Subsidies to Industry and the Environment

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Abstract

Governments support particular firms or sectors by granting low interest financing, reduced regulation, tax relief, price supports, monopoly rights, and a variety of other subsidies. Previous work in partial equilibrium shows that subsidies to environmentally sensitive industries increases output and pollution emissions. We examine the environmental effects of subsidies in general equilibrium. Since all resources are used, whether or not subsidies increase emissions depends on the relative emissions intensity and incentives to emit of the subsidized industry versus the emissions intensity and the incentives to emit of the industry which would otherwise use the resources. Since subsidies must move resources to a less productive use, the economy wide marginal product of emissions falls with an increase in any subsidy, tending to decrease emissions. On the other hand, subsidies tend to move resources to more emissions intensive industries. Thus, subsidies increase pollution emissions if resources are moved to an industry for which emissions intensity is high enough to overcome the reduction in emissions caused by lower overall marginal product of emissions. We show that, under general conditions, subsidies also increase the interest rate, thus causing the economy to over-accumulate capital. Steady state emissions then rise, even if emissions fall in the short run. We also derive an optimal second best environmental policy given industrial subsidies. The results indicate that, under reasonable conditions, subsidies raise the opportunity cost of environmental quality in the long run. Finally, we examine the relationship between growth and the environment with subsidies. Under more restrictive conditions, reducing some subsidies may offer a path to sustainable development by raising income and at the same time improving the environment.

1 Introduction

Nearly all governments support particular firms or sectors by granting low interest financing, reduced regulation, tax relief, price supports, monopoly rights, and a variety of other subsidies. Firms do not receive minimally distorting lump sum payments. Instead, firms receive a complex set of subsides and hidden regulatory relief with significant distortions and offsetting effects. What is the effect of such subsidies on the quality of the environment? This paper develops a model in which one sector receives a variety of subsidies: price supports or output subsides, reduced regulation, low interest financing, and direct cash payments. The other sector, hereafter the *private* sector, receives no subsidies. We prove the existence of an equilibrium in which the subsidies cause the subsidized sector to grow overly large and unproductive. We show first that subsidies increase the economy wide average pollution emissions intensity if subsidized firms are more emissions intensive. On the other hand, since subsidies must move resources to a less productive use, subsidies reduce the economy wide marginal product of emissions under general conditions. Thus, subsidies increase pollution emissions if resources are moved to an industry for which emissions intensity is high enough to overcome the reduction in emissions caused by lower economy wide average marginal product of emissions. We show that, under general conditions, subsidies also increase the rental rate of capital, thus causing the economy to over-accumulate capital. Subsidies cause steady state emissions to rise, even if emissions fall in the current period, due to the over accumulation of emissions causing capital.

We also derive an optimal second best environmental policy given subsidies. The results indicate that since subsidies lower total factor productivity (TFP) and increase the interest rate, which increases the incentive to save, subsidies reduces resources available for consumption and environmental quality. Thus subsidies raise the opportunity cost of environmental quality causing optimal steady state emissions to rise. Finally, we examine the relationship between growth and the environment with subsidies. Under more restrictive conditions, reducing direct cash payments may offer a path to sustainable development by raising income and at the same time improving the environment.

A small literature measures the extent of subsidies in environmentally sensitive industries. Table 1 reports some results from Barde and Honkatukia (2004), based largely on OECD data. From the table agriculture, fishing, energy (especially coal), manufacturing, transport, and water are all environmentally sensitive industries which are heavily subsidized. In many developing countries, subsidies are a significant fraction of GDP. For example Brandt and Zhu (2000) report that subsidies in China amount to 6.8% of GDP in 1993. Further, van Beers and van den Bergh (2001) estimate world wide subsidies to be 3.6% of world GDP in the mid 1990s.

The literature on the effects of subsidies to industry on the environment consists of just a few papers. Barde and Honkatukia (2004) discuss a few channels by which subsidies may affect the quality of the environment. Input and output subsidies, especially in environmentally sensitive industries, encourage the over use of dirty inputs. Bailouts, tax relief, and other cash subsidies prevent the exit from the market of the least efficient producers, which are likely to be the most emissions intensive, which they call a technology lock-in effect. Subsidies in the form of regulatory relief include exemptions from environmental regulation, which directly increase the incentive to emit. Still, their analysis is largely informal. Indeed, they note that "A thorough assessment would require a complex set of general equilibrium analysis (to evaluate the rebound effect on the economy)." This paper provides such a general equilibrium analysis, including all of the above channels.

Subsidies can also be used to protect favored industries against foreign competition. Indeed many trade agreements explicitly call for a reduction in subsidies. For example, subsidies to exporting industries violate WTO rules.[1] Bajona and Kelly (2006) examine the effect on the environment of eliminating the subsidies required for China to enter the WTO and find that elimination of subsidies reduces steady state emissions of three of four pollutants studied. van Beers and van den Bergh (2001) show in a static, partial equilibrium setting how subsidies increase output and therefore emissions in a small open economy. For example, if subsidies are sufficiently large, a country may move from importing to exporting an environmentally sensitive good. The increase in output in turn increases emissions. Further, subsidies worsen the market failure in that subsidies reduce marginal costs below marginal private costs, which are in turn below marginal private plus social costs.

A literature exists on agricultural subsidies and the environment (see for example Antle, Lekakis, and Zanias 1998, Pasour and Rucker 2005). Clearly, price supports and output and input subsidies encourage the use of dirty inputs such as fertilizer and pesticides, and encourage marginal land to be converted from conservation to farming. On the other hand, the USDA in 2003 had over 17 agricultural subsidy programs ($1.9 billion) designed in part to improve environmental quality, primarily by paying farmers to remove environmentally sensitive land from production (Pasour and Rucker 2005). However, such restrictions have an ambiguous effect on erosion and fertilizer and pesticide use, since such restrictions encourage farmers to farmer the remaining land more intensively (Pasour and Rucker 2005, page 110). This effect is magnified by other subsidies, such as output subsidies. It is therefore important to analyze all subsidies together, as they can have offsetting or magnifying effects.

[1] Bagwell and Staiger (2006) argue the criteria for challenging domestic subsidies in the WTO is weak enough so that governments can in principle challenge any positive subsidy.

A related empirical literature exists on heavily subsidized state-owned enterprises (SOEs) and the environment. Wang and Jin (2002) find SOEs in China are up to ten times more emissions intensive than private firms. Gupta and Saksena (2002) find that SOEs in India are monitored for environmental compliance less often than private firms. Wang, Mamingi, Laplante, and Dasgupta (2002) find that SOEs in China enjoy more bargaining power over environmental compliance than private firms. Pargal and Wheeler (1996) find SOEs in Indonesia are more polluting than private firms, even after controlling for age, size, and efficiency. Hettige, Huq, and Pargal (1996) survey studies with similar results. Galiani, Gertler, and Schargrodsky (2005) find that privatization of water services in Argentina improved health outcomes. However, Earnhart and Lizal (2002) find an inverse relationship between emissions intensity and percentage of state ownership among recently partially privatized firms in the Czech Republic in their preferred model. The latter two studies focus on a change in ownership, which does not necessarily imply a change in subsidies.[2]

Previous work, then, has provided an important first step in identifying the extent of subsidies and likely channels by which they effect the environment. Still, the previous literature, with the exception of Bajona and Kelly (2006), does not account for dynamic effects, general equilibrium effects, the effect of multiple subsidies introduced together, and takes environmental policy as exogenous. Further, Bajona and Kelly (2006) consider only two kinds of subsidies, cash payments and interest subsidies, and take environmental policy as exogenous. Since a typical industry receives many subsidies, each of which causing intra-period and dynamic distortions, and since subsidies are uneven across industries, these effects are likely to be important.

To understand emissions in such a setting requires a theory of firms and industry structure with subsidies. Bajona and Kelly (2006) provide a model where private and subsidized firms coexist. Subsidized firms have restrictions on the number of people they can lay off (Yin 2001), which they model as a minimum labor requirement. In exchange, subsidized firms receive low interest loans from the government or state owned banks (modeled as an interest rate subsidy) and receive direct subsidies to cover the negative profits that result from the excess use of labor. Finally, subsidized firms have lower TFP relative to private sector firms. They prove the existence of an equilibrium in which subsidized firms and private firms co-exist with the share of production of subsidized firms determined endogenously by the subsidies, labor requirement, and technology difference. We extend their framework by considering as well output subsidies and regulatory relief. Our model also has endogenous emissions intensity and environmental policy.

[2]It is well known that recently privatized SOEs retain a close relationship to the state and thus possibly their subsidies. Here we examine changes in subsidies, rather than changes in ownership.

We show subsidies affect emissions through three main mechanisms. The first mechanism, the *resource reallocation effect*, is the (static) effect of the reallocation of capital and labor from private to subsidized sectors that subsidies cause. All subsidies cause capital to flow from private to subsidized firms, causing output to become more concentrated in the subsidized sector. In addition, direct cash subsidies cause labor to move from the private to the subsidized sector, further concentrating output in the subsidized sector.[3] Since emissions is a complementary input to labor and capital, emissions rise in the subsidized sector and fall in the private sector. If subsidized firms are more emissions intensive (either because they use a more emissions intensive technology or because they face reduced environmental regulation), aggregate emissions (total emissions of both sectors) tend to rise. On the other hand, subsidies concentrate production in the low productivity subsidized sector. Thus moving a unit of capital from the private sector to the subsidized sector cause the marginal product of emissions to increase less in the subsidized sector than it falls in the private sector (the economy wide marginal product of emissions falls). We derive necessary and sufficient conditions on parameter values for which the rise in emissions intensity more than offsets the fall in the marginal product of emissions.[4]

Subsidies differ in their effect on the marginal product of emissions, thus we derive a ranking of subsidies from most to least harmful to the environment. Output subsidies or price supports are more harmful than interest subsidies, for example, because interest subsidies induce firms to over use capital, which reduces the marginal product of emissions. In contrast, with output subsidies, firms maintain an optimal balance between inputs. Thus output subsidies cause a smaller fall in the marginal product of emissions than interest subsidies (emissions are more productive when firms use inputs in the correct proportion).

A second mechanism, the *capital accumulation effect*, is dynamic in nature and affects intertemporal decisions. On one hand, subsidies to firms directly increase economy-wide average demand for capital. On the other hand, the decline in economy-wide average productivity caused by the concentration of capital in the subsidized sector tends to reduce demand for capital. We show that the former effect is stronger so the return to capital rises, causing the economy to over-accumulate capital, which causes emissions to rise over time with subsidies. The capital accumulation effect is stronger than resource reallocation effect: we show that output subsidies, interest subsidies, and emissions subsidies all cause emissions

[3]Other subsidies do not cause labor to move because subsidized firms already use excess labor.

