

THE MARKET FOR MONEY

I Money Supply

A Money Supply Function

Our definition of M1 is currency HELD BY THE NON BANK PUBLIC, C (this is cash in wallets), and checkable deposits, D . Let $M1 = M$, then we have:

$$M = C + D \tag{5}$$

CURRENCY TO DEPOSIT RATIO: the fraction of deposits that are held as cash, $cr = C/D$. Therefore:

$$M = cr \cdot D + D, \tag{6}$$

$$M = (cr + 1) D, \tag{7}$$

High powered money (H) is the sum of currency and total bank reserves (TR):

$$H = C + TR \tag{8}$$

RESERVE TO DEPOSIT RATIO: the fraction of deposits banks hold in reserve, $rd = TR/D$. So:

$$H = cr \cdot D + rd \cdot D, \tag{9}$$

$$H = (cr + rd) D, \tag{10}$$

Therefore:

$$\frac{M}{H} = \frac{cr + 1}{cr + rd}. \tag{11}$$

Our assumption on reserves is:

$$rd = rrr + e(R) \tag{12}$$

The bank holds the required reserves, plus a little extra in case of a large withdraw. Note that if the interest rate is high, the opportunity cost for the banks to hold excess reserves is

high. Therefore e is decreasing. Hence:

$$\frac{M}{H} = \frac{cr + 1}{cr + rrr + e(R)} \quad (13)$$

The money supply function is then:

$$M = \frac{cr + 1}{cr + rrr + e(R)} H \quad (14)$$

Definition 7 *The money multiplier is the change in money supply that results from a change in high powered money.*

The money multiplier is denoted as $k(R)$ and is given by:

$$k(R) = \frac{cr + 1}{cr + rrr + e(R)} \quad (15)$$

Changes in the money supply are then:

$$\Delta M = k(R) \Delta H \quad (16)$$

B Interpreting the effect of the coefficients on money supply

1 Interest rate

An increase in the interest rate implies banks loan excess reserves. When those loans are deposited, additional checking accounts are created. Thus increasing R increases the money supply.

2 High Powered Money

An increase in high powered money increases both the currency held by the public, and increases checking deposits as the currency is deposited. Thus increasing H increases the money supply.

3 Required Reserves

Increasing the required reserve ratio means banks have less currency to loan, which reduces the checking accounts created. Thus increasing rrr decreases the the money supply.

4 Currency to Deposit Ratio

Take the derivative.

$$\frac{\partial M}{\partial cr} = H \frac{(cr + rrr + e(R)) \cdot 1 - (cr + 1) \cdot 1}{(cr + rrr + e(R))^2} \quad (17)$$

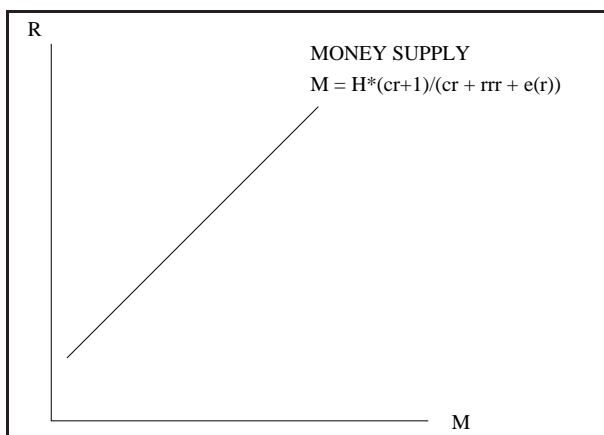
Simplify:

$$\frac{\partial M}{\partial cr} = H \frac{(rrr + e(R)) - 1}{(cr + rrr + e(R))^2} = -H \frac{1 - rd}{(cr + rrr + e(R))^2} \quad (18)$$

This is always less than zero.

Intuition: if more currency is held relative to deposits, the money supply goes up because with currency no reserves are required. However, if more currency is held relative to deposits, the total amount of deposits goes down, hence the money supply goes down. The second factor outweighs the first.

Graph of money supply function:



C Example

Suppose high powered money is \$20 billion, $cr = \frac{1}{2}$, $rrr = \frac{1}{2}$, $e(R) = 0$. Calculate H , $k(R)$, C , D , TR . Calculate the change in H to get $M = 90$.

