

Homework 2: Eco 403, Spring 2009
Solutions

Question 1

- a. The primary cause of hyperinflation in Austria was printing money to use to pay war reparations.
- b. Money supply was still growing, so it must be that money demand rose, due to a fall in inflationary expectations. Allied countries promised to end the printing of money. Because the threat was credible, expectations fell and money demand rose.
- c. Let the three periods be denoted t , $t + 1$, and $t + 2$. The money multiplier is one so $M_t = H_t$, and so since money growth was constant in the first two periods and zero in the last period:

$$H_{t+1} - H_t = H_{t+2} - H_{t+1} > H_{t+3} - H_{t+2} = 0. \quad (1)$$

Hence:

$$\text{seigniorage}_t = \frac{H_{t+1} - H_t}{P_t}, \quad (2)$$

$$\text{seigniorage}_{t+1} = \frac{H_{t+2} - H_{t+1}}{P_{t+1}} = \frac{H_{t+1} - H_t}{P_{t+1}}, \quad (3)$$

$$\text{seigniorage}_{t+2} = \frac{H_{t+3} - H_{t+2}}{P_{t+2}} = 0. \quad (4)$$

Finally, inflation was negative in the second period so $P_{t+1} < P_t$. Thus we have:

$$\text{seigniorage}_{t+1} > \text{seigniorage}_t > \text{seigniorage}_{t+2}. \quad (5)$$

Question 2

- a. We have:

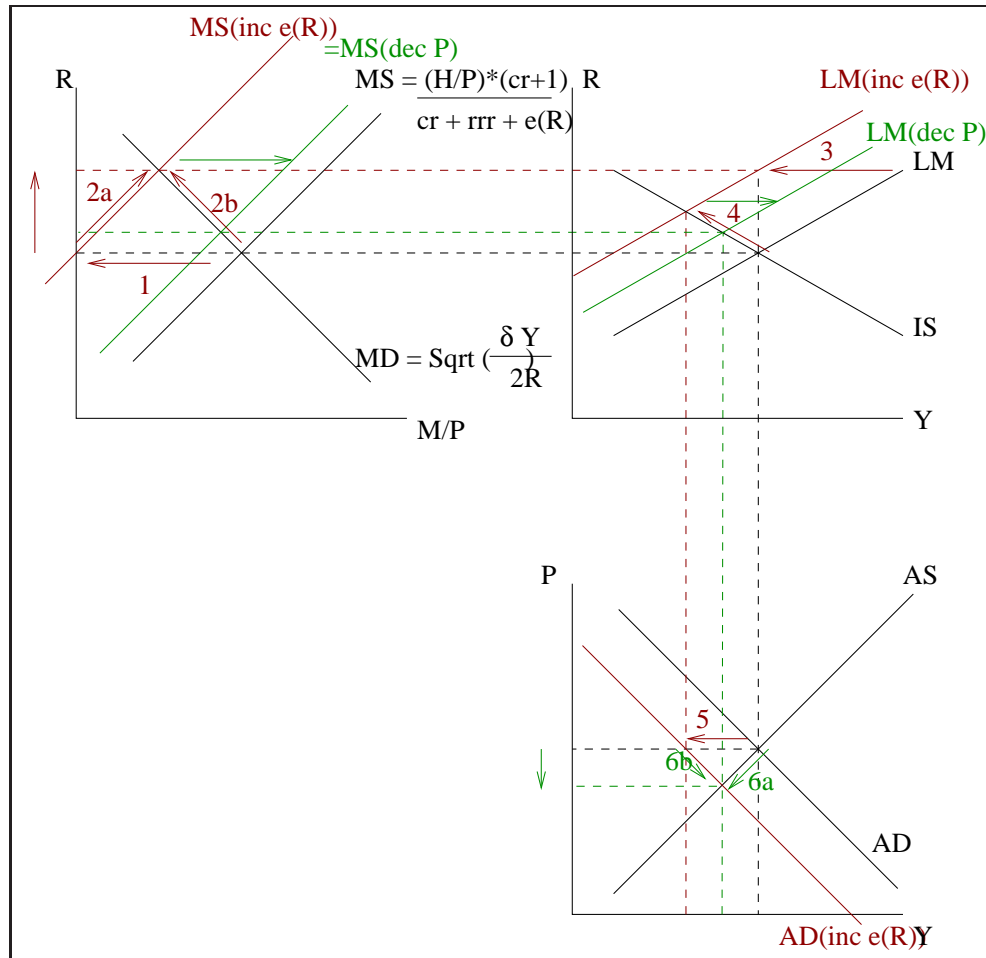
$$\text{seigniorage}_t = \frac{H_{t+1} - H_t}{P_t} = \frac{\$800}{\frac{\$50}{\text{good}}} = 16 \text{ goods}. \quad (6)$$