[4]Note that the technology lock-in effect is incorporated here since, absent subsidies, the subsidized firms who use a low TFP and emissions intensive technology would exit the market. The effects of overuse of dirty inputs by, and higher emissions intensity of, the subsidized sector is also clearly incorporated. Subsidies do not trivially increase emissions because we also consider general equilibrium effects of the reduction in the use of dirty inputs by the private sector.

to rise in the steady state, even if the resource reallocation effect caused emissions to fall in the short run. With direct subsidies, both labor and capital move to the subsidized sector, thus the effect on the interest rate is weaker. Nonetheless, we derive necessary and sufficient conditions for steady state emissions to rise with an increase in direct subsidies.[5]

Our third mechanism, not previously analyzed in the literature, is how subsidies affect the marginal opportunity costs and marginal benefits of environmental quality. All subsidies decrease aggregate resources available and increase the interest rate holding emissions fixed, thus raising the opportunity cost of environmental quality (foregone consumption or saving). Under reasonable conditions, the higher interest rate causes savings to rise to the point where steady state consumption falls, reducing the marginal benefits of environmental quality if environmental quality and consumption are complements. Subsidies do reduce the productivity of emissions, which makes environmental quality more attractive, but this effect is outweighed by the higher opportunity cost of environmental quality for a reasonable risk aversion coefficient. Thus, for reasonable parameter values, subsidies put pressure on governments to relax environmental regulation.

Section 2 posits a theory of emissions and industry with subsidies and Section 3 proves existence of an equilibrium. Section 4 derives the reallocation effects and Section 5 derives the capital accumulation effects. Section 6 shows how subsidies affect environmental policy and Section 7 concludes.

2 A Theory of Emissions and Subsidies

In this Section we derive a competitive equilibrium in which subsidized and private firms coexist, taking environmental policy as given. The environmental policy is a tax on emissions which is constant over time. In Section 6, we allow the government to vary the tax rate in response to changes in income, but still take the subsidies as given.

2.1 Firms

If subsidized and non-subsidized firms co-exist, some cost to receiving subsidies must exist. These costs may include hiring lobbyists, campaign contributions, and/or locating plants or hiring labor in key districts. Following Bajona and Kelly (2006), we model this process in a very simple way. Specifically, subsidized firms have lower TFP than private firms and must hire excess labor to receive subsidies.

Productivity differences are taken as exogenous, with subsidized firms having TFP equal

[5]The capital accumulation effect was first noted by Bajona and Kelly (2005). However, we extend their results to other subsidies, with endogenous emissions.

to $a_G < 1$, while private firms have TFP normalized to one. The TFP difference between the two firms can be thought of as a function of the fraction of the workforce diverted to lobbying activities. Alternatively, the TFP difference could be the result of choosing plant location based on political considerations. Finally, it may simply be that a negative productivity shock (and thus the threat of bankruptcy) is required to receive subsidies.

We assume employment at subsidized firms is constrained to be greater than or equal to a minimum labor constraint, l_G, established by the government. In exchange for using l_G fraction of the total hours per person, the government covers any losses through direct subsidies (cash payments). If the labor constraint binds, the marginal product of labor in subsidized firms falls below the wage rate, causing subsidized firms to earn negative profits. Subsidized and private firms then co-exist if subsidized firms receive enough direct subsidies from the government to earn zero profits.[6] Therefore, let $S = -\pi_G$ be the direct subsidy, where π_G are the (negative) profits of subsidized firms excluding the direct subsidy and $\Pi_G = \pi_G + S = 0$ are the profits including the direct subsidy. To save on notation, we suppress the time t subscripts where no confusion is possible.

Both private and subsidized firms have access to a Cobb-Douglas technology F that produces output Y from capital K, emissions E, and labor l:[7]

$$Y_i = F\left(K_i, E_i, l_i\right) = K_i^{\theta} E_i^{\epsilon} l_i^{1-\theta-\epsilon} \quad i = G, P \tag{2.1}$$

Here K_G and K_P denote the fraction of the aggregate per capita capital stock K allocated to the subsidized and private sectors, respectively. We define l_i and E_i analogously. Hence in equilibrium $K = K_G + K_P$ is the economy wide per capita capital stock and $E = E_P + E_G$ is aggregate emissions. The representative household is endowed with one unit of labor every period, which is supplied inelastically. Therefore, in equilibrium $l_G + l_P = 1$.

Subsidized firms may receive a second subsidy, a discount on their rental rate of capital, which we call an interest subsidy. Let r be the rental rate of capital for private firms, then subsidized firms have rental rate $(1-\gamma)r$, where γ is the subsidy rate. This subsidy can be interpreted as either the government guaranteeing repayment of funds borrowed by subsidized firms, SOEs borrowing at the government's rate of interest, or as the government steering household deposits at state owned banks to subsidized firms at reduced interest

[6] In the absence of subsidies, in a competitive equilibrium only the firm with the highest TFP operates.

[7] It is straightforward to derive a Cobb-Douglas production function with emissions as input to production from a model where emissions can be reduced via a costly abatement technology. See for example Bartz and Kelly (2006). Bartz and Kelly (2006) also calibrate a Cobb-Douglas production function with emissions as an input to production and find the emissions share to be generally less than one percent, and the capital share nearly identical (about 0.4) to a production function without emissions. With a few trivial changes, E can also be thought of as a dirty input.

rates.[8]

Subsidized firms may also receive an output subsidy of λ per unit of output produced from the government. Let the price of output be normalized to one. The output subsidy can then also be interpreted as a price support, where subsidized firms receive a total price of $1+\lambda$ for their output.

Finally, subsidized firms may also receive relief from environmental regulation. Let τ be the tax rate per unit of emissions.[9] Then subsidized firms pay $(1-\mu)\,\tau$ per unit of emissions, where μ is the subsidy rate. One can think of $1-\mu$ as the fraction of emissions by subsidized firms that are reported, or the fraction of emissions that are monitored by the regulators.[10]

Both private and subsidized firms are competitive price takers. The objective of both private and subsidized firms is to maximize profits taking prices and government policies as given. Subsidized firms therefore maximize:

$$\pi_G = \max_{K_G, E_G} (1+\lambda)\, a_G F\left(K_G, E_G, l_G\right) - (1-\gamma)\, r K_G - (1-\mu)\, \tau E_G - w l_G. \tag{2.2}$$

Let subscripts on functions denote partial derivatives and let $A_G \equiv (1+\lambda)\, a_G$. The first order condition which determines the part of the capital stock allocated to the subsidized sector is:

$$(1-\gamma)\, r = A_G F_k\left(K_G, E_G, l_G\right). \tag{2.3}$$

The first order condition which determines emissions by the subsidized sector is:

$$(1-\mu)\, \tau = A_G F_E\left(K_G, E_G, l_G\right). \tag{2.4}$$

The problem for private firms is:

$$\pi_P = \max_{K_P, E_P, l_P} F\left(K_P, E_P, l_P\right) - r K_P - \tau E_P - w l_P. \tag{2.5}$$

The equilibrium rental rate, price of emissions, and wage rate, w are:

$$r = F_k\left(K_P, E_P, l_P\right), \tag{2.6}$$

[8]The latter interpretation is more reasonable for developing countries. All three interpretations are consistent with households renting capital.

[9]One could also think of τ as the price of a tradeable permit allowing one unit of emissions. However, the total permits would have to vary over time in a way that keeps the price constant.

[10]As noted in the introduction, heavily subsidized SOEs in India are monitored less often (Gupta and Saksena 2002) and enjoy more bargaining power over environmental compliance (Wang, Mamingi, Laplante, and Dasgupta 2002).

$$\tau = F_E\left(K_P, E_P, l_P\right), \tag{2.7}$$

$$w = F_l\left(K_P, E_P, l_P\right). \tag{2.8}$$

The subsidies drive wedges between the marginal products of each input in each sector. Many of the conditions derived later depend on the size of the wedges. Let MP_j^i denote the marginal product of input j in sector i, then the wedge η_j between the marginal products are:

$$\frac{MP_k^P}{MP_k^G} = \frac{1+\lambda}{1-\gamma} \equiv \eta_k \tag{2.9}$$

$$\frac{MP_E^P}{MP_E^G} = \frac{1+\lambda}{1-\mu} \equiv \eta_E \tag{2.10}$$

The labor constraint is binding (subsidized firms hire more labor than is efficient) if and only if $w > A_G F_l\left(K_G, E_G, l_G\right)$. In turn, the wage is greater than the marginal product of labor in the subsidized sector if and only if:

$$A_G < \left(1-\gamma\right)^{\theta}\left(1-\mu\right)^{\epsilon}. \tag{2.11}$$

Thus direct subsidies are consistent with the co-existence of subsidized and non-subsidized firms if and only the TFP of subsidized firms is sufficiently less than private firms. If $A_G > \left(1-\gamma\right)^{\theta}\left(1-\mu\right)^{\epsilon}$, then either only subsidized firms exist or the subsidy is a tax. Since this case is not interesting, we assume condition (2.11) holds.

Equations (2.3), (2.4), (2.6), (2.7), and (2.8) have a unique solution:

$$K_G = \frac{1}{1+\Omega}K \tag{2.12}$$

$$\Omega \equiv \frac{1-l_G}{l_G}\left(1-\gamma\right)^{\frac{1-\epsilon}{1-\theta-\epsilon}}\left(1-\mu\right)^{\frac{\epsilon}{1-\theta-\epsilon}} A_G^{\frac{-1}{1-\theta-\epsilon}} \equiv \frac{1-l_G}{l_G}\alpha \tag{2.13}$$

$$E_P = \left(\frac{\Omega K}{1+\Omega}\right)^{\frac{\theta}{1-\epsilon}}\left(1-l_G\right)^{1-\frac{\theta}{1-\epsilon}}\left(\frac{\epsilon}{\tau}\right)^{\frac{1}{1-\epsilon}} \tag{2.14}$$

$$E_G = \left(\frac{K}{1+\Omega}\right)^{\frac{\theta}{1-\epsilon}} l_G^{1-\frac{\theta}{1-\epsilon}}\left(\frac{A_G\epsilon}{\left(1-\mu\right)\tau}\right)^{\frac{1}{1-\epsilon}} \tag{2.15}$$

$$r = \theta \left(\frac{K}{1 - l_G} \frac{\Omega}{1 + \Omega} \right)^{\frac{\theta}{1-\epsilon} - 1} \left(\frac{\epsilon}{\tau} \right)^{\frac{\epsilon}{1-\epsilon}} \tag{2.16}$$

$$w = (1 - \theta - \epsilon) \left(\frac{K}{1 - l_G} \frac{\Omega}{1 + \Omega} \right)^{\frac{\theta}{1-\epsilon}} \left(\frac{\epsilon}{\tau} \right)^{\frac{\epsilon}{1-\epsilon}} \tag{2.17}$$

The subsidies, labor constraint, environmental policy, and productivity differences determine the share of capital in the subsidized sector and emissions in each sector. An increase in any subsidy (γ, λ, μ, or S through an increase in l_G) raises the after-subsidy marginal product of capital in the subsidized sector. To maintain equilibrium the subsidized sector increases in size, causing the after-subsidy marginal product of capital to fall, and the marginal product of capital in the private sector to rise, until the after-subsidy marginal products are equalized. Thus subsidies cause the subsidized sector to grow larger and become more inefficient in that the marginal product of capital absent the subsidies falls.