We have $k(R) = \frac{3}{2}$. Thus $M = \frac{3}{2} \$20 = \60 . Since $M = \left(\frac{1}{2} + 1\right) D$, $D = \$40$. Thus $C = \$20$ and $TR = 20$. To get a \$30 increase in M , we need to buy $\Delta H = \frac{2}{3} \$30 = \20 .

D M2

Suppose we let S denote savings deposits and money market mutual funds. Suppose the savings to deposit ratio is $sr = S/D$. Then:

$$M2 = C + D + S \tag{19}$$

$$= D(cr + 1 + sr) \tag{20}$$

Using the equation (10), we have:

$$\frac{M2}{H} = \frac{cr + 1 + sr}{cr + rd}. \tag{21}$$

We have a different money multiplier for $M2$, because no reserves are required for savings deposits.

$$k_{M2}(R) = \frac{cr + 1 + sr}{cr + rd}. \tag{22}$$

We can see that $M2 > M1$ and $k_{m2} > k$.

II Baumol-Tobin Model of Money Demand (sec 3.6)

People demand money for its use as a medium of exchange. Holding money involves a cost: the forgone interest that could be obtained by putting wealth elsewhere.

The key elements of this model are:

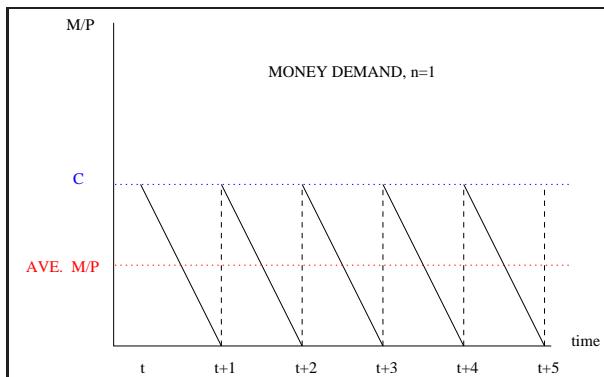
1. At the beginning of each period (say a month), the consumer withdraws some cash for the month. The decision is how many withdraws, n , to make.
2. Purchases, C are spread evenly over the month.
3. At the end of the month, there is zero cash left over.
4. Cash is required for all purchases.
5. There is an interest bearing account which earns real interest rate r , based on average daily balance. Think of a tbill or stock. No purchases can be made from the interest bearing account.
6. There is a cost to withdraw money δ , in real terms. Think of δ as a liquidity cost. Households for example might have to sell their stock at a discount equal to δ in order

to get cash quickly (in less than a month). The cost may also represent time costs associated with calling a broker and depositing the proceeds in the bank. It is best not to think of δ as an ATM fee.

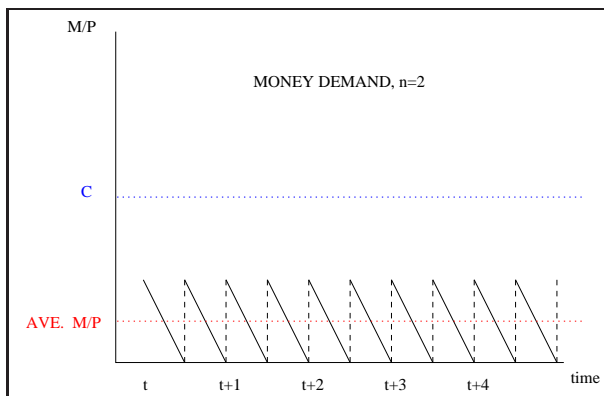
7. We define the money demand as the average daily money holdings.

A Money Demand

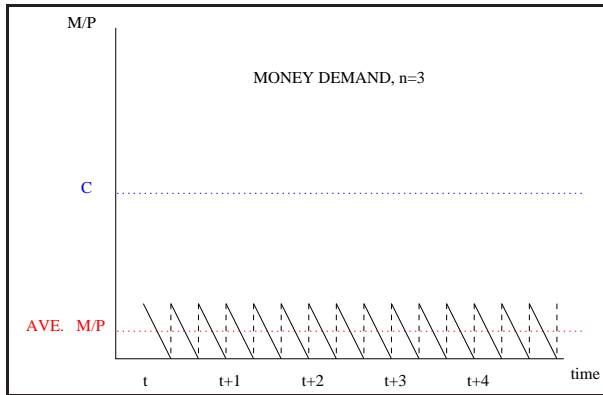
Suppose one withdraw is made, or $n = 1$. Then the average daily balance is: $M = \frac{C \cdot P}{2}$.