- b. A TAF auction is a loan. The FED loans printed money and accepts bills as collateral. However, in a short period of time (usually one month or less), the bank repays the

loan (printed money) to the FED and gets back the collateral. Thus there are no seigniorage revenues here. However, the FED does earn interest on the TAF loan which is revenue for the FED.

Question 3

a. Here is the graph.

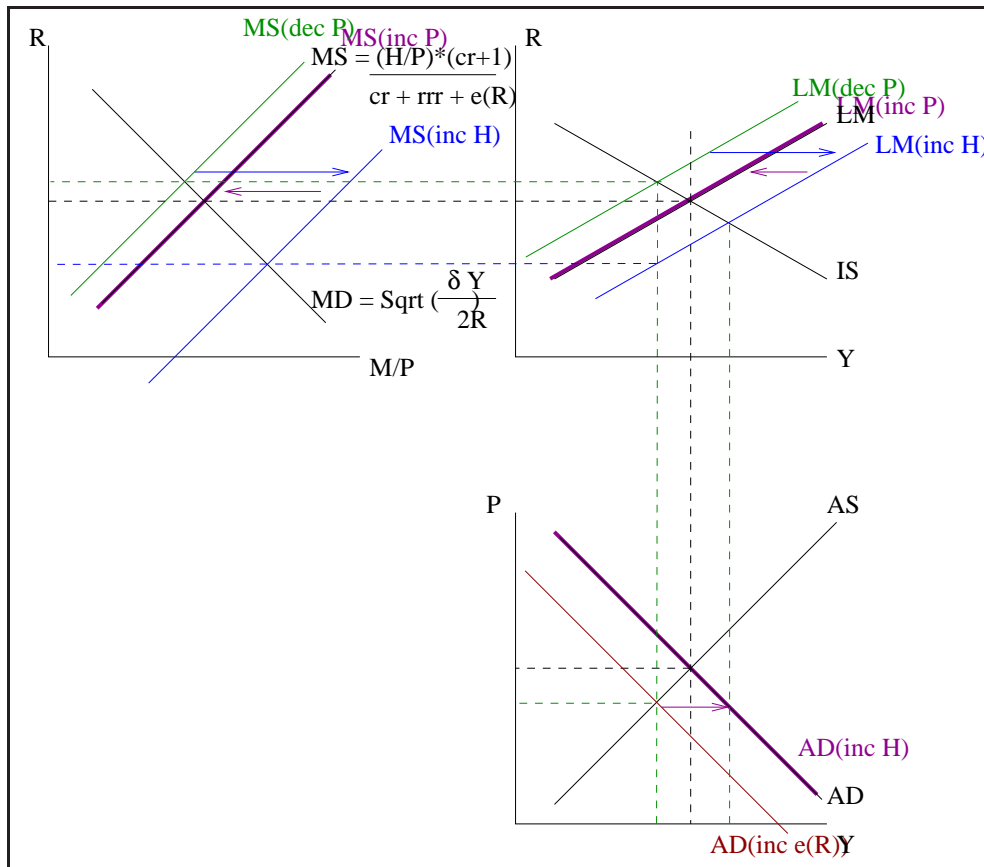


- Initially, we have an increase in excess reserves, which means that high powered money is pulled out of the economy and put into bank reserves. Thus the money supply falls, which is a shift to the left of the money supply curve ('1' on the graph).
- Money supply is now less than money demand which increases the interest rate. In turn, the increase in interest rates increases the opportunity cost of holding excess reserves (banks can loan excess reserves for higher interest rates) which causes excess reserves to fall and the money supply to rise (2a), although overall the money supply is still down.

