Price and Quantity Regulation in General Equilibrium.

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Abstract

We consider a general equilibrium model with a production externality (for example pollution), in which the regulator does not observe firm productivity shocks. We examine quantity regulation in which price taking firms purchase permits issued by the regulator and price regulation in which firms pay a tax per unit of the regulated input. The quantity of permits issued by the regulator are independent of the productivity shock, since shocks are unobserved. Price regulation implies use of the regulated input is an increasing function of the productivity shock because firms take advantage of a good productivity shock by increasing input use. Thus price regulation generates both higher average production (or lower average costs) and more variable production (or more variable costs). Therefore, we show that in general equilibrium the relative advantage of quantity versus price regulation depends not only on the slopes of marginal benefits and costs, but on general equilibrium effects such as risk aversion, the degree of substitution between consumption goods, and the correlation of shocks among the consumption goods. The general equilibrium effects are often more important that the slopes of the marginal benefits and cost curves. For example, in the simplest model, a reasonable risk aversion coefficient implies quantity regulation generates higher welfare regardless of the benefit function.

1 Introduction

What is the optimal way to regulate an externality? When regulators have perfect information about the costs and benefits of regulation, economic theory implies both price (eg. tax) and quantity (eg. permit) regulation can balance marginal costs and benefits and so achieve the welfare maximizing quantity of the good with external costs or benefits. Yet most (but not all) regulation applied to firms is quantity based, from sulfur dioxide permits to government sponsored medical research. For many externalities, general equilibrium effects are likely to be important. For example, sulfur dioxide is a byproduct of electricity production, which is an input to production of a wide variety of goods. Production spillovers affect not just input choices of the firm, but also the return on capital and thus consumption/savings decisions. This paper provides analytic results for the relative advantages of prices versus quantity regulation in general equilibrium when regulators are uncertain about the firm's productivity shock. General equilibrium effects favor quantity regulation in the basic model for reasonable parameters.

In an important paper, Weitzman (1974) argued that the equivalence of price and quantity regulation does not hold when an information asymmetry exists between the firm and the regulator over the firm's costs. Consider an externality of pollution emissions by firms resulting in lower quality of the environment. Suppose firm emissions are very sensitive to the tax on emissions, and the marginal cost to firms of reducing emissions is uncertain. Then a small miscalculation of the appropriate tax rate could result in most firms paying the tax rather than reducing emissions. The quality of the environment then suffers, reducing welfare. In this case, quantity regulation is more attractive. Conversely, if firm behavior is not sensitive to the tax on emissions (for example if few low cost ways exist for the firm to reduce pollution), then a small error in the quantity of permits issued could impose very large costs on the firms, resulting in low production. In this case, a tax is more attractive. Weitzman assumes cost and benefit functions are quadratic in the regulated input and linear in the uncertainty, and a static, partial equilibrium environment.

Thus, many authors have extended Weitzman's framework. With regard to the choice of policy instruments, Roberts and Spence (1976), Weitzman (1978a), McKibbin and Wilcoxen

(1997), and Pizer (2001) consider hybrid polices which combine both price and quantity instruments. Laffont (1977) considers also price instruments applied to the consumer. In the area of uncertainty, Yohe (1978) adds output uncertainty, while Laffont (1977) differentiates between technological uncertainty and information differences. Hoel and Karp (2001) considers multiplicative uncertainty and Costello and Karp (2001) and Karp and Zhang (2001) allows for learning to reduce uncertainty about the costs. Stavins (1996) finds that the often positive correlation of environmental cost and benefit shocks favor quantity regulation for environmental externalities. Recently, Hoel and Karp (1998), Karp and Zhang (2001), and Newell and Pizer (1998) derive results similar to Weitzman (again in a quadratic, partial equilibrium setting) where environmental benefits accrue over time. Koenig (1985) considers both private and external benefits.

Empirical estimates of slopes of marginal cost and benefits are mixed across externalities. Koenig (1984) and Androkovich and Stollery (1991) find that taxes are generally preferred for fisheries, while Kolstad (1986) finds in favor of quantity regulation for air pollutants. Watson and Ridker (1984) allow for nonlinear marginal costs and benefits and obtain mixed results for air and water pollutants with quantity regulation often preferred early when regulation less strict, while price regulation did better later under more strict regulation. Hoel and Karp (1998), Hoel and Karp (2001), Kolstad (1996), Nordhaus (1994), and Pizer (1999) argue that the flatness of marginal benefits of climate change control favor price regulation.

Pizer (1999) and (2001) provides a computational analysis of price versus quantity regulation in general equilibrium for the case of climate change.[1] However, we are not aware of any *theoretical* paper which examines price and quantity regulation in general equilibrium, which is the focus of this paper.

An important, but overlooked, aspect of price regulation is how shocks to marginal costs generate stochastic variation in use of the regulated input and production, and therefore, risk to the consumer. Such risk is present only in general equilibrium, where the consumer side of the economy is modeled explicitly. Suppose a production externality in which use of an input (hereafter the 'regulated input') has external costs or benefits. Assume the regulator

[1] As we will note below, the Pizer model differs in most other respects from the model here, which makes the results difficult to compare.

sets either a quantity of permits to use the regulated input or a tax on the regulated input, prior to the firm productivity shock. Because the regulator does not know the realization of the productivity shock, the quantity of permits is independent of the shock. The permit price, however, depends on the stochastic productivity shock (when productivity is high, demand for permits is high). With taxes, the randomness is reversed: prices are fixed by the regulator, but the quantity of the regulated input is stochastic. We show variation in the regulated input caused by price regulation causes variation in production, leading to consumption risk. Perhaps surprisingly, variation in prices under quantity regulation does not generate *aggregate* consumption risk in general equilibrium. Changes in prices move wealth around the economy, but do not affect the wealth of the economy as a whole. Hence in general equilibrium, the increase in risk associated with tax regulation lowers expected welfare versus quantity regulation. Further, assuming separable utility, quantity regulation has no risk to external benefits derived from variation in the quantity of the regulated input, only risk derived from variation in the benefit function. The tax system adds risk to external benefits since the quantity of the regulated input is stochastic. Price regulation allows more use of the regulated input when productivity of the regulated input is high. Hence an income effect exists which favors tax regulation.[2]

For example, consider a coal fired electric plant which must either pay a tax on sulfur emissions or must buy a permit to emit sulfur. Suppose a positive productivity shock allows the firm to produce more electricity with the same inputs. Permits are in fixed supply, so no additional sulfur is emitted with quantity regulation, but production of electricity rises because the firm is more productive with the same inputs. Tax regulation implies firms emit more sulfur in order to take advantage of the productivity shock. Hence electricity production increases with both types of regulation, but electricity production rises more with tax regulation. Similarly, during a low productivity shock, electricity production falls more under tax regulation. On average, therefore, more electricity is produced with tax regulation, but electricity production is more variable. If households are sufficiently risk averse, desire for smooth consumption under quantity regulation outweighs higher average

[2]The last two considerations are equivalent to issues noted by Weitzman (1974). The first factor is what is new and, we show, important.

production with tax regulation, irrespective of environmental benefits.

One would expect, then, that the risk associated with tax regulation tilts the results in favor of quantity regulation. We provide sufficient conditions for which quantity regulation generates higher welfare than price regulation. The sufficient conditions are generally reasonable: if the coefficient of relative risk aversion is greater than or equal to one half, quantity regulation results in higher welfare *regardless* of the benefit function. The previous literature emerges as a special case when we assume (as in the previous literature) linear uncertainty and risk neutrality.

Our proof technique is new. Most of the literature forms quadratic approximations of the cost and benefit functions, then derives the optimal linear firm decisions, and finally compares the quadratic indirect welfare functions. Instead, we approximate the indirect welfare function (the indirect utility). Because we approximate the solution rather than the problem, we obtain what is usually in practice a more accurate approximation (for example, the approximation is exact given CRR utility and Cobb-Douglas production). A disadvantage of our technique is that we obtain only sufficient conditions for which quantity and price regulation provide higher welfare.

Our specification of the shock as a productivity shock implies price regulation is procyclical, amplifying the effects of shocks on production. We also relax this assumption and consider arbitrary shocks. For the case where a shock simultaneously increases productivity but decreases the productivity of the regulated input, price regulation implies that firms decrease use of the regulated input when productivity is high. Under these conditions, the risk argument favors price regulation. We also consider multiple consumption goods. Assuming the aggregate productivity shock and the productivity shock to the production of the good with the externality are correlated, our results go through essentially unchanged. Independent shocks, however, are more favorable to price regulation. Regardless, general equilibrium effects play a central role.

We also provide an example for which we compute the exact relative advantage as a function of the risk aversion coefficient. In the example, quantity regulation dominates price regulation for risk aversion coefficients greater than or equal to one half. The example also assumes flat marginal benefits. Hence a partial equilibrium analysis would find in favor of price regulation.

The model is sufficiently general for analysis of a variety of production externalities for which general equilibrium effects are important. For example the model could be applied to subsidies for research and development versus direct financing of research or interest rate targeting versus money growth rate targeting in a model where money is an input to production, etc.