In turn, because capital moves from the private sector to the subsidized sector with an increase in any subsidy, emissions in the private sector fall and emissions in the subsidized sector increase with an increase in any subsidy. Similarly, an increase in subsidies γ, λ, or μ increase the after-subsidy marginal product of capital in the subsidized sector and thus increase the economy wide average demand for capital and the interest rate. Thus even though the economy-wide TFP falls as γ, λ, or μ rise, the interest rate rises because this effect is outweighed by the increase in demand for capital by the subsidized sector. When capital falls in the private sector, demand for labor falls, but the the supply of labor is inelastic. Hence the wage falls with an increase in γ, λ, or μ. With an increase in direct subsidies, both labor and capital move from the private sector to the subsidized sector. Thus the overall effect on the wage rate and interest rate depend on whether or not more labor moves than capital. The interest rate is increasing in direct subsidies and the wage is decreasing in direct subsidies if and only if:

$$A_G > (1 - \gamma)^{1-\epsilon} (1 - \mu)^{\epsilon} \tag{2.18}$$

Condition (2.18) holds if and only if the private sector has a smaller capital to labor ratio.

2.2 Households

Households enjoy consumption of an aggregate good c, produced by both subsidized and non-subsidized firms, and environmental quality Q. Let $\mu(c, Q)$ denote the per period utility, which we assume is strictly increasing and concave in each input, twice-continuously differentiable, and satisfies the Inada conditions in c. The objective of households is:

$$\max \sum_{t=0}^{\infty} \beta^t u\left(c_t, Q_t\right). \tag{2.19}$$

Let TR_t denote lump sum transfers (which may be negative and correspond to a tax) and k_t denote the part of the capital stock held by an individual. The maximization is subject to a budget constraint:

$$r_t k_t + w_t + TR_t = c_t + k_{t+1} - (1-\delta) k_t \tag{2.20}$$

Environmental quality is a strictly decreasing function $Q(E)$ of aggregate emissions E, where:

$$E \equiv E_P + E_G. \tag{2.21}$$

2.3 Government

The the government budget constraint sets total subsidy costs plus lump sum transfers TR equal to emissions tax revenue.

$$\gamma r K_G + \lambda Y_G + S + TR = (1-\mu)\tau E_G + \tau E_P. \tag{2.22}$$

Total direct subsidies equal total wage payments of the subsidized sector less the total product of labor of the subsidized sector, that is, direct subsidies equal the total cost of the hiring constraint. Hence:

$$\gamma r K_G + \lambda Y_G + \left(w - A_G F_h\left(K_G, E_G, l_G\right)\right) l_G = -TR + (1-\mu)\tau E_G + \tau E_P. \tag{2.23}$$

The aggregate resource constraint is:

$$Y_t \equiv Y_{P,t} + Y_{G,t} = C_t + K_{t+1} - (1-\delta) K_t. \tag{2.24}$$

Let primes denote next period's value, then the recursive version of the household problem is:

$$\begin{aligned} v(k, K) \;=\; & \max_{k'} \Big\{ u \Big[r(K;\gamma;\lambda;\mu;l_G) k + w(K;\gamma;\lambda;\mu;l_G) + TR(K;\gamma;\lambda;\mu;l_G) - \\ & k' + (1-\delta) k, Q(K;\gamma;\lambda;\mu;l_G) \Big] + \beta v(k', K') \Big\}. \end{aligned} \tag{2.25}$$

3 Equilibrium

We characterize the model by establishing the existence and properties of the equilibrium.

Definition 1 *A Recursive Competitive Equilibrium given individual and aggregate capital stocks k and K and government policies $\{\gamma, \lambda, \mu, l_G, \tau\}$ is a set of individual household decisions $\{c, k'\}$, prices $\{r, w\}$, aggregate household decisions $\{C, K'\}$, an input decision by subsidized firms K_G, input decisions by private firms $\{K_P, l_P\}$, government variables $\{S, TR\}$, and a value function v such that the household's and producers' (private and subsidized) problems are satisfied, all markets clear, subsidized firms earn zero profits, the government budget constraint is satisfied, and the consistency conditions ($k = K$ implies $c = C$ and $k' = K'$) are satisfied.*

Note that an alternative definition of equilibrium is to take the direct subsidy S as given and let l_G be determined in equilibrium. These definitions are equivalent, so we do not distinguish between them, but will occasionally think of S as the given government policy.

Capital accumulation is then determined from the equilibrium first order condition and envelope equation:

$$u_c\left(C\left(K;\gamma;\lambda;\mu;l_G\right), Q\left(K;\gamma;\lambda;\mu;l_G\right)\right) = \beta v_k\left(K', K'\right) \tag{3.1}$$

$$v_k\left(K, K\right) = u_c\left(C\left(K;\gamma;\lambda;\mu;l_G\right), Q\left(K;\gamma;\lambda;\mu;l_G\right)\right)\left(r\left(K;\gamma;\lambda;\mu;l_G\right) + 1 - \delta\right) \tag{3.2}$$

$$C\left(K;\gamma;\lambda;\mu;l_G\right) = Y\left(K;\gamma;\lambda;\mu;l_G\right) - K'\left(K;\gamma;\lambda;\mu;l_G\right) + (1-\delta)K \tag{3.3}$$

$$\begin{aligned} Y\left(K;\gamma;\lambda;\mu;l_G\right) &= F\left(K - K_G\left(K;\gamma;\lambda;\mu;l_G\right), E_P\left(K;\gamma;\lambda;\mu;l_G\right), 1 - l_G\right) \\ &+ A_G F\left(K_G\left(K;\gamma;\lambda;\mu;l_G\right), E_G\left(K;\gamma;\lambda;\mu;l_G\right), l_G\right) \end{aligned} \tag{3.4}$$

Our strategy is to establish some basic properties of the competitive equilibrium, and then use these properties to derive the more complicated results on how emissions changes with changes in subsidies.

THEOREM 1 *Suppose u and F are as described above. Let:*

$$\frac{1}{\eta_k} > \theta + (1-\theta)\,\Omega, \tag{3.5}$$

and

$$0 \leq -u_{cQ}\left(C,Q\right)Q_E\left(E\right) \leq z\left(K\right)u_c\left(C,Q\right) \tag{3.6}$$

for all $k = K > 0$, where $z(K)$ is a function defined in the Appendix. Then a competitive equilibrium exists. Further, the equilibrium gross investment function $K' = H(K)$ is such that:

1. $H_K(K) \geq 0$,

2. $C_K(K) \geq 0$,

3. *$H(K)$ satisfies the Euler equation derived from (3.1) and (3.2), and*

4. *$H(K)$ is concave.*

All proofs are in the Appendix. Assumption (3.5) requires the subsidies not to be so large that taxes exhaust household wages. If so, households would be forced to reduce savings to pay taxes, perhaps unraveling the equilibrium if the capital stock was low enough. Assumption (3.6) ensures that an increase in the capital stock, which lowers the quality of the environment, does not reduce the marginal utility of consumption so much that consumption becomes less attractive, despite the extra income (ie. the second assumption ensures that $C_K > 0$).

Although a complex network of subsidies exists at the household level, the model reduces to a standard capital accumulation problem if the subsidies are not too large. Further, the economy grows at a decreasing rate and capital converges monotonically to the steady state. Hence a change in subsidies affects aggregate emissions within the current period, as capital is reallocated across sectors and dynamically over time as the change in subsidies affects the path of capital accumulation. The former effects we call *reallocation effects* and the later effect we call the *capital accumulation effect*.

4 Reallocation Effects

A small increase in a particular subsidy has two effects. First, some capital moves from the private sector to the subsidized sector. Thus the marginal product of emissions rises in the subsidized sector and falls in the private sector. Because capital and labor are less productive in the subsidized sector, the rise in the marginal product of emissions in the subsidized sector is less than the fall in the private sector. Thus, ignoring emissions subsidies, the decrease in emissions in the private sector exceeds the increase in emissions in the subsidized sector. In addition, a second effect exists in that because of the emissions subsidy, the subsidized sector over uses emissions and is more emissions intensive. Thus an increase in the size of the subsidized sector increases economy-wide average emissions intensity, tending to increase aggregate emissions. The overall effect of subsidies on emissions depends on which of these two effects is greater.

To formalize this argument, let σ_i denote the emissions intensity of output in sector i, then:

$$\sigma_i = \frac{E^i}{Y^i} = \frac{\epsilon}{MP_E^i} \tag{4.1}$$

$$\frac{\sigma_G}{\sigma_P} = \frac{1+\lambda}{1-\mu} = \eta_E \tag{4.2}$$

We then have:

PROPOSITION 2 *Let F and u be as described above, then:*

1. *aggregate emissions are increasing in μ if and only if:*

$$\frac{\sigma_G}{\sigma_P} > \frac{\theta\epsilon\Omega}{\theta\epsilon\Omega + (1-\theta-\epsilon)(1+\Omega)}\eta_k \tag{4.3}$$

2. *aggregate emissions are increasing in λ if and only if:*

$$\frac{\sigma_G}{\sigma_P} > \frac{\theta\Omega}{\theta\Omega + (1-\theta-\epsilon)(1+\Omega)}\eta_k \tag{4.4}$$

3. *aggregate emissions are increasing in S if and only if:*

$$\frac{\sigma_G}{\sigma_P} > \frac{\theta + (1-\theta-\epsilon)(1+\Omega)\, l_G}{\theta + (1-\theta-\epsilon)(1+\Omega)\, l_G/\alpha}\eta_k \tag{4.5}$$

4. *aggregate emissions are increasing in γ if and only if:*

$$\frac{\sigma_G}{\sigma_P} > \eta_k \tag{4.6}$$

Proposition 2 implies a ranking of subsidies from most to least harmful to the environment. Emissions subsidies are the most environmentally harmful (the right hand side of condition 4.3 is smaller than that of 4.4-4.6), since emissions subsidies directly increase the incentive to emit. Surprisingly, allowing subsidized firms to evade environmental regulation can benefit the environment, if the emissions subsidy is not too large. This is because subsidized firms will grow in size and take resources from the highly productive private sector. If the resulting drop in the marginal product if emissions is large enough, aggregate emissions may fall even though emissions intensity rises. Proposition 2 indicates output subsidies are more harmful to the environment than either interest subsidies or direct subsidies. An increase in output subsidies does not further distort the relative input use, so the fall in the marginal product of emissions is relatively small, making the increase in emissions intensity more likely to dominate. Proposition 2 indicates direct subsidies are more harmful to the environment than interest subsidies if and only if condition (2.18) is satisfied, or if and only if an increase in direct subsidies increases the return to capital. In that case, the marginal product of emissions falls less with an increase in direct subsidies. Thus the increase in emissions intensity is more likely to dominate. If direct subsidies decrease the return to capital (condition 2.18 does not hold), then direct subsidies worsen the capital to labor ratio and so result in a larger decrease in the marginal product of emissions. In this case, interest subsidies are more environmentally harmful than direct subsidies.