- The increase in interest rates increases the opportunity cost (interest that could be earned in the interest bearing account) of holding money for households. Households keep money in the interest bearing account longer by making more withdraws, decreasing money demand (2b).
- The interest rate is higher, but we have not yet changed spending. Thus the LM curve must shift to the left (3).
- Higher interest rates means less demand for new houses, factories, and equipment. Investment spending falls. Since investment spending is part of total spending, total spending falls as well (4).
- Prices have not yet changed, but total spending or aggregate demand is lower. So the aggregate demand shifts left (5).
- Aggregate demand is less than aggregate supply so prices fall. A decrease in prices raises real wages since nominal wages are fixed in the labor market. Firms respond by hiring less workers and causing existing workers to work less. Hours worked and therefore production, or aggregate supply, falls (6a).
- Lower prices means the existing money supply buys more goods. The increase in real money supply lowers interest rates, which increases investment spending and total spending (6a and the green arrows), but not enough to offset the initial decrease in spending.

From the graphs and explanations, overall interest rates rise, equilibrium money supply which equals money demand which equals money holdings fall, withdraws rise, investment spending falls, and output falls. Prices fall. The decrease in excess reserves from the increase in interest rates is not enough to offset the initial rise in excess reserves, since overall money supply is down. Thus excess reserves are up overall.

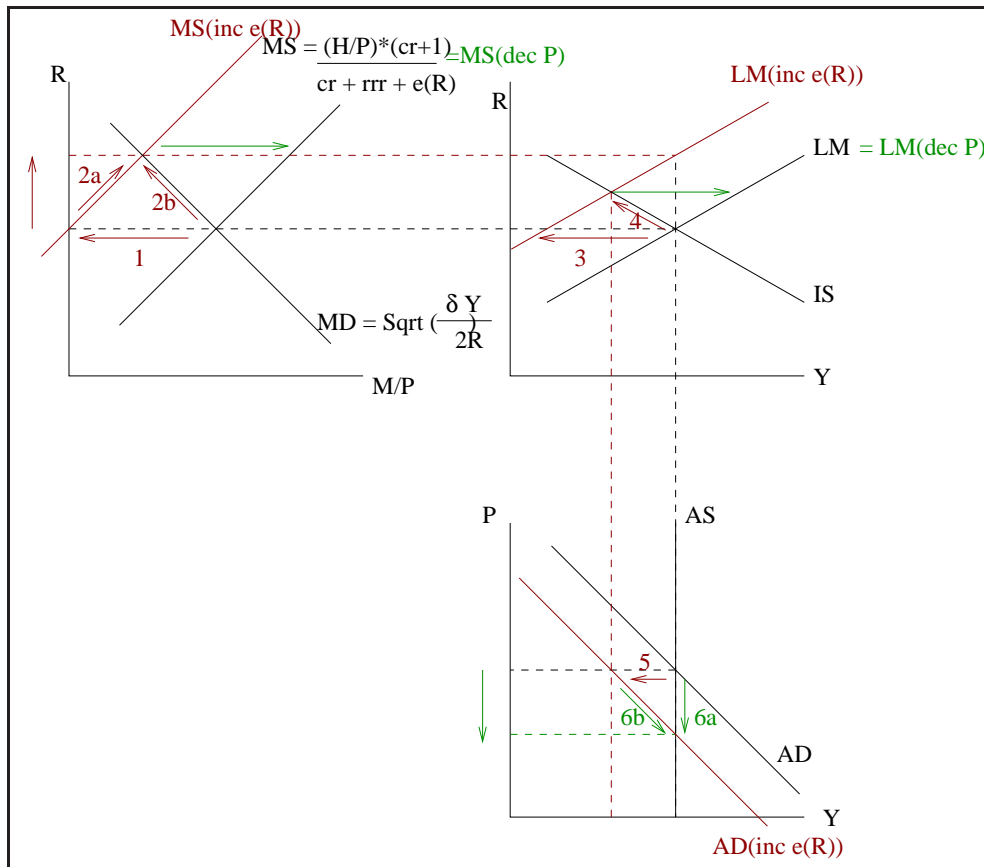
- The two policy variables, or parameters under control of the FED, are high powered money and the required reserve ratio. So the two possible answers are to increase high powered money or decrease the required reserve ratio.
- Here is the graph (I have deleted most of the intermediate red shifts for clarity):



- d. The money supply fell because banks held more excess reserves, which the FED offset by increasing the money supply by increasing high powered money. Since the money supply ends up unchanged, the FED can completely offset any economic effects of the increase in excess reserves.

