Optimal Second Best Taxation of Addictive Goods in Dynamic General Equilibrium: A Revenue Raising Perspective

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Abstract

In this paper we derive conditions under which optimal tax rates for addictive goods exceed tax rates for non-addictive consumption goods within a rational addiction framework where exogenous government spending cannot be financed with lump sum taxes. We reexamine classic results on optimal commodity taxation and find a rich set of new findings. Two dynamic effects exist. First, households anticipating higher future addictive tax rates reduce current addictive consumption, so they will be less addicted when the tax rate increases. Therefore, addictive tax revenue falls prior to the tax increase. Surprisingly, the optimal tax rate on addictive goods is generally decreasing in the strength of tolerance, since strong tolerance strengthens this tax anticipation effect. Second, high current tax rates on addictive goods make households less addicted in the future, affecting all future tax revenues in a way which depends on how elasticities are changing over time. Classic results on uniform commodity taxation emerge as special cases when elasticities are constant and the addiction function is homogeneous of degree one. Finally, we also study features of addictive goods such as complementarity to leisure that, while unrelated to addiction itself, are nonetheless common among addictive goods.

Keywords: Ramsey model, dynamic optimal taxation, addictive goods, habit formation.
JEL Codes: E61, H21, H71.
“People should understand: Those who drink, those who smoke are doing more to help the state...If you smoke a pack of cigarettes, that means you are giving more to help solve social problems such as boosting demographics, developing other social services and upholding birth rates.”

Russian finance minister Alexei Kudrin on September 2, 2010 announcing the Russian government’s plan to raise excise duties on alcohol and cigarettes.

1 Introduction

A popular and increasingly common way for local, state, and federal governments to raise revenue is through taxation of addictive goods, including cigarettes, alcohol, and gambling. According to the Tax Policy Center, in 2008 the US cumulative state and local alcohol, tobacco and lottery tax revenues exceeded 43 billion dollars. What is the optimal excise tax for addictive goods, when the government must raise revenue to finance a stream of exogenous government expenditures? The goal of this paper is to characterize and analyze the conditions under which taxation of addictive goods might differ from taxes on labor and non-addictive consumption goods (hereafter ordinary goods) in a dynamic general equilibrium setting.

This paper extends classic results of optimal commodity taxation (e.g. Atkinson and Stiglitz 1972) to the case of addictive goods and obtains a rich set of new dynamic findings not found in typical models which are either static or assume very specific utility functions or both. For common cases such as homothetic and separable utility, we show that the classic results obtained in the literature on optimal taxation of ordinary goods do not necessarily extend to addictive goods, when addiction is modeled as a rational dynamic process as in Becker and Murphy (1988), hereafter BM.

Two dynamic effects emerge. Both stem from the tolerance property of addictive goods: past consumption decreases current utility by raising the amount of consumption needed to sustain the addiction (BM). The first effect we label the tax anticipation effect. To gain insight, consider an announced increase in the addictive goods excise tax. Households anticipate, prior to the tax increase, that higher addictive taxes will reduce future addictive consumption. This provides an incentive to reduce addictive consumption in the period prior to the increase in taxes, since doing so makes the household less addicted when the tax increase occurs. This reduces addictive tax revenue in the period prior to the increase in addictive taxes and therefore moderates the total revenue raised over time.

We show that, under the mild assumption of homogeneity of the function which maps
addictive consumption into consumption in excess of that required to sustain addiction (hereafter effective consumption), the tax anticipation effect is equivalent to a single current period revenue effect. In particular, the dynamic tax anticipation effect makes a tax on addictive goods equivalent to a smaller tax on effective consumption. This allows us to evaluate the merits of taxing addictive goods in familiar terms, such as the static complementarity between effective consumption and leisure.

A second effect arises since high current tax rates on addictive goods causes households to be less addicted in the future. Future effective consumption rises, affecting future addictive, ordinary, and labor tax revenues. We call this effect the addiction stock effect. Suppose for example that effective consumption is becoming more complementary with leisure over time. Standard public finance theory (Corlett and Hague 1953) suggests that the tax rate on addictive goods should be relatively high, since reduced consumption of addictive goods will increase labor supply, thus raising labor income tax revenues. However, with tolerance, reduced current consumption of addictive goods raises future effective consumption (households are less addicted in the future, and therefore get more effective consumption in the future from a given quantity of addictive goods). But then future labor supply falls, and future labor income tax revenues fall, offsetting some of the revenue gains in the current period. Whether total tax revenues rise or fall depends on how elasticities are changing over time. For example, if the complementarity of addictive consumption with leisure is decreasing over time, labor tax revenues are less sensitive to changes in current addictive consumption than to changes in future addictive consumption. Taxing addictive goods becomes less attractive over time. Optimal addictive taxation should smooth distortions: taxing addictive goods today may make taxing addictive goods more or less distortionary in the future depending, for example, on their relationship with leisure.

The addiction stock effect smoothes intertemporal distortions caused by taxation for revenue raising. In this sense, our results are related to those on capital taxation (Chamley 1986, Chari and Kehoe 1998). A disadvantage of capital taxation is that it reduces the future capital stock and thus the future tax base. Similarly, taxation of addictive goods reduces the future stock of addiction, and through the elasticity, the future tax base. Nonetheless, our results are novel because addictive consumption acts like both a final good (current addictive consumption) and as an intermediate good (current addictive consumption affects future addictive consumption). For instance, optimal steady state tax rates on addictive goods equal tax rates on ordinary consumption in some cases where optimal capital tax rates are zero, because addictive consumption acts like a final good.
Uniform taxation of addictive and ordinary goods emerges as a special cases when elasticities are constant and the effective consumption function is homogeneous of degree one. Constant elasticities ensure no distortion smoothing motivation exists, whereas homogeneity of degree one causes the tax anticipation effect to exactly offset standard static revenue effects.

The literature typically models effective consumption in one of two ways: the subtractive specification (e.g. Campbell and Cochrane 1999, Chugh 2007) and the multiplicative specification (e.g. Abel 1990). These two models differ in terms of their homogeneity properties. In this paper, we show that the optimal tax policy depends crucially on the degree of homogeneity of the addiction function.

In particular, we show that, given separable or homothetic utility with constant relative risk aversion, raising the degree of homogeneity makes addictive consumption more income inelastic in the equivalent static model without addiction. Thus, taxation of addictive goods is more attractive if the addiction model is homogeneous of degree one, as in the subtractive case, than if the addiction model is homogeneous of degree less than one, as in the multiplicative case, since it is optimal to tax necessities at a higher rate. Further, strong tolerance in the multiplicative model decreases the degree of homogeneity, making addictive goods more income elastic, which therefore lowers the optimal tax rate on addictive goods.


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1A good is habit forming if the marginal utility of the good is increasing in past consumption. We use the standard definition of addiction, which is when current consumption is increasing in past consumption, holding fixed the marginal utility of wealth and prices. Habit formation is often used in the macro literature, whereas addiction was introduced by BM. It is straightforward to show that the subtractive model of habit formation implies the good is addictive, and the multiplicative model of habit formation implies the good is addictive with an additional restriction.

2If households are heterogeneous, high addictive consumption may signal high stock of addiction, and therefore inelastic demand, which may imply higher optimal tax rates. While allowing for nonlinear taxes is possible using the Mirrless approach (Kocherlakota 2005), we chose not to adopt that framework for three reasons. First, nonlinear taxation of addictive goods is unlikely to be feasible in practice. Households can evade nonlinear taxes by buying addictive goods in small quantities or over the Internet (Goolsbee, Lovenheim, and Slemrod 2009), or by bootlegging across states, and the government cannot easily aggregate
In the next section we describe the main alternative motives for taxing addictive goods found in the literature. In sections 3 and 4, we develop a dynamic, rational addiction model and determine conditions under which optimal tax rates for addictive goods exceed tax rates for ordinary consumption goods. Sections 5 give results for a general class of preferences and for a specific analytical example. The appendix contains the proofs of all the propositions and extends the analysis to two other preferences classes.

2 Taxing Addiction

Three classical motivations exist in the literature for taxing addictive goods differently than ordinary goods. The first is to lower the external costs often associated with consumption of addictive goods. The second is because some households fail to take into account some private costs and thus over-consume. The motivation which is the focus of this paper is to raise revenue.

2.1 Addictive Goods and Fiscal Concerns

Taxation of many addictive goods, such as lotteries, has an obvious revenue raising component. Taxes on many other addictive goods are dictated by fiscal concerns. For example, the repeal of alcohol Prohibition in the 1930’s was clearly approved for tax revenue considerations as the onset of the Great Depression negatively impacted all other government revenues (Boudreaux and Pritchard 1994). Parry, Laxminarayan, and West (2009) also note that the last two increases in federal alcohol taxes were part of deficit reduction packages.\(^3\)

In November 2010 in California a ballot measure that would have allowed local government to legally sell marijuana was put to vote and rejected with 54% of the voters against it. The “Regulate, Control and Tax Cannabis Act” would have permitted local governments to tax all addictive goods purchases made by the household. Second, the majority of addictive taxes in the US are administered locally and states and municipalities tax addictive goods mostly because they are concerned with raising revenue (see the next paragraph for evidence) rather than optimal redistribution. Indeed, the vast majority of redistribution efforts are carried out at the federal level, since tax competition across states limits the ability of state’s to maintain discriminatory tax rates and since incomes within states and municipalities are more homogeneous than across the US. Lastly, in practice (probably because of the first two reasons), in the US tax system we see almost exclusively linear excise taxes.

\(^3\)For lotteries, external costs are presumably small, but the nationwide average lottery tax ranges from 40% in 1989 (Clotfelter and Cook 1990) to 31% in 2003 (Hansen 2004), accounting for 2% of state tax revenues. States spent about $272 million on lottery advertising in 1989, which is at least a strong indication that states are motivated by revenue concerns, rather than the external costs of lotteries and other forms of gambling. Finally, proposals exist to use higher cigarette taxes to close budget deficits in Florida, Illinois, West Virginia, and elsewhere.
to impose and collect cannabis-related fees and taxes similarly to what is done with alcohol. The revenue potential of taxing marijuana was the major argument in favor of this ballot measure. In an interesting and thought provoking paper, Miron (2010) analyzes the budgetary implications of legalizing drugs. He finds that the savings in government expenditure and the gains in tax revenue (at the federal and local level) that would result from replacing drug prohibition with a regime in which drugs are legal, and taxed like alcohol and tobacco to be $83 billion per year.

Since the poor presumably spend a higher fraction of income on addictive goods, taxing addictive goods for revenue raising must be justified on efficiency grounds, rather than redistribution. A few papers consider the efficient revenue raising motivation by treating addictive goods in a static way as simply goods with external costs, which are possibly complementary with leisure. If so, one can apply the ideas from the “double dividend” literature (e.g. Bovenberg and Goulder 1996). Taxing a good with external costs raises revenues which can be used to reduce taxes on labor income (the “revenue recycling effect”). If taxing addictive goods results in lower dead-weight losses than taxing labor, then the revenue recycling effect is positive and it is optimal to tax addictive goods at a relatively high rate. Moreover, a good with external costs may also be taxed above its Pigouvian rate for revenue raising if it is complementary with leisure, since the tax increases labor supply and labor income tax revenues (the “tax interaction effect”).

However, this literature models addiction in a static way as simply a good with external costs; the dynamic nature of addiction is ignored. It remains unclear how dynamic addictive properties such as tolerance affect optimal revenue raising. This paper fills this gap in the literature by considering a dynamic model of rational addiction while explicitly considering a revenue raising motive. We model addiction using BM’s rational addiction framework. In this approach, addictive consumption is linked to past consumption, but the first welfare theorem holds. If no externalities exist, then the only rationale in the BM setting for taxing addictive goods is a revenue raising one. Unlike static models, in our dynamic framework changes in tax rates on addictive goods affects future revenues, by changing future elasticities.

