Homework 2: Solutions
Eco 403, Spring 2012

Question 1

a. We have:

$$\text{seniorage}_t = \frac{H_{t+1} - H_t}{P_t} = \frac{\$600}{\frac{\$224.9}{\text{good}}} = 2.7 \text{ billion goods.} \tag{1}$$

b. Seniorage is the purchasing of goods or assets with printed money. Here the government buys an asset, which is a mortgage. Alternatively, the government receives an asset without having borrow or use tax revenue to pay for it. Seniorage is:

$$\text{seniorage}_t = \frac{H_{t+1} - H_t}{P_t} = \frac{\$600}{\frac{\$216.6}{\text{good}}} = 2.8 \text{ billion goods.} \tag{2}$$

c. We have:

$$\text{seniorage}_t = \frac{H_{t+1} - H_t}{P_t} = \frac{-\$100}{\frac{\$216.6}{\text{good}}} = -0.4 \text{ billion goods.} \tag{3}$$

The government now has to pay back the extra tbills in circulation. Seniorage is negative.

d. There is no change in H, so seniorage is zero. Now, short and long term interest rates are different so it is possible the government may lose or gain on interest costs. However, taxpayers are ultimately responsible for paying the tbills and they attach equal value to both sets of bonds. So there is no gain or loss in interest here.

e. Here the printed money is repaid to the FED + interest overnight. Overall, the FED does not purchase any goods or assets, so seniorage is zero, but the FED does earn some interest on the loan.

Question 2

a. We have the formula:

$$\text{taxes paid } = k\left(R_{t+1}\right) h_{t+1} \pi_t. \tag{4}$$

We have:

$$h_{t+1} = \frac{H_{t+1}}{P_{t+1}} = \frac{700 + 600}{226.7} = 5.73. \tag{5}$$

Notice that high powered money was \$700, then the government printed \$600 more, so the total in $t + 1$ is \$1300. Further,

$$\pi_t = \frac{P_{t+1} - P_t}{P_t} = \frac{226.7 - 224.9}{224.9} = 0.008. \tag{6}$$

A near doubling of high powered money caused less than 1% inflation! Finally:

$$\text{taxes paid } = k\left(R_{t+1}\right) h_{t+1} \pi_t = 0.82 \cdot 5.73 \cdot 0.008 = 0.038 \tag{7}$$

Almost no inflation taxes were paid, due to the low money multiplier and low inflation rate.

b. (a) Progressive. The inflation tax is a regressive tax since the poor make a higher percentage of transactions in cash. So the inflation tax is still a bad tax on this dimension.

(b) Efficiency in changing behavior. Inflation causes households to make too many withdraws. However, since inflation is only 0.8%, little change in behavior results (all taxes cause changes in behavior, a good tax causes only small or low cost changes in behavior). So the inflation tax is a good tax on this dimension as well.

(c) Efficiency of collection. Usually, the money multiplier is greater than one and more taxes are paid than are collected, as the banks create checking accounts which add to the money supply but do not result in seniorage. Here, however, the money multiplier is less than one, so the inflation tax is a good tax right now.

Question 3

Combining the money demand and inflation expectations equations results in:

$$MD = 20 - 2 \cdot \frac{1}{4}\pi_t = 20 - \frac{1}{2}\pi_t \tag{8}$$

Now plugging the above equation into the inflation response equation results in:

$$\pi_{t+1} = 24 - \left(20 - \frac{1}{2}\pi_t\right) \tag{9}$$

$$\pi_{t+1} = 4 + \frac{1}{2}\pi_t \tag{10}$$

Thus inflation in period one is:

$$\pi_1 = 4 + \frac{1}{2}\pi_0 = 4 + \frac{1}{2} \cdot 16 = 12 \tag{11}$$

Similarly, inflation in period two is $\pi_2 = 4 + \frac{1}{2}\pi_1 = 10$, etc. Using the expectations equation, expectations in period 1 is:

$$\pi_1^e = \frac{1}{4}\pi_0 = \frac{16}{4} = 4. \tag{12}$$

Similarly, $\pi_2^e = \frac{1}{4}\pi_1 = \frac{12}{4} = 3$, etc. Money demand is obtained from equation (8):

$$MD_0 = 20 - \frac{1}{2} \cdot \pi_0 = 20 - \frac{1}{2} \cdot 16 = 12 \tag{13}$$

