Dynamic Regulation Design Without Payments: Timing is Everything*

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Abstract

We consider a two period model of optimal regulation of a firm subject to marginal compliance cost shocks. The regulator faces an asymmetric information problem: the firm knows current compliance costs, but the regulator does not. Both the regulator and the firm are uncertain about future costs. In our basic framework, the regulator may not offer payments to the firm; we show that the regulator can vary the strength of regulation over time to induce the firm to reveal its costs and increase welfare. In the optimal mechanism, the regulator offers stronger (weaker) regulation in the first period and weaker (stronger) regulation in the second period if the firm reports low (high) compliance costs in the first period. Low cost firms expect compliance costs to rise in the future, and thus prefer weaker regulation in the second period. High cost firms expect costs to fall in the future and thus prefer regulation which becomes more strict over time. Thus the regulator offers the low (high) cost firms *slightly* weaker (stronger) regulation in the second period in exchange for *much* stronger (weaker) regulation in the first period, thereby "timing" the regulation. If the regulator can make payments, then the optimal mechanism to some degree times the regulation as long as a positive cost of funds exists. If the cost of funds is high enough, then under the optimal mechanism the regulator will not use payments and use our timing mechanism exclusively.

1 Introduction

We consider a two period model of optimal regulation of a firm subject to marginal compliance cost shocks. The regulator faces an asymmetric information problem: the firm knows the current compliance cost, but the regulator does not. Both the regulator and the firm are uncertain about future compliance costs. Standard economic theory suggests making payments or rebates conditional on the benefits or costs of regulation. Frequently, however, regulators are prohibited or otherwise unable to make monetary payments to firms. Regulators do typically have considerable latitude on how regulations are implemented: regulators may interpret vague statutes weakly or strictly, grant waivers to delay implementation of the regulation, shape future legislation so that regulations become more strict or weak, and/or vary enforcement. We show that the regulator can vary the strength of regulation so as to induce the firm to reveal the cost of compliance and increase welfare, by explicitly characterizing the optimal regulatory policy.

In particular, in the optimal mechanism the regulator offers stronger regulation in the current period and weaker regulation in the future if a firm reports low compliance costs in the current period. Conversely, firms reporting high costs receive regulation that becomes more strict over time. We refer to our mechanism as "timing" the regulation. At first glance, timing the regulation may seem counterintuitive. Since compliance costs are convex, a policy that strengthens regulation in the current period and weakens regulation in the next period by an equal amount is more costly than an average level of regulation in both periods. However, the regulator need only offer firms reporting low costs today slightly weaker regulation in the future in exchange for much stronger regulation today to induce the low cost firms to reveal their type. This is because a firm that receives a below average compliance cost shock in the current period expects higher costs in the future. Thus, low cost firms prefer to be regulated lightly in the future, and so the regulator need only offer slightly weaker future regulation to induce low cost firms to reveal their type. Similarly, firms receiving a higher than average cost shock expect costs to fall over time, and thus prefer regulation that is initially weaker. As will be clear in the paper, timing the regulation not only improves welfare by making regulation stronger when compliance costs are low, but also improves welfare by inducing firms to reveal cost shocks.

A large literature exists which develops mechanisms that induce firms to reveal compliance cost shocks and raise welfare. Standard economic theory (see for example, Roberts and Spence 1976, Kwerel 1977) suggests the first best (full information) level of regulation may be achieved in competitive environments via hybrid tax/subsidy or permit/subsidy mechanisms. For example, Kwerel (1977) suggests a permit/subsidy mechanism whereby competitive firms first purchase permits whose total supply is determined by firms' reported compliance costs. Competitive firms can only exaggerate marginal costs by buying permits at a price above their marginal costs. The government then offers to buy back unused permits at the sale price. If firms exaggerate marginal costs, they will sell back enough permits so that the remaining supply of permits is less than the first best optimum. Firms are therefore better off reporting truthfully, as exaggerating costs leads to fewer permits and higher marginal compliance costs. Kwerel's mechanism, however, requires firms to be competitive price takers and to anticipate that other firms will truthfully reveal their costs.

Dasgupta, Hammond, and Maskin (1980), Kim and Chang (1993), Montero (2008), and Spulber (1988) achieve the first best full information level of regulation with potentially non-competitive firms via tax/subsidy or permit/subsidy mechanisms where the subsidy is linked to the residual marginal benefit of regulating each firm.[1] For example, Montero (2008) proposes an elegant mechanism whereby firms first bid for permits via a uniform-price sealed-bid auction. The regulator then rebates a fraction of the auction revenue to the firm conditional on the residual marginal benefit of regulating each firm. In this way, the benefits of regulation are transferred to the firm, and the firm's problem becomes identical to the regulator's. Firms then optimally choose the first best (full information) level of regulation. Montero's rebate depends only on the marginal damages, therefore firms choose the first best level of regulation as a dominant strategy.[2]

The degree to which each of these mechanisms are used, or could be used, in practice varies. Mechanisms that rely on perfect competition rule out a host of highly regulated industries, such as electricity. Similarly, firms are not typically asked to report each other's costs since cost information is likely private (Wiggins and Libecap 1985). However, Montero's mechanism is consistent with some regulations.[3]

Nearly all mechanisms require that the regulator extract payments from the firm and then credibly commit to make rebates back to the firm.[4] If the regulator has access to a revenue

[1] In Spulber (1988), the regulator cannot always achieve first best since the firm's total tax payments constrain subsidies in the form of tax credits.

[2] Other mechanisms (Varian 1994, Duggan and Roberts 2002) rely on the assumption that firms know each other's marginal costs. Given this unlikely assumption, however, the regulator can simply require firm's to report all other firm's costs, and punish firms if the results do not agree (Cremer and McLean 1988).

[3] NO_x permit allocations in Sweden have a rebate based on market share (Gersbach and Requae 2004). In the US, the EPA holds back 2.8% of grandfathered SO_2 allowances from firms, and then auctions them, rebating the revenue back to the firms (Joskow and Schmalensee 1998).

[4] The exception are those mechanisms requiring firms to know and report each other's costs. Kwerel's mechanism does not use payments to the firm in equilibrium.

stream and legal authority to make payments from that revenue stream, then payments are plausible. For example, sulfur dioxide permit auction revenue provides a plausible funding source and the EPA has the authority to design the auction with a rebate. Mason and Plantinga (2010) also proposes a plausible mechanism whereby payments for carbon offsets are subject to the regulator taking back some payments via a clawback.[5]

Most regulatory environments, however, do not feature payments from the firm to the regulator, nor a legal framework whereby regulators subsidize firms that report low compliance costs. All command-and-control regulation, for example, by definition involves no payments or subsidies. Similarly, permit based regulation in which permits are grandfathered or otherwise freely allocated do not result in truthful information revelation under most of the above mechanisms.[6] Even if freely allocated permits are interpreted as the end result of a payment and a rebate, then initial allocations of permits based on historical pollution emissions is inconsistent with the idea of rewarding firms that report low compliance costs with lower net payments.

In contrast, regulators typically have considerable discretion over the interpretation of vague statutes, the degree to which existing regulations are enforced, granting waivers,[7] the ability to shape future legislation through cost studies, and other decisions affecting the strength of regulation. For example, "New Source Review" regulation requires that modifications to a plant which causes a "significant increase" in a regulated pollutant receive an EPA review that typically forces the plant to adopt the best available pollution control technology ("routine maintenance" is excepted). Both "routine maintenance" and "significant increase" are terms that are not precisely defined, and indeed interpretations of this statute by the EPA has varied over time (Stavins 2006, footnote 90).

In New Source Review and similar command and control legislation, the regulator has no discretion to set up a permit or tax/subsidy mechanism. Our results show that the regulator can improve welfare by timing the regulation: offering firms a choice of regulation that becomes either stronger or weaker over time.

Although our paper is primarily normative, in practice regulators sometimes offer firms a choice of regulation that either becomes stronger or weaker over time. Joskow and Schmalensee (1998) provide a detailed examination of the rules of sulfur dioxide permit trading system created by the 1990 Clean Air Act. One provision gives utilities that install

[5]A legislator may have the freedom to design a bill with a payment of an initial allocation of permits. The allocation would have to be tied to the residual marginal benefits of regulating each firm, however.

[6]See Montero (2008) for a formal argument. Montero's mechanism is an exception.

[7]The provision of the Patient Protection and Affordable Care Act phasing out annual payment limits has been temporarily waived for 729 companies (Department of Health and Human Services 2011).

scrubbers future “bonus” permit allocations. Firms that install scrubbers clearly face more costly regulation up front, and weaker regulation in the future, since at a minimum their allocation of permits rises over time. Conversely, by declining the option, firms save the up front cost of scrubbers, but do not gain bonus permits later. Thus declining the option results in regulation which becomes stronger over time.[8]

Even if the regulation is such that payments to and from the firm are possible, the absence of lump sum taxes means that payments to the firm could instead be used to reduce labor or other distortionary taxes (Bovenberg and Goulder 1996). As shown by Montero (2008), the regulator using payments then faces a tradeoff between information revelation and the distortionary cost of government funds. Therefore, with a distortionary cost of funds, payment-based mechanisms no longer achieve the first best. Our mechanism, which trades off current and future distortions, also does not achieve the first best. Nonetheless, we show that with any positive cost of funds, the optimal regulation involves some degree of timing, even when payments are available. Further, we derive a cutoff cost of funds such that, if the cost of funds is higher than the cutoff, the optimal regulation policy does not use payments at all and instead uses our timing mechanism exclusively.[9]

The timing mechanism takes advantage of firm uncertainty over future cost shocks. Many authors consider time varying compliance cost shocks which fit naturally into our framework. Newell and Pizer (2003) and Karp and Zhang (2005) evaluate tax and permit based regulation with time-varying cost shocks. Kelly (2005) evaluates tax and permit based regulation when firms receive productivity shocks. Heutel (2009) and Fischer and Springborn (2011) evaluate tax and permit based regulation for climate change when firms are subject to productivity shocks which follow an autoregressive process. Productivity shocks fit naturally into our framework since firms know current, but not future, shocks. Our paper extends this literature by deriving the optimal dynamic regulation with dependent cost shocks. A number of other natural interpretations of time varying costs shocks exist. For example, input prices vary randomly over time and future cost saving innovations are uncertain. In section 4, we show that our mechanism extends to general cost shock processes, including correlated shocks,

[8]The Clean Air Act allows pollution permit “banking” (Ellerman and Montero 2007), which also gives firms some control over the strength of regulation over time. However, we show in section 2.2 that our timing mechanism yields higher welfare than permit banking, since the timing mechanism induces firms to reveal cost shocks, while banking does not.

[9]Our result should not be confused with the dynamic moral hazard literature, in which it is optimal for the principal to use both payments and continuation values to reward agents. Here, the gains to the principal from using the continuation value as compensation are not driven by “payment smoothing.” In our mechanism, payments in the form of weaker regulation are not perfect substitutes across time to the agent, which the principal exploits to gain information.

such as productivity shocks.

Our mechanism relies on commitment: the ability of the regulator to commit to weak (strong) regulation in the future for firms that reports low (high) costs today.[10] A number of papers (e.g. Freixas, Guesnerie, and Tirole 1985, Yao 1988) study models in which marginal costs are fixed and not subject to shocks. In this case, the regulator who learns a firm has permanently low costs has an incentive to renege on a commitment to weak regulation and instead impose the optimal regulation given the known low compliance costs in the second period (the "ratchet effect"). In contrast, the incentive to renege is relatively minor in our mechanism. If cost shocks are iid, then the regulator who learns the firm has low costs in period one has only prior information about the firm's costs in period two. The regulator thus does not desire to ratchet up the regulation to the optimal level given known low compliance costs in period two, but instead only desires to strengthen the regulation to the optimal level given the prior.

One way to solve the commitment problem is through contracts. Baron and Besanko (1987) argue that relationships between regulators and public utilities are in practice characterized by contracts whereby the regulator agrees to give the firm a minimum ("fair") profit, and the firm agrees not to withdraw from the relationship as long as the regulator maintains the minimum profit.[11] In addition, if the discount factor is sufficiently high and the regulator and firm have repeated interactions, then commitment is possible (Yao 1988). For this reason, we have in mind repeated interactions between a career regulator and firm, rather than a more temporary political appointee.[12]

Many regulations involve repeated long run interactions between the firm and regulator (Baron and Besanko 1987). Indeed, many studies of such long run relationships argue the result is regulatory capture: because the regulator and the firm have repeated interactions, the regulator is more responsive to the firm's needs and regulation tends to be weak (Besley and Coate 2003).[13] Our model provides an alternative explanation to regulatory capture.

[10] All permit-subsidy schemes require commitment at some level, since otherwise the regulator would renege on the subsidy.

[11] Conversely, Hahn (1989) notes that some permit regulations are written specifically so that the regulator may devalue existing permits without compensation. For example, the sulfur dioxide permit system legislation states that the EPA may abandon the permit system without compensation at any time. However, Joskow and Schmalensee (1998, footnote 4) note that the EPA issued permits several years ahead as a commitment device, making it politically difficult to renege (indeed, the sulfur permit system has now been in place for 20 years and the EPA has not reneged).

[12] Guasch, Laffont, and Straub (2008) show the probability of contract renegotiation between regulators and firms in Latin America decreases significantly when a regulatory agency negotiates the original contract. Besley and Coate (2003) show firms extract more rents from elected than appointed regulators.

[13] Laffont and Tirole (1991) shows that politicians may weaken the power of regulators if regulatory capture

What looks like lax regulation may simply be the regulator following through on a commitment. These two hypothesis can be resolved empirically, due to the model's testable predicted relationship between past and future regulation for different types of firms.

Although the regulator offers weaker regulation in the second period to low cost firms, our mechanism maximizes welfare and is thus preferred by households. Furthermore, we show in section 2.4 that both high and low cost firms weakly prefer our mechanism to the benchmark level of regulation imposed when the regulator has only prior information about firm costs. In addition, we describe in section 7 how our mechanism may be implemented using waivers or credits, which are commonly used in environmental regulation. Therefore, our mechanism is relatively straightforward to implement in practice.[14]

Our basic framework assumes the firm has access only to a static method of regulatory compliance. Suppose the firm can make a dynamic capital investment that complies with the regulation (e.g. install scrubbers) at lower cost than the static method (e.g. switching from high to low sulfur coal). A natural concern is that our timing mechanism may cause firms to under invest in cost-saving capital. We show in section 6 that in fact firms undertake the socially optimal level of investment, and our mechanism is unchanged except that firms are now either above and then below a baseline increasing trend in regulatory stringency or the reverse, depending on the cost shock.[15]

Section 2 solves for the optimal mechanism in the basic model with one firm and determines the properties of the mechanism. Section 3 does the same when the government has a cost of funds. Section 4 characterizes the optimal regulatory policy for more general marginal cost processes, including correlated cost shocks. Section 5 extends the mechanism to n firms and section 6 considers dynamic investment and declining costs.

2 Model: Two period problem with a single firm

Consider a regulator imposing a level of regulation q on a firm whose compliance costs are unknown. The strength of regulation is increasing in q; $q = 0$ represents an unregulated firm. The function $B(q)$ specifies the benefits of regulation, which we assume are increasing and concave. The regulator seeks to maximize expected welfare w over two periods, $t = 1, 2$.

is likely.

[14] Our mechanism, however, does require prior cost information, whereas Montero's requires only the marginal damages.

[15] Yao (1988) points out that firms may under invest in cost saving R&D if the regulator cannot commit to strict regulation in the future.

Let δ be the discount factor, then:

$$\mathcal{W} = \mathrm{E}\left[w\left(q_1, \pi_1\right)\right] + \delta \mathrm{E}\left[w\left(q_2, \pi_2\right)\right], \text{ where} \tag{2.1}$$

$$w\left(q, \pi\right) = B\left(q\right) - C\left(q, \pi\right). \tag{2.2}$$

Here the compliance cost of the regulation to the firm is $C\left(q, \pi\right)$, which is increasing and weakly convex in q and increasing in the cost shock π. Throughout the paper, subscripts on functions denote partial derivatives. We assume π is unknown to the regulator in both periods. The firm knows the cost shock in the first period, and learns π_2 at the beginning of period two. The cost shock follows an iid Bernoulli process: $\pi_t = \pi_L$ with probability γ and π_H otherwise, for $t = 1, 2$. Let $\pi_L < \pi_H$, so π_L indicates low compliance costs.[16] We assume $C_q\left(0, \pi_H\right) < B_q\left(0\right)$, so that some regulation is optimal even if compliance costs are high.

