

## Optimal Second Best Taxation of Addictive Goods

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PRELIMINARY

## **Abstract**

In this paper we derive conditions under which optimal tax rates for addictive goods exceed tax rates for non-addictive consumption goods in an environment where exogenous government spending cannot be financed with lump sum taxes.

Standard static models that consider revenue raising and externalities predict taxing addictive goods at a rate far in excess of that observed in the data. In contrast, our results indicate that, given reasonable parameter values for the strengths of tolerance for the addictive good, homogeneity of the addiction function and the elasticity of substitution, the tax rates are likely to be smaller than the ones implied by the static case. This is the case because high current tax rates on addictive goods tend to reduce future tax revenues, by making households less addicted in the future. Finally, we consider features of addictive goods such as complementarity to leisure that, while unrelated to addiction itself, are nonetheless common among some addictive goods. In general, such effects are weaker in our dynamic setting since if taxing addictive goods has strong positive revenue effects today, then taxing goods has a strong offsetting effect on future tax revenues.

## 1 Introduction

A popular and increasingly common way for local, state, and federal governments to raise revenue is by taxation of addictive goods including, cigarettes, alcohol, and gambling. What is the optimal excise tax for addictive goods, when the government must raise revenue to finance a stream of exogenous government expenditures?

The goal of this paper is to characterize and analyze the conditions under which taxation of addictive goods might differ from taxes on labor and non-addictive consumption goods (hereafter *ordinary goods*). We derive conditions under which tax rates for addictive goods exceeds tax rates for ordinary goods in an environment where the government is motivated by fiscal revenue needs and where exogenous government spending cannot be financed with lump sum taxes. As noted by Becker and Murphy (1988), addictive goods are characterized by tolerance. Tolerance is when past consumption lowers current utility, which is also known as harmful addiction. We show that tolerance makes taxing addictive goods less attractive. Suppose, for example, that consumption in excess of that required to sustain the addiction (hereafter *effective consumption*) is complementary with leisure. Standard public finance theory then implies the tax rate on addictive goods should be relatively high, since reduced consumption of addictive goods will increase labor supply, thus raising labor tax revenues. However, if the good is addictive, then reduced current consumption of addictive goods raises future effective consumption. But then future labor supply falls, and future labor tax revenues falls, offsetting some of the revenue gains in the current period. This type of dynamic effect is not captured by typical static models used to estimate optimal tax rates on addictive goods. Thus ignoring the dynamic nature of addiction when designing optimal fiscal policy may result in lower welfare.

As pointed out by Bossi and Gomis-Porqueras (2006), the two standard models of effective consumption, the subtractive (e.g. Campbell and Cochrane 1999) and the multiplicative specification (e.g. Abel 1990), differ in terms of their homogeneity properties.<sup>1</sup> This paper shows that the optimal tax package for the case when addiction follows the subtractive specification differs significantly from the case where addiction follows the multiplicative one. In particular, under reasonable conditions, we show that if the addiction model is homogeneous of degree one, as in the subtractive case, homotheticity is not broken and a uniform tax across all consumption goods is optimal. On the other hand, if the addiction

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<sup>1</sup>A good is habit forming if the marginal utility of the good is increasing in past consumption. We use the standard definition of addiction, which is when current consumption is increasing in past consumption, holding fixed the marginal utility of wealth and prices. Habit formation is often used in the macro literature, whereas addiction was introduced by Becker and Murphy (1988). It is straightforward to show that the subtractive model of habit formation implies the good is addictive, and the multiplicative model of habit formation implies the good is addictive with an additional restriction.

function is homogeneous of degree less than one, as in the multiplicative case, an equi-proportionate decrease in consumption of addictive and ordinary goods raises the marginal utility of addictive goods more, since the minimum consumption level is higher than that of ordinary goods. Hence, the optimal tax package causes a larger percentage decrease in the consumption of ordinary goods, which in turn implies the optimal tax rate on addictive goods to be smaller. Finally, if we consider separable and CRR utility functions, the addictive good is taxed at a higher rate if and only if the addictive good has a lower income elasticity. We also find that the relative tax rate on the addictive good rises as the income elasticity of the addictive good falls, irrespective of the degree of addiction.

In the next sections we describe the underlying motives and characteristics of taxing addictive goods found in the literature. We then develop a dynamic model in order to determine the conditions under which optimal tax rates for addictive goods exceed tax rates for non-addictive consumption goods in an environment where exogenous government spending cannot be financed with lump sum taxes.

## 2 Taxing Addictive Goods

Three motivations exist in the literature for taxing addictive goods differently than ordinary goods. The first is to lower the external costs often associated with consumption of addictive goods. The second is because some consumers fail to take into account some private costs and thus over-consume. The third motivation is to raise revenue.

### 2.1 Addictive Goods and Externalities

The standard economic rationale for taxation of addictive goods is that consumption of addictive goods are often associated with external costs, such as second-hand smoke, drunk driving, and crime. However, it is well known (Kenkel 1996, Pogue and Sgontz 1989) that taxing an addictive good (e.g. alcohol) whose consumption is imperfectly correlated with an externality is a second-best solution. Taxing the actual behavior causing the externality (e.g. make the punishment for drunk driving more severe) is more efficient. Indeed, Parry, Laxminarayan, and West (2006) show that welfare gains from increasing drunk driving penalties exceed those from raising taxes on alcohol, even when implementation costs and dead-weight losses associated with incarceration are included.

The literature tends to find that most addictive goods are taxed at a rate less than the rate which is second-best in the sense that the government cannot discriminate between consumers which generate external costs and responsible consumers.<sup>2</sup>

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<sup>2</sup>For example Gruber and Koszegi (2001) estimate external costs of smoking at \$0.94 to \$1.75 per pack,

## 2.2 Addictive Goods and Non-market Internal Costs

Another potential source of non-market costs may occur depending on how addiction is modeled. Suppose consumers of addictive goods fail to take into account the adverse health effects caused by consumption of addictive goods, either because they are unaware that consumption of addictive goods has adverse health effects (e.g. Kenkel 1996) or because some consumers are exogenously assumed to be unable to take into account the health gains from reduced consumption (e.g. Pogue and Sgontz 1989). When some consumers are exogenously assumed not to consider some private costs and thus over-consume, the resulting “internality” causes the optimal tax rate to rise considerably. Kenkel (1996) finds the optimal tax rate on alcohol rises to about 106% while Pogue and Sgontz (1989) find the optimal tax rate on alcohol rises to 306%. This approach models addiction as non-rational, excess consumption.

A related, subsequent literature makes excess consumption endogenous and rational by defining “sin goods” as goods for which preferences are time inconsistent (Gruber and Koszegi 2001, Gruber and Koszegi 2004, O’Donoghue and Rabin 2003, O’Donoghue and Rabin 2006). In this approach, consumers optimally choose to consume more now and less in the future. However, next period consumers also optimally choose to consume more now and less in the future. Hence consumers are rational, but over-consume in the sense that consumer welfare increases with a tax that reduces consumption to a level which consumers would choose if they could pre-commit to consume less in the future. O’Donoghue and Rabin (2006) compute numerical examples where the optimal tax on unhealthy foods ranges from 1-72%. Gruber and Koszegi (2001) show that the optimal tax on cigarettes rises to at least \$1 per pack when the time inconsistency problem is included. Thus, including internalities, rational or not, results in optimal taxes generally far above that observed in the data.

This strand of literature then suggests an alternative argument to tax addictive goods by focusing on some aspects of addiction that results in some potential non-market internal costs.

## 2.3 Addictive Goods and Fiscal Concerns

A final motivation for taxation of addictive goods is revenue raising. Taxation of many addictive goods, such as lotteries, have an obvious revenue raising component. Taxes on many other addictive goods have at least a stated goal of raising revenue. For example,

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versus an average excise tax of about \$0.65. For alcohol, Kenkel (1996) finds the optimal tax rate to correct the drunk driving externality is about 42%, while the actual average tax rate ranges from over 50% in 1954 to 20% in the 1980s. Moreover, Grinols and Mustard (2006) estimate external costs of casino gambling are 47% of revenues, thus the optimal tax would be higher than 47% if demand for casino gambling is inelastic, or less than 47% if a significant fraction of casino gamblers do not impose external costs. Anderson (2005) report that casinos pay 16% of gross revenues in taxes.

Parry, Laxminarayan, and West (2006) note that the last two increases in federal alcohol taxes were part of deficit reduction packages. For lotteries, external costs are presumably small, but the nationwide average lottery tax ranges from 40% in 1989 (Clotfelter and Cook 1990) to 31% in 2003 (Hansen 2004),<sup>3</sup> accounting for 2% of state tax revenues. States spent about \$272 million on lottery advertising in 1989, which is at least a strong indication that states are motivated by revenue concerns, rather than the external costs of lotteries and other forms of gambling.<sup>4</sup>

A few papers consider the revenue raising motivation by treating addictive goods in a static way as simply goods with external costs and which are possibly complementary with leisure. If so, one can apply the ideas from the “double dividend” literature (e.g. Bovenberg and Goulder 1996). An increase in taxes on a good with external costs raises revenues which can be used to reduce taxes on labor income (the “revenue recycling effect”). If taxing goods with external costs results in lower dead-weight losses than taxing labor (say if demand for addictive goods was very inelastic), then the revenue recycling effect is positive and it is optimal to tax addictive goods at a relatively high rate. Moreover, a good with external costs may also be taxed above its Pigouvian rate for revenue raising if it is complementary with leisure, since the tax therefore increases labor supply and labor income tax revenues (the “tax interaction effect”).

Sgontz (1993) finds the revenue recycling effect to be positive, and Parry, Laxminarayan, and West (2006) finds both the revenue recycling effect and the tax interaction effect to be positive: alcohol is complementary to leisure and also reduces labor productivity. Therefore, they find it is optimal to tax alcohol above its Pigouvian rate as part of the optimal revenue raising package. Both of these papers treat alcohol in a static way as simply a good with external costs; the dynamic nature of addiction is ignored. It remains unclear how dynamic addictive properties such as tolerance affect optimal revenue raising.

This paper tries to fill this gap in the literature by considering a dynamic model of rational addiction while explicitly considering a revenue raising motive. Throughout the rest of the paper we model addiction using the rational addiction framework of Becker and Murphy (1988) and others. In this approach, consumption of the addictive good is specifically related to past consumption. Although not conclusive, some evidence for rational addiction exists in

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<sup>3</sup>These figures ignore that consumers pay income taxes on winnings.