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4Sgontz (1993) finds the revenue recycling effect to be positive, and Parry, Laxminarayan, and West (2009) finds both the revenue recycling effect and the tax interaction effect to be positive: alcohol is complementary to leisure and also reduces labor productivity. Therefore, they find it is optimal to tax alcohol above its Pigouvian rate as part of the optimal revenue raising package.
2.2 Addictive Goods and Externalities

The standard economic rationale for taxation of addictive goods is that their consumption is often associated with external costs, such as second-hand smoke, drunk driving, and crime. The literature often finds addictive goods are taxed at a rate less than the rate which is second best in the sense that the government cannot discriminate between consumers who generate external costs and responsible consumers.\textsuperscript{5}

If we were to include a negative consumption externality for the addictive good in our model, then the results go through under slightly different conditions. In particular, the conditions for which the optimal tax rate of an addictive good is above the rate which corrects the externality is similar to the conditions derived here for which the optimal tax rate of an addictive good is above the tax rate for ordinary consumption goods. Hence, our results can be simply interpreted as relative to the tax ratio which corrects the externality.

However, we decided not to include an externality in our problem for three reasons: first, it is well known (Kenkel 1996, Pogue and Sgontz 1989) that taxing an addictive good whose consumption is imperfectly correlated with an externality is a second-best solution. Taxing the actual behavior causing the externality (e.g. make the punishment for drunk driving more severe or banning smoking in public places) is more efficient. Indeed, Parry, Laxminarayan, and West (2009) show that welfare gains from increasing drunk driving penalties exceed those from raising taxes on alcohol, even when implementation costs and dead-weight losses associated with incarceration are included. Second, as noted above the results are similar when an externality is included. Finally, our focus is on the effect of addiction on revenue raising, rather than externalities.

\textsuperscript{5}For example Kenkel (1996) finds that a tax rate on alcohol of about 42% is optimal for the drunk driving externality, while the actual average tax rate ranges from over 50% in 1954 to 20% in the 1980s. Moreover, Grinols and Mustard (2006) estimate external costs of casino gambling are 47% of revenues, thus the optimal tax would be higher than 47% if demand for casino gambling is inelastic, or less than 47% if a significant fraction of casino gamblers do not impose external costs. Anderson (2005) reports that casinos pay 16% of gross revenues in taxes. The empirical evidence is, however, mixed for cigarettes taxation: Manning, Keeler, Newhouse, Sloss, and Wasserman (1989) estimated the gross external cost of smoking in the U.S. of approximately $0.43 per pack, but only $0.16 per pack once reductions in health care expenditures stemming from premature deaths were included. Viscusi (1995) finds that after accounting also for the lower nursing home cost and retirement pension savings the net external costs of smoking are negligible for the U.S. Conversely, Gruber and Koszegi (2001) estimate external costs of smoking at $0.94 to $1.75 per pack, versus an average excise tax of about $0.65.
2.3 Addictive Goods and Non-market Internal Costs

Another source of non-market costs occurs if addiction is modeled as non-fully rational excess consumption. Households may fail to take into account the self-adverse health effects caused by consumption of addictive goods, either because they are unaware that addictive goods consumption has adverse health effects (e.g. Kenkel 1996) or because some households are exogenously assumed to be unable to take into account the health gains from reducing addictive goods consumption (e.g. Pogue and Sgontz 1989). Alternatively, households may mistakenly over consume relative to underlying preferences when exposed to environmental cues (Bernheim and Rangel 2004). When some households are exogenously assumed not to consider some private costs, they over-consume. The resulting “internality” causes the optimal second best (again, in the sense that the government cannot distinguish between naive and rational households) tax rate to rise considerably.⁶

A related, subsequent literature makes excess consumption endogenous and rational by modeling addictive goods with preferences that are time inconsistent (Gruber and Koszegi 2001, Gruber and Koszegi 2004, O’Donoghue and Rabin 2003, O’Donoghue and Rabin 2006, Gul and Pesendorfer 2007). In this approach, consumers are rational, but over-consume in the sense that welfare increases with a government policy that reduces consumption to a level which households would choose if they could pre-commit to consume less in the future. For example, O’Donoghue and Rabin (2006) compute numerical examples where the optimal tax on unhealthy foods ranges from 1-72%. Gruber and Koszegi (2001) show that the optimal tax on cigarettes rises to at least $1 per pack when the time inconsistency problem is included. However, Gul and Pesendorfer (2007) show that taxes decrease welfare whereas smoking bans and similar prohibitive policies increase welfare by serving as commitment devices.

Models with internalities match a number of empirical features of addiction, including rehab cycles, environmental cues, commitment devices, and kicking the habit. Empirical evidence also exists for the tax anticipation and addiction stock effect (see section 3.2), features of the rational addiction model. For simplicity, our focus is on the tax anticipation and addiction stock effects, and leave for future research the effect of other interesting features of addiction on revenue raising.

⁶Kenkel (1996) finds the optimal tax rate on alcohol rises to about 106% while Pogue and Sgontz (1989) find the optimal tax rate on alcohol rises to 306%.
3 Model

We consider an infinite horizon closed economy in discrete time. The economy is populated by a continuum of identical households of measure one who maximize the discounted sum of instantaneous utilities. A large number of identical firms produce both addictive and ordinary goods using a constant return to scale technology. Finally, there is a government that needs to finance a constant stream of government expenditures through fiscal policy.

3.1 Firms

A large number of identical firms at time $t$ rent capital $k_t$ and labor $h_t$ from households to produce a composite good using a technology $F(k_t, h_t)$. We assume throughout the paper that:

**Assumption A1** $F(k_t, h_t)$ is constant returns to scale and increasing, concave, and satisfies Inada conditions in each input.

Let $w_t$ denote the wage rate and $r_t$ the rental rate of capital, then the objective of the firm is to maximize profits, which equal:

$$\max_{k_t, h_t} \left\{ F(k_t, h_t) - r_t k_t - w_t h_t \right\}.$$  (3.1)

Let subscripts on functions denote corresponding partial derivatives. The equilibrium rental rate and wage rate are:

$$r_t = F_k(k_t, h_t) ,$$  (3.2)

$$w_t = F_h(k_t, h_t) .$$  (3.3)

For simplicity we assume that the composite good can be used for either addictive or consumption or investment.\(^7\)

\(^7\)It is possible (but cumbersome) to extend the analysis to allow the production technology to differ by consumption goods.
3.2 Households

A representative household derives utility from consumption of an ordinary (non-addictive) good, \( c_t \), the fraction of time allocated to leisure, \( 1 - h_t = l_t \in [0, 1] \), and effective consumption (i.e. consumption in excess of that required to sustain the addiction), \( s_t \), of an addictive good, \( d_t \).

We model addiction using the rational addiction framework of Becker and Murphy (1988). In this approach, consumption of the addictive good is specifically related to past consumption. Evidence for rational addiction exists for cigarettes, alcohol, drugs (Grossman and Chaloupka 1998), food consumption (Carrasco, Labeaga, and Lopez-Salido 2005), carbonated drinks (Liu and Lopez 2009), and caffeine (Olekalns and Bardsley 1996). These papers essentially test and find evidence for the tax anticipation effect, which is a testable implication of the rational addiction model. Dragone (2009) and Levy (2002) give a positive theory for the worldwide diffusion of obesity by proposing a rational addiction model for (junk) food intake behavior that is capable of displaying binges, diets and cycles for food consumption.

Let \( s_t = s(d_t, d_{t-1}) \) map addictive consumption into effective consumption.\(^{11}\) We assume throughout the paper that:

**Assumption A2** Per-period utility, \( u(c_t, s_t, l_t) \), is strictly increasing, concave, and satisfies the Inada conditions in each argument.

Lifetime utility is:

\[
U = \sum_{t=0}^{\infty} \beta^t u(c_t, s_t, l_t); \tag{3.4}
\]

where \( \beta \) is the discount factor with rate of time preference \( \rho = \frac{1-\beta}{\bar{\beta}} \).

For effective consumption, we assume throughout the paper that:

**Assumption A3** \( s(d_t, d_{t-1}) \) is homogeneous of degree \( \alpha \) in \([d_t, d_{t-1}]\) (HD-\( \alpha \)) and satisfies \( s_1 > 0, s_2 < 0, s_{11} \leq 0 \), and \( \sigma_s(c_t, s_t, l_t) = \frac{-u_{ss}(c_t, s_t, l_t) s_t}{u_s(c_t, s_t, l_t)} \geq \frac{s_{22}}{s_2^2} \).

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\(^8\)Endogenous labor supply allows the model to determine the effect of complementarities with leisure on taxation of addictive goods (see section 7.6).


\(^{11}\)Here we follow the literature (e.g. Campbell and Cochrane 1999) and assume \( s \) has only one lag. Becker, Grossman, and Murphy (1994), who assume a utility function of the form \( u(c_t, d_t, d_{t-1}) \), except they assume no preferences for leisure. Our assumption below that \( s \) is homogeneous is the main restriction we impose on their utility specification. However, our specification is considerably more general than the addiction literature which typically assumes one of the two special cases for \( s \) given below.
The first inequality states that households get positive marginal utility from consumption of the addictive good. The second inequality states that the addictive good has the tolerance property, meaning past consumption lowers current utility, which is also known a harmful addiction.\(^{12}\) The third and fourth inequalities are sufficient conditions which ensure that \(U\) is concave in the choice set \([c_t, l_t, d_t]\) if \(u\) is concave when \(s_t = d_t\) (i.e. the standard problem with no addiction is concave, see assumption A2). The role of homogeneity is discussed below.

The two most commonly used specifications of effective consumption, \(s\), in the literature are the subtractive model (see for example Campbell and Cochrane 1999, Chugh 2007), where effective consumption is:

\[
s_t = d_t - \gamma d_{t-1}, \tag{3.9}
\]

and the multiplicative model (see for example Abel 1990), which specifies effective consumption as:

\[
s_t = \frac{d_t}{d_{t-1}^\gamma}. \tag{3.10}
\]

\(^{12}\)Gruber and Koszegi (2004) and others define habit formation as past consumption increasing the taste for current consumption. Therefore, a good is habit forming if and only if:

\[
\frac{\partial^2 u}{\partial d_t \partial d_{t-1}} > 0. \tag{3.5}
\]

From the assumptions on \(s\), a good is habit forming if and only if:

\[
\sigma_s (c_t, s_t, l_t) > \frac{s_t s_{12} (d_t, d_{t-1})}{s_1 (d_t, d_{t-1}) s_2 (d_t, d_{t-1})}. \tag{3.6}
\]

BM and others define addiction as when past consumption increases current consumption, holding fixed prices and the marginal utility of ordinary consumption. Let \(c_t = y_t - p_t d_t\), where \(y_t\) represents income in period \(t\) and \(p_t\) is the price of \(d\) in period \(t\), then \(d\) is addictive if and only if:

\[
\frac{\partial d_t}{\partial d_{t-1}} = \frac{\partial^2 U}{\partial d_t \partial d_{t-1}} > 0, \tag{3.7}
\]

holding fixed the marginal utility of consumption. Using the concavity assumptions, equation (3.7) simplifies to:

\[
\frac{\partial^2 U}{\partial d_t \partial d_{t-1}} = \frac{\partial^2 U}{\partial d_t \partial d_{t-1}} > 0. \tag{3.8}
\]

Thus \(d\) is addictive if and only if \(d\) is habit forming given the one-lag specification of effective consumption, and the concavity assumptions. In general, if \(s\) has more than one lag, addiction is more restrictive than habit formation. Thus, for example, habit formation and addiction are not equivalent in BM, but are equivalent in Becker, Grossman, and Murphy (1994).
In both models \( \gamma \geq 0 \) denotes the strength of tolerance. If \( \gamma = 0 \), then past consumption has no weight at all, in which case the model reduces to the standard time separable model, and utility is fully determined by consumption levels and not by the changes in consumption.

The subtractive specification satisfies assumption A3, and the multiplicative specification satisfies A3 for \( \sigma_s(c_t, s_t, l_t) > (1 + \gamma) / \gamma \forall t \). However, two key differences exist. In the subtractive model, effective consumption is \( H(D-1) \). In the multiplicative model, effective consumption is \( H(D-(1-\gamma)) \), and the degree of homogeneity depends on the degree of tolerance. Moreover, equation (3.8) implies that if \( s \) is subtractive, then \( d \) is additive for all \( \gamma > 0 \). However, if \( s \) is multiplicative, then \( d \) is additive if and only if \( \sigma_s(c_t, s_t, l_t) > 1 \) for all \([c_t, s_t, l_t] \).