The firm incurs the costs, but not the benefits of regulation. Firm profits are negatively affected by compliance costs, and the expected change in firm profits arising from the regulation is:

$$w_f\left(q_1, q_2, \pi_1\right) = -C\left(q_1, \pi_1\right) - \delta \mathrm{E}\left[C\left(q_2, \pi\right)\right]. \tag{2.3}$$

An example is environmental regulation. If E is emissions, with uncontrolled emissions equal to E_0, then $q = E_0 - E$ can be interpreted as regulation implementing an emissions standard of E or a supply of E emissions permits. Similarly, $C\left(E_0 - E, \pi\right)$ is the cost of reducing emissions and $D = D_0 - B\left(E_0 - E\right)$ are the convex damages from emissions.

2.1 Two Period Contract

The regulator requires the firm to report $\hat{\pi}$ in the first period. The firm may report either low ($\hat{\pi} = \pi_L$) or high ($\hat{\pi} = \pi_H$) compliance costs. The regulator commits to a set of policies $q\left(\hat{\pi}\right)$, based on the firm's report. If the firm reports low compliance costs, then the regulator implements q_{1L} in the first period and q_{2L} in the second period, whereas if the firm reports high compliance costs, the regulator implements $\{q_{1H}, q_{2H}\}$. The regulator cannot condition regulation in period two on the firm's report in period two, because the firm would always report the type with the smallest regulation costs. Clearly a firm with low compliance costs has an incentive to report high compliance costs to induce the regulator to implement weaker regulation. We assume that if the firm is indifferent between reporting truthfully or not, the

[16] We relax the iid assumption in section 4.

firm reports truthfully.

Incentive compatibility requires that truthful reporting maximizes profits for both types of firms.

$$w_f\left(q_{1L}, q_{2L}, \pi_L\right) \geq w_f\left(q_{1H}, q_{2H}, \pi_L\right), \tag{2.4}$$

$$w_f\left(q_{1H}, q_{2H}, \pi_H\right) \geq w_f\left(q_{1L}, q_{2L}, \pi_H\right). \tag{2.5}$$

Our strategy is to compute the regulations which maximizes welfare subject to the constraint that the low cost firm not misrepresent itself as a high cost firm. We will then verify that, under mild conditions, the solution implies a high cost firm will not wish to claim costs are low. That is, constraint (2.5) is not binding at the solution of the relaxed problem of maximizing (2.1) subject to (2.4). Therefore, the Lagrangian for the relaxed problem is:

$$\begin{aligned} \mathcal{L} \;\; = \;\; & \gamma \cdot \Big[w\left(q_{1L}, \pi_L\right) + \delta \mathrm{E}\left[w\left(q_{2L}, \pi\right)\right]\Big] + (1-\gamma) \cdot \Big[w\left(q_{1H}, \pi_H\right) + \delta \mathrm{E}\left[w\left(q_{2H}, \pi\right)\right]\Big] + \\ & \lambda \cdot \Big[w_f\left(q_{1L}, q_{2L}, \pi_L\right) - w_f\left(q_{1H}, q_{2H}, \pi_L\right)\Big] \end{aligned} \tag{2.6}$$

Because the mechanism is incentive compatible, the objective function is formulated anticipating truth telling on the part of the firm.

The first order conditions are:

$$\frac{1}{1+\frac{\lambda}{\gamma}} B_q\left(q_{1L}\right) = C_q\left(q_{1L}, \pi_L\right) \tag{2.7}$$

$$\frac{1}{1-\frac{\lambda}{1-\gamma}\frac{C_q\left(q_{1H}, \pi_L\right)}{C_q\left(q_{1H}, \pi_H\right)}} B_q\left(q_{1H}\right) = C_q\left(q_{1H}, \pi_H\right) \tag{2.8}$$

$$\frac{1}{1+\frac{\lambda}{\gamma}} B_q\left(q_{2L}\right) = \mathrm{E}\left[C_q\left(q_{2L}, \pi\right)\right] \tag{2.9}$$

$$\frac{1}{1-\frac{\lambda}{1-\gamma}} B_q\left(q_{2H}\right) = \mathrm{E}\left[C_q\left(q_{2H}, \pi\right)\right] \tag{2.10}$$

$$C\left(q_{1H}, \pi_L\right) - C\left(q_{1L}, \pi_L\right) + \delta\left(\mathrm{E}\left[C\left(q_{2H}, \pi\right)\right] - \mathrm{E}\left[C\left(q_{2L}, \pi\right)\right]\right) = 0 \tag{2.11}$$

Equations (2.7)-(2.11) define the optimal regulatory structure. Since $\lambda > 0$, the regulator is forced to move marginal benefits away from marginal costs in order to induce truth telling.

To complete the solution we must show constraint (2.5) is satisfied:

PROPOSITION 1 *Suppose C is super-modular in $[q, \pi]$. Then the solution to problem (2.6) satisfies condition (2.5).*

All proofs are in the appendix. A twice differentiable function is super modular if and only if the cross partial derivative is positive. Thus we are assuming $C_{q\pi} > 0$, or that π is a positive shock to the firm's marginal costs, which is a standard assumption.

2.2 Properties of the Timing Mechanism

We first derive some properties of the solution, and then use these properties to develop an intuition of the results. Let

$$R(q, \pi_H, \pi_L) \equiv \frac{C_q(q, \pi_H)}{C_q(q, \pi_L)}, \tag{2.12}$$

define the spread between high and low marginal costs. Two natural benchmarks are the full information (first best) and prior information regulation policies. The full information regulations, $\{q_L^*, q_H^*\}$, equate the marginal benefits of regulation with the realized marginal costs:

$$B_q(q_i^*) = C_q(q_i^*, \pi_i), \quad i = L, H. \tag{2.13}$$

The prior information regulation policy, $\bar{q}$, equates the marginal benefits of regulation with the expected marginal cost:

$$B_q(\bar{q}) = \mathrm{E}[C_q(\bar{q}, \pi)]. \tag{2.14}$$

Clearly, $q_H^* < \bar{q} < q_L^*$. Proposition 2 describes the relationship between the optimal dynamic mechanism and these benchmarks.

PROPOSITION 2 *The solution to the two period problem has the following properties:*

2.1. $q_H^* < q_{1H} < \bar{q}$ *and* $q_{1L} < q_L^*$, *and if R is constant in q, then* $\bar{q} < q_{1L}$.

2.2. $q_{2L} < \bar{q} < q_{2H}$.

2.3. $0 \leq \lambda \leq 1 - \gamma$.

Proposition 2 indicates that the optimal second best period one regulation levels lie between the no information regulation levels and their full information counterparts. Thus, first period welfare is higher in the mechanism than under no information regardless of firm type. Proposition 2.2 specifies the incentive cost of the first period welfare gains. In the second period, the ex ante optimal level of regulation for both types is $\bar{q}$, but the low type receives $q_{2L} < \bar{q}$ and the high type receives $q_{2H} > \bar{q}$. These distortions provide the low cost firm with incentives to accept stronger regulation in the first period. However, the low cost firm expects higher costs in period two, and therefore values weaker regulation more in period two. Therefore, the welfare cost of the optimal second period distortions is smaller than the first period gains. Section 2.3 gives a more detailed intuition.

In period two, the regulator has only prior information about costs and prefers $\bar{q}$, regardless of what the firm reported in the first period. The incentive to renege is therefore more moderate here than the typical ratchet effect. If able to renege, the regulator would prefer to set $q = \bar{q}$ rather than the stronger $q = q_L^*$, as in a model with a fixed type.

2.3 Graphical Intuition

Figure 1 illustrates the intuition when $\delta = 1$ for clarity.[17]

[17] A low discount factor means the low cost firm requires more compensation in the form of weaker regulation in the second period. However, the regulator now discounts the welfare loss of weaker regulation for the low cost type in the second period more relative to the gains in the first period. So the qualitative properties of the mechanism do not depend on δ.

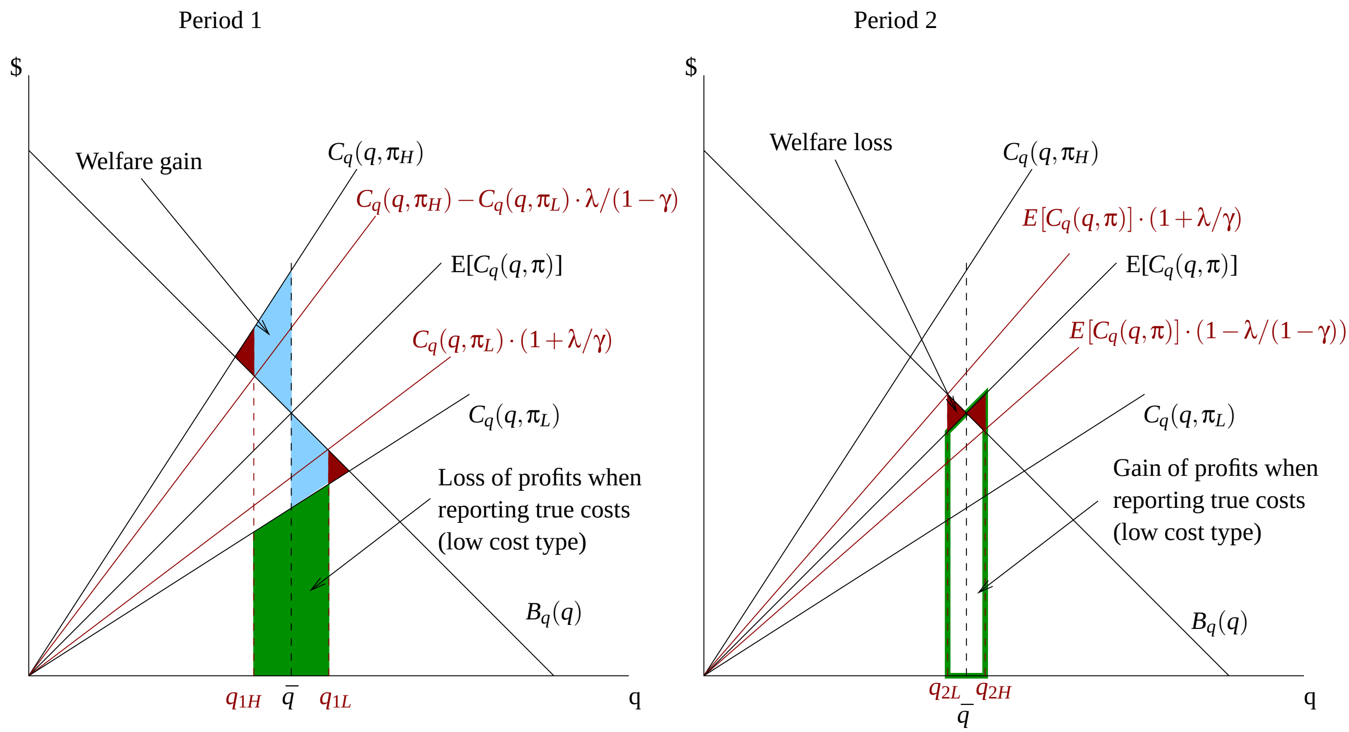

Figure 1: Intuition for the regulator's problem.

The typical welfare loss (e.g. the loss with an emissions standard or tradeable emissions permits) when the regulator has only prior information about firm costs in period one equals the average (weighted by γ) of the red and blue areas. Suppose the regulator sets the level of regulation at $\bar{q}$, which sets marginal benefits equal to expected marginal costs. With probability γ, the firm has low marginal costs, and thus marginal benefits exceed marginal costs, creating a welfare loss equal to the area of the bottom blue and red areas. With probability $1-\gamma$, marginal costs exceed marginal benefits, and welfare loss is the area of the top blue and red areas. Now suppose the regulator imposes q_{1L} or q_{1H} depending on the firm's report. Given truthful reporting, expected welfare loss falls to the weighted average of the area of the two red triangles. However, a firm with low costs now gets higher profits by claiming to be the high cost type. The gain in profits for a low cost firm claiming to be the high cost type in the first period is the green polygon. Thus the regulator must increase the return to reporting low costs in the second period to offset the loss in profits in the first period. Further, the marginal loss to the low cost firm from a marginal increase in q_{1L} is $C_q(q_{1L}, \pi_L)$.

Looking forward to the second period, all firms expect marginal costs equal to $\mathrm{E}[C_q(q,\pi)]$, since the actual period two cost shock is unknown in period one. Thus the low cost firm expects costs to rise in period two. But then the low cost firm values lenient regulation more in the second period more than in the first period (the opposite is true for the high cost firm, which is why the high cost firm is not motivated to report low costs). Conversely, the regulator has no knowledge of firm costs in either period and is thus indifferent as to which period has the stronger regulation. By setting $q_{2L} < q_{2H}$, the low cost firm expects to gain profits in the second period equal to the area of the green polygon by reporting truthfully. Thus the regulator must set the area of the two green polygons in Figure 1 to be equal in order to induce truthful reporting. A marginal decrease in q_{2L} raises expected profits in the second period by $\mathrm{E}\left[C_q\left(q_{2L},\pi\right)\right]$, whereas a marginal decrease in q_{1L} raises expected profits by only $c_q\left(q_{1L},\pi_L\right)$. Therefore, the firm is willing to report truthfully even though the difference in period one regulation, $q_{1L}-q_{1H}$, is larger than the difference in period 2. The regulator can therefore achieve welfare gains in period one at a smaller cost of welfare loss in period two (the red triangles). The regulator continues to raise $q_{1L}-q_{1H}$ and $q_{2H}-q_{2L}$ until the weighted average of the areas of the red triangles in period one equal the weighted average of the red triangles in period two. At this point, welfare gains in period one are small and welfare losses in period two are high enough to offset the fact that the regulator need only decrease q_{2L} by $\delta\rho$ in order to achieve a marginal increase in q_{1L}.

2.4 Quadratic Example

To further illustrate the mechanism, consider the following quadratic example:

$$B\left(q\right)=\theta q-\frac{1}{2}q^2, \tag{2.15}$$

$$C\left(q\right)=\pi q. \tag{2.16}$$

Solving equations (2.7)-(2.11) given the functions (2.15)-(2.16) results in:

$$
\begin{aligned}
q_{1L} &= \bar{q} + (1-\gamma)\,\Delta\pi \frac{\delta\rho^2}{1+\delta\rho^2}, &(2.17)\\
q_{1H} &= \bar{q} - \gamma\Delta\pi \frac{\delta\rho^2}{1+\delta\rho^2}, &(2.18)\\
q_{2L} &= \bar{q} - (1-\gamma)\,\Delta\pi \frac{\rho}{1+\delta\rho^2}, &(2.19)\\
q_{2H} &= \bar{q} + \gamma\Delta\pi \frac{\rho}{1+\delta\rho^2}, &(2.20)\\
\lambda &= \frac{\gamma(\rho-1)}{1+\delta\rho^2}, &(2.21)\\
\Delta\pi &\equiv \pi_H - \pi_L, \quad \rho \equiv \frac{\bar{\pi}}{\pi_L}, \quad \bar{q} = \theta - \bar{\pi}. &(2.22)
\end{aligned}
$$

The degree to which the regulations differ from the no-information benchmark, $\bar{q}$, depends critically on the variability of π in two different ways. First, an increase in the cost difference $\Delta\pi$ moves all four optimal regulations away from $\bar{q}$. As the difference increases, the returns to implementing the mechanism increase. When the gap is large implementing $\bar{q}$ results in large welfare losses, because q_L^* and q_H^* are far from $\bar{q}$. Therefore, the regulator moves q_{1L} and q_{1H} away from $\bar{q}$, and thus q_{2L} and q_{2H} also move away from $\bar{q}$ to maintain incentive compatibility. Second, for large values of ρ,[18] the low cost firm anticipates much higher costs in the second period, and so the regulator need only offer a relatively small weakening of second period regulation to induce the low cost firm to report truthfully. Thus q_{1L} and q_{1H} move away from $\bar{q}$ towards their first best levels for large ρ, as it is cheaper to implement the mechanism. The effect of ρ on q_{2L} and q_{2H} is ambiguous. On one hand the regulator needs to weaken (strengthen) the second period regulation less for the low (high) cost firm to satisfy the incentive constraint, which moves q_{2L} and q_{2H} towards $\bar{q}$. But because the mechanism is overall cheaper to implement, the regulator widens the spread in the first period, which tends to widen the spread in the second period.

The variance of the cost shock also increases the value of the information gained through the mechanism. With no information, the regulator adopts $\bar{q}$ each period and expected welfare is:

$$
\mathcal{W}(\bar{q},\bar{q}) = \frac{1}{2}(1+\delta)\,\bar{q}^2. \tag{2.23}
$$

[18] Here for convenience we use ρ rather than R. Note that $\rho = \gamma + (1-\gamma)\,R$ so the intuition for ρ and R are the same.

