Markets vs. Mechanisms*

Raphael Boleslavsky   Christopher A. Hennessy   David L. Kelly

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Abstract

We demonstrate limitations on usage of direct revelation mechanisms (DRMs) by corporations inhabiting economies with securities markets. We consider a corporation seeking to acquire decision relevant information. In an environment with a securities market, the act of posting a standard DRM endogenously increases the informed agent's outside option value. If the informed agent rejects said DRM, he convinces the market he is uninformed, and he can trade aggressively with low price impact, generating large off-equilibrium trading gains, which increases his outside option. Due to this endogenous outside option value effect, using a DRM to screen out uninformed agents may be impossible. Moreover, even when screening is possible, refraining from posting a mechanism and instead relying on markets for information is optimal if the endogenous change in outside option value is sufficiently large. Finally, even when posting a DRM dominates relying on markets, superior outcomes are achieved by introducing a search friction which randomly limits the agent's ability to observe the DRM, forcing the firm to sometimes rely on markets for information.

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*Boleslavsky: r.boleslavsky@miami.edu, Hennessy: chennessy@london.edu, Kelly: dkelly@miami.edu. We thank seminar participants at EPFL, UNIL, Universite Paris-Dauphine, BI Norwegian Business School, and the University of California at Santa Barbara.

"The greatest trick the devil ever pulled was convincing the world he didn't exist."
—Verbal Kint, *The Usual Suspects*

"The Devil's greatest trick is to persuade you that he does not exist!"
—Charles Baudelaire, *The Generous Gambler*

1 INTRODUCTION

The provision of decision-relevant information to agents is critical for economic efficiency. Hayek (1945) extolled the virtues of markets in this regard, writing, "We must look at the price system as such a mechanism for communicating information..." More specifically, securities markets are commonly viewed as a vital source of information for firms making operating and real investment decisions. For example, Fama and Miller (1972) write, "(an efficient market) has a very desirable feature. In particular, at any point in time market prices of securities provide accurate signals for resource allocation..." Similarly, Fama (1976) writes, "An efficient capital market is an important component of a capitalist system... if the capital market is to function smoothly in allocating resources, prices of securities must be good indicators of value." Baumol (1965) writes, "One has come to look upon the stock market as the allocator of capital resources par excellence."

Notwithstanding the ability of securities markets to convey information, economic theory would seem to suggest that firms have access to a superior source of information: direct revelation mechanisms (DRMs). After all, mechanism design theory informs us that any equilibrium outcome of some indirect revelation mechanism (IRM), can also be achieved by a DRM in which truth-telling is incentive compatible. Thus, one may view a firm using securities markets for information (denoted a *market-reliant* firm) as an IRM for eliciting information from some privately informed trader. But the revelation principle suggests the firm can do at least as well by hiring the trader to work under a DRM. That is, rather than leaving the informed trader outside its boundaries, the firm can bring him inside and provide incentives through a DRM. In fact, economic theory points to another benefit to bringing an informed agent inside firm boundaries: insulation of uninformed shareholders from mispricing due to adverse selection. After all, under *exclusivity* (contractual or legal prohibitions on securities trading by insiders), an informed agent brought in-house cannot trade at the expense of the firm's uninformed shareholders who may be forced to sell due to liquidity shocks.

Phrased differently, economic theory would seem to imply the existence of a Pareto-improving bargain to be struck between the market-reliant firm and the informed trader currently sitting outside its boundaries: The firm should compute what the trader is currently making in market gains and write a contract that pays him equal expected wages, induces him to report truthfully, and screens out uninformed agents. Having done so, the firm should base its decisions directly on his (truthful) reports, as opposed to relying on securities prices. After all, market prices do not generally fully reveal the trader's private information. In fact, it is the very noise in securities prices that is the underlying source of the informed outsider's positive expected trading gains.