2 Model

The timing within the period is such that the regulator moves first, then the shocks are realized, and finally households and firms move. Hence, households and firms know the shock, but the regulator does not. The welfare differences in regulation stem from the information asymmetry between the regulator and the firm subject to the regulation.

In general, we suppose a negative externality in which price regulation is a tax. Clearly, the model applies equally well to a positive externality in which price regulation is a subsidy.

2.1 A Motivating Example

Here we show how to map a simple model of a pollution externality into the framework we are considering. Suppose competitive firms have access to a production technology that uses capital (k) and technology (Z) to produce consumption goods $ZF(k)$. Firms rent the capital from households at rental rate r. Firms may convert n units of output into $q = s(n)$ pollution control units, scrubbers. We assume $q \in [0,1]$ and that scrubber production is increasing and concave: $s_n(n) > 0$ and $s_{nn}(n) < 0$. Firm emissions (E) are proportional to the fraction of production without scrubbers:

$$E = (1 - s(n))\, ZF(k) \tag{2.1}$$

We consider two types of regulation on the firm. Price regulation consists of firms paying a tax τ set by the regulator per unit of emissions. Because the market price of emissions is zero, the price is equal to the tax on emissions. Under quantity (permit) regulation, the regulator issues E permits per firm, which competitive firms purchase on a spot market at price τ. Alternatively, the regulator could give each household E permits, which the households sell on the spot market. Given either tax or permit regulation, the firm problem is:

$$\max_{n,k} (1-n) ZF(k) - rk - \tau (1 - s(n)) ZF(k) \tag{2.2}$$

Alternatively, using the definition of emissions:

$$\max_{E,k} \left(1 - s^{-1}\left(1 - \frac{E}{ZF(k)}\right)\right) ZF(k) - rk - \tau E \tag{2.3}$$

Our assumption is that incentives to merge and/or spin off firms results in firms with constant returns to scale production technology. Further, we assume that the production technology is homogeneous of degree one in $\exp(z) = \phi(Z)$. Let $z \sim G_z$, a symmetric distribution with mean zero and finite variance σ_z^2. Let $e^z f(k, E)$ denote the output net of output converted to scrubbers. It is easy to see that f is strictly concave and strictly increasing in each input. It is straightforward to generate a cost function s satisfying the concavity assumptions which yields a Cobb-Douglas production function, for example.[3] Thus we have mapped a pollution externality problem into a production externality in which pollution is an input to production.

2.2 Firm Behavior

Suppose competitive firms have access to a production technology that uses capital (k), an input E with an externality (the regulated input), and technology (z) to produce $c = e^z f(k, E)$ consumption goods. We assume f is concave, increasing, C^3, satisfies the Inada conditions, and that $f(0, E) = f(k, 0) = 0$.[4] Firms rent capital at rental rate r. Assume without loss of generality that the market price of E in the absence of government regulation

[3] Yohe (1976) and Yohe (1977) also consider emissions as an input to production, but the analysis is partial equilibrium. The use of emissions as an input to production is standard (see for example Copeland and Taylor (2001)).

[4] Like most of the price and quantity literature (a notable exception is Costello and Karp, 2001), we assume an interior solution for the technical reason that the proofs use Taylor series expansions to compute expected welfare. However, John and Pecchenino (1994) and Stokey (1998) point out that for the case of pollution externalities, if income is low, using the dirtiest possible technology may be optimal.

is zero, but firms pay either a tax τ per unit of E or buy government issued permits to use E on a spot market at price τ. Firms maximize profits:

$$\Pi = \max_{E,k} \{e^z f(k, E) - rk - \tau E\} \tag{2.4}$$

Let subscripts on functions denote derivatives. The first order conditions are:

$$r = e^z f_k(k, E) \tag{2.5}$$

$$\tau = e^z f_e(k, E) \tag{2.6}$$

Given our assumption that f_e is continuous and monotonic, a unique inverse exists $f_e^{-1}(e^{-z}\tau)$. We assume that the inverse function has finite mean for all $0 < \tau < \infty$.

Equation (2.5) determines the rental rate of capital, r. Equation (2.6) implies purchases of the regulated input E account for $s(k, E) = \frac{Ef_e(k,E)}{f(k,E)}$ share of output. Tax regulation implies τ is set by the regulator and E is determined from Equation (2.6). Conversely, under quantity regulation E is set by the regulator and τ is determined by Equation (2.6). The regulator does not observe the productivity shock, and hence the policy is set independent of z. Hence, under quantity regulation, τ varies with the productivity shock and E does not, while in the case of price regulation E varies with the productivity shock while τ does not. Note that applying the implicit function theorem to Equation (2.6) implies that, given tax regulation, E is an increasing function of z.

2.3 Households

Under our framework, input decisions are made at the firm level. Households make a consumption decision which is completely determined from the budget constraint. In the example section, we discuss how the results might change if the model was a consumption externality where the government regulation directly affects household decisions.

The household budget constraint sets capital rental income plus government transfers,

TR, equal to spending on consumption.

$$c = rk + TR \tag{2.7}$$

Households have preferences for consumption and receive disutility from E, represented by a separable utility function:

$$U = u(c) - B(y, E) \tag{2.8}$$

Here B is the damage caused by E ($-B$ is the benefit function or the benefit of regulating E). We assume y is an iid shock to damages, with symmetric distribution G_y. We assume B is increasing, strictly convex, and twice differentiable.

Notice that the household sector differentiates our model from previous work on prices versus quantities. In our model, costs associated with producing environmental benefits are translated into reductions in consumption, for which the household has concave preferences. In addition, all government revenue eventually ends up as consumption, and is thus accounted for in utility terms. Another interpretation is that in previous work, the firm bears the entire marginal cost of producing environmental quality (here in terms of lost consumption), while in our work the cost is split between the consumer and firm. If we specify $u(c) = c$, then the consumer side of the economy vanishes and the problem reduces to the previous literature with f as the negative of the cost function and B as the negative of the benefit function (although we do not assume these functions are quadratic).

The household maximizes utility subject to the budget constraint.

$$\max_c \{u(c) - B(y, E)\} \quad \text{subject to:}$$

$$c = rk + TR \tag{2.9}$$

2.4 Government Behavior

The government returns all revenue as lump sum transfers. Under either system:

$$TR = \tau E \tag{2.10}$$

Optimal policy may depend on currently available information (e.g. k). Let $\hat{E}$ denote the quantity regulation policy. For simplicity, we model the case where the regulator observes the shocks (ie where no information asymmetry exists) as quantity regulation, E^*. The price regulation policy τ implies firms choose $\tilde{E}(z,\tau) = f_e^{-1}(e^{-z}\tau, k)$. We assume the regulator chooses a policy and then households and firms choose allocations. Because the regulator needs to predict optimal household and firm behavior in order to set the policy, household and firm behavior are first described by rules which depend on the policy.

Three classes of regulation exist. If no information asymmetry exists, the regulator chooses from a class of functions $E^* \in \mathcal{E}^* = \left\{E(k,z,y) : \Re_+^3 \to \Re_+\right\}$. When the regulator does not observe the productivity shock (ie when the information asymmetry is present), then quantity regulation implies the regulator chooses from a class of functions: $\hat{E} \in \hat{\mathcal{E}} = \{E(k) : \Re_+ \to \Re_+\}$. Similarly, tax regulation given asymmetric information implies the regulator chooses from a class:

$$\tilde{E} \in \tilde{\mathcal{E}} = \left\{E : E = f_e^{-1}\left(e^{-z}\tau, k\right), \tau = \tau(k) : \Re_+ \to \ [0,1]\right\} \tag{2.11}$$

The difference between price and quantity regulation is described by the class of functions the regulator may choose from. If the regulator observes the shocks (if no informational asymmetries exist), the class of functions available include functions which depend on y and z. If the regulator uses quantity regulation and the regulator does not observe the shocks, the class of functions is independent of y and z. If the regulator selects a tax and does not observe the shocks, the resulting class of emissions functions is independent of y and increasing in z.

3 Equilibrium

As is common in the literature, we require households to act optimally for all regulation functions in the class. The requirement is analogous to subgame perfection and rules out some of the equilibria (for example, households could consume zero for all policies except one, and act optimally given the remaining policy. The regulator would then optimally choose the one remaining policy).

Equilibrium in our model varies with the type of regulation. Quantity regulation implies an equilibrium price of permits that is consistent with optimal firm behavior. Conversely, tax regulation equilibrium is an $\tilde{E}$ that is consistent with optimal firm behavior. Regardless of the type of regulation, the competitive equilibrium is characterized by the same set of equations.

Definition 1 *A competitive equilibrium CE^* ($\hat{CE}$, $\tilde{CE}$, respectively) given shocks y and z, policy $E^* \in \mathcal{E}^*$ ($\hat{E} \in \hat{\mathcal{E}}$, $\tilde{E} \in \tilde{\mathcal{E}}$) , and capital k is a set of a price r, transfer TR, allocation c, and price τ (price τ, allocation E) which satisfy: the household budget constraint (2.7), the government budget constraint (2.10), firm maximization (Equations (2.5) and (2.6)), and consumer maximization (2.9) for all $E^* \in \mathcal{E}^*$ ($\hat{E} \in \hat{\mathcal{E}}$, $\tilde{E} \in \tilde{\mathcal{E}}$).*

The definition of competitive equilibrium implies the existence of planning problems which are consistent with competitive equilibrium.