**Household Resources and Optimal Decisions**

The household budget constraint sets after tax wage and rental income and government bond redemptions (equal to \( R^b_t b_t \), where \( b_t \) are bonds issued in \( t - 1 \) and redeemed in \( t \)) equal to after tax expenditures on government bond issues and consumption of addictive, ordinary, and investment goods given by \( i_t = k_{t+1} - (1 - \delta) k_t \), where \( \delta \) is the depreciation rate. Since consumption of ordinary, addictive, and investment goods all have the same production technology, they have the same pre-tax price, which is normalized to one. Let \( \tau_c \) and \( \tau_d \) be the tax rates on consumption of ordinary and addictive goods, respectively and let \( \tau_h \) be the tax rate on labor income. The household budget constraint is then:

\[
R^b_t b_t + r_t k_t + (1 - \tau_{h,t}) w_t h_t = (1 + \tau_{c,t}) c_t + (1 + \tau_{d,t}) d_t + i_t + b_{t+1} . \tag{3.11}
\]

In the next subsection we show that introducing a tax on capital would not change the results. Also, we will be more specific on the tax smoothing role played by government bonds. Let \( \lambda_t \) denote the Lagrange multiplier on the budget constraint. The resulting household first order conditions are:

\[
(1 + \tau_{c,t}) \lambda_t = \beta^t u_c(c_t, s_t, l_t) , \tag{3.12}
\]

\[
(1 - \tau_{h,t}) w_t \lambda_t = \beta^t u_l(c_t, s_t, l_t) , \tag{3.13}
\]

\[
(1 + \tau_{d,t}) \lambda_t = \beta^t u_s(c_t, s_t, l_t) s_1(d_t, d_{t-1}) + \beta^{t+1} u_s(c_{t+1}, s_{t+1}, l_{t+1}) s_2(d_{t+1}, d_t) , \tag{3.14}
\]

\[
\lambda_t R_t = \lambda_{t-1} , t \geq 1 , \tag{3.15}
\]
\[
\lambda_t R_t^b = \lambda_{t-1}, \ t \geq 1,
\]
(3.16)

\[R_t = r_t + 1 - \delta.
\]
(3.17)

Equations (3.12)-(3.16), the budget constraint (3.11), initial conditions \(k_0\) and \(d_{-1}\), and the appropriate transversality conditions characterize the optimal household decisions \(k_t, b_t, h_t, c_t, d_t,\) and \(\lambda_t\). In equation (3.14), the household increases effective consumption by increasing \(d_t\) (first term on the right hand side), but also increases tolerance and therefore reduces future effective consumption (second term on the right hand side). From equations (3.12) and (3.14) we have:

\[
\frac{1 + \tau_{d,t}}{1 + \tau_{c,t}} \frac{u_c(c_t, s_t, l_t) s_1(d_t, d_{t-1}) + \beta u_s(c_{t+1}, s_{t+1}, l_{t+1}) s_2(d_{t+1}, d_t)}{u_c(c_t, s_t, l_t)} \equiv \frac{MU_{d,t}}{MU_{c,t}};
\]
(3.18)

where \(MU_{i,t}\) represents the marginal utility of good \(i\) at time \(t\).

Any difference in tax rates drives a wedge between the marginal utilities of the consumption of ordinary and addictive goods. Thus the optimal tax rate of addictive goods exceeds the tax rate of ordinary consumption goods \((\tau_{d,t} > \tau_{c,t})\) if and only if \(MU_{d,t} > MU_{c,t}\). The goal of this paper is to find conditions under which the marginal utility of addictive goods differs from that of ordinary goods.\(^{13}\)

### 3.3 Government

The government finances an exogenous sequence of expenditures, \(g_t\), with bond issues and consumption and labor income tax revenues. The government budget constraint is:

\[g_t = \tau_{h,t} w_t h_t + \tau_{c,t} c_t + \tau_{d,t} d_t + b_{t+1} - R_t^b b_t.
\]
(3.19)

As will be clear below, three wedges exist in our model: one between the marginal utility of addictive and ordinary consumption, a second between the wage and the marginal rate of substitution between consumption and leisure, and a third between the intertemporal marginal rate of substitution and the rate of interest. Thus we need only three tax instruments for a complete tax system. We therefore set interest taxes equal to zero, noting that the government can affect all three margins by setting a time-varying consumption tax, a

\(^{13}\)Note that \(\tau_c\) is defined as a tax on ordinary goods only, not all consumption goods. Thus, the ordinary consumption tax is not functionally equivalent to a wage tax, as would be the case with a single consumption good. Any change in the wage tax would equally raise the leisure cost of both types of consumption goods, and so does not affect the wedge.
wage tax, and an addictive goods tax. To see this, use equation (3.12) to eliminate $\lambda$ from the household first order conditions so that equation (3.15) becomes:

$$\left( \frac{1 + \tau_{c,t-1}}{1 + \tau_{c,t}} \right) R_t = \frac{u_{c,t-1}}{\beta u_{c,t}}.$$  \hspace{1cm} (3.20)

Given a tax rate $\tau_{c,t-1}$, the planner can create an effective interest tax by varying $\tau_{c,t}$, while using $\tau_{d,t}$ to alter the wedge between the marginal utility of addictive and ordinary consumption, and $\tau_{h,t}$ to alter the wedge between the wage and the marginal rate of substitution between consumption and leisure. Alternatively, suppose we remove the tax on consumption and add a tax $\tau_k$ on interest so that (3.20) becomes:

$$(1 - \tau_{k,t}) R_t = \frac{u_{c,t-1}}{\beta u_{c,t}}.$$ \hspace{1cm} (3.21)

Now the planner can equivalently use $\tau_{k,t}$ to vary the wedge between the intertemporal marginal rate of substitution and the interest rate, while again using $\tau_{d,t}$ and $\tau_{h,t}$ to vary the other wedges. Thus, the optimal allocations derived below can be decentralized using either tax system.

As is common in the literature (see for example Chamley 1986), the government optimally uses bonds to smooth tax burdens over time. In the absence of bonds, the government may favor the tax with better smoothing properties. Changes in current addictive goods tax rates affects both current and future tax revenue. The existence of government bonds enables us to conveniently summarize the effect of a change in current addictive tax rates on all periods as the effect on the infinite horizon version of the government’s budget constraint.

Let $\pi = \left[ (\tau_{c,t})_{t=0}^{\infty}, (\tau_{d,t})_{t=0}^{\infty}, (\tau_{h,t})_{t=0}^{\infty}, (g_t)_{t=0}^{\infty} \right]$ denote an infinite sequence of government policies. As is standard in the literature (e.g. Gruber and Koszegi 2001), we assume the existence of a commitment technology, so that the government commits to all future policies at time zero.

\[14\] We also do not allow a tax on effective consumption, since informational asymmetries rule out taxes on effective consumption in practice.

\[15\] Although labor taxes do not affect the wedge between the marginal utilities of ordinary and addictive consumption, they play an important role in the analysis in that without the tax the tax system would be incomplete and the government would be forced to have the same wedge for consumption/savings as labor/leisure. The government would then have to consider the effect of addictive taxes on this combined wedge, considerably complicating the analysis in a way that is unlikely to matter in practice (state and local governments can equivalently affect all three wedges using property (capital), addictive, and ordinary consumption taxes).

\[16\] However, addictive taxes are common at the state and local level, which frequently have constitutional borrowing restrictions. We leave this interesting case to future research.

\[17\] In principle the government could promise low future taxes on addictive goods, and then find it optimal
4 Equilibrium and Ramsey Problem

Equations (3.2), (3.3), (3.11), (3.12) - (3.15), and (3.19) form a system of nine nonlinear equations that characterize the competitive equilibrium. Hence:

**Definition 1** Given initial values \(k_0\) and \(d_{-1}\), a competitive equilibrium is a set of allocations \(\{c_t, d_t, h_t, k_t\}\), prices \(\{w_t, r_t, R^b_t\}\) and a sequence of policies \(\pi\) that satisfy the household budget constraint (3.11), firm profit maximization (3.1), the government budget constraint (3.19), and household maximization of (3.4) for all \(t\).

We use the primal approach to determine optimal taxation. The primal approach uses household and firm first order conditions to eliminate prices and policies from the equations that define the competitive equilibrium. The planner then chooses allocations which maximize welfare subject to the remaining equations from the competitive equilibrium. These equations are the resource constraint:

\[
F (k_t, h_t) = c_t + d_t + k_{t+1} - (1 - \delta) k_t + g_t, \tag{4.1}
\]

and the implementability constraint (IMC):

\[
\sum_{t=0}^{\infty} \beta^t \left\{ u_c (c_t, s_t, l_t) c_t + \left[ u_s (c_t, s_t, l_t) s_1 (d_t, d_{t-1}) + \beta u_s (c_{t+1}, s_{t+1}, l_{t+1}) \right] \cdot s_2 (d_{t+1}, d_t) \right\} d_t - u_l (c_t, s_t, l_t) h_t \right\} - \frac{u_c (c_0, s_0, l_0) (R_0 k_0 + R^b_0 b_0)}{1 + \tau_{c,0}} = 0. \tag{4.2}
\]

The IMC uses the household first order conditions to substitute out for all prices and policies in the budget constraint and then recursively eliminates \(\lambda_t\). Thus, the IMC is the infinite horizon version of the household budget constraint where all prices and policies have been written in terms of their corresponding marginal utilities. It is immediate from Walras’ Law and the resource constraint that the IMC can also be thought of as the infinite horizon version of the government budget constraint. The Ramsey approach is therefore very convenient in that the planner can, through the IMC, determine the effect of a change in \(d_t\) on government revenues over the infinite horizon.

The first proposition gives the relationship between the competitive equilibrium and the IMC and resource constraint.

**PROPOSITION 1** Let assumptions (A1)-(A3) hold. Given \(k_0, d_{-1}, \tau_{h,0},\) and \(\tau_{c,0}\), the allocations of a competitive equilibrium satisfy (4.1) and (4.2). In addition, given \(k_0, d_{-1}, \tau_{h,0},\) and to renege on the promise once households become addicted.
\( \tau_{c,0} \), and allocations which satisfy (4.1) and (4.2), prices and policies exist which, together with the allocations, are a competitive equilibrium.

All proofs are in the appendix.

The Ramsey Problem (RAM) determines the optimal tax package that maximizes welfare subject to the IMC and resource constraint:

\[
\text{RAM} = \max_{c_t, d_t, h_t, k_t} \left\{ \sum_{t=0}^{\infty} \beta^t \left( u(c_t, s_t, 1-h_t) + \mu \left[ u_c(c_t, s_t, 1-h_t) c_t + (u_s(c_t, s_t, 1-h_t) s_1(d_t, d_{t-1}) + \beta u_s(c_{t+1}, s_{t+1}, h_{t+1}) s_2(d_{t+1}, d_t)) d_t - u_t(c_t, s_t, 1-h_t) h_t \right] - \mu \frac{u_c(c_0, s_0, l_0) (R_0 k_0 + R_0^b b_0)}{1 + \tau_{c,0}} + \phi_t \left[ F(k_t, h_t) - c_t - d_t - k_{t+1} + (1 - \delta) k_t - g_t \right] \right\}
\]

Here, \( \mu \) and \( \phi_t \) are the Lagrange multipliers on the IMC and resource constraints, respectively.

Let IMC with a variable subscript denote the partial derivative of IMC with respect to that variable. The first order conditions that characterize optimal taxation are:

\[
\phi_t = \beta^t MU_{c,t} + \mu IMC_{c_t}, \quad (4.4)
\]

\[
\phi_t = \beta^t MU_{d,t} + \mu IMC_{d_t}, \quad (4.5)
\]

\[
\phi_t F_h(k_t, h_t) = \beta^t u_t(c_t, s_t, 1-h_t) - \mu IMC_{h_t}, \quad (4.6)
\]

\[
\beta \phi_t(F_k(k_t, h_t) + 1 - \delta) = \phi_{t-1}. \quad (4.7)
\]

Conditions (4.4)-(4.6) equate the marginal social welfare of \( c, d, \) and \( l \) with the marginal resource cost \( \phi \). The marginal social value equals the marginal private value, plus the value of the marginal tax revenue. Equation (4.7) equates the return on capital with the intertemporal marginal rate of substitution.