In this paper, we demonstrate an inherent limitation to the use of DRMs by firms inhabiting economies with competitive securities markets. In particular, we show that for firms in such economies the set of feasible mechanisms may actually be empty.[1] Moreover, even when the feasible set is non-empty, the firm may nevertheless find it optimal to refrain from posting a mechanism, instead relying exclusively on the securities market for information. Finally, we show that even in those instances in which posting a mechanism dominates relying exclusively on the market, the firm can always achieve superior outcomes by stochastically limiting agents' ability to observe the posted mechanism, and relying on markets for information when the mechanism is not observed.

In order to understand these results, it is important to first highlight one of the key properties of price formation in securities markets. When an informed agent may be trading the firm's securities, a competitive market maker will lower (raise) price in response to sell (buy) orders. Consequently, when the informed agent trades, selling (buying) on negative (positive) private information, the price moves closer to the fundamental asset value, reducing trading gains. In this way, the competitive securities market, and the price discipline it provides, has a natural tendency to lower the informed agent's rent.

Consider instead the nature of price formation if the firm were to move away from market-reliance by posting a mechanism. In order to induce participation by the informed outsider, should he exist, the firm must offer an expected wage equal to his outside option value–and this outside option value is equal to the expected trading gain he would capture if he were to deviate by foregoing the posted mechanism. But note, since the posted mechanism is designed to induce participation by the informed agent, the market maker believes the informed agent will accept the offer if he exists. Therefore, if the informed agent deviates, and leaves the mechanism sitting, the market maker will form the belief that no informed agent exists. Consequently, the market maker will attribute the arrival of any sell order to uninformed shareholders being hit with a liquidity shock, rather than informed selling. That is, if the informed agent foregoes the posted mechanism, the arrival of any sell order, regardless of its size, will have zero price impact. This dramatically increases the outside option value of the informed agent. The key point here is that the very act of posting a mechanism fundamentally alters the nature of beliefs and price formation in the securities market—by rejecting a posted mechanism, the devil can convince the world he doesn't exist.

The preceding paragraph illustrates starkly why posting a mechanism may be self-defeating. In particular, once the firm posts the mechanism, the informed agent's outside option value rises considerably. In some instances, the endogenous outside option value increases to the point where it is impossible to achieve the dual objectives of screening out incompetents and inducing informed participation. In order to screen out incompetents, the optimal contract rewards the agent when his report proves correct and punishes the agent when his report proves incorrect. When the informed agent's outside option value is sufficiently large, the reward that must be offered for a correct report is so large that even an uninformed agent will be willing to accept the contract and make a guess, hoping to be right.

In other instances, the endogenous increase in the informed agent's outside option may exceed

[1] Here we say a mechanism is feasible if it screens for private information and satisfies the participation constraint.

the value provided by the mechanism, which is the increase in expected cash flow that arises when the firm bases its decision on the direct (truthful) report of the informed agent rather than the noisy stock price. In this case, expected wage costs under the mechanism (equal to the expert's outside option value) exceed the two costs of market-reliance: incorrect firm decisions due to price noise and shareholder trading losses arising from informed trading (adverse selection). In this case, even if the mechanism is feasible, offering it is suboptimal.

Our analysis completely characterizes the conditions under which it is optimal for the firm to refrain from posting a mechanism–instead relying exclusively on the securities market for information. We show market-reliance dominates mechanism reliance when the probability that an informed agent exists is high or the probability of shareholder liquidity shocks is low. In this case, firm decision-making under market-reliance moves closer to first-best since order flow is highly informative about the private information held by the informed outsider, should he exist. Thus the cash flow benefit of offering a mechanism is small. At the same time, in this case, price discipline under market-reliance will be strong, since the market maker will assign a high probability to orders arising from informed trading rather than liquidity trading. Thus, the endogenous increase in the outside option from offering a mechanism will here be especially large, increasing the relative cost of the mechanism compared to the market.