PROPOSITION 1 *Suppose that, given y, z and k:*

$$c = e^z f(k, E) \tag{3.1}$$

$$\tau = e^z f_e(k, E) \tag{3.2}$$

Then prices and policies r and TR exist which, together with the allocations that satisfy (3.1) and (3.2), constitute competitive equilibria CE^*, $\hat{CE}$, and $\tilde{CE}$. Conversely, the allocations given by competitive equilibria CE^*, $\hat{CE}$, or $\tilde{CE}$ satisfy (3.1) and (3.2).

Proof: See Appendix.

4 Optimal Policy

Proposition 1 indicates that for a competitive equilibrium, we need only satisfy the resource constraint. Then a planner may either choose E and let τ be determined from (3.2), or substitute out for E using (3.2) and choose τ. Therefore the planning problem is:

$$\max_{E,\tau} \mathrm{E}\left[u\left(e^{z} f\left(k, E\right)\right) - B\left(y, E\right)\right] \quad \text{subject to: } \tau = e^{z} f_{e}\left(k, E\right) \tag{4.1}$$

In what follows, we suppress the dependence of the production and value functions on k where no confusion is possible. Welfare v under each system is:

$$\hat{v} = \max_{\hat{E}} \mathrm{E}\left[u\left(e^{z} f\left(\hat{E}\right)\right) - B\left(y, \hat{E}\right)\right] \tag{4.2}$$

$$\tilde{v} = \max_{\tau} \mathrm{E}\left[u\left(e^{z} f\left(\tilde{E}\left(z,\tau\right)\right)\right) - B\left(y, \tilde{E}\left(z,\tau\right)\right)\right] \tag{4.3}$$

The expected welfare gain of quantity regulation over price regulation is then:

$$\Delta \equiv \hat{v} - \tilde{v} \tag{4.4}$$

The next result is a sufficient condition for which quantity regulation results in higher welfare. Because shocks are unobserved, quantity regulation is a constant function $\hat{E}$. Tax regulation implies E is an increasing function of the productivity shock, $\tilde{E}\left(z,\tau\right)$. Optimality implies that the optimal constant function is at least as good as the mean of the optimal tax policy, $\bar{E} = \mathrm{E}\left[\tilde{E}\left(z,\tau\right)\right]$. If households are sufficiently risk averse, the mean of the optimal tax policy is preferred to tax regulation which varies positively over the productivity shock. Let R^{u} denote the relative risk aversion at the mean of the optimal tax policy:

$$R^{u}\left(\tilde{c}\right) = \frac{-\tilde{c} u_{cc}\left(\tilde{c}\right)}{u_{c}\left(\tilde{c}\right)}, \quad \tilde{c} = f\left(\bar{E}\right) \tag{4.5}$$

Similarly, let R^{f} denote the curvature of the production function:

$$R^{f}\left(\bar{E}\right) = \frac{-\bar{E} f_{ee}\left(\bar{E}\right)}{f_{e}\left(\bar{E}\right)} \tag{4.6}$$

Finally, let $\sigma^2_{z,\tilde{E}}$ and $\sigma^2_{\tilde{E}}$ denote the covariance between $\tilde{E}(z,\tau)$ and z and the variance of $\tilde{E}(z,\tau)$, respectively. Then:

PROPOSITION 2 *Let:*

$$R^f\left(\bar{E}\right) + R^u(\tilde{c})\, s\left(\bar{E}\right) + \frac{B_{ee}\left(y,\bar{E}\right)\bar{E}}{u_c(\tilde{c})\, f_e\left(\bar{E}\right)} \geq 2\frac{\sigma^2_{z,\tilde{E}}}{\sigma^2_{\tilde{E}}}\left[1 - R^u(\tilde{c})\right]\bar{E} \tag{4.7}$$

Then $\Delta \geq 0$.

Proof: let $v(y,z,E)$ denote the welfare given shocks y and z and policy E. Optimality implies:

$$\mathrm{E}\left[v\left(y,z,\hat{E}\right)\right] \geq \mathrm{E}\left[v\left(y,z,\bar{E}\right)\right] \tag{4.8}$$

Hence it is sufficient to show that the mean of the optimal tax policy yields higher welfare than the actual tax policy:

$$\hat{\Delta} = \mathrm{E}\left[v\left(y,z,\bar{E}\right)\right] - \mathrm{E}\left[v\left(y,z,\tilde{E}\right)\right] \geq 0 \Rightarrow \Delta \geq 0 \tag{4.9}$$

A second order approximation yields:

$$\mathrm{E}\left[v\left(y,z,\bar{E}\right)\right] \approx v\left(y,0,\bar{E}\right) + \frac{\sigma^2_z}{2} v_{zz}\left(y,0,\bar{E}\right) \tag{4.10}$$

$$\begin{aligned}\mathrm{E}\left[v\left(y,z,\tilde{E}\right)\right] \approx\; & v\left(y,0,\bar{E}\right) + \frac{\sigma^2_z}{2} v_{zz}\left(y,0,\bar{E}\right) + \\ & \tfrac{1}{2}\sigma^2_{\tilde{E}} v_{ee}\left(y,0,\bar{E}\right) + \sigma^2_{z,\tilde{E}} v_{ze}\left(y,0,\bar{E}\right)\end{aligned} \tag{4.11}$$

Hence:

$$\hat{\Delta} \approx \tfrac{1}{2}\left(-\sigma^2_{\tilde{E}} v_{ee}\left(y,0,\bar{E}\right) - 2\mathrm{cov}\left(z,\tilde{E}\right) v_{ze}\left(y,0,\bar{E}\right)\right) \tag{4.12}$$

Hence it is sufficient to show:

$$-v_{ee}\left(y,0,\bar{E}\right) - 2\frac{\sigma^2_{z,\tilde{E}}}{\sigma^2_{\tilde{E}}} \cdot v_{ze}\left(y,0,\bar{E}\right) \geq 0 \tag{4.13}$$

Direct calculation of the derivatives implies:

$$v_{ee}\left(y,0,\bar{E}\right)=u_c\left(f\left(\bar{E}\right)\right)f_{ee}\left(\bar{E}\right)+u_{cc}\left(f\left(\bar{E}\right)\right)f_e\left(\bar{E}\right)^2-B_{ee}\left(y,\bar{E}\right) \tag{4.14}$$

$$v_{ze}\left(y,0,\bar{E}\right)=f_e\left(\bar{E}\right)u_c\left(f\left(\bar{E}\right)\right)+u_{cc}\left(f\left(\bar{E}\right)\right)f_e\left(\bar{E}\right)f\left(\bar{E}\right) \tag{4.15}$$

Substituting (4.14) and (4.15) into (4.13) results in:

$$\begin{aligned} - \quad & u_c\left(f\left(\bar{E}\right)\right)f_{ee}\left(\bar{E}\right)-u_{cc}\left(f\left(\bar{E}\right)\right)f_e\left(\bar{E}\right)^2+B_{ee}\left(y,\bar{E}\right)- \\ & 2\frac{\sigma^2_{z,\tilde{E}}}{\sigma^2_{\tilde{E}}}f_e\left(\bar{E}\right)\left[u_c\left(f\left(\bar{E}\right)\right)+u_{cc}\left(f\left(\bar{E}\right)\right)f\left(\bar{E}\right)\right]\geq 0 \end{aligned} \tag{4.16}$$

$$\frac{-f_{ee}\left(\bar{E}\right)}{f_e\left(\bar{E}\right)}+R^u\left(\tilde{c}\right)\frac{f_e\left(\bar{E}\right)}{f\left(\bar{E}\right)}+\frac{B_{ee}\left(y,\bar{E}\right)}{f_e\left(\bar{E}\right)u_c\left(\tilde{c}\right)}\geq 2\frac{\sigma^2_{z,\tilde{E}}}{\sigma^2_{\tilde{E}}}\left[1-R^u\left(\tilde{c}\right)\right] \tag{4.17}$$

Substituting in for R^f and s and simplifying then yields the desired result. □

Equation (4.7) shows how risk aversion determines the relative advantage of prices versus quantities. The first two terms of the left hand side of Equation (4.7) is simply the relative risk aversion, but in terms of the input rather than consumption. As the utility and/or production function becomes more concave, households (at the time government policy is set) desire more smooth input usage, so the left hand side rises and quantity regulation becomes more attractive, since quantity regulation implies E is constant.

The third term represents the convexity of damages. Because damages are convex, households prefer constant damages versus damages which vary with the productivity shock. Hence, the more convex the damages, the more attractive quantity regulation is.

On the right hand side, the covariance between $\tilde{E}\left(z,\tau\right)$ and z is positive with price regulation as firms take advantage of good productivity shocks by using more inputs. If preferences are such that the income effect dominates the risk effect ($R^u<1$), then households prefer regulation that allows firms to take advantage of the good productivity shock and increase production even though consumption risk increases. Then, as the covariance rises, inputs become more productive with a positive productivity shock, so price regulation is more attractive, because price regulation allows firms to increase use of the more productive input E. However, if the risk effect dominates ($R^u > 1$), households prefer regulation that forces firms to smooth input use and thus production to reduce consumption risk. Thus households prefer quantity regulation.