<sup>4</sup>However, some legislation emphasizes that lotteries should maximize revenue subject to constraints such as preservation of “the public good” which could be interpreted as taking into account external costs (Clotfelter and Cook 1990).

that current consumption of cigarettes,<sup>5</sup> alcohol,<sup>6</sup> and caffeine (Olekalns and Bardsley 1996) respond to announced future price changes, as predicted by the rational addiction model. Gruber and Koszegi (2001), however, show that evidence of rational addiction does not preclude time inconsistent preferences.<sup>7</sup> The main alternative, modeling addiction as either rational or irrational excess consumption, has intuitive appeal but also has some practical difficulties. First, it is difficult to determine the degree of excess consumption, especially since it must be heterogeneous across the population, and the optimal tax is sensitive to both the degree of excess consumption and the fraction of the population that suffers from excess consumption. Furthermore, computational difficulties of time inconsistent preferences require separability in addictive and ordinary goods, no savings, and often quadratic utility functions. All of these assumptions affect the optimal tax rates, especially if the government has a revenue raising requirement.

The Becker and Murphy framework has no internality motivation for taxation of addictive goods, but a fiscal motivation can still exist and our framework can be easily extended to examine optimal taxation in the presence of externalities. Thus we examine the revenue-raising motivation, using the long standing tradition of the Ramsey approach (see for example Chari and Kehoe 1998). We show that excessive taxation of addictive goods to raise revenue is closely related to the homogeneity of the addiction function. If the addiction function is homogeneous of degree less than one, as in the multiplicative case, an equi-proportionate decrease in consumption of addictive and ordinary goods raises the marginal utility of addictive goods more, since the minimum consumption level is higher than that of ordinary goods. Hence, the optimal tax package causes a larger percentage decrease in the consumption of ordinary goods, which in turn implies the optimal tax rate on addictive goods is smaller. In other words, taxes on addictive goods impose very high welfare costs on addicts. This effect is missing from static models which assume a constant price elasticity of demand.

The remainder of the paper is organized in four sections. Section 3 develops an addiction model in the context of a simple production economy with capital and labor. Section 4 studies the corresponding Ramsey problem while Section 5 presents the results. Section 6 provides an analytical example and section 7 concludes.

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<sup>5</sup>See for example Gruber and Koszegi (2001), Becker, Grossman, and Murphy (1994), Chaloupka (1991) and Sung, Hu, and Keeler (1994).

<sup>6</sup>See for example, Grossman, Chaloupka, and Sirtalan (1998) Baltagi and Griffin (2002), Bentzen, Eriksson, and Smith (1999), Baltagi and Geishecker (2006), and Waters and Sloan (1995).

<sup>7</sup>Although laboratory evidence of time-inconsistent preferences are strong, little formal econometric evidence exists for or against time inconsistent preferences in actual markets.

### 3 Model

We consider an infinite horizon closed economy in discrete time. The economy is populated by a continuum of identical households of measure one who maximize the discounted sum of instantaneous utilities. A large number of identical firms produce both addictive and ordinary goods using a constant return to scale technology. Finally, there is a government that needs to finance a constant stream of government expenditures through fiscal policy.

#### 3.1 Firms

A large number of identical firms rent capital  $k_t$  and labor  $h_t$  from households to produce a composite good using a constant returns to scale technology  $F(k_t, h_t)$ . Let  $w_t$  denote the wage rate and  $r_t$  the rental rate of capital, then the objective of the firm is to maximize profits, which equal:

$$\max_{k_t, h_t} \{F(k_t, h_t) - r_t k_t - w_t h_t\}. \quad (3.1)$$

Let subscripts on functions denote corresponding partial derivatives. The equilibrium rental rate and wage rate are given by:

$$r_t = F_k(k_t, h_t), \quad (3.2)$$

$$w_t = F_l(k_t, h_t). \quad (3.3)$$

For simplicity we assume that the composite production good can be used for consumption of either the addictive, non-addictive good or investment.<sup>8</sup>

#### 3.2 Households

A large number of identical households derive utility from consumption of an ordinary (non-addictive) good,  $c_t$ , the fraction of time allocated to leisure,  $1 - h_t \equiv l_t \in [0, 1]$ , and consumption of an addictive good,  $d_t$ . Let  $s_t = s(d_t, d_{t-1})$  denote the effective consumption (consumption in excess of that required to sustain the addiction). The per period utility depends on consumption of ordinary goods, effective consumption, and leisure through the utility function  $u(c_t, s_t, l_t)$ .<sup>9</sup> We assume  $u(\cdot)$  is strictly increasing, concave, and satisfies the

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<sup>8</sup>Note that it is straightforward to extend the analysis to allow the production technology to differ by consumption goods.

<sup>9</sup>This specification is clearly equivalent to Becker, Grossman, and Murphy (1994), who assume a utility function of the form  $u(c_t, d_t, d_{t-1})$ , except they assume no preferences for leisure. Our assumption below that  $s$  is homogeneous is the main restriction we impose on their utility specification.

Inada conditions in each argument. Since the household gets positive utility from consumption of the addictive good, we assume  $s_1 > 0$ . We further assume that the addictive good has the *tolerance* property, meaning past consumption lowers current utility, which is also known a harmful addiction, or that  $s_2 < 0$ . Lifetime utility is:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, s_t, l_t); \quad (3.4)$$

where  $\beta$  is the discount factor with rate of time preference  $\rho = \frac{1-\beta}{\beta}$ . Finally, we assume the problem is globally concave in the choice set  $[c_t, l_t, d_t]$ . A simple sufficient condition for concavity is that the return function is concave when  $s_t = d_t$  (i.e. the standard problem with no addiction is concave) and  $s_{11} \leq 0$ .

### Habits versus Addiction

In this paper we use the standard definition of addiction, which is when current consumption is increasing in past consumption, holding fixed the marginal utility of wealth and prices. Habit formation is often used in the macro literature, whereas addiction was introduced by Becker and Murphy (1988).

Gruber and Koszegi (2004) and others define habit formation as past consumption increasing the taste for current consumption.<sup>10</sup> We therefore state that a good is *habit forming* if and only if:

$$\frac{\partial^2 u}{\partial d_t \partial d_{t-1}} > 0. \quad (3.5)$$

From the assumptions on  $s$ , a good is habit forming if and only if:

$$\sigma_s(c_t, s_t, l_t) \equiv \frac{-u_{ss}(c_t, s_t, l_t) s_t}{u_s(c_t, s_t, l_t)} > \frac{s_t s_{12}(d_t, d_{t-1})}{s_1(d_t, d_{t-1}) s_2(d_t, d_{t-1})}. \quad (3.6)$$

Becker and Murphy (1988) and others define *addiction* as when past consumption increases current consumption, holding fixed prices and the marginal utility of ordinary consumption. Let  $c_t = y_t - p_t d_t$ , then  $d$  is addictive if and only if:

$$\frac{\partial d_t}{\partial d_{t-1}} = \frac{\frac{\partial^2 U}{\partial d_t \partial d_{t-1}}}{-\frac{\partial^2 U}{\partial d_t^2}} > 0, \quad (3.7)$$

holding fixed the marginal utility of consumption. Using the concavity assumptions, we can

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<sup>10</sup>Becker and Murphy (1988) define reinforcement as when past consumption increases the taste for current consumption.

simplify the above to:

$$\frac{\partial^2 U}{\partial d_t \partial d_{t-1}} = \frac{\partial^2 u}{\partial d_t \partial d_{t-1}} > 0. \quad (3.8)$$

Thus  $d$  is addictive if and only if  $d$  is habit forming given our one-lag specification of effective consumption, and our concavity assumptions.<sup>11</sup>

The two most commonly used specifications of effective consumption,  $s$ , in the literature are the *subtractive model* (see for example Campbell and Cochrane 1999), where effective consumption is:

$$s_t = d_t - \gamma d_{t-1}, \quad (3.9)$$

and the *multiplicative model* (see for example Abel 1990), which specifies effective consumption as:

$$s_t = \frac{d_t}{d_{t-1}^\gamma}. \quad (3.10)$$

In either model  $\gamma \geq 0$  denotes the strength of tolerance. If  $\gamma = 0$ , then past consumption has no weight at all, in which case the model reduces to the standard time separable model, and utility is fully determined by absolute consumption levels and not by the changes in consumption. It is straightforward to show that if  $s$  is subtractive, then  $d$  is addictive for all  $\gamma > 0$ . Furthermore, if  $s$  is multiplicative, then  $d$  is addictive if and only if  $\sigma_s(c_t, s_t, l_t) > 1$  for all  $[c_t, s_t, l_t]$ . We will assume throughout that  $s$  is homogeneous of degree  $\alpha$  (HD- $\alpha$ ) in  $[d_t, d_{t-1}]$ . In the subtractive model, effective consumption is HD-1. In the multiplicative model, effective consumption is HD- $(1 - \gamma)$ .

## Household Resources and Optimal Decisions

The household budget constraint sets after tax wage and rental income and government bond redemptions (equal to  $R_t^b b_t$ , where  $b_t$  are bonds issued in  $t - 1$  and redeemed in  $t$ ) equal to after tax expenditures on government bond issues and consumption of addictive, ordinary, and investment goods given by  $i_t = k_{t+1} - (1 - \delta) k_t$ , where  $\delta$  is the depreciation rate. Since consumption of ordinary, addictive, and investment goods all have the same production technology, they have the same pre-tax price, which is normalized to one. Let  $\tau_c$  and  $\tau_d$  be the tax rates on consumption of ordinary and addictive goods, respectively and

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<sup>11</sup>In general, if  $s$  has more than one lag, addiction is more restrictive than habit formation. Thus, for example, habit formation and addiction are not equivalent in Becker and Murphy (1988), but are equivalent in Becker, Grossman, and Murphy (1994).

let  $\tau_h$  be the tax rate on labor income. The household budget constraint is then:

$$R_t^b b_t + r_t k_t + (1 - \tau_{h,t}) w_t h_t = (1 + \tau_{c,t}) c_t + (1 + \tau_{d,t}) d_t + i_t + b_{t+1}. \quad (3.11)$$