We extend our analysis by next considering a firm that has access to a technology that can stochastically limit the informed agent's ability to observe the mechanism offer (e.g. scope of marketing). In particular, if the firm offers a mechanism, the agent observes the mechanism with a particular probability, which the firm is able to choose. Here we derive a complementary result: Even when parameters are such that the firm attains higher ex ante value by offering the mechanism (with probability one) than when it does not (market-reliance), ex ante firm value is necessarily increased by introducing some limit on the informed agent's ability to observe the mechanism offer. Intuitively, if the agent does not always observe the mechanism offer, then the market maker can no longer be sure that a mechanism left sitting implies that no informed agent exists. Thus, even when the posted mechanism is left sitting, the market maker will impose some degree of price discipline, which reduces the informed agent's outside option. We show that ex ante firm value necessarily increases by limiting the informed agent's ability to observe the mechanism to some probability less than one. The firm loses valuable information by doing so, but is more than compensated by the endogenous reduction in the informed agent's reservation value.

Related literature. With its focus on understanding the conditions under which corporations will, in equilibrium, rely on securities markets for information, our paper offers a novel contribution to diverse literatures assessing the efficiency of competitive markets versus alternative information acquisition schemes. The key difference between our analytical framework and a large body of existing work is that we analyze the *interaction* between competitive financial markets and alternative information acquisition schemes. By way of contrast, Grossman and Stiglitz (1976) and Dow and Gorton (1997) analyze the allocative efficiency of financial markets when they do not interact with alternative information acquisition schemes. As such, their work is in the spirit of the comparative economic systems and mechanism design literatures, discussed below, both of which effectively put alternative information acquisition systems on different planets, with comparisons being performed.

Formal side-by-side comparisons of markets and mechanisms have been conducted in variety of settings and applications. In general, market-based information systems do not fare well. For example, Lerner (1944), Taylor (1948), and Lange (1967) formally demonstrate the ability of centrally planned economies, left in isolation from market economies, to achieve equally efficient outcomes. More recently, the revelation principle of Hurwicz (1973), Gibbard (1973), Hölmstrom (1979), Dasgupta, Hammond, and Maskin (1979), and Myerson (1979) informs us that indirect schemes, such as securities markets, cannot possibly achieve superior outcomes to a direct revelation mechanism when the latter is implemented in isolation.

Critically, our analysis departs from the standard mechanism design literature by analyzing the *interaction* between markets and mechanisms. In our model, all agents enjoy the option to trade in competitive financial markets, giving rise to endogenous outside option values for informed and uninformed agents. In this setting, we consider whether the principal will want to post a mechanism. In contrast, the standard mechanism design literature assumes that the principal has already decided to offer a mechanism and that the agent's reservation value is identical across all feasible mechanisms that the principal may offer. By focusing on the design of optimal mechanisms under these assumptions, this literature abstracts from interactions between the mechanism and other institutions in the economy. Instead, we embed a firm's decision regarding whether to offer a mechanism within the broader institutional context of an economy with a securities market—fundamentally, in the setting we consider, it is the interaction between the mechanism and the securities market that generates an endogenous reservation value, limiting the mechanism's feasibility and optimality.

Endogenous reservation values have been explored in a variety of other contexts. Tirole (2012), Philippon and Skreta (2012), and Bhattacharya and Nyborg (2013) study government programs to unfreeze markets plagued by adverse selection, where the decision not to participate in the program reveals information about the firm's type to the market. There is a fundamental difference between our setting and other models with endogenous reservation values: mechanisms in other settings provide useful information which enhances the efficiency of the market. In our setting, the offering of a mechanism removes the informed trader from the securities market, so that the market and the mechanism are fundamentally substitute sources of information. Jehiel and Moldovanu (2000) consider auctions in which agents' values for winning and losing depend on their subsequent market interactions. Other mechanism design literature focuses on type-dependent outside options (e.g. Lewis and Sappington 1989, Jullien 2000) or outside options created endogenously from relationship specific investments (Rasula and Sonderegger 2010). To the best of our knowledge, our paper is the first to explore the interaction between the securities market and a mechanism designed to bring expertise inside the firm. Thus, the central choice we analyze, securities markets versus mechanisms as information acquisition schemes, has no analog in the existing endogenous reservation value literature.