Part of Equation (4.7) reflects the convexity of damages, which is often difficult to measure. However, even in the worst case (for quantity regulation) of linear damages, quantity regulation still dominates under reasonable assumptions, as seen by the following corollary.

COROLLARY 3 *Let either:*

1. $R^u(\tilde{c}) \geq \frac{R^f(\bar{E})}{s(\bar{E})+2R^f(\bar{E})}$, *or*
2. $u(c) = c$ *and* $f_{ee}(\bar{E}) + B_{ee}(y, \bar{E}) \geq 0$.

Then $\Delta \geq 0$.

Proof See Appendix.

Corollary 3 indicates the coefficient of relative risk aversion does not have to be very high for quantity regulation to be optimal: a coefficient of relative risk aversion greater than one half is sufficient, and most studies find the coefficient of relative risk aversion between one and three.[5] The concavity of the production function is generally straightforward to check. For example, suppose a Cobb-Douglas production function $f(k,E) = k^{1-\gamma}E^{\gamma}$. Then the first condition requires simply $R^u(\tilde{c}) \geq \frac{1-\gamma}{2-\gamma}$. Finally, any convexity in the damage function further tilts the result toward quantity regulation.

Our prediction that quantities dominate for reasonable risk aversion coefficients is at odds with Pizer (1999), which finds for price regulation in a computational general equilibrium model of climate change. However, it is important to note that the Pizer model differs from ours in most other respects. The Pizer model does not exhibit constant returns to scale, is

[5]Hurd (1989) estimates a value of 1.12. Mehra and Prescott (1980) also conclude that a reasonable number is not too far from one for models that do not explicitly account for leisure. See Kocherlakota (1996) for a thorough discussion.

dynamic, has a non-separable utility function, and has several shocks, most of which are not factor neutral productivity shocks.[6]

Prices wash out of the equilibrium describing consumer welfare under each system. Therefore, the difference between quantity and price regulation is essentially that E are an increasing function of the productivity shock under tax regulation and independent of the productivity shock under quantity regulation. The relative advantage therefore can be summarized by asking if the full information E is sufficiently increasing in the productivity shock. If E^* is constant, decreasing, or slightly increasing in the productivity shock then quantity regulation is preferred. If, however, E^* is sufficiently increasing in the productivity shock, then price regulation comes closer to achieving the full information welfare. In turn, whether or not E^* is increasing in the productivity shock depends on risk aversion. Suppose first that households are very risk averse. With a positive productivity shock, the return to E increases. However, increasing E results in unusually high consumption. Therefore, the social planner prefers to lower E with a positive productivity shock, reducing damages. Consumption remains smooth because the production increase from the productivity shock more than offsets the loss of production due to reduced input usage. If households are not very risk averse, then the planner takes advantage of the higher return given a positive productivity shock and increases input usage. Figure 1 illustrates the idea.

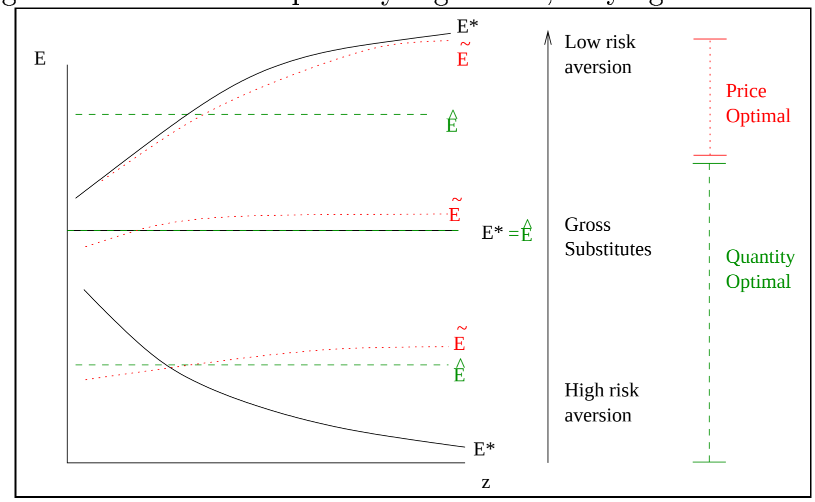
The second part of Corollary 3 compares our results with Weitzman (1974). The essential difference is that in general equilibrium, we model the consumer side of the economy. Corollary 3 makes a linear approximation of the variance and covariance in the correlation coefficient, a key assumption of Weitzman and the subsequent literature. If $u(c) = c$, then the consumer side of the economy vanishes and a sufficient condition for quantities to dominate prices is:

$$f_{ee} + B_{ee} \geq 0 \tag{4.18}$$

Equation (4.18) is the negative of Equation (20) in Weitzman (1974). Thus moving from

[6] Our model is less complicated, which allows us to get analytical results. But mainly the differences arise since Pizer extends the climate change literature, while our model extends the price and quantity literature. These two literatures generally differ in their assumptions.

Figure 1: Price versus quantity regulation, varying risk aversion.



partial equilibrium to general equilibrium adds two terms both of which favor quantity regulation.

A converse of Proposition 2 is available. We can choose a tax policy $\hat{\tau}$ which results in average input usage equal to $\hat{E}$, the optimal quantity regulation. Optimality implies the optimal tax policy yields at least the expected welfare of such a policy. Hence if risk aversion is sufficiently low, the tax policy $\hat{\tau}$ results in higher expected welfare than the policy $\hat{E}$. Let $\hat{c} = f\left(\hat{E}\right)$, then:

PROPOSITION 4 *Let:*

$$R^f\left(\hat{E}\right) + R^u\left(\hat{c}\right) s\left(\hat{E}\right) + \frac{B_{ee}\left(y, \hat{E}\right)\hat{E}}{u_c\left(\hat{c}\right) f_e\left(\hat{E}\right)} \leq 2\frac{\sigma^2_{z,\tilde{E}(z,\hat{\tau})}}{\sigma^2_{\tilde{E}(z,\hat{\tau})}}\left[1 - R^u\left(\hat{c}\right)\right]\hat{E} \tag{4.19}$$

Then $\Delta \leq 0$.

Proof: See Appendix.

5 Generalized Shocks

In the next two sections, we examine two extensions to check for robustness of the results and provide insights into the relationship between general equilibrium and price versus quantity regulation. The two most important assumptions for our results are that the shock is factor neutral and that only one consumption good exists. Because productivity shocks are factor neutral, price regulation amplifies the 'business cycle,' in that an increase in the productivity of inputs directly increases production and also gives an incentive to use additional inputs, which further increases production. In this section, we generalize the model to consider arbitrary production shocks.

Suppose we modify the production technology to $f(z, E)$. We adopt the convention that z is a positive shock, ie that $f_z(z, E) \geq 0$. We assume that f is strictly concave and C^3 in E and z, and satisfies the Inada conditions in E. We also assume again that the inverse function $f_e^{-1}(z, \tau)$ has finite mean for all $0 < \tau < \infty$.

The firm first order condition is:

$$\tau = f_e(z, E) \tag{5.20}$$

The implicit function theorem implies immediately that E is an increasing function of z if and only if an increase in z increases the marginal product of E. Specifically, if and only if $f_{ze}(z, E) > 0$. Hence if technology and E are substitutes, input usage under price regulation is countercyclical. For example, one could imagine two types of capital, a clean and dirty technology. If the productivity shock is factor-specific to the clean capital, emissions are counter-cyclical.

Welfare given price and quantity regulation are:

$$\hat{v} = \max_{\hat{E}} \mathrm{E}\left[u\left(f\left(z, \hat{E}\right)\right) - B\left(y, \hat{E}\right)\right] \tag{5.21}$$

$$\tilde{v} = \max_{\tau} \mathrm{E}\left[u\left(f\left(z, \tilde{E}(z, \tau)\right)\right) - B\left(y, \tilde{E}(z, \tau)\right)\right] \tag{5.22}$$

Reinterpret $\bar{E} = \tilde{E}(\bar{z}, \tau)$ as the input usage given price regulation at the mean shock and optimal policy τ. Reinterpret $\tilde{c} = f\left(\bar{z}, \tilde{E}(\bar{z}, \tau)\right)$ as consumption given price regulation at the mean shock and optimal policy τ. Then:

PROPOSITION 5 *Let* $v_e\left(y,\bar{z},\bar{E}\right)\tilde{E}_{zz}\left(\bar{z},\tau\right)$ *be sufficiently small and:*

$$R^u\left(\bar{c}\right)s\left(\bar{E}\right)+\frac{B_{ee}\left(y,\bar{E}\right)\bar{E}}{u_c\left(\bar{c}\right)f_e\left(\bar{E}\right)}\geq R^f\left(\bar{E}\right)\left[1-2R^u\left(\bar{c}\right)\left(\frac{s\left(\bar{E}\right)f_z\left(\bar{E}\right)}{\bar{E}f_{ze}\left(\bar{E}\right)}\right)\right] \tag{5.23}$$

Then $\Delta\geq 0$.