Whereas, using (2.6), the expected welfare of the optimal mechanism is:

$$\mathcal{L} = \frac{1}{2}(1+\delta)\bar{q}^2 + \frac{1}{2}\gamma(1-\gamma)\left(\frac{\delta\rho^2}{1+\delta\rho^2}\right)\Delta\pi^2. \tag{2.24}$$

Thus the term:

$$\frac{1}{2}\gamma(1-\gamma)\left(\frac{\delta\rho^2}{1+\delta\rho^2}\right)\Delta\pi^2 \tag{2.25}$$

represents the regulator's gains from using the optimal mechanism. With quadratic preferences, $\mathrm{E}[q_1] = \mathrm{E}[q_2] = \bar{q}$, so the welfare gains arise from reducing compliance costs, rather than higher benefits of regulation. Welfare is directly proportional to the variance of the prior, $\gamma(1-\gamma)\Delta\pi^2$, since an increase in the prior variance directly increases the returns to acquiring information about firm type. Welfare is also increasing in ρ, since an increase in ρ makes the mechanism cheaper to implement.

By reporting a low cost shock instead of a high cost shock in period one, the firm expects profits to rise by

$$(1-\gamma)\frac{\rho^2\Delta\pi^2}{(1+\delta\rho^2)(\rho-1)}. \tag{2.26}$$

in the second period. As $\Delta\pi$ increases, the regulator must increase second period compensation to the low cost firm, as the regulator moves q_{1L} and q_{1H} away from $\bar{q}$. The effect of an increase in ρ on compensation to the low cost firm depends on whether the regulator moves q_{1L} and q_{1H} far enough away from $\bar{q}$ so that overall compensation to the low cost firm must increase despite the decrease in the cost of the mechanism.

Consider finally expected firm profits, which from (2.3) satisfy:

$$w_f(q_{1L}, q_{2L}, \pi_L) = w_f(\bar{q}, \bar{q}, \pi_L), \tag{2.27}$$

$$w_f(q_{1H}, q_{2H}, \pi_H) > w_f(\bar{q}, \bar{q}, \pi_H). \tag{2.28}$$

Firms of either type weakly prefer the optimal mechanism over $\bar{q}$. Therefore, the industry will support a transition from the prior information policy to the optimal mechanism either before or after learning the cost shock.

2.5 Other Mechanisms

Other regulation systems also give firms some discretion to choose the strength of regulation over time, but do not reveal firm costs, and thus result in lower welfare than our mechanism. Consider, for example, pollution permit "banking." In our framework, permit banking regulation requires that firms implement a minimum lifetime level of regulation $\hat{q}$, but gives firms the discretion to choose the level of regulation in each period.[19] The firm's problem is then:

$$\max_{q_{1i},q_{2i}} w_f\left(q_{1i}, q_{2i}, \pi_i\right), \quad \text{s.t. } q_{1i} + q_{2i} \geq \hat{q}, \ i = L, H. \tag{2.29}$$

The firm first order condition is:

$$C_q\left(q_{1i}, \pi_i\right) = \mathrm{E}\left[C_q\left(q_{2i}, \pi\right)\right], \ i = L, H. \tag{2.30}$$

Banking regulation equalizes expected marginal costs across time for both firms. Equations (2.7) and (2.9) imply (2.30) holds for the low type, but (2.10) and (2.8) and (2.10) imply (2.30) does not hold for the high type. Therefore, the optimal timing mechanism equalizes marginal costs over time for the low type only. Since costs are convex, both the firm and the regulator desire to equalize marginal costs over time. However, in the optimal timing mechanism marginal costs are not equal for the high type to discourage the low type from claiming to have high costs.[20] Information revelation allows the regulator to better tailor regulation to firm costs. The welfare gains resulting from information revelation outweigh the costs of not equalizing marginal costs over time for the high type, so the timing mechanism yields higher welfare.[21]

In other mechanisms (Montero 2008) the firm receives a monetary payment that is increasing in the strength of regulation. Monetary payments induce the firm to reveal costs, despite the stronger regulation that the low cost firm must endure by reporting low costs. The monetary payment transfers the benefits of regulation to the firm, inducing the firm to choose the first best optimal regulation. Here, the regulator transfers benefits in the form of weaker future regulation to the low cost firm, and the reverse to the firm reporting high costs in the first period. Varying the strength of regulation in the second period to satisfy the

[19] For example, using the notation in the paragraph after equation (2.3), the firm receives $E = E_0 - \frac{1}{2}\hat{q}$ permits in each period and may both save and borrow permits.

[20] The high cost type does not want to claim low costs, which is why the regulator is free to equalize marginal costs over time for the low cost type.

[21] The optimal banking regulation system could be implemented by adding (2.30) as an extra constraint to problem (2.6). Since the unconstrained problem does not equalize the marginal costs, the constrained problem must result in lower welfare.

incentive constraint creates distortions, so the first best level of regulation does not result.

To contrast our results with the literature using monetary payments, we must monetize the value of lenient regulation. Suppose we adopt the proposal of Montero (2008), and specify the following mechanism:

1. The firm reports its type, and specifies a demand for subsidies $\hat{P}(q,\pi)$. The firm must submit demand schedules for both periods in the first period.
2. The regulator sets the subsidy rate to solve $\hat{P}(q,\pi) = B_q(q)$.
3. The firm chooses a level of regulation for both periods prior to learning the second period cost shock, and pays a fraction $\alpha(q,\pi)$ of the total value of the subsidies back to the regulator.

Notice that the subsidy is independent of the report, instead the regulator uses the rebate α to induce truthful reporting. The low cost firm's problem is thus:

$$\begin{aligned} \max_{q_{1L},q_{2L}} \quad & \Big\{(1-\alpha(q_{1L},\pi_L))\,\hat{P}(q_{1L},\pi)\,q_{1L} - C(q_{1L},\pi_L) + \\ & \delta\Big((1-\alpha(q_{2L},\pi_L))\,\hat{P}(q_{2L},\pi)\,q_{2L} - \mathrm{E}\left[C(q_{2L},\pi)\right]\Big)\Big\}. \end{aligned} \tag{2.31}$$

Or:

$$\begin{aligned} \max_{q_{1L},q_{2L}} \quad & \{(1-\alpha(q_{1L},\pi_L))\,B_q(q_{1L})\,q_{1L} - C(q_{1L},\pi_L) + \\ & \delta((1-\alpha(q_{2L},\pi_L))\,B_q(q_{2L})\,q_{2L} - \mathrm{E}\left[C(q_{2L},\pi)\right])\}. \end{aligned} \tag{2.32}$$

Let:

$$f_L \equiv \frac{1}{1+\frac{\lambda}{\gamma}}, \tag{2.33}$$

$$f_{1H} \equiv \frac{1}{1-\frac{\lambda}{1-\gamma}\frac{1}{R}}, \quad f_{2H} \equiv \frac{1}{1-\frac{\lambda}{1-\gamma}}. \tag{2.34}$$

Next, following Montero, we specify:

$$\alpha(q_{1L},\pi_L) = 1 - f_L\frac{B(q)}{B_q(q)\,q}, \tag{2.35}$$

which results in the firm's problem simplifying to:

$$\max_{q_{1L},q_{2L}} \left\{ f_L B\left(q_{1L}\right) - C\left(q_{1L}, \pi_L\right) + \delta\left(f_L B\left(q_{2L}\right) - \mathrm{E}\left[C\left(q_{2L}, \pi_L\right)\right]\right)\right\}. \tag{2.36}$$

The above problem, along with the corresponding rebates if the firm reports high costs, generates a solution identical to equations (2.7)-(2.11).

In general the regulator transfers a fraction of the benefits of regulation to the firm in each period. The fraction is $f_L < 1$ if the firm reports low costs, and $f_{1H} > 1$ and $f_{2H} > 1$ if the firm reports high costs. The regulator cannot transfer the full benefits of regulation to the firm without violating the truth-telling constraint. The fraction in general depends on λ, the shadow price of the incentive constraint, which is endogenous, but bounded by proposition (2.3).[22]

3 Cost of Funds

Suppose now the regulator may offer payments to the firm conditional on the firm's reported type, but such payments are costly for the regulator to offer. Such a cost of funds arises naturally if lump sum taxes are not possible, and the regulator/government obtains funds via distortionary taxation, for example (Bovenberg and Goulder 1996).

The regulator can use the payments to extract information about the firm's costs in the second period. Therefore, the regulator requires the firm to give a cost report in each period, $\{\hat{\pi}_1, \hat{\pi}_2\}$. The regulator may condition regulation in period two on both reports. Let $q_{2ij} \equiv q\left(\hat{\pi}_i, \hat{\pi}_j\right)$ denote the level of regulation in period two if the firm reported type i in period 1 and j in period 2. Since the timing of the problem is such that the regulator implements the regulation in period one before the firm learns the cost shock in the second period, the regulation in period one depends only on the period one report. Similarly, let $t_{1i} \equiv t_1\left(\hat{\pi}_i\right)$ be the first period payment from the regulator to the firm if the firm reports type i in period one and let $t_{2ij} \equiv t_2\left(\hat{\pi}_i, \hat{\pi}_j\right)$ be the payment in the second period if the firm reports type i in period one and type j in period 2.

Incentive compatibility requires that a low cost firm in period two receive profits from

[22] Note that Montero's mechanism requires no prior cost information to set the subsidy or rebate. That is, with $f_L = 1$ in equation (2.35), both the price and the rebate depend only on the benefit function. The firm need not report costs, since the firm essentially chooses the level of regulation. In general f_L depends on prior cost information through λ and γ. So if monetary payments are not possible, the regulator requires more prior information.

reporting low costs which are not less than profits from reporting high costs:

$$-c\left(q_{2iL},\pi_L\right)+t_{2iL}\geq -c\left(q_{2iH},\pi_L\right)+t_{2iH}\ ,\ i=L,H. \tag{3.1}$$

Similarly, a high cost firm in period two must receive higher profits from reporting high costs:

$$-c\left(q_{2iH},\pi_H\right)+t_{2iH}\geq -c\left(q_{2iL},\pi_H\right)+t_{2iL}\ ,\ i=L,H. \tag{3.2}$$

The first period incentive compatibility constraints are:

$$\begin{aligned} &w_f\left(q_{1L},q_2\left(\pi_L,\pi\right),\pi_L\right)+t_{1L}+\delta\mathrm{E}\left(t_2\left(\pi_L,\pi\right)\right)\geq \\ &\quad w_f\left(q_{1H},q_2\left(\pi_H,\pi\right),\pi_L\right)+t_{1H}+\delta\mathrm{E}\left(t_2\left(\pi_H,\pi\right)\right), \end{aligned} \tag{3.3}$$

$$\begin{aligned} &w_f\left(q_{1H},q_2\left(\pi_H,\pi\right),\pi_H\right)+t_{1H}+\delta\mathrm{E}\left(t_2\left(\pi_H,\pi\right)\right)\geq \\ &\quad w_f\left(q_{1L},q_2\left(\pi_L,\pi\right),\pi_H\right)+t_{1L}+\delta\mathrm{E}\left(t_2\left(\pi_L,\pi\right)\right). \end{aligned} \tag{3.4}$$

It is well known (Montero 2008) that the regulator can achieve the first best allocation by imposing a sufficiently large lump sum tax on the firm. The regulator need only make the difference in total payments from the firm to the regulator equal to the benefits of regulation. Because the firm pays the regulator regardless of the firm's choice, the cost of funds is irrelevant. Therefore, similar to Montero (2008), we impose a restriction that payments from the regulator to the firm not be too small. In particular, we impose that lifetime payments are positive, regardless of the firm's reports:[23]

$$t_{1i}+\delta t_{2ij}\geq 0\ ,\ i,j=L,H. \tag{3.5}$$

The regulator's problem is to maximize expected welfare, $\mathcal{W}$, given a cost of funds $\phi>0$. The maximization is subject to (3.1)-(3.5). For this problem, the timing of the payments is irrelevant. That is, the regulator and firm are indifferent between a payment in period one conditional on type i and a payment in period two conditional on reporting type i in period

[23]Suppose condition (3.5) did not hold. Then there exists a t^*_{ij} such that $t_{1i}+\delta t_{2ij}=t^*_{ij}-\bar{T}$, where $\bar{T}=-\min_{ij}\left(t_{1i}+\delta t_{2ij}\right)$ is a lump sum tax on the firm and t^* satisfies (3.5). Therefore, we are ruling out lump sum taxes on the firm. This is sensible since a "lump sum" tax on firms would in fact cause distortions not modeled here: households would reduce savings and increase consumption, and some low profit firms would exit the market. In a more complicated model, the optimal solution would weigh the cost of these distortions against the benefits of government revenue and information revelation.

one.[24] Therefore, let:

$$t_{ij} = \frac{1}{\delta}t_{1i} + t_{2ij}. \tag{3.6}$$

The regulator's problem then simplifies to:

$$\begin{aligned} \max_{q,t} \quad & \gamma w\left(q_{1L}, \pi_L\right) + \left(1-\gamma\right) w\left(q_{1H}, \pi_H\right) + \delta\gamma^2 w\left(q_{1LL}, \pi_L\right) + \\ & \delta\gamma\left(1-\gamma\right) w\left(q_{2LH}, \pi_H\right) + \delta\left(1-\gamma\right)\gamma w\left(q_{2HL}, \pi_L\right) + \delta\left(1-\gamma\right)^2 w\left(q_{2HH}, \pi_H\right) \\ & -\phi\delta \cdot \left(\gamma^2 t_{LL} + \gamma\left(1-\gamma\right)t_{LH} + \left(1-\gamma\right)\gamma t_{HL} + \left(1-\gamma\right)^2 t_{HH}\right), \end{aligned} \tag{3.7}$$

subject to:

$$-c\left(q_{2iL}, \pi_L\right) + t_{iL} \geq -c\left(q_{2iH}, \pi_L\right) + t_{iH}\ ,\ i = L, H, \tag{3.8}$$

$$-c\left(q_{2iH}, \pi_H\right) + t_{iH} \geq -c\left(q_{2iL}, \pi_H\right) + t_{iL}\ ,\ i = L, H, \tag{3.9}$$

$$w_f\left(q_{1L}, q_2\left(\pi_L, \pi\right), \pi_L\right) + \mathrm{E}\left(t\left(\pi_L, \pi\right)\right) \geq w_f\left(q_{1H}, q_2\left(\pi_H, \pi\right), \pi_L\right) + \mathrm{E}\left(t\left(\pi_H, \pi\right)\right), \tag{3.10}$$

$$w_f\left(q_{1H}, q_2\left(\pi_H, \pi\right), \pi_H\right) + \mathrm{E}\left(t\left(\pi_H, \pi\right)\right) \geq w_f\left(q_{1L}, q_2\left(\pi_L, \pi\right), \pi_H\right) + \mathrm{E}\left(t\left(\pi_L, \pi\right)\right), \tag{3.11}$$

$$t_{ij} \geq 0, \quad i, j = L, H. \tag{3.12}$$

Similar to the problem without a cost of funds, we solve a relaxed problem where only some constraints bind. We then show that the solution satisfies the remaining constraints. The non-binding constraints are (3.9), $i = L, H$, and (3.11). Let λ_{LL} and λ_{LH} be the Lagrange multipliers on (3.8), $i = L, H$, respectively. Further, let λ_L be the multiplier for (3.10) and μ_{ij} be the multipliers for (3.12).

Appendix 8.3 gives the first order conditions for problem (3.7). The first order conditions indicate the regulator can use payments to reduce the multipliers on the incentive compatibility constraints, thus moving regulation closer to the first best, but at a cost of funds ϕ. We next ask to what extent the regulator uses payments versus our timing mechanism described in section 2.1. If payments are zero and $q_i \neq q_{2Li} \neq q_{2Hi}$, then the regulator, by non-trivially using the first period report in the second period regulation, is timing the

[24]Typically, with full commitment, the timing of payments is irrelevant.

regulation exclusively. Conversely, if payments are positive and $q_i = q_{2iL} = q_{2iH}$ then the regulator is using payments, but not timing the regulation. It is immediate (see 8.47 and 8.48) that the regulator sets $t_{HH} = 0$, but other payments may be positive or zero depending on ϕ and the size of the multipliers.

Without a cost of funds, we find the well-known result that the regulator achieves first best using only payments.

PROPOSITION 3 *Let C be super modular in $[q, \pi]$ and $\phi = 0$. Then the solution to (3.7) has $q_{1L} = q_{1LL} = q_{1HL} = q_L^*$ and $q_{1H} = q_{2LH} = q_{2HH} = q_H^*$. That is, the regulator achieves first best using payments, and does not use the timing mechanism.*

If the cost of funds is sufficiently high, however, the regulator uses the timing mechanism and no payments.

PROPOSITION 4 *Let C be super modular in $[q, \pi]$ and R be constant in q. Then if:*

$$\phi \geq \left(\frac{1-\gamma}{\gamma}\right) R, \tag{3.13}$$

then the solution to (2.6) solves problem (3.7), with $t_{ij} = 0$ for all i, j. That is, the regulator relies only on the timing mechanism and does not use payments.