Let  $\lambda_t$  denote the Lagrange multiplier on the budget constraint. Then the resulting household first order conditions are then given by:

$$(1 + \tau_{c,t}) \lambda_t = \beta^t u_c(c_t, s_t, l_t), \quad (3.12)$$

$$(1 - \tau_{h,t}) w_t \lambda_t = \beta^t u_l(c_t, s_t, l_t), \quad (3.13)$$

$$(1 + \tau_{d,t}) \lambda_t = \beta^t u_s(c_t, s_t, l_t) s_1(d_t, d_{t-1}) + \beta^{t+1} u_s(c_{t+1}, s_{t+1}, l_{t+1}) s_2(d_{t+1}, d_t), \quad (3.14)$$

$$\lambda_t R_t = \lambda_{t-1}, \quad t \geq 1, \quad (3.15)$$

$$\lambda_t R_t^b = \lambda_{t-1}, \quad (3.16)$$

$$R_t = r_t + 1 - \delta. \quad (3.17)$$

Household decisions,  $k_t$ ,  $b_t$ ,  $h_t$ ,  $c_t$ ,  $d_t$ ,  $\lambda_t$  are thus characterized by the solution to equations (3.12)-(3.16), the budget constraint (3.11), initial conditions  $k_0$  and  $d_{-1}$ , and the appropriate transversality conditions. In equation (3.14), the household increases effective consumption by increasing  $d_t$  (first term on the right hand side), but also increases tolerance and therefore reduces future effective consumption (second term on the right hand side). From equations (3.12) and (3.14) we have:

$$\frac{1 + \tau_{d,t}}{1 + \tau_{c,t}} = \frac{u_s(c_t, s_t, l_t) s_1(d_t, d_{t-1}) + \beta u_s(c_{t+1}, s_{t+1}, l_{t+1}) s_2(d_{t+1}, d_t)}{u_c(c_t, s_t, l_t)} \equiv \frac{MU_{d,t}}{MU_{c,t}}, \quad (3.18)$$

where  $MU_{i,t}$  represents the marginal utility of good  $i$  at time  $t$ .

As we can see, any difference in tax rates drives a wedge between the marginal utilities of the consumption of ordinary and addictive goods. Thus the optimal tax rate of addictive goods exceeds the tax rate of consumption of ordinary goods ( $\tau_{d,t} > \tau_{c,t}$ ) if and only if  $MU_{d,t} > MU_{c,t}$ . The goal of this paper is to find conditions under which the marginal utility of addictive goods exceeds that of ordinary goods.

### 3.3 Government

The government finances an exogenous sequence of expenditures,  $g_t$ , with bond issues and consumption and wage income tax revenues. The government budget constraint is:

$$g_t = \tau_{h,t} w_t h_t + \tau_{c,t} c_t + \tau_{d,t} d_t + b_{t+1} - R_t^b b_t. \quad (3.19)$$

Our model has three basic wedges: one between the marginal rate of substitution between addictive and ordinary consumption, a second between the marginal utility of leisure and marginal utility of working (the after tax wage times the marginal utility of consumption), and a third between the intertemporal marginal rate of substitution and the rate of interest. Thus we need only three tax instruments for a complete tax system. We therefore set interest taxes equal to zero, noting that the government can affect all three margins by setting a time-varying consumption tax (to alter the intertemporal marginal rate of substitution), a wage tax, and an addictive goods tax. The government optimally uses bonds to smooth tax burdens over time, in the absence of bonds, the government may favor the tax with better smoothing properties.<sup>12</sup> Changes in current addictive goods tax rates effects both current and future tax revenue. The existence of government bonds enables us to conveniently summarize the effect of a change in current addictive tax rates on all periods as the effect on the infinite horizon version of the government's budget constraint.

Let  $\pi = [(\tau_{c,t})_{t=0}^\infty, (\tau_{d,t})_{t=0}^\infty, (\tau_{h,t})_{t=0}^\infty, (g_t)_{t=0}^\infty]$  denote an infinite sequence of government policies. We assume the existence of a commitment technology, so that the government commits to all future policies at time zero.<sup>13</sup>

## 4 Equilibrium and Ramsey Problem

Equations (3.2), (3.3), (3.11), (3.12) - (3.15), and (3.19) form a system of nine non-linear equations that fully characterizes the competitive equilibrium. Hence:

**Definition 1** *A competitive equilibrium, given initial values  $k_0$  and  $d_{-1}$ , is a set of allocations  $\{c_t, d_t, h_t, \lambda_t, k_t\}$ , prices  $\{w_t, r_t, R_t^b\}$  and a sequence of policies  $\pi$  that satisfy the household budget constraint (3.11), firm profit maximization (3.1), the government budget constraint (3.19), and household maximization of (3.4) for all  $t$ .*

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<sup>12</sup>However, addictive taxes are common at the state and local level, which frequently have constitutional borrowing restrictions. We leave this interesting case to future research.

<sup>13</sup>Note that in principle the government could promise low future taxes on addictive goods, and then find it optimal to renege on the promise once households become addicted. For example, Bossi and Petkov (2006) show that time inconsistency may occur in the regulation of monopolies which sell addictive goods and examine time consistent policies. All other papers assume the government can pre-commit.

In order to determine optimal taxation we use the primal approach (see for example Chari and Kehoe 1998). In the primal approach, households and firms' first order conditions are used to eliminate prices and policies from the equations that define the competitive equilibrium. The planner then chooses allocations which maximize welfare subject to the remaining equations from the competitive equilibrium. These equations are the resource constraint:

$$F(k_t, h_t) = c_t + d_t + k_{t+1} - (1 - \delta)k_t + g_t, \quad (4.1)$$

and the *implementability constraint* (IMC):

$$\frac{u_c(c_0, s_0, l_0) R_0 (k_0 + b_0)}{1 + \tau_{c,0}} = \sum_{t=0}^{\infty} \beta^t \left\{ u_c(c_t, s_t, l_t) c_t + \left[ u_s(c_t, s_t, l_t) s_1(d_t, d_{t-1}) + \beta u_s(c_{t+1}, s_{t+1}, l_{t+1}) s_2(d_{t+1}, d_t) \right] d_t - u_l(c_t, s_t, l_t) h_t \right\}. \quad (4.2)$$

The IMC uses the household first order conditions to substitute out for all prices and policies in the budget constraint and then recursively eliminates  $\lambda_t$ . Thus, the IMC is the infinite horizon version of the household budget constraint where all prices and policies have been written in terms of their corresponding marginal utilities. It is immediate from Walras Law and the resource constraint that the IMC can also be thought of as the infinite horizon version of the government budget constraint. The Ramsey approach is therefore very convenient in that the planner can, through the IMC, determine the effect of a change in  $d_t$  on government revenues over the infinite horizon.

The first proposition gives the relationship between the competitive equilibrium and the IMC and resource constraint.

**PROPOSITION 1** *Let  $u(\cdot)$ ,  $s(\cdot)$ , and  $F(\cdot)$  be as described above. Given  $k_0$ ,  $d_{-1}$ ,  $\tau_{h,0}$ , and  $\tau_{c,0}$ , the allocations of a competitive equilibrium satisfy (4.1) and (4.2). In addition, given  $k_0$ ,  $d_{-1}$ ,  $\tau_{h,0}$ , and  $\tau_{c,0}$ , and allocations which satisfy (4.1) and (4.2), prices and polices exist which, together with the allocations, are a competitive equilibrium.*

All proofs can be found in the appendix.

The Ramsey Problem (RAM) determines the optimal tax package that maximizes welfare subject to the IMC and resource constraint, is:

$$\text{RAM} = \max_{c_t, d_t, h_t, k_t} \left\{ \sum_{t=0}^{\infty} \beta^t \left( u(c_t, s_t, 1 - h_t) + \mu \left[ u_c(c_t, s_t, 1 - h_t) c_t + (u_s(c_t, s_t, 1 - h_t) s_1(d_t, d_{t-1}) + \beta u_s(c_{t+1}, s_{t+1}, l_{t+1}) s_2(d_{t+1}, d_t)) d_t - \right. \right. \right.$$

$$\begin{aligned}
& \left. u_l(c_t, s_t, 1 - h_t) h_t \right] - \mu \frac{u_c(c_0, s_0, l_0) R_0 (k_0 + b_0)}{1 + \tau_{c,0}} + \\
& \left. \phi_t \left[ F(k_t, h_t) - c_t - d_t - k_{t+1} + (1 - \delta) k_t - g_t \right] \right\}. \tag{4.3}
\end{aligned}$$

The term multiplied by  $\beta^t$  in problem (4.3) is the social welfare in period  $t$  consisting of private welfare  $u(c_t, s_t, 1 - h_t)$  plus public welfare which is discounted tax revenue (expressed as marginal utilities) multiplied by the marginal value of public funds  $\mu$ .

The first order conditions that characterize optimal taxation are given by:

$$\frac{\phi_t}{\beta^t} = MU_{c,t} + \mu \frac{\partial IMC}{\partial c_t} \tag{4.4}$$

$$\frac{\phi_t}{\beta^t} = MU_{d,t} + \mu \frac{\partial IMC}{\partial d_t} \tag{4.5}$$

$$\frac{\phi_t F_h(k_t, h_t)}{\beta^t} = u_l(c_t, s_t, 1 - h_t) - \mu \frac{\partial IMC}{\partial h_t} \tag{4.6}$$

$$\phi_t (F_k(k_t, h_t) + 1 - \delta) = \phi_{t-1}. \tag{4.7}$$

From equations (4.4) and (4.5):

$$MU_{c,t} - MU_{d,t} = \mu \left( \frac{\partial IMC}{\partial d_t} - \frac{\partial IMC}{\partial c_t} \right). \tag{4.8}$$

Hence using equation (3.18), we find that addictive goods are taxed at a higher rate than ordinary goods if and only if the derivative of the IMC with respect to  $d_t$  is smaller than the derivative with respect to  $c_t$ :

$$\tau_{d,t} > \tau_{c,t} \text{ iff } \frac{\partial IMC}{\partial d_t} < \frac{\partial IMC}{\partial c_t}. \tag{4.9}$$

The first order conditions equate the marginal social welfare of  $c$  or  $d$ , with the marginal resource cost  $\phi$ . Since the marginal rate of transformation between  $c$  and  $d$  is one, the marginal rate of substitution is the tax wedge. The Ramsey problem computes the optimal wedges between the marginal utilities as:

$$\text{Wedge} = \frac{MU_{d,t}}{MU_{c,t}} = 1 + \frac{\mu}{MU_{c,t}} \left( \frac{\partial IMC}{\partial c_t} - \frac{\partial IMC}{\partial d_t} \right). \tag{4.10}$$

From equation (4.10), if  $MU_{d,t} > MU_{c,t}$ , reallocating a marginal resource from ordinary to addictive consumption raises private welfare by  $MU_{d,t} - MU_{c,t}$ . Thus, tax revenue must fall by  $\frac{\partial IMC}{\partial c_t} - \frac{\partial IMC}{\partial d_t}$ , resulting in a loss of public welfare of  $\mu$  times the loss of tax revenue. Hence addictive goods are taxed at a higher rate than ordinary goods if and only if moving a resource unit from addictive to ordinary consumption raises revenue, that is, if the marginal tax revenue of ordinary goods exceeds that of addictive goods.

