Homework 3: Solutions
Monetary Theory and Policy: ECO 403

Question 1.

Time inconsistency occurs when short and long run goals are at odds. This happens frequently with monetary policy rules: the optimal long run rule is to keep inflation low but in the short run there is always some crises or other that seems to justify higher inflation.

To make monetary policy more time consistent, one can try to change the institutional structure by for example changing the FED's charter so that low inflation is the only goal. Longer terms for the FED govenors is also helpful.

Question 2.

No: one cannot simultaneously raise interest rates and raise the high powered money stock, for example. Once we set an interest rate, we must adjust H so as to keep the interest rate constant. Conversely, by keeping H constant, we cannot affect the interest rate.

Question 3.

a. Reserves are on deposit at the FED and are thus measurable. In addition, banks must report their reserve positions to the FED so that the FED knows the bank is above the minimum required reserves.

b. The FED, by increasing H, lowers R, which induces banks to hold more excess reserves, raising the reserve to deposit ratio. Similarly, the FED can lower the reserve to deposit ratio by decreasing H. Indeed, the equation $rd = rrr + e(R)$ indicates that to get $rd = rd^*$, the FED can simply compute the R that solves $rd^* = rrr + e(R)$. That is:

$$rd^* = rrr + e(R) \tag{1}$$

$$e(R) = rd^* - rrr \tag{2}$$

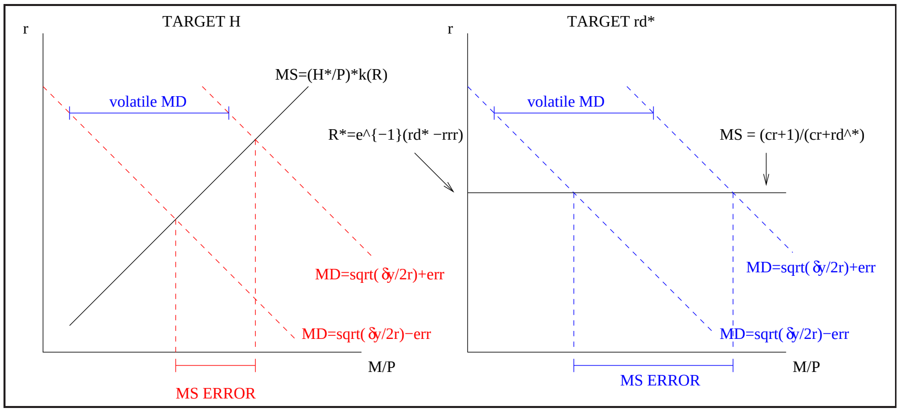
$$R^* = e^{-1}(rd^* - rrr) \tag{3}$$

Here e^{-1} is the inverse of e. Changing rd^* by changing rrr would be possible, but more difficult. Keep in mind that any change in rrr also affects the interest rate. If rrr falls, the interest rate falls as well because banks have more reserves to loan out. That is:

$$rd^* = rrr + e(R(rrr)) \tag{4}$$

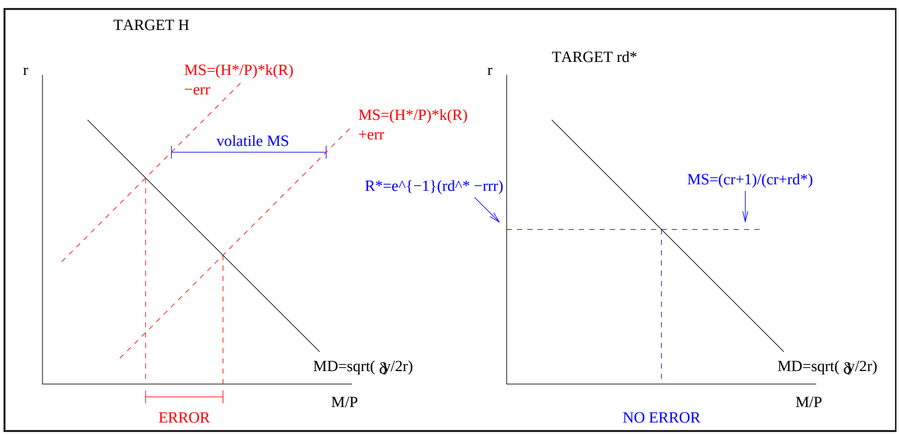
Solving this equation for rrr is more difficult because of the direct and indirect effects of rrr on rd^*. Certainly, though, for rrr high enough banks will hold no excess reserves and rd is easy to control with $rd^* = rrr$.