Suppose two withdraws are made, or $n = 2$. Then the average daily balance is: $M = \frac{C \cdot P}{4}$.



Suppose three withdraws are made, or $n = 3$. Then the average daily balance is: $M = \frac{C \cdot P}{6}$.



And so in any period, the average real balance is

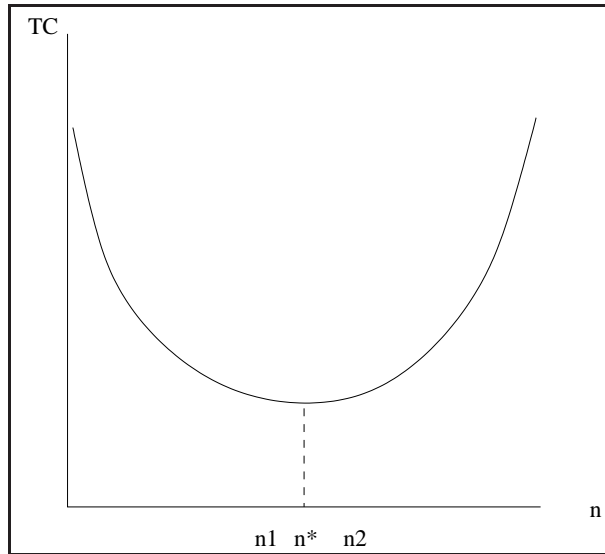
$$\frac{M}{P} = \frac{\frac{C}{n}}{2} = \frac{C}{2n}. \quad (23)$$

Now the opportunity cost of holding money is simply the interest that is lost. Cash held earns a real return of $-\pi$, whereas the interest bearing account earns real return of r . So the opportunity cost is $r - (-\pi) = r + \pi = R$. Total lost real interest is thus $R \frac{M}{P}$. The cost to withdraw n times is a liquidity penalty of δn . So the total cost is:

$$tc = R \cdot \frac{M}{P} + \delta n \quad (24)$$

Using the relationship between the number of withdraws and the amount of money withdrawn:

$$tc = R \frac{\frac{C}{n}}{2} + \delta n \quad (25)$$



As $n \rightarrow \infty$, we have $TC \rightarrow \infty$ and as $n \rightarrow 0$ we also have $TC \rightarrow \infty$. Also n is a discrete variable, but we would expect that over the entire population fractional withdraws are possible. That is, $n = \frac{3}{2}$ means half the population withdraws once per month and half twice.

$$\frac{\partial TC}{\partial n} = 0 = \frac{-R \cdot C}{2n^2} + \delta \quad (26)$$

$$n = \left(\frac{RC}{2\delta} \right)^{\frac{1}{2}} \quad (27)$$

Hence money demand is:

$$\frac{M}{P} = \frac{C}{2n} = \frac{C}{2 \left(\frac{RC}{2\delta} \right)^{\frac{1}{2}}} \quad (28)$$

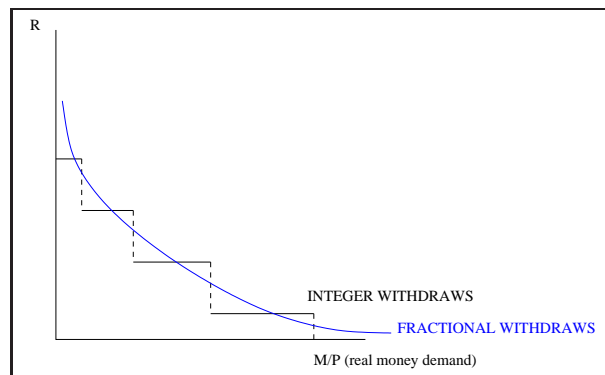
$$\frac{M}{P} = \left(\frac{C\delta}{2R} \right)^{\frac{1}{2}} \quad (29)$$

- As R rises, the opportunity cost of holding money rises and n increases. As n increases, money demand falls.
- As δ increases, the cost of going to the bank increases so n falls and money demand rises.

