Selloffs, Bailouts, and Feedback: Can Asset Markets Inform Policy?*

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Abstract

We present a model in which a policymaker observes trade in a financial asset before deciding whether to intervene in the economy, for example by offering a bailout or monetary stimulus. Because an intervention erodes the value of private information, informed investors are reluctant to take short positions and selloffs are, therefore, less likely and less informative. The policymaker faces a tradeoff between eliciting information from the asset market and using the information so obtained. In general she can elicit more information if she commits to intervene only infrequently. She thus may benefit from imperfections in the intervention process, from delegating the decision to a biased agent, or from being non-transparent about the costs or benefits of intervention.

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Central bankers naturally pay close attention to interest rates and asset prices, in large part because these variables are the principal conduits through which monetary policy affects real activity and inflation. But policymakers watch financial markets carefully for another reason, which is that asset prices and yields are potentially valuable sources of timely information about economic and financial conditions. Because the future returns on most financial assets depend sensitively on economic conditions, asset prices–if determined in sufficiently liquid markets–should embody a great deal of investors' collective information and beliefs about the future course of the economy.

—Ben Bernanke[1]

[1]FRB: Speech, "What Policymakers Can Learn from Asset Prices," April 15, 2004.

1 INTRODUCTION

Asset markets provide informed investors with opportunities to profit from private information about fundamentals. In the process of doing so, the informed investor's trading behavior reveals some of his private information to other market participants and outside observers. Thus, activity in asset markets is a natural source of information about the underlying state. Since the pioneering work of Mitchell and Burns (1938), economists have been aware of the power of asset markets to forecast business cycle fluctuations. In their influential article, Stock and Watson (2003) also found evidence that asset prices predicted inflation and output growth in some historical periods. Indeed the idea that securities prices convey information is a central tenet of the efficient markets hypothesis (Malkiel and Fama 1970), and it is taken as common wisdom that asset prices augur the health of individual firms, industries, and even the entire economy.

Given the prevalence of this idea, it is not surprising that policymakers would rely on information conveyed by asset markets to guide their policy interventions. For instance, since the major financial market selloff of October 19, 1987 (known as Black Monday), it has become common practice for the Federal Reserve to respond to large drops in the stock market by injecting liquidity in to the economy either by reducing the federal funds rate (the so-called Greenspan Put) or by quantitative easing (the so-called Bernanke Put) (Brough 2013).[2]

However, using activity in asset markets to inform policy may be problematic. If privately-informed investors are aware that a selloff will trigger a corrective intervention, then they will have substantially less incentive to take short positions *in the first place*, and selloffs consequently will be smaller and less informative. In other words, relying on the behavior of asset markets to guide policy can undermine the informational content of the very market in question.

To study this dilemma, we construct and analyze a new market micro-structure model, wherein a privately informed investor anticipates that his trade in an Arrow security may trigger an intervention by a policymaker, who can change the unobserved underlying state from *bad* to *good* at some cost. The policymaker relies on trade in the asset market to guide her intervention decision: because intervention is costly, the policymaker would like to intervene only if she is sufficiently convinced that the state is likely to be bad, and she learns about the state by observing market activity. If trade in the asset market is sufficiently noisy, then the model admits a unique Perfect Bayesian equilibrium with some intriguing features. The policymaker never intervenes for small selloffs and intervenes randomly and with increasing probability for large ones. This generates a non-monotonicity in the equilibrium asset price: initially it falls with the magnitude of a selloff as the market becomes more convinced that the bad state will obtain and then it rises as the market anticipates the increasing probability that a corrective intervention will be triggered. Compared with a benchmark setting in which interventions are infeasible, the expected value of the asset price is, therefore, higher. Also, because informed investors are discouraged from taking short positions,

[2]The regulatory case for using stock prices to assess the health of financial firms is documented by Curry, Elmer and Fissel (2003). Bernanke and Woodford (1997) suggest targeting long-run inflation to the level implicit in asset prices.

order flow is less (Blackwell) informative about the state.

Our analysis also delivers a sharp normative result: in this type of equilibrium, the policymaker never benefits from the ability to intervene. Intervention is sequentially rational whenever the policymaker believes that the state is sufficiently likely to be bad. By intervening, the policymaker guarantees that the state is good. Because the probability of intervention is built into the asset price, anticipated interventions undermine the investor's information advantage and reduce his trading profit. Therefore, an informed investor wishes to avoid triggering an intervention with his order; in equilibrium, he adopts a less aggressive trading strategy that truncates beliefs at the policymaker's point of indifference. Thus, the conflict of interest between the trader and policymaker completely undermines the possible gains arising from the policymaker's ability to improve the state.

Motivated by this result, we present three institutional remedies that allow the policymaker to benefit from intervention. We develop intuition for these alternatives by analyzing a benchmark in which the policymaker can commit to any intervention policy that generates a Pareto improvement over the equilibrium arising without commitment. In this benchmark, the optimal intervention policy is *flatter* or a *clockwise rotation* of the equilibrium intervention policy: the optimal policy involves a larger probability of intervening following small selloffs, and a smaller probability of intervening following large ones.

The first institutional remedy that we consider explicitly incorporates this feature of the full-commitment optimal policy. We suppose that imperfections in the political process or intervention technology impose an exogenous cap on the probability with which the policymaker can intervene. For large order flows (where the cap binds), the policymaker is constrained to intervene less often than she would in the equilibrium without the cap, approximating the clockwise rotation inherent in the optimal policy. Because the policymaker intervenes less often, the trader is less concerned about losing rent following a large selloff and is willing to trade more aggressively. Thus, with a binding cap, a large selloff will be more informative and can trigger a strictly beneficial intervention with positive probability. A limited ability to intervene in situations in which it is beneficial *ex post* allows the policymaker to acquire information that is both beneficial and actionable, benefitting her *ex ante*.

Second, we analyze the possibility that the policymaker can delegate the authority to deploy an intervention to an agent who shares her preferences but also places some weight—either positive or negative—on the *ex post* profit of uninformed traders. Because interventions force a seller of the asset to pay on a short position, an agent who weights trading profit positively is less inclined to intervene than the policymaker and must be more convinced that an intervention is beneficial in order to undertake one. Thus if the agent is indifferent about intervening, the policymaker strictly benefits. In equilibrium, an informed trader chokes off information flow at the *agent's* point of indifference, preventing a certain intervention and inducing the agent to mix. Here, the policymaker strictly benefits from an intervention, which takes place with positive probability. Somewhat paradoxically, the agent's effort to protect uninformed traders *ex post* winds up harming them *ex ante* because the lower intervention probability induces more aggressive trade by an informed investor,

increasing the informed investor's ex ante profit and (correspondingly) reducing the *ex ante* profit of the uninformed trader.

Finally, if the policymaker adopts a regime of secrecy rather than transparency regarding the expected benefit or cost of intervening, then a large selloff again will yield valuable information that can guide policy. This finding may seem provocative given the prevailing wisdom that intervention policy should be conducted in an environment of minimal uncertainty (Eusepi 2005, Dincer and Eichengreen 2007). In the prevailing literature, however, no informational feedback exists between trade and policymaker intervention. In the presence of this feedback, the more certain the policymaker is to act on information, the more difficult it is to glean. A lack of transparency increases trader uncertainty about policymaker actions, strengthening the flow of information from the market. In these ways, institutions such as an unpredictable political process or opaque implementation protocol can benefit a policymaker by blurring investors' beliefs about the likelihood that any given selloff will trigger an intervention.

Our paper contributes to several related literatures. From a methodological perspective, our analysis is embedded in a new market micro-structure model that incorporates continuously distributed noise trades and interventions into the single-arrival frameworks of Glosten and Milgrom (1985) and Easley and O'Hara (1987). In these settings, a trader submits an order to a market maker, who adjusts the asset price to equal the asset's expected payoff conditional on observing the trader's order, without knowing whether the order was submitted by an informed investor or a noise trader. The support of the noise trade distribution is binary or discrete, and consequently, an informed trader must also submit an order from the same discrete support in order to avoid being immediately identified by the market maker. In contrast, in our model the noise trader's order flow is distributed continuously, which induces informed traders to adopt mixed strategies (supported on an interval) in equilibrium. Hence, order flow in our single-arrival model is continuously distributed with more extreme orders conveying more information, as in the batch-order model of Kyle (1985). Because order flow is continuous in our model, the equilibrium is continuous in the model parameters, avoiding the discontinuities arising from discrete changes in the trader's equilibrium strategy that can result in a large number of cases. Consequently, our model has only three equilibrium cases, with continuous transitions between them, and one case of primary interest.[3]

Our paper also contributes to an emerging theoretical literature on government bailouts of financial entities. Farhi and Tirole (2012) study a moral hazard setting in which borrowers engage in excessive leverage and banks choose to correlate their risk exposures so as to benefit from a bailout by the monetary authority. They characterize the optimal regulation of banks and the structure of optimal bailouts. Philippon and Skreta (2010) and Tirole (2012) investigate adverse selection in the financial sector, arriving at somewhat different conclusions. Philippon and Skreta (2010) find that simple programs of debt guarantee are optimal in their model and that there is no scope for equity stakes or asset purchases, while in Tirole (2012) the government clears the market

[3]For example, the analysis of Edmans, Goldstein and Jiang (2014) (discussed below), considers feedback between markets and real decisions in a discrete model with a large number of cases.

of its weakest assets through a mixture of buybacks and equity stakes.

Relative to these papers, we take a more agnostic approach both on the need for and implementation of a bailout and focus on a somewhat different set of questions. Rather than moral hazard or adverse selection we suppose (for simplicity) that the need for a bailout arises randomly and exogenously but is unobserved by the policymaker. We also do not model the set of policy instruments available to the authority, assuming only that it has at its disposal an effective but costly intervention technology. Rather than the design of an optimal bailout mechanism, we are concerned with how a policymaker might extract information from asset markets to decide whether a bailout is needed.

In this regard, our paper also relates to the literature on information aggregation in prediction markets (Cowgill, Wolfers and Zitzewitz 2009, Wolfers and Zitzewitz 2004, Snowberg, Wolfers and Zitzewitz 2012, Arrow et al. 2008), which are commonly viewed as efficient means of eliciting information. Our results suggest a caveat: prediction markets aggregate information effectively provided that policymakers (who watch the market in order to learn about the state) cannot take actions that affect the state; otherwise, market informativeness may be compromised.[4] Ottaviani and Sørensen (2007) present a complementary analysis, which shows that the information aggregation properties of prediction markets may be undermined when *traders* can directly affect the state. Also related is Hanson and Oprea (2009) who consider information aggregation in a prediction market with a manipulative trader. Although he cannot affect the state directly, the manipulator can affect the market with his orders, seeking to match the market price to a specific target (unknown to other traders) for exogenous reasons. Paradoxically, these authors find that the presence of the manipulator may increase market informativeness. Unlike Hanson and Oprea (2009), in our model a trader's incentive to manipulate order flow is endogenous. Specifically, he internalizes the impact of his trade on the policymaker's decision, adjusting his trading behavior in an effort to avoid an intervention. In equilibrium, the feedback between trade and intervention in our model *reduces* market informativeness.

Finally, as the title of our paper suggests, we contribute to a burgeoning corporate finance literature on feedback between policy and asset prices. Like ours, these papers consider a decision maker who learns from asset prices in a setting where investors anticipate the impact of the decision maker's ultimate choice of action on asset value. In their insightful paper, Bond, Goldstein and Prescott (2010) consider intervention in a rational expectations framework. These authors also find non-monotonic prices in the presence of intervention, though their model does not feature strategic trade (and sometimes can fail to possess an equilibrium). Bond and Goldstein (2012) consider government interventions in a Grossman and Stiglitz (1980) market framework; interventions change the risk profile of assets and can either generate or dampen trade. The main mechanism underlying

[4]According to Cowgill, Wolfers and Zitzewitz (2009) many companies (e.g., Abbott Labs, Arcelor Mittal, Best Buy, Chrysler, Corning, Electronic Arts, Eli Lilly, Frito Lay, General Electric, Google, Hewlett Packard, Intel, InterContinental Hotels, Masterfoods, Microsoft, Motorola, Nokia, Pfizer, Qualcomm, Siemens, and TNT) incentivize their employees to trade assets in internal prediction markets designed to elicit information on a variety of concerns from forecasting demand to meeting cost and quality targets.

our analysis is different from the ones described in these papers. In our analysis the trader wishes to avoid an intervention because it undermines the value of his private information. It is driven by strategic trade (unlike Bond, Goldstein and Prescott (2010)), and unlike Bond and Goldstein (2012), is not driven by traders' risk preferences.

The analysis of Edmans, Goldstein and Jiang (2014) is more closely related to ours. These authors focus on a firm's decision to abandon or expand an investment in response to trade in a binary version of the Kyle (1985) model, with two states, two trade sizes, a noise trader and an informed investor (who exists with probability less than one), and an exogenous transaction cost for market participation. The authors show that if transaction costs faced by the informed investor and the probability that an informed investor exists are moderate, then an equilibrium exists in which the informed investor buys if he knows that the state is good, but does not participate in the market if he knows that the state is bad. Therefore, whenever the feedback effect arises, sales are less aggressive and negative information is not reflected in asset prices to the same extent as positive information. While our model is substantially different, we also find that the possibility of corrective intervention dampens the informed trader's incentive to sell, and our set of positive results qualitatively complement those of Edmans, Goldstein and Jiang (2014).[5]

However, unlike Edmans, Goldstein and Jiang (2014) which is concerned primarily with characterizing the feedback effect and deriving its implications for asset prices, our analysis also considers normative issues related to intervention policy. As noted above, our normative analysis makes two novel contributions: we characterize the Pareto optimal intervention policy under full commitment by the policymaker and investigate three alternative institutions that approximate the ideal mechanism. This aspect of our contribution has no analogue in Edmans, Goldstein and Jiang (2014).

We present our basic model in the next section. In Section 3 we analyze a benchmark setting where interventions are infeasible or are too expensive to implement. In Section 4 we characterize and analyze the unique Perfect Bayesian equilibrium of the game involving interventions. In Section 5 we characterize the Pareto optimal intervention mechanism for the policymaker assuming that she has full power of commitment. Although the this analysis is illuminating, commitment to the finely tuned optimal intervention mechanism is implausible. Thus, in Section 6 we analyze three cruder forms of commitment: imperfections in the intervention process, delegation of intervention authority to a biased agent, and a regime of secrecy, showing that the policymaker generally benefits

[5]Though there are qualitative similarities, significant differences do exist between the equilibrium characterizations. For example, in the "buy, not sell" equilibrium of Edmans, Goldstein and Jiang (2014), only the noise trader sells in equilibrium. Hence, *large* selloffs (involving 2 rather than 1 unit) are never observed and every sale (of 1 unit) triggers an intervention with certainty. In order to reconcile this result with real-world cases in which firms have taken corrective measures in response to large selloffs, the authors interpret the corrective actions as unanticipated by the investors (see footnote 2 of Edmans, Goldstein and Jiang (2014) and their discussion of Coca-Cola's attempted acquisition of Quaker Oats on page 3). In contrast, in our model an informed trader who knows that the state is bad does participate in the market, even though he rationally anticipates that his sale may trigger a corrective intervention. Small selloffs do not trigger corrective actions, while larger selloffs are increasingly likely to do so. Our result is therefore consistent with Edmans, Goldstein and Jiang (2014)'s examples, without the need to assume that traders are surprised by the intervention.

from these alternative arrangements. We recap our findings in Section 7. Proofs are in Appendix A.

2 THE MODEL

We study a game with two active risk-neutral players, an investor and a policymaker, and a passive market maker who takes the other side of any trade with the investor. Two states of nature are possible: $\omega = 0$, which occurs with prior probability q, and $\omega = 1$, which occurs with $1 - q$. The investor may trade shares of an Arrow security that commits the seller to pay the buyer one if $\omega = 1$ and zero if $\omega = 0$.[6] The asset price is equal to its expected payoff given all publicly available information, and it adjusts instantly to arrival of new public information.[7] Investors can take either positive or negative positions in the asset: a negative position represents a sale and a positive position represents a purchase.

The investor is an *informed trader* with probability a and a *noise trader* with probability $1-a$. An informed trader privately observes a signal realization (his type) $i \in \{0, 1\}$ which is perfectly correlated with ω, and he invests in an effort to maximize his expected return. A noise trader invests for exogenous reasons (e.g., a liquidity shock) and generates a random order flow, uniformly distributed on $[-1, 1]$.[8]

The policymaker cares intrinsically about the state. In particular, she receives payoffs normalized to one if $\omega = 1$ and zero if $\omega = 0$. The policymaker has a costly technology that allows her to "intervene" in the process that generates the state. If she intervenes, then she bears cost c but guarantees that the state is $\omega = 1$ with probability one.[9] We focus on the case in which the intervention is sufficiently costly that the policymaker would not want to intervene under the prior, $c > q$. She may, however, find it optimal to intervene if the investor's trade (e.g. a large sell order) reveals that the state is likely to be $\omega = 0$.[10]

The game proceeds in four stages. In the first stage the state and the trader's type (zero, one or noise) are realized. In the second stage the trader submits an order $t \in \Re$ to the market, observed publicly. The market maker updates his beliefs based on the order and adjusts the price to equal the asset's expected value and then fills the order. In the third stage, the policymaker observes the trade and decides whether to intervene. In the last stage the state is revealed and all payoffs are realized.

In this game, buying the asset is (weakly) dominated for a type-0 trader and selling is (weakly) dominated for a type-1 trader. Therefore, we treat all trades as non-negative numbers with the

[6] It is formally equivalent to suppose that the asset has "fundamental" value ω in state ω and allow short sales.

[7] The standard justification for this argument is Bertrand competition among market makers.

[8] The interval $[-1, 1]$ is a normalization. Investor payoffs are homogenous in the interval length. Also, the uniform distribution is assumed merely for analytic convenience. The model can accommodate any distribution with no qualitative changes in the results.

[9] In section 6 we consider a case in which the intervention does not succeed with certainty.

[10] As presented, the policymaker cannot learn the state before deploying the intervention. If we allow the policymaker to perform a costly audit (based on the order flow), which allows her to learn the state before intervening, the results are formally equivalent. See the analysis in Appendix C.

understanding that a sale of $t > 0$ units results in negative order flow. A mixed strategy by type $i \in \{0, 1\}$ is represented by the probability mass function $\phi_i(\cdot)$, defined over support $S_i \subset \Re_+$, with smallest element m_i.[11] We often refer to m_i as trader i's minimum trade size. The random variable generated by the mixed strategy is τ_i and its realization is t. Denote the belief of the policymaker (and the market maker) that the state is zero conditional on observing order t by $\chi(t) \equiv \Pr\{\omega = 0|\tau_i = t\}$, and denote the probability that the policymaker intervenes after observing order flow t by $\alpha(t)$. The solution concept is Perfect Bayesian equilibrium, which consists of a trading strategy for each type of informed investor, $\phi_i(t)$, an intervention strategy for the policymaker, $\alpha(t)$ and a belief function $\chi(t)$. Each player's strategy must be sequentially rational given the strategy of the other player, and for each order size on the equilibrium path the belief function must be consistent with Bayes' rule applied to equilibrium strategies.[12] Next, we perform some preliminary analysis, deriving expressions for prices, payoff functions, the incentive constraints, and beliefs. We then characterize some simple but significant properties of any equilibrium of the game.[13]

Market Price. The asset pays one if $\omega = 1$ and zero if $\omega = 0$; its price must therefore equal the probability that $\omega = 1$ at the end of the game:

$$p(t) = (1 - \chi(t))(1 - \alpha(t)) + \alpha(t) = 1 - \chi(t) + \alpha(t)\chi(t) \tag{1}$$

If an intervention takes place, the asset is worth one for certain, but if the intervention does not take place, then the expected payoff of the asset is equal to $1-\chi(t)$, the probability that the state is one, given the observed order. The price therefore incorporates information about both the "fundamental" (it is decreasing in $\chi(t)$) and about the anticipated intervention policy (it is increasing in $\alpha(t)$).

Trader Payoffs. Whether or not an intervention takes place, the type-1 trader expects the asset to be worth one. Thus, the type one trader's expected profit on each share purchased is just one minus the price $p(t)$. A type-1 trader's expected payoff from purchasing t shares of the asset is thus

$$\begin{aligned} u_1(t) &= t(1 - p(t)) \\ &= t\chi(t)(1 - \alpha(t)) \end{aligned} \tag{2}$$

Meanwhile, a type-0 trader collects the sale price, $p(t)$ on each share that he sells. In the absence of

[11] With the inclusion of the Dirac $\delta(.)$ function, this definition also allows for pure strategies. This issue is irrelevant, because in equilibrium traders always play mixed strategies with no mass points.

[12] Because all trades inside $[-1, 1]$ may be submitted by the noise trader, all possible orders in this interval are on the equilibrium path. No order outside this interval is submitted by a trader (noise or informed) in any equilibrium.

[13] To understand why trading on the opposite side of the market is weakly dominated (claimed above), note that if the type-1 investor sells t units, his expected payoff from the trade is $t(p(t) - 1)$, because he collects $p(t)$ on each share but will have to pay back one one each share sold, whether or not intervention takes place. If the type-0 investor buys, his expected payoff from the trade is $t(\alpha(t) - p(t))$ because he pays $p(t)$ for an asset that is worth 1 only if the policymaker intervenes. Because $p(t) \in [\alpha(t), 1]$ (consult (1)), both of these expected payoffs are negative.

intervention he knows that the asset will be worth zero, allowing him to cover his short position at zero cost. If intervention occurs, however, he will owe one per share sold. Therefore, on each share sold the type-0 trader expects to collect a payoff equal to $p(t) - \alpha(t)$. A type-0 trader's expected payoff from selling t shares of the asset is thus

$$\begin{aligned} u_0(t) &= t(p(t) - \alpha(t)) \\ &= t(1 - \chi(t))(1 - \alpha(t)) \end{aligned} \tag{3}$$

In the absence of interventions ($\alpha(t) = 0$), either type of informed trader expects a positive rent, unless the order fully reveals his private information. Whenever the market maker is uncertain about the true state, the market maker mis-prices the asset, selling it too cheaply to a type-1 trader ($p(t) < 1$), and buying it too expensively from a type-0 trader ($p(t) > 0$). The possibility of intervention does not change the mis-pricing that arises from asymmetric information. However, if an intervention takes place, the state is known to be $\omega = 1$; asymmetric information vanishes, and with it, the trader's rent. Thus the trader's expected payoff when interventions occur with positive probability is simply his expected payoff in the absence of interventions, multiplied by the probability that no intervention takes place.

Policymaker Payoff. The policymaker's expected payoff from intervening with probability α is

$$(1 - \alpha)(1 - \chi(t)) + \alpha(1 - c) = 1 - \chi(t) + \alpha(\chi(t) - c).$$

If the policymaker intervenes, she ensures $\omega = 1$, but loses c; if she does not intervene she receives payoff one whenever $\omega = 1$. From this, it follows that the policymaker's equilibrium intervention strategy must satisfy the following sequential rationality condition:

$$\alpha(t) = \begin{cases} 0, & \text{if} \quad \chi(t) < c \\ [0, 1] & \text{if} \quad \chi(t) = c \\ 1, & \text{if} \quad \chi(t) > c \end{cases} \tag{4}$$

Thus, the policymaker intervenes whenever the probability that the state is low exceeds the intervention cost. This condition highlights the dilemma facing an informed trader. If he executes a trade t that reveals too much information, so that $\chi(t) > c$, then the market maker will anticipate an intervention and will set $p(t) = 1$, resulting in a payoff of zero for the trader. The investor must therefore be cognizant of precisely how his trades impact beliefs.