Proof: See Appendix.

Notice that the results reduce to Corollary 3 for the factor neutral productivity shock $f(z,E)=e^zf(E)$ and $\bar{z}=0$ or $f(z,E)=zf(E)$ and $\bar{z}=1$. The theorem also reduces to the familiar Weitzman condition for the case $u(c)=c$. If $f_{ze}<0$, the advantage of quantities over prices is drastically reduced, as the second term on the right hand side becomes positive.

Propositions 2 and 4 require the production function to have factor neutral shocks (which implies $f_{ze}>0$), but allow the production function to be such that $\tilde{E}_{zz}$ is arbitrary, since the approximation is around $\bar{E}$, rather than $\bar{z}$. On the other hand, Proposition 5 allows the production function to have possibly factor specific shocks (f_{ze} is arbitrary), but requires the production function to be such that $\tilde{E}_{zz}$ is small. Both improve upon previous literature with quadratic costs and linear shocks, which imply both $\tilde{E}_{zz}=0$ and $f_{ze}>0$.

A converse is also available for Proposition 5. Reinterpret $\hat{c}=f\left(\bar{z},\hat{E}\right)$, then:

PROPOSITION 6 *Let* $v_e\left(y,\bar{z},\hat{E}\right)\tilde{E}_{zz}\left(\bar{z},\hat{\tau}\right)$ *be sufficiently small and:*

$$R^u\left(\hat{c}\right)s\left(\hat{E}\right)+\frac{B_{ee}\left(y,\hat{E}\right)\hat{E}}{u_c\left(\hat{c}\right)f_e\left(\hat{E}\right)}\leq R^f\left(\hat{E}\right)\left[1-2R^u\left(\hat{c}\right)\left(\frac{s\left(\hat{E}\right)f_z\left(\hat{E}\right)}{\hat{E}f_{ze}\left(\hat{E}\right)}\right)\right] \tag{5.24}$$

Then $\Delta\leq 0$.

Proof: See Appendix.

6 Multiple Goods

A second extension involves adding additional consumption goods. One criticism of our basic result might be that the consumption good associated with the externality is a small share of a composite consumption good. If so, then the additional risk to total consumption under price regulation might be small. Here we show that the risk effect of price regulation in an environment with multiple goods depends not only on the consumption share, but also the correlation between shocks across sectors and the elasticity of substitution between consumption goods.

We consider two cases, which vary the correlation of shocks across sectors. First, suppose an aggregate consumption good x exists which is a perfect substitute for the consumption good associated with the externality. We assume consumption of x is exogenous for simplicity. Therefore, equilibrium consumption of a composite commodity c is given by:

$$c = x + e^{z} f(E) \tag{6.1}$$

All other assumptions do not change. Let $s^c\left(\bar{E}\right) \equiv \frac{f\left(\bar{E}\right)}{c\left(\bar{E}\right)}$ denote the share of the consumption good at average use of the regulated input. Corollary 3 then becomes:

PROPOSITION 7 *Suppose a perfect substitute consumption good x exists, and let:*

$$R^u(\tilde{c}) \geq \frac{1}{s^c\left(\bar{E}\right)} \frac{R^f\left(\bar{E}\right)}{s\left(\bar{E}\right) + 2R^f\left(\bar{E}\right)} \tag{6.2}$$

Then $\Delta \geq 0$.

Proof: Analogous to Corollary 3.□

Hence if the consumption good associated with the externality is a small share of the composite consumption good, the shock affects only the consumption good associated with the externality, and the consumption goods are perfect substitutes then price regulation likely dominates.

If the goods are not perfect substitutes, then the risk to consumption is not as easily eliminated by substitution. An extension to the above setup with a CES utility function implies a complicated non-linear relationship between the relative advantage of quantities versus prices and the elasticity of substitution. However, one can show that if the elasticity of substitution is small enough (if the goods are not very good substitutes), then quantity regulation dominates regardless of $s^c\left(\bar{E}\right)$.

We also consider the case where the shock is an aggregate shock which affects all sectors. Consumption of the composite commodity is thus $c = e^z (x + f(E))$. Here Corollary 3 goes through almost unchanged.

PROPOSITION 8 *Suppose a perfect substitute consumption good x exists, let shocks be perfectly correlated across sectors, and let:*

$$R^u(\tilde{c}) \geq \frac{R^f\left(\bar{E}\right)}{s^c\left(\bar{E}\right) s\left(\bar{E}\right) + 2R^f\left(\bar{E}\right)} \tag{6.3}$$

Then $\Delta \geq 0$.

Proof: Analogous to Corollary 3.□

Thus if the shock is an aggregate shock, then quantity regulation still dominates for a coefficient of relative risk aversion greater than or equal to one half even when the consumption goods are perfect substitutes and the consumption good is a small part of the composite consumption good. An extension with a CES utility function also implies a complicated relationship between the elasticity of substitution and the relative advantage of quantities versus prices. For a sufficiently small elasticity of substitution, quantities again dominate.

The multiple good cases are kept simple to fit into the overall framework. The multiple good cases do however illustrate the importance of general equilibrium effects.

7 Example 1: Pollution Externality

Suppose pollution emissions increase the probability of incurring some environmental damage. The probability of incurring damage given emissions E is $\psi = \frac{E}{\Psi}$. If emissions reach Ψ, damage occurs with probability one, however, we assume Ψ is sufficiently large so that $\psi < 1$ for all feasible emissions. If damage occurs, households suffer a utility loss of μ. An example might be greenhouse gas emissions and hurricanes. As temperatures rise from emissions, the probability of a hurricane rises, although a hurricane need not occur.

Household expected utility is thus:

$$\mathrm{E}U = \psi \Big[u(c) - \mu\Big] + (1 - \psi)\Big[u(c) - 0\Big] \tag{7.1}$$

$$= u(c) - \psi\mu = u(c) - bE \tag{7.2}$$

Here $b \equiv \frac{\mu}{\Psi}$. The production technology f is:

$$Zf(k, E) = Zk^{1-\gamma}E^{\gamma} \tag{7.3}$$

Here $0 < \gamma < 1$ and Z is iid log-normal, with:

$$\log Z \sim \mathrm{N}\left(0, \sigma_z^2\right) \tag{7.4}$$

Finally, let utility be of the constant relative risk aversion class:

$$u(c) = \frac{c^{1-\delta} - 1}{1-\delta} \tag{7.5}$$

Here $\delta \geq 0$ is the coefficient of relative risk aversion and the limiting case of $\delta = 1$ is log utility. Finally, without loss of generality set $k = 1$.

Let y denote a stochastic shock to benefits, with symmetric distribution G. One can think of y as an absorption shock, so that a random fraction of emissions is absorbed. We assume $y \sim G$, a symmetric distribution, with y normalized so that $\mathrm{E}(y) = 1$.

The problem when no information asymmetry exists is:

$$v^*(y, Z) = \max_E \left\{ \frac{(ZE^{\gamma})^{1-\delta} - 1}{1-\delta} - byE \right\} \tag{7.6}$$

Let $\eta = 1 - \gamma(1-\delta)$ and let $\gamma^{\frac{1}{1-\delta}} Z = z^{\gamma}$. Then we have:

$$v^*(y, z) = \max_E \left\{ \frac{(zE)^{1-\eta} - \gamma}{1-\eta} - byE \right\} \tag{7.7}$$

The first order condition is:

$$z^{1-\eta}E^{-\eta} = by \tag{7.8}$$

Let: $\alpha = \frac{1-\eta}{\eta}$ and $\theta = \frac{1}{b^{\alpha}\alpha}$. Then:

$$E^* = z^{\alpha} y^{\frac{-1}{\eta}} b^{\frac{-1}{\eta}} \tag{7.9}$$

Hence:

$$v^*(y,z) = \theta z^\alpha y^{-\alpha} - \frac{\gamma}{1-\eta} \tag{7.10}$$

The optimal emissions depends on on both z and y for $\eta \neq 1$, (which is the log utility case). Notice that the combination of risk neutrality over environmental damage and log utility (in which the income and substitution effects cancel), implies optimal absorbed emissions is constant (see Figure 1).

7.1 Quantity Regulation

We assume households and firms anticipate a constant emissions policy. Because the capital stock is fixed and the shock is unobserved, no state variables exist for the regulator to condition on. Hence the constant emissions policy is the unique rational expectations equilibrium. Given households anticipate a constant number of emissions permits, the regulator's problem is:

$$\max_{\hat{E}} \mathrm{E}\left\{\frac{\left(z\hat{E}\right)^{1-\eta} - \gamma}{1-\eta} - by\hat{E}\right\} \tag{7.11}$$

The first order condition implies a constant emissions policy:

$$\hat{E} = b^{\frac{-1}{\eta}}\hat{z}, \quad \hat{z} = \mathrm{E}\left(z^{1-\eta}\right)^{\frac{1}{\eta}} \tag{7.12}$$

Hence we have a rational expectations equilibrium. Welfare is:

$$\hat{v}(y,z) \quad = \quad b^{-\alpha}\left[\frac{z^{1-\eta}\hat{z}^{1-\eta}}{1-\eta} - y\hat{z}\right] - \frac{\gamma}{1-\eta} \tag{7.13}$$

It is straightforward to show that welfare loss of quantity regulation under uncertainty versus perfect information is a positive, quadratic function of y and z.