All subsidies increase emissions if and only if:

$$\frac{\sigma_G}{\sigma_P} > \max\left(1, \frac{\theta + (1-\theta-\epsilon)(1+\Omega)\, l_G}{\theta + (1-\theta-\epsilon)(1+\Omega)\, l_G/\alpha}\right)\eta_k \tag{4.7}$$

If $\alpha < 1$, then we need only (4.6) to hold, which in turn holds if and only if $\mu > s$, if the emissions subsidy exceeds the interest subsidy. This illustrates the importance of accounting for all subsidies together, as they have significant interaction effects.

Previous work (e.g. van Beers and van den Bergh 2001) show how subsidies increase output and thus emissions in the subsidized sector. Here, the results are more complicated because we consider the general equilibrium results on other sectors as well. Even though output in the subsidized sector always rises with the subsidies, aggregate emissions might still fall if emissions fall in the sector which would otherwise use the resources (ie. if conditions of Proposition 2 do not hold).

Aggregate output falls as emissions subsidies rise if and only if:

$$\eta_k > 1 + \frac{(1-\theta-\epsilon)}{\theta}\frac{1+\Omega}{\Omega} \tag{4.8}$$

Aggregate output falls as output subsidies rise if and only if:

$$\eta_k > 1 + \frac{\epsilon(1-\theta-\epsilon)}{\theta}\frac{1+\Omega}{\Omega} \tag{4.9}$$

Although capital and labor resources move to the low productivity subsidized sector, emissions is not a fixed resource. Output or emissions subsidies may increase output if firms increase emissions enough to compensate for using capital and labor in a less productive manner.

Combining the above with the results of Proposition 2, in some cases subsidies may increase emissions even though aggregate output falls.

COROLLARY 3 *Suppose the conditions of Proposition 2 hold. Then current output falls and emissions rise:*

1. *with an increase in direct subsidies,*
2. *with an increase in interest subsidies,*
3. *with an increase in emissions subsidies if in addition condition (4.9) holds.*
4. *with an increase in output subsidies if in addition condition (4.8) holds.*

Further, if current output rises with any subsidy, emissions rise as well.

5 The Capital Accumulation Effect

An increase in γ, λ, or μ raises the rate of return to capital, causing the economy to over accumulate capital. An increase in S raises the return to capital if and only if condition (2.18) holds. When the economy over accumulates capital, output and emissions rise, which we call the capital accumulation effect. From Theorem 1, the economy follows a standard path of capital accumulation in that capital increases at a decreasing rate as it transitions to a steady state. Thus, starting at time 0 at the steady state capital stock, $\bar{K}$, an increase in any subsidy causes capital to increase at decreasing rate to a new steady state capital stock. Since emissions are increasing in the capital stock, emissions will also increase from the previous steady state $\bar{E}$ to a new steady state. If the reallocation effect was positive (that is, if the conditions in Proposition 2 are satisfied), then emissions are above the initial level for all $t \geq t_0$. If the reallocation effect is negative, then emissions initially fall but then increase for all $t > t_0$. The next proposition gives conditions for which emissions eventually rise above their initial drop so that the new steady state emissions exceeds the initial emissions.

PROPOSITION 4 *Let F and u be as described above, let conditions (3.5) and (3.6) hold, and let $K_0 = \bar{K}$. Then:*

1. *if μ is increased at time 0, then if condition (4.3) holds, $E_t > \bar{E}$ for all $t \geq 0$. If condition (4.6) does not hold, then $E_t > \bar{E}$ for all $t \geq t^*$ for some finite t^*.*
2. *if λ is increased at time 0, then if condition (4.4) holds, $E_t > \bar{E}$ for all $t \geq 0$. If condition (4.6) does not hold, then $E_t > \bar{E}$ for all $t \geq t^*$ for some finite t^*.*

3. *if γ is increased at time 0, then if condition (4.6) holds, $E_t > \bar{E}$ for all $t \geq 0$. If condition (4.6) does not hold, then $E_t > \bar{E}$ for all $t \geq t^*$ for some finite t^*.*

4. *if S is increased at time 0, then if condition (4.5) holds, $E_t > \bar{E}$ for all $t \geq 0$. If condition (4.6) does not hold, then $E_t > \bar{E}$ for all $t \geq t^*$ for some finite t^* if and only if $A_G > (1-\gamma)^{\theta}(1-\mu)^{1-\theta}$.*

Hence subsidies γ, λ, and μ raise emissions in the long run regardless of the ratio of emissions intensities. Even if no emissions subsidy exists and both the private and subsidized firms have identical emissions intensities, the over accumulation of emissions causing capital raises emissions enough to offset the fall in emissions caused by moving resources to the less productive subsidized sector. For an increase in direct subsidies, whether or not steady state emissions rise depends on whether or not the interest rate and output rise.

Figures 1 and 2 show the possible dynamic paths of capital and emissions after an increase in interest subsidies (emissions and output subsidies are analogous). It is interesting to note that if the conditions of Proposition 2 do not hold, then increases in subsidies γ, λ, and μ temporarily decrease emissions, only to see emissions rise eventually as capital accumulates.

Steady state output rises with an increase in γ, λ, or μ. However, steady state output falls with an increase in direct subsidies. Thus for direct subsidies conditions arise for which reducing subsidies may enable a government to increase output and reduce emissions.

COROLLARY 5 *Let F and u be as described above, let conditions (3.5) and (3.6) hold, and let $K_0 = \bar{K}$. Then if S falls at time 0 and if $A_G > (1-\gamma)^{\theta}(1-\mu)^{1-\theta}$, steady state output rises and steady state emissions fall.*

Thus reducing direct subsidies may create a path for sustainable development. A developing country, by reducing direct payments to firms, can increase output and decrease emissions in the long run.

6 Optimal Policy

We next consider optimal second best emissions tax policy given subsidies. Here we follow a large literature which takes some sub-optimal policies as given and then determines the optimal environmental policy.[11] We give conditions below for which subsidies decrease the optimal tax on emissions and increase optimal emissions.

[11]For example, Bovenberg and Goulder (1996) and Parry (1997) calculate optimal environmental taxes taking other distorting taxes as given.

A valid criticism of this analysis and this literature is that it is unlikely that a government which cannot implement the optimal policy in one dimension would be able to implement the optimal policy in another dimension. Here it is unlikely that a government which, due to political/institutional constraints, gives welfare decreasing subsidies would face no political/institutional constraints in designing environmental policy. However, we argue that an increase in subsidies is likely to lower the tax rate on emissions even in a world with environmental policy political/institutional constraints. Subsidies reduce resources available for consumption and environmental quality and governments must at some level take into account available resources, irrespective of the exact mechanism that determines policy.

To determine the optimal second best emissions policy, we create a set of constraints such that, given the constraints, prices and policies exist which are consistent with the competitive equilibrium. The planner then need only maximize utility subject to the constraints. Therefore, from equilibrium equations (2.4) and (2.7):

$$E^G = E^p \frac{1-\gamma}{1-\mu} \frac{1}{\Omega} \tag{6.1}$$

Since $E = E^G + E^P$, emissions of each sector are a constant fraction of aggregate emissions:

$$E^G = \frac{1-\gamma}{1-\gamma+\Omega\left(1-\mu\right)} E \tag{6.2}$$

$$E^P = \frac{\Omega\left(1-\mu\right)}{1-\gamma+\Omega\left(1-\mu\right)} E \tag{6.3}$$

Next, given equation (2.12) aggregate income as a function of aggregate emissions is:

$$Y = \psi K^{\theta} E^{\epsilon} \tag{6.4}$$

$$\psi \equiv \frac{a_G\left(\left(1+\lambda\right)\Omega + 1 - \gamma\right) l_G^{1-\theta-\epsilon}}{\left(\Omega\left(1-\mu\right)+1-\gamma\right)^{1-\epsilon}\left(1+\Omega\right)^{\theta}\left(1-\gamma\right)^{\epsilon}} \tag{6.5}$$

Here $a_G \leq \psi \leq 1$ is a weighted average of the TFP in the subsidized sector (a_G) and the private sector (1). The weights reflect the amount of each input allocated to each sector, which is determined by the subsidies. It follows that the resource constraint is:

$$C = \psi K^{\theta} E^{\epsilon} + \left(1-\delta\right) K - K' \tag{6.6}$$

Thus the subsidies affect the resources the planner has available for consumption, investment, environmental quality. However, subsidies generally raises the interest rate above the economy wide marginal product of capital ($\theta\psi K^{\theta-1}E^{\epsilon}$). The most straightforward way to account for this is to create a planning problem with a discount factor closer to one, so the planner is induced to save in a way which is consistent with the high interest rate in the competitive equilibrium. Since the planner determines optimal emissions, we henceforth assume Q is convex and u satisfies the Inada conditions in Q ($u_Q(c,0)=\infty$ and $u_Q\left(c,\hat{E}\right)=0$, where $\hat{E}$ is the maximum sustainable emissions). Consider the following planning problem:

$$\nu(K)=\max_{K',E}\left\{u\left[\psi K^{\theta}E^{\epsilon}+(1-\delta)K-K',Q(E)\right]+\phi\beta\nu(K')\right\} \tag{6.7}$$

$$\phi\equiv\frac{1+\Omega}{\Omega+\frac{1-\gamma}{1+\lambda}} \tag{6.8}$$

The first order conditions and envelope equations are:

$$u_c(C,Q)=\phi\beta\nu_k(K') \tag{6.9}$$

$$u_c(C,Q)\,\epsilon\psi K^{\theta}E^{\epsilon-1}=-u_Q(C,Q)\,Q_E(E) \tag{6.10}$$

$$\nu_k(K)=u_c(C,Q)\left(\theta\psi K^{\theta-1}E^{\epsilon}+1-\delta\right) \tag{6.11}$$

We then have:

PROPOSITION 6 *Let F and u be as described above, and let conditions (3.5) and (3.6) hold. Then any competitive equilibrium satisfies (6.6). Conversely, assume also either $\gamma=\lambda=0$ or $\delta=1$. Then, there exists prices and policies such that the allocations given by (6.6) and (6.9)-(6.11) are a competitive equilibrium.*

Recall that the interest rate net of depreciation is subsidized; households receives no subsidy for nondepreciating capital. Thus the competitive equilibrium yields identical allocations as a planning problem with a larger discount factor only if all capital depreciates. Alternatively, if no interest or output subsidies exists, then the interest rate will equate with the economy wide marginal product of capital.