Equilibrium Beliefs. In equilibrium, beliefs are determined by Bayes' Rule applied to strategies. Because trading on the opposite side of the market is weakly dominated, a buy order either

comes from a type-1 informed trader or a noise trader, so

$$\chi(t) = \frac{q(1-a)\frac{1}{2}}{(1-q)a\phi_1(t) + (1-a)\frac{1}{2}} \text{ if buy order } t \text{ is observed.} \tag{5}$$

Likewise, a sell order either comes from a type-0 informed trader or a noise trader, so

$$\chi(t) = \frac{q\left(a\phi_0(t) + (1-a)\frac{1}{2}\right)}{qa\phi_0(t) + (1-a)\frac{1}{2}} \text{ if sell order of } t \text{ is observed} \tag{6}$$

Additional Notation. For ease of exposition, we define some additional notation. The following multivariate function turns out to be instrumental in the analysis:

$$Q(m, x|j, k) \equiv \left(\frac{xj^2}{1-x} + 1\right) m^2 - 2(k+1)m + 1 - x. \tag{7}$$

It is also helpful to define the following transformations of the parameters of the model:

$$J(c, q) \equiv \frac{1-q}{1-c} \quad (c, q) \in [0, 1]^2$$

$$K_i(a) \equiv \frac{2a}{1-a}((1-i)q + i(1-q)) \quad (a, q) \in [0, 1]^2 \text{ and } i \in \{0, 1\}$$

We suppress arguments (c, q) and a, writing J, K_i whenever doing so does not create confusion.

Simple Observations. A number of observations follow from the simple analysis presented so far, described in Lemma A.2 in appendix A. We highlight the most significant of these here. In any equilibrium in which a type-i informed trader expects a positive payoff, he must employ a mixed strategy representable by a probability density function $\phi_i(\cdot)$ with no mass points or gaps, supported on interval $[m_i, 1]$. In order to avoid revealing his private information to the market maker (or policymaker) the informed trader must "hide in the noise" generated by the uninformed trader's order flow. Furthermore, any order that triggers an intervention for certain results in a zero payoff for the trader, and it therefore cannot be in the support of an equilibrium strategy with positive trader payoff. This observation underlies the central tension of our setting: if the trader expects to make money by participating in the market, he will never submit an order that is expected to trigger an intervention for certain. At the same time if the expected benefit of the intervention is positive, the policymaker would like to undertake it for certain; thus no beneficial intervention could be triggered by a trader's order.

3 NO INTERVENTIONS

As a benchmark, we first present the case in which interventions are either infeasible or prohibitively expensive (an exact bound on the intervention cost will be derived). The following proposition

describes the equilibrium and its comparative statics. In equilibrium, both types of trader must be indifferent over all trades inside the support of their mixed strategies. Using this observation together with the expressions for trader payoffs (see equations (2) and (3)) allows us to determine the mixing densities, parametrized by the minimum trade size m_i. We then invoke Lemma A.2 to determine the connection between the trader's equilibrium payoff and his minimum trade size. Finally, we solve for the equilibrium minimum trade size by ensuring that the trader's mixing density integrates to one (see appendix A for details).

Proposition 3.1 *(No interventions). The unique equilibrium when interventions are not possible is characterized as follows.*

- ***Strategies****. A type-i trader submits an order of random size t, described by probability density function $\phi_i(t)$ over support $[m_i^*, 1]$, where*

$$\phi_1(t) = \frac{t - m_1^*}{K_1 m_1^*} \text{ and } \phi_0(t) = \frac{t - m_0^*}{K_0 m_0^*},$$

  and m_i^* is the smaller value satisfying $Q(m_i^*, 0|J, K_i) = 0$ (recall equation (7)).

- ***Payoffs****. Informed trader payoffs are $u_1 = m_1^* q$ and $u_0 = m_0^*(1 - q)$*

- ***Comparative Statics.***
  - *An increase in a causes m_i^* to decrease, both types to trade less aggressively in the sense of first order stochastic dominance, and a fall in type i's equilibrium expected payoff u_i.*
  - *An increase in q causes an increase in m_1^* and decrease in m_0^*, type 1 to trade more aggressively and type 0 to trade less aggressively in the sense of first order stochastic dominance, a rise in type 1's equilibrium expected payoff u_1, and a fall in type 0's equilibrium expected payoff u_0.*

The informed investors' equilibrium mixing densities are increasing linear functions supported in an interval from a strictly positive trade m_i^* to one. Other things equal, an informed investor stands to gain more from extreme trades. Other things are, of course, not equal. Because they are tempting for the informed trader, large orders cause large movements in the beliefs of the market maker and hence, in the transaction price. Because the investor must be indifferent over all trades $t \in [m_i^*, 1]$, in equilibrium the effect on the price must exactly offset the informed trader's benefit of trading larger volume. The trader's payoff functions (equations (2) and (3)) reveal the exact connection between equilibrium beliefs and order flow over the support of the trader's mixed strategy:

$$\text{Sell order } t \geq m_0^* \Rightarrow \chi(t) = 1 - \frac{m_0^*(1-q)}{t} \qquad \text{Buy order } t \geq m_1^* \Rightarrow \chi(t) = \frac{m_1^* q}{t}$$

Meanwhile, any other order of size $t \leq 1$ must be submitted by a noise trader, and thus has no effect on beliefs: $\chi(t) = q$. In other words, large sell (buy) orders are associated with a relatively

high (low) belief that $\omega = 0$ and correspondingly low (high) asset price. The most extreme beliefs derive from trades of size $t = 1$:

$$\overline{\chi} \equiv 1 - m_0^*(1-q) \text{ and } \underline{\chi} \equiv m_1^* q.$$

Figure 1 illustrates the equilibrium beliefs as a function of order flow.

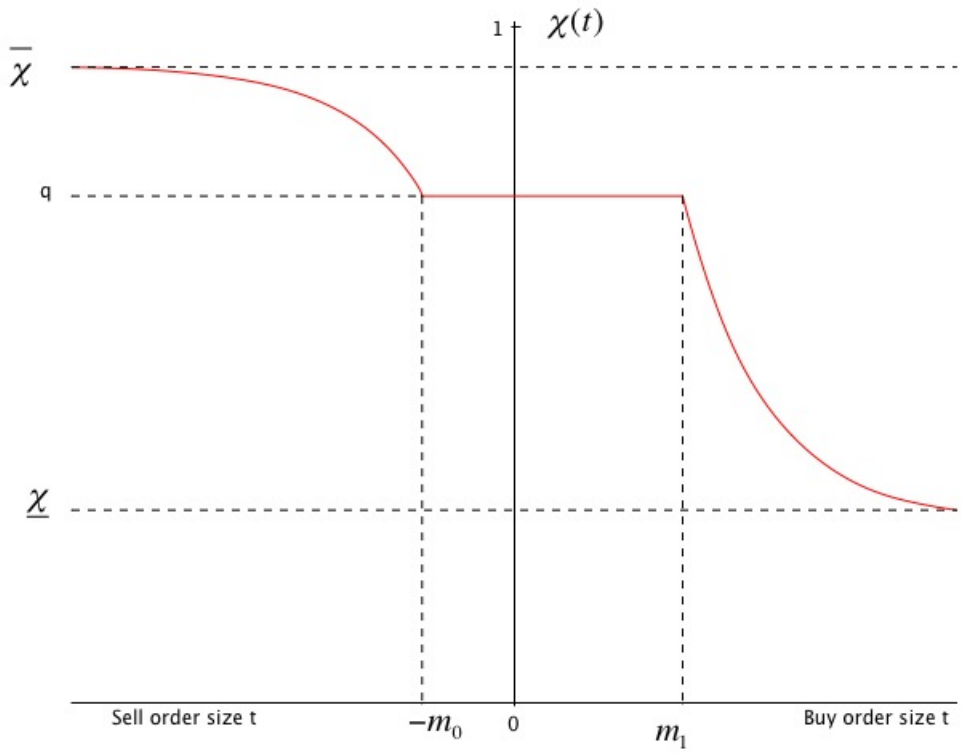

Figure 1: The posterior belief with no possibility of intervention

Values m_1^* and m_0^* represent respectively the largest buy and sell orders that a trader can make "for free"—that is, without revealing information that causes beliefs to change from the prior. Since an informed trader must be indifferent between all trades over which he mixes, it follows from (2) and (3) that equilibrium payoffs are $u_1 = m_1^* q$ and $u_0 = m_0^*(1-q)$.

The comparative statics in Proposition 3.1 are intuitive. When the probability that the trader is informed, a, is high, the market maker's beliefs (and hence prices) are very sensitive to order flow. Both types of informed trader mix over a wide range of orders using a relatively flat density approximating the noise trader's uniform one. Because the noise trader is unlikely to be present, this possibility provides weak "cover" for the informed trader, yielding him meager information rents. By contrast, when a is low, order flow is most likely generated by a noise trader and prices are, therefore, relatively insensitive. Hence, an informed investor can make large trades without

causing large movements in the price, thereby securing substantial information rents. Similarly, when the prior is more biased toward state zero, i.e. q is high, a sell order is relatively more likely to have come from an informed trader and a buy order is relatively more likely to have come from a noise trader. Thus, sell orders – which confirm the prior – generate large downward movements in the price and buy orders – which contradict the prior – generate small upward movements. Hence, an informed buyer can trade more aggressively without revealing his information and secure correspondingly higher rents than an informed seller. Of course, the reverse comparative statics (and intuition) hold when $1 - q$ is high.

4 INTERVENTIONS

With the benchmark of the preceding section in hand, we now turn to a setting in which interventions are possible. First, observe that the no intervention equilibrium of Proposition 3.1 remains the unique equilibrium when $c \geq \overline{\chi}$ (review (4)). This can happen for one of three reasons: either c is too high, a is too low, or q is too low. In the first case interventions are simply too costly and in the second and third cases even a maximal selloff is not sufficiently informative to justify intervening. Hence, a necessary condition for interventions to occur in equilibrium is $c < \overline{\chi}$. Next, note that the possibility of intervention does not affect the type-1 trader's equilibrium behavior. Part 3 of Lemma A.2 shows that beliefs following a buy order must be (weakly) more optimistic, $\chi(t) \leq q$. Hence, a buy order never triggers an intervention and a type-1 trader's equilibrium behavior and payoff are as described in Proposition 3.1. For this reason, we focus in what follows on the equilibrium strategies of the type-0 trader and the policymaker.

Imagine for a moment that when it is time for her to act, the policymaker observes that a buy order was *not* placed, but she has no other information about the order. Because type-1 investors always buy in equilibrium, this limited information reveals the trader to be either a noise trader (with probability $1 - a$) or a type-0 (with probability a), indicating that the state is more likely to be bad. In this scenario, the policymaker's posterior belief that the state is zero is given by the following expression, derived from Bayes' rule.

$$\hat{\chi} \equiv \frac{q\left(a + (1-a)\frac{1}{2}\right)}{qa + (1-a)\frac{1}{2}} = \frac{K_0 + q}{K_0 + 1} > q.$$

If a is high enough or c is low enough that $c < \hat{\chi}$, then simply knowing that a buy order was not submitted is sufficient to induce the policymaker to intervene. In Appendix A (see Lemma A.1), we show that in this case in every equilibrium the policymaker intervenes after observing *any* sell order, and the type-0 trader's equilibrium payoff is zero. We also show the inverse: whenever $c > \hat{\chi}$, the policymaker does not intervene for certain after every sell order and the type-0 trader's equilibrium payoff must be strictly positive. For the remainder of the paper, we focus on the case in which $c \in (\hat{\chi}, \overline{\chi})$, so that interventions take place with positive probability in equilibrium but

are not triggered by every sell order.[14]

The higher the trader's mixing density $\phi_0(t)$, the more information is revealed by order t, and the higher is $\chi(t)$.[15] If $\chi(t) > c$, then the policymaker intervenes with probability 1 (see (4)). As just noted, this cannot happen in equilibrium when $c \in (\hat{\chi}, \overline{\chi})$. Thus, it must be the case that $\chi(t) \leq c$, and this implies an upper bound on the type-0 trader's equilibrium mixing density:

$$\chi(t) \leq c \iff \phi_0(t) \leq f \equiv \frac{J-1}{K_0} \tag{8}$$

The following proposition characterizes the unique equilibrium in our main case of interest. It is derived in an analogous way to Proposition 3.1, imposing condition (8) as an additional constraint.

Proposition 4.1 *(Stochastic Interventions). When $c \in (\hat{\chi}, \overline{\chi})$ and interventions are possible, the game has a unique equilibrium, characterized as follows.*

- ***Strategies.***
  - *The type-0 trader places a sell order distributed according to continuous probability density function $\phi_0(t)$ over support $[m_0^\dagger, 1]$ defined piecewise:*

$$\phi_0(t) = \begin{cases} \frac{t - m_0^\dagger}{K_0 m_0^\dagger} & \text{if} \quad t \in [m_0^\dagger, \theta^\dagger] \\ f & \text{if} \quad t \in [\theta^\dagger, 1], \end{cases}$$

  where $m_0^\dagger$ is the non-zero value satisfying $Q(m^\dagger, 1 - Jm^\dagger | J, K_0) = 0$, and $\theta^\dagger = Jm_0^\dagger$.
  - *The policymaker intervenes with probability*

$$\alpha(t) = \begin{cases} 0 & \text{if} \quad t \in [0, \theta^\dagger] \\ 1 - \frac{\theta^\dagger}{t} & \text{if} \quad t \in [\theta^\dagger, 1]. \end{cases}$$

- ***Payoffs.*** *The type-0 trader's expected payoff is $u_0 = m_0^\dagger(1-q)$. Policymaker's expected payoff is the same as if interventions were not possible, $1-q$.*
- ***Comparative Statics.*** *$m_0^\dagger$, $\theta^\dagger$, and u_0 are increasing in c and decreasing in a and q. In addition, $c = \hat{\chi}$ implies $m_0^\dagger = \theta^\dagger = 0$, and $c = \overline{\chi}$ implies $m_0^\dagger = m_0^*$ and $\theta^\dagger = 1$.*

When $c \in (\hat{\chi}, \overline{\chi})$, sell orders are partitioned into two intervals, a *safe* zone of modest trades ($t \leq \theta^\dagger$) that never trigger an intervention and a *risky* zone of larger trades ($t > \theta^\dagger$) that trigger an intervention with positive probability. Over the safe zone the type-0 trader mixes with an increasing linear density similar to Proposition 3.1, and larger trades reveal more information, increasing the posterior belief $\chi(t)$. At the critical trade $t = \theta^\dagger$, the policymaker is just indifferent

[14] Point 1 of Lemma A.3 in appendix shows $\hat{\chi} < \overline{\chi} \iff a > 0,\ q > 0$.

[15] Because noise trades are uniformly distributed, higher $\phi_i(t)$ corresponds to higher likelihood that trade t was submited by and informed investor.

about intervening, $\chi(\theta^\dagger) = c$ (see Figure 2). At this point beliefs must stop increasing with order flow because higher beliefs would induce the policymaker to intervene with certainty. In order to truncate beliefs at this level, the informed trader pools with the noise trader, mixing uniformly over the risky zone of trades. Although her beliefs are constant over the risky zone, the probability that the policymaker intervenes increases with order flow to offset the temptation of the investor to make larger trades. A degree of unpredictability is often inherent in government intervention policies; examples include the differential treatments of Bear Stearns and Lehman Brothers, and the government's repeated refusal to delineate an explicit bailout policy for Fannie Mae and Freddie Mac (Frame and White 2005).

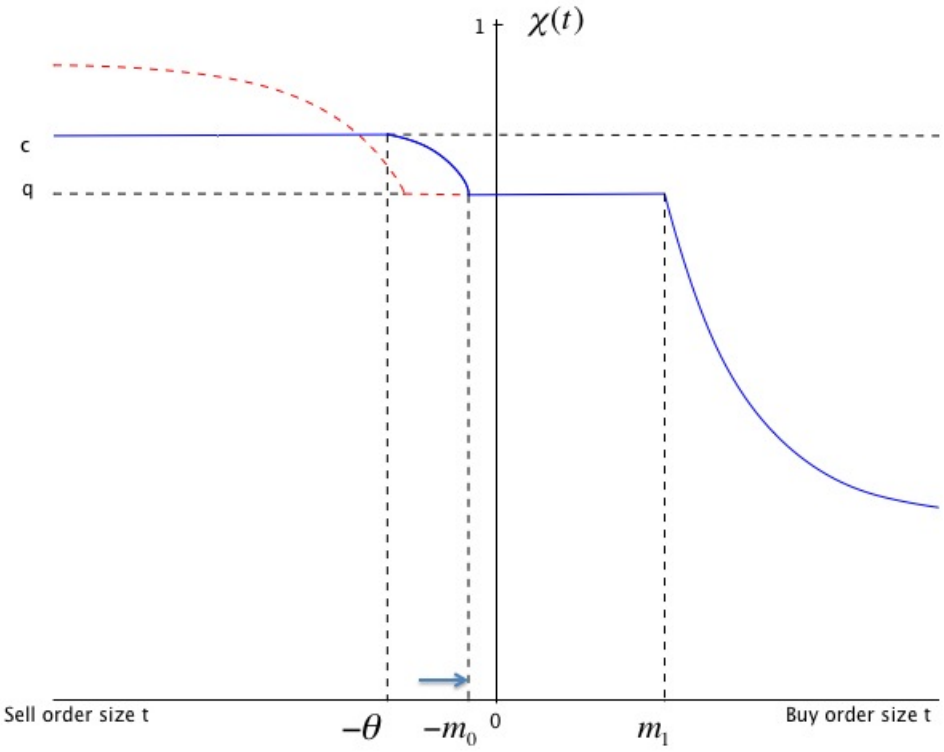

Figure 2: The posterior belief for the equilibrium with stochastic interventions

As $c \to \overline{\chi}$, the equilibrium approaches the no-intervention benchmark presented in Proposition 3.1: the risky zone shrinks; the probability of an intervention goes to zero, and the investor's payoff approaches $m_0^*(1-q)$. By contrast, as $c \to \hat{\chi}$ the risky zone expands; the probability of intervention goes to one, and the investor's payoff tends to zero.

Unfortunately for the policymaker, when $c \in (\hat{\chi}, \overline{\chi})$, her expected equilibrium payoff is the same as if interventions were not possible. In the unique equilibrium, the policymaker either does not intervene, or mixes and is therefore indifferent between intervening and not. This is a direct consequence of the fundamental conflict of interest between the trader and the policymaker:

the trader makes positive profit in equilibrium only if the policymaker does not benefit from the ability to intervene. In Section 5, we investigate a version of the model in which the policymaker can commit to an intervention policy that violates her sequential rationality condition (4). This commitment mitigates the conflict of interest, allowing the policymaker to benefit from intervening while preserving positive profit for the investor.

The type-0 trader is worse off in the equilibrium of Proposition 4.1 than in the benchmark setting of Proposition 3.1: $m_0^\dagger(1-q) < m_0^*(1-q)$. In order to avoid triggering an intervention for certain with his order, the informed trader's mixing density is capped over the risky zone: he must put less mass on larger orders. Consequently his mixing distribution must be shifted towards smaller, less profitable trades. We show in the next proposition that the possibility of intervention induces him to trade less aggressively (in a formal sense) than he would if interventions were not possible. Because the equilibrium order flow acts as a public signal of the (pre-intervention) fundamental, less aggressive trade by the type-0 investor reduces the Blackwell-informativeness of this signal.

Proposition 4.2 *(Equilibrium Properties).*

- *(Less aggressive trade). The type-0 investor trades less aggressively in the equilibrium with stochastic interventions than in the no-intervention benchmark: his equilibrium mixed strategy in the absence of interventions first order stochastic dominates his equilibrium mixed strategy in the stochastic intervention equilibrium.*
- *(Less information). Order flow is less Blackwell informative about the underlying state in the equilibrium with stochastic interventions than in the no-intervention benchmark.*
- *(Non-monotonic price). In the stochastic intervention equilibrium the asset price is a non-monotonic function of the order flow.*
- *(Higher mean price). The expected asset price is higher in the equilibrium with stochastic interventions than in the no-intervention benchmark.*

The last two points of this proposition describe properties of the equilibrium price. The asset price is a non-monotonic function of the order flow (review (1)). Over the safe zone, the price falls with larger sell orders as the market maker becomes more convinced that $\omega = 0$—in this case $\chi(t)$ increases while $\alpha(t) = 0$. Over the risky zone, however, the price rises as the market maker becomes ever more convinced that an intervention is forthcoming—here $\chi(t) = c$ but $\alpha(t)$ increases.

To understand why the expected asset price is higher, note that the equilibrium price (see equation (1)) is composed of two distinct terms: the first term $1 - \chi(t)$ is the posterior belief that $\omega = 1$ in the absence of an intervention, while the second term $\alpha(t)\chi(t)$ reflects the impact of a possible intervention. Because $1 - \chi(t)$ is the posterior belief that $\omega = 1$ conditional on order flow t, which is itself random (as it derives from a mixed strategy), the Law of Iterated Expectations implies that the expected value of $1 - \chi(t)$ is equal to $1 - q$. This is true whether or not interventions occur with positive probability. In the equilibrium with stochastic interventions,

however, $\alpha(t)$ is positive for $t > \theta^\dagger$, so the second term in the expected price, $E[\alpha(t)\chi(t)]$ is also positive. Interestingly, the increase in the asset price cannot be attributed to a less aggressive selling strategy on the part of the type-0 investor: changes in the trader's selling strategy have no effect on the expected fundamental, which is equal to $1 - q$. Rather, the increase in price is due to the expectation of corrective intervention, $E[\alpha(t)\chi(t)]$, which is positive and is built into the price.

A number of empirical papers document a connection between the expectation of corrective intervention and a high asset price. Frame and White (2005) survey several investigations estimating that debt issued by Fannie Mae and Freddie Mac trades at interest rates 0.35-0.40% below its risk rating, resulting in a higher asset price. According to these authors "financial markets treat [Fannie and Freddie's] obligations as if those obligations are backed by the federal government" despite the fact that the government is under no legal obligation to intervene in the event of trouble. In fact, financial markets correctly forecasted government policy: in September 2008 the federal government intervened to stabilize Fannie Mae and Freddie Mac (see Frame (2009) for more information). Thus, while the connection between interventions and Fannie Mae and Freddie Mac's asset price is empirically documented, it is less well understood in the literature that the possibility of corrective action reduced the informativeness of Fannie Mae's stock price about its fundamental value. Hence, the intervention was deployed based on less-reliable information than is commonly understood. Other empirical analyses also link corrective actions to high asset prices. O'Hara and Shaw (1990) show that congressional testimony by the Comptroller of the Currency that some banks are "too big to fail" caused equity prices to increase at several large banks. Interestingly, the increases were most significant for eleven banks named in media coverage of the story, which was not identical to the set of banks covered by the policy. Indeed, certain banks that the Comptroller intended to cover experienced price drops because they were excluded from the list reported in the media. Gandhi and Lustig (2010) also show that announcements in support of bailouts increase bank equity prices and present a broad range of evidence.