7.2 Tax Regulation

As with quantity regulation, there are no observable states to condition on, so the optimal constant tax is the unique rational expectations equilibrium. Given households and firms expect a constant tax, the regulator's problem is:

$$\max_{\tau} \mathrm{E}\left\{\frac{\left(z\tilde{E}\left(z,\tau\right)\right)^{1-\eta}-\gamma}{1-\eta}-by\tilde{E}\left(z,\tau\right)\right\} \tag{7.14}$$

The first order condition for the firm implies:

$$\tilde{E}\left(z,\tau\right)=z^{\frac{\gamma}{1-\gamma}}\gamma^{\frac{1-\eta-\gamma}{(1-\eta)(1-\gamma)}}\tau^{\frac{-1}{\gamma}} \tag{7.15}$$

Substituting (7.15) into (7.14) and simplifying gives:

$$\max_{\tau}\left\{\frac{\tau^{\frac{-1}{\gamma}}}{1-\eta}\gamma^{\frac{1-\eta-\gamma}{1-\gamma}}\mathrm{E}\left[z^{\frac{1-\eta}{1-\gamma}}\right]-b\gamma^{\frac{1-\eta-\gamma}{(1-\eta)(1-\gamma)}}\mathrm{E}\left[z^{\frac{\gamma}{1-\gamma}}\right]\tau^{\frac{-1}{1-\gamma}}\right\} \tag{7.16}$$

The first order condition for the optimal tax is:

$$\tau=b^{\frac{1-\gamma}{\eta}}\gamma^{\frac{1-\eta-\gamma}{(1-\eta)}}\tilde{z}^{\frac{-(1-\gamma)}{1-\eta}},\quad \tilde{z}=\mathrm{E}\left[z^{\frac{1-\eta}{1-\gamma}}\right]^{\alpha}\mathrm{E}\left[z^{\frac{\gamma}{1-\gamma}}\right]^{-\alpha} \tag{7.17}$$

Hence we have a rational expectations equilibrium. Welfare is:

$$\tilde{v}\left(y,z\right)=b^{-\alpha}\left[\frac{z^{\frac{1-\eta}{1-\gamma}}}{1-\eta}\tilde{z}-yz^{\frac{\gamma}{1-\gamma}}\tilde{z}^{\frac{1}{1-\eta}}\right]-\frac{\gamma}{1-\eta} \tag{7.18}$$

It is straightforward to show that the welfare loss of price regulation under uncertainty versus perfect information is a positive, quadratic function of y and z.

7.3 Welfare Loss From Taxes Versus Quantities

The primary results for the example are as follows:

PROPOSITION 9 *The welfare gain from quantity versus price regulation is:*

$$\Delta=\hat{v}-\tilde{v} \;=\; b^{-\alpha}\left[\frac{z^{1-\eta}\hat{z}^{1-\eta}}{1-\eta}-y\hat{z}-\frac{z^{\frac{1-\eta}{1-\gamma}}}{1-\eta}\tilde{z}+yz^{\frac{-\gamma}{1-\gamma}}\tilde{z}^{\frac{1}{1-\eta}}\right] \tag{7.19}$$

Further, expected welfare gain is:

$$E(\Delta) = \theta \left[\hat{z} - \tilde{z} E \left[z^{\frac{1-\eta}{1-\gamma}}\right]\right] \tag{7.20}$$

and $E(\Delta) > 0$ *if and only if* $\delta > \frac{1-\gamma}{2-\gamma}$.

Proof: The first two parts are a matter of combining Equations (7.18) and (7.13) and simplifying. The proof of the last claim is in the Appendix.

Notice that the condition is identical, given the functional forms, as Corollary 3, except that the proof requires no approximations. For $\delta > \frac{1}{2}$, quantity regulation is preferred regardless of γ. However, there exists an open set of γ around zero and δ around zero such that tax regulation is preferred.

Let $\delta = 0$, then Example 1 falls into the framework of Weitzman (1974). In this case, price regulation is preferred for all $\gamma < 1$, because marginal benefits are perfectly flat.[7]

8 Example 2: Production Spillovers

Suppose use of capital k enhances productivity, and that a production externality of perfect spillovers exist so that the productivity of each firm is enhanced by observing other firm's production techniques. Specifically, let productivity $A\left(\bar{k}\right) = \bar{k}^{\zeta}$ be a function of the economy wide capital use per firm $\bar{k}$. Production Q is a function of capital, hours (h), and the productivity shock, z:

$$Q = A\left(\bar{k}\right) e^{z} k^{\gamma} h^{1-\gamma} \tag{8.21}$$

Note that the model has constant returns in the private inputs, and thus the number of firms is indeterminate, hence we assume without loss of generality a large number of firms with mass equal to one and an identical number of households. Hours worked are inelastically

[7] One can also try to use the Weitzman formula with correlated shocks by deriving linear approximations of a marginal benefit function in terms of dollars: $MB = -\frac{b}{u_c(c(Z,E))}$ and a marginal cost function $MC = -Zf_e$. However, since the shock in general is not a pure shift of the marginal cost and benefit functions, the results are not generally accurate.

supplied and normalized to one. We assume $\theta \equiv \zeta + \gamma \leq 1$, with the equality case being the well-known AK-model framework.

To apply our static theory, we use a two-period framework (the conclusions of the example generalize readily to the infinite horizon case). The government moves prior to the first period, then the shock is realized, then the households and firms move. Price regulation corresponds to a subsidy t per unit of capital rented by the firm, while quantity regulation corresponds to a quota on capital usage.

The problem of the representative firm is thus:

$$\max \Pi = A\left(\bar{k}\right) e^{z} k^{\gamma} h^{1-\gamma} - rk - wh + tk \tag{8.22}$$

The first order conditions for the firm evaluated at the equilibrium where $\bar{k} = k$ and $h = 1$ are:

$$r - t \equiv R = e^{z} \gamma k^{\theta - 1} \tag{8.23}$$

$$w = e^{z} \left(1 - \gamma\right) k^{\theta} \tag{8.24}$$

Hence R is the rental price of capital net of subsidies.

Let c_i correspond to consumption in each period. The household problem is to maximize utility subject to two budget constraints.

$$\max_{k} u = \log c_1 + \beta \log c_2 \tag{8.25}$$

Let ω be the fixed endowment in the first period, then the budget constraints are:

$$c_1 = \omega - k \tag{8.26}$$

$$c_2 = rk + w - TR \tag{8.27}$$

Here TR are the lump sum taxes used to finance the subsidies. The government budget constraint is thus:

$$TR = t\bar{k} = (r - R)\,k \tag{8.28}$$

It is easy to see that the optimal policy under certainty is to set a subsidy equal to the marginal social product of knowledge, $t = \zeta e^z k^{\theta-1}$, or simply specify a quota equal to the solution to the planner's problem: $k = \frac{\beta\theta}{1+\beta\theta}\omega$.

Our solution given uncertainty requires first characterizing the equilibrium as a set of constraints on the planning problem. A *competitive equilibrium* given a policy R (price regulation) or k (quantity regulation) is a set of prices r, w, policies TR, allocations c_1 and c_2, and either an aggregate per capita capital stock k (price regulation) or a price R (quantity regulation), such that firms maximize (Equations (8.23) and (8.24)) for all possible policies, households maximize (8.25) subject to budget constraints (8.26) and (8.27) for all possible policies, and the government budget constraint (8.28) holds.

PROPOSITION 10 *Suppose that:*

$$c_1 = \omega - k \tag{8.29}$$

$$c_2 = e^z k^\theta \tag{8.30}$$

$$R = \gamma e^z k^{\theta-1} \tag{8.31}$$

Then prices and policies constituting a competitive equilibrium satisfy (8.29)-(8.31). Conversely, given (8.29)-(8.31), prices and policies exist which constitute a competitive equilibrium.

Proof: Analogous to Proposition 1.□

Hence we can write the problem under uncertainty as:

$$\max_{k,R} \mathrm{E}\left\{\log(\omega - k) + \beta \log\left(e^z k^\theta\right)\right\} \text{ subject to } R = \gamma e^z k^{\theta-1} \tag{8.32}$$

The problem is now in the framework of the model with $\tau = \frac{\theta R}{\gamma}$, $E = k$, $f(E) = E^{\theta}$, $u(c) = \log c$, $B(y, E) = -\beta \log(\omega - E)$. Thus, since the utility function is logarithmic ($\delta = 1$), Corollary 3 applies and quantity regulation is optimal. The optimal quantity with perfect information is independent of the productivity shock. Thus asymmetric information causes no welfare loss with quantity regulation. Conversely, the optimal tax cannot depend on the productivity shock, as is the case with perfect information, and thus the tax system yields lower welfare.[8]

9 Conclusions

Our main result is that the relative advantage of quantities versus prices depends on risk aversion. Tax regulation implies firms increase inputs in response to a good productivity shock, generating more variance in output relative to quantity regulation, where the regulated input is constant in the productivity shock. Hence if households are very risk averse, quantity regulation is preferred. If households prefer regulation which allows firms to take advantage of the high return to increasing the regulated input, then price regulation has higher welfare.