Combining equations (4.4) and (4.5), we see that since the marginal rate of transformation is one, the first best (\( \mu = 0 \)) sets the marginal rate of substitution equal to one. In the second best, equation (3.18) implies the tax ratio is greater than one if and only if the marginal rate
of substitution of addictive goods for ordinary goods is greater than one, and so:

$$\tau_{d,t} > \tau_{c,t} \text{ iff } IMC_{d,t} < IMC_{c,t}. \quad (4.8)$$

If the marginal rate of substitution of addictive goods for ordinary goods is greater than one, then moving resources from ordinary to addictive consumption raises private utility. Therefore, it must be that moving resources from ordinary to addictive consumption results in the loss of tax revenue.

The marginal tax revenue of ordinary consumption depends on how a small change in ordinary consumption affects ordinary consumption tax revenue, addictive tax revenue in periods $t$ and $t-1$, and labor income tax revenue:

$$\frac{IMC_{c,t}}{\beta^t} = u_{c,t} + u_{cc,t}c_t + s_1,t d_t u_{cs,t} + s_2,t d_{t-1} u_{cs,t} - u_{ct} h_t. \quad (4.9)$$

An increase in $c_t$ directly increases ordinary consumption tax revenue (first term), but decreases the marginal utility of consumption and thus requires the planner to lower the ordinary consumption tax rate in order to maintain equilibrium, which lowers tax revenues (second term). The third and fourth term are offsetting effects. Suppose for example that $u_{cs}>0$. Then an increase in $c_t$ raises $MU_{d,t}$, so the planner must raise the tax on $d_t$ to maintain equilibrium, which increases addictive tax revenues. However, an increase in $c_t$ with $u_{cs}>0$ lowers $MU_{d,t-1}$: consuming $d_{t-1}$ is less attractive because it causes effective consumption to fall in $t$ (tolerance), when the marginal utility of effective consumption is relatively high due to the increase in $c_t$. Thus, the planner must also lower $\tau_{d,t-1}$, reducing revenues. Thus an increase in $c_t$ has offsetting addictive tax revenue effects, but both work through the $u_{cs}$ term. Given the homogeneity assumption, these two dynamic effects can be combined into a single effect, $\alpha u_{cs}$, as if a smaller tax on $s_t$, rather than $d_t$, existed. Finally, the fourth term implies that an increase in $c_t$ increases preferences for leisure, and thus causes the planner to decrease the labor income tax rate to maintain equilibrium, if and only if $u_{ct}>0$.

Let $im_t$ denote the period $t$ part of the implementability constraint:

$$im_t = \beta^t \left( u_c(c_t, s_t, l_t) c_t + \left[ u_s(c_t, s_t, l_t) s_1(d_t, d_{t-1}) + \beta u_s(c_{t+1}, s_{t+1}, l_{t+1}) \cdot 
\begin{bmatrix}
 s_2(d_{t+1}, d_t) & d_t - u_l(c_t, s_t, l_t) h_t
\end{bmatrix} \right] \right) \quad (4.10)$$
The marginal tax revenue of addictive goods depends on revenue effects in periods \( t - 1, t, \) and \( t + 1 \):

\[
\frac{IMC_{dt}}{\beta^t} = \beta^{-1}im_{t-1,dt} + im_{t,dt} + \beta im_{t+1,dt}.
\] (4.11)

The first term of (4.11) indicates that if, for example, the planner lowers \( d_t \) by raising the addictive tax rate in period \( t \), then households anticipate higher taxes in period \( t \) and respond by changing behavior in period \( t - 1 \), changing tax revenue in period \( t - 1 \). We call this the tax anticipation effect.\(^{18}\)

The second term of (4.11) are the revenue effects on ordinary, addictive, and labor income tax revenue in the current period. The current period revenue consist of static effects as in equation (4.9), but also a dynamic stock effect for \( d_t \). For example, households are more reluctant to increase \( d_t \) following a decrease in the addictive tax rate, since an increase in \( d_t \) raises the future stock of \( d_t \).

The third term indicates that if, for example, the planner lowers \( d_t \) by raising the addictive tax rate, then households are less addicted in period \( t + 1 \) which changes ordinary, addictive, and labor tax revenues in period \( t + 1 \). We denote both the current and future revenue effects caused by a change in the addiction stock the addiction stock effect. Therefore, comparing (4.9) and (4.11), we see that addictive consumption has two additional dynamic revenue effects, the tax anticipation effect and the addiction stock effect, which are not present with ordinary consumption.

Using the homogeneity of \( s \) and it’s first derivatives, equation (4.11) becomes:

\[
\frac{IMC_{dt}}{\beta^t} = \alpha MU_{d,t} \left( \alpha u_{ss,t}s_{1,t}s_t + \alpha \beta u_{ss,t+1}s_{2,t+1}s_{t+1} + u_{cs,t}s_{1,t}c_t + \beta u_{cs,t+1}s_{2,t+1}c_{t+1} - u_{sl,t}s_{1,t}h_t - \beta u_{sl,t+1}s_{2,t+1}h_{t+1} \right).
\] (4.12)

The first two terms on the right and side show the tax anticipation effect and some of the current period revenue effects and addiction stock effects are equivalent to a smaller (for \( \alpha < 1 \)) revenue effect in period \( t \). The revenue effect is smaller because the dynamic and current period effects tend to offset. For example, raising the addictive tax rate in period \( t \)

\(^{18}\)Strictly speaking, given no uncertainty, all addictive consumption levels over time are endogenously determined by the planner through the tax rates at time 0, and the first order condition for \( d_t \) is written with all other values of \( d_t \) over time held at their optimal levels. Therefore, the terms in equation (4.11) indicate that a change in the tax rate in \( t \) affects incentives in \( t - 1 \) and \( t + 1 \), forcing the planner to alter the tax rates in \( t - 1 \) and \( t + 1 \) to maintain equilibrium with the same optimal values of \( d_{t-1} \) and \( d_{t+1} \). The change in tax rates is what affects revenues in \( t - 1 \) and \( t + 1 \). Rates in other periods are unaffected since the economy remains on the same equilibrium path.
may raise revenue in period $t$, but by the tax anticipation effect causes households to reduce addictive consumption in $t - 1$, offsetting some of the revenue gains.

The last five terms show that the remaining addiction stock effects and current period revenue effects are not equivalent to a current period revenue effect. Nonetheless, since $s_1$ and $s_2$ are of opposite signs, the revenue effects in $t$ and $t + 1$ are in the opposite direction. For example, an increase in $d_t$ raises effective consumption $s$ in period $t$, but lowers effective consumption in period $t + 1$ holding all other decisions equal.

In summary then, simple static results and intuition might indicate that taxing addictive goods is a good revenue raiser because addictive goods tend to be complementary to leisure. However, dynamic considerations are likely to yield more moderate results. To see this consider, for example, the last two terms of (4.12). If $u_{sl} > 0$, then a decrease in $d_t$ raises labor income tax revenues in period $t$ (second to last term), but increases $s_{t+1}$, reducing labor income tax revenues in period $t + 1$ (last term). Thus the dynamic and static terms offset. In addition, the stronger the tolerance, the stronger is the dynamic effect. A similar intuition holds for the other terms. To obtain further results requires more specific preference assumptions.

5 Results for General Preference Classes

As in the literature on optimal commodity taxation, characteristics of the utility function play an important role in determining any deviations from uniform taxation. In this section we first establish theoretical results for the well-known case of homothetic utility; we then proceed to numerically investigate the linear quadratic utility case. In the appendix we study the separable utility function and a utility function which is not weakly separable in leisure.

5.1 Homothetic Utility with weak separability of leisure

In this section we assume the utility function takes the form:

$$u(c_t, s_t, l_t) = q(v(c_t, s_t), l_t),$$

(5.1)

where $v(.)$ is homothetic and $q(.)$ is an increasing function. Weak separability in leisure implies that the more income inelastic good is also more complementary with leisure. Further, homotheticity implies $c$ and $s$ are equally income elastic, and thus equally substitutable with leisure. Thus, homotheticity and weak separability imply uniform taxation of $c$ and $s$. To what extent does this result change for the more realistic case where $d$ is taxed rather than
To find out, we combine equations (4.8), (4.9), and (4.12), assuming (5.1). Let us define the following elasticities:

\[ \sigma_{cs,t} = \frac{u_{cs}(c_t, s_t, l_t)}{u_s(c_t, s_t, l_t)} c_t, \quad \sigma_{sc,t} = \frac{u_{cs}(c_t, s_t, l_t)}{u_c(c_t, s_t, l_t)} s_t, \quad \sigma_{hs,t} = \frac{u_{sl}(c_t, s_t, l_t)}{u_s(c_t, s_t, l_t)} h_t, \]

(5.2)

and let \( \sigma_{hc,t} \) be defined analogously. Then we have the following result.

**PROPOSITION 2** Let assumptions (A1)-(A3) hold. In addition, let \( u(.) \) be of the form given in equation (5.1). Then \( \tau_{d,t} > \tau_{c,t} \) if and only if:

\[ (1 - \alpha)(1 - \sigma_{s,t} - \sigma_{sc,t}) > J(\alpha \Delta \sigma_s - \Delta \sigma_{cs} + \Delta \sigma_{hs}), \]

(5.3)

where

\[ J \equiv -\beta u_{s,t+1}s_{2,t+1} + MU_{d,t}, \quad \Delta \sigma_i \equiv \sigma_{i,t+1} - \sigma_{i,t}, \quad i = s, cs, hs. \]

Homotheticity of \( c \) and \( s \) does not generally result in uniform taxation of addictive and ordinary goods. Three effects determine the intuition behind condition (5.3). From (4.9) and (4.12), the dynamic tax anticipation and addiction stock effects are present for addictive consumption but not for ordinary consumption. In addition, even ignoring the dynamic effects taxes are not uniform since homotheticity of \( c \) and \( s \) does not necessarily imply homotheticity of \( c \) and \( d \).

To see the effects in isolation, first assume that \( s_2 = 0 \), which eliminates both dynamic effects. Condition (5.3) reduces to:

\[ -\frac{s_{11,t}d_t}{s_{1,t}} + \left(1 - \frac{s_{1,t}d_t}{s_t}\right)(-\sigma_{s,t} - \sigma_{sc,t}) > 0. \]

(5.4)

Condition (5.4) is a static condition, which holds with equality if and only if \( u \) is homothetic in \( c \) and \( d \).

To see the tax anticipation effect, consider only the first term of (4.11) and the fourth term of (4.9). Condition (5.3) becomes:

\[ -\frac{s_{12,t}d_{t-1}}{s_{1,t}} + \frac{s_{2,t}d_{t-1}}{s_t} \sigma_{s,t} > -\frac{s_{2,t}d_{t-1}}{s_t} \sigma_{sc,t}. \]

(5.5)

The left (right) hand side of condition (5.5) gives the effect on addictive (ordinary) tax

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We are following the addiction literature here which models utility as a function of effective consumption.
revenues in period $t-1$ from a change in $d_t$ ($c_t$). By condition (3.6), the left hand side is negative, which favors taxing $c_t$ at a higher rate. Households anticipate that an increase in taxes in period $t$ will reduce addictive consumption in period $t$. This reduces the marginal utility of addictive consumption in $t-1$ as addictive consumption in $t-1$ will raise the stock of addiction in $t$, reducing effective consumption when it is already low due to the tax. Therefore households desire to reduce addictive consumption in $t-1$, so the planer must decrease the addictive tax rate in $t-1$ to maintain equilibrium, reducing revenues gained from raising the tax in $t$. Thus, the tax anticipation effect tends to reduce optimal addictive tax rates (unless $\sigma_{sc}$ is sufficiently negative).

Combining (5.4) and (5.5) results in the left hand side of (5.1). Thus the tax anticipation and static current period effects reduce to a single current period term which depends only on the elasticities and homogeneity, and should therefore be straightforward to check in empirical applications.