5 PARETO IMPROVING INTERVENTIONS

As noted in Proposition 4.1, when $c \in (\hat{\chi}, \overline{\chi})$, the policymaker intervenes stochastically, but does not benefit relative to the no-intervention benchmark. The reason for this is her lack of commitment power. Specifically, it is sequentially rational for the policymaker to never intervene if $\chi(t) < c$ and to always intervene if $\chi(t) > c$. The type-0 trader therefore chokes off the information content of order flow for $t > \theta^\dagger$ to avoid triggering a certain intervention.

In this section we consider a hypothetical setting in which *ex ante* commitment to an intervention policy $\alpha(t)$ is possible. In particular, we derive the *ex ante* optimal intervention plan for the policymaker holding the type-0 investor's expected payoff at its equilibrium level, $u_0 = m_0^\dagger(1 - q)$. This restriction ensures that the expected payoffs of all market participants are unchanged from their equilibrium values so that the policy we derive represents a Pareto improvement over the equilibrium.[16] Although we do not regard full commitment to an optimal random intervention

[16]Recall that the market maker's payoff is zero, and informed trader rents come at the expense of noise traders.

policy as especially realistic, it is helpful to understand how the policymaker's optimal plan differs from her no-commitment equilibrium strategy. Further, this comparison paves the way for exploring more plausible *institutional* remedies in the next section.

To formulate the policymaker's optimal policy with commitment as a constrained programming problem, we adopt the standard approach of the principal-agent literature, allowing the policymaker to select both her own strategy and the type-0 trader's, imposing the equilibrium conditions for the trader's strategy as incentive compatibility constraints. We therefore imagine that the policymaker chooses $\alpha(t)$ and $\phi_0(t)$ in order to maximize her *ex ante* expected payoff

$$(9)\quad v = (1-q)\left(a + (1-a)\frac{1}{2}\right) + \int_0^1 \left(aq\phi_0(t) + (1-a)\frac{1}{2}\right)(\alpha(t)(1-c) + (1-\alpha(t))(1-\chi(t)))\,\mathrm{d}t$$

The first term in this expression is the contribution to the policymaker's expected payoff from a buy order (after which it is never optimal to intervene). The first term of the integrand is the density of sell orders and the second term is the policymaker's expected payoff given the chosen intervention probability at order flow t. This maximization is subject to feasibility conditions that ensure that the intervention probability and mixing density are valid, and incentive constraints, that ensure that the trader is willing to comply with the policymaker's recommended mixing density (see the proof of Proposition 5.1 for details).

Proposition 5.1 *(Pareto Optimal Interventions and Trade). If* $\max\{\frac{1+q}{2}, \hat{\chi}\} < c < \overline{\chi}$*, then a constant* $\lambda \in (0,1)$ *exists such that the policymaker's optimal choices of intervention policy and investor mixing density are characterized as follows.*

- *The type-0 trader places a sell order distributed according to continuous probability density function* $\phi_0(t)$ *over support* $[m_0^\dagger, 1]$ *defined piecewise:*

$$\phi_0(t) = \begin{cases} \frac{t-m_0^\dagger}{K_0 m_0^\dagger} & \text{if} \quad t \in [m_0^\dagger, \frac{\theta^\dagger}{1+\lambda}] \\ f + \frac{(1-\lambda)t-\theta^\dagger}{2K_0 m_0^\dagger} & \text{if} \quad t \in [\frac{\theta^\dagger}{1+\lambda}, 1] \end{cases}$$

- *Policymaker intervenes with probability*

$$\alpha(t) = \begin{cases} 0 & \text{if} \quad t \in [0, \frac{\theta^\dagger}{1+\lambda}] \\ \frac{1}{2}\left(1+\lambda-\frac{\theta^\dagger}{t}\right), & \text{if} \quad t \in [\frac{\theta^\dagger}{1+\lambda}, 1]. \end{cases}$$

When designing the *ex ante* optimal policy, the policymaker faces a delicate tradeoff. In order to induce the type-0 trader to reveal information, she must commit not to use the information too aggressively. On the other hand, acquiring information is pointless if she cannot act on it by

Other Pareto optimal policies exist (indexed by the informed trader's payoff) but these cannot be Pareto ranked.

making a beneficial intervention. To highlight this tension, note that (9) simplifies to:[17]

$$v = (1-q) + aq(1-c)\int_0^1 \alpha(t)(\phi_0(t) - f)\,\mathrm{d}t \tag{10}$$

Two things are evident from this formulation of the policymaker's objective. First, it is clear why she does not benefit in an equilibrium without commitment – namely, $\alpha(t) = 0$ for $t < \theta^\dagger$ and $\phi_0(t) = f$ for $t \geq \theta^\dagger$: so $v = 1 - q$ in this case. Second, an intervention at t is beneficial if and only if $\phi_0(t) > f$. The incentive constraints of the trader, however, imply that a high value of $\phi_0(t)$ necessitates a low value of $\alpha(t)$ and *vice versa*: acquiring valuable information requires a commitment to intervene with relatively low probability.

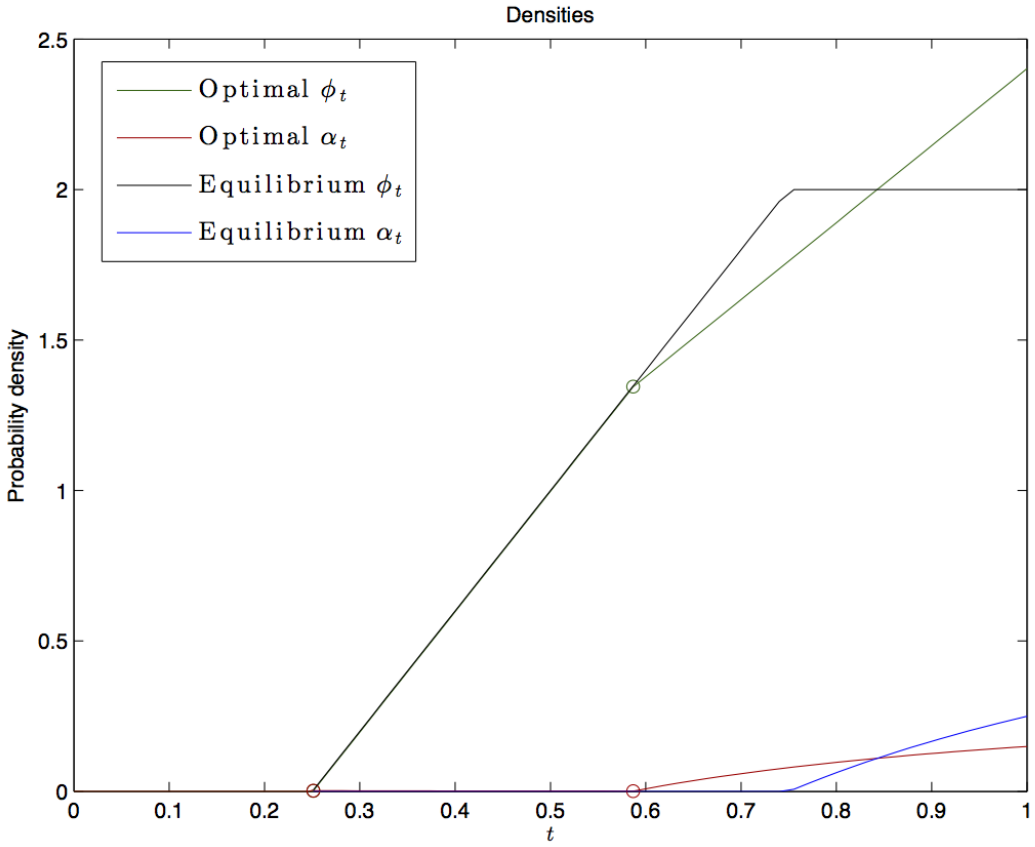

Figure 3: The Pareto optimal intervention policy

As illustrated in Figure 3, compared with the policymaker's strategy in the stochastic interventions equilibrium of Proposition 4.1, her *ex ante* optimal intervention policy (viewed as a function of the order flow) has a smaller intercept and is less steep.[18] The policymaker therefore optimally

[17] See the proof of proposition 5.1 for the derivation.

[18] Figure 3 corresponds to the following parameter values: $f = 2$, $k = 1$, $q = 1/10$. From these the following values are implied: $a = 1/12$, $\hat{\chi} = 11/20$, $c = 7/10$, $m_0^\dagger = 1/4$, $\theta^\dagger = 3/4$, $u_0 = 9/40$, $m_0^* = 2 - \sqrt{3} \approx 0.27$, $\overline{\chi} \approx 0.76$. This

begins intervening sooner (i.e. for smaller sell orders), but uses an intervention rule that is flatter and therefore less sensitive to increases in the trader's order. The two intervention policies intersect at order flow $t = \theta^\dagger/(1-\lambda)$; for larger order flows the optimal policy lies below the equilibrium one (and is weakly above for smaller orders).

Facing the policymaker's *ex ante* optimal intervention policy, the type-0 trader mixes according to a piecewise linear density. In fact, this density is the same as his no-commitment equilibrium strategy over the commitment safe zone, $[m_0^\dagger, \theta/(1+\lambda)]$. However, the commitment safe zone is smaller than the safe zone without commitment, ending at order flow $t = \theta^\dagger/(1+\lambda)$ instead of $t = \theta^\dagger$. At $t = \theta^\dagger/(1+\lambda)$ the type-0 trader's optimal density becomes flatter so that the policymaker's belief increases less rapidly as the order flow increases. The optimal density does not become completely flat (as it does without commitment), but continues increasing until it crosses the flat equilibrium density f at $\theta^\dagger/(1-\lambda)$. Thus, for order flow $t \in \big(\theta^\dagger/(1+\lambda), \theta^\dagger/(1-\lambda)\big)$ beliefs satisfy $\chi(t) < c$ and interventions – which occur with positive probability under the optimal policy – actually harm the policymaker in expectation. On the other hand, for order flow $t \in \big(\theta^\dagger/(1-\lambda), 1\big]$, beliefs satisfy $\chi(t) > c$ and interventions strictly benefit the policymaker in expectation.

The *ex ante* Pareto optimal policy therefore requires *both* kinds of commitment from the policymaker: for an intermediate range of order flow she must intervene with positive probability when she would prefer not to intervene at all, and for high order flow she must refrain from intervening with certainty, although she would benefit from doing so. In essence, the optimal intervention policy is a *clockwise rotation* of the equilibrium policy that induces a *counter-clockwise rotation* in the trader's mixing density over the region $[\theta^\dagger/(1+\lambda), 1]$. Thus, the optimally induced mixing density for the trader conveys more information on large orders, increasing the policymaker's payoff from an intervention. For this to be incentive compatible, the policymaker must intervene less often on large orders and slightly more often for an interval of moderate orders.

Of course, committing to an intervention policy that randomizes with precisely the correct probabilities at each order flow is implausible because it requires verification by the type-0 trader or some impartial third party. In the absence of such verification, the policymaker could simply implement her *ex post* preferred policy and claim that she randomized according to the *ex ante* optimal one. Even if the policymaker cannot commit to the precise *ex ante* optimal intervention policy, she may be able to verifiably commit to an institution that – while suboptimal – still raises her expected payoff. We investigate this possibility in the next section.

6 REMEDIES

In the previous section we derived the Pareto optimal intervention policy supposing that the policymaker could verifiably commit not to abide by her sequential rationality condition (4). However, commitment to the finely-tuned optimal intervention policy presented in Proposition 5.1 seems very implausible. Nevertheless, the policymaker may have access to other less precise, but more easily

figure was generated from a numerical computation, not from an analytical solution.

verified forms of commitment that allow her to relax sequential rationality at least to some extent. We explore three possibilities below.

IMPERFECT INTERVENTIONS

We consider an alternative environment in which the institution or technology that executes interventions is imperfect. The institution caps the intervention probability from above at some commonly known level $\overline{\alpha} < 1$. In a political context it could be that any attempted bailout is blocked with probability $1 - \overline{\alpha}$, so that the probability of actually intervening given that a bailout is attempted with probability $\beta(t)$ is $\alpha(t) = \overline{\alpha}\beta(t)$. Alternatively, the policymaker could employ a policy instrument that does not guarantee that the state is one for certain, but effects a transition to state one with some probability $\overline{\alpha} < 1$. Because the cap is a feature of the institution or technology, its existence is likely to be common knowledge among all parties.

As noted in Proposition 5.1, the Pareto optimal policy is flatter than the equilibrium policy. a cap on the intervention policy approximates this by forcing the policymaker to use a *completely* flat intervention policy for large orders (where the cap is binding). With the cap imposed, sequential rationality for the policymaker requires:

$$\alpha(t) = \begin{cases} 0 & \text{if} \quad \chi(t) < c \\ [0, \overline{\alpha}] & \text{if} \quad \chi(t) = c \\ \overline{\alpha} & \text{if} \quad t > c \end{cases}$$

This condition suggests why the cap might be desirable for the policymaker. Without the cap, whenever the policymaker believes that intervention is strictly beneficial, she intervenes with probability one, depriving the trader of all rent (see equations (2, 3)). The trader therefore mixes in a way that chokes off information and avoids a certain intervention. With the cap, the policymaker cannot intervene with probability one and leave the trader with zero payoff. Therefore, the trader may be willing to place more weight on large orders, even if this leads the policymaker to believe that an intervention is strictly beneficial. This intuition is formalized in the following proposition.

Proposition 6.1 *(Imperfect Interventions). If $c \in (\hat{\chi}, \overline{\chi})$ and $\overline{\alpha} < 1 - \theta^{\dagger}$, the game has a unique equilibrium that is characterized below. If $\overline{\alpha} > 1 - \theta^{\dagger}$, the intervention cap is non-binding, and the equilibrium is identical to the one in Proposition 4.1.*

- ***Strategies.***
  - *The type-0 trader places a sell order distributed according to continuous probability density function $\phi_0(t)$ over support $[\overline{m}_0, 1]$ defined piecewise:*

$$\phi_0(t) = \begin{cases} \frac{t - \overline{m}_0}{K_0 \overline{m}_0} & \text{if} \quad t \in [\overline{m}_0, \theta_1] \\ f & \text{if} \quad t \in [\theta_1, \theta_2] \\ \frac{(1-\overline{\alpha})t - \overline{m}_0}{K_0 \overline{m}_0} & \text{if} \quad t \in [\theta_2, 1] \end{cases}$$

where $\overline{m}_0$ *is the smaller value that solves* $Q(\overline{m}_0, \overline{\alpha}|J, K_0) = 0$*, and* $\theta_1 = J\overline{m}_0$*, and* $\theta_2 = \frac{\theta_1}{1-\overline{\alpha}}$.

– *The policymaker intervenes with probability*

$$\alpha(t) = \begin{cases} 0 & \text{if} \quad t \in [0, \theta_1] \\ 1 - \frac{\theta_1}{t} & \text{if} \quad t \in [\theta_1, \theta_2] \\ \overline{\alpha} & \text{if} \quad t \in [\theta_1, 1] \end{cases}$$

- ***Payoffs****. The type-0 trader's expected payoff is* $u_0 = \overline{m}_0(1-q)$*. The policymaker's expected payoff is strictly greater than her payoff in the stochastic intervention equilibrium.*

When interventions are imperfect, the risky zone of trades is split into two segments. For $t \in [\theta_1, \theta_2)$, the cap on the intervention probability does not bind, and equilibrium behavior for both players is similar to that given in Proposition 4.1. The investor chokes off information by mixing uniformly, and the policymaker intervenes with increasing probability. For the larger range of orders, $t \in [\theta_2, 1]$, the cap on intervention probability binds. Over this range, the type-0 investor mixes using an increasing density, thereby releasing more information than in the case with no cap; indeed over this range $\chi(t) > c$, and because interventions take place with positive probability over this range, the policymaker expects to benefit (see Figure 4).

The imperfect technology commits the policymaker not to intervene with certainty, and this induces the investor to reveal more information over the interval of extreme trades where the cap binds. Because large orders simultaneously convey more information and trigger interventions in expectation, the policymaker benefits relative to the case of stochastic interventions with no cap. In fact, both the policymaker and the type-0 trader prefer that the policymaker uses *any* (sufficiently) imperfect intervention technology

Corollary 6.2 *(Gains From Imperfect Interventions). If* $c \in (\hat{\chi}, \overline{\chi})$*, then the equilibrium payoffs of both the type-0 trader and the policymaker are strictly higher with a binding cap,* $\overline{\alpha} \in (0, 1-\theta^\dagger)$ *than without a binding cap,* $\overline{\alpha} \in [1-\theta^\dagger, 1]$*. Moreover, the trader's payoff is decreasing in* $\overline{\alpha}$*, while the policymaker's payoff is single peaked.*

As $\overline{\alpha}$ ranges from 0 to $1-\theta^\dagger$, the equilibrium moves continuously from the no-intervention benchmark of Proposition 3.1 to the stochastic intervention setting of Proposition 4.1. Since the type-0 trader prefers the former environment to the latter one, it is not surprising that his welfare increases as the cap decreases. Recall, however, that the policymaker's expected payoff is $1-q$ in both the no-intervention and stochastic-intervention settings. Nevertheless, her expected equilibrium payoff is strictly higher for any intermediate case. The reason is that the cap $\overline{\alpha}$ influences her payoff in two ways. With a tighter cap, the investor reveals more information through his trades, but the policymaker is less able to use this information to execute a beneficial intervention. At one extreme $\overline{\alpha} = 0$, the investor reveals the most information, but the policymaker's hands are tied. At the other extreme $\overline{\alpha} = 1-\theta^\dagger$, the policymaker is unconstrained, but the investor reveals

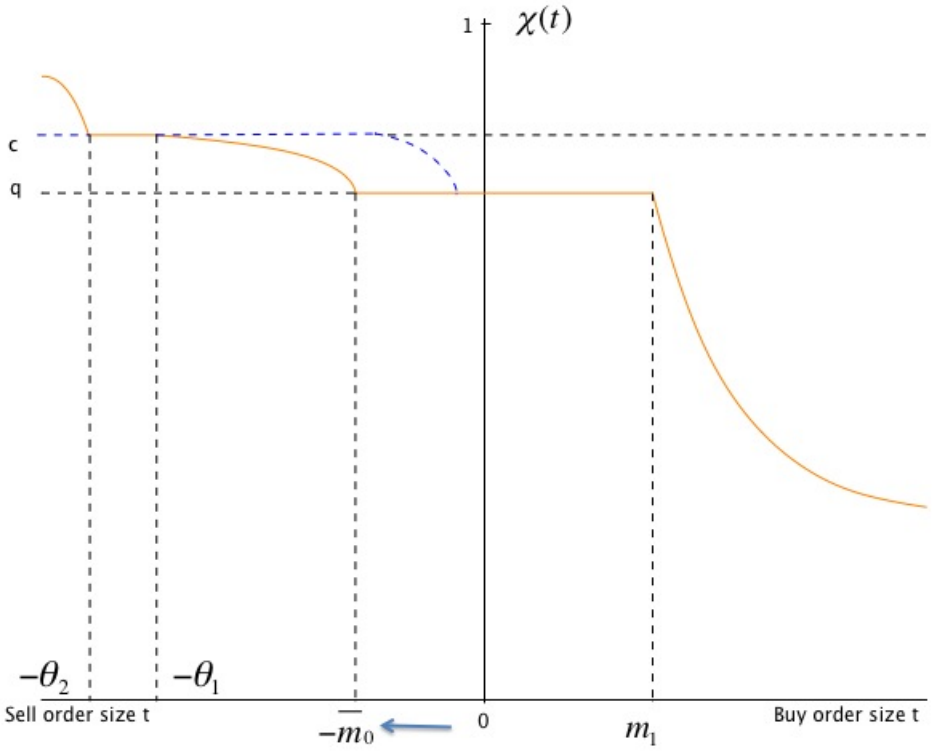

Figure 4: The posterior belief in the equilibrium with a binding intervention cap.

no valuable information. For all intermediate values of the cap, the investor reveals *some* valuable information which the policymaker can *sometimes* exploit.

Unlike the Pareto optimal policy of Proposition 5.1 which improves the policymaker payoff while keeping the market participants' payoffs at their equilibrium levels, imperfect interventions benefit the informed trader as well as the policymaker. This benefit to the informed trader comes at the expense of the noise trader, however, so that the capped equilibrium does not Pareto dominate the stochastic intervention equilibrium. Imperfections in the political process or intervention technology benefit the policymaker and increase utilitarian social welfare but also effect a "transfer" of utility from the noise trader to the informed trader.

DELEGATING AUTHORITY

Imagine that the policymaker can delegate the decision to deploy the intervention to an agent. The agent shares the policymaker's preferences for the fundamental (receiving payoff one if the terminal fundamental is $\omega = 1$) and also internalizes the full cost of the intervention, c, (which we assume is borne by the policymaker). Unlike the policymaker, whose preferences depend only on the fundamental and on the intervention cost, the agent's preferences also place some weight on

the noise trader's realized trading profit.[19] We show that if the agent weights noise trader profits positively, then the policymaker benefits from delegation; however, if this weight is negative, then the policymaker is hurt by delegation.

Consider an agent whose preferences weight the noise trader's realized (ex post) trading profit by b. We allow for both positive and negative values of this weight (bounds will be presented), which we refer to as the agent's bias. The agent has exactly the same information as the policymaker: she does not know the agent's type at the time that she decides whether to intervene, though she draws inferences about the agent's type from the observed order. We characterize the resulting equilibrium, showing that the policymaker benefits from delegation if and only if the agent positively weights ex post uninformed investor profit.

To begin the analysis, define the agent's posterior belief that the trader is informed conditional on observing sell order t by

$$\delta(t) \equiv \frac{aq\phi_0(t)}{aq\phi_0(t) + (1-a)\frac{1}{2}} = \frac{K_0\phi_0(t)}{K_0\phi_0(t) + 1}$$

Like $\chi(t)$, the belief that the trader is informed is higher when $\phi_0(t)$ is higher: because noise trades are uniformly distributed, the likelihood that a sell order of t was submitted by an informed trader is proportional to his trading density, namely $K_0\phi_0(t)$.

To write the agent's expected payoff, observe that if the agent intervenes with probability α following an executed sale of size t transacted at price $p(t)$, then the expected value of the noise trader's terminal profit is

$$\pi_{NS}(t) \equiv t(p(t) - \alpha - (1-\alpha)(1-q))$$

If the noise trader submits a sell order, he also collects $p(t)$ on each share, but expects to pay back one whenever the terminal fundamental is equal to one. Because the noise trader does not have private information about the state, he believes that in the absence of intervention, the state is good with probability $1-q$. However, interventions (assumed to occur with probability α), also generate a good terminal fundamental. Thus, when selling, the noise trader will collect $p(t)$ on each share *ex ante* but will pay back 1 on each share *ex post* with probability $\alpha + (1-\alpha)(1-q)$. Therefore, intervening (after the sell order has been executed) reduces the noise trader's profit, because it forces him to pay back on his short position for certain.[20]

The agent's expected payoff from intervening with probability α after observing a sale of size t

[19] Although we model the agent as weighting only the noise trader's profit, similar results hold if the agent weights the informed trader's profit, or weights both. We focus on bias in favor of the noise trader for ease of exposition and because protecting uninformed traders is sometimes suggested as a public policy objective.