The properties of the optimal environmental policy E^* follow from (6.6) and (6.9)-(6.11).

PROPOSITION 7 *Let F and u be as described above, let conditions (3.5) and (3.6) hold, and either $\gamma=\lambda=0$ or $\delta=1$. Suppose $R\left(\bar{C}\right)\geq 1$, where R is the relative risk aversion and $\beta\phi(1+\theta)>1$. Then the optimal second best steady state state environmental policy $\bar{E}^*$ is increasing in μ, γ, λ, and S.*

Viewing the problem as one in which investment is chosen for fixed emissions and then choosing emissions is instructive. With emissions fixed, an increase in any subsidy decreases aggregate resources available for current consumption and raises the effective discount factor. Hence the cases in the previous sections in which increasing subsidies increase output or decrease interest rates depended solely on increasing emissions.

The change in optimal emissions given a decrease in resources and an increased incentive to save is a complicated mixture of offsetting effects. First, an increase in any subsidy decreases aggregate resources available and thus raises the opportunity cost of environmental quality. An increase in any subsidy decreases steady state consumption holding emissions fixed, since with $\beta\phi(1+\theta) > 1$, the reduction in consumption caused by increased savings outweighs the increase in steady state resources caused by higher savings.[12] Since steady state consumption falls and $u_{CQ} \geq 0$, the marginal utility of environmental quality also falls, increasing optimal emissions. Subsidies do reduce productivity and thus the marginal product of emissions, which makes emissions less attractive. For $R\left(\bar{C}\right) \geq 1$, however, this income effect is outweighed by the increase in the marginal utility of consumption caused by the decrease in aggregate resources.

7 Conclusions

We have derived conditions under which subsidies increase emissions in the short run and in the steady state. In the short run, we show that subsidies can be ranked from most to least harmful to the environment. Emissions subsidies directly increase the incentive to emit and are thus most harmful. Output subsidies allow the firm to use an optimal balance of inputs, so the marginal product of emissions falls very little. Thus emissions are likely to rise because output subsidies move production to the more emissions intensive subsidized sector. Interest subsidies and direct subsidies are less harmful, because they distort individual inputs, thus reducing the marginal product of emissions.

Regardless of the short run effect, emissions, output, and interest subsidies all increase emissions in the steady state. We have also shown that direct payments can increase emissions in the steady state even if emissions fall in the short run. Thus we have shown the capital accumulation effect is the most important channel by which subsidies affect emissions, although it has not received much attention in the literature.

Finally, we have shown that if the emissions policy is endogenous, subsidies increase the opportunity cost of environmental quality and thus put pressure on governments to weaken environmental policy. Although our result depends on the fictional assumption that

[12]Since the annual discount factor is in the range of 0.9 and the capital share is around 0.4 and $\phi > 1$, this condition is not too restrictive.

governments can choose an optimal environmental policy but not an optimal subsidy policy, at some level our results are likely to be qualitatively relevant since regardless of political constraints governments must pay some attention to the cost of environmental policy.

Thus, allowing for general equilibrium, dynamic, and policy effects, subsidies tend to reduce environmental quality in the long run, even if the short run effect depends on the parameters and the size of the subsidies. Thus subsidies are welfare reducing and reduce the quality of the environment. It is not clear if subsidies can be easily reduced, as they create a few vocal winners and have diverse costs. Perhaps free trade agreements, which create new sets of winners and losers, can change the political dynamic (see for example Bajona and Kelly 2005). Perhaps the environmental lobby can bring new political incentives to reduce subsidies. Regardless of whether or not subsidies are easily reduced, it is important for policy makers and other stakeholders to understand their negative consequences.

8 Appendix: Proof of theorems

For many of the results of the paper, it is useful to derive aggregate emissions and output. From equations (2.14) and (2.15):

$$\begin{aligned} E &= E_P + E_G \\ &= (1+\Omega)^{\frac{-\theta}{1-\epsilon}} \left(\frac{\epsilon}{\tau}\right)^{\frac{1}{1-\epsilon}} \left[(1-l_G)^{1-\frac{\theta}{1-\epsilon}} \Omega^{\frac{\theta}{1-\epsilon}} + A_G^{\frac{1}{1-\epsilon}} l_G^{1-\frac{\theta}{1-\epsilon}} (1-\mu)^{\frac{-1}{1-\epsilon}} \right] K^{\frac{\theta}{1-\epsilon}}. \quad (8.1) \end{aligned}$$

The definition of Ω implies:

$$E = (1+\Omega)^{\frac{-\theta}{1-\epsilon}} \left(\frac{\epsilon}{\tau}\right)^{\frac{1}{1-\epsilon}} l_G^{1-\frac{\theta}{1-\epsilon}} A_G^{\frac{1}{1-\epsilon}} (1-\mu)^{\frac{-\epsilon}{1-\epsilon}} \left[\frac{1}{1-\mu} + \frac{\Omega}{1-\gamma}\right] K^{\frac{\theta}{1-\epsilon}}. \quad (8.2)$$

For output, substituting equations (2.12)-(2.14) into (3.4) and again using the definition of Ω gives:

$$Y = Y_P + Y_G = (1+\Omega)^{\frac{-\theta}{1-\epsilon}} \left(\frac{\epsilon}{\tau}\right)^{\frac{\epsilon}{1-\epsilon}} l_G^{1-\frac{\theta}{1-\epsilon}} A_G^{\frac{1}{1-\epsilon}} (1-\mu)^{\frac{-\epsilon}{1-\epsilon}} \left[\frac{1}{1+\lambda} + \frac{\Omega}{1-\gamma}\right] K^{\frac{\theta}{1-\epsilon}} \quad (8.3)$$

$$\equiv \Gamma K^{\frac{\theta}{1-\epsilon}}. \quad (8.4)$$

8.1 Proof of Theorem 1

The proof is an extension of the proof given in Greenwood and Huffman (1995) (hereafter GH). The difference is that here the utility function has a second argument, Q. Further restrictions than those given in GH are needed so that an increase in the capital stock, which reduces environmental quality, does not reduce the marginal utility of consumption so much that consumption falls in response to the increase in the capital stock.

The first step is to verify the assumptions of GH. To map the problem into the general framework of GH, we write the budget constraint (2.20) in the form:

$$c + k' = G(k, K) \tag{8.5}$$

$$G(k, K) \equiv (r(K) + 1 - \delta) k + w(K) + TR(K). \tag{8.6}$$

Here $r(K)$ is given by equation (2.16), $w(K)$ by equation (2.17), and $TR(K)$ by equation (2.23) and G represents household wealth.

The assumptions are in terms of G. For assumption (i), note:

$$G_1 = r(K) + 1 - \delta > 0, \tag{8.7}$$

$$\lim_{K \to 0} G_1(K, K) = \lim_{K \to 0} r(K) + 1 - \delta = \infty, \tag{8.8}$$

where the last inequality follows from equation (2.16). Further, $G(k, K) > 0$ for all k if and only if $w(K) > -TR(K)$. That is, the subsidies cannot be so large as to require lump sum taxes which more than exhaust wages, otherwise the savings rate would be affected. Substituting equation (2.23) for TR gives:

$$w(1 - l_G) + \tau E_P > -(1 - \mu)\tau E_G - A_G F_h(K_G, E_G, l_G) l_G + \lambda Y_G + \gamma r K_G. \tag{8.9}$$

Exploiting constant returns to scale and $K = K_G + K_P$ implies $w > -TR$ if and only if:

$$Y > rK. \tag{8.10}$$

Using equation (8.3) and equation (2.16) to substitute for aggregate output and the interest rate and canceling gives:

$$\frac{1}{\eta_k} + \Omega > \theta(1 + \Omega), \tag{8.11}$$

$$\eta_k < \frac{1}{\theta + (1 - \theta)\Omega}, \tag{8.12}$$

which holds by assumption (3.5). For assumption (ii), note that the maximum sustainable capital stock $\hat{K}$ is the solution to:

$$G\left(\hat{K}, \hat{K}\right) = \hat{K}. \tag{8.13}$$

Equation (8.4) implies:

$$G\left(\hat{K}, \hat{K}\right) = \Gamma \hat{K}^{\theta 1-\epsilon} + (1-\delta)\hat{K} = \hat{K}. \tag{8.14}$$

Hence:

$$\hat{K} = \left(\frac{\Gamma}{\delta}\right)^{\frac{1-\epsilon}{1-\theta-\epsilon}}, \tag{8.15}$$

and so the maximum sustainable capital stock is finite. For assumption (iii), note that $G_{11} = 0$ and $G_{12} = r_K(K) < 0$ and so $G_{11} + G_{12} < 0$. Next

$$G_2(K, K) = r_K(K)K + \frac{\partial(w + TR)}{\partial K}, \tag{8.16}$$

and from equation (8.10), $w + TR = Y - rK$. Hence, using equation (8.4):

$$G_2(K, K) = r_K(K)K + \frac{\theta}{1-\epsilon}\Gamma K^{\frac{\theta}{1-\epsilon}-1} - r_K(K)K - r(K) \tag{8.17}$$

$$= \frac{\theta}{1-\epsilon}\Gamma K^{\frac{\theta}{1-\epsilon}-1} - r(K). \tag{8.18}$$

Finally, using equation (8.7):

$$G_1 + G_2 = \frac{\theta}{1-\epsilon}\Gamma K^{\frac{\theta}{1-\epsilon}-1} + 1 - \delta > 0 \tag{8.19}$$

We have thus verified the wealth assumptions of GH.

It remains to show the methodology of GH applies here. Combining the first order condition (3.1) and envelope equation (3.2) gives the Euler equation:

$$u_c(C(K), Q(E(K))) = \beta u_c(C(K'), Q(E(K')))(r(K') + 1 - \delta) \tag{8.20}$$

$$C(K) = G(K, K) - K' \tag{8.21}$$

If $u_{cQ} = 0$, the Euler equation is in the form of GH and an equilibrium exists. If $u_{cQ} > 0$, then all aspects of the proof in GH go through analogously except for showing $c_K > 0$.