In turn, the marginal tax revenue of ordinary consumption depends on how a small change in ordinary consumption affects ordinary consumption tax revenue, addictive tax revenue, and labor tax revenue:

$$\frac{\partial IMC}{\partial c_t} = u_{c,t} + u_{cc,t}c_t + \alpha u_{cs,t}s_t - u_{cl,t}h_t. \quad (4.11)$$

An increase in  $c_t$  directly increases ordinary consumption tax revenue (first term), but decreases the marginal utility of consumption and thus requires the planner to lower the ordinary consumption tax rate in order to maintain equilibrium, which lowers tax revenues (second term). The third term contains two offsetting effects. Suppose for example that  $u_{cs} > 0$ . Then an increase in  $c_t$  raises  $MU_{d,t}$ , so the planner must raise the tax on  $d_t$  to maintain equilibrium, which increases addictive tax revenues. In addition, an increase in  $c_t$  lowers  $MU_{d,t-1}$ : consuming  $d_{t-1}$  is less attractive because it causes effective consumption to fall in  $t$  (tolerance), which lowers utility since  $u_{cs} > 0$ . Thus, the planner must also lower  $\tau_{d,t-1}$ , reducing revenues. Thus an increase in  $c_t$  has offsetting addictive tax revenue effects, but both work through the  $u_{cs}$  term. Given the homogeneity assumption, these two effects can be combined into a single effect, as if a smaller tax on  $s_t$ , rather than  $d_t$ , existed. Finally, an increase in  $c_t$  increases preferences for leisure, and thus causes the planner to decrease the labor tax rate to maintain equilibrium, if and only if  $u_{cl} > 0$ .

The marginal tax revenue of addictive goods depends on how a change in addictive consumption affects ordinary consumption tax revenue, addictive consumption tax revenue, and labor tax revenue:

$$\begin{aligned} \frac{\partial IMC}{\partial d_t} = & \alpha (MU_{d,t} + u_{ss,t}s_{1,t}s_t + \beta u_{ss,t+1}s_{2,t+1}s_{t+1}) + \\ & u_{cs,t}s_{1,t}c_t + \beta u_{cs,t+1}s_{2,t+1}c_{t+1} - u_{sl,t}s_{1,t}h_t - \beta u_{sl,t+1}s_{2,t+1}h_{t+1} \end{aligned} \quad (4.12)$$

A small increase in  $d_t$  directly increases addictive goods tax revenue, but reduces the marginal utility of effective consumption, which requires the planner to decrease  $\tau_{d,t}$ . An increase in  $d_t$  increases the marginal utility of ordinary consumption and thus increases  $\tau_{c,t}$  if and only if  $u_{cs} > 0$ . An increase in  $d_t$  increases the marginal utility of leisure and thus decreases  $\tau_{h,t}$  if and only if  $u_{sl} > 0$ . An increase in  $d_t$  also increases tolerance in period  $t + 1$ , thus reducing

$s_{t+1}$ . Thus an increase in  $d_t$  affects all three types of tax revenue in period  $t + 1$  as well but *in the opposite direction*.

In summary then, simple static results and intuition might indicate that taxing addictive goods is a good revenue raiser because addictive goods tend to be income inelastic and complementary to leisure. However, the dynamic results are likely to be more moderate. For example, if leisure and effective consumption are highly complementary, then a decrease in  $d_t$  raises labor tax revenues in period  $t$ , but increases  $s_{t+1}$ , reducing labor tax revenues in period  $t + 1$ . In addition, the stronger the tolerance, the stronger is the dynamic effect. To go further requires more specific preference assumptions. These assumptions shed further light on the optimal tax rates on addictive goods in a dynamic setting.

## 5 Results for Specific Preferences

As in the literature on optimal commodity taxation, characteristics of the utility function play an important role in determining any deviations from a uniform tax rate across consumption goods. Two well-known cases are when utility is homothetic and when utility is separable. In the next sections we explore these two cases as well as the possible interaction between the consumption of addictive goods and leisure.

### 5.1 Homothetic Utility

In this section we assume utility takes the form:

$$u(s_t, d_t, l_t) = q(v(c_t, s_t), l_t), \quad (5.1)$$

where  $v(\cdot)$  is homothetic and  $q(\cdot)$  is an increasing function.

To determine whether or not addictive goods should be taxed at a higher rate than ordinary goods, we combine equations (4.4) and (4.5). Let us define the following elasticities:

$$\sigma_{cs,t} \equiv \frac{u_{cs}(c_t, s_t, l_t) c_t}{u_s(c_t, s_t, l_t)}, \quad \sigma_{sc,t} \equiv \frac{u_{cs}(c_t, s_t, l_t) s_t}{u_c(c_t, s_t, l_t)}, \quad \sigma_{hs,t} \equiv \frac{u_{sl}(c_t, s_t, l_t) h_t}{u_c(c_t, s_t, l_t)} \quad (5.2)$$

and let  $\sigma_{hc,t}$  be defined analogously. Then we have the following result.

**PROPOSITION 2** *Let  $q(\cdot)$ ,  $s(\cdot)$ , and  $F(\cdot)$  be as described above. In addition, let  $u(\cdot)$  be of the form given in equation (5.1). Then  $\tau_{d,t} > \tau_{c,t}$  if and only if:*

$$(1 - \alpha)(1 - \sigma_{s,t} - \sigma_{sc,t}) > -\frac{\beta u_{s,t+1} s_{2,t+1}}{MU_{d,t}} \left( \begin{array}{l} \alpha(\sigma_{s,t+1} - \sigma_{s,t}) - (\sigma_{cs,t+1} - \sigma_{cs,t}) \\ + (\sigma_{hs,t+1} - \sigma_{hs,t}) \end{array} \right) \quad (5.3)$$

The left hand side of condition (5.3) is a generalization of the standard condition for optimal taxation when utility is homothetic. To see this, suppose  $s_2=0$  (continue to let  $s$  be HD- $\alpha$ ), which implies no addiction. Then the left hand side is positive if and only if  $d$  has a lower income elasticity than  $c$ .<sup>14</sup> This is just the standard result that it is optimal to tax necessities at a higher rate than luxuries. Alternatively, the left hand side is zero if and only if  $v(c_t, s(d_t, d_{t-1}))$  is homothetic in  $[c_t, d_t]$ , which is just the standard result that optimal commodity taxation is uniform if utility is homothetic.<sup>15</sup> Tolerance adds an extra term which is the term on the right hand side. In effect tolerance can break the homotheticity depending on the current state of addiction (that is, past addictive consumption).

Thus tolerance and homogeneity play a key role in optimal taxation of addictive goods. Reinforcement, a situation when past consumption increases the taste for current consumption, does not play a direct role, but may be present in that all of the  $\sigma$  terms are at least potentially endogenous. That is, reinforcement may (by changing the  $\sigma$  terms) alter the income elasticities and therefore cause the income expansion path to become nonlinear. As shown in the following corollaries, however, some common specifications for  $v$  and  $s$  result in uniform taxation.

**COROLLARY 3** *Let the conditions of Proposition 2 hold, and let  $q(\cdot) = z(l) + \frac{(c^\delta s^{1-\delta})^{1-\sigma} - 1}{1-\sigma}$ , and  $z(\cdot)$  be concave, then  $\tau_d = \tau_c$  for all  $t$ .*

Although we have assumed here that  $v(\cdot)$  is constant relative risk aversion (CRR), this corollary is considerably more realistic than the existing literature which either assumes a static utility function or separable quadratic utility for tractability. Hence, ignoring labor supply effects and making utility CRR in  $c$  and  $s$  means it is optimal not to tax addictive goods at a higher rate than ordinary goods. However, if labor supply effects are present then the results could shift in favor of taxing addictive goods at a higher rate.

Let  $\bar{x}$  denote the steady state value of any variable  $x$ , then we have the following steady state result.

**COROLLARY 4** *Let  $u(\cdot)$ ,  $s(\cdot)$ , and  $F(\cdot)$  be as described above and let  $u(\cdot)$  be of the form given in equation (5.1). Then  $\bar{\tau}_d > \bar{\tau}_c$  if and only if:*

$$(1 - \alpha)(1 - \bar{\sigma}_s - \bar{\sigma}_{sc}) > 0. \tag{5.4}$$

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<sup>14</sup>That is, let  $M$  denote non-labor income and  $M + h = p_c c + p_d d$  be the budget constraint for a static version of the model. Then the left hand side of (5.3) is positive if and only if  $d$  has a lower elasticity with respect to non-labor income than  $c$  in the static version of the model.

<sup>15</sup>Note that the simple partial equilibrium intuition that goods that are price inelastic should be taxed at a greater rate does not hold in general equilibrium, unless utility is separable and no income effects exist (see for example Chari and Kehoe 1998).

Tolerance plays no direct role in the steady state. Homogeneity does matter, however. For HD-1 addiction functions, including the subtractive model, Corollary 4 indicates that the steady state tax rates are uniform. On the other hand when the degree of homogeneity is less than one, as is the case for the multiplicative model, then Corollary 4 reduces to:

$$\bar{\sigma}_s - 1 < -\bar{\sigma}_{sc} \quad (5.5)$$

Since with multiplicative habits addiction and reinforcement occur if and only if  $\bar{\sigma}_s > 1$ , it is optimal to tax addictive goods at a higher rate only if addictive and ordinary consumption goods are strong substitutes. Further, the stronger the reinforcement and addiction, the more likely it is to tax addictive goods at a *lower* rate. The intuition for this result is that a proportional decrease in  $c$  and  $d$  raises the marginal utility of  $d$  more because past addictive goods consumption has raised the taste for current addictive goods consumption.

## 5.2 Additively Separable Utility

In this section we consider the case in which utility is additively separable. Following a similar procedure as with Proposition 2, we have the following result:

**PROPOSITION 5** *Let  $s(\cdot)$ , and  $F(\cdot)$  be as described above. In addition, let  $u(\cdot)$  be additively separable in  $c$ ,  $s$ , and  $l$ . Then  $\tau_{d,t} > \tau_{c,t}$  if and only if:*

$$\alpha(\sigma_{s,t} - 1) - (\sigma_{c,t} - 1) > -\frac{\beta\alpha u_{s,t+1} s_{2,t+1}}{MU_{d,t}} (\sigma_{s,t+1} - \sigma_{s,t}) \quad (5.6)$$

The interpretation of condition (5.6) is analogous to that of condition (5.3). The left hand side is positive if and only if  $d$  has a lower income elasticity in the static version of the model with no addiction. Thus the left hand side simply reflects that it is optimal to tax necessities at a higher rate than luxuries. The right hand side is the direct effect of tolerance on the tax ratio.

