Homework 3: Solutions
Environmental Economics: ECO 345

Question 1

The best option would be to introduce a Pigouvian tax on total BOD levels equal to the marginal damage and charge the tax to both firms, regardless of which firm emitted.

Question 2

Marginal damages are flat if marginal costs are low, favoring taxes. Small errors in emissions cause little extra damages while small changes in the price of permits results in relatively large costs to firms. Conversely, if marginal costs are low marginal damages are steep, favoring permits. Since permits fixes emissions and therefore damages, while with taxes there is a risk that too many firms pay the tax, resulting in large emissions and therefore large damages.

Since taxes are favored in one case and permits in the other, a hybrid system is best. A price floor fixes the price in the low case, like taxes, and fixes the quantity in the high case, like permits.

Question 3

a. The government should tighten the standard (raise the MPG standard).

b. The goverment should weaken the standard.

c. To avoid (a), (the ratchet effect) firms may wish to keep the MPG low and not receive subsidies, so as not to reveal to the government that the cost of compliance is low.

Question 4

a. We have:

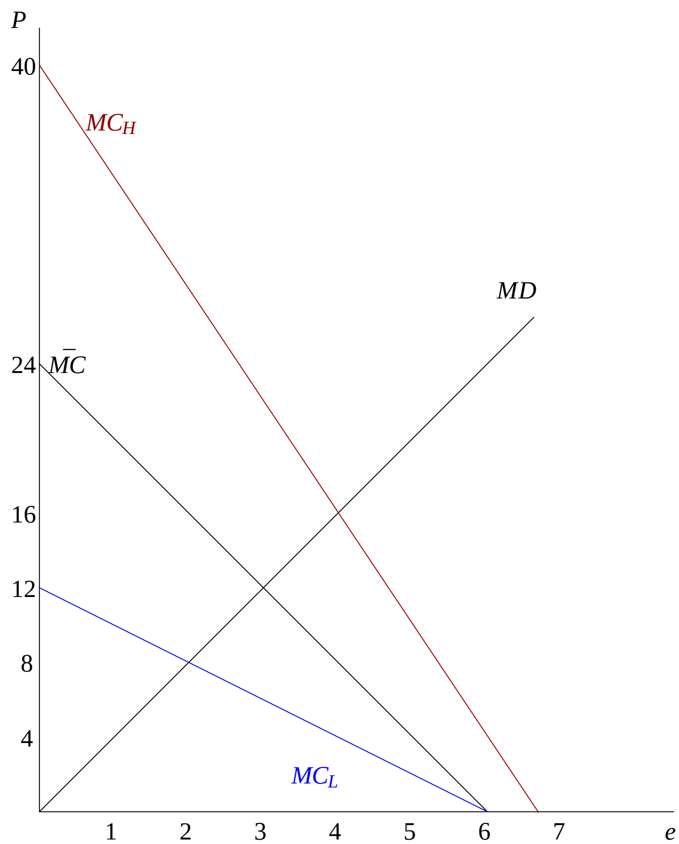

Figure 1: High marginal cost case is steep.

b. For the tax, we have:

$$MD = 4e = \bar{MC} = 24 - 4e, \tag{1}$$

$$8e = 24 \quad \rightarrow \quad e = 3. \tag{2}$$

$$t = MD = 4e = 4 \cdot 3 = 12. \tag{3}$$

Now if marginal costs are low, we have:

$$t = MC_L = 12 = 12 - 2e, \tag{4}$$

$$e_L = 0. \tag{5}$$

$$MD_L = 4e_L = 0. \tag{6}$$

Conversely, if marginal costs are high, we have:

$$t = MC_H = 12 = 40 - 6e, \tag{7}$$

$$6e_H = 28 \quad \rightarrow \quad e_H = \frac{14}{3} = 4\frac{2}{3}. \tag{8}$$

$$MD_H = 4e_H = \frac{56}{3} = 18\frac{2}{3}. \tag{9}$$

c. For the welfare loss, we need to know how close the tax policies e_L and e_H come to e_L^* and e_H^*, which are the efficient emissions. We have:

$$MD = 4e = MC_L = 12 - 2e, \tag{10}$$

$$6e = 12, \quad \rightarrow \quad e_L^* = 2. \tag{11}$$

$$MD = 4e = MC_H = 40 - 6e, \tag{12}$$

$$10e = 40, \quad \rightarrow \quad e_H^* = 4. \tag{13}$$

So the welfare loss in the low and high cases are:

$$\text{loss}_L = \frac{1}{2}(2-0)(12-0) = 12. \tag{14}$$

$$\text{loss}_H = \frac{1}{2}\left(4\frac{2}{3} - 4\right)\left(18\frac{2}{3} - 12\right) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{20}{3} = \frac{20}{9}. \tag{15}$$