For a cost of funds sufficiently large, the regulator does not use any payments, but instead relies on the timing mechanism to induce incentive compatibility in the first period. As $\gamma \to 1$, the critical threshold approaches zero.

Intuitively, as $\gamma \to 1$, timing the regulation becomes less costly since the regulator can simply impose very high penalties in the second period if the firm reports it is the high cost type. Low cost firms are then motivated to report truthfully, but because firms are unlikely to be of the high type, the regulator is unlikely to bear the cost of overly stringent regulation of the high cost type in the second period.[25] In contrast, with payments the firm reporting low costs must receive a higher payment than the firm reporting high costs. For γ near one, almost all firms are low type, so the regulator incurs the cost of funds with high probability. Therefore, payments become more costly (and therefore less effective) as the regulator pays the low cost firm more often. The presence of R in equation (3.13) occurs since a large spread between marginal costs of the high and low type implies a large welfare gain from moving to

[25]This intuition can also be seen in that the bound of shadow cost of the incentive constraint $\lambda < 1 - \gamma$ approaches zero as $\gamma \to 1$. Equations (2.19) and (2.20) also show that as $\gamma \to 1$, the mechanism only punishes the high type in period 2, but does not reward the low type.

the first best regulation. The regulator is therefore more motivated to use payments even if ϕ is large.

Thus the timing mechanism tends to work well when payments do not and vice versa. It is even possible that the choice of payments versus timing would vary across industries depending on γ and R. However, the regulator uses the timing mechanism to some degree, as long as $\phi > 0$:

PROPOSITION 5 *Let C be super modular in $[q, \pi]$ and $\phi > 0$. Then $q_{1H} = q_{2LH} = q_{2HH}$ does not hold. That is, the regulator relies at least in part on the timing mechanism.*

Proposition 5 is most interesting because in practice the absence of lump sum taxes implies the cost of funds to the regulator is positive. Therefore, in practice using the timing mechanism is optimal.

4 General Marginal Cost Processes

The previous sections assume marginal cost shocks are iid. Here we explicitly characterize the optimal regulatory policy for more general marginal cost processes, including correlated shocks. In the iid case, the firm and the regulator have identical information regarding marginal costs in the second period. Independent shocks might be appropriate if, for example, input prices fluctuate around a stationary value. For example, the marginal cost of stricter sulfur regulations might rise as the price of low sulfur coal rises. However, for some applications, correlated costs are more appropriate (Stavins 1996, Heutel 2009). Further, for some applications, asymmetric information may be persistent (that is, asymmetric information may exist regarding second period expected costs). Finally, a more general stochastic process for the marginal costs shocks yields additional insights as to the nature of the mechanism.

Let $c(q, \pi) = \pi c(q)$. Suppose first period costs are unchanged ($Pr(\pi_1 = \pi_L) = \gamma$), but second period costs are now:

$$E(\pi_2|\pi_1 = \pi_L) = \bar{\pi}_L \text{ and } E(\pi_2|\pi_1 = \pi_H) = \bar{\pi}_H. \tag{4.14}$$

Thus, $\bar{\pi}_i$ represents a firm's expectations about period two costs, given cost realization π_i in period one. Equation (4.14) allows for the most general dependent cost structures possible in a two period model. Because contracting happens at time one, from the perspective of both regulator and firm only the conditional expectations $\bar{\pi}_i = E[\pi_2|\pi_1]$ enter into the objective function and the constraints. Therefore, equation (4.14) supports any dependent distribution $F\left(x\right|\pi_1)$. Below, we describe a number of natural cases of this specification.

- **Permanent shocks.** If costs are permanent, then $\bar{\pi}_i = \pi_1$; that is, period two's expected cost is identical to period one's realized cost.[26]

- **Persistent shocks.** Persistence in the cost process implies that a low cost firm in period one is more likely to have a low cost in period two, relative to a high cost firm in period one, or $\bar{\pi}_L < \bar{\pi}_H$.

- **Productivity shocks.** Suppose costs are a fraction of GDP (y), so that $c\left(q\right) = \hat{c}\left(q\right) y$, where $y = \pi k$ is subject to a productivity shock that follows a discrete Markov process. Suppose further the transition matrix is:

$$\Pi = \begin{bmatrix} \gamma & 1-\gamma \\ 1-\beta & \beta \end{bmatrix}. \tag{4.15}$$

  Then if we assume $\pi_0 = \pi_L$, productivity shocks fit our framework with $\bar{\pi}_L = \bar{\pi}$ and $\bar{\pi}_H = \left(1-\beta\right)\pi_L + \beta\pi_H$.

- **Multiplicative shocks.** Let $\bar{\pi}_i = \beta_i\chi_i\pi_i + \left(1-\beta_i\right)\pi_i$, then with probability β the firm experiences a multiplicative marginal cost shock χ. This may be the result of an uncertain innovation, etc. If the shock represents the discovery of a cost reducing innovation, because of learning by doing for example, then it is natural to think that the probability and size of the innovations depend significantly on the firms current technology/type.

Optimal regulation given no information may now be different in period two. Let $\bar{q}_1 = \bar{q}$ be the no information level of regulation in period one, and let $\bar{q}_2$ satisfy:

$$B_q\left(\bar{q}_2\right) = \mathrm{E}\left[\pi_2\right] c_q\left(\bar{q}_2\right), \tag{4.16}$$

where $\mathrm{E}\left[\pi_2\right] = \gamma\bar{\pi}_L + \left(1-\gamma\right)\bar{\pi}_H$ is the unconditional expected cost shock in period two.

The expected change in firm profits from the regulation is now:

$$w_f\left(q_1, q_2, \pi_1, \bar{\pi}_1\right) = -\pi_1 C\left(q_1\right) - \delta\bar{\pi}_1 C\left(q_2\right). \tag{4.17}$$

[26] Given that only conditional expectations matter, the optimal regulation policy is identical for permanent shocks and shocks which are only expected not to change.

The incentive constraints are now:

$$w_f(q_{1L}, q_{2L}, \pi_L, \bar{\pi}_L) \geq w_f(q_{1H}, q_{2H}, \pi_L, \bar{\pi}_L), \tag{4.18}$$

$$w_f(q_{1H}, q_{2H}, \pi_H, \bar{\pi}_H) \geq w_f(q_{1L}, q_{2L}, \pi_H, \bar{\pi}_H), \tag{4.19}$$

The Lagrangian for the relaxed problem is:

$$\mathcal{L}_r = \gamma(B(q_{1L}) - \pi_L c(q_{1L}) + \delta(B(q_{2L}) - \bar{\pi}_L c(q_{2L}))) + \tag{4.20}$$
$$(1-\gamma)(B(q_{1H}) - \pi_H c(q_{1H}) + \delta(B(q_{2H}) - \bar{\pi}_H c(q_{2H}))) + \tag{4.21}$$
$$\lambda(w_f(q_{1L}, q_{2L}, \pi_L, \bar{\pi}_L) - w_f(q_{1H}, q_{2H}, \pi_L, \bar{\pi}_L)). \tag{4.22}$$

Proposition 6 shows the mechanism is unchanged, given a simple condition,

$$\frac{\bar{\pi}_H}{\bar{\pi}_L} \leq \frac{\pi_H}{\pi_L}, \tag{4.23}$$

which we discuss below.

PROPOSITION 6 *Suppose $c(q,\pi) = \pi c(q)$ and let $\bar{\pi}_H > \bar{\pi}_L$. Then the solution to the relaxed problem (4.22) solves the original problem subject to (4.18) and (4.19).*

Suppose further that the inequality in condition (4.23) is strict. Then the solution to the problem (4.22) has the following properties:

6.1. $\gamma(1-\gamma)\left(\frac{\bar{\pi}_H}{\bar{\pi}_L} - 1\right) = \lambda_L < \lambda < \lambda_R = \gamma(1-\gamma)\left(\frac{\pi_H}{\pi_L} - 1\right)$

6.2. $q_{1H} < \bar{q}_1 < q_{1L}$.

6.3. $q_{2L} < \bar{q}_2 < q_{2H}$.

Conversely, if condition (4.23) holds with equality, then the solution is the no information solution: $q_{1L} = q_{1H} = \bar{q}_1$ and $q_{2L} = q_{2H} = \bar{q}_2$.

To understand the intuition of condition (4.23), let us focus on the special case where $\delta = 1$. The regulator would like to move regulation closer to the first best level in the period where the expected difference in marginal costs is widest. The regulator will use the other period to satisfy incentive compatibility. Condition (4.23) states that the growth rate of π_H is less than that of π_L. If the inequality in (4.23) is strict, then marginal costs are wider in the first period,[27] and the mechanism is qualitatively unchanged from the previous sections. If (4.23) holds with equality, then the difference in marginal costs is identical in the first and second period. In this case, moving regulation toward first best in one period requires an equal move away from first best in the other period. Thus in this special case the regulator does not gain from varying the regulation over time, and uses the no information level of regulation.[28] Finally, if (4.23) does not hold, then the timing of the mechanism would reverse: the regulator would move regulation towards first best in period two, and use period one to satisfy incentive compatibility. Therefore, the mechanism (or its reverse) applies for all but the knife edge case of identical expected growth rates.

The above intuition suggests that it is possible to achieve the first best regulation in both periods if $\bar{\pi}_H < \bar{\pi}_L$. If $\bar{\pi}_H$ is less than $\bar{\pi}_L$ then under the first-best mechanism a firm reporting L is regulated strongly in period one, but receives weak regulation in period two and the reverse timing for a firm reporting H. In this case, a low-cost firm's incentive to exaggerate its cost is mitigated. By lying it reduces its regulation in the first period, when its compliance cost is low, but increases it in the second period, when its cost is high. In this case the benefit of lying is small while the cost of lying is large. If this effect is sufficiently strong, the first-best mechanism may be incentive compatible.[29] In order to ensure that the first-best mechanism violates incentive compatibility, we focus on the case of $\bar{\pi}_H \geq \bar{\pi}_L$. Here, the firm with lower cost in period one also expects a lower cost in period two and, under the first-best mechanism, would face higher regulation in both periods.

We can get an idea about how the expected growth rates of the cost shocks affects welfare and the optimal policy by looking at the quadratic example for this case. Let $\rho_L = \bar{\pi}_L/\pi_L$, then resolving the model of section 2.4 with the more general shocks yields the solution:

$$q_{1L} = \bar{q} + (1-\gamma)\frac{\delta\rho_L^2}{1+\delta\rho_L^2}\pi_L\left(\frac{\pi_H}{\pi_L} - \frac{\bar{\pi}_H}{\bar{\pi}_L}\right), \tag{4.24}$$

$$q_{1H} = \bar{q} - \gamma\frac{\delta\rho_L^2}{1+\delta\rho_L^2}\pi_L\left(\frac{\pi_H}{\pi_L} - \frac{\bar{\pi}_H}{\bar{\pi}_L}\right), \tag{4.25}$$

[27]Note that the theorem also requires $\bar{\pi}_H > \bar{\pi}_L$. If the growth rate of π_H is very negative, the difference in marginal costs can be wider in the second period even if (4.23) holds.

[28]Since costs are convex, the regulator prefers a constant level of regulation over time relative to regulation that is varied by equal amounts in each period.

[29]This effect can be observed in the proposition. If $\bar{\pi}_H < \bar{\pi}_L$ then λ_L can be negative, which allows for a zero Lagrange multiplier on the incentive constraint.

$$q_{2L} = \bar{q}_2 - (1-\gamma)\frac{\rho_L}{1+\delta\rho_L^2}\pi_L\left(\frac{\pi_H}{\pi_L} - \frac{\bar{\pi}_H}{\bar{\pi}_L}\right), \tag{4.26}$$

$$q_{2H} = \bar{q}_2 + \gamma\frac{\rho_L}{1+\delta\rho_L^2}\pi_L\left(\frac{\pi_H}{\pi_L} - \frac{\bar{\pi}_H}{\bar{\pi}_L}\right), \tag{4.27}$$

$$\lambda = \frac{1}{1+\delta\rho_L^2}\lambda_R + \frac{\delta\rho_L^2}{1+\delta\rho_L^2}\lambda_L, \tag{4.28}$$

$$W = \frac{1}{2}\left(\bar{q}_1^2 + \delta\bar{q}_2^2\right) + \frac{1}{2}\gamma(1-\gamma)\frac{\delta\rho_L^2}{1+\delta\rho_L^2}\pi_L^2\left(\frac{\pi_H}{\pi_L} - \frac{\bar{\pi}_H}{\bar{\pi}_L}\right)^2. \tag{4.29}$$

As shown in Proposition 6, the solution reduces to the no information case if the growth rates are identical, and reduces to the solution of section 2.4 if $\bar{\pi}_H = \bar{\pi}_L = \bar{\pi}$. Further, the solution is analogous to that of section (2.4), with $\Delta\pi$ being replaced by the difference in growth rates. The welfare gains are convex in the difference in growth rates between π_H and π_L, as the regulator gains more by differentially regulating in the first period if the first best regulation levels are very far apart.

5 Multiple firms

In this section, we allow for multiple firms and show that the qualitative results continue to hold. Suppose now n firms exist, each of which receives an independently distributed cost shock equal to π_L with probability γ and π_H otherwise. We assume the timing is such that the regulator collects all reports, and then assigns regulation to each firm in each period based on all reports. All firms that report low costs are identical to the regulator, and thus receive identical regulation. Let $0 \leq m \leq n$ be the number of firms reporting the low cost shock. If m firms report low costs, firms receive regulation $q_{ij,m} = q_i(\pi_j, m)$, $i = 1, 2$, $j = L, H$. We further assume that regulation of one firm is a perfect substitute for regulating another in the benefit function:

$$B_i = B\left(mq_{iL,m} + (n-m)\,q_{iH,m}\right), \quad i = 1, 2. \tag{5.1}$$

Let $Pr\,(m|i)$ denote the probability that m firms received the low cost shock, conditional on one firm receiving shock $i \in \{L, M\}$. The incentive constraints (2.4)-(2.5) for low and high cost firms are now:

$$\sum_{m=1}^{n} Pr\left(m|L\right)\left[w_f\left(q_{1L,m},q_{2L,m},\pi_L\right)\right] \geq \sum_{m=1}^{n} Pr\left(m|L\right)\left[w_f\left(q_{1H,m-1},q_{2H,m-1},\pi_L\right)\right]. \quad (5.2)$$

$$\sum_{m=0}^{n-1} Pr\left(m|H\right)\left[w_f\left(q_{1H,m},q_{2H,m},\pi_H\right)\right] \geq \sum_{m=0}^{n-1} Pr\left(m|H\right)\left[w_f\left(q_{1L,m+1},q_{2L,m+1},\pi_H\right)\right]. \quad (5.3)$$

Here the firm takes expectations since other firms' costs are unknown. Mechanisms that satisfy (5.2) and (5.3) imply truthful revelation of information is a Bayesian-Nash equilibrium.

The Lagrangian of the regulator's problem is:

$$\begin{aligned}
\mathcal{L}_n \;=\; & \sum_{m=0}^{n} Pr\left(m\right)\Big[B\left(mq_{1L,m}+\left(n-m\right)q_{1H,m}\right)-mC\left(q_{1L,m},\pi_L\right)-\left(n-m\right)C\left(q_{1H,m},\pi_H\right) \\
& +\delta B\left(mq_{2L,m}+\left(n-m\right)q_{2H,m}\right)-m\delta \mathrm{E}\left[C\left(q_{2L,m},\pi\right)\right]-\left(n-m\right)\delta\mathrm{E}\left[C\left(q_{2H,m},\pi\right)\right]\Big] \\
& +n\lambda\sum_{m=1}^{n} Pr\left(m|L\right)\Big[w_f\left(q_{1L,m},q_{2L,m},\pi_L\right)-w_f\left(q_{1H,m-1},q_{2H,m-1},\pi_L\right)\Big] \quad (5.4)
\end{aligned}$$

All low cost firms have the same incentive constraints and thus λ does not vary by firm.