- As C rises, we must at each trip withdraw more cash. This does not affect the total cost of withdraws, since we can withdraw any amount at cost δ . However, we are holding more cash and thus the opportunity cost rises. So the solution is to make a few more withdraws, so n increases and M increases.

variable	effect on n	effect on M
R	increasing	decreasing
δ	decreasing	increasing
c	increasing	increasing

Table 1: Money Demand



B Velocity of money

VELOCITY OF MONEY: how many times each dollar is spent in a given period.

CONSUMPTION VELOCITY OF MONEY: how many times each dollar is spent on consumption.

Consumption velocity is determined from the exchange equation:

$$MV = PC \tag{30}$$

The right hand side is the dollar value of all goods purchased (price times quantity purchased) and the left hand side is number of dollars spent (dollars times number of times each dollar is spent). Velocity is thus:

$$V = \frac{C}{\frac{M}{P}} \tag{31}$$

There is an equivalent private sector velocity which uses $C + I$, and income velocity which uses $Y = C + I + G$.

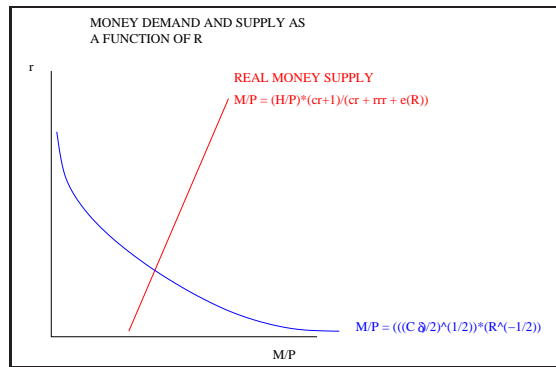
Notice that:

$$V = \frac{C}{\frac{C}{2n}} = 2n = 2\sqrt{\frac{RC}{2\delta}} = \sqrt{\frac{2RC}{\delta}} \quad (32)$$

The BT model predicts velocity is not constant but is instead related to the interest rate. As interest rates rise, money is spent more quickly as the cost of holding money rises.

III Equilibrium in the money market

Putting money demand and supply together looks like this:



Consider the effect of two changes on the equilibrium money balances.

1. A rise in the currency-to-deposit ratio (shifts the money supply to the left).
2. A rise in real consumption (shift money demand to the right).

Tests of the model with real data do fairly well during times with low inflation. An important result from the data is that money demand is not very sensitive to R and Y . Most people apparently do not make more than one withdraw.

IV Constant Velocity Model of Money Demand

A simpler model is to assume velocity is constant, in which case money demand can be derived directly from the quantity equation:

$$\frac{M}{P} = \frac{C}{V} \quad (33)$$

This is thus a special case of the BT model in which n is constant. This model is sometimes used in Eco 212, but the evidence is that velocity does respond to changes in interest rates.

V The Cagan Model of Money Demand

A Modification to BT-Model to get Cagan Model

It is important to remember that it is the nominal interest rate, not the real rate, that we use in the BT model. What matters is the difference between the return on money and the return on the interest bearing account. The real return on the interest bearing account is r_t . The return on money is calculated as follows. Recall that P_t dollars held as money buys one good today. Tomorrow, each of the P_t dollars buys $\frac{1}{P_{t+1}}$ goods. So the gross return is $\frac{P_t}{P_{t+1}}$. Net rate of return is:

$$\text{net return on money} = \frac{P_t}{P_{t+1}} - 1 = -\pi_{t+1} \quad (34)$$

The Cagan model is primarily applied to high inflation countries (at least 50% per year). In such countries, changes in r are small relative to changes in inflation. Cagan fixed r and C and took a linear approximation of the model so that:

$$MD = a - b\pi_{t+1} \quad (35)$$

For those that are interested, the formula is derived as follows. Take logs:

$$\log M - \log P = MD = \frac{1}{2} \log(\delta C) - \frac{1}{2} \log 2 - \frac{1}{2} \log R_t \quad (36)$$

Use a Taylor approximation:

$$\log R_t \approx \log 1 + (R_t - 1) \frac{1}{1} \approx R_t - 1 \quad (37)$$

So:

$$\log M - \log P = MD = \frac{1}{2} \log(\delta C) - \frac{1}{2} \log 2 + \frac{1}{2} - \frac{1}{2} r - \frac{1}{2} \pi \quad (38)$$

$$\log M - \log P = MD = a - b\pi_{t+1} \quad (39)$$

MD is negatively related to the inflation rate. Further, Cagan realized that in high inflation countries the inflation rate can change quickly, so he relied on expected inflation,