Risk aversion is an important consideration in addition to the slopes of the marginal benefit and cost curves. Even when marginal benefits are flat, a situation which favors price regulation, quantity regulation is preferred if the coefficient of relative risk aversion is greater than one half.

Our results are relatively free of strong parameter assumptions, such as the slopes of marginal benefit and cost curves, but the nature of the uncertainty and the number of consumption goods play an important role. Extensions allowing multiple goods show that a positive correlation of shocks across consumption sectors favors quantity regulation. If the good associated with the externality is a large component of a composite consumption good, then the results also favor quantity regulation. On the other hand if consumption goods are good substitutes, or if the shock reduces the productivity of the regulated input, then general equilibrium considerations favor price regulation. Regardless, general equilibrium

[8] If the regulation were instead on households, then an optimal interest subsidy of $t = \frac{\zeta}{\gamma}$ also results in the full information optimum. This illustrates that an important assumption of our framework is that regulation is applied to producers.

considerations play an important role.

Our framework and techniques may be extended a variety of ways and to a number of new applications. The techniques could be applied to consumption externalities and other price versus quantity issues such as tariffs versus quotas in international trade. The model could be extended to dynamic problems, the presence of other distortionary taxes, and hybrid instruments.

10 Appendix: Proofs

10.1 Proof of Proposition 1

Substituting (2.10) into (2.7) gives:

$$rk + \tau E = c \tag{10.1}$$

Substituting (2.5) and (2.6) into the above equation and using constant returns to scale implies (3.1). Equation (2.6) implies Equation (3.2). Thus (3.1) and (3.2) are necessary for competitive equilibrium of any type.

Conversely, let:

$$r = e^z f_k (k, E) \tag{10.2}$$

$$TR = e^z f_e (k, E) E \tag{10.3}$$

Then (3.2) and (10.2) imply (2.6) and (2.5) hold. Further, substituting (3.2) into (10.3) implies (2.10). Finally, constant returns to scale implies we can write the resource constraint (3.1) as:

$$e^z f_k (k, E) k + e^z f_e (k, E) E = c \tag{10.4}$$

Substituting in (10.3) gives:

$$e^z f_k (k, E) k + TR = c \tag{10.5}$$

Substituting in equation (10.2) then yields (2.7). Hence the prices, policies, and allocations constitute a competitive equilibrium of type CE^*, $\hat{CE}$, or $\tilde{CE}$. □

10.2 Proof of Corollary 3

The Generalized Mean Value Theorem implies there exists a $\tilde{z}$ such that:

$$\bar{E} = \int \tilde{E} (z) G (z) dz = \tilde{E} (\tilde{z}) \int G (z) dz = \tilde{E} (\tilde{z}) \tag{10.6}$$

Approximating the variance and covariance around $\tilde{z}$ gives:

$$\text{var}\left(\tilde{E}(z)\right) \approx \tilde{E}_z(\tilde{z})^2 \sigma_z^2 \tag{10.7}$$

$$\text{cov}\left(\tilde{E}(z), z\right) \approx \tilde{E}_z(\tilde{z}) \sigma_z^2 \tag{10.8}$$

Using the implicit function theorem on the firm first order condition implies:

$$\tilde{E}_z(\tilde{z}) = \frac{f_e\left(\tilde{E}(\tilde{z})\right)}{-f_{ee}\left(\tilde{E}(\tilde{z})\right)} = \frac{f_e\left(\bar{E}\right)}{-f_{ee}\left(\bar{E}\right)} \tag{10.9}$$

Substituting (10.7) and (10.8) into (4.7) and using (10.9) implies that (4.7) holds if:

$$R^f\left(\bar{E}\right) + R^u(\tilde{c})\, s\left(\bar{E}\right) \geq 2\left[1 - R^u(\tilde{c})\right] \frac{-\bar{E} f_{ee}\left(\bar{E}\right)}{f_e\left(\bar{E}\right)} \tag{10.10}$$

Or:

$$R^f\left(\bar{E}\right) + R^u(\tilde{c})\, s\left(\bar{E}\right) \geq 2\left[1 - R^u(\tilde{c})\right] R^f\left(\bar{E}\right) \tag{10.11}$$

The first result follows by solving the above equation for R^u. □

For the second result, note that the above approximation implies $R^f \approx \frac{\sigma^2_{z,\tilde{E}}}{\sigma^2_{\tilde{E}}} \bar{E}$. Substituting R^f for the correlation in Equation (4.7) implies:

$$-R^f + R^u s + 2R^f R^u + \frac{B_{ee}\bar{E}}{f_e u_c} \geq 0 \tag{10.12}$$

Letting $u(c) = c$ implies $R^u = 0$ and $u_c = 1$. Substituting the utility function into (10.12) and simplifying gives the second result. □

10.3 Proof of Proposition 4

The Inada conditions and the continuity of the production function imply there exists a $\hat{\tau}$ such that given price regulation:

$$\mathrm{E}\left[\tilde{E}(z, \hat{\tau})\right] = \mathrm{E}\left[f_e^{-1}\left(\hat{\tau} e^{-z}\right)\right] = \hat{E} \tag{10.13}$$

Further, the optimal tax generates at least the expected welfare of $\hat{\tau}$:

$$\mathrm{E}\left[v\left(y, z, \tilde{E}(z, \tau)\right)\right] \geq \mathrm{E}\left[v\left(y, z, \tilde{E}(z, \hat{\tau})\right)\right] \tag{10.14}$$

Hence it is sufficient to show that the tax policy with mean $\hat{E}$ generates higher expected welfare than the optimal quantity:

$$\mathrm{E}\left[v\left(y,z,\tilde{E}\left(z,\hat{\tau}\right)\right)\right]\geq\mathrm{E}\left[v\left(y,z,\hat{E}\right)\right] \tag{10.15}$$

A second order approximation yields:

$$\mathrm{E}\left[v\left(y,z,\hat{E}\right)\right]\approx v\left(y,0,\hat{E}\right)+\frac{\sigma_z^2}{2}v_{zz}\left(y,0,\hat{E}\right) \tag{10.16}$$

$$\begin{aligned}\mathrm{E}\left[v\left(y,z,\tilde{E}\left(z,\hat{\tau}\right)\right)\right] \approx\ & v\left(y,0,\hat{E}\right)+\frac{\sigma_z^2}{2}v_{zz}\left(y,0,\hat{E}\right)+\tfrac{1}{2}\sigma^2_{\tilde{E}(z,\hat{\tau})}v_{ee}\left(y,0,\hat{E}\right)+\\ & \sigma^2_{z,\tilde{E}(z,\hat{\tau})}\left(z,\tilde{E}\left(z,\hat{\tau}\right)\right)v_{ze}\left(y,0,\hat{E}\right)\end{aligned} \tag{10.17}$$

Hence we need only show:

$$\tfrac{1}{2}\sigma^2_{\tilde{E}(z,\hat{\tau})}\left(-v_{ee}\left(y,0,\hat{E}\right)-2\frac{\sigma^2_{z,\tilde{E}(z,\hat{\tau})}}{\sigma^2_{\tilde{E}(z,\hat{\tau})}}v_{ze}\left(y,0,\hat{E}\right)\right)\leq 0 \tag{10.18}$$

Notice the derivatives are identical to the proof of Proposition 2, except for the function arguments. Hence calculation of the derivatives and the resulting simplification are identical to the proof of Proposition 2, except for the function arguments, yielding the desired result.□

10.4 Proof of Proposition 5

Recall that $\bar{E}=\tilde{E}\left(\bar{z},\tau\right)$. Optimality implies:

$$\mathrm{E}\left[v\left(y,z,\hat{E}\right)\right]\geq\mathrm{E}\left[v\left(y,z,\bar{E}\right)\right] \tag{10.19}$$

Hence it is sufficient to show that the tax policy at the mean shock yields higher welfare than the actual tax policy:

$$\hat{\Delta}=\mathrm{E}\left[v\left(y,z,\bar{E}\right)\right]-\mathrm{E}\left[v\left(y,z,\tilde{E}\right)\right]\geq 0\Rightarrow\Delta\geq 0 \tag{10.20}$$

A second order approximation yields:

$$\mathrm{E}\left[v\left(y,z,\bar{E}\right)\right]\approx v\left(y,\bar{z},\bar{E}\right)+\frac{\sigma_z^2}{2}v_{zz}\left(y,\bar{z},\bar{E}\right) \tag{10.21}$$

$$\begin{aligned}\mathrm{E}\left[v\left(y,z,\tilde{E}\right)\right] \approx\ & v\left(y,\bar{z},\bar{E}\right)+\frac{\sigma_z^2}{2}\left[v_{zz}\left(y,\bar{z},\bar{E}\right)+v_{ee}\left(y,\bar{z},\bar{E}\right)\tilde{E}_z^2\left(\bar{E}\right)+\right.\\ & \left.2v_{ze}\left(y,\bar{z},\bar{E}\right)\tilde{E}_z\left(\bar{E}\right)+v_e\left(y,\bar{z},\bar{E}\right)\tilde{E}_{zz}\left(\bar{E}\right)\right]\end{aligned} \tag{10.22}$$