To show $c_K \geq 0$, let $H^j(K)$ be a given investment function which satisfies:

$$H_K^j(K) \leq G_1(K,K) + G_2(K,K), \tag{8.22}$$

and let $x = H^{j+1}(K)$ be the solution to:

$$u_c(G(K,K) - x, Q(E(K))) = \beta u_c\left(G(x,x) - H^j(x), Q(E(x))\right)(r(x) + 1 - \delta) \tag{8.23}$$

We then must show that:

$$H_K^{j+1}(K) \leq G_1(K,K) + G_2(K,K) \tag{8.24}$$

Let prime superscripts on a utility or wealth function denote that the function is evaluated at $[c(x), Q(E(x))]$ or $[x,x]$, respectively and let the utility or wealth function be otherwise evaluated at $[K,K]$. Taking the derivative of equation (8.23) with respect to K (interpreting the derivatives as finite differences if necessary), implies condition (8.24) becomes:

$$\begin{aligned} H_K^{j+1}(K) &= \frac{-u_{cc}(G_1 + G_2) + u_{cQ}Q_E(E)E_K(K)}{-u_{cc} + \beta u_c'(G_{11}' + G_{12}') - \beta u_{cc}'\left(G_1' + G_2' - H_K^j(x)\right) - \beta u_{cQ}'G_1'Q_E(E(x))E_K(x)} \\ &\leq G_1 + G_2. \end{aligned} \tag{8.25}$$

Using the Euler equation and that $G_{11} = 0$ reduces the above condition to:

$$\frac{u_{cQ}Q_E(E)}{u_c} \leq \frac{G_1 + G_2}{E_K(K)}\left(-\frac{G_{12}'}{G_1'} - \frac{u_{cc}'}{u_c'}\left(G_1' + G_2' - H_K^j(x)\right) - \frac{u_{cQ}'Q_E(E(x))E_K(x)}{u_c'}\right) \tag{8.26}$$

Given our assumptions, all three parts of the second term on the right hand side are positive. Hence it is sufficient to show:

$$\frac{u_{cQ}Q_E}{u_c} \leq \frac{G_1(K,K) + G_2(K,K)}{E_K(K)}\left(-\frac{G_{12}(x,x)}{G_1(x,x)}\right). \tag{8.27}$$

Next since G_{12}/G_1 is a decreasing function, it is sufficient to show:

$$\frac{u_{cQ}Q_E}{u_c} \leq \frac{G_1(K,K) + G_2(K,K)}{E_K(K)}\left(-\frac{G_{12}\left(\hat{K},\hat{K}\right)}{G_1\left(\hat{K},\hat{K}\right)}\right) \tag{8.28}$$

Define the right hand side as $z(K)$. Then, by assumption (3.6), $H_K^{j+1} \leq G_1 + G_2$ as required in the existence proof of GH.

As noted above, by following steps analogous to GH, we can thus establish that an equilibrium exists which has the desired properties.

8.2 Proof of Proposition 2

We will prove part (1), since parts (2)-(4) are analogous. After taking the derivative of equation (8.2) with respect to μ, we see that the derivative of aggregate emissions with respect to μ is positive if and only if:

$$-\theta\left[1-\gamma+\Omega\left(1-\mu\right)\right]\Omega_u+\left(1+\Omega\right)\left[\frac{1-\gamma}{1-\mu}+\epsilon\Omega-\Omega_u\left(1-\epsilon\right)\left(1-\mu\right)\right]>0. \tag{8.29}$$

Equation (2.12) implies:

$$\Omega_u=-\frac{\epsilon}{1-\theta-\epsilon}\frac{\Omega}{1-\mu}. \tag{8.30}$$

Hence, the above condition reduces to:

$$\frac{1-\gamma}{1-\mu}\left(\theta\epsilon\Omega+\left(1-\theta-\epsilon\right)\left(1+\Omega\right)\right)-\theta\epsilon\Omega>0. \tag{8.31}$$

Next, equations (2.9) and (2.10) imply:

$$\frac{\eta_E}{\eta_k}=\frac{1-\gamma}{1-\mu}. \tag{8.32}$$

Hence the above condition becomes:

$$\frac{\sigma_G}{\sigma_P}>\frac{\theta\epsilon\Omega}{\theta\epsilon\Omega+\left(1-\theta-\epsilon\right)\left(1+\Omega\right)}\eta_k, \tag{8.33}$$

as required.

8.3 Proof of Corollary 2

We will prove part (1), since parts (2)-(4) are analogous. Recall from the definition of equilibrium that we can treat either S or l_G as the given direct subsidy parameter, with the other determined from the zero profit condition in the subsidized sector. Hence, taking the derivative of equation (8.3) with respect to l_G implies that the derivative of aggregate output with respect to l_G is negative if and only if:

$$\begin{aligned}-\quad&\theta l_G\left(\frac{1-\gamma}{1+\lambda}+\Omega\right)\Omega_{l_G}+\left(1+\Omega\right)\left(1-\epsilon\right)l_G\Omega_{l_G}+\\&\left(1-\theta-\epsilon\right)\left(1+\Omega\right)\left(\frac{1-\gamma}{1+\lambda}+\Omega\right)<0.\end{aligned} \tag{8.34}$$

Equation (2.12) implies:

$$\Omega_{l_G} = \frac{-\Omega}{l_G\left(1-l_G\right)}. \tag{8.35}$$

Hence the above condition reduces to:

$$\theta\Omega\left(\frac{1-\gamma}{1+\lambda}+\Omega\right)+\left(1-\theta-\epsilon\right)\left(1+\Omega\right)\left(\frac{1-\gamma}{1+\lambda}+\Omega\right)\left(1-l_G\right)<\left(1-\epsilon\right)\left(1+\Omega\right)\Omega. \tag{8.36}$$

Since $1-\gamma<1+\lambda$, it is sufficient to show:

$$\theta\Omega\left(1+\Omega\right)+\left(1-\theta-\epsilon\right)\left(1+\Omega\right)\left(\frac{1-\gamma}{1+\lambda}+\Omega\right)\left(1-l_G\right)<\left(1-\epsilon\right)\left(1+\Omega\right)\Omega, \tag{8.37}$$

which holds if and only if:

$$\left(\frac{1-\gamma}{1+\lambda}+\Omega\right)\left(1-l_G\right)<\Omega \tag{8.38}$$

$$\frac{1-\gamma}{1+\lambda}<\frac{l_G}{1-l_G}\Omega. \tag{8.39}$$

Now from equation (2.12):

$$\frac{l_G}{1-l_G}\Omega=\left(\frac{\left(1-\gamma\right)^{1-\epsilon}\left(1-\mu\right)^{\epsilon}}{A_G}\right)^{\frac{1}{1-\theta-\epsilon}}>1 \tag{8.40}$$

Here the inequality follows from condition (2.11). Hence the right hand side of (8.39) is greater than one, which implies immediately that condition (8.39) holds. Hence aggregate output is decreasing in the level direct subsidies. From Proposition 2, aggregate output decreases and emissions rise with an increase in direct subsidies if and only if the condition (4.5) holds.

8.4 Proof of Proposition 4

Proposition 4 and Corollary 4 require calculation of the steady state emissions. Evaluating equation (8.20) at the steady state capital stock $\bar{K}$ implies:

$$1=\beta\left(r\left(\bar{K}\right)+1-\delta\right). \tag{8.41}$$

Let $\beta=\frac{1}{1+\rho}$, then:

$$r\left(\bar{K}\right)-\delta=\rho, \tag{8.42}$$

which is the modified golden rule for this economy. Equation (2.16) implies:

$$\bar{K} = \left(\frac{\theta}{\rho+\delta}\right)^{\frac{1-\epsilon}{1-\theta-\epsilon}} \left(\frac{\epsilon}{\tau}\right)^{\frac{\epsilon}{1-\theta-\epsilon}} \frac{1+\Omega}{\Omega} \left(1-l_G\right). \tag{8.43}$$

Substituting equation (8.43) into equation (8.3) gives the steady state output:

$$\bar{Y} = \left(\frac{\theta}{\rho+\delta}\right)^{\frac{\theta}{1-\theta-\epsilon}} \left(\frac{\epsilon}{\tau}\right)^{\frac{\epsilon}{1-\theta-\epsilon}} \left(1-l_G\right) \left(\frac{1-\gamma}{1+\lambda}\frac{1}{\Omega}+1\right). \tag{8.44}$$

Substituting equation (8.43) into equation (8.2) gives the steady state emissions:

$$\bar{E} = \left(\frac{\theta}{\rho+\delta}\right)^{\frac{\theta}{1-\theta-\epsilon}} \left(\frac{\epsilon}{\tau}\right)^{\frac{1-\theta}{1-\theta-\epsilon}} \left(1-l_G\right) \left(\frac{1-\gamma}{1-\mu}\frac{1}{\Omega}+1\right). \tag{8.45}$$

We will prove part (1), since parts (2)-(4) are analogous. It is immediate from equation (8.45) and (2.12) that $\bar{E}$ is an increasing function of μ. Further, from Proposition 2, period 0 emissions rise if and only if condition (4.3) holds. For periods between 0 and the steady state, note that from equation (8.43), $\bar{K}$ is an increasing function of μ. Further, from Theorem 1, $H(K)$ is strictly increasing and concave in K. Hence, K will converge monotonically from $K_0 = \bar{K}$ to a new steady state $\bar{\bar{K}}$ from below. Given that emissions are an increasing in the capital stock, emissions will also increase monotonically to a new steady state $\bar{\bar{E}} > \bar{E}$. If condition (4.3) does not hold, then we have shown that $E_0 < \bar{E}$, $\bar{\bar{E}} > \bar{E}$, and E_t is converging monotonically to $\bar{\bar{E}}$. Hence there exists a t^* such that for all $t \geq t^*$, $E_t > \bar{E}$. If condition (4.3) holds, then it is immediate that $E_t > \bar{E}$ for all t since $E_0 > \bar{E}$ and E_t is monotonically increasing.

8.5 Proof of Corollary 4

Taking the derivative of (8.45) with respect to l_G implies steady state emissions is increasing in S if and only if:

$$\frac{1-\gamma}{1-\mu}\frac{1-l_G}{l_G\Omega} - 1 > 0. \tag{8.46}$$

Using the definition of Ω from equation (2.12) results in:

$$\frac{1-\gamma}{1-\mu} > (1-\gamma)^{\frac{1-\epsilon}{1-\theta-\epsilon}} (1-\mu)^{\frac{\epsilon}{1-\theta-\epsilon}} A_G^{\frac{-1}{1-\theta-\epsilon}}. \tag{8.47}$$

Simplifying then gives

$$A_G > (1-\gamma)^{\theta} (1-\mu)^{1-\theta}, \tag{8.48}$$

as required. Hence $\bar{E}$ is increasing in S if and only if condition (8.48) holds.

Taking the derivative of (8.44) with respect to l_G implies steady state output is decreasing in S if and only if:

$$\frac{1-\gamma}{1+\lambda}\frac{1-l_G}{l_G\Omega}-1<0 \tag{8.49}$$

$$\frac{1-\gamma}{1+\lambda}<\frac{l_G\Omega}{1-l_G}, \tag{8.50}$$

which we have shown holds in Section 8.3. Hence $\bar{Y}$ is decreasing in S.

Combining the results for $\bar{E}$ and $\bar{Y}$ establishes the Corollary.