π_{t+1}^e .

$$MD = a - b\pi_{t+1}^e \tag{40}$$

B Expected Inflation

1 Adaptive Expectations

An important part of the Cagan model is how households anticipate inflation. Cagan said that people formed expectations ADAPTIVELY, by looking at past values of inflation and other information such as the money supply. For example, one might suppose that:

$$\pi_{t+1}^e = d\pi_t \tag{41}$$

Another example is:

$$\pi_{t+1}^e = d\pi_t + (1 - d)m_t \qquad m_t = \log M_t \tag{42}$$

As inflationary expectations rise, money demand falls. It may be the case even with a small rise in inflation, inflationary expectations rise significantly and money demand falls. This may become self-fulfilling as a decrease in money demand results in a lowering of the value of money or inflation.

2 Rational Expectations

RATIONAL EXPECTATIONS: Consumers do not make consistent mistakes, and use all available information.

Adaptive Cagan model is not rational. If inflation is rising, consumers will be consistently too low.

Also, adaptive expectations does not use such valuable information as government debt levels. Example of hyperinflating economies: $\pi_t = 0$ before M^s fell!

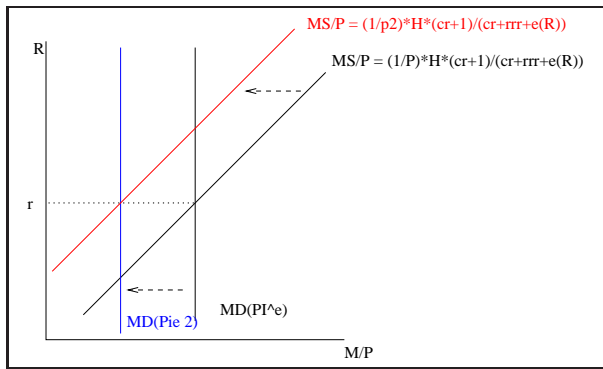
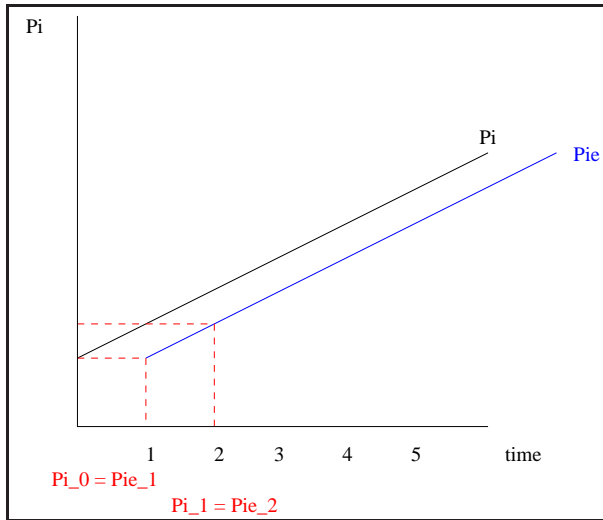
C Self fulfilling nature of the Cagan Model

1 Self-Fulfilling Adaptive Expectations

Suppose beliefs are that inflation will rise. What happens? Money demand falls.

Money demand falls, so the value of money falls, money is exchanged for less goods, so inflation rises. So beliefs that inflation will rise are self-fulfilling.

SELF FULFILLING EXPECTATIONS: An event occurs because people expected it to happen.



FED's and central banks take self-fulfilling nature of expectations very seriously.

- Constantly talk down inflation. “We will be ready to decrease M at the slightest sign of inflation.”
- But... talk is cheap. If consumers believe FED and then FED increases the money supply (and inflation rises), consumers will not believe talk next time.
- Inflation targets help.
- Large inflation reduction is free of unemployment if promise to reduce inflation is viewed as credible.

To determine if inflationary expectations are self generating, note that if inflationary expectations rise, money demand must fall and thus inflation must rise. Therefore, if inflationary expectations respond positively to past inflation, expectations are self-fulfilling.