The addiction stock effect is the impact on current and future tax revenues from a change in $s_{t+1}$ caused by a change in $d_t$:

$$0 > 1 - \alpha + \alpha \sigma_{s,t+1} - \sigma_{cs,t+1} + \sigma_{hs,t+1}.$$  

(5.6)

It is important to note that addiction stock effect is weighted differently as it occurs in a different period with a different marginal utility of effective consumption. In particular, the tax anticipation and static revenue effects for a change in $d$ (the left hand sides of equations 5.4 and 5.5) have weight $u_{s,t}s_{1,t}/MU_{d,t}$, whereas the addiction stock effect has weight $\beta u_{s,t+1}s_{2,t+1}/MU_{d,t}$. Normalizing the period $t$ weight to one and adding (5.4), (5.5), and (5.6) gives (5.3). The right hand side of (5.3) shows that the addiction stock effect tends to offset some of the tax anticipation and current period revenue effects.

In general the addiction stock effect implies addictive goods should be taxed at a higher rate when responses to tax changes are becoming more elastic over time. Suppose that $\sigma_{hs}$ is falling, so that $J \cdot \Delta \sigma_{hs}$ is becoming more negative. Then taxing $d_t$ raises labor tax revenues today, but decreases labor tax revenues less in $t+1$, when $\sigma_{hs}$ is lower. Because the hours response is more elastic in period $t+1$, the planner must raise the tax rate by a relatively small amount in $t+1$ to maintain equilibrium, whereas in period $t$ the planner must lower the tax rate by a relatively large amount. In this case, taxing addictive goods is more attractive than taxing ordinary consumption goods.

As shown in the following propositions, however, some common specifications for $v$ and $s$ induce constant elasticities, in which case the addiction stock effect exactly offsets static
PROPOSITION 3  Let the conditions of Proposition 2 hold, and let \( u(.) = z(l)\left(\frac{(s^{1-\frac{1}{\sigma}})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\right) \), and \( z(.) \) be concave, then \( \tau_d = \tau_c \) for all \( t \).

Although we have assumed here that \( v(.) \) is constant relative risk aversion (CRR), this proposition is considerably more realistic than the existing literature which assumes a static utility function and/or separable quadratic utility for tractability. If utility is CRR in \( c \) and \( s \) and separable in \( l \), then we obtain the classic result of uniform commodity taxation.

The above result, however, is not a special case of the previous static literature. Consider only the terms that contain \( d_t \) in the infinite horizon utility function:

\[
U = \ldots + \beta^t u(c_t, s(d_t, d_{t-1}), 1 - h_t) + \beta^{t+1} u(c_t, s(d_t, d_{t-1}), 1 - h_t) + \ldots
\]

Treating (5.7) as a static utility function with goods \([c_t, c_{t+1}, d_{t-1}, d_t, d_{t+1}, l_t, l_{t+1}]\), then the result in the static literature (e.g., Chari and Kehoe 1998, pages 19-20) is that uniform taxation results if \( U \) is homothetic in \( c_t \) and \( d_t \) and weakly separable in other goods \((U = G(H(c_t, d_t), c_{t+1}, d_{t-1}, d_{t+1}, l_t, l_{t+1}))\), with \( H \) homothetic), which does not hold for the CRR utility function in Proposition 3. Instead, the uniform taxation result in Proposition 3 arises because the current period revenue and dynamic effects exactly offset.\(^{20}\)

Consider for example the utility function:

\[
u = \frac{c^\sigma}{1-\sigma} + \frac{s^{1-\sigma}}{1-\sigma}, \quad s = \frac{d_t^{1+\alpha}}{d_{t-1}}.\]

We then have \( \sigma_s = \sigma_c = \sigma, \sigma_{cs} = \sigma_{hs} = \sigma_{sc} = 0 \), and \( s \) is HD-\( \alpha \). Equation (5.3) then reduces to:

\[(1-\alpha)(1-\sigma) > 0.\]

In contrast, the condition which considers only static effects, (5.4), reduces to:

\[\alpha\sigma > \alpha(1+\alpha).\]

So for \( \alpha = 1 \) we have uniform taxation in the dynamic model but not in the static model, and the reverse if \( \alpha = 0 \). Further, for \( \alpha \in (0, 1) \), the static and dynamic models have the

\(^{20}\)Uniform taxation does not occur generally even when \( u \) and \( s \) are both homogeneous to the same degree. However, if both \( u \) and \( s \) are HD-1, the dynamic and static effects also exactly cancel, resulting in uniform taxation.
opposite predictions about which good to tax at a higher rate, for $1 < \sigma < 1 + \alpha$.

Elasticities are also constant in the steady state. Let $\bar{y}$ denote the steady state value of any variable $y$, then:

**PROPOSITION 4** Let the conditions of Proposition 2 hold. Then $\bar{\tau}_d > \bar{\tau}_c$ if and only if:

$$(1 - \alpha)(1 - \bar{\sigma}_s - \bar{\sigma}_{sc}) > 0.$$  \hfill (5.11)

In the steady state with homotheticity in $c$ and $s$, an increase in addictive tax revenues in $t$ has an equal and opposite decrease in addictive tax revenues in $t + 1$. In contrast, whether or not the tax anticipation effect fully offsets the remaining current revenue effects depends on the homogeneity of the addiction function. For HD-1 addiction functions, including the subtractive model, Proposition 4 indicates that the steady state tax rates are uniform. For the multiplicative case, the degree of homogeneity is decreasing in the strength of tolerance. Condition (5.11) becomes:

$$\gamma (1 - \bar{\sigma}_s - \bar{\sigma}_{sc}) > 0.$$  \hfill (5.12)

Therefore, an increase in tolerance causes a decrease in the steady state addictive tax rate if and only if $1 - \bar{\sigma}_s - \bar{\sigma}_{sc} < 0$. Therefore, addictive goods are taxed at a lower rate in the steady state if and only if higher tolerance reduces the steady state addictive tax rate.

Propositions 2 and 4 indicate that the choice of addiction function is not innocuous when designing optimal tax policies. In particular, condition (5.11), implies uniform steady state taxation for the subtractive model, but not necessarily for the multiplicative model.

### 5.2 An Analytical Example: The Quadratic Case

In this section, we consider a linear-quadratic utility function. The linear-quadratic utility, a common specification in the literature (e.g., Becker, Grossman, and Murphy 1994, Gruber and Koszegi 2001), offers several advantages. First, we obtain an analytic solution, which allows us to study how the dynamics of addictive consumption change optimal taxation. Second, since this specification has no income effects to complicate the dynamics, we can derive a more precise relationship between tolerance and addictive taxation. In the appendix we consider classes of utility functions with non-trivial income and labor supply effects.

Suppose the subtractive specification (3.9) for effective consumption and that the utility
and production function are:

\[ u(c_t, s_t, l_t) = \omega c_t + \nu s_t - \frac{s_t^2}{2} + e l_t - \frac{l_t^2}{2}, \quad e < 1, \quad \nu > \frac{\omega}{1 - \beta \gamma}. \]  

(5.1)

\[ F(k_t, h_t) = k_t^{\theta} h_t^{1-\theta}. \]  

(5.2)

Here, the assumption on \( e \) ensures positive steady state hours and the assumption on \( \nu \) is required for positive steady state addictive consumption (see Proposition 6).

Inspection of equations (4.4) and (4.5), given the utility function (5.1), reveals that the marginal utility of \( d_t \) is constant in the optimal second best allocation. In particular:

\[ MU_{d,t} = \frac{\omega + \mu (\omega + \nu (1 - \beta \gamma))}{1 + 2\mu}. \]  

(5.3)

The marginal utility of \( d_t \) divided by \( MU_{c,t} = \omega \) equals the tax ratio given by equation (3.18). Hence the tax ratio is constant over time. Furthermore, inspection of equations (3.12), (3.15), (4.4), and (4.7) indicates that \( \tau_c \) is constant over time. Therefore, \( \tau_d \) and \( \tau_h \) are also constant over time. Thus the implicit interest tax rate is zero for all \( t \).

Equation (5.3) implies \( MU_{d,t} > MU_{c,t} \) and thus \( \tau_{d,t} > \tau_{c,t} \) for all \( t \). Hence we have shown:

**Proposition 5** Let \( u(., ., .) \) and \( F(., .) \) be given by equations (5.1) and (5.2) and let effective consumption be given by the subtractive model. Then \( \tau_{d,t} > \tau_{c,t} \) for all \( t \) and the ratio of tax rates \( \frac{1 + \tau_{d,t}}{1 + \tau_{c,t}} \) is constant over time.

In the static version of the model without addiction, \( d_t = s_t \) has an income elasticity equal to zero whereas the income elasticity of \( c_t \) is positive. Further, ordinary consumption and leisure are substitutes, whereas \( \frac{\partial s_t}{\partial w} \frac{\partial w}{s} = 0 \). Thus, it is optimal to tax \( d_t \) at a higher rate because, regardless of \( k_t \) or \( d_{t-1} - c \), \( c \) is more substitutable with leisure.\(^{21}\)

It is also clear from equation (5.3) that the second best optimal \( d_t \) is the solution to a linear second order difference equation,

\[ -\beta \gamma d_{t+1} + (1 + \beta \gamma^2) d_t - \gamma d_{t-1} = \frac{1 + \mu}{1 + 2\mu} (\nu (1 - \beta \gamma) - \omega), \]  

(5.4)

and that the second best optimal \( s_t \) is the solution to a linear first order difference equation. However, before computing the solution to \( d_t \), we must verify that a solution exists for \( \mu \). In

\(^{21}\)The quadratic case with identical elasticities, \( \nu c_t - \frac{1}{2} e c_t^2 + \nu s_t - \frac{1}{2} s_t^2 + v (1 - h_t) \), cannot be solved analytically. Nonetheless, we can show using proposition 8 in the appendix that for this case \( \tau_d \leq \tau_c \), with equality if and only if \( \gamma = 0 \).
appendix 7.4, we prove that a unique, positive solution exists if government spending is not so large as to exhaust the maximum feasible revenue in the economy, and not so small that given initial tax rates are sufficient to pay for all current and future government expenditures.

Given a unique solution for $\mu$, $d_t$ is the solution to the second order difference equation (5.3).

**PROPOSITION 6** Let the conditions for Proposition 5 hold. Then the explicit solution for $d_t$ is:

$$d_t = \frac{\nu (1 - \beta \gamma) - \omega}{(1 - \gamma) (1 - \beta \gamma)} \left( \frac{1 + \mu}{1 + 2\mu} \right) (1 - \gamma t + 1) + d_{-1} \gamma t + 1.$$  (5.5)

The solution for $d_t$, given by equation (5.5), allows us to derive some interesting properties of the second best solution, both over time and as compared to the first best solution ($\mu = 0$). First, optimal consumption of $d_t$ increases over time, assuming $d_{-1}$ is less than the steady state. The planner decreases $d_t$ relative to the first best solution through the tax. However, since $0 < \mu < \infty$, equation (5.5) implies $d_t$ in the second best optimum is at least half of the first best level in the steady state. The planner also decreases the growth rate of $d_t$ since:

$$\text{gr}_t = \frac{d_t - d_{t-1}}{d_{t-1}} = \frac{\gamma t (1 - \gamma) \left( \hat{dx} - d_{-1} \right)}{dx (1 - \gamma t) + \gamma t d_{-1}}, \quad \hat{d} = \frac{\nu (1 - \beta \gamma) - \omega}{(1 - \gamma) (1 - \beta \gamma)}, \quad x = \frac{1 + \mu}{1 + 2\mu},$$  (5.6)

which is decreasing in $\mu$ because:

$$\frac{\partial \text{gr}_t}{\partial \mu} = \frac{-\gamma t (1 - \gamma) d_{-1}}{d_{-1}^2 (1 + 2\mu)^2} < 0.$$  (5.7)

We can also explore how the strength of tolerance affects second best addictive consumption. Since the solution for $\mu$ is unique, we can use the implicit function theorem to derive comparative statics using equation (5.3). Our intuition is that strong tolerance should moderate the optimal tax ratio, as gains in current tax revenue from taxation of addictive goods are offset by losses in future tax revenues. If $d_{-1}$ is sufficiently large, it is indeed true that the optimal tax ratio is inversely related to the degree of tolerance. In particular, we have:

$$d_{-1} > \frac{\beta \omega}{(1 - \beta \gamma) (1 - \beta)} \Rightarrow \frac{\partial \frac{1 + \gamma t}{1 + \gamma t}}{\partial \gamma} < 0.$$  (5.8)

Condition (5.8), derived in appendix 7.4, is a sufficient condition calculated assuming $\mu = \infty$. In practice, for $\mu$ small, the optimal tax ratio is decreasing in the degree of tolerance under much less restrictive conditions.
Table 1 gives parameter values for a numerical example. Table 1 indicates that the optimal tax ratio is decreasing in the degree of tolerance, even though condition (5.8) is violated, since the parameter $G$, set to 30% of GDP for all $t$, generates at most a value of only $\mu = 4.74$. The planner relies increasingly on labor taxes and less on addictive taxes as the degree of tolerance increases. For $\gamma = 0.55$, taxation is nearly uniform. Figure 1 shows the time path of the first and second best levels of $d$ for various values of $\gamma$. Increasing the level of tolerance severely reduces addictive consumption since the future costs of current consumption are higher. As expected, the difference between first and second best addictive consumption is widest at the steady state.