[20] An intervention following a buy order is helpful to the noise trader, because it guarantees that the asset that he bought for $p(t)$ is worth 1 for certain. If an intervention is expected with probability α, then following a buy order the noise trader's expected profit is given by $\pi_{NB}(t) \equiv t(\alpha + (1-\alpha)(1-q) - p(t))$. Thus, if the agent positively weights noise trader profit, with a large bias, interventions may be triggered after buy orders. We impose bounds on the agent's bias to ensure that this does not happen in equilibrium.

executed at price $p(t)$ is

$$\{1-\chi(t)+\alpha(\chi(t)-c)\}+b(1-\delta(t))\pi_{NS}(t)$$

The bracketed term in this expression represents the policymaker's payoff and the final term represents the agent's weight on the noise trader's *ex post* payoff. Belief function $\delta(t)$ represents the agent's uncertainty about whether the trader is informed conditional on the observed order flow and $\pi_{NS}(t)$ represents the trader's expected profit, conditional on the trader being uninformed.

By maximizing the agent's expected payoff with respect to α, we find her sequentially rational intervention strategy:

$$\alpha(t)=\begin{cases} 0 & \text{if} \quad \chi(t)<c+t(1-\delta(t))qb \\ [0,1] & \text{if} \quad \chi(t)=c+t(1-\delta(t))qb \\ 1 & \text{if} \quad \chi(t)>c+t(1-\delta(t))qb \end{cases}$$

When the agent is biased positively, she is less inclined to intervene than the policymaker after a sell order: the agent must be more certain that an intervention is needed in order to be willing to undertake one. Thus, if the agent positively weights the noise trader's profit and chooses to intervene following a sale in equilibrium, the policymaker benefits. In the following proposition, we characterize the equilibrium of the game when the authority to deploy interventions is delegated to a biased agent. To simplify the analysis, it is necessary to bound the bias from above and below:

$$\max\{-(c-q),-\frac{2}{Jq}K_0(1-q)(f-1)\}\equiv\underline{B}\leq b\leq\overline{B}\equiv\min\{\frac{\overline{\chi}-c}{qm_0^*},\frac{c-q}{qm_1^*}\} \tag{11}$$

When $c\in(\hat{\chi},\overline{\chi})$, $f>1$ (see point 3 of Lemma in appendix A) , hence $\underline{B}<0<\overline{B}$. The upper bound ensures that the agent does intervene after some sell orders and does not intervene after any buy order. The lower bound ensures that the structure of the equilibrium is unchanged.

Proposition 6.3 *(Biased Interventions). Suppose that $c\in(\hat{\chi},\overline{\chi})$ and that the bounds described in (11) are satisfied. A value $\widetilde{m}_0$ exists, such that the unique equilibrium of the delegated intervention game is characterized as follows:*

- ***Strategies.***
  - *The type-0 trader places a sell order distributed according to continuous probability density function $\phi_0(t)$ over support $[\widetilde{m}_0,1]$ defined piecewise:*

$$\phi_0(t)=\begin{cases} \frac{t-\widetilde{m}_0}{K_0\widetilde{m}_0} & \text{if} \quad t\in[\widetilde{m}_0,\widetilde{\theta}] \\ f+\frac{Jbqt}{K_0(1-q)} & \text{if} \quad t\in[\widetilde{\theta},1] \end{cases}$$

  where $\widetilde{\theta}\equiv\frac{J\widetilde{m}_0(1-q)}{1-q-Jbq\widetilde{m}_0}$.

– *The (biased) agent intervenes with probability*

$$\alpha(t) = \begin{cases} 0 & \text{if} \quad t \in [0, \widetilde{\theta}] \\ 1 - \frac{J\widetilde{m}_0}{t} - \frac{J\widetilde{m}_0 bq}{1-q} & \text{if} \quad t \in [\widetilde{\theta}, 1] \end{cases}$$

a continuous and increasing function.

- ***Relationships.***
  - *When $b > 0$, the minimum trade size $\widetilde{m}_0 \in (m_0^\dagger, m_0^*)$, the intervention threshold $\widetilde{\theta} \in (\theta^\dagger, 1)$, and the intervention probability is weakly smaller than in the undelegated equilibrium, with strict inequality whenever either is non-zero.*
  - *When $b < 0$, the minimum trade size $\widetilde{m}_0 \in (0, m_0^\dagger)$, the intervention threshold $\widetilde{\theta} \in (\widetilde{m}_0, \theta^\dagger)$, and the intervention probability is weakly larger than in the undelegated equilibrium, with strict inequality whenever either is non-zero.*
- ***Payoffs.*** *The type-0 trader's ex ante expected payoff is $u_0 = \widetilde{m}_0(1-q)$.*

As in the equilibrium with no bias, the sell orders are broken into a safe zone, in which no intervention occurs, and a risky zone, in which intervention may take place. When the agent is positively biased, she has less incentive than the policymaker to intervene following sell orders (because interventions harm sellers of the asset). Consequently, the agent must be more convinced than the policymaker that an intervention is justified. Thus, inside the risky zone, the informed trader can trade more aggressively when the positively biased agent controls the intervention technology, because he is less concerned that his order will induce the agent to intervene: indeed, his mixing density over the risky zone strictly exceeds f, leaving the agent indifferent but benefiting the policymaker (review (10)). This equilibrium therefore approximates the counter-clockwise rotation of the trader's strategy arising in the case of policymaker commitment. The preceding logic applies in reverse to a negatively biased agent, who is more willing than the policymaker to deploy interventions, in order to hurt the noise trader *ex post* after a sell, forcing him to pay back on his short position for certain.

Proposition 6.4 *(Beneficial and Harmful Delegation) When $c \in (\hat{\chi}, \overline{\chi})$ and $b \in \underline{B}, \overline{B})$,delegating the decision to deploy an intervention to a positively (resp. negatively) biased agent increases (resp. decreases) the policymaker's expected payoff.*

The equilibrium with a positively biased agent reveals an interesting commitment dilemma in its own right. Somewhat paradoxically, when the agent positively weights the *ex post* profit of the noise trader, the noise trader's *ex ante* expected profit is reduced. To understand this, recall that by intervening after a sale, the agent hurts the noise trader *ex post*, forcing him to cover his short position. The positively biased agent, therefore, has less incentive to intervene *ex post*, giving rise to a lower equilibrium intervention probability $\alpha(t)$. However, a lower intervention probability

benefits the *informed* trader ex ante. If he used the same strategy as in the equilibrium without delegation, a smaller value of $\alpha(t)$ would (weakly) increase his payoff of submitting any sell order (review (3)). Thus, the lower intervention probability arising under delegation induces the informed investor to trade more aggressively and more profitably in equilibrium. Therefore the market maker must lower the price received by the noise trader on sell orders in order to maintain zero profits, reducing the noise trader's profit whenever he sells the asset. Thus, in an effort to benefit the noise trader *ex post* by intervening less often, the agent winds up hurting him *ex ante*.

LACK OF TRANSPARENCY

Finally, we consider a setting in which the policymaker possesses private information, so that some aspect of her decision environment is not transparent to the market. Formally, we model this as private information about her cost of intervening. Observe, however, that because the policymaker's benefit is normalized to equal one, c is actually the cost-to-benefit ratio, and hence, modeling private information about the cost of intervening is equivalent to modeling private information about the benefit.[21]

Because of the lack of transparency, both the trader and market maker are uncertain about the intervention cost. Suppose both believe that it is c_L with probability γ and c_H with probability $1-\gamma$. We focus on the case in which $\hat{\chi} < c_L < c_H < \overline{\chi}$, so that an equilibrium with stochastic interventions (in which the ability to intervene is worthless) would obtain under either cost realization; any benefits for the policymaker in the resulting equilibrium can therefore be attributed to the lack of transparency. For $i \in \{L, H\}$ define

$$J_i \equiv \frac{1-q}{1-c_i},\ f_i \equiv \frac{J_i - 1}{K_0},\ \text{and}\ R \equiv \frac{J_H^2 - 1}{J_L^2 - 1}$$

The first two parameters are analogous to the case of transparency allowing for different intervention costs, while the third parameter is a measure of the relative difference between the possible intervention costs (note that $R > 1$). Let $m_i^\dagger$ and $\theta_i^\dagger$ be type-0 minimum trade size and intervention threshold that would obtain in the equilibrium with transparency if it were common knowledge that the policymaker's intervention cost was c_i (described in Proposition 4.1); let $\overline{m}_i(\overline{\alpha})$ be the value of type-0's minimum trade size that would obtain in an equilibrium with intervention cap $\bar{\alpha}$ and cost c_i (as described in Proposition 6.1). Finally, define

$$\overline{\gamma} = (1 - \theta_H^\dagger)R \text{ and } \widehat{m}_0 \equiv \frac{R(1-\gamma)}{R-\gamma} m_H^\dagger \quad \text{for } \gamma \in [0, \overline{\gamma}]. \tag{12}$$

The equilibrium – while always unique – can be one of three types, depending on the prior belief about the policymaker's cost. We characterize the three possible equilibrium configurations in the

[21] In Appendix B we show that private information about cost is analogous to the policymaker receiving a private (imperfect) signal about the underlying state ω. Thus, learning that she has low intervention cost is the same as learning that she has high intervention benefit or learning that state zero is more likely to occur than under the prior.

following propositions.

Proposition 6.5 *(Privately Informed Policymaker–I). Suppose $\hat{\chi} < c_L < c_H < \overline{\chi}$ and the policymaker is likely to have a high cost, $\gamma \in [0, \overline{\gamma}]$, then the unique equilibrium is characterized as follows.*

- ***Strategies***.
  - *The type-0 trader places a sell order distributed according to probability density function $\phi_0(t)$ over support $[\widehat{m}_0, 1]$ defined piecewise:*

$$\phi_0(t) = \begin{cases} \frac{t-\widehat{m}_0}{K_0\widehat{m}_0} & \text{if} \quad t \in [\widehat{m}_0, \theta_1] \\ f_L & \text{if} \quad t \in [\theta_1, \theta_2] \\ \frac{(1-\gamma)t-\widehat{m}_0}{K_0\widehat{m}_0} & \text{if} \quad t \in [\theta_2, \theta_3] \\ f_H & \text{if} \quad t \in [\theta_3, 1] \end{cases}$$

  where $\theta_1 = J_L\widehat{m}_0$, $\theta_2 = \frac{J_L\widehat{m}_0}{1-\gamma}$, $\theta_3 = \frac{J_H\widehat{m}_0}{1-\gamma}$.
  - *Each type of policymaker intervenes with probability*

$$\alpha_L(t) = \begin{cases} 0 & \text{if} \quad t \in [0, \theta_1] \\ \frac{1}{\gamma}\left(1 - \frac{\theta_1}{t}\right) & \text{if} \quad t \in [\theta_1, \theta_2] \\ 1 & \text{if} \quad t \in (\theta_2, 1], \end{cases} \qquad \alpha_H(t) = \begin{cases} 0 & \text{if} \quad t \in [0, \theta_3] \\ 1 - \frac{\theta_3}{t} & \text{if} \quad t \in [\theta_3, 1]. \end{cases}$$

- ***Payoffs***. *The type-0 trader's expected payoff is $u_0 = \widehat{m}_0(1-q)$. The type-H policymaker's expected payoff is $v_H = 1-q$, and the type-L policymaker's expected payoff is $v_L > 1-q$.*

If the policymaker is likely to have the high intervention cost, then sell orders are broken into four regions. The first region ($t < \theta_1$) is completely safe: neither type of policymaker intervenes. Over this region increasing trade size reveals information, but not enough for even the low cost policymaker to intervene. In the second region, ($t \in (\theta_1, \theta_2)$) the low cost policymaker intervenes with increasing probability, but the high cost policymaker does not intervene. Information is choked off to keep the low cost policymaker from intervening for certain. At the right endpoint of the second region, the low cost policymaker intervenes for certain, while the high cost policymaker does not intervene. In the third region, (θ_2, θ_3), the low cost policymaker intervenes for certain ($\chi(t) > c_L$), but the high cost policymaker does not intervene ($\chi(t) < c_H$). In this region an increase in the trade size cannot increase the intervention probability because the low cost policymaker is already intervening with certainty and it is too expensive for the high cost policymaker to intervene. Therefore to offset the benefit of larger volume the trader's order must reveal more information to the market. Over this region both the trader density and the belief function grow. Because γ is relatively low, the posterior belief function reaches c_H at $t = \theta_3$. In the fourth region ($t \in (\theta_3, 1)$), the low cost policymaker intervenes for certain, and the high cost policymaker intervenes with

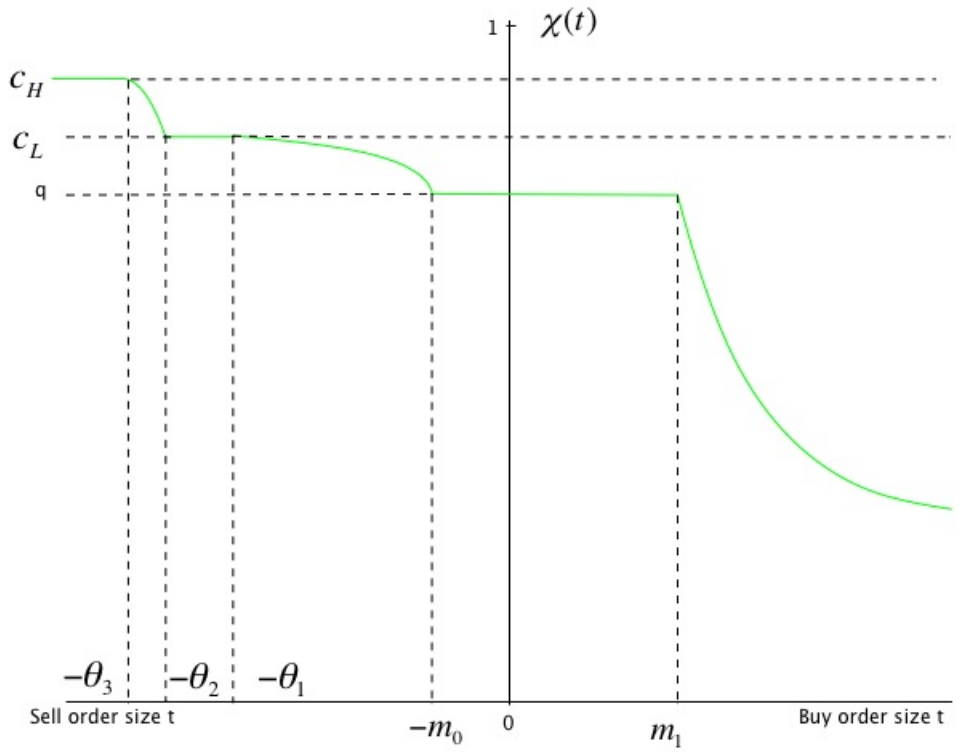

Figure 5: Posterior beliefs in the Privately Informed Policymaker–I equilibrium.

increasing probability. Information flow is choked off to prevent the high cost policymaker from intervening for certain. Figure 5 depicts posterior beliefs in this equilibrium

When $\gamma = 0$ – so that the policymaker's high cost is common knowledge – region two is empty ($\theta_1 = \theta_2$), the posterior belief increases smoothly from q to c_H, and the equilibrium is identical to the case of transparency with $c = c_H$. As γ increases, region two ($t \in (\theta_1, \theta_2)$) expands and region four ($t \in (\theta_3, 1)$) shrinks until $\gamma = \overline{\gamma}$. At this point, region four disappears entirely, so that the high cost policymaker never intervenes. Indeed, for $\gamma \in [\overline{\gamma}, 1 - \theta_L^\dagger]$, the equilibrium closely resembles the case of a cap on the probability of intervention with $\overline{\alpha} = \gamma$.

Proposition 6.6 *(Privately Informed Policymaker–II). For $\hat{\chi} < c_L < c_H < \overline{\chi}$, if the probability of low cost is intermediate, $\gamma \in [\overline{\gamma}, 1 - \theta_L^\dagger]$, the unique equilibrium is characterized as follows. The high cost policymaker never intervenes. The type-0 trader plays the same strategy as in Proposition 6.1 (the equilibrium with an intervention cap) with $c = c_L$ and $\overline{\alpha} = \gamma$. The low cost policymaker's intervention policy is equal to the intervention policy of Proposition 6.1, multiplied by $1/\gamma$. The trader's expected payoff is $u_0 = \overline{m}_L(\gamma)(1 - q)$. The high cost policymaker's expected payoff is $v_H = 1 - q$, and the low cost policymaker's expected payoff is strictly greater than $1 - q$.*

For intermediate values of γ, the high cost policymaker never intervenes. Therefore, the proba-

bility of having low cost γ acts as a cap on the intervention probability. Up to a transformation (or relabeling) of the low cost policymaker intervention probability, the second equilibrium with private cost is identical to the capped equilibrium, in which the intervention cost is c_L and the intervention cap is γ. As γ continues to increase, it becomes more likely that the policymaker's cost is low, and the region in which the "cap" binds shrinks, disappearing at $\gamma = 1 - \theta_L^\dagger$. For larger values of γ, the "cap" never binds and the equilibrium parallels the case of transparency with $c = c_L$. Here, both types of policymaker expect payoff $1 - q$ and do not benefit from interventions. We characterize this equilibrium formally in the Appendix (see Proposition A.4), but omit it here for brevity.

Whenever costs are likely to be low, $\gamma > 1 - \theta^\dagger$, secrecy does not benefit the policymaker. In contrast, whenever costs are sufficiently likely to be high, (when $\gamma < 1 - \theta_L^\dagger$), there is a positive probability that $\chi(t) > c_L$ in equilibrium. In this case, the low cost policymaker may receive strictly beneficial information from a large selloff, something that never occurs in the absence of private information.

Corollary 6.7 *(Benefit of Private Information). If $\widetilde{\chi} < c_L < c_H < \overline{\chi}$ and $\gamma \in (0, 1 - \theta_L^\dagger)$, then the low cost policymaker strictly benefits in expectation from her private information, $v_L > 1 - q$.*

Suppose that – before learning her type – the policymaker could choose a disclosure regime. That is, she could commit either to reveal her private information (transparency) or not to reveal it (secrecy). An implication of Corollary 6.7 is that she would strictly prefer secrecy if $\gamma < 1 - \theta_L^\dagger$ and would never strictly prefer transparency. Under transparency the resulting equilibrium would involve stochastic interventions and an expected payoff of $1 - q$ for either cost realization. Under secrecy, a positive probability exists that a low cost policymaker would observe a strong selloff, allowing her to make a strictly beneficial (in expectation) intervention.

One interesting aspect of this result is that merely having a low intervention cost does not necessarily benefit the policymaker; rather, the policymaker benefits from the trader's *uncertainty* about her cost. Indeed, the policymaker benefits when her cost is low *ex post*, but the trader believes it is likely to be high ($\gamma < 1 - \theta_L^\dagger$). Thus, if the policymaker's cost of intervention is initially known to be c, a policy that stochastically lowers the intervention cost to $c_L < c$ can be beneficial for the policymaker *ex ante*, provided it is not too likely to succeed. Surprisingly, a policy that *increases* the intervention cost can be beneficial for the policymaker *ex ante*, provided it is sufficiently likely to succeed.

Finally, briefly consider a setting in which n cost realizations are possible, satisfying $\hat{\chi} < c_1 < \cdots < c_n < \overline{\chi}$. Arguments analogous to those used to prove Propositions 6.5 and 6.6 show that a regime of secrecy can strictly benefit all policymaker types except type-n. Much like an intervention cap, private information acts as a commitment on the probability of intervening and induces the type-0 trader to reveal more information than he would if costs were publicly disclosed.

7 CONCLUSION

In this paper we explore a setting in which privately informed investors trade an asset in an effort to profit from their knowledge of the underlying state and in which a policymaker who cares intrinsically about the state observes trading activity and decides whether to take a preemptive costly intervention; e.g., a bailout. We completely characterize the set of perfect Bayesian equilibria of the game and derive a number of results.

We show that there exists a region of the parameter space in which sales orders are partitioned into two sets: a safe zone of small trades that never trigger an intervention and a risky zone of large trades that induce random interventions by the policymaker in equilibrium. Thus our analysis provides an explanation for apparently random interventions (e.g. the differential treatment of Bear Stearns and Lehman Brothers). Although the probability of an intervention increases with sell volume over the risky zone, the policymaker does not expect to benefit from intervening. This occurs because investors employ equilibrium trading strategies that choke off information at the point where the policymaker is *just* indifferent about acting. The asset price is non-monotonic in sell volume, falling over the safe zone as the market becomes more convinced that the 'bad' state will obtain and rising over the risky zone as the market becomes more convinced that a bailout will be triggered resulting in the 'good' state. Indeed, the expected price of the asset is higher in equilibrium than if interventions were not possible. Moreover, to mitigate the probability of an intervention, informed sellers trade less aggressively and trades are, therefore, less Blackwell informative about the underlying state.

The primary tension facing the policymaker is that in order to induce investors to reveal information through their trades, she must commit not to intervene with high probability. Absent such commitment, traders employ strategies that reveal no useful information to her at all. While committing to a finely tuned intervention plan is generally not plausible for the policymaker, she may have access to institutions that provide some degree of commitment. For instance, we show that if the political process or intervention technology places a potentially binding cap on the probability of a successful bailout, then extreme selloffs do reveal valuable information. Similarly, the policymaker may induce investors to reveal useful information if she adopts a regime of secrecy regarding the actual cost or benefit of a particular bailout or her own imperfect signal of the underlying state. That is, the policy maker should *not* be transparent about revealing her own private information to investors.

Policymakers such as the Federal Reserve frequently advocate using financial markets to inform policy. Our results are less sanguine: interventions caused by short sales tend to cause less selling in the first place, reducing the information provided by the market to the point where the policymaker does not benefit. Our results do show that financial markets can inform policy if partial commitment devices, non-transparency, delegation and institutional gridlock are present, reducing the probability of an intervention (especially after large selloffs, when the temptation to intervene is strongest). Unless the policymaker commits to intervene less often than she would like, large selloffs by informed traders will occur so rarely that the policymaker will not gain enough informa-

tion to justify an intervention. Our result that a regime of secrecy can benefit the policymaker calls into question the prevailing wisdom that transparency should be implemented whenever possible in order to mitigate risk. While risk considerations are certainly important, official non-transparency policies by the Federal Home Financing Agency and (until recently) the FED suggest that there may also be significant informational benefits from being somewhat unpredictable.

While the model we present and analyze in this paper is quite stylized, it does identify some key tradeoffs and deliver novel insights regarding the use of financial markets to inform policy. A number of avenues remain open for future research. For instance, it would be edifying to study the incentives for information acquisition by investors. Also, the methods employed here could be used to investigate a variety of similar settings such as a seller who learns the value of her object by observing bids in an auction and who may decide not to sell if the object is revealed to be highly valuable. Indeed, situations in which strategies of agents both inform and anticipate policy interventions are quite ubiquitous, and the question of how policymakers should act in such settings has never been more relevant.

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A APPENDIX, FOR ONLINE PUBLICATION

This appendix contains the proofs of all the propositions presented in the text as well as several technical lemmas and their proofs.

Lemma A.1 *(Zero Payoff Equilibrium). A Perfect Bayesian equilibrium in which the type-0 trader's expected payoff is zero exists if and only if* $c \in (q, \hat{\chi})$. *In any equilibrium in which the type-0 trader's expected payoff is zero, the policymaker intervenes following any sell order* $t > 0$, $\alpha(t) = 1$. *In addition, for all sell orders,* $t > 0$, $\phi_0(t) \geq f$.