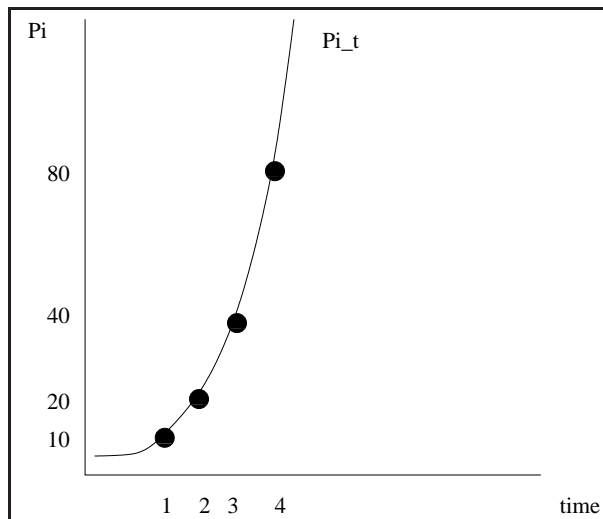
Here is an example. Suppose money demand is $MD = 1 - 2\pi_{t+1}^e$. Suppose further that equilibrium adjusts in the money market according to $\pi_{t+1} = 2 - MD$. Finally let expectations be $\pi_{t+1}^e = d\pi_t$. Then combining all three equations gives:

$$\pi_{t+1} = 1 + 2d\pi_t \tag{43}$$

So if $d > 0$ then expectations are self-fulfilling since an increase in beliefs causes an increase in actual inflation.

D Self generating nature of the Cagan Model

Suppose beliefs about inflation imply so little money demand that prices must rise by more or equal to the rise in beliefs. For example, suppose $\pi_{t+1}^e = 10$ and money demand falls to a very low level which implies a large increase in P so that $\pi_{t+1} = 20$. Then next period $\pi_{t+2}^e = 20$ which implies money demand falls even more and so prices rise even more which gives $\pi_{t+2} = 40$ and so on. In this case, inflation spirals out of control or is unstable.



Conversely, if inflation is stable then the self fulfilling nature of expectations is not that harmful as self fulfilling inflation will quickly fade away.

Cagan estimated axis countries in Europe after WWI. He found unstable inflation in Germany and Russia, but stable inflation in Austria, Greece, Hungary, and Poland. His econometric methods were suspect, however. Conclusion: 2 countries inflation caused primarily by expectations, 4 primarily by increased money growth.

To determine if inflationary expectations are self-generating, we again combine the equa-

tions for expectations, equilibrium, and money demand. In the above example,

$$\pi_{t+1} = 1 + 2d\pi_t. \tag{44}$$

Inflationary expectations are self-generating if the coefficient on π_t is greater than or equal to one, so we need $d \geq \frac{1}{2}$. In this case increases in inflationary expectations cause inflation to rise without bound.

1 Example 1

Let expectations be given by $\pi_{t+1}^e = \pi_t$, $MD = 1 - 2\pi_{t+1}^e$, and let equilibrium be restored in the money market according to: $\pi_{t+1} = 1 - MD$. Hence we have:

$$\pi_{t+1} = 2\pi_t \tag{45}$$

So expectations are both self-generating and self-fulfilling.

period	inflation	expectations
0	10	10
1	20	10
2	40	20
3	80	40
4	160	80
5	320	160

2 Example 2

A second example is $MD = 20 - \frac{1}{2}\pi_{t+1}^e$, $\pi_{t+1}^e = \pi_t$, and $\pi_{t+1} = 70 - MD$. Hence:

$$\pi_{t+1} = 50 + \frac{1}{2}\pi_t \tag{46}$$

Here since the coefficient is positive but less than one, expectations are self-fulfilling but not self-generating. In fact, the inflation rate converges to 100, as the effect of the expectations die out over time. Note that the response of money demand to changes in inflationary expectations are much weaker in example 2, so the response of actual inflation is also weaker.