Our paper also provides a new perspective within the extant literature on the boundaries of the firm.[2] Williamson (1985) emphasizes the firm as a device for avoiding transaction costs. Grossman and Hart (1986) and Hart and Moore (1990) argue that firm boundaries allocate residual control rights optimally given the need for relationship specific investments. Other ideas include resolving incentive problems (e.g. Holmstrom 1999) and minimizing rent seeking (e.g. Klein 2000). In contrast, we analyze a corporation's decision regarding whether to bring informed expertise inside the firm, via the mechanism, or to refrain from posting a mechanism and instead rely on market provision of information. Our analysis reveals a significant limitation on the use of mechanisms in market economies: the very act of offering the mechanism increases the agent's reservation value. In light of this limitation, we derive conditions under which it is either infeasible or suboptimal for the firm to offer a mechanism that brings the informed agent inside its boundaries.

A growing feedback-effect literature analyzes the interplay between the information contained in securities prices and economic decisions (Kahn and Winton 1998, Bond, Goldstein, and Prescott 2009, Bond and Goldstein 2015, Boleslavsky, Kelly, and Taylor 2017, BKT). This literature establishes limitations on the information that may be gleaned from securities prices. As in this literature, our model has the feature that anticipated feedback between order flow and firm decisions causes less aggressive informed trading and limited price informativeness. Therefore, our key finding that market-reliance may be optimal—even if the mechanism is feasible—is even more striking. The feedback-effect literature simply assumes that the informed agent resides outside the firm. In contrast, we treat the firm as making an endogenous choice between outside (market-based) information production versus inside (managerial) information production.

In the setting we consider, if the agent's potential liability is sufficiently large (relative to his reservation value), then the firm can design a contract to screen for the agent's expertise, elicit the agent's private information, and act on this information efficiently, while just meeting the agent's reservation value. This result is reminiscent of Riordan and Sappington (1988),[3] who consider a contracting problem with a verifiable public signal of the agent's private information, deriving conditions under which the principal can implement the efficient production plan without paying the agent in excess of his reservation value. Cremer and McLean (1988) derive similar results in an auction with correlated valuations, deriving conditions under which the seller can design an auction mechanism that implements any feasible allocation rule, including one which fully extracts the buyers' expected surplus. Of course, our primary focus is on the interaction between the mechanism and the securities market and its implications for the mechanism's feasibility and optimality, which is absent in these analyses. Given the power of the mechanism in the setting we consider, our finding that market-reliance may nevertheless dominate mechanism-reliance is all the more surprising.

Section 2 provides an overview of the economic setting. Section 3 develops a microstructure model of the securities market in which market activity is the sole source of information for the firm. Section 4 derives conditions under which there exists a mechanism that screens for expertise

[2]See Williamson (2002) and Gibbons (2005) for review articles.
[3]See also Gromb and Martimort (2007), who study a related model with collusion.

and characterizes the optimal mechanism, treating the expert's outside option value as exogenous. Section 5 derives conditions under which posting a mechanism is optimal given the existence of a securities market, allowing the expert's outside option value to change endogenously when the mechanism is posted. Section 6 considers an extension in which the firm uses a technology which limits an informed agent's ability to observe the mechanism offer. Section 7 discusses the robustness of results to alternative assumptions. Section 8 concludes. Proofs are in the appendix.

2 MODEL

We analyze the interaction between markets and mechanisms in the context of a canonical firm-level decision problem with an information asymmetry.