The first order condition for $q_{1L,m}$ is:

$$B_q\left(mq_{1L,m}+\left(n-m\right)q_{1H,m}\right)=\left(1+\frac{\lambda nPr\left(m|L\right)}{mPr\left(m\right)}\right)C_q\left(q_{1L,m},\pi_L\right). \quad (5.5)$$

Note that, from the properties of the binomial distribution, the probability that m of n firms are low type conditional on one known low type equals the probably that $m-1$ of $n-1$ remaining firms are the low type:

$$Pr\left(m|L\right)=\left[\begin{array}{c} n-1 \\ m-1 \end{array}\right]\gamma^{m-1}\left(1-\gamma\right)^{n-m}=\frac{m}{\gamma n}Pr\left(m\right). \quad (5.6)$$

Thus, the first order condition reduces to:

$$B_q\left(mq_{1L,m}+\left(n-m\right)q_{1H,m}\right)=C_q\left(q_{1L,m},\pi_L\right)\left(1+\frac{\lambda}{\gamma}\right). \quad (5.7)$$

Next, via a similar calculation:

$$B_q\left(mq_{1L,m}+(n-m)\,q_{1H,m}\right)=C_q\left(q_{1H,m},\pi_H\right)\left(1-\frac{\lambda nPr\left(m+1|L\right)}{(n-m)\,Pr\left(m\right)}\frac{C_q\left(q_{1H,m},\pi_L\right)}{C_q\left(q_{1H,m},\pi_H\right)}\right),\tag{5.8}$$

$$Pr\left(m+1|L\right)=\frac{n-m}{n\left(1-\gamma\right)}Pr\left(m\right),\tag{5.9}$$

$$B_q\left(mq_{1L,m}+(n-m)\,q_{1H,m}\right)=C_q\left(q_{1H,m},\pi_H\right)\left(1-\frac{\lambda}{(1-\gamma)}\frac{C_q\left(q_{1H,m},\pi_L\right)}{C_q\left(q_{1H,m},\pi_H\right)}\right).\tag{5.10}$$

The second period first order conditions are:

$$B_q\left(mq_{2L,m}+(n-m)\,q_{2H,m}\right)=\mathrm{E}\left[C_q\left(q_{2L,m},\pi\right)\right]\left(1+\frac{\lambda}{\gamma}\right).\tag{5.11}$$

$$B_q\left(mq_{2L,m}+(n-m)\,q_{2H,m}\right)=\mathrm{E}\left[C_q\left(q_{2H,m},\pi\right)\right]\left(1-\frac{\lambda}{(1-\gamma)}\right).\tag{5.12}$$

The first order conditions for firms reporting low costs revert back to those of section 2.1 for $n=m=1$, and the first order conditions for firms that report high costs revert to those of section 2.1 for $n=1$ and $m=0$. Indeed, the results change only in that the marginal benefits are lower with more firms since costs increase linearly with the number of firms but benefits are concave.[30]

Equations (5.7), (5.10), and (5.11)-(5.12) imply that the equi-marginal principle is violated in both periods. The regulator cannot equalize marginal costs across types without violating the incentive constraint.

We define the optimal regulation with only prior information for n firms, $\bar{q}_n$ as the solution to:

$$\max B\left(n\bar{q}_n\right)-n\mathrm{E}\left[C\left(\bar{q}_n,\pi\right)\right],\tag{5.13}$$

$$B_q\left(n\bar{q}_n\right)=\mathrm{E}\left[C_q\left(\bar{q}_n,\pi\right)\right].\tag{5.14}$$

[30] For the limiting case, normalize the size of each firm to $1/n$, then as $n\to\infty$, total regulation approaches $\gamma q_{jL}+(1-\gamma)\,q_{jH},\ j=1,2$. This case differs from section 2.1 only in that here the regulator faces no aggregate uncertainty.

The first best policies, $q^*_{L,m}$ and $q^*_{H,m}$, satisfy:

$$B_q\left(mq^*_{L,m} + (n-m)\,q^*_{H,m}\right) = C_q\left(q^*_{L,m}, \pi_L\right) = C_q\left(q^*_{H,m}, \pi_H\right). \tag{5.15}$$

Proposition 7 shows that optimal mechanism is analogous with n firms.

PROPOSITION 7 *Let C be super modular and R be constant in q. Then the solution to the two period problem with n firms has the following properties:*

7.1. $q_{1H,m} < \bar{q}_n < q_{1L,m}$.

7.2. $q_{2L,m} < \bar{q}_n < q_{2H,m}$.

7.3. $q_{1H,m}$ *and* $q_{1L,m}$ *are increasing functions of* m.

With n firms, the optimal mechanism is to offer each firm a choice of regulation which becomes either more stringent or more lax over time. The high cost firms all select regulation which is initially more lax and the low cost firms all select regulation which is initially more stringent.

6 Endogenous Investment and Declining Costs

Frequently, firms undertake investment or R&D which reduces compliance costs over time. In response, regulation often becomes more strict over time. Here we suppose that firms may undertake endogenous investment which reduces compliance costs and show that our basic result continues to hold. In particular, if regulation becomes more strict over time in expectation, then the regulator offers one regulation which is above the expected trend in regulation in the first period and is below the expected trend in regulation in the second period, and a second regulation which is initially below, and subsequently above, the trend.

We consider the model of section 2.1, but assume costs are also a function of investment ζ in a cost saving technology: $C = C(q, \pi, \zeta)$. We assume $C_\zeta < 0$, so investment reduces costs and $C_{\zeta\zeta} > 0$ so the firm's investment problem is concave. We also assume that $C_{\zeta q} < 0$, so that investment reduces marginal costs of compliance as well. Investment is increasing in regulatory stringency if and only if $C_{\zeta q} < 0$. Finally, we assume that the cost function is such that the second order conditions for the regulator's problem continue to hold. Let δP_ζ denote the price of investment paid in the first period and we normalize the stock of investment in the first period to 0.

The firm chooses a level of investment after reporting first period costs to the regulator. Because the regulator announces regulation for both periods in the first period, the firm anticipates the level of regulation in the second period when the investment decision is made. The firm's investment problem conditional on regulation q_{2i} is then:

$$\max_{\zeta} -C\left(q_{1i}, \pi, 0\right) - \delta \mathrm{E}\left[C\left(q_{2i}, \pi, \zeta\right)\right] - \delta P_{\zeta} \zeta. \tag{6.1}$$

The firm's first order condition is:

$$P_{\zeta} = -\mathrm{E}\left[C_{\zeta}\left(q_{2i}, \pi, \zeta\right)\right], \quad i = L, H. \tag{6.2}$$

We assume a function $\zeta_i = \zeta\left(q_{2i}\right)$, $i = L, H$, satisfying (6.2) exists which maps the level of regulation the firm receives upon reporting costs to the regulator into an investment decision.

The change in firm profits from the regulation becomes:

$$w_f\left(q_1, q_2, \pi_1, \zeta\left(q_2\right)\right) = -C\left(q_1, \pi_1, 0\right) - \delta \mathrm{E}\left[C\left(q_2, \pi, \zeta\left(q_2\right)\right)\right] - \delta P_{\zeta} \zeta\left(q_2\right). \tag{6.3}$$

The welfare function includes the resource costs of investment:

$$w\left(q, \pi, \zeta\left(q\right)\right) = B\left(q\right) - C\left(q, \pi, \zeta\left(q\right)\right) - P_{\zeta} \cdot \zeta\left(q\right). \tag{6.4}$$

The incentive constraints are then:

$$w_f\left(q_{1L}, q_{2L}, \pi_L, \zeta\left(q_{2L}\right)\right) \geq w_f\left(q_{1H}, q_{2H}, \pi_L, \zeta\left(q_{2H}\right)\right), \tag{6.5}$$

$$w_f\left(q_{1H}, q_{2H}, \pi_H, \zeta\left(q_{2H}\right)\right) \geq w_f\left(q_{1L}, q_{2L}, \pi_H, \zeta\left(q_{2L}\right)\right). \tag{6.6}$$

The problem in Lagrange form is then:

$$\begin{aligned}\mathcal{L}_{\zeta} \;=\; & \gamma \cdot \Big[w\left(q_{1L}, \pi_L, 0\right) + \delta \mathrm{E}\left[w\left(q_{2L}, \pi, \zeta\left(q_{2L}\right)\right)\right]\Big] + (1-\gamma) \cdot \Big[w\left(q_{1H}, \pi_H, 0\right) + \\ & \delta \mathrm{E}\left[w\left(q_{2H}, \pi, \zeta\left(q_{2H}\right)\right)\right]\Big] + \lambda \cdot \Big[w_f\left(q_{1L}, q_{2L}, \pi_L, \zeta\left(q_{2L}\right)\right) - \\ & w_f\left(q_{1H}, q_{2H}, \pi_L, \zeta\left(q_{2H}\right)\right)\Big]\end{aligned} \tag{6.7}$$

The investment decision does not affect the first order conditions in the first period:

$$B_q\left(q_{1L}\right) = C_q\left(q_{1L}, \pi_L, 0\right)\left(1 + \frac{\lambda}{\gamma}\right), \tag{6.8}$$

$$B_q(q_{1H}) = C_q(q_{1H}, \pi_H, 0)\left(1 - \frac{\lambda}{1-\gamma}\frac{C_q(q_{1H}, \pi_L, 0)}{C_q(q_{1H}, \pi_H, 0)}\right). \tag{6.9}$$

The first order condition with respect to q_{2L} is:

$$B_q(q_{2L}) = (\mathrm{E}[C_q(q_{2L}, \pi, \zeta_L)] + (\mathrm{E}[C_\zeta(q_{2L}, \pi, \zeta_L) + P_\zeta)\zeta_q(q_{2L})])\left(1 + \frac{\lambda}{\gamma}\right). \tag{6.10}$$

However, using (6.2), we see that:

$$B_q(q_{2L}) = \mathrm{E}[C_q(q_{2L}, \pi, \zeta_L)]\left(1 + \frac{\lambda}{\gamma}\right). \tag{6.11}$$

Similarly, using (6.2), the first order condition with respect to q_{2H} is:

$$B_q(q_{2H}) = \mathrm{E}[C_q(q_{2H}, \pi, \zeta_H)]\left(1 - \frac{\lambda}{1-\gamma}\right). \tag{6.12}$$

The incentive constraint binds:

$$\begin{aligned} &C(q_{1H}, \pi_L, 0) - C(q_{1L}, \pi_L, 0) = \\ &\quad \delta(\mathrm{E}[C(q_{2L}, \pi, \zeta_L)] - \mathrm{E}[C(q_{2H}, \pi, \zeta_H)] + P_\zeta(\zeta_L - \zeta_H)). \end{aligned} \tag{6.13}$$

From the first order conditions, the regulator knows that after assigning a second period level of regulation, the firm chooses the optimal level of investment given the regulation. Because the second period regulation is suboptimal relative to the first best level of regulation in the second period, investment is also not first best. But investment is optimal (for both the firm and the regulator) conditional on q_2, and thus the mechanism is qualitatively unchanged.

For the properties of the mechanism, let $\bar{q}_\zeta$ satisfy $B_q(\bar{q}_\zeta) = \mathrm{E}[C_q(\bar{q}_\zeta, \pi, \zeta(\bar{q}_\zeta))]$. Then:

PROPOSITION 8 *Let R be constant in q. Then the solution to problem (6.7) has the following properties:*

8.1. $q_{1H} < \bar{q} < q_{1L}$.

8.2. $q_{2L} < \bar{q}_\zeta < q_{2H}$.

Note $\bar{q}_\zeta > \bar{q}$, so if the regulator has only prior information about firm costs, regulation becomes more stringent over time since costs fall. Under our mechanism, the regulator offers one contract that is initially above and subsequently below the trend line of regulatory stringency in the prior information case. The other regulation option starts out below the trend in regulatory stringency, and then is above the trend in the second period.

Thus the mechanism is essentially unchanged. One difference is that, with declining costs, the regulator may offer regulation which strengthens over time, but at different rates depending on the firm's report. From proposition (8), the high cost firm is offered regulation which becomes more stringent over time since $q_{1H} < \bar{q} < \bar{q}_{\zeta} < q_{2H}$. For the firm reporting low costs, if costs decline enough, $q_{1L} < q_{2L}$ is possible. The regulator offers the low cost firm regulation which becomes more stringent over time, just not as stringent as when the firm reports high costs. Thus the results are consistent with the empirical observation that regulation tends to strengthen over time as costs decline.

7 Conclusions

We have shown, in an environment where marginal compliance costs are subject to random shocks, that the regulator can induce firms to reveal their costs shocks and increase welfare by varying the strength of regulation over time. In particular, the optimal mechanism is to offer the firm two regulation choices. The first starts out weak and becomes stronger, while the second does the opposite. Firms currently facing high cost shocks know their costs are likely to decline over time, and chose regulation which is initially weak. Firms with low cost shocks choose the opposite. In this way, firms reveal their cost shocks to the regulator. Welfare improves both because firms choose strict regulation only when marginal costs are low, *and* because doing so reveals information to the regulator.

To implement our mechanism in practice, the regulator could combine a default regulation that becomes more stringent over time (as is the case for most regulations), with a program whereby firms exceeding the regulation standard in the current period receive waivers or credits for use in the future. Such waiver and credit programs are common. For example, a provision of the corporate average fuel economy standards allows companies exceeding the fuel economy standard in the current period to receive credits which allow the companies to be below the standard in the future. Low cost firms take advantage of the credit program, in order to better equalize marginal costs over time. High cost firms do not, delaying costly regulation as costs are expected to fall. Our mechanism differs from existing waiver/credit programs in that the waiver program must be implemented so as to reveal information. The regulator must set the appropriate intertemporal price (the rate at which exceeding the current standard is exchanged for future credits), which trades off the benefits of equalizing marginal costs with higher average regulation.

Our mechanism is robust to a number of extensions. If the regulator may make payments to the firm, then for any positive cost of funds, the optimal mechanism varies regulation over time to some degree. Further, if the cost of funds is high enough, the optimal mechanism does not use payments, but instead relies exclusively on the timing of regulation. In general, varying regulation over time is more effective than payments when the probability of receiving a low cost shock is high.

Our mechanism comes with several caveats. First, our mechanism essentially trades off current for future distortions, and thus cannot achieve first best. Using payments results in the first best allocation, but only under the assumption that no cost of funds exists (that is, that lump sum taxes are available).

Second, our mechanism relies on commitment. The regulator has an incentive to renege on promised regulation in the second period, and revert to the optimal level of regulation given no cost shock information. Nonetheless, the incentive to renege here is more mild than in models where firm types do not change over time, since the regulator only desires to return to the no-information level of regulation, not the optimal regulation given the firm has a low cost shock (that is, no "ratchet effect" exists). For the example of the 1990 amendment to the Clean Air Act, the EPA offered firms bonus permits in the future for installing scrubbers. The EPA kept the commitment and allocated the bonus permits, despite specific clauses in the law stating that the EPA could revoke any part of the permit system at any time.

Third, the mechanism breaks down if shocks have identical expected growth rates across time, for example if marginal costs were constant over time. The mechanism relies on differentiating regulation in the period where marginal costs are most different, and satisfying incentive compatibility in the period where marginal costs are most similar. A deeper issue arises, however, here and in some of the literature which takes cost heterogeneity across firms as constant. In the long run only firms with the lowest cost technologies survive in a competitive market. Thus it is not clear that shocks which are constant over time are consistent with a long run competitive equilibrium.[31] In contrast, differential expected growth rates has natural interpretations. For example, the more regulated firm could see a more negative expected growth rate in costs since as it adapts to more stringent regulation.[32].

A number of further extensions are possible, but are unlikely to change the main results. The most interesting extension is to make the number of periods infinite. In this case, our hypothesis is that the regulator would start with some promised level of total future profits generated from past reports, and then offer a promise of future profits that are either higher

[31] In addition, it is not clear why a regulator would be uncertain about shocks which are constant in time, since the regulator could simply invert the cost function after one observation and learn the unobserved cost parameter.

[32] Conversely, one might suppose that the high cost firm has a more negative growth rate as it observes the technological choices of the low cost firm (spillovers)

or lower depending on the firm's report. Still, an infinite horizon version of the model may be less realistic than our two period model, since commitment into the infinite future implies the regulator will never be replaced.

Despite these caveats, a robust result is that our timing mechanism improves regulatory efficiency. In recent years the public's appetite for increased regulation has grown. Regulations are becoming increasingly complex, with compliance costs that are increasingly difficult to forecast, for both firms and regulators. Therefore, it is clear that more efficient regulations is an important policy goal, and will only become more so in the future.

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8 Appendix

8.1 Proof of Proposition 1

Let $X \equiv \delta \left(\mathrm{E}\left[C\left(q_{2L}, \pi\right)\right] - \mathrm{E}\left[C\left(q_{2H}, \pi\right)\right]\right)$. Then the solution to the problem (2.6) satisfies:

$$C\left(q_{1H}, \pi_L\right) - C\left(q_{1L}, \pi_L\right) = X. \tag{8.1}$$

Thus, condition (2.5) holds if and only if:

$$X \geq C\left(q_{1H}, \pi_H\right) - C\left(q_{1L}, \pi_H\right). \tag{8.2}$$

Combining equations (8.1) and (8.2), condition (2.5) holds if and only if:

$$C(q_{1H},\pi_L) - C(q_{1L},\pi_L) \geq C(q_{1H},\pi_H) - C(q_{1L},\pi_H), \tag{8.3}$$

Next, let $a = [q_{1H},\pi_H]$ and $b = [q_{1L},\pi_L]$, then since $q_{1L} > q_{1H}$ (see Proposition 2) and $\pi_H > \pi_L$:

$$C(a \wedge b) - C(b) \geq C(a) - C(a \vee b). \tag{8.4}$$

Condition (8.4) holds if and only if C is supermodular.