8.6 Proof of Proposition 6

For the first part, let conditions (3.5) and (3.6) hold so that an equilibrium exists. Substituting equilibrium conditions (2.4)-(2.8), and transfers (2.23) into the budget constraint gives:

$$\begin{aligned} c+k'-(1-\delta)k &= F_k(K_P,E_P,l_P)k+F_l(K_P,E_P,l_P)+F_E(K_P,E_P,l_P)E_P+\\ &\quad A_GF_E(K_G,E_G,l_G)E_G-r\gamma K_G-\\ &\quad l_G(F_l(K_P,E_P,l_P)-A_GF_l(K_G,E_G,l_G))-\lambda Y_G. \end{aligned} \tag{8.51}$$

Evaluating at $k=K$ and using $K=K_G+K_P$ results in:

$$\begin{aligned} C+K'-(1-\delta)K &= F_k(K_P,E_P,l_P)K_P+F_l(K_P,E_P,l_P)+F_E(K_P,E_P,l_P)E_P+\\ &\quad A_GF_E(K_G,E_G,l_G)E_G+r(1-\gamma)K_G-\\ &\quad l_G(F_l(K_P,E_P,l_P)-A_GF_l(K_G,E_G,l_G))-\lambda Y_G. \end{aligned} \tag{8.52}$$

Exploiting constant returns to scale and equation (2.12) results in:

$$C+K'-(1-\delta)K=Y_P+Y_G=Y. \tag{8.53}$$

Now we have shown in Section 6 that in equilibrium $Y=\psi K^{\theta}E^{\epsilon}$ and hence equation (8.53) is equivalent to the resource constraint (6.6).

For the second part, consider any solution to the planning problem E and K' given K. Let E_G and E_P be given by equations (6.2) and (6.3), respectively. Note that $E_G+E_P=E$. Further, let $K_G=\frac{1}{1+\Omega}K$ and $K_P=\frac{\Omega K}{1+\Omega}$, noting also that $K_P+K_G=K$. Given these allocations, let r, τ, and w be defined according to equations (2.6)-(2.8). Let

$S = l_G\left(w - A_G F_l\left(K_G, E_G, l_G\right)\right)$. For equation (2.3), the above definitions of r, E_G, E_P, K_G, and K_P imply:

$$\begin{aligned}
&(1-\gamma)\,\theta\left(\frac{K\Omega}{1+\Omega}\right)^{\theta-1}\left(\frac{\Omega\,(1-\mu)\,E}{1-\gamma+\Omega\,(1-\mu)}\right)^{\epsilon}(1-l_G)^{1-\theta-\epsilon} = \\
&A_G\theta\left(\frac{K}{1+\Omega}\right)^{\theta-1}\left(\frac{(1-\gamma)\,E}{1-\gamma+\Omega\,(1-\mu)}\right)^{\epsilon} l_G^{1-\theta-\epsilon}. \qquad (8.54)
\end{aligned}$$

After canceling, we see that the above equation holds. It is easy to see that equation (2.4) also holds given the above allocations. Thus the allocations of the planning problem are consistent with firm maximization and zero profits in the subsidized sector, as required by the competitive equilibrium. Further, let TR be defined by equation (2.23), then the government budget constraint is also satisfied.

It remains to show that the allocations are consistent with household maximization. First, using the definitions of ψ and the allocations for K and E imply:

$$\begin{aligned}
\psi K^{\theta} E^{\epsilon} &= \frac{a_G\left((1+\lambda)\,\Omega+1-\gamma\right) l_G^{1-\theta-\epsilon}}{\left(\Omega\,(1-\mu)+1-\gamma\right)^{1-\epsilon}(1+\Omega)^{\theta}(1-\gamma)^{1-\epsilon}}\left(\frac{1+\Omega}{\Omega}K_P\right)^{\theta}\cdot \\
&\quad \left(\frac{1-\gamma+\Omega\,(1-\mu)}{\Omega\,(1-\mu)}E_P\right)^{\epsilon} \qquad (8.55)
\end{aligned}$$

$$= \frac{a_G\left((1+\lambda)\,\Omega+1-\gamma\right) l_G^{1-\theta-\epsilon}}{(1-\gamma)^{1-\epsilon}\,\Omega^{\theta+\epsilon}\,(1-\mu)^{\epsilon}} K_P^{\theta} E_P^{\epsilon} \qquad (8.56)$$

$$= \frac{A_G l_G^{1-\theta-\epsilon}\Omega^{1-\theta-\epsilon}}{(1-\gamma)^{1-\epsilon}(1-\mu)^{\epsilon}} K_P^{\theta} E_P^{\epsilon} + \frac{a_G\,(1-\gamma)^{\epsilon}}{\Omega^{\theta+\epsilon}\,(1-\mu)^{\epsilon}} K_P^{\theta} E_P^{\epsilon} l_G^{1-\theta-\epsilon}. \qquad (8.57)$$

Using the definition of Ω, equation (6.1), and that $K_G\Omega = K_P$:

$$= K_P^{\theta} E_P^{\epsilon}(1-l_G)^{1-\theta-\epsilon} + \frac{a_G\,(1-\gamma)^{\epsilon}}{\Omega^{\theta+\epsilon}\,(1-\mu)^{\epsilon}}\left(\Omega K_G\right)^{\theta}\left(\frac{\Omega\,(1-\mu)}{1-\gamma}E_G\right)^{\epsilon} l_G^{1-\theta-\epsilon} \qquad (8.58)$$

$$= K_P^{\theta} E_P^{\epsilon}(1-l_G)^{1-\theta-\epsilon} + a_G K_G^{\theta} E_G^{\epsilon} l_G^{1-\theta-\epsilon} = Y. \qquad (8.59)$$

Hence, given the allocations, production of the planning problem equals income in the competitive equilibrium. Applying constant returns to scale and the above price and policy definitions to the resource constraint then yields the budget constraint.

Finally, combining equations (6.9) and (6.11) gives the Euler equation for the planning problem:

$$u_c(C,Q) = \phi\beta u_c(C',Q')\left(\theta\psi K^{\theta-1}E^{\epsilon} + 1 - \delta\right). \tag{8.60}$$

Substituting in the definition of ϕ from equation (6.8) and ψ from (6.5) results in:

$$\begin{aligned} u_c(C,Q) &= \frac{1+\Omega}{\Omega + \frac{1-\gamma}{1+\lambda}}\beta u_c(C',Q') \cdot \\ & \left(\theta\frac{a_G\left((1+\lambda)\Omega + 1 - \gamma\right) l_G^{1-\theta-\epsilon}}{\left(\Omega(1-\mu) + 1 - \gamma\right)^{\epsilon}(1+\Omega)^{\theta}(1-\gamma)^{1-\epsilon}} K^{\theta-1}E^{\epsilon} + 1 - \delta\right). \end{aligned} \tag{8.61}$$

Given either $\delta = 1$ or $\gamma = \lambda = 0$, we have:

$$u_c(C,Q) = \beta u_c(C',Q')\left(\theta\frac{A_G(1+\Omega)^{1-\theta} l_G^{1-\theta-\epsilon}}{\left(\Omega(1-\mu) + 1 - \gamma\right)^{\epsilon}(1-\gamma)^{1-\epsilon}} K^{\theta-1}E^{\epsilon} + 1 - \delta\right). \tag{8.62}$$

Inspection of equations (8.59), (6.6), and (8.62), which define the allocations of the planning problem, and equations (8.20) and (3.3), which define the aggregate allocations of the competitive equilibrium, reveals that the planning problem is consistent with household maximization in the competitive equilibrium if and only if:

$$r = \theta\frac{A_G(1+\Omega)^{1-\theta} l_G^{1-\theta-\epsilon}}{\left(\Omega(1-\mu) + 1 - \gamma\right)^{\epsilon}(1-\gamma)^{1-\epsilon}} K^{\theta-1}E^{\epsilon}. \tag{8.63}$$

Using equation (6.2), we see that the above relation holds if and only if:

$$r = \theta\frac{A_G(1+\Omega)^{1-\theta} l_G^{1-\theta-\epsilon}}{(1-\mu)^{\epsilon}(1-\gamma)^{1-\epsilon}\Omega^{\epsilon}} K^{\theta-1}E_P^{\epsilon}. \tag{8.64}$$

Using equation (2.12), the above holds if and only if:

$$r = \theta\frac{A_G l_G^{1-\theta-\epsilon}\Omega^{1-\theta-\epsilon}}{(1-\mu)^{\epsilon}(1-\gamma)^{1-\epsilon}} K_P^{\theta-1}E_P^{\epsilon}. \tag{8.65}$$

The above equation holds by the definition of Ω and equation (2.6). Hence the allocations of the planning problem are consistent with household maximization in the competitive equilibrium and thus the allocations of the planning problem can be supported by a competitive equilibrium.

8.7 Proof of Proposition 7

Imposing $K' = K = \bar{K}$ on the Euler equation (8.60) and resource constraint (6.6), using that either $\delta = 1$ or $\gamma = \lambda = 0$ results in:

$$1 = \beta\left(\theta\psi\phi\bar{K}^{\theta-1}\bar{E}^{\epsilon} + 1 - \delta\right) \tag{8.66}$$

$$\bar{K} = \left(\frac{\phi\psi\theta}{\rho+\delta}\right)^{\frac{1}{1-\theta}}\bar{E}^{\frac{\epsilon}{1-\theta}} \equiv \zeta\bar{E}^{\frac{\epsilon}{1-\theta}} \tag{8.67}$$

$$\bar{C} = \psi\bar{K}^{\theta}\bar{E}^{\epsilon} - \delta\bar{K} \tag{8.68}$$

$$= \psi\left(\frac{\phi\psi\theta}{\rho+\delta}\right)^{\frac{\theta}{1-\theta}}\bar{E}^{\frac{\epsilon}{1-\theta}} - \delta\left(\frac{\phi\psi\theta}{\rho+\delta}\right)^{\frac{1}{1-\theta}}\bar{E}^{\frac{\epsilon}{1-\theta}} \tag{8.69}$$

$$= \frac{\psi^{\frac{1}{1-\theta}}(\theta\phi)^{\frac{\theta}{1-\theta}}(\rho+\delta(1-\phi\theta))}{(\rho+\delta)^{\frac{1}{1-\theta}}}\bar{E}^{\frac{\epsilon}{1-\theta}} \equiv \xi\bar{E}^{\frac{\epsilon}{1-\theta}}. \tag{8.70}$$

Imposing the steady state conditions on equation (6.10) implies:

$$u_c\left(\bar{C},\bar{Q}\right)\epsilon\psi\bar{K}^{\theta}\bar{E}^{\epsilon-1} = -u_Q\left(\bar{C},\bar{Q}\right)Q_E\left(\bar{E}\right). \tag{8.71}$$

Using the values of $\bar{C}$ and $\bar{K}$ gives a single equation which determines $\bar{E}$:

$$\begin{aligned} 0 \quad = \quad & \epsilon(\rho+\delta)u_c\left(\xi\bar{E}^{\frac{\epsilon}{1-\theta}}, Q\left(\bar{E}\right)\right)\xi\bar{E}^{\frac{-(1-\theta-\epsilon)}{1-\epsilon}} + \\ & (\rho+\delta(1-\phi\theta))u_Q\left(\xi\bar{E}^{\frac{\epsilon}{1-\theta}}, Q\left(\bar{E}\right)\right)Q_E\left(\bar{E}\right). \end{aligned} \tag{8.72}$$

It is easy to see that the derivative of the left hand side of equation (8.72) with respect to $\bar{E}$ is negative. Hence an increase in any subsidy increases $\bar{E}$ if and only if the derivative of the left hand side of equation (8.72) with respect to the subsidy is positive.