For some special cases, tolerance plays no role in the optimal tax rate as illustrated by the following corollary:

**COROLLARY 6** *Let the conditions of Proposition 5 hold, and let  $u(\cdot) = v^1(c) + v^2(s) + v^3(l)$ , with  $v^1$  and  $v^2(\cdot)$  CRR, and  $v^3(\cdot)$  concave. Then  $\tau_{d,t} > \tau_{c,t}$  for all  $t$  if and only if:*

$$\alpha(\sigma_s - 1) > \sigma_c - 1 \quad (5.7)$$

That is, if preferences are separable and CRR, the addictive good is taxed at a higher rate if and only if the addictive good has a lower income elasticity. The steady state tax rates also depend only on the income elasticities.

The moderating effect that addiction has on optimal taxation can be seen by collecting the  $\sigma_s$  terms in equation (5.6), which gives:

$$\alpha \frac{u_{s,t} s_{1,t} (\sigma_{s,t} - 1) + \beta u_{s,t+1} s_{2,t+1} (\sigma_{s,t+1} - 1)}{MU_{d,t}} > \sigma_{c,t} - 1 \quad (5.8)$$

If an addictive good had a relatively low income elasticity or a relatively high value of  $\sigma_s$ , then the conclusion from a static analysis would be to tax addictive goods at a relatively high rate. However, equation (5.8) shows that the dynamic nature of addiction moderates this conclusion. Since the second term on the left hand side is negative, if  $\sigma_{s,t}$  and  $\sigma_{s,t+1}$  are close, then the benefits of a high value of  $\sigma$  are reduced. Specifically, if  $d$  is a necessity in period  $t$ , then increasing  $\tau_{d,t}$  is a good revenue raiser in period  $t$ , but will nonetheless reduce  $s_{t+1}$  and therefore  $d_{t+1}$  and future tax revenues. Furthermore, the stronger the effect in period  $t$ , in general the stronger the effect in period  $t + 1$ .

In summary, in general the optimal tax on addictive goods depends on the degree of tolerance and the relative income elasticities. The relative tax rate on the addictive good rises as the income elasticity of the addictive good falls, irrespective of the degree of addiction. Since effective consumption increases along the transition path, high tolerance implies a higher tax rate on addictive goods reduces current consumption, which increases future effective consumption, which in turn lowers the future income elasticity (if  $\sigma_s$  is increasing), which makes taxing addictive goods more attractive. However, since  $\sigma_s$  is constant for CRR utility functions and at the steady state, in these cases tolerance and addiction play no role and the optimal policy is simply to tax the good with the lower income elasticity or to maintain a uniform tax if the goods have an identical income elasticity.

### 5.3 Labor Supply Effects

The homothetic and separable cases assume that changes in  $c$  and  $s$  have identical effects on labor supply. However, some addictive goods are highly correlated with leisure. For example, alcohol is correlated with leisure and reduces productivity (Parry, Laxminarayan, and West 2006).<sup>16</sup> In this section we consider the possibility that ordinary and addictive goods differ in their labor supply effects. Addiction, as defined here, is related to reinforcement and tolerance, neither of which depends on the correlation with leisure. Thus we should expect to see extra terms which reflect how addictive goods consumption affects labor tax revenues weighed in addition to the relative income elasticity and degree of tolerance.

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<sup>16</sup>On the other hand, goods like lotteries do not seem to exhibit strong labor supply effects.

To see this in a concise way, let us consider the following class of utility functions:

$$u(s_t, d_t, l_t) = q(c_t) + v(s_t, l_t). \quad (5.9)$$

The specification (5.9) allows addictive consumption to be potentially complementary with leisure. For this specification, we find:

**PROPOSITION 7** *Let  $s(\cdot)$ , and  $F(\cdot)$  be as described above. In addition, let  $u(\cdot)$  be of the form given by (5.9). Then  $\tau_{d,t} > \tau_{c,t}$  if and only if:*

$$1 - \alpha + \alpha\sigma_{s,t} + \sigma_{hs,t} > \sigma_{c,t} - \frac{\beta u_{s,t+1} s_{2,t+1}}{MU_{d,t}} (\sigma_{s,t+1} - \sigma_{s,t} + \alpha(\sigma_{s,t+1} - \sigma_{s,t})) \quad (5.10)$$

The left hand side of equation (5.10) indicates that, ignoring the dynamic effect caused by tolerance, strong complementarity between the addictive good and leisure tends to increase the relative tax on addictive goods.

However, tolerance may change this effect. To see how, note that equation (5.10) can be rewritten as:

$$\frac{u_{s,t} s_{1,t} (\alpha(\sigma_{s,t} - 1) + \sigma_{hs,t}) + \beta u_{s,t+1} s_{2,t+1} (\alpha(\sigma_{s,t+1} - 1) + \sigma_{hs,t+1})}{MU_{d,t}} > \sigma_{c,t} - 1 \quad (5.11)$$

Since utility is separable in  $c$ , the right hand side of equation (5.11) incorporates only two effects: an increase in  $c$  directly raises revenue but requires the planner to lower the tax rate on  $c$  to maintain equilibrium. Conversely, the tax on  $d$  incorporates both a direct and an indirect labor supply effect, both of which are affected by addiction. First, taxing  $d$  is not as attractive as taxing  $s$ , since taxing  $d$  creates an offsetting effect on next period's revenue. The homogeneity assumption causes the tax on  $d$  to be equivalent to a smaller tax on  $s$ , and hence the  $\alpha$  term. Second, the left hand side is increasing in  $s_{2,t+1}$  if and only if  $\sigma_{hs,t+1} + \alpha\sigma_{s,t+1} > \sigma_{hs,t} + \alpha\sigma_{s,t}$ , in which case tolerance makes taxing  $d$  more attractive.

Equation (5.10) simplifies in the steady state or for constant relative risk aversion preferences.

**COROLLARY 8** *Let the conditions of Proposition 7 hold, and let  $v(\cdot) = \frac{(s^\delta l^{1-\delta})^{1-\sigma} - 1}{1-\sigma}$ , with  $\sigma \geq 1$ . Then  $\tau_{d,t} < \tau_{c,t}$  for all  $t$ .*

This interesting case shows that introducing complementarities between leisure and consumption of addictive goods can actually reduce the optimal tax rate on  $d_t$ .

**COROLLARY 9** *Let the conditions of Proposition 7 hold. Then  $\bar{\tau}_d > \bar{\tau}_c$  if and only if:*

$$\alpha(\bar{\sigma}_s - 1) + \bar{\sigma}_{hs} > \bar{\sigma}_c - 1 \quad (5.12)$$

Here only the revenue effects matter.

## 6 An Analytical Example: The Quadratic Case

To understand the effect of taxation on the dynamics of addictive good consumption, we focus on an example with an analytical solution. Suppose the subtractive specification for the effective consumption and the following utility and production function:

$$u(c_t, s_t, l_t) = c_t + bs_t - \frac{s_t^2}{2} + el_t - \frac{l_t^2}{2}, \quad e < 1, b > \frac{1}{1 - \beta\gamma}, \quad (6.1)$$

$$F(k_t, h_t) = k_t^\theta h_t^{1-\theta}. \quad (6.2)$$

Inspection of equations (4.4) and (4.5), given the utility function (6.1), reveals that the marginal utility of  $d_t$  is constant in the optimal second best allocation. In particular, we have

$$(1 + \mu) = (1 + 2\mu) MU_{d,t} - \mu b(1 - \beta\gamma). \quad (6.3)$$

Hence:

$$MU_{d,t} = \frac{1 + \mu(1 + b(1 - \beta\gamma))}{1 + 2\mu}. \quad (6.4)$$

Since the marginal utility of  $c_t$  is unity, the marginal utility of  $d_t$  equals the tax ratio given by equation (3.18). Hence the tax ratio is constant over time. Furthermore, given the assumptions on  $b$  (necessary for consumption of  $d_t$  to be positive at the steady state), equation (6.4) implies  $MU_{d,t} > 1$  and thus  $\tau_{d,t} > \tau_{c,t}$  for all  $t$ . Hence we have shown:

**PROPOSITION 10** *Let  $u(\cdot)$  and  $F(\cdot, \cdot)$  be given by equations (6.1) and (6.2) and let effective consumption be given by the subtractive model. Then  $\tau_{d,t} > \tau_{c,t}$  for all  $t$  and the ratio of tax rates  $\frac{1+\tau_{d,t}}{1+\tau_{c,t}}$  is constant over time.*

In the static version of the model without taxes  $d_t = s_t$  has an income elasticity equal to zero whereas the income elasticity of  $c_t$  is positive. Since no cross price or labor supply effects exist and since  $c_t$  is everywhere more income elastic than  $d_t$ , it is optimal to tax  $d_t$  at a higher rate regardless of  $k_t$  or  $d_{t-1}$ , because  $s_t$  is a necessity.

It is also clear from equation (6.4) that the second best optimal  $d_t$  is the solution to a linear second order difference equation and the second best optimal  $s_t$  is the solution to a linear first order condition. However, before computing the solution to  $d_t$ , we must verify that a solution exists for  $\mu$ . As the next proposition shows, a unique solution exists if government

spending is not too large as not to exhaust the maximum feasible revenue in the economy, and not so small that given initial tax rates are sufficient to pay for all current and future government expenditures.

**PROPOSITION 11** *Let  $u(\cdot)$  and  $F(\cdot, \cdot)$  be given by equations (6.1) and (6.2) and let effective consumption be given by the subtractive model. Let  $g_t$  be a stationary sequence with limiting value  $\bar{g}$ , then there exists an interval  $[\zeta_l, \zeta_h]$ , with  $0 < \zeta_l < \zeta_h < \infty$  such that if  $G \equiv \sum_{t=0}^{\infty} \beta^t g_t \in [\zeta_l, \zeta_h]$ , then a unique positive solution for  $\mu$  exists.*

Given a solution for  $\mu$ ,  $d_t$  is the solution to the second order difference characterized by equation (6.4).