Proof. (i) We first show that the type-0 trader's equilibrium expected profit is zero if and only if $\alpha(t) = 1$ for all sell orders t. If the trader's equilibrium payoff is zero then there does not exist any order that would generate non-zero profit: for all t, $t(1 - \chi(t))(1 - \alpha(t)) = 0$. Hence, for any $t > 0$, either $\chi(t) = 1$ or $\alpha(t) = 1$ or both. However, from the policymaker's incentive constraint we find that $\chi(t) = 1 \Rightarrow \alpha(t) = 1$. Hence, for all sell orders $t > 0$, $\alpha(t) = 1$. (ii) An immediate consequence of the point (i) is that in any equilibrium in which the type-0 trader's expected profit is zero, $\phi_0(t) \geq f$ for all t. Indeed, from the policymaker's incentive constraint, $\alpha(t) = 1 \Rightarrow \chi(t) \geq c \iff \phi_0(t) \geq f$.
Next, observe that by point 2 of Lemma A.3, $c < \hat{\chi} \Rightarrow f < 1$. Therefore, whenever $c < \hat{\chi}$ it is possible to find a density that integrates to one that also satisfies $\phi_0(t) \geq f$ for all non-zero sell orders. Therefore, if $c < \hat{\chi}$ an equilibrium with zero trader payoff exists, and the points (i) and (ii) above establish that such an equilibrium must have $\alpha(t) = 1$ and $\phi_0(t) \geq f$ for all non-zero sell orders. Finally, we show that an equilibrium in which the type-0 trader's profit is zero exists only if $c < \hat{\chi}$. If such an equilibrium exists, then (ii) implies that $\phi_0(t) \geq f$ for all non-zero sell orders, and hence

$$\int_0^1 \phi_0(t)\,\mathrm{d}t \geq \int_0^1 f\,\mathrm{d}t = f \Rightarrow f \leq 1.$$

Point 2 of Lemma A.3, implies that $f \leq 1 \iff c \leq \hat{\chi}$. ■

Lemma A.2 *(Equilibrium Properties). When* $c > \hat{\chi}$, *any equilibrium must satisfy the following properties.*

1. The type-0 trader's equilibrium payoff must be non-zero.
   Proof. This is an immediate consequence of Lemma A.1. ■

2. Trader i plays a mixed strategy with no mass point.
   Proof. If type-i's equilibrium strategy contains a mass point on order size $\hat{t}$ (or consists entirely of a mass point, i.e. a pure strategy), then the posterior belief $\chi(\hat{t})$ is equal to $1 - i$ (because the distribution of noise trades has no mass points). This implies that the payoff associated with pure strategy $\hat{t}$ is zero for type i. Because $\hat{t}$ is inside the support, type i's equilibrium payoff must therefore be zero, which violates the result that the trader's expected payoff is strictly positive. ■

3. If an order triggers an intervention for certain, then it must be outside the support of the trader's mixed strategy.

   Proof. If order $\hat{t}$ triggers an intervention for certain then type i's expected payoff of submitting order $\hat{t}$ is zero (see (2) and (3). If this order were inside the support of i's mixed strategy, then i's equilibrium payoff would also be zero, violating the result that the trader's equilibrium payoff is positive. ■

4. The posterior belief associated with any sell (buy) order is weakly above (below) q.

   Proof. Consider the posterior belief following a buy order of size t, $\chi(t) = \frac{(1-a)q/2}{a(1-q)\phi_1(t)+(1-a)/2}$. Observe that this posterior belief is decreasing in $\phi_1(t)$, and because $\phi_1(t) \geq 0$, the posterior belief is bounded from above by q, i.e. $\chi(t) \leq q$. The proof of the reverse case is analogous. ■

5. Equilibrium payoffs for the traders are $u_1 = m_1 q$ and $u_0 = m_0(1-q)$.

   Proof. Because the equilibrium payoff of a type-1 trader is the same as the payoff from playing m_1 for certain, we find (from (2)) that $u_1 = m_1\chi(m_1)(1-\alpha(m_1))$. Applying point 2. gives, $u_1 \leq m_1 q(1-\alpha(m_1))$. Next, consider an order just below m_1. Because m_1 is the smallest element of the support of type 1's mixed strategy, an order just below m_1 could only be submitted by a noise trader and is therefore associated with posterior belief q. Hence, by submitting an order less than m_1 (and outside of the support), the type one trader can guarantee a payoff arbitrarily close to $m_1 q$. In order for such a deviation to be unprofitable, $u_1 \geq m_1 q$. Combining these inequalities gives $m_1 q \leq u_1 \leq m_1 q(1-\alpha(m_1))$. These inequalities imply that $\alpha(m_1) = 0$ and $u_1 = m_1 q$. The second part, $u_0 = m_0(1-q)$ is established in the same way. ■

6. There exists some $\epsilon > 0$ such that probability of intervention inside $[m_0, m_0 + \epsilon]$ is zero.

   Proof. The proof of part 5 establishes that $u_0 = m_0(1-q)$ and part 1 establishes that $u_0 > 0$ and $m_0 > 0$. Suppose that for any $\epsilon > 0$, the intervention probability $\alpha(m_0+\epsilon) > 0$. Because certain intervention cannot take place, policymaker's incentive constraint implies that $\chi(m_0+\epsilon) = c$, and hence, $u_0 = (m_0+\epsilon)(1-c)(1-\alpha(m_0+\epsilon))$. This equation contradicts the trader's incentive constraint as shown below. Note that:

   $$u_0 = m_0(1-q) > m_0(1-c)$$

   Hence, a small $\epsilon > 0$ can be found such that

   $$u_0 = m_0(1-q) > m_0(1-c) > (m_0+\epsilon)(1-c) > (m_0+\epsilon)(1-c)(1-\alpha(m_0+\epsilon))$$

   Thus, it cannot be that for all $\epsilon > 0$, $\alpha(m_0+\epsilon) > 0$. ■

7. The maximum element of S_i is 1.

   Proof. Suppose that $t = 1$ is outside the support of type-i's mixed strategy. Because $t = 1$

is only submitted by a noise trader $\chi(1) = q$. Hence, by deviating outside the support to $t = 1$ a type 1 trader can guarantee payoff q and a type 0 trader can guarantee payoff $1 - q$. Because the equilibrium payoff for the type 1 trader must be no less than the payoff of this deviation, $m_0(1 - q) \geq 1 - q \Rightarrow m_0 \geq 1$ and $m_1 q \geq q \Rightarrow m_1 \geq 1$. Because any order above 1 reveals the trader's type, the maximum element of the support must be less than or equal to one, hence $m_0 = m_1 = 1$. If, however, the minimum and maximum of the support are both one, then the trader is employing a pure strategy on 1, ruled out in part 1. ■

8. The support of each type of trader's mixed strategy contains no gaps.
   Proof. The argument is very similar to the one in part 4. Suppose a gap exists inside of the support. This means that intervals $[m_i, x]$ and $[y, 1]$ are inside the support of type i mixed strategy (where $m_i < x < y < 1$) but (x, y) is not inside the support. Because it is the center of $[x, y]$, $\hat{t} = \frac{x+y}{2}$ is outside the support. Hence by deviating to $\hat{t}$, a type 1 trader can ensure a payoff of $\hat{t}q$ and a type 0 trader can ensure a payoff of $\hat{t}(1 - q)$. Because $\hat{t} > x > m_i$, this is a profitable deviation $\hat{t}(1 - q) > m_0(1 - q) = u_0$ and $\hat{t}q > m_1 q = u_1$. ■

9. If the probability of intervention is constant for all sell (buy) orders in an interval inside S_i, then $\chi(t)$ is increasing (decreasing) on this interval.
   Proof. Suppose $[t_L, t_H] \subset S_i$, and for $t \in [t_L, t_H]$ the probability of intervention $\alpha(t) = \alpha \in [0, 1)$. By point 1. of this Lemma, $\alpha < 1$. Type 1's indifference condition requires that $u_1 = t\chi(t)(1 - \alpha)$, which immediately implies that $\chi(t)$ is decreasing. Similarly, type 0 indifference condition requires that $u_0 = t(1 - \chi(t))(1 - \alpha)$, which immediately implies that $\chi(t)$ is increasing. ■

10. If the posterior belief is constant for all sell (buy) orders in an interval inside S_i then $\alpha(t)$ is increasing on this interval.
    Proof. Analogous to 7. ■

Proof. Proposition 3.1. If the trader uses a mixed strategy in equilibrium, he must be indifferent between all orders inside the support of his mixed strategy. In the absence of interventions, this condition requires (see equations (2) and (3))

$$u_1 = t\chi(t), \quad \forall\, t \in [m_1, 1] \text{ and } u_0 = t(1 - \chi(t)) \quad \forall\, t \in [m_0, 1].$$

Substituting the equilibrium payoffs derived in part 4 of Lemma A.2, $u_1 = m_1(1 - q)$ and $u_0 = m_0(1 - q)$ and the posterior beliefs given in (5) and (6) and then solving these equations gives expressions for the equilibrium mixing densities:

$$\phi_1(t) = \frac{t - m_1}{K_1 m_1} \text{ and } \phi_0(t) = \frac{t - m_0}{K_0 m_0}$$

The equilibrium value of m_i is computed by ensuring that the densities integrate to one. For the type-1 trader

$$\int_{m_1^*}^{1} \frac{t - m_1^*}{K_1 m_1^*} \, \mathrm{d}t = \frac{(1 - m_1^*)^2}{2K_1 m_1^*}$$

Setting this equal to 1 yields equation $Q(m_1^*, 0|K_1, J) = 0$ (see equation (7)). Hence,

$$m_1^* = K_1 + 1 - \sqrt{(K_1 + 1)^2 - 1}$$

which is evidently always inside $(0, 1)$.[22] Moreover, replacing $(1-q)$ with q in the above calculation yields

$$m_0^* = K_0 + 1 - \sqrt{(K_0 + 1)^2 - 1} \tag{A1}$$

We prove the comparative static claims for q; those for a are proven analogously. Define

$$F(k) = k + 1 - \sqrt{(k + 1)^2 - 1}.$$

Observe that

$$F'(k) = -\frac{F(k)}{\sqrt{(k+1)^2 - 1}} < 0$$

$$\frac{\partial K_1}{\partial q} = -\frac{2a}{1 - a} < 0 \tag{A2}$$

1. Note that $\frac{\partial m_1^*}{\partial q} = F'(K_1)\frac{\partial K_1}{\partial q} > 0$.

2. We just established that the left end of the support of type-1's equilibrium mixed strategy increases with q. The right endpoint of the support is 1 for all q. First order stochastic dominance will follow if a rise in q causes $\phi_1(t)$ to become steeper at every t for which it is non-zero. Because $\phi_1(t)$ is linear in t, this amounts to demonstrating that its slope increases with q. Hence, showing that

   $$\frac{d}{dq}\left(\frac{1}{K_1 m_1^*}\right) > 0$$

   will prove the claim.

   $$\frac{d}{dq}\left(\frac{1}{K_1 m_1^*}\right) = -\frac{\frac{\partial K_1}{\partial q} m_1^* - \frac{\partial m_1^*}{\partial q} K_1}{(K_1 m_1^*)^2}$$

   This expression is positive if the numerator of the fraction is negative

   $$\frac{\partial K_1}{\partial q} m_1^* - \frac{\partial m_1^*}{\partial q} K_1 < 0$$

[22] It is straightforward to verify that the larger root is greater than 1.

Because $\frac{\partial K_1}{\partial q} < 0$ and $\frac{\partial m_1^*}{\partial q} > 0$, as well as $m_1^* > 0$ and $K_1 > 0$, the result is evident.

3. Note that $u_1 = m_1^* q$. Because m_1^* and q both increase with q, the result is evident.

This completes the proof of Proposition 3.1 ■

Lemma A.3 *(Useful Relationships) The following relationships turn out to be analytically useful.*

1. $a > 0$ and $q > 0$ imply $\hat{\chi} < \overline{\chi}$.
   Proof. Consider the following string of equivalent expressions:

$$\begin{aligned}
&\hat{\chi} < \overline{\chi} \\
&\Leftrightarrow \quad \frac{K_0 + q}{K_0 + 1} < 1 - (1 - q) m_0^* \\
&\Leftrightarrow \quad m_0^* < \frac{1}{K_0 + 1} \\
&\Leftrightarrow \quad K_0 + 1 - \sqrt{(K_0 + 1)^2 - 1} < \frac{1}{K_0 + 1} \\
&\Leftrightarrow \quad (K_0 + 1)^2 - \sqrt{(K_0 + 1)^4 - (K_0 + 1)^2} < 1
\end{aligned}$$

   Let $z = (K_0 + 1)^2$. Then, the last line above holds iff

$$\begin{aligned}
z - 1 < \sqrt{z^2 - z} \\
&\Leftrightarrow \quad 1 < z \\
&\Leftrightarrow \quad K_0 > 0 \\
&\Leftrightarrow \quad a > 0 \text{ and } q > 0.
\end{aligned}$$

   ■

2. $\phi_0(t) < f \Leftrightarrow \chi(t) < c$
   Proof. Consider the following string of equivalent expressions:

$$\begin{aligned}
&\chi(t) < c \\
&\Leftrightarrow \quad \frac{q\left(a\phi_0(t) + (1-a)\frac{1}{2}\right)}{qa\phi_0(t) + (1-a)\frac{1}{2}} < c \\
&\Leftrightarrow \quad \frac{K_0\phi_0(t) + q}{K_0\phi_0(t) + 1} < c \\
&\Leftrightarrow \quad K_0\phi_0(t)(1-c) < c - q \\
&\Leftrightarrow \quad \phi_0(t) < f
\end{aligned}$$

   ■

3. $\hat{\chi} < c \Leftrightarrow f > 1$ and $J - 1 > K_0$.
   Proof. Consider the following string of equivalent expressions:

$$\begin{aligned}
&\hat{\chi} < c \\
&\quad \Leftrightarrow \frac{K_0 + q}{K_0 + 1} < c \\
&\quad \Leftrightarrow 1 - \frac{1 - q}{K_0 + 1} < c \\
&\quad \Leftrightarrow 1 - c < \frac{1 - q}{K_0 + 1} \\
&\quad \Leftrightarrow K_0 + 1 < J \\
&\quad \Leftrightarrow 1 < f.
\end{aligned}$$

   ■

4. $c < \overline{\chi} \Leftrightarrow J < \frac{1}{m_0^*}$
   Proof. Consider the following string of equivalent expressions:

$$\begin{aligned}
&c < \overline{\chi} \\
&\Leftrightarrow \quad c < 1 - m_0^*(1 - q) \\
&\Leftrightarrow \quad J < \frac{1}{m_0^*}
\end{aligned}$$

   ■

Proof. Proposition 4.1. Because $c > \hat{\chi}$, the type-0 trader's equilibrium payoff is non-zero, and is equal to $u_0 = m_0(1 - q)$, where m_0 represents the type-0 trader's minimum trade size. Furthermore, because $c < \overline{\chi}$ the equilibrium with no interventions does not exist, and therefore, following a non-negligible set of possible order flow the intervention probability is non-zero. Because the intervention probability cannot be one for any order flow, the policymaker must employ a random mixed intervention strategy.

Sequential rationality for the policymaker (i.e., (4) requires

$$\chi(t) = c \text{ if } \alpha(t) \in (0, 1)$$

Substituting for $\chi(t)$ from (6) and solving yields

(A3) $$\phi_0(t) = f, \text{ if } \alpha(t) \in (0, 1).$$

Moreover, the same argument as in the proof of Proposition 3.1 yields

$$\phi_0(t) = \frac{t - m_0}{K_0 m_0}, \text{ if } \alpha(t) = 0. \tag{A4}$$

Denote θ to be the intersection point between the mixing density for those order where $\alpha(t) = 0$ (given in equation (A4)) and the mixing density for those orders where $\alpha(t) > 0$ (given in equation (A3)):

$$\frac{\theta - m_0}{K_0 m_0} = f \iff \theta = J m_0$$

Next, we establish that $\alpha(t) > 0$ if and only if $t > \theta$.

Consider an order t strictly greater than θ. Note that because $\theta > m_0 > 0$, and the support has no gaps, t belongs to the support of the mixed strategy. Suppose that the policymaker does not intervene at t, $\alpha(t) = 0$. In this case the trader's mixing density at t is given by (A3):

$$\frac{t - m_0}{K_0 m_0} > \frac{\theta - m_0}{K_0 m_0} = f$$

Hence, for $t > \theta$, the assumption that $\alpha(t) = 0$ implies $\phi_0(t) > f$, but sequential rationality for the policymaker requires $\alpha(t) = 1$ in this case, a contradiction. Because $t > \theta$ therefore implies $\alpha(t) > 0$, but $\alpha(t) = 1$ cannot be part of an equilibrium, it must be that $\alpha(t) \in (0, 1)$.

Next, consider order $t \leq \theta$, and suppose that $\alpha_\theta > 0$. Policymaker incentive compatibility would require that $\chi(t) = c$. Thus, the type-0 trader's expected payoff of submitting order flow t is equal to $t(1 - c)(1 - \alpha(t)$. However,

$$t(1-c)(1-\alpha(t)) \leq \theta(1-c)(1-\alpha(t)) = J m_0 (1-c)(1-\alpha(t)) = m_0(1-q)(1-\alpha(t)) < m_0(1-q),$$

which violates the type-0 trader's indifference condition. Thus, for $t \leq \theta$, it must be that $\alpha(t) = 0$.

We have therefore established that in equilibrium no intervention takes place inside interval $[m_0, \theta]$ and consequently the mixing density over this interval is given by A3, while intervention must take place stochastically inside interval $(\theta, 1]$ and the mixing density over this interval is $\phi_0(t) = f$. Because $\phi_0(t)$ is a density, it must integrate to 1, and therefore the equilibrium value of m_0, represented by $m_0^\dagger$ (and the equilibrium value of θ, represented by $\theta^\dagger$) can be found from the following:

$$\int_{m_0^\dagger}^{\theta^\dagger} \frac{t - m_0^\dagger}{K_0 m_0^\dagger} \, \mathrm{d}t + (1 - \theta^\dagger) f = 1.$$

Recalling that $\theta = Jm_0^\dagger$ and $f = (J-1)/K_0$ gives:

$$\int_{m_0^\dagger}^{Jm_0^\dagger} \frac{t - m_0^\dagger}{K_0 m_0^\dagger}\,\mathrm{d}t + (1 - Jm_0^\dagger)\frac{J-1}{K_0} = 1.$$

Integrating yields equation $Q(m_0^\dagger, 1 - Jm_0^\dagger | K_0, J) = 0$. Two solutions exist. One is zero (violating point 1 of Lemma A.2) and the other is

$$m_0^\dagger \equiv \frac{2(J - K_0 - 1)}{J^2 - 1}. \tag{A5}$$

This is a valid equilibrium only if $m_0^\dagger \geq 0$ and $\theta^\dagger \leq 1$. Part 3 of Lemma A.3 shows $m^\dagger > 0$ when $c > \hat{\chi}$. To verify $\theta^\dagger \leq 1$, we show that $\theta^\dagger = 1$ for $c = \overline{\chi}$ and that $\frac{\partial m_0^\dagger}{\partial c} > 0$. Note that by part 4 of Lemma A.3, $c < \overline{\chi} \Rightarrow J = 1/m_0^*$. In this instance

$$\theta^\dagger = Jm_0^\dagger = \frac{1}{m_0^*}\frac{2(\frac{1}{m_0^*} - K_0 - 1)}{\frac{1}{(m_0^*)^2} - 1}$$

Subtracting one from this expression and simplifying yields, $\theta^\dagger - 1 = Q(m_0^*, 0 | K_0, J)/(1 - m_0^2) = 0$ by the definition of m_0^*. Hence $c = \overline{\chi} \Rightarrow \theta^\dagger = 1$. Note that an immediate consequence is $c = \overline{\chi} \Rightarrow m_0^\dagger = m_0^*$. Next, we show $\frac{\partial m_0^\dagger}{\partial c} > 0$. Differentiation yields

$$\frac{\partial m_0^\dagger}{\partial c} = \left(\frac{2(1-q)}{\left((1-c)(J^2-1)\right)^2}\right)\left(-J^2 + 2J(K+1) - 1\right)$$

The first term in parentheses is evidently positive, so we must show

$$-J^2 + 2J(K+1) - 1 > 0 \tag{A6}$$

Observe that

$$\frac{\partial}{\partial J}\left(-J^2 + 2J(K+1) - 1\right) = -2(J - K - 1)$$

This is negative by part 3 of Lemma A.3. Hence, the left side of (A6) is smallest when J is as large as possible. By part 4 of Lemma A.3 that $J < \frac{1}{m_0^*}$. Thus, the result will follow if

$$-(m_0^*)^{-2} + 2(m_0^*)^{-1}(K+1) - 1 \geq 0,$$

or equivalently if

$$(m_0^*)^2 - 2(K+1)m_0^* + 1 \leq 0.$$

But, the left side of this expression is zero by definition of m_0^* (i.e., it is $Q(m_0^*, 0 | K_0, J)$ (see equation (7)). Thus, $m_0^\dagger$, and $\theta^\dagger$ are increasing in c. Thus, $c \in (\hat{\chi}, \overline{\chi})$ implies $m_0^\dagger \in (0, m_0^*)$ and $\theta^\dagger \in (m_0^\dagger, 1)$.

(Showing $\frac{\partial m_0^\dagger}{\partial q} < 0$ follows the same lines as showing $\frac{\partial m_0^\dagger}{\partial c} > 0$. Also, $\frac{\partial m_0^\dagger}{\partial a} < 0$ follows directly from $\frac{\partial K}{\partial a} > 0$.)

Next, we derive policymaker's equilibrium strategy. The trader indifference conditions and policymaker's sequential rationality conditions (3) and (4) on $[\theta^\dagger, 1]$ require:

$$u_0 = t(1 - \chi(t))(1 - \alpha(t)) \text{ and } \chi(t) = c$$

Solving these gives

$$\alpha(t) = 1 - \frac{u_0}{t(1-c)}$$

Because $u_0 = m_0^\dagger(1-q)$ (by part 4 of Lemma A.2), we find that $u_0 = \theta^\dagger(1-c)$.

$$\alpha(t) = 1 - \frac{\theta^\dagger}{t}$$

This is obviously a valid probability for $t \in [\theta^\dagger, 1]$. For $t \in [0, \theta^\dagger)$ $\chi(t) < c$ and so $\alpha(t) = 0$ (see equation (4)).

Finally, note that because policymaker does not intervene for $t \in [0, \theta^\dagger)$ and is indifferent about intervening for $t \in [\theta^\dagger, 1]$, her expected payoff must be the same as if she never intervened, namely $1 - q$.

Proof. Proposition 4.2.