Firm ownership. We consider a widely-held public corporation with tradable shares.[4] Initially, a set of ex ante identical risk neutral atomistic shareholders owns all outstanding equity. For brevity, these original shareholders are referred to as "the shareholders." Each atomistic share entitles its holder to an infinitesmal share of the firm's cash flow. The measure of outstanding shares and the measure of original shareholders are both normalized to 1, with each atomistic shareholder owning an atomistic share. The corporation is unlevered, ruling out distortions in decisions arising from conflicts of interest between debt and equity. Ex ante, the objective of the firm is to maximize the expected payoff the shareholders derive from their share, i.e. the ex ante value of the firm.

Ex ante firm value consists of two parts. First, a share held to maturity entitles the shareholder to the firm's cash flow which accrues at the terminal date. Second, shareholders may be hit by liquidity shocks which force them to sell their stock in a competitive secondary market. Because markets may mistake uninformed liquidity selling for informed trading, the shareholders face potential adverse price impact causing shares to sell for less than expected cash flow ("fundamental value"). The expected underpricing of shares in the secondary market reduces the ex ante value of a share. In other words, underpricing is capitalized into the ex ante value of the firm's equity as in Holmstrom and Tirole (1993) and Maug (1998), for example.

Firm decision. As is standard, we consider a firm with a stock of assets in place and a growth option. To conserve notation, we normalize the terminal cash flow from assets in place at 0. The firm must choose between a risky investment (R) and a safe investment (S), and this decision must be sequentially rational. Sequential rationality can be understood as arising from a corporation's fiduciary responsibility to its shareholders, which requires it to select actions that are optimal given the information available to it.

The two competing investments have equal cost which we normalize at 0 for simplicity. The terminal cash flow of the firm under investment R is a binary random variable ω drawn from $\{0, 1\}$. Below, we refer to ω as the *economic state*. All agents have the common prior $\Pr(\omega = 0) = q$,

[4] With advanced securities markets, bets can even be placed on unlisted firms using private share trading platforms or by trading debt or credit default swaps (CDS).

where $q \in (0,1)$. If the firm instead implements investment S, it is insulated from risk, receiving a sure terminal cash flow equal to $1-c$, where $c \in (0,1)$. Intuitively, one can interpret c as the cost of insulating the firm from the consequences of the bad economic state. Throughout the analysis, we assume $c > q$. Notice, given this maintained assumption, investment R would be optimal if the firm's decision were to be based solely on prior beliefs. For ease of exposition, we will think of investment R as the default, and speak of the firm "switching" to investment S.

Shareholders. Our treatment of the firm's original shareholders is a tractable variation of the noise-trader setup commonly-adopted in the market microstructure literature, e.g. Kyle (1985) and Glosten and Milgrom (1985). In particular, it is assumed that each original shareholder will hold their stock until the terminal date unless forced to liquidate. The probability of a liquidity shock is $l \in (0,1)$. The arrival of the liquidity shock is observed only by the atomistic shareholders, creating noise in the market.

If a liquidity shock does indeed arrive, the fraction of original shareholders forced to sell is itself a random variable assumed to be uniformly distributed on $[0,1]$. Aside from assuming a compact support for the liquidity shocks, the specific density function assumed has no bearing on the results other than simplifying the algebra. As in Maug (1998) and Faure-Grimaud and Gromb (2004), it is assumed that the shareholders' liquidity sales arrive in the market as a batch (block).

Outsider Agents. A countably infinite number of identical agents exist, each of whom will never acquire any information. These agents are labeled *uninformed outsiders*. With probability $a \in (0,1)$ an additional outsider agent exists, and this agent is labeled as the *expert outsider* (or "expert" for brevity).[5] Should he exist, the expert outsider privately observes the economic state ω at the time it is determined by nature. For brevity, we shall refer to an expert who learns that the economic state is ω as the "type-ω expert." The expert's existence is his private information: to others, the expert is indistinguishable from an uninformed outsider. We refer to the existence of the expert as the *information state*, denoted $\eta \in \{0,1\}$: if $\eta = 1$, then the expert exists, and if $\eta = 0$, then the expert does not exist. The information state, the economic state, and the liquidity shocks are statistically independent.