**PROPOSITION 12** *Let the conditions for Proposition 11 hold. Then the explicit solution for  $d_t$  is:*

$$d_t = \frac{b(1 - \beta\gamma) - 1}{(1 - \gamma)(1 - \beta\gamma)} \left( \frac{1 + \mu}{1 + 2\mu} \right) (1 - \gamma^{t+1}) + d_{-1}\gamma^{t+1}. \quad (6.5)$$

The solution for  $d_t$  given by equation (6.5) allows us derive some interesting properties of the second best solution, both over time and as compared to the first best solution ( $\mu = 0$ ). First, from equation (6.5), optimal consumption of  $d_t$  increases over time, assuming  $d_{-1}$  is less than the steady state. The planner decreases  $d_t$  and the growth rate of  $d_t$  through the tax. However, in the second best optimum  $d_t$  falls by at most one third in the steady state, and falls by a smaller fraction on the transition.

We can also explore how the strength of tolerance affects second best addictive consumption. Since the solution for  $\mu$  is unique, we can use the implicit function theorem to derive comparative statics using equation (6.4). Our intuition is that strong tolerance should moderate the optimal tax ratio, as gains in current tax revenue from taxation of addictive goods are offset by losses in future tax revenues. If  $d_{-1}$  is sufficiently large, it is indeed true that the optimal tax ratio is inversely related to the degree of tolerance. In particular, we have:

$$d_{-1} \geq \frac{\beta}{(1 - \beta\gamma)(1 - \beta)} \Rightarrow \frac{\partial \frac{1 + \tau_d}{1 + \tau_c}}{\partial \gamma} < 0. \quad (6.6)$$

Furthermore, for  $G$  sufficiently large, the optimal tax ratio is inversely related to the degree of tolerance if and only if condition (6.6) holds. Thus the inverse relationship holds unless both the household is initially not very addicted and government revenue requirements are small.

## 7 Conclusions

This paper is the first attempt in the literature to characterize and analyze the conditions under which taxation of addictive goods might differ from taxes on leisure and other consumption goods in a dynamic setting. In particular, we derive conditions under which tax rates for addictive goods exceeds tax rates for non-addictive goods in an environment where exogenous government spending cannot be financed with lump sum taxes.

One of the properties of addictive goods that we find in this paper is that if taxing them has strong positive revenue effects today, then there also exist a strong offsetting effect on future tax revenues. This dynamic property of addictive goods might explain why static models, which do not model addiction, but incorporate revenue raising and externalities motives, tend to derive tax rates far in excess of that observed in the data.

In the simplest case, where consumption of addictive goods is a small fraction of income and has no labor supply effects, then the optimal tax rate for addictive goods must be larger than that of other consumption goods because addictive goods are more price inelastic. However, this intuition quickly breaks down if general equilibrium effects are present. If preferences are homothetic in the consumption of ordinary and addictive goods above the consumption level needed to maintain the addiction, then the optimal tax rate depends on or how the addictive property breaks the homotheticity. In particular, we show that how the addictive property breaks homotheticity depends crucially on the degree of homogeneity of the addictive good. More precisely, we show that if habits are homogeneous of degree one, as in the subtractive case, homotheticity is not broken and it is optimal to have uniform taxation. On the other hand, if habits are homogeneous of degree less than one, multiplicative case, the optimal tax rate on addictive goods is smaller than ordinary goods. Thus, the choice of how to model habits is not as innocuous as it might seem.

When we consider separable utility functions, the addictive good is taxed at a higher rate if and only if the addictive good has a lower income elasticity. We also find that the relative tax rate on the addictive good rises as the income elasticity of the addictive good falls, irrespective of the degree of addiction.

Finally, we also consider features of addictive goods such as complementarity to leisure that, while unrelated to addiction itself, are nonetheless common among some addictive goods. In general, such effects are weaker in our dynamic setting. One common feature of addictive goods, the presence of externalities, has not been considered in this paper. Still, the “double dividend” literature (Bovenberg and Goulder 1996)e.g. indicates that second best taxation of goods with external effects is not generally optimal to tax such goods above the first best level which corrects the externalities to raise extra revenue. Thus here we

would expect to see the optimal tax to increase if externalities are included, but not to the first best level which corrects the externality.

## 8 Appendix: Proofs

### 8.1 Proof of Proposition 1

To see that a competitive equilibrium satisfies the IMC and resource constraint, we substitute the factor prices (3.2) and (3.3) into the budget constraint (3.11). Using constant returns to scale, we then have:

$$R_t^b b_t + F(k_t, h_t) - \tau_{h,t} F_h(k_t, h_t) h_t = (1 + \tau_{c,t}) c_t + (1 + \tau_{d,t}) d_t + i_t + b_{t+1}. \quad (8.1)$$

Combining the above equation with the government budget constraint (3.19) gives the resource constraint (4.1).

To derive the IMC from the budget constraint, we substitute the household first order conditions (3.12)-(3.14) into the budget constraint (3.11), eliminating the tax rates, so that:

$$\lambda_t R_t k_t + \lambda_t R_t^b b_t - \lambda_t k_{t+1} - \lambda_t b_{t+1} = \beta^t (u_c(c_t, s_t, l_t) c_t + MU_{d,t} d_t + u_l(c_t, s_t, l_t) h_t). \quad (8.2)$$

Next using the first order conditions (3.15) and (3.16), we have:

$$\lambda_t R_t (k_t + b_t) - \lambda_{t+1} R_{t+1} (k_{t+1} + b_{t+1}) = \beta^t (u_c(c_t, s_t, l_t) c_t + MU_{d,t} d_t + u_l(c_t, s_t, l_t) h_t). \quad (8.3)$$

The above equation characterizes a sequence of budget constraints that can be used to recursively eliminate  $\lambda_t R_t (k_t + b_t)$ , yielding:

$$\lambda_0 R_0 (k_0 + b_0) - \lim_{t \rightarrow \infty} \lambda_{t+1} R_{t+1} (k_{t+1} + b_{t+1}) = \sum_{t=0}^{\infty} \beta^t (u_c(c_t, s_t, l_t) c_t + MU_{d,t} d_t - u_l(c_t, s_t, l_t) h_t). \quad (8.4)$$

The transversality condition implies the second term on the left hand side equals zero. Again using the household first order conditions at period zero gives:

$$\frac{u_{c,0}}{1 + \tau_{c,0}} (F_{k,0} + 1 - \delta) (k_0 + b_0) = \sum_{t=0}^{\infty} \beta^t (u_c(c_t, s_t, l_t) c_t + MU_{d,t} d_t - u_l(c_t, s_t, l_t) h_t) \quad (8.5)$$

which is the IMC.

We next show that, given allocations which satisfy the IMC and resource constraint, prices and policies exist which, along with the allocations, are a competitive equilibrium. Let  $\{c_t, k_t, h_t, d_t\}$  be a sequence which satisfies the IMC and resource constraint. Then  $r_t$  and  $w_t$  are defined using equations (3.2) and (3.3). Since  $\tau_{c,0}$  is given, we can define  $\lambda_0$  using equation (3.12). Then  $\lambda_t$  can be defined recursively using equation (3.15). Then  $R_t^b$  is defined using equation (3.16). Next, we define the government policies:

$$(1 + \tau_{c,t}) = \frac{\beta^t u_c(c_t, s_t, l_t)}{\lambda_t}, \quad (8.6)$$

$$(1 - \tau_{h,t}) = \frac{\beta^t u_l(c_t, s_t, l_t)}{\lambda_t F_h(k_t, h_t)}, \quad (8.7)$$

$$(1 + \tau_{d,t}) = \frac{\beta^t MU_{d,t}}{\lambda_t}, \quad (8.8)$$

$$g_t = \tau_{h,t} w_t h_t + \tau_{c,t} c_t + \tau_{d,t} d_t \quad (8.9)$$

Given the above prices and policies, all equations which define a competitive equilibrium are satisfied except the household and government budget constraints. We use  $b_t$  to satisfy the household budget constraint:

$$b_t = \frac{1}{R_t^b} (-r_t k_t - (1 - \tau_{h,t}) w_t h_t + (1 + \tau_{c,t}) c_t + (1 + \tau_{d,t}) d_t + i_t + b_{t+1}). \quad (8.10)$$

We can multiply the above equation by  $\lambda_t$  and recursively eliminate  $b_{t+1}$  from the above equation. After eliminating prices and policies using the household first order conditions (3.12)-(3.14),  $b_t$  is a function of the allocations:

$$b_t = \left( \prod_{i=0}^{t-1} (F_k(K_i, H_i) + 1 - \delta) \right) \frac{1 + \tau_{c,0}}{\tau_{c,0}} \sum_{i=t}^{\infty} \beta^i (u_{c,i} c_i + MU_{d,i} d_i - u_{l,i} h_i) - k_t \quad (8.11)$$

The above equation is the debt allocation which implies the household budget constraint is satisfied.

Since the budget constraint is satisfied, we simply substitute the resource constraint into the budget constraint to see that the government budget constraint is satisfied. Finally, by substituting the prices and policies into the IMC and reversing the derivation of the IMC,

we see that the transversality conditions are satisfied.