Less Aggressive Trade. Let $\phi_0^*(\cdot)$ and $\phi_0^\dagger(\cdot)$ be the mixing densities of the type-0 trader given respectively in Propositions 3.1 and 4.1. Recall that $c \in (\widetilde{\chi}, \overline{\chi})$ implies $m_0^\dagger < m_0^*$. Thus, $\phi_0^\dagger(t)$ starts sooner and is steeper than $\phi_0^*(t)$ for $t < \theta^\dagger$. Because both densities end at $t = 1$ and both must integrate to 1, $\phi_0^*(t)$ must cross the flat portion of $\phi_0^\dagger(t)$ at some $\hat{t} \in (\theta^\dagger, 1)$. That is

$$t \in [m_0^\dagger, \hat{t}) \Rightarrow \phi_0^\dagger(t) - \phi_0^*(t) > 0$$

$$t \in (\hat{t}, 1] \Rightarrow \phi_0^\dagger(t) - \phi_0^*(t) < 0$$

The first inequality immediately implies:

$$t \in [m_0^\dagger, \hat{t}] \Rightarrow \int_{m_0^\dagger}^{t} \phi_0^\dagger(t) - \phi_0^*(t)\,\mathrm{d}t > 0$$

Next observe that because the top of the support of either mixed strategy is one,

$$\int_{m_0^\dagger}^{1} \phi_0^\dagger(t) - \phi_0^*(t)\,\mathrm{d}t = 0$$

and thus

$$t \in [\hat{t}, 1] \Rightarrow \int_{m_0^\dagger}^{t} \phi_0^\dagger(t) - \phi_0^*(t)\,\mathrm{d}t + \int_{t}^{1} \phi_0^\dagger(t) - \phi_0^*(t)\,\mathrm{d}t = 0$$

and thus

$$t \in [\hat{t}, 1] \Rightarrow \int_{m_0^\dagger}^{t} \phi_0^\dagger(t) - \phi_0^*(t)\,\mathrm{d}t = \int_{t}^{1} \phi_0^*(t) - \phi_0^\dagger(t)\,\mathrm{d}t$$

For $t > \hat{t}$ the right side is positive. Hence,

$$t \in [m_0^\dagger, 1] \Rightarrow \int_{m_0^\dagger}^{t} \phi_0^\dagger(t) - \phi_0^*(t)\,\mathrm{d}t \geq 0$$

and therefore, equilibrium order flow in the no intervention equilibrium first order stochastic dominates equilibrium order flow in the stochastic intervention equilibrium.

(*Less Information*). We begin by constructing the posterior belief random variables for the equilibria with and without interventions. We adopt a convention of labeling sell orders as negative and buy orders as positive.

No-intervention Construction. In the benchmark case, each trader type mixes according to densities:

$$\phi_1^*(t) = \frac{t - m_1^*}{K_1 m_1^*} \text{ and } \phi_0^*(t) = \frac{t - m_0^*}{K_0 m_0^*}$$

over supports $S_0 = [m_0^*, 1]$ and $S_1 = [m_1^*, 1]$. Viewed *ex ante*, the equilibrium order flow is a random variable t^*, with support on $[-1, 1]$, and the following density:

$$f^*(t) = \begin{cases} aq\phi_0^*(-t) + (1-a)/2 & \text{if} \quad t \in [-1, -m_0^*] \\ (1-a)/2 & \text{if} \quad t \in [-m_0^*, m_1^*] \\ a(1-q)\phi_1^*(t) + (1-a)/2 & \text{if} \quad t \in [m_1^*, 1] \end{cases}$$

which simplifies in the following way:

$$f^*(t) = \begin{cases} \frac{(1-a)|t|}{2m_0^*} & \text{if} \quad t \in [-1, -m_0^*] \\ (1-a)/2 & \text{if} \quad t \in [-m_0^*, m_1^*] \\ \frac{(1-a)t}{2m_1^*} & \text{if} \quad t \in [m_1^*, 1] \end{cases}$$

In the interval $[-1, -m_0^*]$ the order flow could be generated either by a noise trader or by a negatively informed trader. Thus the density is a weighted average of the densities of the negatively informed trader and the noise trader. In the interval $[-m_0^*, m_1^*]$ order flow is generated only by the noise trader, and therefore follows his density, and in $[m_1^*, 1]$ the order flow could again be generated by either the positively informed trader or the noise trader. The posterior belief as a function of the

observed order flow is as follows:

$$\chi(t) = \begin{cases} 1 + \frac{m_0^*(1-q)}{t} & \text{if} \quad t \in [-1, -m_0^*] \\ q & \text{if} \quad t \in [-m_0^*, m_1^*] \\ \frac{m_1^* q}{t} & \text{if} \quad t \in [m_1^*, 1] \end{cases}$$

Let x^* represent the posterior belief random variable for the no intervention equilibrium, i.e. $x^* = \chi(t^*)$. The support of x^* is clearly $[\underline{\chi}, \overline{\chi}] \equiv [m_1^* q, 1 - (1-q)m_0^*]$. Furthermore x^* clearly has a mass point on q, taking on this realization with probability $(m_0^* + m_1^*)(1-a)/2$. To calculate the density of x^* on intervals $[\underline{\chi}, q)$ and $(q, \overline{\chi}]$ apply the standard formula for transformation of density to obtain $g^*(x)$, the density of x^*.

$$x \in [\underline{\chi}, q) \Rightarrow g^*(x) = f^*(\frac{m_1^* q}{x}) || \frac{d}{dx}(\frac{m_1^* q}{x}) || = \frac{m_1^* q^2 (1-a)}{2x^3}$$

$$x \in (q, \overline{\chi}] \Rightarrow g^*(x) = f^*(\frac{m_0^* q}{1-x}) || \frac{d}{dx}(\frac{m_0^* q}{1-x}) || = \frac{m_0^* (1-q)^2 (1-a)}{2(1-x)^3}$$

Integrating this density function (and remembering the mass point on q) gives the distribution function of x^*, $G^*(x)$

$$G^*(x) = \begin{cases} \frac{(1-a)(x^2 - q^2 m_1^{*2})}{4 m_1^* x^2} & \text{if} \quad x \in [\underline{\chi}, q) \\ \frac{(1-a)(m_1^{*2} + 2 m_0^* m_1^* + 1)}{4 m_1^*} + \frac{m_0^*(1-a)(x-q)(2-x-q)}{4(1-x)^2} & \text{if} \quad x \in (q, \overline{\chi}] \\ 1 & \text{if} \quad x \in [\overline{\chi}, \infty) \end{cases}$$

Stochastic Intervention Construction. Denote the posterior belief random variable for the stochastic intervention equilibrium as $x^\dagger$. In this equilibrium the type-1 trader places an order distributed in an identical fashion to the no-intervention equilibrium. Thus the distribution of $x^\dagger$ over interval $[\underline{\chi}, q)$ is unaffected. The type-0 trader places a sell order distributed according to probability density function $\phi_0^\dagger(t)$ over support $[m_0^\dagger, 1]$ defined piecewise:

$$\phi_0^\dagger(t) = \begin{cases} \frac{t - m_0^\dagger}{m_0^\dagger K_0} & \text{if} \quad t \in [m_0^\dagger, \theta^\dagger] \\ \frac{(c-q)}{(1-c)K_0} & \text{if} \quad t \in [\theta^\dagger, 1] \end{cases}$$

Thus the order flow in an equilibrium with stochastic interventions has the following density:

$$f^\dagger(t) = \begin{cases} aq\frac{(c-q)}{(1-c)K_0} + (1-a)/2 & \text{if} \quad t \in [-1, -\theta^\dagger] \\ aq(\frac{-t-m_0^\dagger}{m_0^\dagger K_0}) + (1-a)/2 & \text{if} \quad t \in [-\theta^\dagger, -m_0^\dagger] \\ (1-a)/2 & \text{if} \quad t \in [-m_0^\dagger, m_1^*] \\ a(1-q)\phi_1^*(t) + (1-a)/2 & \text{if} \quad t \in [m_1^*, 1] \end{cases}$$

which simplifies to

$$f^\dagger(t) = \begin{cases} \frac{(1-a)(1-q)}{2(1-c)} & \text{if} \quad t \in [-1, -\theta^\dagger] \\ \frac{(1-a)|t|}{2m_0^\dagger} & \text{if} \quad t \in [-\theta^\dagger, -m_0^\dagger] \\ (1-a)/2 & \text{if} \quad t \in [-m_0^\dagger, m_1^*] \\ \frac{(1-a)t}{2m_1^*} \text{if} & t \in [m_1^*, 1] \end{cases}$$

The posterior belief as a function of order t is given by

$$\chi(t) = \begin{cases} c & \text{if} \quad t \in [-1, -\theta^\dagger] \\ 1 + \frac{m_0^\dagger(1-q)}{t} & \text{if} \quad t \in [-\theta^\dagger, -m_0^\dagger] \\ q & \text{if} \quad t \in [-m_0^\dagger, m_1^*] \\ \frac{m_1^* q}{t} & \text{if} \quad t \in [m_1^*, 1] \end{cases}$$

From here, it is clear that the distribution of posterior beliefs greater than q is affected in a number of ways. First, a mass point on c exists.

$$\Pr(x^\dagger = c) = (1-\theta^\dagger)\left(\frac{c-q}{(1-c)K(a,q)}aq + (1-a)\frac{1}{2}\right) = (1-\theta^\dagger)\frac{(1-a)(1-q)}{2(1-c)}$$

The mass point on q is smaller with intervention, because the bottom of the support with intervention $m_0^\dagger < m_0^*$, as the following calculation illustrates:

$$\Pr(x^\dagger = q) = (m_0^\dagger + m_1^*)\frac{1-a}{2}$$

Inside interval (q, c) the density is defined by the same expression as with no intervention, however, the minimum trade size is different: instead of m_0^* substitute $m_0^\dagger = \theta^\dagger\frac{1-c}{1-q}$. For $x \in (q, c)$ the density of $x^\dagger$, denoted $g^\dagger(x)$ is as follows:

$$x \in (q, c) \Rightarrow g^\dagger(x) = \theta^\dagger(1-c)\frac{(1-q)(1-a)}{2(1-x)^3}$$

Hence, for $x \in (q, c)$ the distribution of the posterior belief is

$$x \in (q, c) \Rightarrow G^\dagger(x) = \frac{(1-a)(1-m_1^{*2})}{4m_1^*} + \frac{1-a}{2}(m_0^\dagger + m_1^*) + \int_q^x \theta^\dagger(1-c)\frac{(1-q)(1-a)}{2(1-s)^3}\,\mathrm{d}s$$

$$G^{\dagger}(x) = \frac{(1-a)(1-m_1^{*2})}{4m_1^*} + \frac{1-a}{2}(m_0^{\dagger} + m_1^*) + \frac{m_0^{\dagger}(1-a)(x-q)(2-x-q)}{4(1-x)^2}$$

Thus the distribution of the posterior belief random variable for the equilibrium with stochastic interventions is as follows:

$$G^{\dagger}(x) = \begin{cases} \frac{(1-a)(x^2-q^2m_1^{*2})}{4m_1^*x^2} & \text{if} \quad t \in [\underline{\chi}, q) \\ \frac{(1-a)(1-m_1^{*2})}{4m_1^*} + \frac{1-a}{2}(m_0^{\dagger} + m_1^*) + \frac{m_0^{\dagger}(1-a)(x-q)(2-x-q)}{4(1-x)^2} & \text{if} \quad x \in (q, c) \\ 1 & \text{if} \quad x \in [c, \infty) \end{cases}$$

Comparison. Our goal is to show that the signal with interventions is less Blackwell informative than the signal without interventions. Because both posterior belief random variables must have the same mean (namely q) by the law of iterated expectations, to establish a ranking by Blackwell informativeness it is sufficient to establish that the posterior random variable with no intervention x^* is second order stochastic dominated by the posterior belief random variable with intervention, $x^{\dagger}$. See Theorem 2 of Ganuza and Penalva (2010) for a proof that this property is equivalent to Blackwell informativeness for binary states. To establish this result, we will show that $x \leq c \Rightarrow G^*(x) \geq G^{\dagger}(x)$, with strict inequality for $x \in (q, c)$, and that $x > c \Rightarrow G^*(x) < G^{\dagger}(x)$. Thus, the cumulative distribution functions of these random variables can be ranked in the rotation order, with $G^*(x)$ flatter than $G^{\dagger}(x)$. Standard results imply the required stochastic dominance relationship (A proof that uses the standard integral condition is available upon request).

Consider $x < q$. In this case $G^*(x) = G^{\dagger}(x)$, and the required property is trivially satisfied. Next, consider $x \in (q, c)$.

$$x \in (q, c) \Rightarrow G^*(x) - G^{\dagger}(x) = (m_0^* - m_0^{\dagger})\left(\frac{1-a}{2} + \frac{(1-a)(x-q)(2-x-q)}{4(1-x)^2}\right)$$

This is positive because $m_0^* > m_0^{\dagger}$, $x > q$, and $x + q < 2$. Hence $G^*(s) > G^{\dagger}(s)$ for all $x \in (q, c)$. Next, consider $x \in (c, \overline{\chi})$. At $x = c$ distribution $G^{\dagger}(x)$ jumps to 1, while $G^*(x)$ attains 1 at $x = \bar{\chi} > c$. Hence, for $x \in (c, \overline{\chi}]$, $G^*(x) < G^{\dagger}(x)$. Hence, $G^*(x)$ is flatter than $G^{\dagger}(x)$ in the rotation order, implying that order flow without intervention is more Blackwell informative (as described above).

(*Higher Expected Price*). Taking expectations in (1) gives

$$E[p(t)] = 1 - E[\chi(t)] + E[\chi(t)\alpha(t)].$$

Because $\chi(t)$ is the posterior belief that the state is zero *conditional on observed order flow* t, the law of iterated expectations guarantees that the expected value of $\chi(t)$ with respect to the order flow t is equal to the prior. Thus,

$$E[p(t)] = 1 - q + E[\chi(t)\alpha(t)].$$

In the no-intervention benchmark, $\alpha(t) = 0$ for all t, while in the equilibrium with stochastic interventions, $\alpha(t) > 0$ for sell orders $t \in (\theta^\dagger, 1]$. Since these orders occur with positive probability, the result follows.

(*Non-monotonic prices*). The equilibrium price (see (1)) following a sell order is equal to

$$p(t) = 1 - \chi(t) \quad \text{for } t \leq \theta^\dagger$$

$$p(t) = 1 - c + c\alpha(t) \quad \text{for } t > \theta^\dagger$$

Substituting the equilibrium intervention probability and belief function from Proposition 4.1, we find that

$$p(t) = \frac{m_0^\dagger(1-q)}{t} \quad \text{for } t \leq \theta^\dagger$$

$$p(t) = 1 - c + c(1 - \frac{\theta^\dagger}{t}) = 1 - \frac{c\theta^\dagger}{t} \quad \text{for } t > \theta^\dagger$$

Note that $p(t)$ is a continuous function, decreasing for $t \leq \theta^\dagger$, and increasing for $t > \theta^\dagger$. ■

Proof. Proposition 5.1. PM chooses $\alpha(t)$ and $\phi_0(t)$ to maximize her *ex ante* expected payoff:

(A7)

$$v = (1-q)\left(a + (1-a)\frac{1}{2}\right) + \int_0^1 \left(aq\phi_0(t) + (1-a)\frac{1}{2}\right)(\alpha(t)(1-c) + (1-\alpha(t))(1-\chi(t)))\,\mathrm{d}t$$

Associated with these choices are feasibility constraints:

$$0 \leq \alpha(t) \leq 1 \quad \text{and} \quad \phi_0(t) \geq 0$$

The trader's equilibrium indifference condition requires that he is indifferent between all order flows inside the support of his mixed strategy. Therefore, if $\phi_0(t) > 0$ then $t(1-\chi(t))(1-\alpha(t)) = u_0$. We write this constraint in the following way:

$$\phi_0(t)[t(1-\chi(t))(1-\alpha(t)) - u_0] = 0.$$

A second equilibrium condition requires that no pure strategy outside of the support of the trader's mixed strategy would give the trader an expected payoff greater than u_0. We write this condition

in the following way:

$$t(1-\chi(t))(1-\alpha(t)) \leq u_0$$

The last equilibrium condition requires that the trader's density integrates to one over the unit interval.

$$\int_0^1 \phi_0(t)\,\mathrm{d}t = 1$$

When formulating the policymaker's problem, we will include Lagrange multipliers for the following constraints:

$$-\alpha(t) \leq 0 \quad \text{multiplier:} \quad \mu_{1t}$$

$$-\phi_0(t) \leq 0 \quad \text{multiplier:} \quad \mu_{2t}$$

$$\phi_0(t)[t(1-\chi(t))(1-\alpha(t)) - u_0] = 0 \quad \text{multiplier:} \quad \mu_{3t}$$

$$\int_0^1 \phi_0(t)\,\mathrm{d}t - 1 = 0 \quad \text{multiplier:} \quad \lambda$$

To proceed, we now simplify the policymaker's objective function:

$$v = (1-q)\left(a + (1-a)\frac{1}{2}\right)\int_0^1 \left(aq\phi_0(t) + (1-a)\frac{1}{2}\right)\left((1-\chi(t)) + \alpha(t)(\chi(t)-c)\right)\,\mathrm{d}t$$

Using the definition of $\chi(t)$, the first term of the product in the integrand can be reduced:

$$\left(aq\phi_0(t) + (1-a)\frac{1}{2}\right)(1-\chi(t)) = (1-a)(1-q)\frac{1}{2}$$

Hence,

$$v = (1-q) + \int_0^1 \alpha(t)\left(aq\phi_0(t) + (1-a)\frac{1}{2}\right)(\chi(t)-c)\,\mathrm{d}t$$

Next, simplify by substituting the definition of $\chi(t)$.

$$v = (1-q) + \int_0^1 aq(1-c)\alpha(t)\phi_0(t) - \frac{1}{2}(c-q)(1-a)\alpha(t)\,\mathrm{d}t$$

(A8)
$$v = (1-q) + aq(1-c)\int_0^1 \alpha(t)(\phi_0(t) - f)\,\mathrm{d}t$$

Because the constant terms in front of the integral do not affect the choice of $(\alpha(t), \phi_0(t))$ we omit these from the Lagrangian:

$$\mathcal{L} = \int_0^1 \alpha(t)(\phi_0(t)-f) + \mu_{1t}\alpha(t) + \mu_{2t}\phi_0(t) + \mu_{3t}\phi_0(t)[t\frac{(1-q)}{1+K\phi_0(t)}(1-\alpha(t)) - u_0] - \lambda(\phi_0(t)-1)\,\mathrm{d}t$$

The stationarity condition for $\alpha(t)$ requires:

$$\frac{\partial \mathcal{L}}{\partial \alpha(t)} = 0 \Rightarrow \phi_0(t) - f + \mu_{1t} - \mu_{3t}(1-q)\frac{t\phi_0(t)}{1+K\phi_0(t)} = 0 \tag{A9}$$

The stationarity condition for $\phi_0(t)$ requires

$$\frac{\partial \mathcal{L}}{\partial \phi_0(t)} = 0 \Rightarrow \alpha(t) + \mu_{2t} + \mu_{3t}[t\frac{(1-q)}{1+K\phi_0(t)}(1-\alpha(t)) - u_0 - \phi_0(t)t(1-\alpha(t))\frac{K(1-q)}{(1+K\phi_0(t))^2}] - \lambda = 0$$

Observe that

$$t\frac{(1-q)}{1+K\phi_0(t)}(1-\alpha(t)) - u_0 - \phi_0(t)t(1-\alpha(t))\frac{K(1-q)}{(1+K\phi_0(t))^2} = \frac{t(1-\alpha(t))(1-q)}{(1+K\phi_0(t))^2} - u_0$$

Simplifying this stationarity condition gives:

$$\alpha(t) + \mu_{2t} + \mu_{3t}[\frac{t(1-\alpha(t))(1-q)}{(1+K\phi_0(t))^2} - u_0] - \lambda = 0 \tag{A10}$$

In addition to the primal feasibility conditions and the two stationarity conditions, dual feasibility requires that

$$\mu_{1t} \geq 0 \quad \mu_{2t} \geq 0$$

and complementary slackness requires that

$$\mu_{1t}\alpha(t) = 0 \quad \mu_{2t}\phi_0(t) = 0.$$

In an equilibrium with $u_0 > 0$, the trader cannot mix over the entire interval $[0, 1]$. A set of order flows must exist for which $\phi_0(t) = 0$. Consider some such order flow. The stationarity condition for $\alpha(t)$ implies that

$$\mu_{1t} = f > 0$$

Because $\mu_{1t} > 0$, complementary slackness implies that $\alpha(t) = 0$. Therefore $\phi_0(t) = 0 \Rightarrow \alpha(t) = 0$, that is, no intervention takes place outside of the support of the trader's mixed strategy. Therefore from a primal constraint, for any t outside of the support of the trader's mixed strategy it must be that

$$t(1-q) \leq u_0$$

Recalling that $u_0 = m_0^\dagger(1-q)$, this becomes $t \leq m_0^\dagger$ Hence if some t is outside of the support then all smaller t are also outside of the support. Thus, the set of order flows outside the support forms an interval. The supremum of points outside of the support is $m_0^\dagger$. Part 5 of Lemma A.2 implies that no intervention takes place at $m_0^\dagger$. This also implies that an interval exists in which $\phi_0(t) > 0$ and $\alpha(t) = 0$.

Consider an order flow for which $\phi_0(t) > 0$ and $\alpha(t) = 0$. Here the stationarity conditions require

$$\phi_0(t) - f + \mu_{1t} - \mu_{3t}(1-q)\frac{t\phi_0(t)}{1+K\phi_0(t)} = 0$$

$$\mu_{3t}[\frac{t(1-q)}{(1+K\phi_0(t))^2} - m_0^\dagger(1-q)] - \lambda = 0$$

Primal feasibility then implies that

$$\phi_0(t) = \frac{t - m_0^\dagger}{Km_0^\dagger}$$

Solving these equations gives that

$$\mu_{3t} = -\frac{\lambda t}{m_0^\dagger(t-m_0^\dagger)(1-q)}$$

$$\mu_{1t} = f + \frac{1}{K} - \frac{1+\lambda}{Km_0^\dagger}t$$

Thus (recalling $J = 1 + Kf$ and $\theta^\dagger = Jm_0^\dagger$)

$$\mu_{1t} \geq 0 \Leftrightarrow t \leq \frac{\theta^\dagger}{1+\lambda}$$

$$\phi_t \geq 0 \Leftrightarrow t \geq m_0$$

If no region in which $\alpha(t) > 0$ exists, then the policymaker never benefits from intervention. Hence, under the optimal policy, a region must exist with $\alpha(t) > 0$. In this region the stationarity conditions are

$$\phi_0(t) - f - \mu_{3t}(1-q)\frac{t\phi_0(t)}{1+K\phi_0(t)} = 0$$

$$\alpha(t) + \mu_{3t}[\frac{t(1-\alpha(t))(1-q)}{(1+K\phi_0(t))^2} - m_0^\dagger(1-q)] - \lambda = 0$$

and primal feasibility implies that

$$\phi_0(t) = \frac{t(1-\alpha(t)) - m_0^\dagger}{K_0 m_0^\dagger}$$

Solving these conditions gives that

$$\alpha(t) = \frac{1}{2}\left(1 + \lambda - \frac{\theta^\dagger}{t}\right)$$

$$\alpha(t) > 0 \Rightarrow t > \frac{\theta^\dagger}{1+\lambda}$$

$$\alpha(1) < 1 \Rightarrow \lambda < 1 + \theta^\dagger$$

Observe that when $\lambda = 0$ the commitment policy is exactly half of the equilibrium intervention policy (and starts from the same intervention threshold $\theta^\dagger$). Therefore, if λ were zero, the density would integrate to more than one: hence, $\lambda > 0$ (this can also be established directly by differentiating the constraint and signing its derivative). When PM intervenes,

$$\phi_0(t) = \frac{t(1 - \frac{1}{2}(1 + \lambda - \frac{\theta^\dagger}{t})) - m_0^\dagger}{Km_0^\dagger} = f + \frac{(1-\lambda)t - (1 + Kf)m_0}{2Km_0}$$

Observe first that at the intervention threshold, $t = \frac{\theta^\dagger}{1+\lambda}$ the value of $\phi_0(t)$ is

$$\phi_0(t) = f - \frac{\lambda J}{K(1+\lambda)}$$

which (because $\lambda > 0$) implies that in the constrained optimal program the policymaker begins to intervene at a *smaller order* than is profitable (i.e. when intervention negatively impacts her payoff). This result also implies that $\lambda < 1$, otherwise $\phi_0(t)$ is decreasing in the intervention region, which implies that $\alpha(t) > 0 \Rightarrow \phi_0(t) < f$, but this is dominated by the equilibrium intervention policy. An immediate consequence of $\lambda < 1$ is $\alpha(1) < 1$, so the upper feasibility constraint for $\alpha(t)$ never binds. In fact $\alpha(1)$ must be less than the corresponding equilibrium value. Because the intervention policies are functions of the form $k_1 - \frac{k_2}{t}$ they cross at most once (as a function of t). For low t the optimal policy lies above the equilibrium policy; If they did not cross at all, then the optimal policy would always be above, generating a (weakly) smaller value of $\phi_0(t)$ for all t, certainly worse than the equilibrium policy. The value of λ can be found by solving the constraint:

$$\int_{m_0^\dagger}^{\frac{\theta^\dagger}{1+\lambda}} \frac{t - m_0^\dagger}{Km_0^\dagger} \mathrm{d}t + \int_{\frac{\theta^\dagger}{1+\lambda}}^{1} f + \frac{(1-\lambda)t - \theta^\dagger}{2Km_0} \mathrm{d}t = 1$$

We know that the left side exceeds one when $\lambda = 0$ and that optimality requires $\lambda < 1$. In order for this solution to also be consistent with inequality constraints for order flows outside the support, it is necessary and sufficient that the intervention threshold exceeds m_0, or that $\lambda < J - 1$. This condition is implied by $\lambda < 1$ whenever $J > 2$ or $c > \frac{1+q}{2}$. Otherwise, it is possible that interventions take place at $m_0^\dagger$, violating the inequality conditions for order flows below $m_0^\dagger$. These inequalities therefore bind in this case. As this possibility is quite technical and not especially illuminating, we omit it here, and assume $c > \frac{1+q}{2}$. ■

Proof. Proposition 6.1.
In the equilibrium with stochastic interventions the maximum probability of intervention is $\alpha(1) = 1 - \theta^\dagger$. Hence, a necessary condition for the cap to bind is $\overline{\alpha} < 1 - \theta^\dagger$.