The uninformed outsiders and the expert are risk neutral and seek to maximize their expected wealth. Each outsider (uninformed or expert) has wealth $W \geq 1$ which is sufficient to cover any feasible short or long position. In addition, each outsider has the ability to post a "bond" worth $B \geq 0$ as part of any incentive contract (mechanism) signed with the firm. The bond represents the maximum value the legal system can actually extract from an agent hired by the firm, inclusive of reputation costs. In practice, B is a function of the legal system, the value attached to reputation, wealth, and the financial structure of a bonded agent.[6] In the interest of generality, we remain agnostic regarding the size of B aside from assuming it to be non-negative. This treatment allows

[5] Competition amongst multiple experts is considered in our extension section.

[6] The parameter B can be less than one for a variety of reasons, e.g. limited liability and frictions in contract enforcement.

us to analyze parametrically whether and how the existence of a securities market, as an outside option, serves to test the limits of bonding capability, undermining an otherwise viable mechanism.

Mechanism. The firm has an opportunity to publicly offer a mechanism to the outsider agents in an attempt to elicit information about the economic state ω. Formally, the mechanism specifies a transfer to the firm's advisor as a function of his report of the economic state and the realized terminal cash flow.

The mechanism is offered on a first-come first-serve basis. An outsider who takes up the firm's offer becomes an inside advisor. To fix ideas, one can think of the mechanism as being a consulting contract or an employment contract. It is assumed that, as a firm insider, the advisor would be barred from trading the firm's stock–an assumption consistent with standard insider trader laws.

We show in section 7 that this "exclusivity assumption" is made without loss of generality. In particular, we show that even if it were legal to do so, the firm could not increase its ex ante value under a mechanism by granting its advisor the freedom to trade its shares. Intuitively, what dictates the required total payoff (wages plus trading gains at the expense of shareholders) to an advisor who takes up a given contract is the trading gain they stand to make if they leave the contract sitting. This off-equilibrium trading gain is invariant across all contracts acceptable to the expert outsider.

Information Channels. We separately consider two different channels by which the firm can acquire information about the economic state ω. A firm will be said to be *mechanism-reliant* if it offers a mechanism to the outsider agents. The goal of a mechanism-reliant firm is to elicit direct reporting of the private information of the expert outsider, should such an agent exist. In contrast, a firm will be said to be *market-reliant* if it refrains from offering a mechanism to the outsider agents. Such a firm relies only on the market for information about the economic state.

Sections 3 and 4 provide more-detailed information about the sequence of events in each setting. After characterizing the equilibrium in each setting, we determine whether the firm's shareholders achieve higher ex ante value by posting a mechanism (mechanism-reliance) or by refraining from posting a mechanism (market-reliance).

3 THE CASE FOR MARKETS: FIRM VALUE GIVEN MARKET-RELIANCE

This section considers a market-reliant firm–a firm that refrains from offering a mechanism to the outsider agents. Such a firm relies exclusively on the market for the provision of information.

As will be shown formally below, market-reliance suffers two weaknesses relative to mechanism-reliance. First, the expert outsider anticipates that if his trade reveals some information to the market, the stock price will move closer to its fundamental value, reducing his trading gain. This effect is exacerbated by the fact that the expert also anticipates that the firm may adjust its investment in response to his trade. In particular, if the expert sells too aggressively after observing that the economic state is bad ($\omega = 0$), then the firm may infer this information and switch to the safe investment S. But note, if the firm switches to the safe investment, the expert's private knowledge about the economic state becomes worthless since the firm's terminal cash flow will be $1-c$ for sure. The expert's incentive to mask his private information implies that securities trading generally provides only a noisy signal of the economic state. Consequently, the market-reliant firm may implement the incorrect investment even if the expert outsider exists.