## 8.2 Proof of Proposition 2

First, we rewrite equations (4.11) and (4.12), using the  $\sigma$  definitions, so that:

$$\frac{\partial IMC}{\partial c_t} = u_{c,t} (1 - \sigma_c + \alpha \sigma_{sc,t} - \sigma_{hc,t}), \quad (8.12)$$

$$\begin{aligned} \frac{\partial IMC}{\partial d_t} = & MU_{d,t} (\alpha - \alpha \sigma_{s,t} + \sigma_{cs,t} - \sigma_{hs,t} + \\ & \frac{\beta u_{s,t+1} s_{2,t+1}}{MU_{d,t}} (\alpha \sigma_{s,t} - \alpha \sigma_{s,t+1} - \sigma_{cs,t} + \sigma_{cs,t+1} + \sigma_{hs,t} - \sigma_{hs,t+1})). \end{aligned} \quad (8.13)$$

Now since  $v$  is homothetic, we know that:

$$\frac{v_c(\psi c, \psi s)}{v_s(\psi c, \psi s)} \equiv \frac{v_c(c, s)}{v_s(c, s)}, \quad (8.14)$$

which implies:

$$\frac{v_{cc}(c, s) c}{v_c(c, s)} + \frac{v_{cs}(c, s) s}{v_c(c, s)} = \frac{v_{ss}(c, s) s}{v_s(c, s)} + \frac{v_{cs}(c, s) c}{v_s(c, s)}, \quad (8.15)$$

which, using the definition of  $u(\cdot)$  in equation (5.1), implies:

$$\sigma_{sc} - \sigma_c = \sigma_{cs} - \sigma_s. \quad (8.16)$$

It is also immediate from the definition of  $u(\cdot)$  that  $\sigma_{hc} = \sigma_{hs}$ . These facts and equations (4.4), (4.5), (8.12), and (8.13) together imply:

$$\frac{MU_{d,t}}{u_{c,t}} = \frac{1 + \lambda (1 - \sigma_{s,t} + \sigma_{cs,t} - (1 - \alpha) \sigma_{sc,t} - \sigma_{hs,t})}{1 + \lambda \left( \alpha - \alpha \sigma_{s,t} + \sigma_{cs,t} - \sigma_{hs,t} + \frac{\beta u_{s,t+1} s_{2,t+1}}{MU_{d,t}} j_t \right)}, \quad (8.17)$$

where  $j_t \equiv \alpha \sigma_{s,t} - \alpha \sigma_{s,t+1} - \sigma_{cs,t} + \sigma_{cs,t+1} + \sigma_{hs,t} - \sigma_{hs,t+1}$ . Hence,  $\tau_{d,t} > \tau_{c,t}$  if and only if the right hand side is greater than one, or:

$$\begin{aligned} 1 - (1 - \alpha) \sigma_{sc,t} - \sigma_{s,t} > \\ \alpha - \alpha \sigma_{s,t} + \frac{\beta u_{s,t+1} s_{2,t+1}}{MU_{d,t}} (\sigma_{s,t} - \sigma_{s,t+1} - \sigma_{cs,t} + \sigma_{cs,t+1} + \sigma_{hs,t} - \sigma_{hs,t+1}), \end{aligned} \quad (8.18)$$

which simplifies to the desired result.

### 8.3 Proof of Corollaries 3-4

For the CRR case, note that  $\sigma_{sl} = 0$ ,  $\sigma_s = 1 - \delta(1 - \sigma)$ , and  $\sigma_{sc} = (1 - \delta)(1 - \sigma)$ , which implies the left hand side of condition (5.3) is zero. The right hand side of (5.3) is also zero since  $\sigma_{cs}$  and  $\sigma_s$  are constant.

For the steady state case,  $\sigma_{i,t} = \sigma_{i,t+1}$  for all  $i \in \{s, sc, cs, hs\}$ , so the result follows immediately from condition (5.3).

### 8.4 Proof of Proposition 5

If utility is separable, equations (8.12) and (8.13) become:

$$\frac{\partial IMC}{\partial c_t} = u_{c,t}(1 - \sigma_c), \quad (8.19)$$

$$\frac{\partial IMC}{\partial d_t} = MU_{d,t} \left( \alpha - \alpha\sigma_{s,t} + \frac{\beta u_{s,t+1} s_{2,t+1}}{MU_{d,t}} (\alpha\sigma_{s,t} - \alpha\sigma_{s,t+1}) \right). \quad (8.20)$$

Equations (4.4), (4.5), (8.19), and (8.20) together imply:

$$\frac{MU_{d,t}}{u_{c,t}} = \frac{1 + \lambda(1 - \sigma_{c,t})}{1 + \lambda \left( \alpha(1 - \sigma_{s,t}) + \frac{\beta \alpha u_{s,t+1} s_{2,t+1}}{MU_{d,t}} (\sigma_{s,t} - \sigma_{s,t+1}) \right)}. \quad (8.21)$$

Hence,  $d$  is taxed at a higher rate if and only if the right hand side is greater than one, or:

$$1 - \sigma_{c,t} > \alpha(1 - \sigma_{s,t}) + \frac{\beta \alpha u_{s,t+1} s_{2,t+1}}{MU_{d,t}} (\sigma_{s,t} - \sigma_{s,t+1}), \quad (8.22)$$

which simplifies to the desired result.

### 8.5 Proof of Corollary 6

For CRR preferences,  $\sigma_{i,t} = \sigma_{i,t+1}$  for all  $i \in \{s, c\}$ , so the results follow immediately from condition (5.6).

### 8.6 Proof of Proposition 7

If utility is given by equation (5.9), equations (8.12) and (8.13) become:

$$\frac{\partial IMC}{\partial c_t} = u_{c,t}(1 - \sigma_{c,t}), \quad (8.23)$$

$$\frac{\partial IMC}{\partial d_t} = MU_{d,t} (\alpha - \alpha\sigma_{s,t} - \sigma_{hs,t} + \frac{\beta u_{s,t+1} s_{2,t+1}}{MU_{d,t}} (\alpha (\sigma_{s,t} - \sigma_{s,t+1}) + \sigma_{hs,t} - \sigma_{hs,t+1})). \quad (8.24)$$

Equations (4.4), (4.5), (8.23), and (8.24) together imply:

$$\frac{MU_{d,t}}{u_{c,t}} = \frac{1 + \lambda(1 - \sigma_{c,t})}{1 + \lambda \left( \alpha(1 - \sigma_{s,t}) - \sigma_{hs,t} + \frac{\beta u_{s,t+1} s_{2,t+1}}{MU_{d,t}} (\alpha (\sigma_{s,t} - \sigma_{s,t+1}) + \sigma_{hs,t} - \sigma_{hs,t+1}) \right)} \quad (8.25)$$

Hence,  $d$  is taxed at a higher rate if and only if the right hand side is greater than one, or:

$$1 - \sigma_{c,t} > \alpha(1 - \sigma_{s,t}) - \sigma_{hs,t} + \frac{\beta u_{s,t+1} s_{2,t+1}}{MU_{d,t}} (\alpha (\sigma_{s,t} - \sigma_{s,t+1}) + \sigma_{hs,t} - \sigma_{hs,t+1}), \quad (8.26)$$

which simplifies to the desired result.

## 8.7 Proof of Corollaries 8 and 9

For the CRR case, note that  $\sigma_s = 1 - \delta(1 - \sigma)$ , and  $\sigma_{sl} = (1 - \delta)(1 - \sigma) \frac{h}{1-h}$ . Substituting in these conditions into equation (5.10) gives:

$$\delta(1 - \alpha)(1 - \sigma) - \sigma_{c,t} > -\frac{\beta(1 - \delta)(1 - \sigma) u_{s,t+1} s_{2,t+1}}{MU_{d,t}} \left( \frac{u_{s,t} s_{1,t}}{l_t} - \frac{u_{s,t} s_{1,t}}{l_t} \right). \quad (8.27)$$

For  $\sigma \geq 1$ , the left hand side is negative. Further the right hand side is positive if:

$$\frac{u_{s,t} s_{1,t}}{l_t} + \frac{u_{s,t+1} s_{2,t+1}}{l_{t+1}} > 0, \quad (8.28)$$

$$u_{s,t} s_{1,t} l_{t+1} > -u_{s,t+1} s_{2,t+1} l_t. \quad (8.29)$$

If  $l_{t+1} \geq l_t$ , then the above inequality is satisfied since marginal utility is positive. But stability analysis indicates  $l_t$  is an increasing sequence along the transition path. Therefore, the right hand side is indeed positive, and so the inequality (5.10) cannot be satisfied, and hence  $\tau_{c,t} \geq \tau_{d,t}$ .

For the steady state case,  $\sigma_{i,t} = \sigma_{i,t+1}$  for all  $i \in \{s, hs\}$ , so the result follows immediately from condition (5.10).

## 8.8 Proof of Propositions 10-12

Proposition 10 was proved in the text. For Proposition 11, we derive the solution for  $\mu$  as follows. First, for the quadratic case, the first order conditions for the Ramsey problem (4.4)-(4.7) are now:

$$\frac{\phi_t}{\beta^t} = 1 + \mu, \quad (8.30)$$

$$\frac{\phi_t}{\beta^t} = b(1 - \beta\gamma)(1 + \mu) - (1 + 2\mu)(s_t - \beta\gamma s_{t+1}), \quad (8.31)$$

$$\frac{\phi_t(1 - \theta) \left(\frac{k_t}{h_t}\right)^\theta}{\beta^t} = (e - 1)(1 + \mu) + h_t, \quad (8.32)$$

$$\phi_t \left( \theta \left(\frac{k_t}{h_t}\right)^{\theta-1} + 1 - \delta \right) = \phi_{t-1}. \quad (8.33)$$

Using equation (8.30) to eliminate  $\phi_t$  gives:

$$1 + \mu = (b(1 + \mu) - \mu)(s_t - \beta\gamma s_{t+1}), \quad (8.34)$$

$$\frac{(1 + \mu)(1 - \theta) \left(\frac{k_t}{h_t}\right)^\theta}{\beta^t} = (e - 1)(1 + \mu) + h_t, \quad (8.35)$$

$$\beta \left( \theta \left(\frac{k_t}{h_t}\right)^{\theta-1} + 1 - \delta \right) = 1. \quad (8.36)$$

For the subtractive model equations (8.30) and (8.34) imply:

$$1 + \mu = b(1 - \beta\gamma)(1 + \mu) - (1 + 2\mu)(d_t - \gamma d_{t-1} - \beta\gamma(d_{t+1} - \gamma d_t)), \quad (8.37)$$

$$-\beta\gamma d_{t+1} + (1 + \beta\gamma)d_t - \gamma d_{t-1} = \frac{1 + \mu}{1 + 2\mu}(b(1 - \beta\gamma) - 1), \quad (8.38)$$

which has general solution given by (6.5). Proposition (12) thus holds if a non-zero and finite solution for  $\mu$  exists, which we now show.