Similarly, $MD_1 = 20 - \frac{1}{2} \cdot 12 = 14$, etc. The money multiplier is $k = 2$ so $m = 2h$ or $h = \frac{1}{2}m$. So $h_0 = \frac{1}{2}m_0 = \frac{1}{2}12 = 6$, etc. For the seniorage, a formula in the notes that works is

$$\text{seniorage } = h_{t+1}\left(1 + \pi_t\right) - h_t \tag{14}$$

Thus:

$$\text{seniorage}_0 \ = h_1\left(1 + \pi_0\right) - h_0 = 7\left(1 + .16\right) - 6 = 2.12 \tag{15}$$

$$\text{seniorage}_0 \ = h_1\left(1 + \pi_0\right) - h_0 = 7\left(1 + 16\right) - 6 = 113 \tag{16}$$

Similarly, $\text{seniorage}_1 = 7.5 \cdot (1 + 0.12) - 7 = 1.4$, etc. Overall,

period	inflation	expectations	MD	h	seniorage revenue	using 16
0	16	16	12	6	2.12	113
1	12	4	14	7	1.40	90
2	10	3	15	7.5	1.03	70
3	9	2.5	15.5	7.75	na	na

Note that seniorage revenues are falling as inflation falls.

Question 4

a. The government in the first period does not borrow, and therefore must print \$2 trillion to cover the deficit. This is high powered money. Therefore:

$$M = k\,(R)\,H \quad \rightarrow \quad M_1 = 2 \cdot 2 = 4. \tag{17}$$

Further using the equation given in the problem:

$$\pi_0 = \frac{M_1 - M_0}{M_0} = \frac{5-1}{1} = 4. \tag{18}$$

The next period, the government starts again with no debt and must print another \$2 trillion to cover the deficit. Therefore, we have:

year	debt	H	M	π
0	0	0.5	1	4
1	0	2.5	5	4/5
2	0	4.5	9	4/9
3	0	6.5	13	4/13
4	0	8.5	17	na

Notice that the inflation rate is declining. This is an artifact of the problem since the deficit is not really constant. The price level is going up and the NOMINAL deficit is a constant \$2 trillion, so in fact the deficit buys less goods each period and is smaller in real terms.

b. Here we print no money, but instead borrow the whole amount. Therefore H and M do not change, and inflation is zero. In the first period we borrow \$2, so in the second period the government must borrow enough to refinance the principle plus interest, plus the new deficit:

$$\text{debt}_1 = 2. \tag{19}$$

$$\text{debt}_2 = 2 \cdot 1.1 + 2 = 4.2. \tag{20}$$

$$\text{debt}_3 = 4.2 \cdot 1.1 + 2 = 6.6. \tag{21}$$

Therefore:

year	debt	H	M	π
0	0	0.5	1	0
1	2	0.5	1	0
2	4.2	0.5	1	0
3	6.6	0.5	1	0
4	9.3	0.5	1	na