Question 4

Here is the graph:



- Initially, we have an increase in excess reserves, which means that high powered money is pulled out of the economy and put into bank reserves. Thus the money supply falls, which is a shift to the left of the money supply curve ('1' on the graph).
- Money supply is now less than money demand which increases the interest rate. In turn, the increase in interest rates increases the opportunity cost of holding excess reserves (banks can loan excess reservers for higher interest rates) which causes excess reserves to fall and the money supply to rise (2a), although overall the money supply is still down.
- The increase in interest rates increases the opportunity cost (interest that could be earned in the interest bearing account) of holding money for households. Households keep money in the interest bearing account longer by making more withdraws, decreasing money demand (2b).
- The interest rate is higher, but we have not yet changed spending. Thus the LM curve must shift to the left (3).
- Higher interest rates means less demand for new houses, factories, and equipment. Investment spending falls. Since investment spending is part of total spending, total spending falls as well (4).

- Prices have not yet changed, but total spending or aggregate demand is lower. So the aggregate demand shifts left (5).
- Aggregate demand is less than aggregate supply so prices fall. A decrease in prices has no effect on hours worked in the labor market, since the nominal wage adjusts. Thus hours and aggregate supply are unchanged (6a).
- Lower prices means the existing money supply buys more goods. The increase in real money supply lowers interest rates to their original level, which increases investment spending and total spending to their original levels (6a and the green arrows).

From the graphs and explanations, overall interest rates, equilibrium money supply which equals money demand which equals money holdings, withdraws, investment spending, output are all unchanged. Prices fall. Interest rates have returned to their original level, so there is no overall effect on excess reserves due to changes in interest rates. However, excess reserves rose at the beginning for reasons other than the change in rates. Thus excess reserves rose overall.

Question 5

- a. At the maximum seigniorage, the slope or derivative of the seigniorage equation is zero. The derivative is:

$$\text{seigniorage} = \alpha + \beta\pi + \gamma\pi^2 \quad (7)$$

$$\frac{d\text{seigniorage}}{d\pi} = 0 + \beta + 2\gamma\pi \quad (8)$$

Now we set the derivative equal to zero:

$$\beta + 2\gamma\pi = 0 \quad (9)$$

From the above equation, only the value of π which solves this equation will make the slope equal to zero. Thus:

$$2\gamma\pi^* = -\beta \quad (10)$$

$$\pi^* = \frac{-\beta}{2\gamma} \quad (11)$$

If the inflation rate is equal to π^* , seigniorage will be as high as possible. Beyond this inflation rate, prices rise too quickly. Printing money causes prices to go up so much that the government can actually buy less with the printed money than if they had printed a smaller amount.

b. If $\alpha = 0$, equation (7) implies:

$$\text{seigniorage} = \beta\pi + \gamma\pi^2 = \pi(\beta + \gamma\pi). \quad (12)$$

Notice that $\pi = 0$ generates zero seigniorage since no money is being printed. But we can also have zero seigniorage revenues if the economy becomes a barter economy. Then since money is not accepted as payment, the government cannot buy anything regardless of how much money is printed. This occurs when:

$$\text{seigniorage} = 0 = \pi(\beta + \gamma\pi). \quad (13)$$

$$\pi = \frac{\beta}{-\gamma}. \quad (14)$$

c. For all countries we have:

$$\pi^* = \frac{-\beta}{2\gamma} = \frac{-9.563}{-2 \cdot 4.691} = 1.02 \quad (15)$$

That is seigniorage is maximized at an inflation rate of 102%. For only high inflation countries:

$$\pi^* = \frac{-9.775}{-2 \cdot 1.586} = 3.08 \quad (16)$$

d. Any country with an inflation rate higher than 1.02 can increase welfare by decreasing inflation (i.e. by printing less money). That way inflation goes down, benefiting holders of money and seigniorage revenues rise, benefiting the government. The country on the graph has an inflation rate higher than 1.02, and so should reduce its money supply growth.