6 Conclusions

This paper is the first attempt in the literature to characterize and analyze the conditions under which taxation of addictive goods might differ from taxes on labor and ordinary consumption goods in a dynamic general equilibrium setting. We derive a rich set of results. In particular, we derive conditions for a variety of classes of utility functions for which tax rates for addictive goods exceeds tax rates for ordinary goods in an environment where exogenous government spending cannot be financed with lump sum taxes.

We find that two effects differentiate taxation of addictive goods from ordinary goods. The first is the tax anticipation effect tends to moderate the optimal tax: households anticipate future increases in addictive tax rates and reduce addictive consumption, decreasing addictive tax revenues. Given the empirical literature finds evidence for the tax anticipation effect, it is important to account for it.

Second, an addiction stock effect exists. Current addictive taxes change the future tax base. Households will be less addicted in the future following an addictive tax increase. Thus future addictive, ordinary, and labor tax revenues are affected by a change in the current addictive tax rate. Whether the addiction stock effect increases revenues or not depends on whether or not elasticities are rising or falling. In the steady state, or with CRR utility, elasticities are constant and thus the addiction stock effect and current period revenue effects cancel. Surprisingly, the tax anticipation effect makes taxing addictive goods in a dynamic setting similar to taxing effective consumption, in which the addictive good is less complementary with leisure to a degree which depends on the homogeneity of the addiction function. Therefore, classic results on uniform taxation re-emerge in some special cases such as CRR utility.

22See section 3.2.
We consider homogeneous addiction functions, an improvement upon the literature which typically assumes only either subtractive or multiplicative addiction functions. Higher tolerance strengthens the tax anticipation effect and therefore reduces the optimal addictive tax. This result is quite surprising as the textbook intuition suggests that stronger addiction would make taxing addictive goods more attractive.

We also consider features such as complementarity to leisure that, while unrelated to addiction itself, are nonetheless common among addictive goods. In general, such effects are weaker in our dynamic setting. Examples are constructed where failing to account for the dynamic effects results in taxing addictive goods at a higher rate, when the optimal addictive tax is actually less than that of ordinary goods, due to the tax anticipation effect.

Our results come with a few caveats. First, one common feature of addictive goods, the presence of externalities, has not been considered in this paper. However, it is unclear whether externalities are not better dealt with by regulating the exact behavior that causes the externality, such as banning smoking in public places, rather than the second best solution of taxing consumption of the addictive good. In addition, if we include an externality, then our results go through under slightly different conditions. That is, our results can be interpreted as relative to the tax ratio which corrects the externality.

Second, the rational addiction framework does not capture all features of addiction. The main alternative, modeling addiction as either rational or irrational excess consumption, has intuitive appeal but also some drawbacks. First, in that framework the degree of excess consumption must be heterogeneous across the population. The optimal tax is sensitive to both the degree of excess consumption and the fraction of the population that suffers from excess consumption. Furthermore, time inconsistent preferences require separability in addictive and ordinary goods, no savings, and often \textit{ad hoc} utility functions for tractability purposes. Since we show here that properties of the utility function such as homotheticity, tolerance, and homogeneity of the addiction function are crucial in establishing optimal taxation results, our modeling choice is in some dimensions less restrictive than alternatives, especially if the government has a revenue raising motive. Nonetheless, the effect of other features of addiction, such as rehab cycles and commitment devices, on optimal revenue raising is an interesting subject of future research.

Third, we consider only the optimal tax package, not the optimal addictive goods tax taking as given other taxes. Certain features of the tax code such as balanced budget rules and positive capital tax rates, if added as extra constraints, may change our results. Third, we have no heterogeneity in addictive consumption or wealth. However, if we assume that
the poor are more likely to consume addictive goods, then our results would likely strengthen, because consumers of addictive goods would have a higher marginal utility of income.

Finally, by choosing to adopt the Ramsey approach, we are subject to the typical criticisms made to this framework: it takes the set of possible fiscal instruments as given and it requires linear taxes. Our results might change if we allowed for heterogeneous households and nonlinear tax rates within a Mirrless framework. However, while capital and income taxes are nonlinear in the data, addictive taxes are typically linear. In addition, it would be very hard, in particular at the state level, to monitor addictive consumption for each household. Most importantly, the majority of redistributive fiscal policy is carried out at the federal level in the US. Excise taxes at the state level are not typically set with a redistributive intent, as in a Mirrless framework, but rather to generate fiscal revenues and balance the state budgets. We leave the interesting theoretical issues concerning taxation of addictive goods in the Mirrless framework for further research as well.
References


7 Appendix: Proofs and Extras

7.1 Proof of Proposition 1

To see that a competitive equilibrium satisfies the IMC and resource constraint, we substitute the factor prices (3.2) and (3.3) into the budget constraint (3.11). Using constant returns to scale, we then have:

\[ R_t b + F(k_t, h_t) - \tau_h F(k_t, h_t) h_t = (1 + \tau c) c_t + (1 + \tau d) d_t + i_t + b_{t+1}. \]  

(7.1)

Combining the above equation with the government budget constraint (3.19) gives the resource constraint (4.1).

To derive the IMC from the budget constraint, we substitute the household first order conditions (3.12)-(3.14) into the budget constraint (3.11), eliminating the tax rates, so that:

\[ \lambda_t R_t k + \lambda_t R^b b_t - \lambda_t k_{t+1} - \lambda_t b_{t+1} = \beta^t (u_c (c_t, s_t, l_t) c_t + MU_d d_t + u_l (c_t, s_t, l_t) h_t). \]  

(7.2)

Next using the first order conditions (3.15) and (3.16), we have:

\[ \lambda_t R_t (k_t + b_t) - \lambda_{t+1} R_{t+1} (k_{t+1} + b_{t+1}) = \beta^t (u_c (c_t, s_t, l_t) c_t + MU_d d_t + u_l (c_t, s_t, l_t) h_t). \]  

(7.3)
The above equation characterizes a sequence of budget constraints that can be used to recursively eliminate $\lambda_t R_t (k_t + b_t)$, yielding:

$$
\lambda_0 \left( R_0 k_0 + R^b_0 b_0 \right) - \lim_{t \to \infty} \lambda_{t+1} R_{t+1} (k_{t+1} + b_{t+1}) = \\
\sum_{t=0}^{\infty} \beta^t \left( u_c (c_t, s_t, l_t) c_t + MU_{d,t} d_t - u_l (c_t, s_t, l_t) h_t \right). \tag{7.4}
$$

The transversality conditions imply the second term on the left hand side equals zero. Again using the household first order conditions at period zero gives:

$$
\frac{u_{c,0} \left( R_0 k_0 + R^b_0 b_0 \right)}{1 + \tau_{c,0}} = \sum_{t=0}^{\infty} \beta^t \left( u_c (c_t, s_t, l_t) c_t + MU_{d,t} d_t - u_l (c_t, s_t, l_t) h_t \right), \tag{7.5}
$$

which is the IMC.

We next show that, given allocations which satisfy the IMC and resource constraint, prices and policies exist which, along with the allocations, are a competitive equilibrium. Let $\{c_t, k_t, h_t, d_t\}$ be a sequence which satisfies the IMC and resource constraint. Then $r_t$ and $w_t$ are defined using equations (3.2) and (3.3). Since $\tau_{c,0}$ is given, we can define $\lambda_0$ using equation (3.12). Then $\lambda_t$ can be defined recursively using equation (3.15). Then $R^b_t$ is defined using equation (3.16). Next, we define the government policies:

$$
(1 + \tau_{c,t}) = \frac{\beta^t u_c (c_t, s_t, l_t)}{\lambda_t}, \tag{7.6}
$$

$$
(1 - \tau_{h,t}) = \frac{\beta^t u_l (c_t, s_t, l_t)}{\lambda_t F_h (k_t, h_t)}, \tag{7.7}
$$

$$
(1 + \tau_{d,t}) = \frac{\beta^t MU_{d,t}}{\lambda_t}. \tag{7.8}
$$

Given the above prices and policies, all equations which define a competitive equilibrium are satisfied except the household and government budget constraints. We use $b_t$ to satisfy the household budget constraint:

$$
b_t = \frac{1}{R^b_t} \left( -r_t k_t - (1 - \tau_{h,t}) w_t h_t + (1 + \tau_{c,t}) c_t + (1 + \tau_{d,t}) d_t + i_t + b_{t+1} \right). \tag{7.9}
$$

We can multiply the above equation by $\lambda_t$ and recursively eliminate $b_{t+1}$ from the above
equation. After eliminating prices and policies using the household first order conditions (3.12)-(3.14), \( b_t \) is a function of the allocations:

\[
b_t = \left( \prod_{i=0}^{t-1} (F_k (k_i, h_i) + 1 - \delta) \right) \frac{1 + \tau_c}{\tau_c} \sum_{i=t}^{\infty} \beta^i \left( u_{c,t} c_i + MU_{d,t} d_i - u_{t,i} h_i \right) - k_t \quad (7.10)
\]

The above equation is the debt allocation which implies the household budget constraint is satisfied.

Since the budget constraint is satisfied, we simply substitute the resource constraint into the budget constraint to see that the government budget constraint is satisfied. Finally, by substituting the prices and policies into the IMC and reversing the derivation of the IMC, we see that the transversality conditions are satisfied.

### 7.2 Proof of Proposition 2

First, we rewrite equations (4.9) and (4.12), using the \( \sigma \) definitions, so that:

\[
IMC_{c_t} = \beta^t u_{c,t} \left( 1 - \sigma_c + \alpha \sigma_{sc,t} - \sigma_{hc,t} \right), \quad (7.11)
\]

\[
IMC_{d_t} = \beta^t MU_{d,t} \left( \alpha - \alpha \sigma_{s,t} + \sigma_{cs,t} - \sigma_{hs,t} + \frac{\beta u_{s,t+1}s_{s,t+1}}{MU_{d,t}} \left( \alpha \sigma_{s,t} - \alpha \sigma_{s,t+1} - \sigma_{cs,t} + \sigma_{cs,t+1} + \sigma_{hs,t} - \sigma_{hs,t+1} \right) \right). \quad (7.12)
\]

Now since \( v \) is homothetic, we know that:

\[
\frac{v_c(\psi c, \psi s)}{v_s(\psi c, \psi s)} \equiv \frac{v_c(c, s)}{v_s(c, s)}, \quad (7.13)
\]

which implies:

\[
\frac{v_{cc}(c, s)}{v_c(c, s)} + \frac{v_{cs}(c, s)}{v_c(c, s)} = \frac{v_{ss}(c, s)}{v_s(c, s)} + \frac{v_{cs}(c, s)}{v_s(c, s)}, \quad (7.14)
\]

which, using the definition of \( u(.) \) in equation (5.1), implies:

\[
\sigma_{sc} - \sigma_c = \sigma_{cs} - \sigma_s. \quad (7.15)
\]

It is also immediate from the definition of \( u(.) \) that \( \sigma_{hc} = \sigma_{hs} \). These facts and equations
(4.4), (4.5), (7.11), and (7.12) together imply:

\[
\frac{MU_{d,t}}{u_{c,t}} = \frac{1 + \mu \left(1 - \sigma_{s,t} + \sigma_{cs,t} - (1 - \alpha) \sigma_{sc,t} - \sigma_{hs,t}\right)}{1 + \mu \left(\alpha - \alpha \sigma_{s,t} + \sigma_{cs,t} - \sigma_{hs,t} + J \left(\alpha \Delta \sigma_{s} - \Delta \sigma_{cs} + \Delta \sigma_{hs}\right)\right)},
\]

(7.16)

Hence, \(\tau_{d,t} > \tau_{c,t}\) if and only if the right hand side is greater than one, or:

\[
1 - (1 - \alpha) \sigma_{sc,t} - \sigma_{s,t} > \alpha - \alpha \sigma_{s,t} + J \left(\alpha \Delta \sigma_{s} - \Delta \sigma_{cs} + \Delta \sigma_{hs}\right),
\]

(7.17)

which simplifies to the desired result.