VI Inflation Taxes

Why do countries print so much money if the end result is inflation? After the hyperinflations in Europe after WWI, Axis countries had their factories destroyed, were forced to pay war

period	inflation	expectations
0	40	40
1	70	40
2	85	70
3	92.5	85
4	96.3	92.5
5	98.1	96.3
6	99.1	98.1

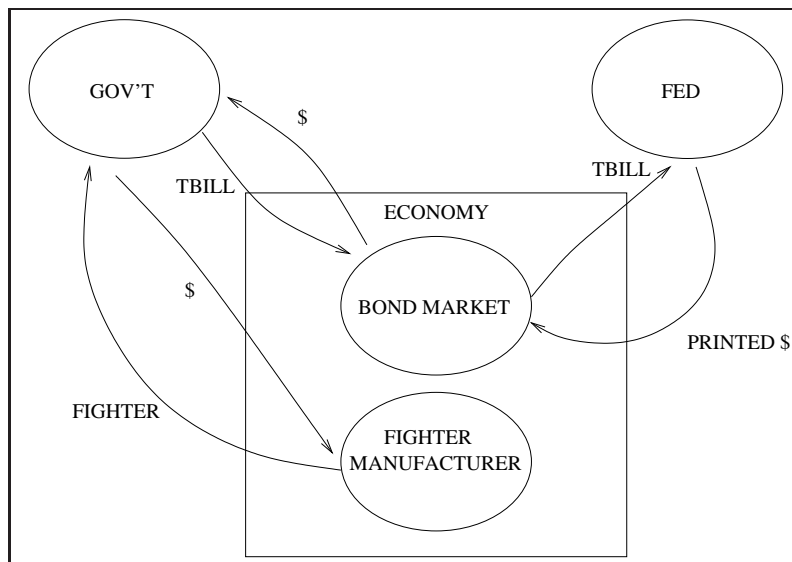
reparations, and had their populations reduced. Thus, required spending was high, but the tax base was low. How were the Axis countries to pay the war reparations? The answer is that the Axis countries printed money and used printed money for expenditures, until the Allied countries basically shut down the printing presses with tanks. Other examples include Latin America in the 1980's.

To think of inflation as a tax, we must know who pays and how the government gets the revenue. First the revenue side.

A Tax Revenue of Inflation

1 How the Government Raises Revenue

Graphically:



Net is that the government has 'paid' for the fighter with printed money since the FED is part of the government. Note that no new bonds remain in the economy at the end of the period, so we may think of it as the government has not issued bonds at all and just paid

for the fighter with printed money. The FED can raise as much as \$20 billion per year in this manner.

The government budget constraint is:

$$G - T = \text{Deficit} = \text{borrowing} + \text{seigniorage} \quad (47)$$

Spending less taxes is the deficit, which can be financed either by borrowing or via seigniorage, which is just printing money.

2 Seigniorage

SEIGNIORAGE: the value of printed money less printing costs.

Seigniorage is thus the tax revenue that goes to the government for the inflation tax. Typically, printing costs are near zero, but there are exceptions such as the strain on printers in Europe after WWI and the gold dollar.

Next we calculate the seigniorage revenue. Note that printed money is equal to:

$$\text{printed } \$ = H_{t+1} - H_t \quad (48)$$

This money is worth:

$$\text{value of printed } \$ = (\$H_{t+1} - \$H_t) \frac{1 \text{ good}}{\$P_t} = \frac{H_{t+1} - H_t}{P_t} \quad (49)$$

$$= \frac{H_{t+1}}{P_{t+1}} \frac{P_{t+1}}{P_t} - \frac{H_t}{P_t} \quad (50)$$

$$= \frac{H_{t+1}}{P_{t+1}} \left(\frac{P_{t+1} - P_t}{P_t} + 1 \right) - \frac{H_t}{P_t} \quad (51)$$

$$\text{value of printed } \$ = \text{seigniorage} = h_{t+1} (1 + \pi_t) - h_t \quad (52)$$

Here $h_t = \frac{H_t}{P_t}$ which is the real high powered money stock. Notice that in the long run, $h_{t+1} = h_t = h$ so that:

$$\text{Long run seigniorage} = \pi h \quad (53)$$

Remember that if we have a tax rate t per unit of h the revenue raised is $t \cdot h$ (think of a cigarette tax). So here we can think of π as the tax rate on real money holdings (actually just the real high powered money).

B Who pays the tax?