c. A target variable must be kept constant. To keep rd constant requires R to be kept constant. Specifically, we have shown in part (b) that $R^* = e^{-1}(rd^* - rrr)$. So the graphs are identical to the case where R is targeted. For volatile money demand, we have:



Here H is a better target and has a more predicatable effect on m.

d. Here we have: So rd is a better target. When money suppy drifts upward, banks have



more reserves and so the interest rate for reserve loans, the FED Funds rate, falls. To counter the increase in reserves, the FED reduces H, reducing reserves and increasing the interest rate. Thus, m goes up because of the increase in m, but then falls due to the decrease in H. So m stays constant, reducing volatility.

Question 4.

a. Substituting in the definition of unemployment results in:

$$\min \left\{ \frac{1}{2} \left(2 + \pi_t^e - \pi_t\right)^2 + \pi_t^2 \right\}$$

The FED cares about both inflation and unemployment, but inflation has greater weight ($1 > \frac{1}{2}$). Further, the squared terms indicate the FED prefers an average inflation and unemployment to one being very high and the other low (e.g. the FED prefers $\pi = u = 5\%$ to $\pi = 10$ and $u = 0$).

The rule is set once for the long run so $\pi^e = \pi$. Therefore, we have:

$$\min \left\{ \frac{1}{2} \cdot 2^2 + \pi_t^2 \right\}$$

$$\min \left\{ 2 + \pi_t^2 \right\}$$

Clearly the optimal rule is $\pi_t = 0$. The FED cannot affect unemployment, so the FED might as well keep inflation as low as possible. Mathematically, take the derivative with respect to π and set the derivative equal to zero:

$$0 + 2\pi_t = 0 \;\Rightarrow\; \pi_t = 0$$

The optimal discretion is set for the short run, so that expectations do not necessarily equal inflation. We have:

$$\min \left\{ \frac{1}{2} \left(2 + \pi_t^e - \pi_t\right)^2 + \pi_t^2 \right\}$$

Here we can apply the chain rule: the derivative of the first term equals $\frac{d \text{ first term}}{du} \cdot \frac{du}{d\pi}$:

$$\frac{1}{2} \cdot 2 \cdot \left(2 + \pi_t^e - \pi_t\right)(-1) + 2\pi_t = 0$$

Solving for π gives:

$$\pi_t = \frac{2 + \pi_t^e}{3}$$

b. In the long run, inflationary expectations equal actual inflation, so the $u = NR$ for both rules and discretion. Under the rule, the long run inflation is also zero. For the discretion, substitute $\pi_t = \pi_t^e$ to get:

$$\pi_t = \frac{2 + \pi_t}{3}$$

$$3\pi_t = 2 + \pi_t$$

$\pi_t = 1$

Plugging the results into the objective function results in:

$$\text{obj}_{\text{rule}} = \frac{1}{2}2^2 + 0^2 = 2 \tag{5}$$

$$\text{obj}_{\text{discretion}} = \frac{1}{2}2^2 + 1^2 = 3 \tag{6}$$

The discretionary policy results in long run inflation greater than zero. Thus the rule does equally well at keeping unemployment down and has lower inflation. Thus the rule is better.

c. When expectations rise, the FED raises inflation. Recall, however, that the FED cares more about keeping inflation low than unemployment. So the FED does not fully eliminate unemployment caused by an increase in expectations by setting $\pi_t = \pi_t^e$, as this would result in unacceptably high inflation. Instead, it will only increase inflation by $\frac{1}{3}$ the increase in π_t^e.

Question 5.

a. If a is large, most firms misperceive. In this case, a decrease in inflation would result in most firms attributing low demand as a relative change and responding by reducing output and reducing employment which would cause unemployment to go way up. This is consistent with a flat Phillips curve: small decreases in inflation cause large changes in unemployment. It also makes sense. Since inflation has been low for two decades, it is a waste of time for firms to monitor CNBC daily for changes in FED policy. It saves time to simply set price increases equal to 2%, which is very close to actual inflation for most of the last 20 years.

