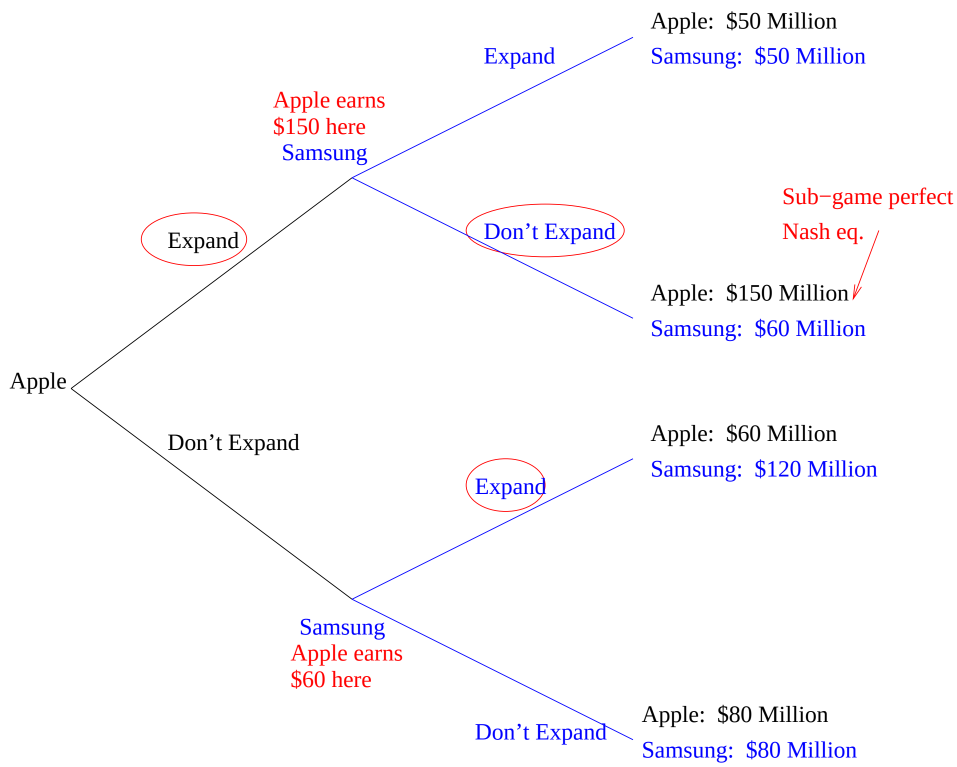

Figure 10: Anti-coordination game in sequential form.

Definition 35 *A* **Sub-Game Perfect Equilibrium** *requires each sub-tree to be a Nash Equilibrium.*

Sub-game perfect requires beliefs to be correct, even off the equilibrium path. The equilibrium path here is Apple expands, and then Samsung doesn't expand. Sub-game perfect requires beliefs to be correct even off the equilibrium path: Apple must correctly believe Samsung will expand if Apple doesn't, even though we never actually see this part of the game.

Consider an alternative equilibrium definition:

Definition 36 A Nash Equilibrium (not sub-game perfect) *requires only that beliefs are correct along the equilibrium path. That is, only the sub-games actually observed must be Nash equilibria.*

In the above example, a Nash equilibrium which is not sub-game perfect exists. Suppose Samsung threatens to expand if Apple expands. If Apple believes this threat, then Apple does best by not expanding. If Apple does not expand, then Apple's belief that Samsung will expand if Apple expands is never put to the test. So the equilibrium satisfies the definition of a Nash equilibrium since all beliefs are correct along the equilibrium path.