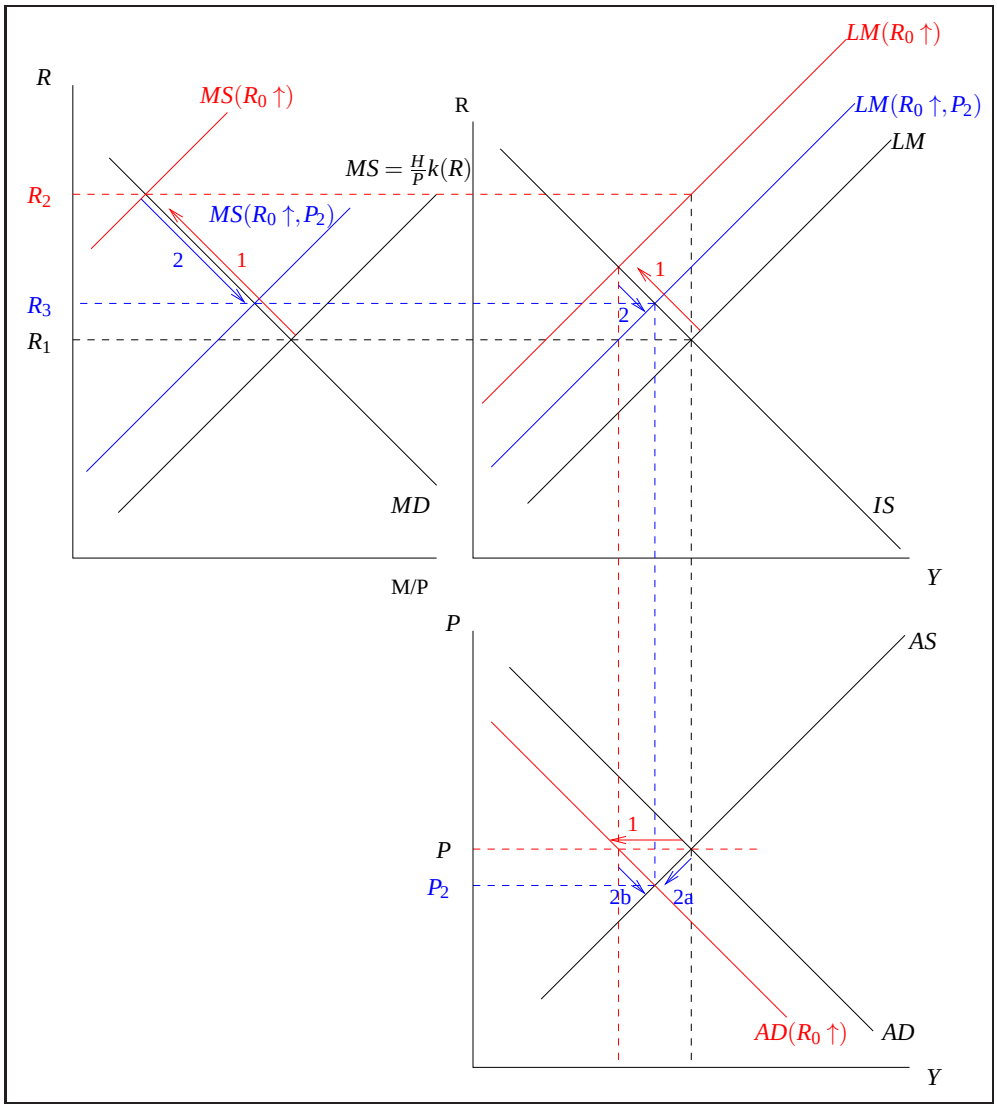
c. Yes. The debt is growing in part (b) at an unsustainable rate and the government will quickly lose the ability to borrow since investors will know they will never be repaid. At that point, seniorage becomes the only option. From the table, if the government used seniorage in period 4, we would have $H_4 = 9.3$ and so $M_4 = 18.6$ and inflation would be:

$$\pi_4 = \frac{18.6 - 1}{1} = 17.6. \tag{22}$$

This is more inflation than all periods combined in part (a), since the government also prints money to pay for the interest on the debt, as well as all the accumulated deficits.

Question 5

Here is the graph.



- The FED pays more interest on reserves, and so banks increase excess reserves, which means less money is circulating, and so loans and checking deposits fall. Thus the money supply falls, which is a shift to the left of the money supply curve ('1' on the graph).
- Money supply is now less than money demand which increases the interest rate.
- The higher interest rate increases the opportunity cost of holding money, and so the number of withdraws increases and money demand falls. On the supply side, the higher interest rate gives banks an incentive to reduce excess reserves and increase lending, which increases deposits.