Remember, money carried forward from period t to period $t + 1$ is M_{t+1} . Now money held buys $\frac{1 \text{ good}}{\$P_t}$ goods today and $\frac{1 \text{ good}}{\$P_{t+1}}$ goods next period. So the loss of value is equal to:

$$\text{loss} = M_{t+1} \left(\frac{1}{P_t} - \frac{1}{P_{t+1}} \right) = \frac{M_{t+1}}{P_{t+1}} \left(\frac{P_{t+1}}{P_t} - 1 \right) \quad (54)$$

$$= m_{t+1} \pi_t = k(R_{t+1}) h_{t+1} \pi_t \quad (55)$$

The loss of value is the ‘taxes paid’ to the government.

$$\text{taxes paid} = k(R_{t+1}) h_{t+1} \pi_t \quad (56)$$

Note that since the money multiplier is in general greater than one, the government does not collect all of the tax revenue. This is an inefficiency in the tax system. For example, if $k(R) = 2$, for every 1% of money holdings that the government collects, households pay 2%. The tax system is inefficient, half the revenue is lost.

In the long run, we have:

$$\text{taxes paid} = k(R) \cdot \text{Long run seniorage} \quad (57)$$

So holders of money pay the tax, the tax rate on h_t is π_t , and the government does not collect all taxes paid.

C Welfare Cost of inflation

Is inflation tax a good tax? Two criteria:

- Progressive: A progressive tax taxes the rich at a higher rate than the poor.
- Efficiency: People do not engage in costly behavior simply to avoid paying the tax.

1 Progressive

It turns out the inflation tax is regressive: taxes poor at a higher rate. The poor hold a higher percentage of wealth in cash, whereas the rich evade the inflation tax by using credit rather than holding money. The rich have better access to interest bearing accounts (many overseas).

2 Efficiency

People do evade the inflation tax by holding more wealth in interest bearing accounts and paying δ cost of withdraw more times.

As we have written the model, this is not a horrible problem. So low inflation is generally considered not very costly. But if the monetary system is abandoned and people resort to barter, the costs are much worse. No country has been able to live with hyperinflation for longer than a few years without a major change of government or a major change in the monetary system.

D Budget Deficits and Inflation

Note that long run budget deficits are closely related to inflation:

$$G - T = \text{deficit} = \text{borrowing} + \pi h \quad (58)$$

UNPLEASANT MONETARIST ARITHMETIC: Government deficits financed by inflation are eventually more inflationary than deficits financed with seniorage (Sargent and Wallace).

Suppose the government decides to be ‘good’ and not use the inflation tax, but instead borrow. If the deficit is large the government must pay another deficit plus interest payments next period. Eventually interest payments pile up and the government has no choice but to default or use a large amount of seniorage.

E Laffer Curve

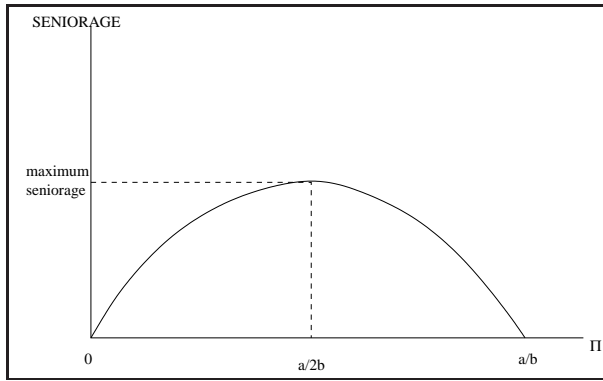
Does the government raise more revenue by increasing the inflation tax? Not necessarily. It could be the case that prices rise by more than the increase in high powered money. In that case: $h_t = \frac{H_t}{P_t}$ falls. So $\pi_t h_t$ may fall.

Suppose we plot seniorage as a function inflation. Clearly zero inflation gives no tax revenue. Further, as the inflation rate gets large people will hold no money. It just loses too much value and bartering is more attractive. Since bartering has finite cost, people hold zero money at some finite inflation rate, where the tax revenue is again zero. Thus an inflation rate which maximizes tax revenue exists.

Consider for example, the Cagan model with $\pi_{t+1}^e = \pi_t$. It follows that:

$$\text{Long Run seniorage: Cagan} = \frac{\pi}{k(R)} (a - b\pi) \quad (59)$$

We have:



Interestingly, Cagan estimated that the revenue maximizing inflation rate was ‘only’ about 20% per month in Germany, whereas the German hyperinflation was as large as 322% per month.