8.2 Proof of Proposition 2

2.1. By definition of q_L^*,

$$B_q(q_L^*) = C_q(q_L^*,\pi_L). \tag{8.5}$$

Given B is concave and C is convex, $q_{1L} < q_L^*$ if and only if:

$$B_q(q_{1L}) > C_q(q_{1L},\pi_L). \tag{8.6}$$

Using equation (2.7),

$$C_q(q_{1L},\pi_L)\left(1+\frac{\lambda}{\gamma}\right) > C_q(q_{1L},\pi_L). \tag{8.7}$$

Equation (8.7) holds since $\lambda > 0$. We prove $\bar{q} < q_{1L}$ in 2.3 below.

2.2. By definition of q_H^*,

$$B_q(q_H^*) = C_q(q_H^*,\pi_H). \tag{8.8}$$

Given B is concave and C is convex, $q_{1H} > q_H^*$ if and only if:

$$B_q(q_{1H}) < C_q(q_{1H},\pi_H). \tag{8.9}$$

Using equation (2.8),

$$C_q\left(q_{1H},\pi_H\right)\left(1-\frac{\lambda}{1-\gamma}\frac{C_q\left(q_{1H},\pi_L\right)}{C_q\left(q_{1H},\pi_H\right)}\right)<C_q\left(q_{1L},\pi_H\right). \tag{8.10}$$

Equation (8.10) holds since $\lambda > 0$. We prove $q_{1H} < \bar{q}$ in 2.3 below.

2.3. For the second period regulations, by definition:

$$B_q\left(\bar{q}\right)=\mathrm{E}\left[C_q\left(\bar{q},\pi\right)\right]. \tag{8.11}$$

Since B is concave and C is convex, $q_{2L} < \bar{q}$ if and only if:

$$B_q\left(q_{2L}\right)>\mathrm{E}\left[C_q\left(q_{2L},\pi\right)\right]. \tag{8.12}$$

Equation (2.9) implies the above inequality holds since $\lambda > 0$. Similarly, $q_{2H} > \bar{q}$ if and only if:

$$B_q\left(q_{2H}\right)<\mathrm{E}\left[C_q\left(q_{2H},\pi\right)\right]. \tag{8.13}$$

Equation (2.10) implies the above inequality holds since $\lambda > 0$.

For $q_{1H} < \bar{q}$, we first show that $q_{1L} > q_{1H}$. To see this, we suppose not. Suppose $q_{1H} \geq q_{1L}$ and note that the Kuhn-Tucker condition for the incentive constraint (2.4) is:

$$\lambda\left[C\left(q_{1H},\pi_L\right)-C\left(q_{1L}\pi_L\right)+\delta\mathrm{E}\left[C\left(q_{2H},\pi\right)\right]-\delta\mathrm{E}\left[C\left(q_{2L},\pi\right)\right]\right]=0. \tag{8.14}$$

We have shown $q_{2H} > q_{2L}$, which along with $q_{1H} \geq q_{1L}$ implies the second term in (8.14) is positive and thus that $\lambda = 0$. But from the first order conditions, $\lambda = 0$ implies $q_{2L} = q_L^*$ which contradicts $q_{2L} < q_{2H}$, for example. Thus $q_{1H} < q_{1L}$.

With $q_{2L} < q_{2H}$ in hand, to show $q_{1H} < \bar{q}$, we suppose not and then construct a regulation set which is feasible and provides higher welfare, thus contradicting that $q_{1H} \geq \bar{q}$ is an optimum. Suppose $\{q_{1L}, q_{1H}, q_{2L}, q_{2H}\}$ is optimal with $q_{1H} \geq \bar{q}$. Consider an alternative policy $\{q_{1L}-\epsilon, q_{1H}-\epsilon, q_{2L}, q_{2H}\}$, with $\epsilon > 0$ sufficiently small (i.e. small enough to make a first order approximation of B and C accurate enough so as to not change the signs of any of the inequalities). The alternative policy is feasible if and only if:

$$C\left(q_{1H}-\epsilon,\pi_L\right)-C\left(q_{1L}-\epsilon,\pi_L\right)\geq X=C\left(q_{1H},\pi_L\right)-C\left(q_{1L},\pi_L\right). \tag{8.15}$$

Approximating $C\left(q_{1L}-\epsilon,\pi_L\right)$ around q_{1L} and $C\left(q_{1H}-\epsilon,\pi_L\right)$ around q_{1H}, implies for ϵ small the inequality reduces to:

$$C_q\left(q_{1H},\pi_L\right)\left(-\epsilon\right)-C_q\left(q_{1L},\pi_L\right)\left(-\epsilon\right)\geq 0, \tag{8.16}$$

$$C_q\left(q_{1L},\pi_L\right)\geq C_q\left(q_{1H},\pi_L\right). \tag{8.17}$$

Equation (8.17) holds since $q_{1L}>q_{1H}$. Thus the alternative policy is feasible.

We next compare the welfare of the alternative policy with the optimal policy. Since the second period policies are identical, the alternative policy generates higher welfare if and only if:

$$\begin{aligned}\gamma\Big[B\left(q_{1L}-\epsilon\right)-C\left(q_{1L}-\epsilon,\pi_L\right)\Big]+(1-\gamma)\Big[B\left(q_{1H}-\epsilon\right)-C\left(q_{1H}-\epsilon,\pi_H\right)\Big]>\\ \gamma\Big[B\left(q_{1L}\right)-C\left(q_{1L},\pi_L\right)\Big]+(1-\gamma)\Big[B\left(q_{1H}\right)-C\left(q_{1H},\pi_H\right)\Big].\end{aligned} \tag{8.18}$$

Performing first order approximations reduces the inequality to:

$$\gamma\Big[B_q\left(q_{1L}\right)-C_q\left(q_{1L},\pi_L\right)\Big]+(1-\gamma)\Big[B_q\left(q_{1H}\right)-C_q\left(q_{1H},\pi_H\right)\Big]<0. \tag{8.19}$$

Next, since we assumed $q_{1H}\geq\bar{q}$, and B is concave and C is convex:

$$B_q\left(q_{1H}\right)-C_q\left(q_{1H},\pi_H\right)\leq B_q\left(\bar{q}\right)-C_q\left(\bar{q},\pi_H\right). \tag{8.20}$$

Further, since $q_{1L}>q_{1H}\geq\bar{q}$,

$$B_q\left(q_{1L}\right)-C_q\left(q_{1L},\pi_L\right)<B_q\left(\bar{q}\right)-C_q\left(\bar{q},\pi_L\right). \tag{8.21}$$

We multiply (8.20) by $1-\gamma$ and (8.21) by γ, and sum the two resulting equations. Comparing the result with (8.19), it is sufficient to show:

$$\gamma\left[B_q\left(\bar{q}\right)-C_q\left(\bar{q},\pi_L\right)\right]+\left(1-\gamma\right)\cdot\left[B_q\left(\bar{q}\right)-C_q\left(\bar{q},\pi_H\right)\right]\leq 0, \tag{8.22}$$

$$B_q\left(\bar{q}\right)-\mathrm{E}\left[C_q\left(\bar{q},\pi_L\right)\right]=0\leq 0. \tag{8.23}$$

Thus we have a contradiction that $\{q_{1L},q_{1H},q_{2L},q_{2H}\}$ is optimal as the alternative policy is feasible and generates higher welfare.

To show $q_{1L}>\bar{q}$, we use the previous results. First, since $q_{1H}<\bar{q}$, we have:

$$\frac{B_q\left(q_{1H}\right)}{\mathrm{E}\left[c_q\left(q_{1H},\pi\right)\right]}>\frac{B_q\left(\bar{q}\right)}{\mathrm{E}\left[c_q\left(\bar{q},\pi\right)\right]}=1. \tag{8.24}$$

Using equation (2.8):

$$\left(1-\frac{\lambda}{1-\gamma}\frac{1}{R}\right)C_q\left(q_{1H},\pi_H\right)>\mathrm{E}\left[C_q\left(q_{1H},\pi\right)\right], \tag{8.25}$$

$$\left(1-\frac{\lambda}{1-\gamma}\frac{1}{R}\right)R>\gamma+\left(1-\gamma\right)R, \tag{8.26}$$

$$\lambda<\gamma\left(1-\gamma\right)\left(R-1\right). \tag{8.27}$$

Thus λ cannot be too big. Finally, $q_{1L}>\bar{q}$ if and only if:

$$\frac{B_q\left(q_{1L}\right)}{\mathrm{E}\left[c_q\left(q_{1L},\pi\right)\right]}<\frac{B_q\left(\bar{q}\right)}{\mathrm{E}\left[c_q\left(\bar{q},\pi\right)\right]}=1. \tag{8.28}$$

Using equation (2.7):

$$\left(1+\frac{\lambda}{\gamma}\right)C_q\left(q_{1L},\pi_L\right)>\mathrm{E}\left[C_q\left(q_{1H},\pi\right)\right], \tag{8.29}$$

$$\left(1+\frac{\lambda}{\gamma}\right) > \gamma + (1-\gamma)\,R, \tag{8.30}$$

$$\lambda < \gamma\,(1-\gamma)\,(R-1)\,. \tag{8.31}$$

The above equation is identical to (8.27). Therefore $q_{1L} > \bar{q}$.

2.4. To see that $\lambda < 1-\gamma$, suppose not, suppose $\{q_{1L}, q_{1H}, q_{2L}, q_{2H}\}$ is an optimum with $\lambda \geq (1-\gamma)$. Then from condition (2.10), we have a corner solution of $q_{2H} = 0$ since for all $q_{2H} \geq 0$,

$$B_q\,(q_{2H}) > 0 \geq \mathrm{E}\left[C_q\,(q_{2H}, \pi)\right]\left(1-\frac{\lambda}{1-\gamma}\right), \tag{8.32}$$

which violates the first order condition (2.10).

Next, the incentive constraint (2.4) with $q_{2H} = 0$ implies:

$$C\,(q_{1H}, \pi_L) - C\,(q_{1L}, \pi_L) \geq \mathrm{E}\left[C\,(q_{2L}, \pi)\right]. \tag{8.33}$$

Thus $q_{1H} \geq q_{1L}$ is required for incentive compatibility, with equality if and only if $q_{2L} = 0$.

Further, combining the two incentive constraints yields (8.3):

$$C\,(q_{1H}, \pi_L) - C\,(q_{1L}, \pi_L) \geq C\,(q_{1H}, \pi_H) - C\,(q_{1L}, \pi_H)\,. \tag{8.34}$$

Let $a = [q_{1H}, \pi_L]$ and $b = [q_{1L}, \pi_H]$, then since $\pi_H > \pi_L$, if $q_{1L} < q_{1H}$:

$$C\,(a) - C\,(a \wedge b) > C\,(a \vee b) - C\,(b)\,, \tag{8.35}$$

which contradicts that C is super modular. Thus we have a contradiction unless $q_{1H} = q_{1L}$ and $q_{2H} = q_{2L} = 0$. In this case, the firm is unregulated in the second period, regardless of type.

Consider now an alternative policy with identical first period policies of $q_{1L} = q_{1H}$, but positive regulation of $q_{2j} = q^*_H$ for all $j = L, H$ in the second period. From (2.4) and (2.5), such a policy is clearly incentive compatible. We also have $q_H^* > 0$ since $B_q(0) > C_q(0, \pi_H)$ by assumption and B_q is decreasing and C_q is increasing in q.

No welfare difference exists in the first period. In the second period, the alternative policy incurs no welfare loss if the firm is the high cost type, so the alternative policy generates higher welfare than the unregulated economy if the firm is the high cost type. If the firm is the low cost type, then the difference in welfare loss between the alternative policy and unregulated firm is:

$$\Delta \text{ Loss } = \int_{q_H^*}^{q_L^*} \left(B_q(q) - C_q(q, \pi_L)\right) dq - \int_0^{q_L^*} \left(B_q(q) - C_q(q, \pi_L)\right) dq, \tag{8.36}$$

$$= -\int_0^{q_H^*} \left(B_q(q) - C_q(q, \pi_L)\right) dq. \tag{8.37}$$

Note that the function being integrated in (8.37) is positive over the domain of integration, since $B_q(q_H^*) = C_q(q_H^*, \pi_L)$, B_q is decreasing, and C_q is increasing. Thus the integral is positive, the difference in welfare losses is negative, and the alternative policy gives higher welfare regardless of firm type. Thus no regulation in the second period cannot be an optimum. Thus we have a contradiction and so $\lambda < 1 - \gamma$.

8.3 First Order Conditions With a Cost of Funds and Proof of Proposition 3

The first order conditions for problem (3.7) are:

$$B_q(q_{1L}) = c_q(q_{1L}, \pi_L)\left(1 + \frac{\lambda_L}{\gamma}\right), \tag{8.38}$$

$$B_q(q_{1H}) = C_q(q_{1H}, \pi_H)\left(1 - \frac{\lambda_L}{1-\gamma}\frac{C_q(q_{1H}, \pi_L)}{C_q(q_{1H}, \pi_H)}\right), \tag{8.39}$$

$$B_q(q_{2LL}) = c_q(q_{2LL}, \pi_L)\left(1 + \frac{\lambda_L}{\gamma} + \frac{\lambda_{LL}}{\gamma^2}\right), \tag{8.40}$$

$$B_q(q_{2LH}) = C_q(q_{2LH}, \pi_H)\left(1 + \frac{\lambda_L}{\gamma} - \frac{\lambda_{LL}}{\gamma(1-\gamma)}\frac{C_q(q_{2LH}, \pi_L)}{C_q(q_{2LH}, \pi_H)}\right), \tag{8.41}$$

$$B_q(q_{2HL}) = c_q(q_{2HL}, \pi_L)\left(1 - \frac{\lambda_L}{1-\gamma} + \frac{\lambda_{LH}}{\gamma(1-\gamma)}\right), \tag{8.42}$$

$$B_q(q_{2HH}) = C_q(q_{2HH}, \pi_H)\left(1 - \frac{\lambda_L}{1-\gamma} - \frac{\lambda_{LH}}{(1-\gamma)^2}\frac{C_q(q_{2HH}, \pi_L)}{C_q(q_{2HH}, \pi_H)}\right), \tag{8.43}$$

$$\frac{\mu_{LL}}{\gamma^2} = \phi - \frac{\lambda_L}{\gamma} - \frac{\lambda_{LL}}{\gamma^2}, \tag{8.44}$$

$$\frac{\mu_{LH}}{\gamma(1-\gamma)} = \phi - \frac{\lambda_L}{\gamma} + \frac{\lambda_{LL}}{\gamma(1-\gamma)}, \tag{8.45}$$

$$\frac{\mu_{HL}}{\gamma(1-\gamma)} = \phi + \frac{\lambda_L}{1-\gamma} - \frac{\lambda_{LH}}{\gamma(1-\gamma)}, \tag{8.46}$$

$$\frac{\mu_{HH}}{(1-\gamma)^2} = \phi + \frac{\lambda_L}{1-\gamma} + \frac{\lambda_{LH}}{(1-\gamma)^2}, \tag{8.47}$$

$$\mu_{ij}t_{ij} = 0, \quad i,j = L,H. \tag{8.48}$$

Imposing the solution $q_{1L} = q_{1LL} = q_{1HL} = q_L^*$ and $q_{1H} = q_{2LH} = q_{2HH} = q_H^*$ on (8.38)-(8.43), we see that the solution satisfies the first order conditions (8.38)-(8.43) if and only if $\lambda_L = \lambda_{LL} = \lambda_{LH} = 0$. Imposing this and $\phi = 0$ on (8.44)-(8.48), we see that the solution satisfies the first order conditions (8.44)-(8.48) if and only if $\mu_{ij} = 0$ for all i,j.

We next show a set of positive payments exists that satisfies all constraints. First, let $t_{HH} = 0$, then constraint (3.8), $i = H$, is satisfied for:

$$t_{HL} = \Delta C_L \equiv C(q_L^*, \pi_L) - C(q_H^*, \pi_L) > 0. \tag{8.49}$$

Next, we let $t_{LH} = \frac{\Delta C_L}{\delta}$ and $t_{LL} = \frac{1+\delta}{\delta}\Delta C_L$. Substituting these conditions and $q_{2LL} = q_L^*$ and $q_{2LH} = q_H^*$ into (3.8), $i = L$, implies constraint (3.8) is satisfied with equality. Substituting the proposed solution for t_{LH} and t_{LL} and the first best solutions for all q's into (3.10) implies (3.10) holds with equality.

Finally, substituting the proposed solution for all t_{ij} and the first best solution for all q's into (3.9), $i = L,H$ and (3.11), we see that all three constraints hold if and only if C

is super modular, which holds by assumption. Therefore, since all payments are positive and all constraints and first order conditions are satisfied, the first best level of regulation is optimal for $\phi = 0$.