- The interest rate is higher, but we have not yet changed spending. Thus the LM curve must shift to the left.
- Higher interest rates means less demand for new houses, factories, and equipment. Investment spending falls. Since investment spending is part of total spending, total spending falls as well (1 on the IS-LM graph).
- Prices have not yet changed, but total spending or aggregate demand is lower. So the aggregate demand shifts left (1 on the AD-AS graph).
- Aggregate demand is less than aggregate supply so prices fall. A decrease in prices raises real wages since nominal wages are fixed in the labor market. Firms respond by hiring less workers and causing existing workers to work less. Hours worked and therefore production, or aggregate supply, falls (2a).
- Lower prices means the existing money supply buys more goods. Real money supply is now higher than money demand, so the interest rate falls. Lower interest rates means banks have an incentive to keep more excess reserves and decrease lending, which decreases deposits. The opportunity cost of holding money decreases for households, who decrease the number of withdraws, which increases money demand (2 in the money market).
- Lower interest rates also increases investment spending and total spending (2 on the IS-LM graph and 2a on the AD-AS graph).

Overall, from the graphs:

a. R rises.

b. M/P falls.

c. P falls.

d. Y falls.

Overall interest rates are up. In the explanation, these effects depended only on R:

e. n is up.

f. I is down.

Overall the price level is down. These effects depended only on the price level:

g. real wages rise.

h. labor demand and therefore hours worked falls.

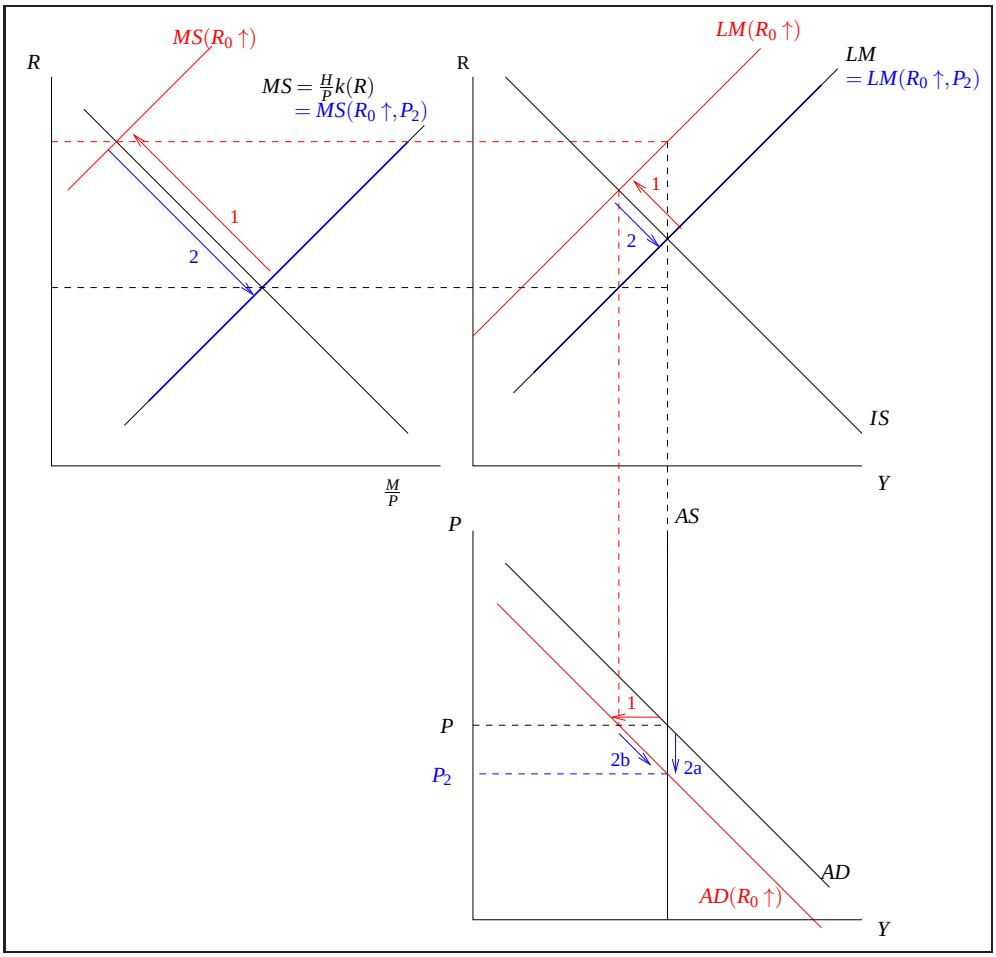
Finally we have:

i. Excess reserves initially rose since R_0 rose, then we had 2 interest rate effects. The interest rate is overall up so this effect causes ER to fall. We know, however, that overall the money supply is down and so there must be more dollars in excess reserves and less dollars circulating. Therefore ER is overall up.

j. Similarly, less dollars are circulating so overall deposits and lending are down.