To get the average loss, we know we are in the low case with probability one half:

$$\text{ave loss} = \frac{1}{2}\ \text{loss}_L + \frac{1}{2}\ \text{loss}_H = \frac{1}{2} \cdot 12 + \frac{1}{2}\frac{20}{9} = 7\frac{1}{9}. \tag{16}$$

Graphically:

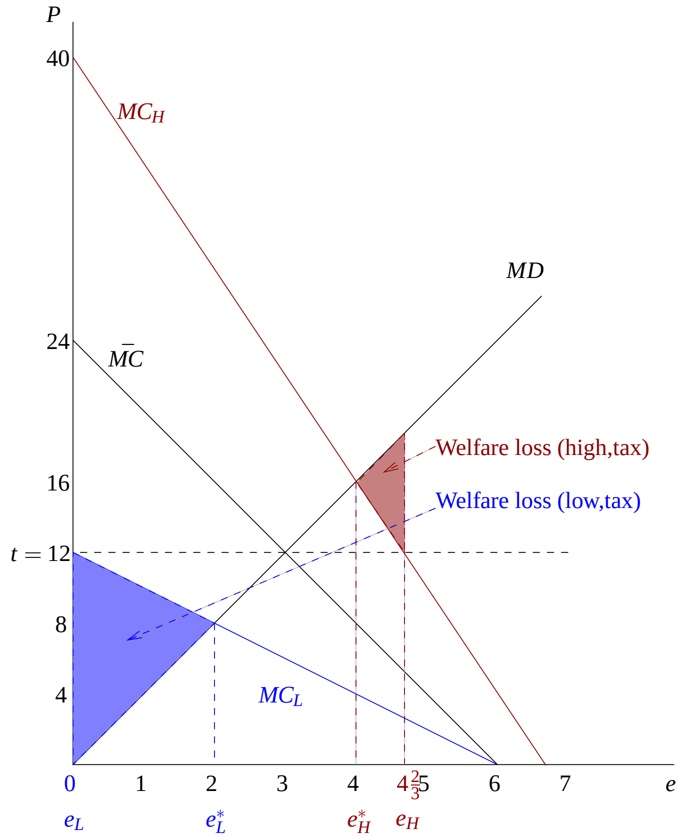

Figure 2: Welfare losses: taxes.

d. For permits:

$$MD = 4e = \bar{MC} = 24 - 4e, \tag{17}$$

$$8e = 24 \quad \rightarrow \quad e = 3. \tag{18}$$

So we issue 3 permits. Now if marginal costs are low, we have:

$$MC_L = 12 - 2e = 12 - 2 \cdot 3 = 6, \tag{19}$$

$$MD = 4e = 12. \tag{20}$$

Conversely, if marginal costs are high, we have:

$$MC_H = 40 - 6e = 40 - 6 \cdot 3 = 22, \tag{21}$$

e. For the welfare loss, we need to know how close the permit policy $e = 3$ comes to e_L^* and e_H^*, which are the efficient emissions. The welfare loss in the low and high cases are:

$$\text{loss}_L = \frac{1}{2}(3-2)(12-6) = 3. \tag{22}$$

$$\text{loss}_H = \frac{1}{2}(4-3)(22-12) = 5. \tag{23}$$

To get the average loss, we know we are in the low case with probability one half:

$$\text{ave loss} = \frac{1}{2}\,\text{loss}_L + \frac{1}{2}\,\text{loss}_H = \frac{1}{2}\cdot 5 + \frac{1}{2}3 = 4. \tag{24}$$

Graphically:

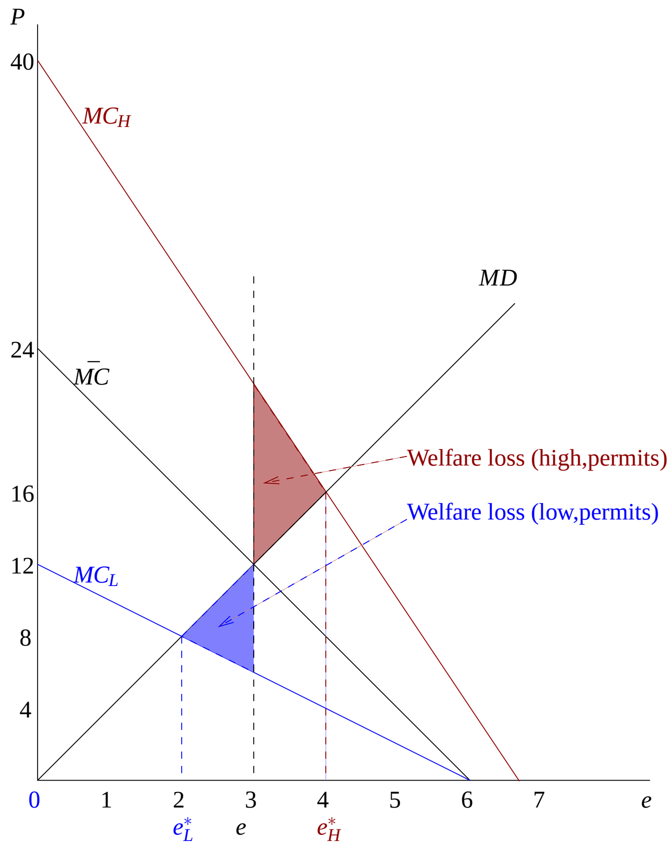

Figure 3: Welfare losses: permits.