8.4 Proof of Proposition 4

We must show the solution to (2.7)-(2.11) with $t_{ij} = 0$ for all i, j satisfies all first order conditions and constraints for (3.7). Comparing (2.7) and (8.38), we see that (8.38) is satisfied if and only if $\lambda_L = \lambda$, where λ is the multiplier for problem (2.6). Condition (2.8) implies condition (8.39) is also satisfied for $\lambda_L = \lambda$.

Next, imposing $\lambda_L = \lambda$ and $q_{2LL} = q_{2L}$ on (8.40), we see that (8.40) holds if and only if:

$$\frac{B_q(q_{2L})}{c_q(q_{2L}, \pi_L)} = \left(1 + \frac{\lambda}{\gamma} + \frac{\lambda_{LL}}{\gamma^2}\right). \tag{8.50}$$

Using (2.9), this is equivalent to:

$$\frac{\mathrm{E}\left[c_q(q_{2L}, \pi)\right]}{c_q(q_{2L}, \pi_L)} \left(1 + \frac{\lambda}{\gamma}\right) = \left(1 + \frac{\lambda}{\gamma} + \frac{\lambda_{LL}}{\gamma^2}\right). \tag{8.51}$$

Using the definition of R:

$$(\gamma + (1-\gamma) R) \left(1 + \frac{\lambda}{\gamma}\right) = \left(1 + \frac{\lambda}{\gamma} + \frac{\lambda_{LL}}{\gamma^2}\right), \tag{8.52}$$

$$\lambda_{LL} = \gamma^2 (1-\gamma)(R-1) \left(1 + \frac{\lambda}{\gamma}\right). \tag{8.53}$$

So if (8.53) holds, condition (8.40) is satisfied. Further, imposing $q_{2LH} = q_{2L}$ on (8.41) and using (2.9) to eliminate the marginal benefit function, we see that (8.41) holds if and only if (8.53) holds.

For (8.42), we impose $q_{2HL} = q_{2H}$ and $\lambda_L = \lambda$, yielding:

$$\frac{B_q(q_{2H})}{c_q(q_{2H}, \pi_L)} = 1 - \frac{\lambda}{1-\gamma} + \frac{\lambda_{LH}}{\gamma(1-\gamma)}. \tag{8.54}$$

Imposing (2.10) gives:

$$\frac{\mathrm{E}\left[C_q(q_{2H}, \pi)\right]}{c_q(q_{2H}, \pi_L)} \left(1 - \frac{\lambda}{1-\gamma}\right) = 1 - \frac{\lambda}{1-\gamma} + \frac{\lambda_{LH}}{\gamma(1-\gamma)}. \tag{8.55}$$

Finally, using the definition of R:

$$(\gamma + (1-\gamma)\, R)\left(1 - \frac{\lambda}{1-\gamma}\right) = 1 - \frac{\lambda}{1-\gamma} + \frac{\lambda_{LH}}{\gamma\,(1-\gamma)}, \tag{8.56}$$

$$\lambda_{LH} = \gamma\,(1-\gamma)^2\,(R-1)\left(1 - \frac{\lambda}{1-\gamma}\right). \tag{8.57}$$

So condition (8.42) is satisfied if and only if (8.57) holds. Further, an analogous argument shows that (8.43) holds if and only if (8.57) holds.

Next, our solution requires all payments to be zero, and thus all of the multipliers μ_{ij} to be positive. From (8.44) given $\lambda_L = \lambda$, this requires:

$$\frac{\mu_{LL}}{\gamma^2} = \phi - \frac{\lambda}{\gamma} - \frac{\lambda_{LL}}{\gamma^2} > 0. \tag{8.58}$$

Given (8.53), the above inequality holds if and only if:

$$\phi > \frac{\lambda}{\gamma} + (1-\gamma)\,(R-1)\left(1 + \frac{\lambda}{\gamma}\right), \tag{8.59}$$

$$\phi > (1-\gamma)\,(R-1) + \frac{\lambda}{\gamma}\,(\gamma + (1-\gamma)\,R)\,. \tag{8.60}$$

So for ϕ sufficiently big, $t_{LL} = 0$ as required, with μ_{LL} defined by substituting (8.53) and $\lambda_L = \lambda$ into (8.44). Recall λ is the multiplier from problem (2.6), and thus is independent of ϕ, so a ϕ sufficiently large always exists.

For t_{LH}, (8.45) requires:

$$\frac{\mu_{LH}}{\gamma\,(1-\gamma)} = \phi - \frac{\lambda}{\gamma} + \frac{\lambda_{LL}}{\gamma\,(1-\gamma)} > 0, \tag{8.61}$$

Using (8.53) and simplifying gives:

$$\phi > \frac{\lambda}{\gamma} - \gamma\,(R-1)\left(1 + \frac{\lambda}{\gamma}\right). \tag{8.62}$$

So for ϕ satisfying (8.62), $t_{LH} = 0$ as required, with μ_{LH} defined by substituting (8.53) and $\lambda_L = \lambda$ into (8.45).

Condition (8.60) is more restrictive than condition (8.62) if and only if:

$$(1-\gamma)(R-1)+\frac{\lambda}{\gamma}(\gamma+(1-\gamma)R)>\frac{\lambda}{\gamma}-\gamma(R-1)\left(1+\frac{\lambda}{\gamma}\right). \tag{8.63}$$

Simplifying gives:

$$(R-1)\left(1+\frac{\lambda}{\gamma}\right)>0, \tag{8.64}$$

which holds since $R>1$.

For t_{HL}, (8.46) requires:

$$\frac{\mu_{HL}}{\gamma(1-\gamma)}=\phi+\frac{\lambda}{1-\gamma}-\frac{\lambda_{LH}}{\gamma(1-\gamma)}>0. \tag{8.65}$$

Substituting in (8.57) and simplifying gives:

$$\phi>(1-\gamma)(R-1)\left(1-\frac{\lambda}{1-\gamma}\right)-\frac{\lambda}{1-\gamma}. \tag{8.66}$$

Given (8.66), $t_{HL}=0$ and μ_{HL} is defined by substituting (8.57) into (8.46). Clearly (8.66) is less restrictive than (8.60).

Finally, note that for t_{HH}, the multiplier is always positive and so $t_{HH}=0$, with the multiplier defined by (8.47) and (8.57).

Therefore, given (8.60), the proposed solution satisfies the first order conditions. Clearly, for C supermodular, the solution satisfies the first period incentive compatibility constraints, which are identical to the incentive compatibility constraints from the problem without payments. The second period incentive compatibility constraints are also satisfied since payments are zero and $q_{2LH}=q_{2LL}=q_{2L}$ and $q_{2HL}=q_{2HH}=q_{2H}$. Therefore the proposed solution satisfies all constraints and first order conditions given (8.60).

Finally, we can bound (8.60) using Proposition 2.3. Since $\lambda<1-\gamma$, a sufficient condition for (8.60) is:

$$\phi>(1-\gamma)(R-1)+\frac{1-\gamma}{\gamma}(\gamma+(1-\gamma)R). \tag{8.67}$$

The above equation simplifies to:

$$\phi>\frac{(1-\gamma)R}{\gamma}, \tag{8.68}$$

which is the desired result.

8.5 Proof of Proposition 5

Suppose not, suppose that $q_{1H} = q_{2LH} = q_{2HH}$ and $\phi > 0$. Then since the cost function is convex and the benefit function is concave:

$$\frac{B_q(q_{1H})}{C_q(q_{1H}, \pi_H)} = \frac{B_q(q_{2HH})}{C_q(q_{2HH}, \pi_H)}. \tag{8.69}$$

Substituting equations (8.39) and (8.43) and simplifying gives:

$$1 - \frac{\lambda_L}{1-\gamma}\frac{1}{R} = 1 - \frac{\lambda_L}{1-\gamma} - \frac{\lambda_{LH}}{(1-\gamma)^2}\frac{1}{R}, \tag{8.70}$$

$$\frac{\lambda_{LH}}{1-\gamma}\frac{1}{R} = -\lambda_L\left(\frac{R-1}{R}\right). \tag{8.71}$$

Since λ_L and λ_{LH} are non-negative and $R > 1$, the above equation is satisfied if and only if $\lambda_L = \lambda_{LH} = 0$.

Further, we have:

$$\frac{B_q(q_{2LH})}{C_q(q_{2LH}, \pi_H)} = \frac{B_q(q_{1H})}{C_q(q_{1H}, \pi_H)}. \tag{8.72}$$

Using equations (8.39) and (8.41) and simplifying gives:

$$\frac{\lambda_L}{\gamma} - \frac{\lambda_{LL}}{\gamma(1-\gamma)}\frac{1}{R} = -\frac{\lambda_L}{1-\gamma}\frac{1}{R}. \tag{8.73}$$

Since $\lambda_L = 0$, we must have $\lambda_{LL} = 0$. Thus, $q_{1H} = q_{2LH} = q_{2HH}$ implies all incentive constraints are non-binding. The first order conditions (8.38)-(8.43) then imply the regulator achieves the first best allocation $q_{1L} = q_{2HL} = q_{2LL} = q_L^*$ and $q_{1H} = q_{2LH} = q_{2HH} = q_H^*$.

Plugging in the first best decisions for all i, j into for example, the incentive constraint (3.8), $i = L$, implies:

$$t_{LL} - C(q_L^*, \pi_L) \geq t_{LH} - C(q_H^*, \pi_L), \tag{8.74}$$

which implies $t_{LL} > t_{LH} \geq 0$ since $q_L^* > q_H^*$.

Next, plugging in $\lambda_L = \lambda_{LL} = \lambda_{LH} = 0$ into (8.44) and evaluating (8.48) at LL gives:

$$\mu_{LL} = \gamma^2 \phi, \tag{8.75}$$

$$\mu_{LL} t_{LL} = 0. \tag{8.76}$$

The above two equations are satisfied only if $\phi = 0$ since $t_{LL} > 0$, which contradicts $\phi > 0$. Thus $q_{1H} = q_{2LH} = q_{2HH}$ cannot hold for $\phi > 0$.

8.6 Proof of Proposition 6

We first show that upward incentive compatibility (4.19) is always satisfied if constraint (4.18) is satisfied with equality and the level of regulation in period one is monotonic non-increasing in cost $q_{1L} \geq q_{1H}$. Given the low (high) cost firm expects costs to rise (fall):

$$\frac{\bar{\pi}_H}{\pi_H} \leq \frac{\bar{\pi}_L}{\pi_L}. \tag{8.77}$$

Hence given $q_{1L} \geq q_{1H}$:

$$\frac{\bar{\pi}_H}{\bar{\pi}_L} \pi_L \left(c\left(q_{1L}\right) - c\left(q_{1H}\right)\right) \leq \pi_H \left(c\left(q_{1L}\right) - c\left(q_{1H}\right)\right). \tag{8.78}$$

Next, since (4.18) holds with equality:

$$\pi_L \left(c\left(q_{1L}\right) - c\left(q_{1H}\right)\right) = \delta \bar{\pi}_L \left(c\left(q_{2H}\right) - c\left(q_{2L}\right)\right). \tag{8.79}$$

Substituting (8.79) into (8.78) gives:

$$\frac{\bar{\pi}_H}{\bar{\pi}_L} \delta \bar{\pi}_L \left(c\left(q_{2H}\right) - c\left(q_{2L}\right)\right) \leq \pi_H \left(c\left(q_{1L}\right) - c\left(q_{1H}\right)\right), \tag{8.80}$$

$$\delta \bar{\pi}_H \left(c\left(q_{2H}\right) - c\left(q_{2L}\right)\right) \leq \pi_H \left(c\left(q_{1L}\right) - c\left(q_{1H}\right)\right), \tag{8.81}$$

$$-\pi_H c\left(q_{1L}\right) - \delta \bar{\pi}_H c\left(q_{2L}\right) \leq -\pi_H c\left(q_{1H}\right) - \delta \bar{\pi}_H c\left(q_{2H}\right), \tag{8.82}$$

which is just (4.19). Provided the solution to a relaxed problem which ignores constraint (4.19) satisfies $q_{1L} \geq q_{1H}$, it also solves the original problem.

The first order conditions for the relaxed problem are:

$$B_q(q_{1L}) = \pi_L \left(1 + \frac{\lambda}{\gamma}\right) c_q(q_{1L}), \tag{8.83}$$

$$B_q(q_{1H}) = \pi_H \left(1 - \frac{\lambda}{1-\gamma}\frac{\pi_L}{\pi_H}\right) c_q(q_{1H}), \tag{8.84}$$

$$B_q(q_{2L}) = \bar{\pi}_L \left(1 + \frac{\lambda}{\gamma}\right) c_q(q_{2L}), \tag{8.85}$$

$$B_q(q_{2H}) = \bar{\pi}_H \left(1 - \frac{\lambda}{1-\gamma}\frac{\bar{\pi}_L}{\bar{\pi}_H}\right) c_q(q_{2H}), \tag{8.86}$$

$$\pi_L \left(c(q_{1L}) - c(q_{1H})\right) = \delta \bar{\pi}_L \left(c(q_{2H}) - c(q_{2L})\right). \tag{8.87}$$

Define $\lambda_L = \gamma(1-\gamma)\left(\frac{\bar{\pi}_H}{\bar{\pi}_L} - 1\right)$ and $\lambda_R = \gamma(1-\gamma)\left(\frac{\pi_H}{\pi_L} - 1\right)$. Observe that:

$$\lambda < (=)\lambda_R \Leftrightarrow q_{1L} > (=) q_{1H}, \tag{8.88}$$

$$\lambda > (=)\lambda_L \Leftrightarrow q_{2H} > (=) q_{2L}, \tag{8.89}$$

$$\frac{\bar{\pi}_H}{\bar{\pi}_L} < (=) \frac{\pi_H}{\pi_L} \Leftrightarrow \lambda_L < (=) \lambda_R. \tag{8.90}$$

Consider first the case in which condition (8.90) holds with equality. Consider the solution $\lambda = \lambda_L = \lambda_R$. Equations (8.88) and (8.89) imply that $q_{1L} = q_{1H}$ and $q_{2L} = q_{2H}$ satisfy the first order conditions. Therefore the incentive constraints (4.18) and (4.19) are satisfied. Plugging in the solution for λ into any of the first period first order conditions, we see that $q_{1L} = q_{1H} = \bar{q}_1$. Further, plugging in the solution for λ into either of the second period first order conditions implies $q_{2L} = q_{2H} = \bar{q}_2$. Therefore, the regulator offers the no information regulation level to both types in both periods.

Next, consider the case where the inequality in (8.90) is strict. Consider $\lambda = \lambda_L < \lambda_R$. Since $q_{2H} = q_{2L}$, the incentive constraint (4.18) is satisfied if and only if $q_{1H} \geq q_{1L}$, but this contradicts (8.88) since $\lambda < \lambda_R$.

Conversely, consider $\lambda = \lambda_R > \lambda_L$. Since $q_{1H} = q_{1L}$, the incentive constraint (4.18) is satisfied if and only if $q_{2H} \geq q_{2L}$, which holds by (8.89) since $\lambda > \lambda_L$.

Therefore, the incentive constraint is slack at $\lambda = \lambda_R$, and is violated at $\lambda = \lambda_L$. By the intermediate value theorem, there exists $\lambda^* \in (\lambda_L, \lambda_R)$ for which the incentive constraint (4.18) holds with equality. From (8.88)-(8.89), at λ^*, $q_{1L} > q_{1H}$ and $q_{2H} > q_{2L}$. Therefore the solution to the relaxed problem solves the original problem.

We next compute the properties of the solution. First $q_{1L} > \bar{q}_1$ if and only if:

$$\frac{B_q(q_{1L})}{c_q(q_{1L})} < \frac{B_q(\bar{q}_1)}{c_q(\bar{q}_1)}. \tag{8.91}$$

From the first order conditions and the definition of $\bar{q}_1$, equation (8.91) holds if and only if:

$$\pi_L\left(1+\frac{\lambda}{\gamma}\right) < \bar{\pi}. \tag{8.92}$$

Using the definition of $\bar{\pi}$, we have:

$$\frac{\lambda}{\gamma}\pi_L < (1-\gamma)(\pi_H - \pi_L), \tag{8.93}$$

$$\lambda < \gamma(1-\gamma)\left(\frac{\pi_H}{\pi_L} - 1\right) = \lambda_R, \tag{8.94}$$

which holds by (8.88).