Question 6

Here is the graph:



- The FED pays more interest on reserves, and so banks increase excess reserves, which means that high powered money is pulled out of the economy and put into bank reserves. Thus the money supply falls, which is a shift to the left of the money supply curve ('1' on the graph).
- Money supply is now less than money demand which increases the interest rate.
- The interest rate is higher, but we have not yet changed spending. Thus the LM curve must shift to the left.
- Higher interest rates means less demand for new houses, factories, and equipment. Investment spending falls. Since investment spending is part of total spending, total spending falls as well (1 on the IS-LM graph).
- Prices have not yet changed, but total spending or aggregate demand is lower. So the aggregate demand shifts left (1 on the AD-AS graph).
- Prices have not yet changed, but total spending or aggregate demand is lower. So the aggregate demand shifts left (1 on the AD-AS graph).
- Aggregate demand is less than aggregate supply so prices fall. A decrease in prices has no effect on hours worked in the labor market, since the nominal wage adjusts. Thus hours and aggregate supply are unchanged (2a).
- Lower prices means the existing money supply buys more goods. The increase in real money supply lowers interest rates to their original level, which increases investment spending and total spending to their original levels (2b and the blue arrows).

From the graphs:

a. R, M/P, and Y are all unchanged.

b. Overall interest rates are unchanged so n and I are unchanged.

c. Overall P is down but the real wage is unaffected by P in the classical model. Therefore, n^* is also unchanged.

d. Excess reserves initially went up and then we have 2 interest rate effects which exactly offset since R is unchanged. Therefore ER is up. The price level is lower so the real value of ER is also up. Deposits and lending fell initially and then we had 2 interest rate effects which offset so the nominal deposits and lending are down. However, the remaining deposits and lending buy more goods since the price level is down. Overall, since the real money supply is unchanged, the real deposits and lending are also unchanged.

Question 7

a. At the maximum seniorage, the slope or derviative of the seniorage equation is zero. The derivative is:

$$\text{seniorage } = \alpha + \beta\pi + \gamma\pi^2 \tag{23}$$

$$\frac{d\text{seniorage}}{d\pi} = 0 + \beta + 2\gamma\pi \tag{24}$$

Now we set the derivative equal to zero:

$$\beta + 2\gamma\pi = 0 \tag{25}$$

From the above equation, only the value of π which solves this equation will make the slope equal to zero. Thus:

$$2\gamma\pi^* = -\beta \tag{26}$$

$$\pi^* = \frac{-\beta}{2\gamma} \tag{27}$$

If the inflation rate is equal to π^*, seniorage will be as high as possible. Beyond this inflation rate, prices rise too quickly. Printing money causes prices to go up so much that the government can actually buy less with the printed money than if they had printed a smaller amount.

b. If $\alpha = 0$, equation (23) implies:

$$\text{seniorage } = \beta\pi + \gamma\pi^2 = \pi\left(\beta + \gamma\pi\right). \tag{28}$$

Notice that $\pi = 0$ generates zero seniorage since no money is being printed. But we can also have zero seniorage revenues if the economy becomes a barter economy. Then since money is not accepted as payment, the government cannot buy anything regardless of how much money is printed. This occurs when:

$$\text{seniorage } = 0 = \pi\left(\beta + \gamma\pi\right). \tag{29}$$

$$\pi = \frac{\beta}{-\gamma}. \tag{30}$$

c. For all countries we have:

$$\pi^* = \frac{-\beta}{2\gamma} = \frac{-9.563}{-2 \cdot 4.691} = 1.02 \tag{31}$$

That is seniorage is maximized at an inflation rate of 102%.

d. The country has an inflation rate of 1.1 (110%) which is higher than 1.02. So the country can increase welfare by decreasing inflation (i.e. by printing less money). That way inflation goes down, benefiting holders of money and seniorage revenues rise, benefiting the government.