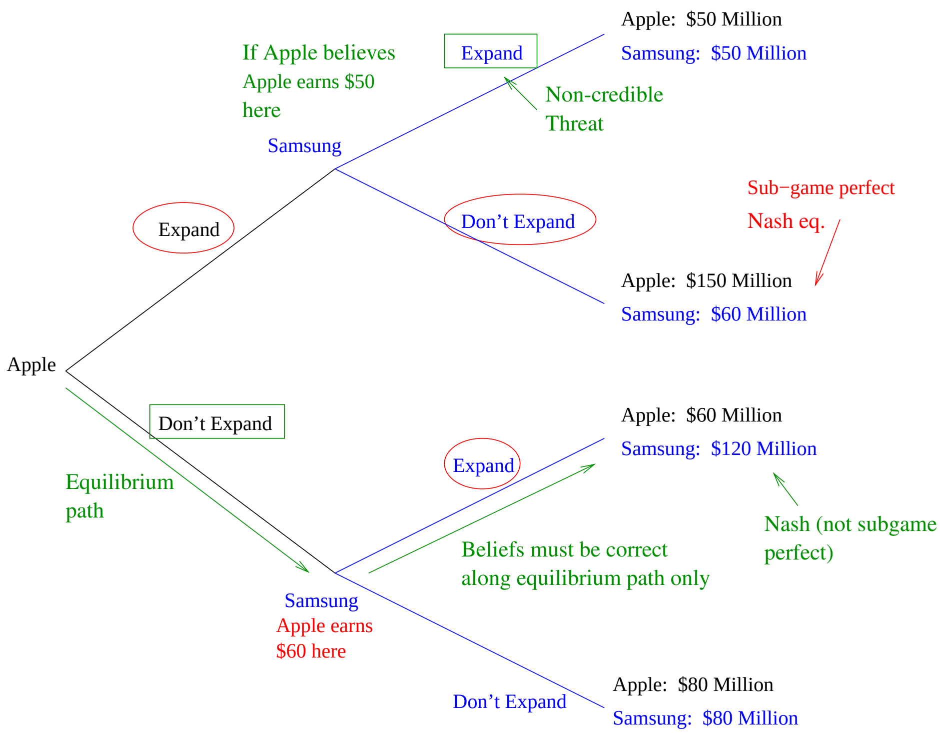

Figure 11: Anti-coordination game in sequential form.

However, the Nash equilibrium which is not sub-game perfect is less realistic in that the threat is not credible (Apple should call Samsung's bluff here).

In the simultaneous game, two Nash Equilibria exist and both players prefer a different equilibria.

		Samsung	
		Expand	Don't Expand
Apple	Expand	50,50	150,60
	Don't Expand	60,120	80,80

Table 34: Simultaneous first mover game.

Apple prefers the Nash equilibrium in the upper right corner of table 34, whereas Samsung prefers the Nash equilibrium in the lower left corner. The first mover gets an advantage which results in the selection of the first mover's preferred Nash Equilibrium. That is, the preferred Nash equilibrium of the first mover in the simultaneous game is sub-game perfect.

Definition 37 First Mover Advantage*: Advantage to the first mover in sequential games when the simultaneous game has multiple Nash Equilibria.*

We see in this example that sub-game perfect equilibria are more stringent than Nash Equilibria (only 1 equilibrium exists, not two).

B Preemption and Deterrence

An important strategy for the market leader in an industry is to deter entry by potential rivals. Suppose Intel dominates in terms of market share the market for computer memory chips. Intel knows a number of other makers of electronic parts could enter the market at any time, for example AMD. The payoff matrix is:

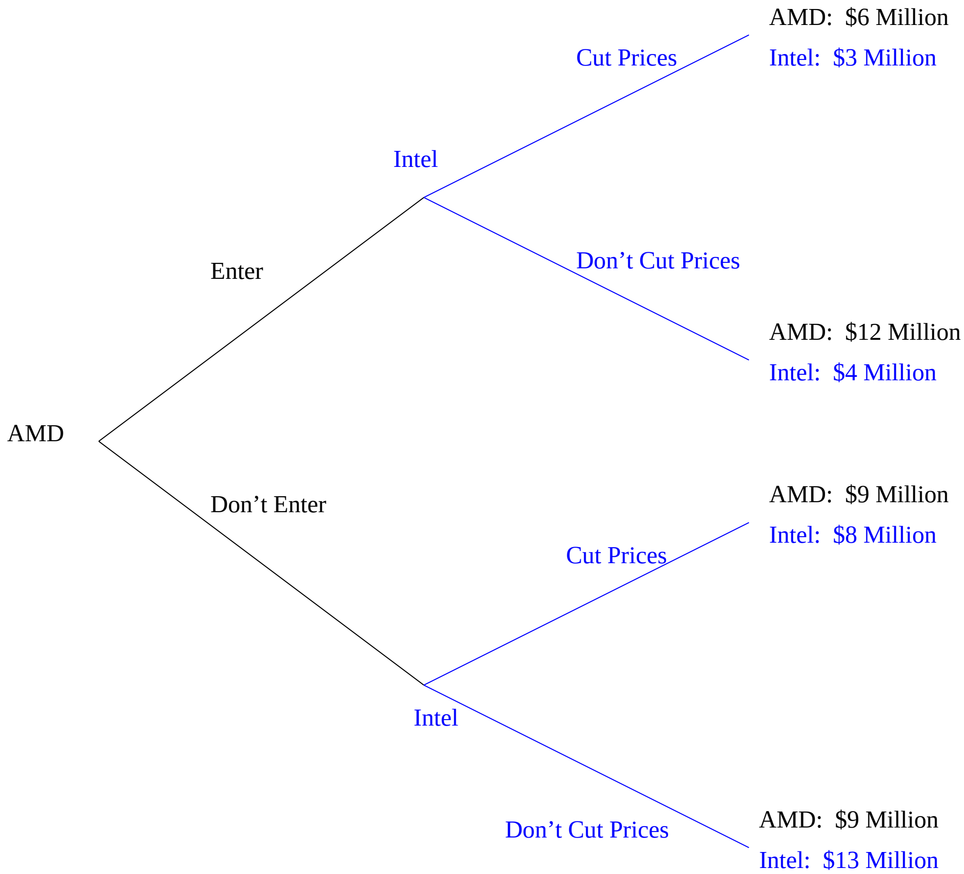

Figure 12: Entry game.

AMD moves first, by deciding whether or not to enter. Then Intel decides whether to respond by cutting prices and expanding output, or ignoring AMD and sharing the market. We solve the problem by working backward. First, if AMD enters, then an ensuing price war would mean that Intel's profits are lower than if it instead kept prices high, making money from the most loyal customers, while ceding some of the market to AMD. If AMD does not enter, than Intel of course does much better holding prices high. So AMD has the choice of entering, knowing that Intel will not cut prices or not entering. Thus AMD enters.

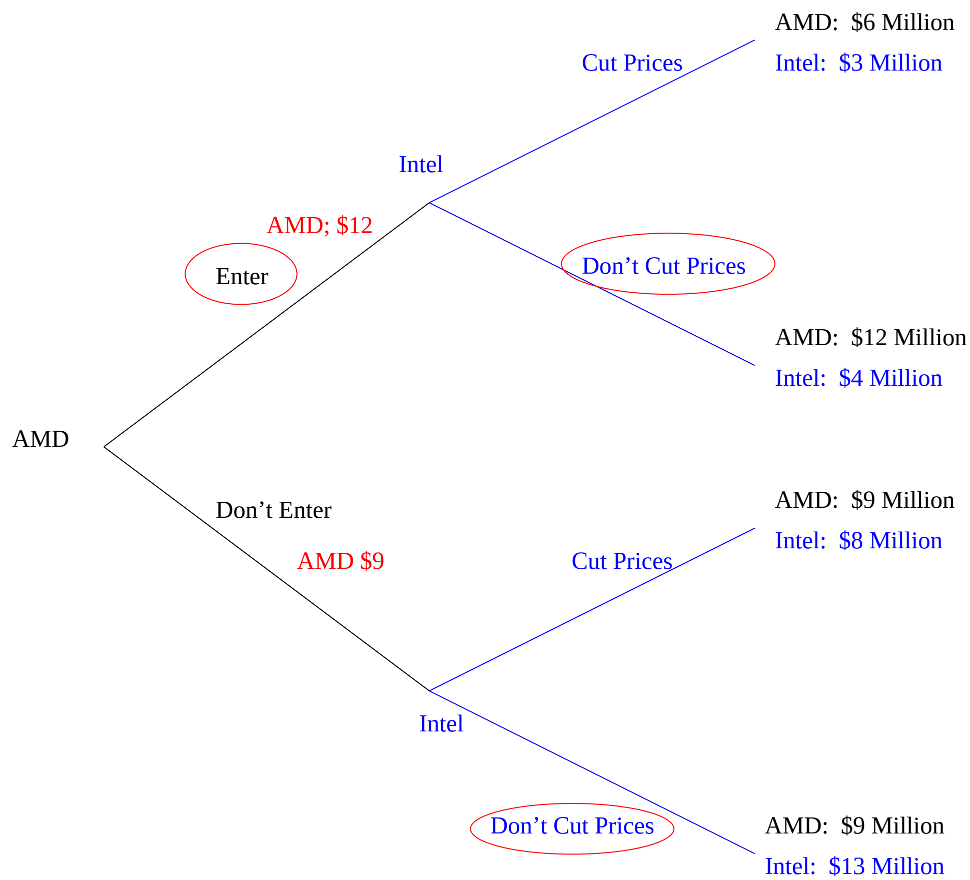

Figure 13: Entry game.