The second weakness of market-reliance is that with an expert outsider left free to trade in the securities market, the firm's shareholders are exposed to adverse selection. In particular, if shareholders are forced to sell due to a liquidity shock, then the market maker considers the possibility that the order reflects trading by an expert outsider with knowledge that the true economic state is 0. This results in secondary market stock underpricing which is capitalized into the ex ante share value.

Timing. If the firm is market-reliant then the game unfolds in the following sequence.

1. *Information State.* The expert outsider's existence (information state η) is realized.
2. *Economic State.* The economic state ω is realized and is privately observed by the expert outsider if he exists.
3. *Liquidity Shock.* Any liquidity shock is realized.
4. *Market.* The firm's equity is traded in a market (described below) with orders observed by a competitive market maker and the firm.[7]
5. *Decision.* The firm chooses an investment, S or R.
6. *Cash flows.* The firm's cash flows are revealed.

Market. The firm's stock is traded in an anonymous competive market. As discussed above, the firm's shareholders submit a uniformly distributed sell order if hit with a liquidity shock. In addition, each outsider agent i has discretion to submit a single sell order of size $t_i \in [-1, 1]$. Since the focus of the formal analysis below is largely on the sell side, we denote positive values of t as sell orders and negative values as buy orders. Of course, each outsider agent is free to submit a sell order of size 0. We shall label an order of 0 as *inactivity*. Indeed, we will be particularly interested in the ability of a competitive securities market to deter trading by the uninformed outsiders, thus screening out incompetents.

A competitive market maker observes the countably infinite vector of submitted orders without observing each order's source. Thus, the market maker cannot observe whether a particular order in the vector is the result of liquidity selling by shareholders, a trade by an uninformed outsider, or a trade by the expert outsider. The market maker updates her beliefs about the economic state based on the vector of observed orders, adjusts the price of the stock to equal its expected terminal

[7]BKT (2017) show in a related model that payoffs and information flow are equivalent if the firm observes securities prices, but not order flow.

cash flow, and simultaneously fills all orders at this one price. Intuitively, one can think of the market maker as engaging in Bertrand competition with other market makers in order to fill the submitted order vector. As is standard, we abstract from institutional details of market making, and assume a frictionless securities market in which the market maker is willing and able take the opposite side of any order vector.

Notation. Let T denote the countably infinite vector consisting of the orders submitted by the shareholders and each outsider agent. Let $T = \vec{0}$ denote an order vector consisting entirely of zeros, i.e. complete inactivity by all agents. Let $T = \vec{t}$ denote an order vector consisting of zeros along with a single non-zero sell order of size t. Such an order vector is an important case in the model, as only in this case is information not perfectly revealed by the order vector.

Let $\chi(T)$ denote the probability assessment of the market maker and firm that the economic state is bad ($\omega = 0$) given observed order vector T. Let $\alpha(T)$ denote the probability that the firm switches to the safe investment S after observing order vector T.

Recall, the type-ω expert knows the economic state is ω. With this in mind, let $\Phi_\omega(\cdot)$, denote the cumulative distribution function from which the type-ω expert draws his order. We denote the associated probability density function by $\phi_\omega(\cdot)$, using the Dirac $\delta(\cdot)$ function to denote mass points. An uninformed outsider's trading strategy is denoted by the distribution function $\Phi_U(\cdot)$. Finally, let u^*_ω denote the expected trading profit for the expert outsider in economic state ω.

PRELIMINARY ANALYSIS

We derive conditions and present a number of simplifying results ahead of the main equilibrium characterization in the next section.

Consider the firm's choice between the safe and risky investment following the observation of order vector T. If the firm chooses safe, its terminal cash flow is $1 - c$ for sure. If the firm chooses risky, the expected cash flow is $1 - \chi(T)$. Therefore, any sequentially rational strategy for the firm must entail:

$$\alpha(T) = \begin{cases} 0 & \text{if} \quad \chi(T) < c \\ \in [0,1] & \text{if} \quad \chi(T) = c \\ 1 & \text{if} \quad \chi(T) > c \end{cases} \tag{1}$$

Intuitively, the firm will find it optimal to switch to the safe investment only if it assesses a sufficiently high probability that the true economic state is $\omega = 0$. Recalling that the risky investment is optimal based upon prior information ($q < c$), it is apparent that the firm will only switch to the safe investment if the observed order vector brings about a sufficiently large negative revision of beliefs.