One could also approach this question mathematically. Recall the Phillips curve and the parameter a are related according to:

$$u_t = NR + ka\left(\pi_t^e - \pi_t\right)$$

However, the Phillips curve actually has π as a function of u (π is on the vertical axis). So:

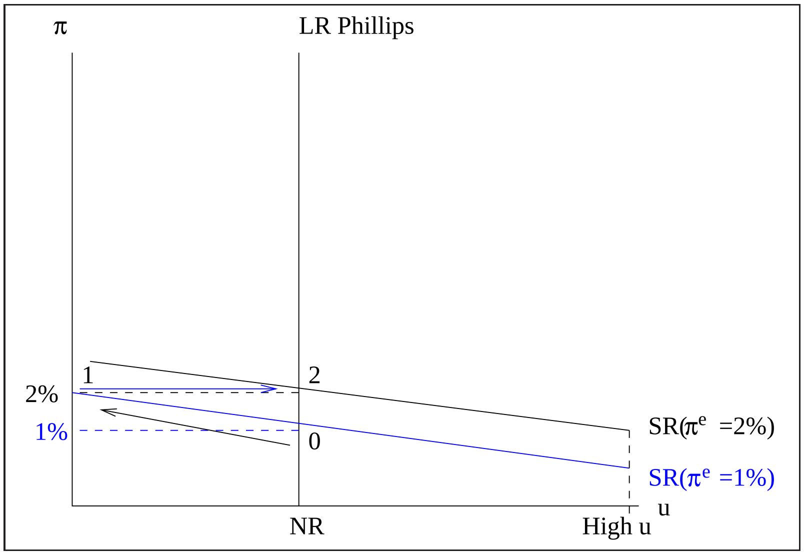
$$\frac{u_t - NR}{ka} = \pi_t^e - \pi_t$$

$$\pi_t = \pi_t^e - \frac{u_t - NR}{ka}$$

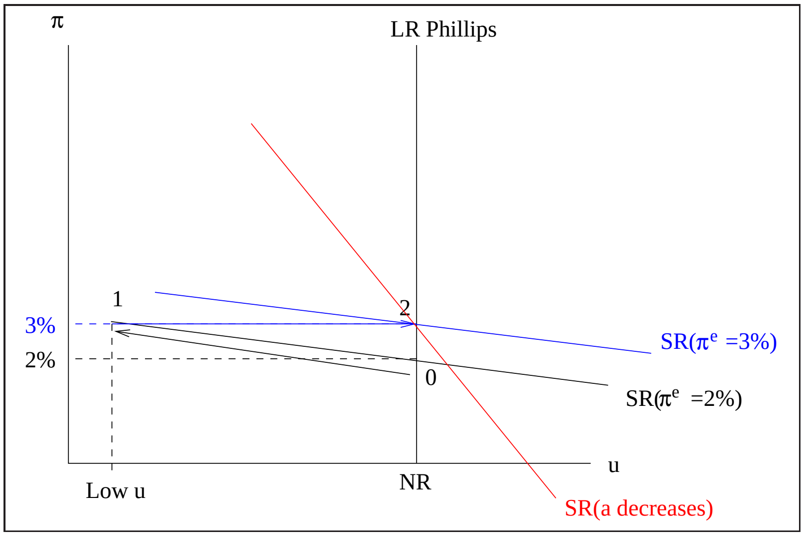
$$\pi_t = \pi_t^e - \frac{u_t}{ka} + \frac{NR}{ka}$$

So we have a linear equation ($y = ax + b$) for π (the y) as a function of u (the x). The slope term is $b = \frac{-1}{ka}$. Hence a large a corresponds to a smaller slope or flatter Phillips curve.

b. The advantage of a flat Phillips curve is that large decreases in unemployment can be brought about with minimal inflation:



c. An overly aggressive response to the credit market problems corresponds to a big decrease in interest rates or a big increase in the money supply. Graphically: As seen



in the graph, the increase in inflation and expectations will be minimal: most firms misperceive and are thus unlikely to increase prices very much. The change in demand will be viewed by most firms as relative and thus result in low unemployment. The credit market could problems could thus be solved with a minimal increase in inflation and expectations, if the Phillips curve is very flat.

The danger is if after the whole ordeal the small increase in inflation causes more firms to pay attention, which in turn makes the Phillips curve more steep (red line).