Intel can threaten to cut prices, but AMD knows the threat is not credible and enters. Sub-game perfection requires threats to be credible while the Nash Equilibrium does not. To see this, suppose AMD predicts Intel will cut prices if AMD enters. Then AMD should not enter. Then Nash Equilibria requires us to check whether or not our prediction is correct. But in this case the prediction does not matter, because AMD has decided not to enter. So we can say the prediction is correct.

Next suppose we add another move for Intel. Intel can add capacity or not add capacity before AMD decides whether or not to enter. If Intel adds capacity, then Intel can efficiently produce a large volume of chips if AMD enters. However, the expansion results in $2 million of extra fixed costs if the factory is not used. The payoff matrix then changes as shown below, and AMD decides not to enter. Building capacity is one way to make threats more credible and deter entry. Building excess capacity is of course costly, however.

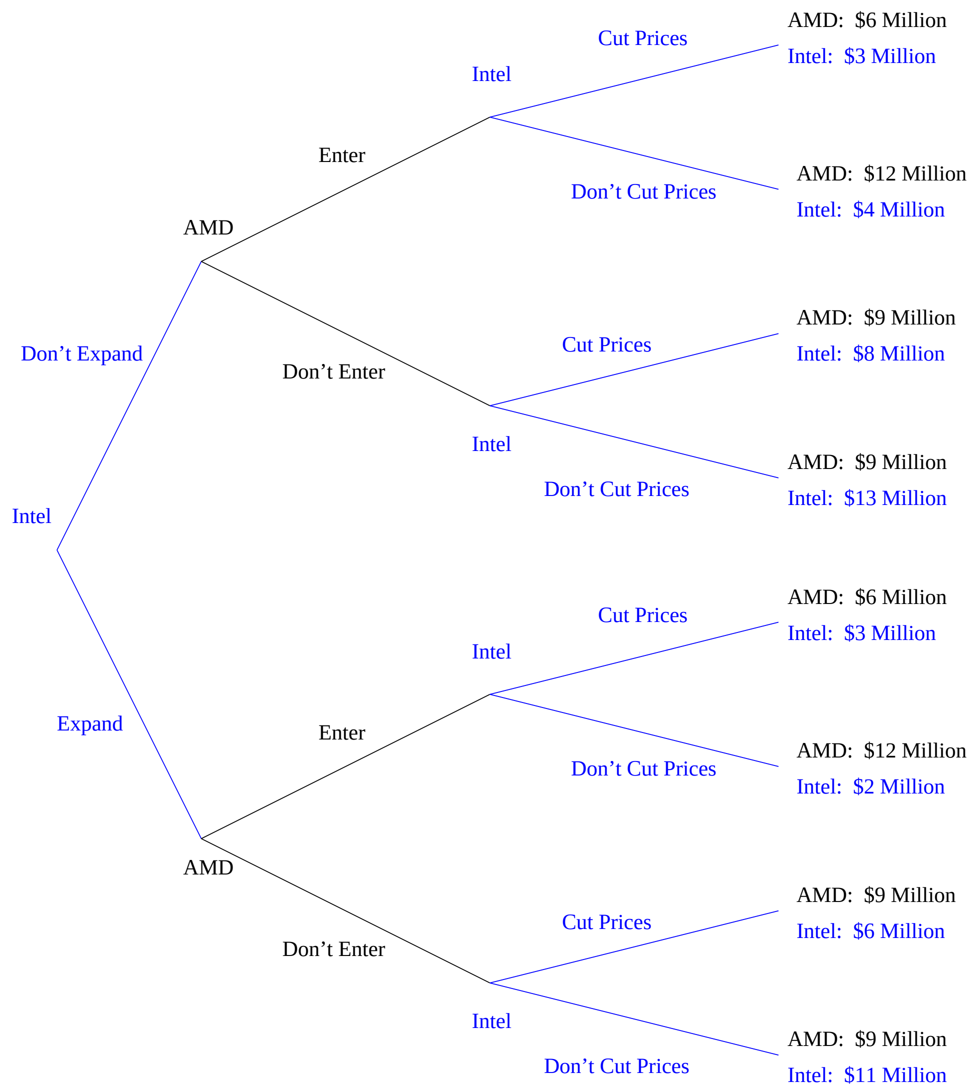

Figure 14: Entry game.