By part 4 of Lemma A.2 the trader's expected payoff is $u_0 = m_0(1 - q)$. Hence, his indifference

requires the following:

$$\alpha(t) = 0 \Rightarrow m_0(1-q) = t(1-\chi(t))$$

$$\alpha(t) \in (0, \overline{\alpha}) \Rightarrow m_0(1-q) = t(1-\chi(t))(1-\alpha(t))$$

$$\alpha(t) = \overline{\alpha} \Rightarrow m_0(1-q) = t(1-\chi(t))(1-\overline{\alpha}).$$

Solving these equations gives the following expressions:

$$\phi_0(t) = \frac{t - m_0}{K_0 m_0} \text{ if } \alpha(t) = 0$$

$$\phi_0(t) = f \text{ if } t \in [\theta_1, \theta_2] \text{ if } \alpha(t) \in (0, \overline{\alpha})$$

$$\phi_0(t) = \frac{(1-\overline{\alpha})t - m_0}{K_0 m_0} \text{ if } \alpha(t) = \overline{\alpha}$$

Define the following thresholds θ_1, θ_2 by the intersections of these functions:

$$\frac{\theta_1 - m_0}{K_0 m_0} = f \iff \theta_1 = J m_0$$

$$\frac{\theta_2(1-\overline{\alpha}) - m_0}{K_0 m_0} = f \iff \theta_2 = \frac{J m_0}{1-\overline{\alpha}}$$

Analogous arguments to those in the proof of Proposition 4.1 establish that, in equilibrium, (i) for all $t \in [m_0, \theta_1]$, no intervention takes place, $\alpha(t) = 0$; (ii) for all $t \in [\theta_1, \theta_2]$, intervention takes place with probability less than $\overline{\alpha}$, $\alpha(t) \in (0, \overline{\alpha})$. (iii) for all $t \in [\theta_2, 1]$, intervention takes place with probability $\overline{\alpha}$, $\alpha(t) = \overline{\alpha}$.

Thus we have specified equilibrium strategies up to constant m_0. To determine this constant, integrate $\phi_0(t)$ from m_0 to 1 and set the result equal to 1. This yields an equation $Q(\overline{m}, \overline{\alpha} | K_0, J) = 0$ (see eq. 7). The larger root of this equation is greater than 1 (proof available on request). Consider the smaller root

$$\bar{m}_0 = \frac{1-\overline{\alpha}}{\overline{\alpha}J^2 + 1 - \overline{\alpha}} \left(K_0 + 1 - \sqrt{(K_0+1)^2 - (\overline{\alpha}J^2 + 1 - \overline{\alpha})} \right) \tag{A11}$$

Clearly if $\overline{m}_0$ is real, then it is positive. Hence to verify existence of the equilibrium we must show $\overline{m}_0$ is real and that $\theta_2 < 1$.
Now, $\overline{m}_0$ is real if

$$(K_0+1)^2 \geq \overline{\alpha}J^2 + 1 - \overline{\alpha}.$$

Because $J > 1$ the right side is largest when $\overline{\alpha}$ is as big as possible, namely when $\overline{\alpha} = 1 - \theta^\dagger$. Hence $\overline{m}_0$ is real iff

$$(K+1)^2 \geq (1-\theta^\dagger)J^2 + \theta^\dagger = J^2 - J m_0^\dagger \left(J^2 - 1\right).$$

Substituting the definition of $m_0^\dagger$ from (A5) gives

$$(K+1)^2 \geq J^2 - 2J(J-K-1)$$

or

$$(J-(K+1))^2 \geq 0.$$

Hence, $\overline{m}_0$ is real and positive.
Now, $\theta_2 \leq 1$ iff $\frac{J\overline{m}_0}{1-\overline{\alpha}} \leq 1$. Letting $Z \equiv \overline{\alpha}J^2 + 1 - \overline{\alpha}$ and substituting from (A11) we have

$$\begin{aligned}
&\frac{J}{Z}\left(K+1-\sqrt{(K+1)^2 - Z}\right) \leq 1 \\
&\qquad \Leftrightarrow \quad J(K+1) - Z \leq J\sqrt{(K+1)^2 - Z} \\
&\qquad \Leftrightarrow \quad J^2(K+1)^2 - 2J(K+1)Z + Z^2 \leq J^2(K+1)^2 - J^2 Z \\
&\qquad \Leftrightarrow \quad Z \leq 2J(K+1) - J^2 \\
&\qquad \Leftrightarrow \quad \overline{\alpha}J^2 + 1 - \overline{\alpha} \leq 2J(K+1) - J^2.
\end{aligned}$$

Again, the left side of the last line is largest when $\overline{\alpha} = 1 - \theta^\dagger$ or

$$\begin{aligned}
&J^2 - Jm_0^\dagger(J^2-1) \leq 2J(K+1) - J^2 \\
&\qquad \Leftrightarrow \quad m_0^\dagger \geq \frac{2(J-K-1)}{J^2-1}.
\end{aligned}$$

The last line holds with equality by definition of $m_0^\dagger$ (A5) Hence, the specified strategies and beliefs constitute a valid equilibrium.
The type-0 trader's expected equilibrium payoff is $\overline{m}_0(1-q)$ by part 4 of Lemma A.2. PM's expected payoff is found by using (A7)

$$v = 1 - q + aq(1-c)\int_0^1 \alpha(t)(\phi_0(t) - f)\,\mathrm{d}t$$

For $t \in [m_0(\overline{\alpha}), \theta_1]$, $\alpha(t) = 0$ and for $t \in [\theta_1, \theta_2]$, $\phi_0(t = f$. Thus

$$v = 1 - q + aq(1-c)\int_{\theta_2}^1 \overline{\alpha}\left(\frac{(1-\overline{\alpha})t - m_0(\overline{\alpha})}{Km_0(\overline{\alpha})} - f\right)\,\mathrm{d}t.$$

Performing the integration and collecting terms yields the claim.
To prove the comparative static claims, implicitly differentiate (7) to get

$$\frac{d\overline{m}_0}{d\overline{\alpha}} = \frac{1-\theta_2^2}{2\left(\left(\frac{\overline{\alpha}J^2}{1-\overline{\alpha}}+1\right)m_0(\overline{\alpha}) - K - 1\right)}$$

Note that the numerator on the right is positive because $\theta_2 < 1$ for $\overline{\alpha} < 1 - \theta^\dagger$ was established above. Hence,

$$\frac{d\overline{m}_0}{d\overline{\alpha}} < 0 \Leftrightarrow \left(\frac{\overline{\alpha}J^2}{1-\overline{\alpha}} + 1\right)\overline{m}_0 < K + 1.$$

Substituting from (A11), this is true iff

$$K_0 + 1 - \sqrt{(K_0+1)^2 - (\overline{\alpha}J^2 + 1 - \overline{\alpha})} < K_0 + 1$$

which clearly holds. To prove the last part of the claim, observe that the smaller root of $Q(m, 0|K_0, J)$ (in m) is m_0^*. Whereas the non-zero root of $Q(m, 1-\theta^\dagger|K_0, J) = 0$ is $m_0^\dagger$. ■

Proof. Corollary 6.2. Applying the results of Proposition 6.1 we have the following.

Trader. $m_0^\dagger(1-q) < \overline{m}_0(1-q) < m_0^*(1-q)$. Moreover $\overline{m}_0$ is decreasing in $\overline{\alpha}$.

Policymaker. Recall that

$$aq(1-c)\overline{\alpha}\left(\frac{1-\theta_1-\overline{\alpha}}{2Km_0(\overline{\alpha})}\right) = aq(1-c)\overline{\alpha}\int_{\theta_2}^1 \phi_0(t) - f\,\mathrm{d}t.$$

Because $\phi_0(\theta_2) = f$ the integrand is strictly positive for $t > \theta_2$. Moreover, $\overline{\alpha} = 1 - \theta^\dagger$ implies $\theta_2 = 1$. ■

Below we present the proposition characterizing the equilibrium with non-transparency that was skipped in the text. Note that the proof that follows it covers all three non-transparency cases.

Proposition A.4 *(Privately Informed Policymaker–III). For $\hat{\chi} < c_L < c_H < \overline{\chi}$, if the probability of low cost is high, $\gamma \in (1-\theta_L^\dagger, 1]$, the unique equilibrium is characterized as follows. The high cost policymaker never intervenes. The type-0 trader plays the same strategy as in Proposition 4.1 (the equilibrium with stochastic intervention) with $c = c_L$. The low cost policymaker's intervention strategy is the intervention strategy of Proposition 4.1, multiplied by $1/\gamma$, and is always strictly less than one. The trader's expected payoff is $u_0 = m_L^\dagger(1-q)$. Both types of policymaker expect payoff $1-q$ and do not benefit from interventions.*

Proof. Propositions 6.5, 6.6, A.4.
First we establish $0 < \overline{\gamma} < 1 - \theta_L^\dagger$. That $\bar{\gamma} = R(1-\theta_H^\dagger) > 0$ is obvious. To show $\gamma \bar{<} 1$, consider

the following string of equivalent expressions.

$$\begin{aligned}
\overline{\gamma} < 1 - \theta_L^\dagger & \\
\Leftrightarrow \quad & R(1-\theta_H^\dagger) < 1 - \theta_L^\dagger \\
\Leftrightarrow \quad & \frac{J_H^2-1}{J_L^2-1}(1 - J_H\frac{2(J_H-K_0-1)}{J_H^2-1}) < 1 - J_L\frac{2(J_L-K_0-1)}{J_L^2-1} \\
\Leftrightarrow \quad & \frac{J_H-J_L}{J_L^2-1}(2(K_0+1) - J_H - J_L) < 0
\end{aligned}$$

where the last inequality holds by point 3 of Lemma A.3.

To continue, note that all parts of Lemma A.2 apply. Hence, letting m_0 represents the minimum equilibrium trade size, then

$$\begin{aligned}
\alpha_L(t) = \alpha_H(t) = 0 \Rightarrow \phi_0(t) &= \frac{t-m_0}{K_0 m_0} \\
\alpha_L(t) \in (0,1),\, \alpha_H(t) = 0 \Rightarrow \phi_0(t) &= f_L \\
\alpha_L(t) = 1,\, \alpha_H(t) = 0 \Rightarrow \phi_0(t) &= \frac{t(1-\gamma)-m_0}{K_0 m_0} \\
\alpha_L(t) = 1,\, \alpha_H(t) \in (0,1) \Rightarrow \phi_0(t) &= f_H
\end{aligned}$$

Define the following points of intersection:

$$\begin{aligned}
\frac{\theta_1 - m_0}{K_0 m_0} = f_L \iff \theta_1 &= J_L m_0 \\
f_L = \frac{\theta_2(1-\gamma)-m_0}{K_0 m_0} \iff \theta_2 &= \frac{J_L m_0}{1-\gamma} \\
\frac{\theta_3(1-\gamma)-m_0}{K_0 m_0} = f_H \iff \theta_3 &= \frac{J_H m_0}{1-\gamma}
\end{aligned}$$

Note that $m_0 < \theta_1 < \theta_2 < \theta_3$. Similar arguments to those in the proof of Proposition 4.1, establish four results about the equilibrium structure. We omit the proofs for the sake of brevity (they are available upon request).

1. $t \in [m_0, \theta_1] \Rightarrow \alpha_i(t) = 0$ and $\phi_0(t) = \frac{t-m_0}{K_0 m_0}$.
2. $t \in (\theta_1, \theta_2) \Rightarrow \alpha_L(t) \in (0,1)$, $\alpha_H(t) = 0$ and $\phi_0(t) = f_L$.
3. $t \in [\theta_2, \theta_3] \Rightarrow \alpha_L(t) = 1$, $\alpha_H(t) = 0$ and $\phi_0(t) = \frac{t(1-\gamma)-m_0}{K_0 m_0}$.
4. $t > \theta_3 \Rightarrow \alpha_L(t) = 1$, $\alpha_H(t) \in (0,1)$ and $\phi_0(t) = f_H$

Case I. Consider the possibility of an equilibrium in which $\theta_3 < 1$. We will show that such an equilibrium exists if and only if $\gamma \in [0, \overline{\gamma}]$.

In this type of equilibrium, the trader's mixing densities are given by points 1-4 above. Thus we have specified the trader's equilibrium strategy up to constant m_0. To determine this constant, integrate $\phi_0(t)$ from m_0 to 1 and set the result equal to 1. Collecting terms then yields the quadratic equation

$$\left(\frac{J_H - \gamma J_L}{1-\gamma} - 1\right) m_0^2 - 2(J_H - K_0 - 1)m_0 = 0$$

The non-zero root of this is

$$\begin{aligned}
\widehat{m}_0 &= \frac{2(J_H - K_0 - 1)}{\left(\frac{J_H - \gamma J_L}{1-\gamma} - 1\right)} \\
&= \left(\frac{(1-\gamma)(J_H^2 - 1)}{J_H^2 - 1 - \gamma(J_L^2 - 1)}\right) \frac{2(J_H - K_0 - 1)}{J_H^2 - 1} \\
&= \frac{(1-\gamma)R}{R-\gamma} m_H^\dagger
\end{aligned}$$

Because $c_H > \hat{\chi}$, $m_H^\dagger$ is positive; the denominator of the fraction positive because $R > 1 > \gamma$. Thus $\widehat{m}_0 > 0$.

Because $0 < \widehat{m}_0 < \theta_1 < \theta_2 < \theta_3$, to verify validity of the strategies we need only to show $\theta_3 < 1$. Consider the following string of equivalent expressions

$$\begin{aligned}
&\theta_3 < 1 \\
&\quad \Leftrightarrow \quad \frac{J_H}{1-\gamma} \widehat{m}_0 < 1 \\
&\quad \Leftrightarrow \quad \frac{J_H}{1-\gamma} \frac{R(1-\gamma)}{R-\gamma} m_H^\dagger < 1 \\
&\quad \Leftrightarrow \quad \frac{J_H(J_H^2 - 1)}{J_H^2 - 1 - \gamma(J_L^2 - 1)} m_H^\dagger < 1 \\
&\quad \Leftrightarrow \quad \gamma < (1 - J_H m_H^\dagger) \frac{J_H^2 - 1}{J_L^2 - 1} \\
&\quad \Leftrightarrow \quad \gamma < (1 - \theta_H^\dagger) R = \bar{\gamma}
\end{aligned}$$

Note also that $\gamma = \overline{\gamma}$ implies $\theta_3 = 1$.

The policymaker's strategy is specified by points 1-4 above, except for the mixing probability $\alpha_L(t)$ in interval (θ_2, θ_3) and $\alpha_H(t)$ in interval $(\theta_3, 1)$. These are calculated from the trader's indifference conditions, with the following results:

$$t \in [\theta_1, \theta_2] \Rightarrow \alpha_L(t) = \frac{1}{\gamma}\left(1 - \frac{\theta_1}{t}\right)$$

$$t \in [\theta_3, 1] \Rightarrow \alpha_H(t) = 1 - \frac{\theta_3}{t}$$

Notice that for the intervals in which the expressions given above hold, $\alpha_i(t) \in [0, 1]$.

The type-0 trader's expected equilibrium payoff is $\widehat{m}_0(1-q)$ by part 4 of Lemma A.2. The type-H PM does not intervene for $t \leq \theta_3$ and mixes for $t > \theta_3$ and hence has expected payoff $1 - q$. The type-L PM's expected payoff is found by using (A7)

$$v_L = 1 - q + aq(1-c) \int_0^1 \alpha_L(t)(\phi_0(t) - f_L)\, \mathrm{d}t$$

For $t \in [0, \theta_1]$, $\alpha_L(t) = 0$; for $t \in [\theta_1, \theta_2]$, $\phi_0(t) = f_L$; and for $t > \theta_2$, $\alpha_L(t) = 1$. Thus

$$v_L = 1 - q + aq(1-c) \int_{\theta_2}^{\theta_3} \left(\frac{(1-\gamma)t - \widehat{m}_0}{K_0 \widehat{m}_0} - f_L \right) \mathrm{d}t + aq(1-c) \int_{\theta_3}^{1} (f_H - f_L)\, \mathrm{d}t. \tag{A12}$$

Performing the integration and collecting terms yields the claim.

Case II. Consider an equilibrium in which $\theta_2 < 1 < \theta_3$. We will show that such an equilibrium exists if and only if $\gamma \in [\overline{\gamma}, 1 - \theta_L^\dagger]$. Because $\gamma > \bar{\gamma}$ an equilibrium with $\theta_3 < 1$ does not exist. Hence, in this case, the type-H PM never intervenes. The trading densities in this type of equilibrium are given by points 1-3 above. Integrating $\phi_0(t)$ from m_0 to 1, setting the result equal to 1, and collecting terms, shows that m_0 is the root in m of $Q(m, \gamma | K_0, J_L)$,. Hence, the trader plays exactly as if it were common knowledge that $c = c_L$ and there was an intervention cap of γ. Hence, from Proposition 6.1, $\theta_2 < 1$ if and only if the "intervention cap" γ is less than $1 - \theta^\dagger$. Inside (θ_1, θ_2), the low type policymaker intervenes with probability $\alpha_L(t) = \frac{1}{\gamma}(1 - \frac{\theta_1}{t})$, and with probability 1 inside $(\theta_2, 1)$. Letting $\overline{m_0}(\gamma)$ represent the minimum trade size in this equilibrium, trader's expected payoff is thus $\overline{m}_0(\gamma)(1-q)$. The type-$H$ PM's expected payoff is obviously $1 - q$. The type-L PM's expected payoff is found by using (A7)

$$v_L = 1 - q + aq(1-c) \int_0^1 \alpha_L(t)(\phi_0(t) - f_L)\, \mathrm{d}t$$

For $t \in [0, \theta_1]$, $\alpha_L(t) = 0$; for $t \in [\theta_1, \theta_2]$, $\phi_0(t) = f_L$; and for $t > \theta_2$, $\alpha_L(t) = 1$. Thus

$$v_L = 1 - q + aq(1-c) \int_{\theta_2}^{1} \left(\frac{(1-\gamma)t - \bar{m}_0(\gamma)}{K_0 m_0(\gamma)} - f_L \right) \mathrm{d}t$$

Performing the integration and collecting terms yields the claim.

Case III. Finally, consider an equilibrium in which $\theta_1 < 1 < \theta_2$. We will show that such an equilibrium exists if and only if $\gamma \in [1 - \theta_L^\dagger, 1]$. In this type of equilibrium, points 1 and 2 above establish that given some value of the minimum trade size m_0, the mixing density for the trader is

identical to the one in the stochastic intervention equilibrium:

$$t < J_L m_0 \Rightarrow \phi_0(t) = \frac{t - m_0}{K_0 m_0} \quad t > J_L m_0 \Rightarrow \phi_0(t) = f_L$$

Integrating $\phi_0(t)$ from m_0 to 1, setting the result equal to 1, and collecting terms yields $Q(m_0, 0|K-0, J_L) = 0$, hence the solution is $m_L^\dagger$. The trader plays exactly as if it were common knowledge that $c = c_L$. Thus, provided $\theta_2 = J_L m_0^\dagger/(1-\gamma) > 1$ such an equilibrium exists. This condition simplifies to $\gamma > 1 - J_L m_0^\dagger \iff \gamma > 1 - \theta_L^\dagger$. Thus the type-0 trader's expected payoff is $(m_L^\dagger(1-q)$ and the expected payoff of both types of PM is $1-q$. The type-L PM mixes according to $\alpha_L(t) = \frac{1}{\gamma}(1 - \frac{\theta_L^\dagger}{t})$ over $[\theta_L^\dagger, 1]$, and the intervention probability is less strictly than one over this interval. ■

Proof. Corollary 6.7. Observe that $\phi_0(\theta_2) = f_L$. Hence the integrand in (A12) is positive at every point $t \in (\theta_2, 1]$. Moreover, $\gamma < 1 - \theta_L^\dagger$ implies $\theta_2 < 1$. ■

Proof. Proposition 6.3. We first show that the bounds on b in (11) ensure that interventions take place in equilibrium after some sell order but do not take place in equilibrium after any buy order. Recall (as in Proposition 3.1) that in the absence of intervention

$$\phi_i(t) = \frac{t - m_i^*}{K_i m_i^*}$$

Thus, following a sell order

$$\chi(t) = 1 - \frac{m_0^*(1-q)}{t} \qquad \delta(t) = 1 - \frac{m_0^*}{t}$$

For no-intervention after sells to be incentive compatible, it must be that given the mixing density for the type-0, the agent's sequential rationality condition calls for no intervention for all $t \in [m_0^*, 1]$:

$$\chi(t) < c + qb(1-\delta(t))t \iff 1 - \frac{m_0^*(1-q)}{t} < c + qbm_0^* \iff 1 - m_0^*(1-q) < c + qbm_0^* \iff b > \frac{\overline{\chi} - c}{bm_0^*}$$

Hence whenever

$$b < \frac{\overline{\chi} - c}{bm_0^*} = \frac{(1-q)(1 - Jm_0^*)}{Jqm_0^*}$$

no-intervention following sells violates the agent's sequential rationality condition.