7.3 Proof of Propositions 3-4

For the CRR case, note that \(\sigma_{hs} = 0\), \(\sigma_{s} = 1 - (1 - \xi)(1 - \sigma)\), and \(\sigma_{sc} = (1 - \xi)(1 - \sigma)\), which implies the left hand side of condition (5.3) is zero. The right hand side of (5.3) is also zero since \(\sigma_{cs}\) and \(\sigma_{s}\) are constant.

For the steady state case, \(\sigma_{i,t} = \sigma_{i,t+1}\) for all \(i \in \{s, sc, cs, hs\}\), so the result follows immediately from condition (5.3).

7.4 Details of the linear quadratic case: Proof of existence and of Propositions 5-6 and of condition (5.8)

Proposition 5 was proved in the text. Now we prove that a unique positive solution for \(\mu\) exists.

PROPOSITION 7 Let the conditions for Proposition 5 hold. Let \(g_t\) be a stationary sequence with limiting value \(\bar{g}\). Then there exists an interval \([\zeta_l, \zeta_h]\), with \(0 < \zeta_l < \zeta_h < \infty\) such that if \(G \equiv \sum_{t=0}^{\infty} \beta^t g_t \in [\zeta_l, \zeta_h]\), then a unique positive solution for \(\mu\) exists.

For Proposition 7, we derive the solution for \(\mu\) as follows. First, for the quadratic case, the first order conditions for the Ramsey problem (4.4)-(4.7) are now:

\[
\frac{\phi_t}{\beta^t} = \omega \left(1 + \mu\right),
\]

(7.18)

\[
\frac{\phi_t}{\beta^t} = \nu \left(1 - \beta \gamma\right) \left(1 + \mu\right) - (1 + 2\mu) \left(s_t - \beta \gamma s_{t+1}\right),
\]

(7.19)

\[
\frac{\phi_t (1 - \theta) \left(\frac{k_t}{h_t}\right)^{\theta}}{\beta^t} = (e - 1) \left(1 + \mu\right) + (1 + 2\mu) h_t,
\]

(7.20)
\( \phi_t \left( \theta \left( \frac{k_t}{h_t} \right)^{\theta-1} + 1 - \delta \right) = \phi_{t-1}. \) \hfill (7.21)

Using equation (7.18) to eliminate \( \phi_t \) gives:

\[
\omega (1 + \mu) = (\nu (1 + \mu) - \mu) (s_t - \beta \gamma s_{t+1}), \tag{7.22}
\]

\[
\omega (1 + \mu) (1 - \theta) \left( \frac{k_t}{h_t} \right)^\theta = (e - 1) (1 + \mu) + (1 + 2\mu) h_t, \tag{7.23}
\]

\[
\beta \left( \theta \left( \frac{k_t}{h_t} \right)^{\theta-1} + 1 - \delta \right) = 1. \tag{7.24}
\]

For the subtractive model equation (7.22) implies:

\[
\omega (1 + \mu) = \nu (1 - \beta \gamma) (1 + \mu) - (1 + 2\mu) (d_t - \gamma d_{t-1} - \beta \gamma (d_{t+1} - \gamma d_t)), \tag{7.25}
\]

which simplifies to (5.4), and which we show in the next proof has general solution given by (5.5). Proposition 6 thus holds if a non-zero and finite solution for \( \mu \) exists, which we now show.

Equation (7.24) implies the capital to labor ratio, denoted by \( A \), is constant:

\[
A \equiv \left( \frac{\theta}{\rho + \delta} \right)^{1/\theta}. \tag{7.26}
\]

Thus, equation (7.23) implies

\[
h_t = \frac{1 + \mu}{1 + 2\mu} \hat{h}, \quad \hat{h} \equiv 1 - e + \omega (1 - \theta) A^\theta, \tag{7.27}
\]

is constant. Thus, \( k_t = Ah_t \) is constant and equation (5.5) implies \( s_t \) and thus \( MU_{d,t} \) is constant as well. Since elasticity of substitution of consumption over time is infinite, the planner absorbs all changes in \( g_t \) by varying \( c_t \). Combining these results with resource constraint (4.1) yields a solution for \( c_t \):

\[
c_t = \frac{1 + \mu}{1 + 2\mu} \left( \hat{h} (A^\theta - \delta A) - \hat{d} \left( 1 - \gamma^{t+1} \right) \right) - \gamma^{t+1} d_{-1} - g_t, \tag{7.28}
\]
\[
\hat{d} \equiv \frac{\nu (1 - \beta \gamma) - \omega}{(1 - \gamma) (1 - \beta \gamma)}. \tag{7.29}
\]

Now since \( h_0 \) enters into the left hand side of the IMC (4.2) and \( k_0 \) is given, the solutions for \( h_0, k_0 \) and therefore \( c_0 \) generally differ from the solutions for \( t \geq 1 \). Therefore, we let \( x \equiv \frac{1 + \mu}{1 + 2\mu} \) and insert the solutions for \( c_t, d_t, \) and \( h_t \) into the IMC (4.2) for \( t \geq 1 \), so that:

\[
\begin{align*}
R_0 k_0 + R^b_0 b_0 & = \sum_{t=1}^{\infty} \beta^t \left[ x \omega \left( \hat{h} (A^p - \delta A) - \hat{d} \left( 1 - \gamma^{t+1} \right) \right) - \omega \gamma^{t+1} d_{-1} + \\
(1 - \beta \gamma) \left( \nu - (1 - \gamma) \hat{d} x \right) \left( \hat{d} x (1 - \gamma_{t+1}) + \gamma^{t+1} d_{-1} \right) - (e - 1 + \hat{h} x) \hat{h} x \right] + \\
\omega c_0 + M U_{d,0} d_0 - u_{t,0} h_0 + \omega g_0 - \omega G. \tag{7.30}
\end{align*}
\]

Next, recall from Proposition 1 that \( \tau_{c,0} \) and \( \tau_{h,0} \) are given. It follows from equations (3.12) and (3.13) that the planner cannot choose \( h_0 \) in this example, and instead takes the solution for \( h_0 \) from the competitive model as given. Further, the terms inside the summation depend on time only through \( \gamma^{t+1} \) and \( \beta^t \), and the equation is quadratic in \( x \). Therefore, after evaluating the summation we can write equation (7.30) as:

\[
\begin{align*}
ch (x) & \equiv -\zeta_1 x^2 + (\zeta_1 - \zeta_2) x + \zeta_2 - \zeta_3 = 0, \tag{7.31} \\
\zeta_1 & \equiv \hat{h}^2 + \frac{(1 - \gamma)^2}{\beta} d^2, \tag{7.32} \\
\zeta_2 & \equiv \left( \frac{1 - \beta}{\beta} \right) (1 - \gamma) \gamma \hat{d} d_{-1}, \tag{7.33} \\
\zeta_3 & \equiv \frac{1 - \beta}{\beta} \left( \omega G - \frac{(\omega (1 + \tau_{c,0}) - 1) (1 - \delta) k_0 - R^b_0 b_0}{1 + \tau_{c,0}} - \right. \\
& \left. \omega (1 + \tau_{c,0}) - \theta \right) \frac{k_0^\theta h_{0}^{1-\theta} + u_{t,0} h_0}{1 + \tau_{c,0}}. \tag{7.34}
\end{align*}
\]

A solution such that \( \mu > 0 \) is a solution in the range \( \frac{1}{2} < x < 1 \). Note that equation (7.31) can be written as:

\[
\begin{align*}
ch (x) = (\zeta_1 x + \zeta_2) (1 - x) - \zeta_3 = 0. \tag{7.36}
\end{align*}
\]
Now \( ch(x) \) attains a maximum at \( x^* = (\zeta_1 - \zeta_2) / (2\zeta_1) < 1/2 \) and \( ch(0) > ch(1) \). Hence it is immediate that \( \zeta_3 > 0 \) is necessary for \( x < 1 \).

From equation (7.35), \( \zeta_3 > 0 \) if and only if:

\[
G > \zeta_l \equiv \frac{(\omega(1 + \tau_{c,0}) - 1)(1 - \delta) k_0 - R_0^h b_0}{\omega(1 + \tau_{c,0})} + \frac{\omega(1 + \tau_{c,0}) - \theta h_0 h_1^{1 - \theta} - u_{l,0} h_0}{\omega}. \tag{7.37}
\]

This is the lower bound for \( G \).

Condition (7.37) implies that \( \zeta_3 > 0 \) which implies both roots have modulus less than one. It remains to show that the roots are real and that one root is greater than one half. Since \( x^* < 1/2 \), the smaller root has modulus less than one half. The larger root is real and greater than one half if and only if:

\[
ch \left( \frac{1}{2} \right) = -\frac{\zeta_1}{4} + \frac{\zeta_1 - \zeta_2}{2} + \zeta_2 - \zeta_3 > 0, \tag{7.38}
\]

\[
\zeta_3 < \frac{\zeta_1 - 2\zeta_2}{4}. \tag{7.39}
\]

Using equation (7.35), condition (7.39) holds if and only if:

\[
G < \zeta_h \equiv \zeta_l + \frac{\beta}{1 - \beta} \left( \frac{\zeta_1 + 2\zeta_2}{4\omega} \right). \tag{7.40}
\]

Defining \( \zeta_l \) and \( \zeta_h \) using equations (7.37) and (7.40) completes the proof. \( \square \)

For Proposition 6, we solve the second order difference equation (5.4):

\[
d_{t+1} - \left( \frac{1 + \beta \gamma^2}{\beta \gamma} \right) d_t + \frac{1}{\beta} d_{t-1} = -\frac{1 + \mu}{1 + 2\mu} \left( \frac{\nu(1 - \beta \gamma) - \omega}{\beta \gamma} \right). \tag{7.41}
\]

It is straightforward to show the general solution of the above difference equation is:

\[
d_t = D_p + A_0 \gamma^t + A_1 (\beta \gamma)^{-t}, \tag{7.42}
\]

\[
D_p \equiv \frac{1 + \mu}{1 + 2\mu} \left( \frac{\nu(1 - \beta \gamma) - \omega}{(1 - \beta \gamma)(1 - \gamma)} \right). \tag{7.43}
\]

Following convention, we rule out the explosive, bubble solutions which requires \( A_1 = 0 \). Letting \( t = -1 \) implies \( A_0 = \gamma (d_{-1} + D_p) \). Substituting for \( A_0 \) and simplifying gives the desired solution. \( \square \)
Finally, to derive equation (5.8), we rewrite equation (5.3) using the definition of $x$, which implies:

$$\frac{1 + \tau_{d,t}}{1 + \tau_{c,t}} = \frac{MU_{d,t}}{\omega} = x + \frac{\nu (1 - \beta \gamma)}{\omega} (1 - x).$$

(7.44)

Hence:

$$\frac{\partial}{\partial \gamma} \left( \frac{1 + \tau_{d,t}}{1 + \tau_{c,t}} \right) = \frac{\partial x}{\partial \gamma} \cdot \left( 1 - \frac{\nu (1 - \beta \gamma)}{\omega} - \frac{\nu \beta}{\omega} (1 - x) \right) < 0 \text{ iff,}$$

(7.45)

$$\frac{\partial x}{\partial \gamma} > \frac{\beta \nu (1 - x)}{\nu (1 - \beta \gamma) - \omega}.$$  

(7.46)