If the last term of (10.22) is small, then it is sufficient to show:

$$-v_{ee}\left(y,\bar{z},\bar{E}\right)-2v_{ze}\left(y,\bar{z},\bar{E}\right)\left(\tilde{E}_{z}\left(\bar{E}\right)\right)^{-1}\geq 0 \tag{10.23}$$

The relevant derivatives are:

$$v_{ee}\left(y,\bar{z},\bar{E}\right)=u_{c}\left(f\left(\bar{z},\bar{E}\right)\right)f_{ee}\left(\bar{z},\bar{E}\right)+u_{cc}\left(f\left(\bar{z},\bar{E}\right)\right)f_{e}\left(\bar{z},\bar{E}\right)^{2}-B_{ee}\left(y,\bar{E}\right) \tag{10.24}$$

$$v_{ze}\left(y,\bar{z},\bar{E}\right)=f_{ze}\left(\bar{z},\bar{E}\right)u_{c}\left(f\left(\bar{z},\bar{E}\right)\right)+u_{cc}\left(f\left(\bar{z},\bar{E}\right)\right)f_{e}\left(\bar{z},\bar{E}\right)f_{z}\left(\bar{z},\bar{E}\right) \tag{10.25}$$

From the implicit function theorem:

$$\tilde{E}_{z}\left(\bar{z},\tau\right)=-\frac{f_{ze}\left(\bar{z},\bar{E}\right)}{f_{ee}\left(\bar{z},\bar{E}\right)} \tag{10.26}$$

Substituting (10.24), (10.25), and (10.26) into (10.23) results in:

$$\begin{aligned}
&-\quad u_{c}\left(f\left(\bar{z},\bar{E}\right)\right)f_{ee}\left(\bar{z},\bar{E}\right)-u_{cc}\left(f\left(\bar{z},\bar{E}\right)\right)f_{e}\left(\bar{z},\bar{E}\right)^{2}+B_{ee}\left(y,\bar{E}\right)\geq\\
&-\quad 2\frac{f_{ee}\left(\bar{z},\bar{E}\right)}{f_{ze}\left(\bar{z},\bar{E}\right)}\left[u_{cc}\left(f\left(\bar{z},\bar{E}\right)\right)f_{e}\left(\bar{z},\bar{E}\right)f_{z}\left(\bar{z},\bar{E}\right)+u_{c}\left(f\left(\bar{z},\bar{E}\right)\right)f_{ze}\left(\bar{z},\bar{E}\right)\right]
\end{aligned} \tag{10.27}$$

$$\begin{aligned}
&-\quad u_{cc}\left(f\left(\bar{z},\bar{E}\right)\right)f_{e}\left(\bar{z},\bar{E}\right)^{2}+B_{ee}\left(y,\bar{E}\right)\geq\\
&-\quad 2u_{cc}\left(f\left(\bar{z},\bar{E}\right)\right)\frac{f_{e}\left(\bar{z},\bar{E}\right)f_{z}\left(\bar{z},\bar{E}\right)f_{ee}\left(\bar{z},\bar{E}\right)}{f_{ze}\left(\bar{z},\bar{E}\right)}-u_{c}\left(f\left(\bar{z},\bar{E}\right)\right)f_{ee}\left(\bar{z},\bar{E}\right)
\end{aligned} \tag{10.28}$$

Substituting in the definitions of s, R^{u}, and R^{f} yields:

$$R^{u}\left(\tilde{c}\right)s\left(\bar{E}\right)+\frac{B_{ee}\left(y,\bar{E}\right)\bar{E}}{u_{c}\left(\tilde{c}\right)f_{e}\left(\bar{E}\right)}\geq R^{f}\left(\bar{E}\right)-2R^{u}\left(\tilde{c}\right)R^{f}\left(\bar{E}\right)s\left(\bar{E}\right)\frac{f_{z}\left(\bar{E}\right)}{\bar{E}f_{ze}\left(\bar{E}\right)} \tag{10.29}$$

Simplifying then gives the desired result.□

10.5 Proof of Proposition 6

Consider the tax policy $\hat{\tau}=f_{e}\left(\bar{z},\hat{E}\right)$. The first order condition of the firm implies:

$$\hat{\tau}=f_{e}\left(\bar{z},\hat{E}\right)=f_{e}\left(z,\tilde{E}\right) \tag{10.30}$$

Hence since f_{e} is continuous and monotonic:

$$\tilde{E}\left(\bar{z},\hat{\tau}\right)=\hat{E} \tag{10.31}$$

Further, the optimal tax generates at least the expected welfare of $\hat{\tau}$:

$$\mathrm{E}\left[v\left(y,z,\tilde{E}\left(z,\tau\right)\right)\right]\geq\mathrm{E}\left[v\left(y,z,\tilde{E}\left(z,\hat{\tau}\right)\right)\right] \tag{10.32}$$

Hence it is sufficient to show:

$$\mathrm{E}\left[v\left(y,z,\tilde{E}\left(z,\hat{\tau}\right)\right)\right]\geq\mathrm{E}\left[v\left(y,z,\hat{E}\right)\right] \tag{10.33}$$

A second order approximation yields:

$$\mathrm{E}\left[v\left(y,z,\hat{E}\right)\right]\approx v\left(y,\bar{z},\hat{E}\right)+\frac{\sigma_z^2}{2}v_{zz}\left(y,\bar{z},\hat{E}\right) \tag{10.34}$$

$$\begin{aligned}\mathrm{E}\left[v\left(y,z,\tilde{E}\left(z,\hat{\tau}\right)\right)\right] \approx\ & v\left(y,\bar{z},\hat{E}\right)+\frac{\sigma_z^2}{2}\left[v_{zz}\left(y,\bar{z},\hat{E}\right)+v_{ee}\left(y,\bar{z},\hat{E}\right)\tilde{E}_z^2\left(\hat{E}\right)+\right.\\ & \left.2v_{ze}\left(y,\bar{z},\hat{E}\right)\tilde{E}_z\left(\hat{E}\right)+v_e\left(y,\bar{z},\hat{E}\right)\tilde{E}_{zz}\left(\hat{E}\right)\right]\end{aligned} \tag{10.35}$$

Hence if the last term of (10.35) is sufficiently small, it is sufficient to show:

$$-v_{ee}\left(y,\bar{z},\hat{E}\right)-2v_{ze}\left(y,\bar{z},\hat{E}\right)\left(\tilde{E}_z\left(\hat{E}\right)\right)^{-1}\leq 0 \tag{10.36}$$

Notice the derivatives are identical to Proposition 5, except for the function arguments. Hence calculation of the derivatives and the resulting simplification are identical to Proposition 5, except for the function arguments, yielding the desired result.□

10.6 Proof of the third claim in Proposition 9

Recall that $b>0$ and $\eta>0$, but α may be negative. Given the log-normal distribution of Z, we have:

$$\mathrm{E}\left[z^x\right]=\mathrm{E}\left[\gamma^{\frac{x}{1-\eta}}Z^{\frac{x}{\gamma}}\right]=\gamma^{\frac{x}{1-\eta}}\exp\left[\frac{\sigma_z^2x^2}{2\gamma^2}\right] \tag{10.37}$$

From Equation (7.20), we then have $\mathrm{E}\left(\Delta\right)>0$ if and only if:

$$\alpha^{-1}\exp\left[\frac{\sigma_z^2\left(1-\eta\right)^2}{2\eta\gamma^2}\right]>\alpha^{-1}\exp\left[\frac{\sigma_z^2}{2}\left(\frac{\left(1-\eta\right)^2}{\eta\gamma^2\left(1-\gamma\right)^2}-\frac{\gamma^2\left(1-\eta\right)}{\eta\gamma^2\left(1-\gamma\right)^2}\right)\right] \tag{10.38}$$

Because the exponential is a strictly increasing function, $\mathrm{E}\left(\Delta\right)>0$ if and only if:

$$\alpha^{-1}\left(\frac{\sigma_z^2\left(1-\eta\right)^2}{2\eta\gamma^2}\right)>\alpha^{-1}\left(\frac{\sigma_z^2}{2}\left(\frac{\left(1-\eta\right)^2}{\eta\gamma^2\left(1-\gamma\right)^2}-\frac{\gamma^2\left(1-\eta\right)}{\eta\gamma^2\left(1-\gamma\right)^2}\right)\right) \tag{10.39}$$

Simplifying and using the definition of α and η gives:

$$\frac{\gamma\left(1-\delta\right)^2}{1-\gamma\left(1-\delta\right)}\left(1-\left(1-\delta\right)\left(2-\gamma\right)\right)>0 \tag{10.40}$$

$$\delta\,(2-\gamma) > 1-\gamma \tag{10.41}$$

Solving for δ then gives the desired result. $\square$

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