f. Overall, permits are better since $4 < 7\frac{1}{9}$. The result does depend on which case we are in, however. Permits are better in the low case ($3 < 12$), since MC_L is flatter than MD. Taxes are better in the high case ($\frac{20}{9} < 5$) since MC_H is steeper than MD.

g. For a safety valve, we issue initially 3 permits, just like in the permit case. For the price of additional permits, we use $MC_H(e_H^*)$:

$$p_H = MC_H(e_H^*) = 40 - 6 \cdot 4 = 16. \tag{25}$$

Since we have initially 3 permits and if we have MC_H the safety valve results in $e_H^* = 4$, we know that firms buy 1 extra permit at the safety valve price. Welfare loss is the same as permits for the low case and zero for the high case so:

$$\text{ave loss} = \frac{1}{2}\,\text{loss}_L + \frac{1}{2}\,\text{loss}_H = \frac{1}{2} \cdot 3 + \frac{1}{2} 0 = \frac{3}{2}. \tag{26}$$

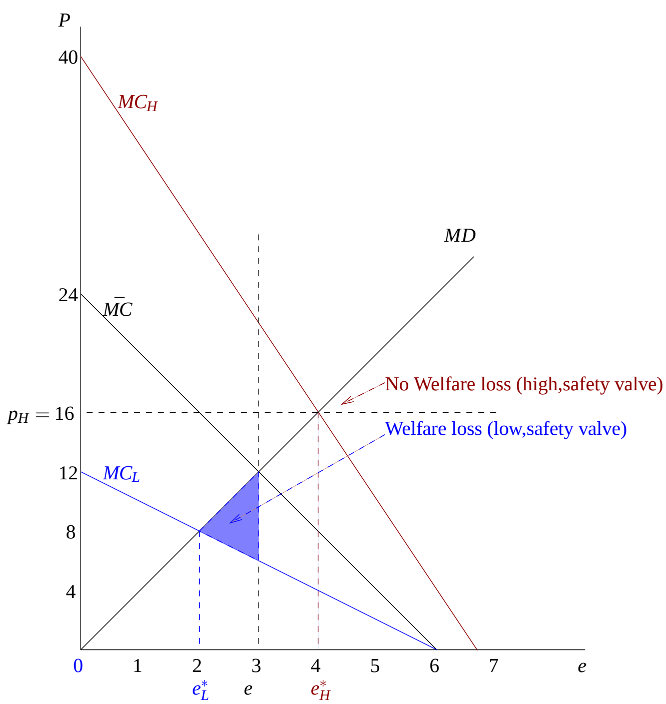

Figure 4: Welfare losses: Safety Valve.

h. For a price floor equal to 8, we again initially issue 3 permits. Then we have in the low case:

$$8 = p_L = 12 - 2e, \tag{27}$$

$$e_L = 2. \tag{28}$$

So the price floor results in emissions of 2, which is the efficient emissions. Since we intially issued 3 permits and end up with 2, we can infer that firms sold 1 permit to the government. The welfare loss is zero in the low case, and equal to the permit loss in the high case:

$$\text{ave loss} = \frac{1}{2}\,\text{loss}_L + \frac{1}{2}\,\text{loss}_H = \frac{1}{2}\cdot 0 + \frac{1}{2}5 = \frac{5}{2}. \tag{29}$$