Next, using the implicit function theorem on equation (7.36), we see that:

$$\frac{\partial x}{\partial \gamma} = \frac{(\zeta_1 \gamma + \zeta_2 \gamma) (1 - x)}{2 \zeta_1 x - \zeta_1 + \zeta_2}.$$  

(7.47)

We have $x \in \left( \frac{1}{2}, 1 \right)$, and so the denominator of equation (7.47) is positive. Thus, substituting equation (7.47) into condition (7.46) and cross multiplying results in:

$$\left[ \zeta_1 \gamma (\nu (1 - \beta \gamma) - \omega) + 2 \beta \nu \zeta_1 \right] x + \zeta_2 \gamma (\nu (1 - \beta \gamma) - \omega) + \nu \zeta_2 > \beta \nu \zeta_1.$$  

(7.48)

The coefficient on $x$ equation (7.48) is positive. Thus, it is sufficient to show:

$$\left[ \zeta_1 \gamma (\nu (1 - \beta \gamma) - \omega) + 2 \beta \nu \zeta_1 \right] \frac{1}{2} + \zeta_2 \gamma (\nu (1 - \beta \gamma) - \omega) + \beta \nu \zeta_2 > \beta \nu \zeta_1,$$  

(7.49)

$$\frac{1}{2} \zeta_1 \gamma (\nu (1 - \beta \gamma) - \omega) + \zeta_2 \gamma (\nu (1 - \beta \gamma) - \omega) + \beta \nu \zeta_2 > 0.$$  

(7.50)

Finally substituting in the definitions of $\zeta$ using equations (7.32) and (7.33) and the derivatives of $\zeta$:

$$\zeta_{1,\gamma} = \frac{-2 \omega (\nu (1 - \beta \gamma) - \omega)}{(1 - \beta \gamma)^3}, \quad \zeta_{2,\gamma} = \frac{1 - \beta}{\beta} \left( \frac{\nu (1 - \beta \gamma)^2 - \omega}{(1 - \beta \gamma)^2} \right) d_{-1},$$  

(7.51)

and simplifying yields the desired result.
7.5 Additively Separable Utility

In this section we consider the case in which utility is additively separable. Following a similar procedure as with Proposition 2, we have:

**PROPOSITION 8** Let assumptions (A1)-(A3) hold. In addition, let \( u(\cdot) \) be additively separable in \( c, s, \) and \( l \). Then \( \tau_{d,t} > \tau_{c,t} \) if and only if:

\[
\alpha \sigma_{s,t} + 1 - \alpha - \sigma_{c,t} > J(\alpha \Delta s).
\] (7.52)

**Proof:** If utility is separable, equations (7.11) and (7.12) become:

\[
IMC_{c_t} = \beta_t u_{c,t} (1 - \sigma_c),
\] (7.53)

\[
IMC_{d_t} = \beta_t MU_{d,t} (\alpha - \alpha \sigma_{s,t} + J(\alpha \Delta s)).
\] (7.54)

Equations (4.4), (4.5), (7.53), and (7.54) together imply:

\[
\frac{MU_{d,t}}{u_{c,t}} = \frac{1 + \mu (1 - \sigma_{c,t})}{1 + \mu (\alpha (1 - \sigma_{s,t}) + J(\alpha \Delta s))}.
\] (7.55)

Hence, \( d \) is taxed at a higher rate if and only if the right hand side is greater than one, or:

\[
1 - \sigma_{c,t} > \alpha (1 - \sigma_{s,t}) + J(\alpha \Delta s),
\] (7.56)

which simplifies to the desired result. \( \square \)

For the intuition of (7.52), first assume \( s_2 = 0 \), which removes all dynamic effects. Condition (7.52) becomes:

\[
\sigma_{s,t} s_{1,t} d_t - \frac{s_{1,t} d_t}{s_t} - \sigma_{c,t} > 0
\] (7.57)

This condition is satisfied if and only if \( d_t \) is more complementary with leisure (or income inelastic) than \( c_t \) in the model without dynamic effects. For example, assume again that \( s_{1,t} = 0 \) and \( s_{1,t} d_t = s_t \). Then condition (7.57) holds if and only if \( s \) is more complementary with leisure than \( c \). If these conditions do not hold, then \( d_t \) and \( s_t \) have different income elasticities in the model without dynamic effects, and thus condition (7.57) results.
The difference between the left hand side of conditions (7.57) and (7.52) is the tax anticipation effect:

\[-\frac{s_{12,t}d_{t-1}}{s_{1,t}} + \frac{s_{2,t}d_{t-1}}{s_{t}}\sigma_{s,t} > 0.\] (7.58)

Equation (3.6) implies the tax anticipation effect reduces the optimal addictive tax if and only if \(d_t\) is addictive.

As in Proposition 2, the dynamic effects in \(t - 1\) and \(t\) reduce to a single current period term which depends only on the elasticities and homogeneity, and should therefore be straightforward to check in empirical applications.

The addiction stock effect equals \(\alpha\sigma_{s,t+1}\), which offsets tax anticipation and current period revenue effects.

For the CRR case, we have:

**PROPOSITION 9** Let the conditions of Proposition 8 hold, and let \(u(.) = v^1(c) + v^2(s) + v^3(l)\), with \(v^1\) and \(v^2(.)\) CRR. Then \(\tau_{d,t} > \tau_{c,t}\) for all \(t\) if and only if:

\[\alpha\sigma_s + 1 - \alpha > \sigma_c.\] (7.59)

**Proof:** For CRR preferences, \(\sigma_{i,t} = \sigma_{i,t+1}\) for all \(i \in \{s,c\}\), so the result follows immediately from condition (7.52). \(\Box\)

Condition (7.59) combines tax anticipation effect with any difference in income elasticities between \(d_t\) and \(c_t\). Condition (7.59) requires that \(d_t\) and leisure to be sufficiently strong complements relative to \(c\) in the model without dynamic effects to overcome the dynamic tax anticipation effect.

The homogeneity of the addiction function affects the strength of the tax anticipation effect. For \(\alpha = 1\), as in the subtractive model, the dynamic tax anticipation effect exactly offsets the difference in income elasticities between \(d\) and \(s\) in the model without dynamic effects. In this case, proposition (9) says \(\tau_{d,t} > \tau_{c,t}\) if and only if \(s_t\) is more complementary with leisure in a static model where \(s\) could be taxed. However, in the multiplicative model, higher tolerance implies a lower \(\alpha\) and thus makes addictive goods less complementary with leisure, lowering the optimal tax rate, for \(\sigma_{s,t} > 1\).

Thus, under the conditions outlined above, neglecting the dynamic tax anticipation effect results in overly high tax rates for addictive goods, relative to the optimum, especially for addictive goods that exhibit strong tolerance.
7.6 Non-weakly separable utility

For the homothetic and separable cases studied above, weak separability implies that the static income elasticity and the substitutability with leisure are identical. Here we consider a utility function for which the substitutability with leisure does not necessarily equal the income elasticity.

To see this in a concise way, let us consider the following class of utility functions:

\[ u(s_t, d_t, l_t) = q(c_t) + v(s_t, l_t). \]  

(7.60)

For this specification, we find:

**Proposition 10** Let assumptions (A1)-(A3) hold. In addition, let \( u(\cdot) \) be of the form given by (7.60). Then \( \tau_{d,t} > \tau_{c,t} \) if and only if:

\[ 1 - \alpha + \alpha \sigma_{s,t} + \sigma_{hs,t} - \sigma_{c,t} > J(\alpha \Delta \sigma_s + \Delta \sigma_{hs}). \]  

(7.61)

**Proof:** If utility is given by equation (7.60), equations (7.11) and (7.12) become:

\[ IMC_{c_t} = \beta^t u_{c,t} (1 - \sigma_{c,t}), \]  

(7.62)

\[ IMC_{d_t} = \beta^t MU_{d,t} (\alpha - \alpha \sigma_{s,t} - \sigma_{hs,t} + J(\alpha \Delta \sigma_s + \Delta \sigma_{hs})). \]  

(7.63)

Equations (4.4), (4.5), (7.62), and (7.63) together imply:

\[ \frac{MU_{d,t}}{u_{c,t}} = \frac{1 + \mu (1 - \sigma_{c,t})}{1 + \mu (\alpha (1 - \sigma_{s,t}) - \sigma_{hs,t} + J(\alpha \Delta \sigma_s + \Delta \sigma_{hs}))}. \]  

(7.64)

Hence, \( d \) is taxed at a higher rate if and only if the right hand side is greater than one, or:

\[ 1 - \sigma_{c,t} > \alpha (1 - \sigma_{s,t}) - \sigma_{hs,t} + J(\alpha \Delta \sigma_s + \Delta \sigma_{hs}), \]  

(7.65)

which simplifies to the desired result. \( \square \)

The intuition for (7.61) is identical to that of Propositions 2 and 8, with the exception of the additional current period effect on labor tax revenues, reflecting the lack of weak separability.

For constant elasticities, we have:

**Proposition 11** Let the conditions of Proposition 10 hold, and let \( v(\cdot) = \left( \frac{s^t(1-t)^{1-\xi} 1-\sigma - 1}{1-\sigma} \right) \),
with \( 1 - \sigma < \frac{-1}{1-2\xi} \) and \( \xi < \frac{1}{2} \) to ensure concavity. Then \( \tau_{d,t} < \tau_{c,t} \) if and only if:

\[
1 + (1 - \sigma)(1 - \xi (1 + \alpha)) > \sigma_c. \tag{7.66}
\]

**Proof:** note that \( \sigma_s = 1 - \xi (1 - \sigma) \), and \( \sigma_{hs} = (1 - \xi) (1 - \sigma) \). Since both terms are constant, we have \( J = 0 \). Substituting in these conditions into equation (7.61) gives (7.66). □

Note that the concavity restrictions imply the left hand side of (7.66) is less than one. Since \( \sigma > 1 \) and \( \xi < \frac{1}{2} \) (both conditions are necessary for concavity) and \( \alpha \leq 1 \), the second term on the left hand side is negative and so \( \sigma_{c,t} \geq 1 \) is sufficient for the condition to be violated, and thus \( \tau_{d,t} \leq \tau_{c,t} \).

**PROPOSITION 12** Let the conditions of Proposition 10 hold. Then \( \tilde{\tau}_d > \tilde{\tau}_c \) if and only if:

\[
\alpha (\tilde{\sigma}_s - 1) + \tilde{\sigma}_{hs} > \tilde{\sigma}_c. \tag{7.67}
\]

**Proof:** \( \sigma_{i,t} = \sigma_{i,t+1} \) for all \( i \in \{s, hs\} \) in the steady state, so the result follows immediately from condition (7.61). □

It is possible to construct examples for which \( d_t \) is more complementary with leisure and yet the tax anticipation effect implies a lower tax rate for addictive goods. Suppose subtractive model, then in the static model with utility as in Proposition 11, \( s \) is more complementary with leisure than \( c \). Yet if (7.66) is violated it is optimal to tax \( d_t \) at a lower rate.

Finally, given the subtractive model, the static effects for condition (7.66) are:

\[
\frac{1 - \xi (1 - \sigma)}{1 - \gamma} + (1 - \xi) (1 - \sigma) > \sigma_c, \tag{7.68}
\]

and condition (7.67) becomes:

\[
1 - \xi (1 - \sigma) + (1 - \xi) (1 - \sigma) > \sigma_c. \tag{7.69}
\]

So for \( \sigma_c \) satisfying (7.68), but not (7.69), \( \tilde{\tau}_d > \tilde{\tau}_c \) in the model without dynamic effects but \( \tilde{\tau}_d < \tilde{\tau}_c \) when the tax anticipation effect is accounted for. Note the range of values satisfying (7.68) but not (7.69) is increasing in \( \gamma \). Strong tolerance tends to decrease the optimal addictive tax, by strengthening the tax anticipation effect.

8 Appendix: Tables and Figures
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Table 1: Parameter values and results for variables which are constant over time. The parameters $h_0$ and $k_0$ are set equal to $h_t$ using equation (7.27) and $k_t = Ah_t$, respectively. The parameter $g_t$ is set equal to 30% of GDP for all $t$.

Figure 1: Dynamics of first and second best addictive consumption for various values of $\gamma$. 

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