Recall that any subsidy s impacts equation (8.72) only through ϕ and ψ. Thus:

$$\begin{aligned} \frac{\partial r.h.s.}{\partial s} \quad = \quad & \epsilon(\rho+\delta)\xi\bar{E}^{\frac{\epsilon}{1-\theta}}u_{cc}(.,.)\bar{E}^{-\frac{1-\epsilon-\theta}{1-\theta}}\xi_s + \epsilon(\rho+\delta)u_c(.,.)\bar{E}^{-\frac{1-\epsilon-\theta}{1-\theta}}\xi_s + \\ & (\rho+\delta(1-\phi\theta))u_{cQ}(.,.)\xi_s\bar{E}^{\frac{\epsilon}{1-\theta}}Q_E\left(\bar{E}\right) - \delta\theta u_Q(.,.)Q_E\left(\bar{E}\right)\phi_s. \end{aligned} \tag{8.73}$$

From equation (8.72) and the definition of relative risk aversion, the above equation is positive if and only if:

$$\left(1 - R\left(\bar{C}\right)\right)\xi_s + \frac{(\rho + \delta(1-\phi\theta))\, u_{cQ}(.,.)\, Q_E\left(\bar{E}\right)\bar{E}\xi_s}{\epsilon(\rho+\delta)\, u_c(.,.)} + \frac{\delta\theta\xi}{\rho+\delta(1-\phi\theta)}\phi_s > 0. \quad (8.74)$$

It is straightforward to establish that ϕ is increasing in all subsidies λ, μ, S, and γ. Therefore, a sufficient condition for the above inequality is $\xi_s < 0$: that subsidies decrease steady state consumption. From equation (8.70), $\xi_s < 0$ if and only if:

$$\phi(\rho + \delta(1-\phi\theta))\psi_s + \phi_s\psi\theta(\rho + \delta(1-\theta\phi) - \phi\delta) < 0 \quad (8.75)$$

It is straightforward, but tedious, to show that ψ is a decreasing function of all subsidies λ, μ, S, and γ. Hence, if $\gamma = \lambda = 0$ so that $\phi = 1$, the result is immediate. If instead $\delta = 1$, the above simplifies to:

$$\phi(1-\beta\phi\theta)\psi_s + \phi_s\psi\theta(1-\beta\phi\theta-\beta\phi) < 0, \quad (8.76)$$

which holds since $\beta\phi(1+\theta) > 1$, by assumption. We have therefore established that the derivative of $\bar{E}$ with respect to any subsidy is positive.

9 Appendix: Tables and Figures

Industry	total OECD Subsidies	Year	Source
Agriculture	$318 Billion	2002	OECD (2003)
Fishing	$6 Billion	1999	OECD (2001)
Energy	$20-30 Billion	1999	Barde and Honkatukia (2004)
Manufacturing	$43.7 Billion	1993	OECD (1998b)
Transport	$40 Billion	1998	Nash, Bickel, Rainer, Link, and Steward (2002)
Water	$10 Billion	n.a.	Barde and Honkatukia (2004)

Table 1: Total OECD subsidies in environmentally sensitive industries, in the most recent year available, as reported in Table 7.2 and text of Barde and Honkatukia (2004).

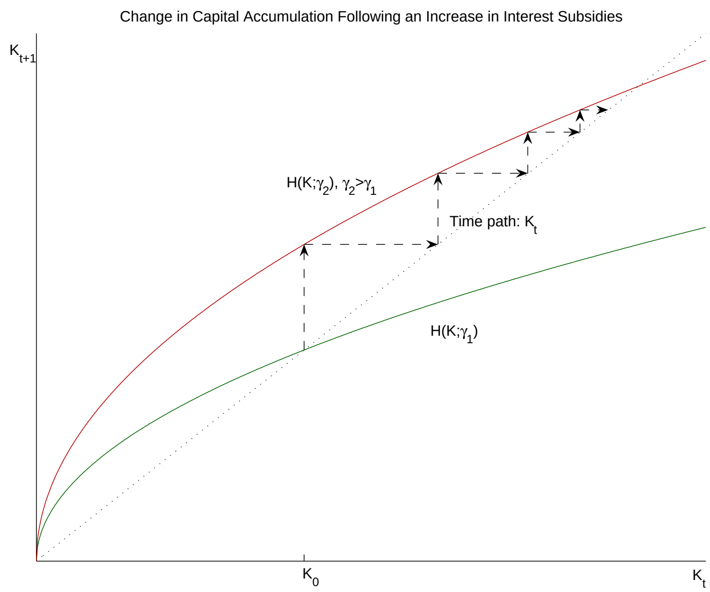

Figure 1: Dynamics of capital following an increase in subsidies γ, λ, or μ.

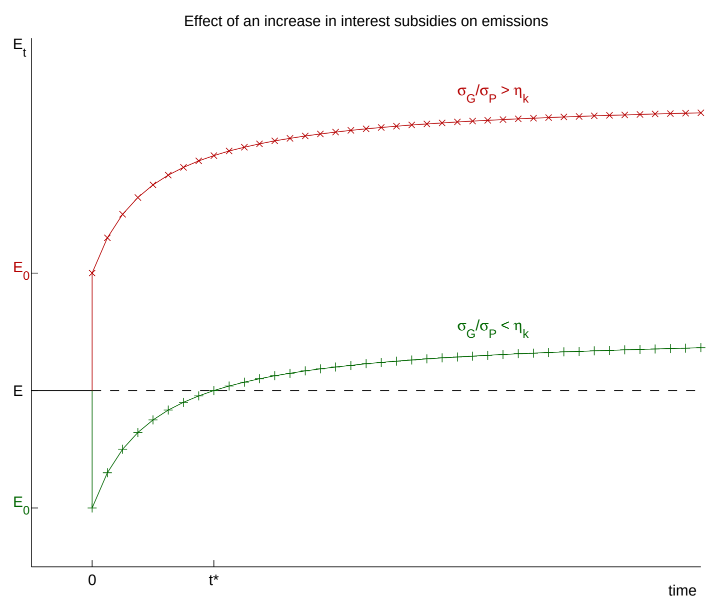

Figure 2: Possible dynamics of emissions following an increase in subsidies γ, λ, or μ.

References

Antle, J., J. Lekakis, and G. Zanias (eds.), 1998, *Agriculture, Trade and the Environment: The Imapct of Liberalization on Sustainable Development*, Elgar, Northhampton, MA.

Bagwell, K., and R. W. Staiger, 2006, "Will International Rules on Subsidies Disrupt the World Trading System?," *American Economic Review*, 96, 877–95.

Bajona, C., and D. L. Kelly, 2005, "Trade and the Environment With Pre-Existing Subsidies," University of Miami working paper.

Bajona, C., and D. L. Kelly, 2006, "Trade and the Environment with Pre-Existing Subsidies: A Dynamic General Equilibrium Analysis," University of Miami Department of Economics Working Paper.

Barde, J.-P., and O. Honkatukia, 2004, "Environmentally Harmful Subsidies," in Tom Tietenberg, and Henk Folmer (ed.), *The International Yearbook of Environmental and Resource Economics 2004/2005* . chap. 7, pp. 254–288, Edward Elgar, Northampton, MA.

Bartz, S., and D. L. Kelly, 2006, "Economic Growth and the Environment: Theory and Facts," University of Miami Working Paper.

Bovenberg, A., and L. H. Goulder, 1996, "Optimal Environmental Taxation in the Presence of Other Taxes: General-Equilibrium Analyses," *American Economic Review*, 86, 985–1000.

Brandt, L., and Z. Zhu, 2000, "Redistribution in a Decentralized Economy: Growth and Inflation in China Under Reform," *Journal of Political Economy*, 108, 422–439.

Earnhart, D., and L. Lizal, 2002, "Effects of Ownership and Financial Status on Corporate Environmental Performance," Discussion Paper 3557, Center for Economic Policy Research, CEPR Discussion Paper.

Galiani, S., P. Gertler, and E. Schargrodsky, 2005, "Water for Life: The Impact of the Privization of Water Services on Child Mortality," *Journal of Political Economy*, 113, 83–120.

Greenwood, J., and G. Huffman, 1995, "On the Existence of Nonoptimal Equilibria in Dynamic Stochastic Economies," *Journal of Economic Theory*, 65, 611–23.

Gupta, S., and S. Saksena, 2002, "Enforcement of Pollution Control Laws and Firm Level Compliance: A Study of Punjab, India," paper presented at 2nd World Congress of Environmental and Resource Economics, Monterey CA.

Hettige, H., M. Huq, and S. Pargal, 1996, "Determinants of Pollution Abatement in Developing Countries: Evidence from South and Southeast Asia," *World Development*, 24, 1891–1904.

Nash, C., P. Bickel, F. Rainer, H. Link, and L. Steward, 2002, "The Environmental Impact of Transport Subsidies," OECD Workshop on Environmentally Harmful Subsidies, 7-8 November.

OECD, 1998b, *Spotlight on Public Support to Industry*, OECD, Paris.

OECD, 2001, *Review of Fisheries in OECD Countries: Issues and Strategies*, OECD, Paris.

OECD, 2003, *Agricultural Policies in OECD Countries, Monitoring and Evaluation*, OECD, Paris.

Pargal, S., and D. Wheeler, 1996, "Informal Regulation of Industrial Pollution in Developing Countries: Evidence from Indonesia," *Journal of Political Economy*, 104, 1314–27.

Parry, I. W. H., 1997, "Environmental Taxes and Quotas in the Presence of Distorting Taxes in Factor Markets," *Resource and Energy Economics*, 19, 203–220.

Pasour, E. C. J., and R. R. Rucker, 2005, *Plowshares and Pork Barrels: The Political Economy of Agriculture*, The Independent Institute, Oakland, CA.

van Beers, C., and J. C. van den Bergh, 2001, "Perseverance of Perverse Subsidies and their Impact on Trade and Environment," *Ecological Economics*, 36, 475–86.

Wang, H., and Y. Jin, 2002, "Industrial Ownership and Environmental Performance: Evidence from China," Discussion Paper 2936, World Bank Policy Research Working Paper.

Wang, H., N. Mamingi, B. Laplante, and S. Dasgupta, 2002, "Incomplete Enforcement of Pollution Regulation: Bargaining Power of Chinese Factories," World Bank working paper.

Yin, X., 2001, "A Dynamic Analysis of Overstaff in China's State-Owned Enterprises," *Journal of Development Economics*, 66, 87–99.