Next, we prove that the bounds on bias guarantee that no intervention takes place in equilibrium following a buy order. Consider the agent's sequential rationality condition following a buy order. The agent's posterior belief that the trader is informed conditional on buy order t by $\delta_B(t)$:

$$\delta_B(t) \equiv \frac{a(1-q)\phi_1(t)}{a(1-q)\phi_1(t) + (1-a)/2} = \frac{K_1\phi_1(t)}{K_1\phi_1(t) + 1}$$

To write the agent's expected payoff, observe that if the policymaker intervenes with probability α following an executed buy order of size t transacted at price $p(t)$, then the expected value of the investor's terminal profit given that he is a noise trader is given by the following:

$$\pi_{NB}(t) \equiv t(\alpha + (1-q)(1-\alpha) - p(t))$$

When buying, the noise trader pays $p(t)$ on each share for an asset that is expected to be worth one with probability $\alpha + (1-\alpha)(1-q)$. The agent's expected payoff from intervening with probability α after observing a trade of size t executed at price $p(t)$ is given by:

$$\alpha(1-c) + (1-\alpha)(1-\chi(t)) + b(1-\delta_B(t))\pi_{NB}(t)$$

By maximizing the agent's expected payoff with respect to α, we find her sequentially rational intervention strategy following a buy order:

$$\alpha(t) = \begin{cases} 0 & \text{if} \quad \chi(t) < c - t(1-\delta_B(t))qb \\ [0,1] & \text{if} \quad \chi(t) = c - t(1-\delta_B(t))qb \\ 1 & \text{if} \quad \chi(t) > c - t(1-\delta_B(t))qb \end{cases}$$

In the no intervention equilibrium of Proposition 3.1, following a buy order

$$\chi(t) = \frac{m_1^*(1-q)}{t} \qquad \delta(t) = 1 - \frac{m_1^*}{t}$$

For no intervention to be incentive compatible after all buy orders:

$$\chi(t) < c - qb(1-\delta(t))t \iff \frac{m_1^* q}{t} < c - qbm_1^* \iff q < c - qbm_1^* \iff b < \frac{c-q}{qm_1^*}$$

Hence whenever

$$b < \frac{c-q}{qm_1^*} = \frac{J - 1 + q(1 - Jm_1^*)}{Jqm_1^*}$$

no intervention following buy orders in $[m_1^*, 1]$ is incentive compatible. Furthermore, for any buy order in $[0, m_1^*]$, $\chi(t) = q$ and $\delta(t) = 0$. Thus no intervention over this interval requires $q < c - qbt$. Thus, if this inequality is satisfied at $t = m_1^*$ it is satisfied inside the interval. The inequality at m_1^* is implied by $b < (c-q)/(qm_1^*)$. Hence, for all buy orders no intervention takes place, and the type-1 trader mixes as in the case of no intervention, described in Proposition 3.1.

The argument above implies that following sell orders, intervention must take place in equilibrium. When intervention takes place, the agent's incentive constraint must hold:

$$\chi(t) \geq c + qb(1-\delta(t))t \iff \frac{K_0\phi_0 + q}{K_0\phi_0 + 1} \geq c + qb(1 - \frac{K_0\phi_0}{1 + K\phi_0})t \iff \phi_0(t) \geq \frac{c - q + bqt}{K(1-c)}$$

Note that whenever $b > -(c-q)$, this value of $\phi_0(t)$ is non-negative. Hence,

$$\alpha(t) > 0 \Rightarrow \phi_0(t) \geq \frac{c-q+bqt}{K(1-c)} = \frac{c-q}{K(1-c)} + \frac{bqt}{K(1-c)} = f + \frac{Jbqt}{K_0(1-q)}$$

Next we show that under the bounds on the agent bias the type-0 trader's expected equilibrium payoff must be positive. This payoff can be zero only if the policymaker intervenes for all order flow (see the proof of Lemma A.1). Sequential rationality for the agent therefore requires that for all t,

$$\phi_0(t) \geq f + \frac{Jbqt}{K_0(1-q)} \Rightarrow 1 = \int_0^1 \phi_0(t)\,\mathrm{d}t \geq f + \frac{Jbq}{2K_0(1-q)}$$

Because $c > \hat{\chi} \Rightarrow f > 1$ the right hand side clearly exceeds 1 for positive bias, $b > 0$. When b is sufficiently negative, the right hand side may be less than one, requiring:

$$f + \frac{Jbq}{2K_0(1-q)} \leq 1 \iff b < -\frac{2}{Jq}K_0(f-1)(1-q)$$

which evidently contradicts $b > \underline{B}$ in condition (11). Hence, when the bounds in (11) hold and $c \in (\hat{\chi}, \overline{\chi})$, the type-0 trader's expected profit is non-zero.

The type-0 trader's equilibrium expected profit is non-zero, and by point 5. of Lemma A.2 (which still holds), the type-0 trader's equilibrium expected profit is $u_0 = \widetilde{m}_0(1-q)$, where $\widetilde{m}_0$ represents the minimum trade size for the type-0 trader. Because the type-0 trader's expected equilibrium profit is strictly positive, no order inside $[\widetilde{m}_0, 1]$ triggers intervention for certain. Because interventions sometimes take place, they must be due to a mixed strategy. We know that a set of order flows exists which triggers a stochastic intervention. A set of order flows also exists which definitely does not trigger an intervention (if an intervention could be triggered for all strictly positive trades then $\phi_0(t) \geq f + Jbtq/(K_0(1-q))$, but in this case it cannot integrate to one). Following the argument in the proof of Proposition 4.1, we see that:

$$\alpha(t) = 0 \iff \phi_0(t) = \frac{t - \widetilde{m}_0}{K_0\widetilde{m}_0}$$

and we have already argued

$$\alpha(t) > 0 \iff \phi_0(t) = f + \frac{Jbqt}{K_0(1-q)}$$

Note that both of these are linear functions; the mixing density for t with $\alpha(t) = 0$ has positive slope and intersects the t axis at $\widetilde{m}_0$; meanwhile the mixing density for t with $\alpha(t) > 0$ has a strictly positive intercept with the y axis and a slope that can be positive or negative depending on the agent's bias. When interventions take place in equilibrium following sells, $b < \overline{B}$, these functions

have a unique intersection inside $(\widetilde{m}_0, 1)$. Let $\widetilde{\theta}$ represent this intersection:

$$\frac{\widetilde{\theta} - \widetilde{m}_0}{K_0 \widetilde{m}_0} = f + \frac{Jbq\widetilde{\theta}}{K_0(1-q)} \iff \widetilde{\theta} = J\widetilde{m}_0 \frac{1-q}{1-q-bJ\widetilde{m}_0}.$$

where

$$t \gtrless \widetilde{\theta} \iff f + \frac{Jbq\widetilde{\theta}}{K_0(1-q)} \gtrless \frac{\widetilde{\theta} - \widetilde{m}_0}{K_0 \widetilde{m}_0}$$

A similar argument to the one used in Proposition 4.1 establishes that in equilibrium, order flow below $\widetilde{\theta}$ leads to no intervention, while order flow above $\widetilde{\theta}$ leads to stochastic intervention:

$$t \in [\widetilde{m}_0, \widetilde{\theta}] \Rightarrow \phi_0(t) = \frac{t - \widetilde{m}_0}{K_0 \widetilde{m}_0} \tag{A13}$$

$$t \in [\widetilde{\theta}, 1] \Rightarrow \phi_0(t) = f + \frac{Jbqt}{K_0(1-q)}$$

To determine properties of the minimum trade size, define the following function, representing the area under function $\phi_0(t)$ defined by (A13).

$$G(m) \equiv \int_m^{Jm\frac{1-q}{1-q-bJm}} \frac{t-m}{Km}\,\mathrm{d}t + \int_{Jm\frac{1-q}{1-q-bJm}}^1 f + \frac{Jbqt}{K_0(1-q)}\,\mathrm{d}t$$

The equilibrium value of the minimum trade size, $\widetilde{m}_0$ satisfies $G(\widetilde{m}_0) = 1$. Consider the monotonicity of $G(\cdot)$. With some calculations, it is possible to show

$$G'(m) < 0 \iff (Jb)^2 m^2 - 2Jbq(1-q)m - (J^2-1)(1-q)^2 < 0$$

Which holds whenever m lies in between the roots of the quadratic, which are

$$x_1 \equiv \frac{(J+1)(1-q)}{Jbq} \qquad x_2 \equiv -\frac{(J-1)(1-q)}{Jbq}$$

Consider first $b > 0$. Here the smaller roots is x_2, and it is negative. Hence for $m \in [0, x_1)$ function $G(m)$ is decreasing. Next, we show that given the bounds $0 < b < \overline{B}$, the larger root x_1 exceeds m_0^*. Indeed

$$x_1 \equiv \frac{(J+1)(1-q)}{Jbq} > \frac{(J+1)(1-q)}{J\overline{B}q} \geq \frac{(J+1)(1-q)}{Jq\left(\frac{(1-q)(1-Jm_0^*)}{Jqm_0^*}\right)} = m_0^* + \frac{Jm_0^*(1+m_0^*)}{1-Jm_0^*}$$

The last term is positive because $c < \overline{\chi} \Rightarrow Jm_0^* < 1$ (consult point 4 of Lemma A.3). Hence, for $b > 0$, function $G(m)$ is strictly decreasing on $[0, m_0^*]$. Now consider $b < 0$. Here root x_1 is strictly

negative, hence, $G(m)$ is strictly decreasing on $[0, x_2]$. Furthermore:

$$x_2 \equiv -\frac{(J-1)(1-q)}{Jbq} > -\frac{(J-1)(1-q)}{J\underline{B}q} \geq -\frac{(J-1)(1-q)}{Jq(-(c-q))} > \frac{(J-1)(1-q)}{Jq(c-q))} = \frac{1}{q} > 1$$

Hence, when $b < 0$, for all $m \in [0, 1]$ function $G(m)$ is decreasing. Thus, whether b is positive or negative, function $G(m)$ is decreasing on $[0, m_0^*]$.

Next, we show that $G(m_0^*) < 1$. The following inequalities

$$t > \frac{Jm^*(1-q)}{1-q-bJm_0^*} \iff f + \frac{Jbqt}{K_0(1-q)} < \frac{t-m_0^*}{K_0 m_0} \quad \text{and} \quad \frac{Jm^*(1-q)}{1-q-bJm_0^*} \in (m_0^*, 1)$$

imply that

$$G(m_0^*) = \int_{m_0^*}^{Jm_0^* \frac{1-q}{1-q-bJm_0^*}} \frac{t-m_0^*}{K_0 m_0^*}\,\mathrm{d}t + \int_{Jm_0^* \frac{1-q}{1-q-bJm_0^*}}^{1} f + \frac{Jbqt}{K_0(1-q)}\,\mathrm{d}t < \int_{m_0^*}^{1} \frac{t-m_0^*}{K_0 m_0^*}\mathrm{d}t = 1.$$

We also establish that for $b > 0 \Rightarrow G(m_0^\dagger) > 1$ and that $b <\Rightarrow G(m_0^\dagger) < 1$. Consider $G(m_0^\dagger)$ (remembering that $\theta^\dagger = Jm_0^\dagger$):

$$G(m_0^\dagger) = \int_{m_0^\dagger}^{\theta^\dagger \frac{1-q}{1-q-b\theta^\dagger}} \frac{t-m_0^\dagger}{K_0 m_0^\dagger}\,\mathrm{d}t + \int_{\theta^\dagger \frac{1-q}{1-q-b\theta^\dagger}}^{1} f + \frac{Jbqt}{K_0(1-q)}\,\mathrm{d}t$$

Observe that

$$b > 0 \Rightarrow \theta^\dagger \frac{1-q}{1-q-b\theta^\dagger} > \theta^\dagger \frac{1-q}{1-q-b\theta^\dagger} \quad \text{and} \quad t > \theta^\dagger \Rightarrow \frac{t-m_0^\dagger}{K_0 m_0^\dagger} > f$$

Hence,

$$G(m_0^\dagger) = \int_{m_0^\dagger}^{\theta^\dagger} \frac{t-m_0^\dagger}{K_0 m_0^\dagger}\,\mathrm{d}t + \int_{\theta^\dagger}^{\theta^\dagger \frac{1-q}{1-q-b\theta^\dagger}} \frac{t-m_0^\dagger}{K_0 m_0^\dagger}\,\mathrm{d}t + \int_{\theta^\dagger \frac{1-q}{1-q-b\theta^\dagger}}^{1} f + \frac{Jbqt}{K_0(1-q)}\,\mathrm{d}t$$

$$> \int_{m_0^\dagger}^{\theta^\dagger} \frac{t-m_0^\dagger}{K_0 m_0^\dagger}\,\mathrm{d}t + \int_{\theta^\dagger}^{\theta^\dagger \frac{1-q}{1-q-b\theta^\dagger}} f\,\mathrm{d}t + \int_{\theta^\dagger \frac{1-q}{1-q-b\theta^\dagger}}^{1} f\,\mathrm{d}t$$

$$= \int_{m_0^\dagger}^{\theta^\dagger} \frac{t-m_0^\dagger}{K_0 m_0^\dagger}\,\mathrm{d}t + \int_{\theta^\dagger}^{1} f\,\mathrm{d}t = 1$$

To see the last equation, consult the proof of Proposition 4.1. Hence, $b > 0 \Rightarrow G(m_0^\dagger) > 1$. The proof that $b < 0 \Rightarrow G(m_0^\dagger) < 1$ is analogous.

To prove that $b > 0 \Rightarrow m_0^\dagger < \widetilde{m}_0 < m_0^*$, observe (from above) that $b > 0 \Rightarrow G(m_0^*) < 1 < G(m_0^\dagger)$

and that $G(\cdot)$ is a strictly decreasing (and continuous) function. Hence, by the intermediate value theorem, there exists some value $\widetilde{m}_0$, such that $m_0^\dagger < \widetilde{m}_0 < m_0^*$ and $G(\widetilde{m}_0) = 1$. as required. Because $G(\cdot)$ is strictly decreasing, on the relevant interval, this value is unique.

As shown, $b < 0 \Rightarrow G(m_0^\dagger) < 1$. As $G(\cdot)$ is decreasing and continuous, if $\widetilde{m}_0$ exists for which $G(\widetilde{m}_0) = 1$, then $\widetilde{m}_0 < m^\dagger$. Existence of such $\widetilde{m}_0$ is guaranteed because the trader's expected profit is non-zero given the bounds on the agent's bias.

From the trader's incentive constraint, it is easy to see that

$$t \in [\widetilde{\theta}, 1] \Rightarrow \alpha(t) = 1 - \frac{\widetilde{m}_0(1 + K_0\phi_0(t))}{t} = 1 - \frac{J\widetilde{m}_0}{t} - \frac{Jbq\widetilde{m}_0}{1-q}$$

Because $b > 0 \Rightarrow \widetilde{m}_0 > m_0^\dagger$ the intervention probability derived above is smaller than the corresponding intervention probability in the stochastic intervention equilibrium, given by $1 - Jm_0^\dagger$. Similarly, $b < 0 \Rightarrow \widetilde{m}_0 < m_0^\dagger$ the intervention probability derived above is larger than the corresponding intervention probability in the stochastic intervention equilibrium. ■

Proof. Corollary 6.4. When the bounds on bias are satisfied, $c \in (\hat{\chi}, \overline{\chi})$ and the agent's bias is positive, the agent intervenes after a non-negligible set of sell orders and never after buy orders. Whenever the positively agent intervenes in equilibrium, $\phi_0(t) > f$, so these interventions are valuable for the policymaker. Conversely, when the agent's bias is negative, (even if it exceeds the bounds in the previous proposition), the agent intervenes in equilibrium with non-zero probability. Any such intervention is damaging to the policymaker, because for any t which triggers the negatively biased agent to intervene, $\phi_0(t) < f$. ■

APPENDIX B

In this appendix we consider a case in which PM observes a private signal in addition to the order flow before making an intervention decision and show that this is formally equivalent to privately known intervention cost.

Although it does not impact the results, we focus on the following timing: PM observes the order flow, and then observes the realization $\sigma \in \{L, H\}$ of the following signal

$$\Pr(\Sigma = L|\omega = 0) = \Pr(\Sigma = H|\omega = 1) = b$$

$$\Pr(\Sigma = H|\omega = 0) = \Pr(\Sigma = L|\omega = 1) = 1 - b$$

This signal structure is a straightforward garbling of the true state. Parameter $b \geq 1/2$ represents the probability that the signal realization is a true indicator of the state: when b is high the signal is more-likely to realize H in state one and L in state 0. We refer to the privately observed signal realization as PM's type.

Given an interim belief for PM that the state is 0, $\chi(t)$, the Bayesian update given realization σ is

$$\chi_L(t) = \frac{\chi(t)b}{\chi(t)b + (1 - \chi(t))(1 - b)}$$

$$\chi_H(t) = \frac{\chi(t)(1 - b)}{\chi(t)(1 - b) + (1 - \chi(t))b}$$

Each type of PM intervenes when her private belief exceeds the intervention cost, $\chi_i(t) \geq c$, therefore

$$\chi(t) \geq c_L \equiv \frac{c(1 - b)}{c(1 - b) + b(1 - c)} \Leftrightarrow \chi_L(t) \geq c$$

$$\chi(t) \geq c_H \equiv \frac{cb}{cb + (1 - b)(1 - c)} \Leftrightarrow \chi_H(t) \geq c$$

Sequential rationality for each type of PM, $i \in \{L, H\}$ requires

$$\alpha_i(t) = \begin{cases} 0 & \text{if} \quad \chi(t) < c_i \\ [0, 1] & \text{if} \quad \chi(t) = c_i \\ 1 & \text{if} \quad \chi(t) > c_i \end{cases}$$

Given prior $\chi(t)$ that the state is zero, the probability of observing either signal realization is

$$\Pr(\Sigma = L|\chi(t)) \equiv \gamma(\chi(t)) = \chi(t)b + (1 - \chi(t))(1 - b)$$

$$\Pr(\Sigma = H|\chi(t)) = 1 - \gamma(\chi(t)) = \chi(t)(1 - b) + (1 - \chi(t))b$$

The market maker sets the price of the asset equal to its expected payoff conditional on the

observed order:

$$\begin{aligned} p(t) &= \gamma(\chi(t))((1-\alpha_L(t))(1-\chi_L(t))+\alpha_L(t)) + (1-\gamma(\chi(t)))((1-\alpha_H(t))(1-\chi_H(t))+\alpha_H(t)) \\ &= \gamma(\chi(t))(1-\chi_L(t)+\alpha_L(t)\chi_L(t)) + (1-\gamma(\chi(t)))(1-\chi_H(t)+\alpha_H(t)\chi_H(t)) \\ &= 1-\chi(t)(1-b\alpha_L(t)+(1-b)\alpha_H(t)) \end{aligned}$$

The model therefore reduces to the model with private costs, in which the intervention "costs" are given by the expressions above, and the probability of having a low cost is b instead of γ. In order for the analysis to apply we must have $\widehat{\chi} < c_L < c_H < \overline{\chi}$. If $c \in (\widehat{\chi}, \overline{\chi})$ and b is not too large, then both "costs" satisfy these requirements.

APPENDIX C

In the main text we consider interventions that must be deployed at an interim stage, without PM knowledge of the state. In a variety of instances, it is conceivable that through some type of auditing mechanism, the PM could determine the state before deploying the intervention. We focus on the case in which the principal can choose to either audit or intervene at the interim stage. We show that through a simple transformation of parameters, the equilibrium of the model with auditing reduces to the equilibrium of the model without auditing.

Imagine that after observing orderflow, the policymaker can intervene to guarantee state one at cost c, or can conduct an audit to determine the true state at cost k; following the audit, it is sequentially rational for the policymaker to intervene in state zero. Let $\beta(t)$ represent the probability of auditing following orderflow t. The policymaker's interim payoff of intervening with probability $\alpha(t)$ and auditing with probability $\beta(t)$ is

$$(1-\alpha(t)-\beta(t))(1-\chi(t))+\alpha(t)(1-c)+\beta(t)(1-\chi(t)+\chi(t)(1-c)-k)=$$

$$1-\chi(t)+\alpha(t)(\chi(t)-c)+\beta(t)(\chi(t)(1-c)-k)$$

Comparing these alternatives gives the policymaker incentive constraint. Observe that auditing gives higher expected payoff than intervening if and only if

$$\chi(t)<1-\frac{k}{c}.$$

Meanwhile the expected payoff of intervening is positive

$$\chi(t)>c$$

and expected payoff of auditing is positive whenever

$$\chi(t)>\frac{k}{1-c}$$

Note that when $k \geq c(1-c)$, the expected payoff of auditing is strictly less than the expected payoff of intervening for any order flow.

$$\chi(t)(1-c)-k \leq \chi(t)(1-c)-c(1-c) \leq (1-c)(\chi(t)-c) < \chi(t)-c$$

Hence, when $k \geq c(1-c)$, auditing is never preferred to intervening, and is therefore irrelevant. Next, suppose that $k < c(1-c)$. In this case the above thresholds are ordered as follows:

$$\frac{k}{1-c}<c<1-\frac{k}{c}<1.$$

Hence sequentially rational behavior for the policymaker is

$$\begin{array}{rll} \alpha(t) = \beta(t) = 0 & \text{if} & \chi(t) < \frac{k}{1-c} \\ \alpha(t) = 0,\ \beta(t) = 1 & \text{if} & \frac{k}{1-c} < \chi(t) < 1 - \frac{k}{c} \\ \alpha(t) = 1,\ \beta(t) = 0 & \text{if} & \chi(t) > 1 - \frac{k}{c} \end{array}$$

Next, consider the market price as a function of the order flow:

$$p(t) = (1 - \alpha(t) - \beta(t))(1 - \chi(t)) + (\alpha(t) + \beta(t))$$

From the perspective of the market, following either an intervention or an audit, the state will be one. Indeed, an intervention (without audit) guarantees state one, while an audit ensures that an intervention will take place whenever the state would be zero otherwise. Thus, following either audit or intervention, the state is guaranteed to be one. This immediately implies the following payoff functions for the trader:

$$u_0(t) = t(1 - \chi(t))(1 - \alpha(t) - \beta(t)) \text{ and } u_1(t) = t\chi(t)(1 - \alpha(t) - \beta(t))$$

Therefore, consider a "generalized intervention" to be either a direct intervention or an audit. The probability of a generalized intervention, $\alpha'(t) \equiv \alpha(t) + \beta(t)$. The incentive constraint for the policymaker for generalized intervention requires

$$\alpha'(t) = \begin{cases} 0 & \text{if} \quad \chi(t) < \frac{k}{1-c} \\ [0,1] & \text{if} \quad \chi(t) = \frac{k}{1-c} \\ 1 & \text{if} \quad \chi(t) > \frac{k}{1-c} \end{cases}$$

which is identical to the PM incentive constraint in the basic model with a cost of generalized intervention of $k/(1-c)$. The trader incentive constraints become

$$u_0(t) = t(1 - \chi(t))(1 - \alpha'(t)) \text{ and } u_1(t) = t\chi(t)(1 - \alpha'(t))$$

also identical to their counterparts in the model without auditing. Hence, for equilibrium analysis, the model with auditing is outcome equivalent to the model without auditing, with a transformed cost.