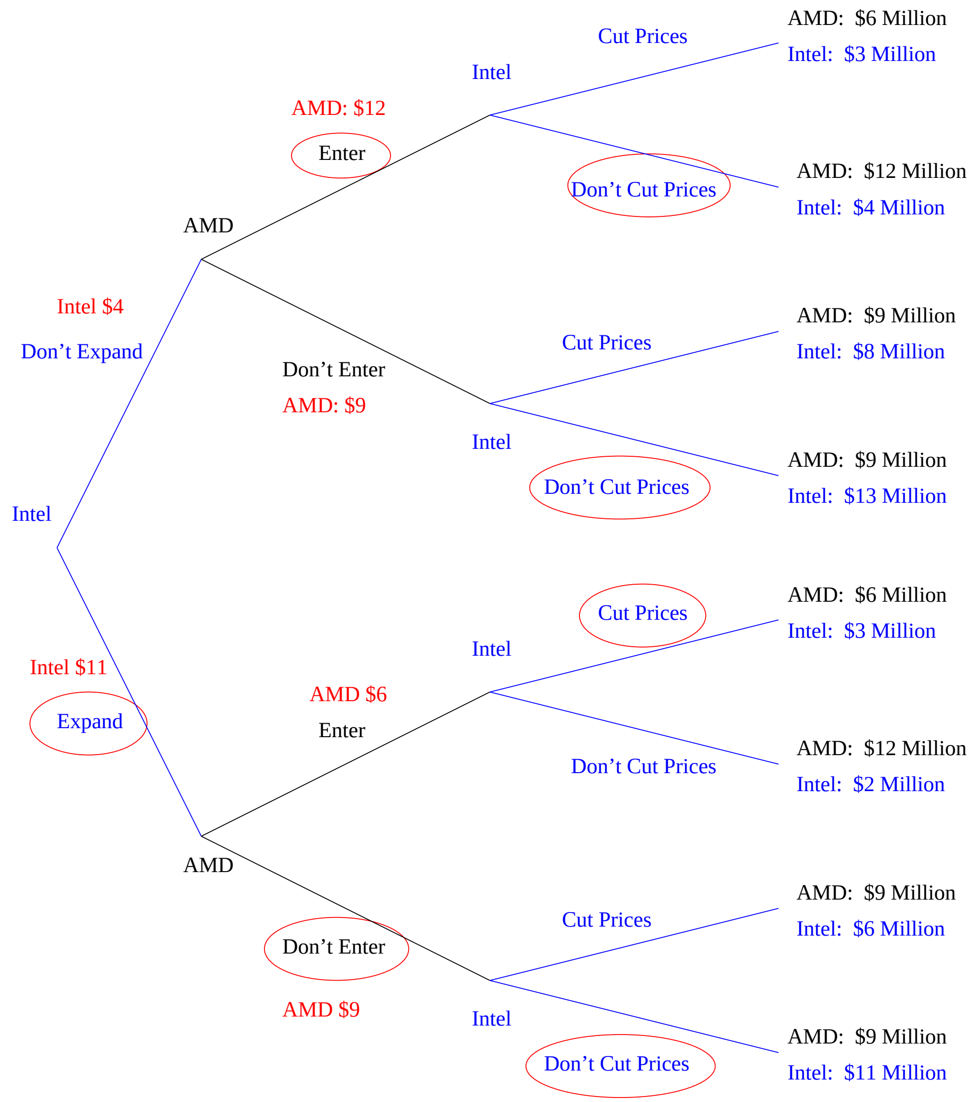

Figure 15: Entry game with capacity decision.

The excess capacity means it is more profitable for Intel to cut prices if AMD enters. Therefore, the expansion by Intel deters entry by AMD. Intel takes a preemptive action, to deter AMD from entering. Examples of entry deterrence:

1. DuPont in 1979 invests $0.33 billion in titanium dioxide plants to deter rivals with older plants from updating to newer, more competitive plants.

2. Walmart's strategy is to build large stores in smaller towns first in order to deter others from building similar stores, since each town can support at most one store.

Definition 38 ***Preemption*** *is an action taken to deter later actions by other players.*

- Preemption allows the firm to gain a first mover advantage.
- The preemptive action must be permanent (irreversible) action. It is not a threat.
- **Rule from game theory: it is better to anticipate and preempt than to react.** It is easy for a market leader to become complacent and not take action until after a rival has entered. This is often too late.