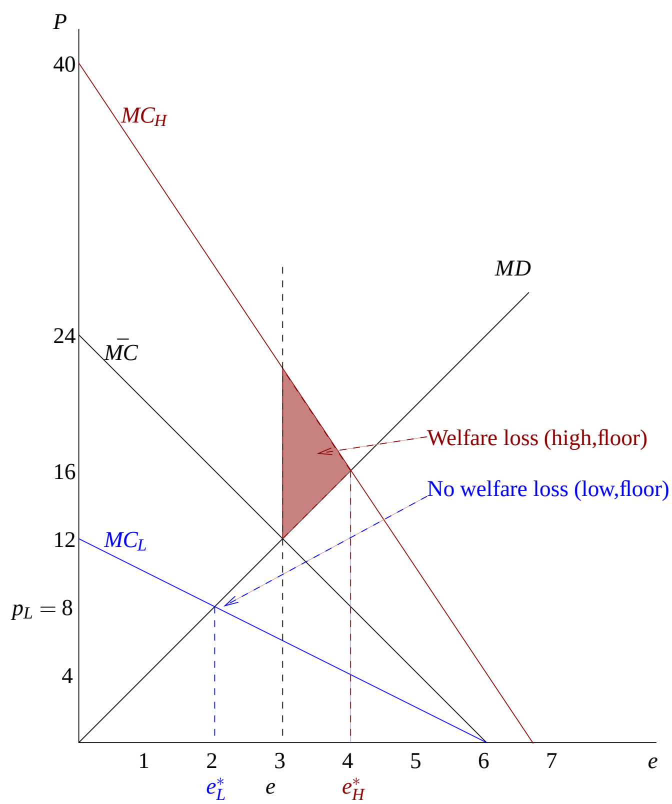

Figure 5: Welfare losses: Price Floor.

i. For the hybrid tax/subsidy, we set the subsidy price to get e_L^*:

$$s = MC_L\left(e_L^*\right) = 12 - 2\cdot 2 = 8. \tag{30}$$

We set the tax to get e_H^*:

$$t = MC_H\left(e_H^*\right) = 40 - 6\cdot 4 = 16. \tag{31}$$

Welfare loss is zero. In the high case, firms reduce until they are indifferent between paying the tax and reducing, which occurs at the efficient emissions. In the low case firms reduce until they are indifferent between reducing and getting subsidies, and doing nothing which occurs at the efficient emissions. Graphically:

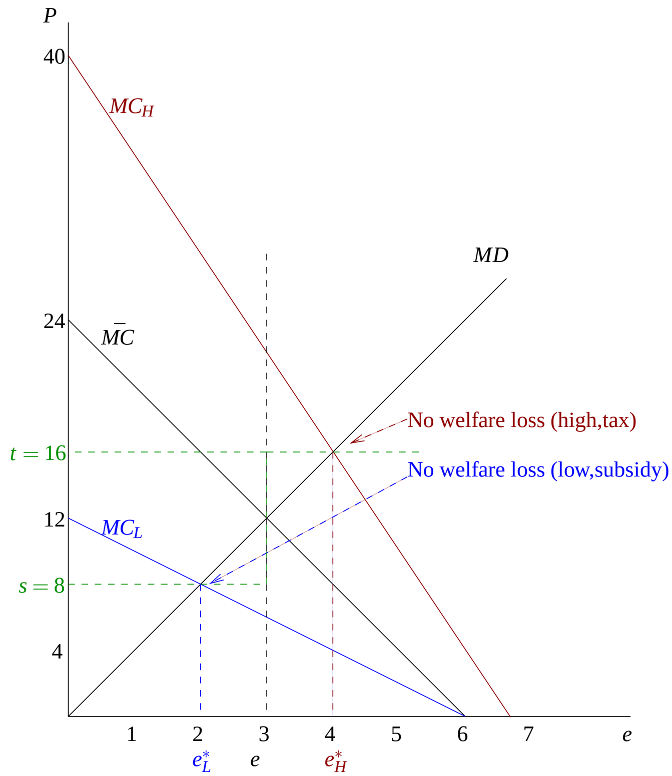

Figure 6: Welfare losses: Tax/subsidy.

j. We have from best to worst: hybrid tax/subsidy, safety valve, floor, permits, taxes.

Question 5

a. Efficient emissions are:

$$MC = 10 - e = MD = 8 \tag{32}$$

$$10 - e = 8 \quad \rightarrow \quad e = 2 \tag{33}$$

The firm would emit 10 in the absence of any regulation, so the total cost to the firm of reducing emissions from 10 to two is:

$$CC = \frac{1}{2}(10 - 2)\, 8 = \$32 \tag{34}$$

b. If the firm does not comply, it might as well emit 10 as the penalty is the same regardless of emissions. The penalty is a fixed cost. It therefore pays \$10 only if it is caught. The average cost of not complying is then:

$$\text{Cost } = 0.4 \cdot \$10 + MC\,(10) = \$4 + 0 = \$4. \tag{35}$$

It is cheaper to cheat than it is to comply with the regulation.

c. Now the fine depends on the size of the emissions. The firm will emit until the expected fine equals the marginal cost. So we set the expected fine equal to the marginal damages:

$$\pi f = MD = 8, \tag{36}$$

$$0.4f = 8, \ \ \rightarrow \ \ f = 20. \tag{37}$$

d. They system is inefficient because firms have a strong incentive to not comply. Emisions will be 10 rather thna the efficient 2. Complying costs \$32 in compliance costs, but cheating costs only \$4 in expected fines.