Next, $q_{2L} < \bar{q}_2$ if and only if:

$$\frac{B_q(q_{2L})}{c_q(q_{2L})} > \frac{B_q(\bar{q}_2)}{c_q(\bar{q}_2)}, \tag{8.95}$$

$$\bar{\pi}_L\left(1+\frac{\lambda}{\gamma}\right) > \mathrm{E}[\pi_2] = \gamma\bar{\pi}_L + (1-\gamma)\bar{\pi}_H, \tag{8.96}$$

$$\lambda > \gamma(1-\gamma)\left(\frac{\bar{\pi}_H}{\bar{\pi}_L} - 1\right) = \lambda_L, \tag{8.97}$$

which holds by (8.89).

Analogous arguments show that $q_{1H} < \bar{q}_1$ and $q_{2H} > \bar{q}_2$.

8.7 Proof of Proposition 7

7.2. Starting with the second period policies, we combine (5.11) and (5.12), so that:

$$\mathrm{E}\left[C_q\left(q_{2L,m},\pi\right)\right]\left(1+\frac{\lambda}{\gamma}\right)=\mathrm{E}\left[C_q\left(q_{2H,m},\pi\right)\right]\left(1-\frac{\lambda}{1-\gamma}\right). \tag{8.98}$$

Since C_q is an increasing function, it is immediate that $q_{2L,m} < q_{2H,m}$.

Next, $q_{2L,m} < \bar{q}_n$ if and only if:

$$\frac{B_q\left(nq_{2L,m}\right)}{\mathrm{E}\left[C_q\left(q_{2L,m},\pi\right)\right]} > \frac{B_q\left(n\bar{q}_n\right)}{\mathrm{E}\left[C_q\left(\bar{q}_n,\pi\right)\right]}. \tag{8.99}$$

Equation (5.11) and the definition of $\bar{q}_n$ implies:

$$\left(1+\frac{\lambda}{\gamma}\right)\frac{B_q\left(nq_{2L,m}\right)}{B_q\left(mq_{2L,m}+(n-m)\,q_{2H,m}\right)} > 1. \tag{8.100}$$

Therefore, it is sufficient to show:

$$B_q\left(nq_{2L,m}\right) > B_q\left(mq_{2L,m}+(n-m)\,q_{2H,m}\right), \tag{8.101}$$

which holds if and only if:

$$nq_{2L,m} > mq_{2L,m}+(n-m)\,q_{2H,m}, \tag{8.102}$$

$$q_{2H,m} > q_{2L,m}, \tag{8.103}$$

which holds as shown above. Thus, $q_{2L,m} < \bar{q}_n$.

Similarly, $q_{2H,m} > \bar{q}_n$ if and only if:

$$\frac{B_q\left(nq_{2H,m}\right)}{\mathrm{E}\left[C_q\left(q_{2H,m},\pi\right)\right]} < \frac{B_q\left(n\bar{q}_n\right)}{\mathrm{E}\left[C_q\left(\bar{q}_n,\pi\right)\right]}. \tag{8.104}$$

Equation (5.12) and the definition of $\bar{q}_n$ implies:

$$\left(1-\frac{\lambda}{1-\gamma}\right)\frac{B_q\left(nq_{2H,m}\right)}{B_q\left(mq_{2L,m}+\left(n-m\right)q_{2H,m}\right)}<1. \tag{8.105}$$

Therefore, it is sufficient to show:

$$B_q\left(nq_{2H,m}\right)<B_q\left(mq_{2L,m}+\left(n-m\right)q_{2H,m}\right), \tag{8.106}$$

$$nq_{2H,m}<mq_{2L,m}+\left(n-m\right)q_{2H,m}, \tag{8.107}$$

$$q_{2L,m}<q_{2H,m}, \tag{8.108}$$

which holds as shown above. Thus, $q_{2L,m}<\bar{q}_n$.

7.1. For the first period policies, we begin by showing $q_{1H,m}<q_{1L,m}$, which requires several steps. The first step is to show that the sign of $q_{1L,m}-q_{1H,m}$ does not vary with m. To see this, we combine (5.7) and (5.10):

$$\left(1+\frac{\lambda}{\gamma}\right)C_q\left(q_{1L,m},\pi_L\right)=\left(1-\frac{\lambda}{1-\gamma}\frac{1}{R}\right)C_q\left(q_{1H,m},\pi_H\right). \tag{8.109}$$

By the definition of R:

$$\left(1+\frac{\lambda}{\gamma}\right)C_q\left(q_{1L,m},\pi_L\right)=\left(R-\frac{\lambda}{1-\gamma}\right)C_q\left(q_{1H,m},\pi_L\right), \tag{8.110}$$

$$\frac{C_q\left(q_{1L,m},\pi_L\right)}{C_q\left(q_{1H,m},\pi_L\right)}=\frac{R-\frac{\lambda}{1-\gamma}}{1+\frac{\lambda}{\gamma}}. \tag{8.111}$$

Since C is convex, $q_{1L,m}-q_{1H,m}>0$ if and only if the right hand side of (8.111) is greater than one or if and only if:

$$\lambda < \gamma (1-\gamma)(R-1). \tag{8.112}$$

Since (8.112) is independent of m, the sign of $q_{1L,m} - q_{1H,m}$ is independent of m.

The next step is to show $q_{1i}(m, \lambda)$, $i = L, H$ is an increasing function of m if and only if $q_{1L,m} > q_{1H,m}$. Totally differentiating (5.7) and (5.10) with respect to m and solving for $q'_{1L}(m)$ gives:

$$\begin{aligned} q'_{1L}(m) &= \frac{B_{qq} \cdot (q_{1L,m} - q_{1H,m})}{-B_{qq} \cdot (m + (n-m) z) + C_{qq}(q_{1L,m}, \pi_L)\left(1 + \frac{\lambda}{\gamma}\right)} \\ q'_{1L}(m) &= z q'_{1L,m}, \qquad z \equiv \frac{C_{qq}(q_{1L,m}, \pi_L)\left(1 + \frac{\lambda}{\gamma}\right)}{C_{qq}(q_{1L,m}, \pi_L)\left(R - \frac{\lambda}{1-\gamma}\right)}. \end{aligned} \tag{8.113}$$

Here we have suppressed the function arguments for B_{qq}. Hence both derivatives are positive if and only if $q_{1L,m} > q_{1H,m}$.

Finally, to prove $q_{1L,m} > q_{1H,m}$ we suppose not, so that $q_{1L,m} \leq q_{1H,m}$. If so, then step one implies the inequality holds for all m and step two implies both derivatives are decreasing functions of m. From the incentive constraint (5.2):

$$\sum_{m=1}^{n} Pr(m|L)\left[C(q_{1H,m-1}, \pi_L) - C(q_{1L,m}, \pi_L) - \delta \mathrm{E}\left[C(q_{2H,m-1}, \pi) - C(q_{2L,m}, \pi)\right]\right] \geq 0,$$

with strict inequality only if $\lambda = 0$. Rewriting results in:

$$\begin{aligned} &\sum_{m=1}^{n-1} Pr(m|L)\Big[(C(q_{1H,m-1}, \pi_L) - C(q_{1H,m}, \pi_L)) + (C(q_{1H,m}, \pi_L) - C(q_{1L,m}, \pi_L)) \\ &\quad + \delta \mathrm{E}\left[(C(q_{2H,m-1}, \pi) - C(q_{2H,m}, \pi)) + (C(q_{2H,m}, \pi) - C(q_{2L,m}, \pi))\right]\Big] + \\ &\quad Pr(n|L)\Big[(C(q_{1L,n-1}, \pi_L) - C(q_{1L,n}, \pi_L)) + (C(q_{1H,n-1}, \pi_L) - C(q_{1L,n-1}, \pi_L)) + \\ &\quad \delta \mathrm{E}\left[(C(q_{2L,n-1}, \pi) - C(q_{2L,n}, \pi)) + (C(q_{2H,n-1}, \pi) - C(q_{2L,n-1}, \pi))\right]\Big] > 0. \end{aligned} \tag{8.114}$$

Notice that the second term in rows one and three are non-negative, since we have supposed $q_{1L,m} \leq q_{1H,m}$, which, if true holds for all m. The second terms in rows two and four are strictly positive, since we have shown $q_{2L,m} < q_{2H,m}$. The first term in each row is non-negative since $q_{1L,m}$ and $q_{1H,m}$ are non-increasing in m, given $q_{1L,m} \leq q_{1H,m}$. Thus the incentive constraint is strictly positive.

However, the Kuhn-Tucker condition then implies $\lambda = 0$. Then, from the first order conditions (5.7) and (5.10), the first best solution results: $q_{1L,m} = q^*_{Lm} > q_{1H,m} = q^*_{Hm}$, which contradicts that $q_{1L,m} \leq q_{1H,m}$. Thus $q_{1L,m} > q_{1H,m}$.

With this result in hand, we now show that $q_{1L,m} > \bar{q}_n$. First, multiplying the first order conditions (5.7) and (5.10) by γ and $1-\gamma$, respectively, and using the definition of R gives:

$$\gamma B_q \left(mq_{1L,m} + (n-m)\, q_{1H,m}\right) = (\gamma + \lambda)\, C_q \left(q_{1L,m}, \pi_L\right), \tag{8.115}$$

$$(1-\gamma)\, B_q \left(mq_{1L,m} + (n-m)\, q_{1H,m}\right) = \left(R\,(1-\gamma) - \lambda\right) C_q \left(q_{1H,m}, \pi_L\right). \tag{8.116}$$

Since $q_{1L,m} > q_{1H,m}$:

$$(1-\gamma)\, B_q \left(mq_{1L,m} + (n-m)\, q_{1H,m}\right) < \left(R\,(1-\gamma) - \lambda\right) C_q \left(q_{1L,m}, \pi_L\right). \tag{8.117}$$

Adding equations (8.115) and (8.117) gives:

$$B_q \left(mq_{1L,m} + (n-m)\, q_{1H,m}\right) < \left(\gamma + R\,(1-\gamma)\right) C_q \left(q_{1L,m}, \pi_L\right) = \mathrm{E}\left[C_q \left(q_{1L,m}, \pi\right)\right]. \tag{8.118}$$

Here the last equality follows from the definition of R. Now since $q_{1L,m} < q_{1H,m}$:

$$B_q \left(nq_{1L,m}\right) < B_q \left(mq_{1L,m} + (n-m)\, q_{1H,m}\right) < \mathrm{E}\left[C_q \left(q_{1L,m}, \pi\right)\right]. \tag{8.119}$$

Finally, from the definition of $\bar{q}_n$.

$$\frac{B_q \left(nq_{1L,m}\right)}{\mathrm{E}\left[C_q \left(q_{1L,m}, \pi\right)\right]} = 1 = \frac{B_q \left(n\bar{q}_n\right)}{\mathrm{E}\left[C_q \left(\bar{q}_n, \pi\right)\right]}. \tag{8.120}$$

Since the right and left hand sides are decreasing functions, $q_{1L,m} > \bar{q}_n$ as desired.

The proof that $q_{1H,m} < \bar{q}_n$ follows an identical logic.

7.3. That $q_{1L,m}$ and $q_{1H,m}$ are increasing functions of m follows immediately from $q_{1L,m} > q_{1H,m}$ and (8.113).

8.8 Proof of Proposition 8

Starting with the second claim, first note that for the second order conditions for the regulator's problem to hold, a necessary condition is that the derivative of equation (6.10) with respect to q_{2L} is negative:

$$B_{qq} - \left(\mathrm{E}\left[C_{qq}\right] + \mathrm{E}\left[C_{q\zeta}\right]\zeta_L'\right)\left(1+\frac{\lambda}{\gamma}\right) - \left(1+\frac{\lambda}{\gamma}\right)\left(\mathrm{E}\left[C_{\zeta}\right] + P_{\zeta}\right)\zeta_L'' - \left(1+\frac{\lambda}{\gamma}\right)\left(\mathrm{E}\left[C_{qq}\right]\zeta_L' + \mathrm{E}\left[C_{q\zeta}\right]\right)\zeta_L'. \tag{8.121}$$

Differentiating the firm first order condition (6.2) gives:

$$\zeta_q\left(q\right) = \frac{\mathrm{E}\left[C_{\zeta q}\left(q,\pi,\zeta\left(q\right)\right)\right]}{\mathrm{E}\left[C_{qq}\left(q,\pi,\zeta\left(q\right)\right)\right]}. \tag{8.122}$$

Substituting (8.122) and the firm first order condition (6.2) into (8.121) and simplifying gives:

$$B_{qq} - \left(\mathrm{E}\left[C_{qq}\right] + \mathrm{E}\left[C_{q\zeta}\right]\zeta_L'\right)\left(1+\frac{\lambda}{\gamma}\right). \tag{8.123}$$

Thus, $B'\left(q\right) - \mathrm{E}\left[C_q\left(q,\pi,\zeta\left(q\right)\right)\right]$ is decreasing in q. Therefore $q_{2L} < \bar{q}_\zeta$ if and only if:

$$B_q\left(q_{2L}\right) - \left(1+\frac{\lambda}{\gamma}\right)\mathrm{E}\left[C_q\left(q_{2L},\pi,\zeta_L\right)\right] > B_q\left(\bar{q}_\zeta\right) - \left(1+\frac{\lambda}{\gamma}\right)\mathrm{E}\left[C_q\left(\bar{q}_\zeta,\pi,\zeta\left(\bar{q}_\zeta\right)\right)\right]. \tag{8.124}$$

From the definition of $\bar{q}_\zeta$ and (6.11):

$$0 > \mathrm{E}\left[C_q\left(\bar{q}_\zeta,\pi,\zeta\left(\bar{q}_\zeta\right)\right)\right] - \left(1+\frac{\lambda}{\gamma}\right)\mathrm{E}\left[C_q\left(\bar{q}_\zeta,\pi,\zeta\left(\bar{q}_\zeta\right)\right)\right], \tag{8.125}$$

which holds since $\lambda > 0$.

Similarly, for $q_{2H} > \bar{q}$, the second or order conditions imply $q_{2H} > \bar{q}$ if and only if:

$$
\begin{aligned}
&B_q\left(q_{2H}\right) - \left(1 - \frac{\lambda}{1-\gamma}\right) \mathrm{E}\left[C_q\left(q_{2H}, \pi, \zeta_H\right)\right] < \\
&\quad B_q\left(\bar{q}_\zeta\right) - \left(1 - \frac{\lambda}{1-\gamma}\right) \mathrm{E}\left[C_q\left(\bar{q}_\zeta, \pi, \zeta\left(\bar{q}_\zeta\right)\right)\right]. \qquad (8.126)
\end{aligned}
$$

Using the definition of $\bar{q}_\zeta$ and equation (6.12):

$$
0 < \mathrm{E}\left[C_q\left(\bar{q}_\zeta, \pi, \zeta\left(\bar{q}_\zeta\right)\right)\right] - \left(1 - \frac{\lambda}{1-\gamma}\right) \mathrm{E}\left[C_q\left(\bar{q}_\zeta, \pi, \zeta\left(\bar{q}_\zeta\right)\right)\right], \qquad (8.127)
$$

which holds since $\lambda > 0$.

For the first claim, we first show that $q_{1L} > q_{1H}$. The incentive constraint (6.13) implies:

$$
\begin{aligned}
&C\left(q_{1H}, \pi_L, 0\right) - C\left(q_{1L}, \pi_L, 0\right) = \\
&\quad \delta\left(\mathrm{E}\left[C\left(q_{2L}, \pi, \zeta_L\right)\right] + P_\zeta \zeta_L - \mathrm{E}\left[C\left(q_{2H}, \pi, \zeta_H\right)\right] - P_\zeta \zeta_H\right). \qquad (8.128)
\end{aligned}
$$

A first order Taylor expansion of $\mathrm{E}\left[C\left(q_{2L}, \pi, \zeta\left(q_{2L}\right)\right)\right] + P_\zeta \zeta\left(q_{2L}\right)$ around q_{2H} implies the right and side is approximately:

$$
r.h.s \approx \delta\left(\mathrm{E}\left[C_q\left(q_{2H}, \pi, \zeta_H\right)\right] + \left(\mathrm{E}\left[C_\zeta\left(q_{2H}, \pi, \zeta_H\right)\right] + P_\zeta\right) \zeta_q\left(q_{2H}\right)\right)\left(q_{2L} - q_{2H}\right), \qquad (8.129)
$$

$$
= \delta \mathrm{E}\left[C_q\left(q_{2H}, \pi, \zeta_H\right)\right]\left(q_{2L} - q_{2H}\right) < 0. \qquad (8.130)
$$

Here the last equality uses the firm first order condition (6.2). Since the right hand side is negative, equation (8.128) implies $q_{1L} > q_{1H}$.

Given $q_{1L} > q_{1H}$, the proof that $q_{1H} < \bar{q} < q_{1L}$ is identical to the proof of proposition (2). The proof of proposition (2) given $q_{1H} < \bar{q} < q_{1L}$ depends only on equations (2.7) and (2.8). In turn, these equations are identical to equations (6.8) and (6.9). Hence the proof is identical and $q_{1H} < \bar{q} < q_{1L}$.