Equation (8.36) implies the capital to labor ratio, denoted by  $A$ , is a constant equal to:

$$A \equiv \left( \frac{\theta}{\rho + \delta} \right)^{\frac{1}{1-\theta}}. \quad (8.39)$$

Thus, equation (8.35) implies

$$h_t = \frac{1 + \mu}{1 + 2\mu} \hat{h}, \quad \hat{h} \equiv 1 - e + (1 - \theta) A^\theta, \quad (8.40)$$

is constant. Thus,  $k_t = Ah_t$  is constant and equation (6.5) implies  $s_t$  and thus  $MU_{d,t}$  is constant as well. Since elasticity of substitution of consumption over time is infinite, the planner absorbs all changes in  $g_t$  by varying  $c_t$ . Combining these results with resource constraint (4.1) yields a solution for  $c_t$ :

$$c_t = \frac{1 + \mu}{1 + 2\mu} \left( \hat{h} (A^\theta - \delta A) - \hat{d} (1 - \gamma^{t+1}) \right) - \gamma^{t+1} d_{-1} - g_t. \quad (8.41)$$

Now since  $h_0$  enters into the left hand side of the IMC (4.2) and  $k_0$  is given, the solutions for  $h_0$ ,  $k_0$  and therefore  $c_0$  generally differ from the solutions for  $t \geq 1$ . Therefore, we let  $x \equiv \frac{1+\mu}{1+2\mu}$  and insert the solutions for  $c_t$ ,  $d_t$ , and  $h_t$  into the IMC (4.2) for  $t \geq 1$ , so that:

$$\begin{aligned} \frac{R_0 (k_0 + b_0)}{1 + \tau_{c,0}} &= \sum_{t=1}^{\infty} \beta^t \left[ x \left( \hat{h} (A^\theta - \delta A) - \hat{d} (1 - \gamma^{t+1}) \right) - \gamma^{t+1} d_{-1} + \right. \\ &\quad \left. (1 - \beta\gamma) \left( b - (1 - \gamma) \hat{d}x \right) \left( \hat{d}x (1 - \gamma_{t+1}) + \gamma^{t+1} d_{-1} \right) - (e - 1 + \hat{h}x) \hat{h}x \right] + \\ &\quad c_0 + MU_{d,0} d_0 - u_{l,0} h_0 + g_0 - G, \end{aligned} \quad (8.42)$$

$$\hat{d} \equiv \frac{b(1 - \beta\gamma) - 1}{(1 - \gamma)(1 - \beta\gamma)}. \quad (8.43)$$

Next, recall from Proposition 1 that  $\tau_{c,0}$  and  $\tau_{h,0}$  are given. It follows from equations (3.12) and (3.13) that the planner cannot choose  $h_0$  in this example, and instead takes the solution for  $h_0$  from the competitive model as given. Further, the terms inside the summation depend on time only through  $\gamma^{t+1}$  and  $\beta^t$ , and the equation is quadratic in  $x$ . Therefore, after evaluating the summation we can write the equation as:

$$-\zeta_2 x^2 + \zeta_1 x + \zeta_0 = 0, \quad (8.44)$$

$$\zeta_2 \equiv \hat{h}^2 + \frac{(1 - \gamma)^2}{\beta} \hat{d}^2, \quad (8.45)$$

$$\zeta_1 \equiv \zeta_2 - \left( \frac{1-\beta}{\beta} \right) (1-\gamma) \gamma \hat{d} d_{-1}, \quad (8.46)$$

$$\zeta_0 \equiv \zeta_2 - \zeta_1 + \zeta_3, \quad (8.47)$$

$$\zeta_3 \equiv \frac{1-\beta}{\beta} \left( \frac{\tau_{c,0} (1-\delta) k_0 + R_0 b_0}{1+\tau_{c,0}} + \frac{1+\tau_{c,0}-\theta}{1+\tau_{c,0}} k_0^\theta h_0^{1-\theta} + u_{l,0} h_0 - G \right). \quad (8.48)$$

A solution such that  $\mu > 0$  is a solution in the range  $\frac{1}{2} < x < 1$ . Note that equation (8.44) can be written as:

$$(\zeta_2 x + \zeta_2 - \zeta_1) (1-x) = -\zeta_3. \quad (8.49)$$

Hence it is immediate that  $\zeta_3 < 0$  is necessary for  $x < 1$ . From equation (8.48),  $\zeta_3 < 0$  if and only if:

$$G > \zeta_l \equiv \frac{\tau_{c,0} (1-\delta) k_0 + R_0 b_0}{1+\tau_{c,0}} + \frac{1+\tau_{c,0}-\theta}{1+\tau_{c,0}} k_0^\theta h_0^{1-\theta} + u_{l,0} h_0. \quad (8.50)$$

This is the lower bound for  $G$ .

Equation (8.49) implies that  $\zeta_3 < 0$  which implies both roots are less than one. It remains to show that the roots are real and that one root is greater than one half. The roots are real if and only if:

$$\zeta_1^2 + 4\zeta_2 (\zeta_2 - \zeta_1 + \zeta_3) > 0, \quad (8.51)$$

$$(2\zeta_2 - \zeta_1)^2 > -4\zeta_2 \zeta_3, \quad (8.52)$$

$$-\zeta_3 < \frac{(2\zeta_2 - \zeta_1)^2}{4\zeta_2}, \quad (8.53)$$

$$G < \zeta_l + \frac{1-\beta}{\beta} \left( \frac{(2\zeta_2 - \zeta_1)^2}{4\zeta_2} \right). \quad (8.54)$$

Finally, the roots are greater than one half if and only if:

$$\frac{2\zeta_2 - \zeta_1}{2\zeta_2} \pm \frac{\sqrt{(2\zeta_2 - \zeta_1)^2 + 4\zeta_2 \zeta_3}}{2\zeta_2} > \frac{1}{2}, \quad (8.55)$$

$$\pm\sqrt{(2\zeta_2 - \zeta_1)^2 + 4\zeta_2\zeta_3} > \zeta_2 - \zeta_1. \quad (8.56)$$

The right hand side is positive, so the smaller root cannot be bigger than one half. The larger root is bigger than one half if and only if:

$$(2\zeta_2 - \zeta_1)^2 + 4\zeta_2\zeta_3 > (\zeta_2 - \zeta_1)^2, \quad (8.57)$$

$$-\zeta_3 < \frac{3\zeta_2 - 2\zeta_1}{4\zeta_2}, \quad (8.58)$$

$$G < \zeta_l + \frac{1 - \beta}{\beta} \left( \frac{3\zeta_2 - 2\zeta_1}{4\zeta_2} \right). \quad (8.59)$$

The final bound for a unique solution is thus the intersection of conditions (8.50), (8.54), and (8.59):

$$\zeta_l < G < \zeta_l + \frac{1 - \beta}{\beta} \frac{1}{4\zeta_2} \min \left( (2\zeta_2 - \zeta_1)^2, 3\zeta_2 - 2\zeta_1 \right). \quad (8.60)$$

Defining  $\zeta_h$  as the right hand side of equation (8.60) completes the proof.

## References

- Abel, A., 1990, "Asset Prices Under Habit Formation and Catching up with the Joneses," *American Economic Review*, 80, 38–42.
- Anderson, J., 2005, "Casino Taxation in the United States," *National Tax Journal*, 58, 303–24.
- Baltagi, B., and I. Geishecker, 2006, "Rational Alcohol Addiction: Evidence from the Russian Longitudinal Monitoring Survey," *Health Economics*, 15, 893–914.
- Baltagi, B., and J. Griffin, 2002, "Rational Addiction to Alcohol: Panel Data Analysis of Liquor Consumption," *Health Economics*, 11, 485–91.
- Becker, G., M. Grossman, and K. Murphy, 1994, "An Empirical Analysis of Cigarette Addiction," *American Economic Review*, 84, 396–418.
- Becker, G., and K. Murphy, 1988, "A Theory of Rational Addiction," *Journal of Political Economy*, 96, 675–700.
- Bentzen, J., T. Eriksson, and V. Smith, 1999, "Rational Addiction and Alcohol Consumption: Evidence from Nordic Countries," *Journal of Consumer Policy*, 22, 257–79.
- Bossi, L., and P. Gomis-Porqueras, 2006, "Consequences of Modeling Habit Persistence," Discussion Paper 2006-5, University of Miami, Department of Economics Working Paper Series.
- Bossi, L., and V. Petkov, 2006, "Habits, Market Power, and Policy Selection," University of Miami Working Paper.
- Bovenberg, A., and L. H. Goulder, 1996, "Optimal Environmental Taxation in the Presence of Other Taxes: General-Equilibrium Analyses," *American Economic Review*, 86, 985–1000.
- Campbell, J., and J. Cochrane, 1999, "By Force of Habit: a Consumption Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107, 205–51.
- Chaloupka, F., 1991, "Rational Addictive Behavior and Cigarette Smoking," *Journal of Political Economy*, 99, 722–42.
- Chari, V. V., and P. J. Kehoe, 1998, "Optimal Fiscal and Monetary Policy," Discussion Paper 251, Federal Reserve Bank of Minneapolis Research Department Staff Report.
- Clotfelter, C., and P. Cook, 1990, "On the Economics of State Lotteries," *Journal of Economic Perspectives*, 4, 105–119.
- Grinols, E., and D. Mustard, 2006, "Casinos, Crime, and Community Costs," *Review of Economics and Statistics*, 88, 28–45.

- Grossman, M., F. Chaloupka, and I. Sirtalan, 1998, "An Empirical Analysis of Alcohol Addiction: Results From Monitoring the Future Panels," *Economic Inquiry*, 36, 39–48.
- Gruber, J., and B. Koszegi, 2001, "Is addiction rational? Theory and Evidence," *Quarterly Journal of Economics*, 116, 1261–1303.
- Gruber, J., and B. Koszegi, 2004, "Tax Incidence When Individuals are Time Inconsistent: The Case of Cigarette Excise Taxes," *Journal of Public Economics*, 88, 1959–88.
- Hansen, A., 2004, "Lotteries and State Fiscal Policy," Discussion Paper 46, The Tax Foundation Background Paper.
- Kenkel, D., 1996, "New Estimates of the Optimal Tax on Alcohol," *Economic Inquiry*, 34, 296–319.
- O'Donoghue, T., and M. Rabin, 2003, "Studying Optimal Paternalism, Illustrated by a Model of Sin Taxes," *American Economic Review*, 83, 186–191.
- O'Donoghue, T., and M. Rabin, 2006, "Optimal Sin Taxes," *Journal of Public Economics*, forthcoming.
- Olekalns, N., and P. Bardsley, 1996, "Rational Addiction to Caffeine: An Analysis of Coffee Consumption," *Journal of Political Economy*, 104, 1100–04.
- Parry, I., R. Laxminarayan, and S. West, 2006, "Fiscal and Externality Rationales for Alcohol Taxes," Discussion Paper 06-51, Resources For the Future Discussion Paper.
- Pogue, T., and L. Sgontz, 1989, "Taxing to Control Social Costs: The Case of Alcohol," *American Economic Review*, 79, 235–43.
- Sgontz, L., 1993, "Optimal Taxation: The Mix of Alcohol and Other Taxes," *Public Finance Quarterly*, 21, 260–75.
- Sung, H., T. Hu, and T. Keeler, 1994, "Cigarette Taxation and Demand: An Empirical Model," *Contemporary Economic Policy*, 12, 91–100.
- Waters, T., and F. Sloan, 1995, "Why Do People Drink? Tests of the Rational Addiction Model," *Applied Economics*, 27, 727–36.