Given order vector T, the competitive secondary market stock price must be equal to the expected terminal cash flow,

$$p(T) = [1 - \alpha(T)][1 - \chi(T)] + \alpha(T)(1 - c). \tag{2}$$

With probability $\alpha(T)$, the firm implements the safe investment, with terminal cash flow equal to $1 - c$ with certainty. If the firm instead implements the risky investment, expected cash flow is $1 - \chi(T)$. Notice, the secondary market stock price reflects information about the economic state contained in the order flow, as well as the firm's optimal investment given the order flow.

Consider now the type-0 expert's expected trading gain in economic state $\omega = 0$, assuming he indeed exists. Here the expert knows that if the firm implements the risky investment a share will be worth 0. Thus, if the type-0 expert outsider submits order t, and the realized order vector is T, then his realized trading gain will be

$$\begin{aligned} u_0(t,T) &= t[p(T) - \alpha(T)(1-c)] \\ &= t[1 - \chi(T)][1 - \alpha(T)]. \end{aligned} \tag{3}$$

Consider next the type-1 expert's expected trading gain in economic state $\omega = 1$, assuming he indeed exists. If $\omega = 1$, the type-1 expert knows that if the firm implements the risky investment a share will be worth 1. Thus, if he submits order t, and the realized order vector is T, then his realized trading gain will be

$$\begin{aligned} u_1(t,T) &= t[p(T) - (1 - \alpha(T)) - \alpha(T))(1-c)] \\ &= -t\chi(T)[1 - \alpha(T)]. \end{aligned} \tag{4}$$

From equations (3) and (4) we see that, all else equal, the expert's trading gain increases with his trade size (type-0 for sell orders and type-1 for buy orders). Conversely, his trading gain decreases when order flow reveals more information to the market maker about the true economic state. Finally, we see that the expert's trading profit decreases with the probability of the firm implementing the safe investment. By implementing the safe investment, the firm severs the link between the economic state and the firm's terminal cash flow, rendering the expert's private knowledge of the economic state worthless.

Next, note that equations (3) and (4) imply the following:

Lemma 3.1 *(Dominance). Given any belief function $\chi(T) \in [0,1]$ for the market maker and the firm, any firm strategy $\alpha(T) \in [0,1]$, and any realization of the liquidity shock, in state zero (state one) the expert's profit from submitting a sell order t_S (buy order t_B) is weakly larger than his payoff from inactivity, which is weakly larger than his payoff from submitting a buy order (sell order):*

$$t_B < 0 < t_S \Rightarrow u_0(t_B,T) \le 0 \le u_0(t_S,T) \quad \textit{and} \quad u_1(t_S,T) \le 0 \le u_1(t_B,T).$$

In other words, for the type-0 expert, selling a positive amount is always weakly better than inactivity or buying, and for the type-1 expert, buying a positive amount is always weakly better than inactivity or selling.

Equilibrium Trading Patterns. We characterize equilibria in which trading patterns satisfy three intuitive conditions. First, we conjecture equilibria in which each uninformed outsider finds it optimal to be inactive, and verify in Lemma 3.4 that inactivity is indeed an optimal strategy for such agents. Intuitively, since uninformed outsiders have no private information, they should have no incentive to trade, especially given that equilibrium prices tend to move against an uninformed trader.

Second, since Lemma 3.1 establishes that for the type-0 expert, selling a positive amount is always weakly better than inactivity or buying, we characterize equilibria in which the type-0 expert, should he exist, always sells ($\Phi_0(0) = 0$). Third, since Lemma 3.1 establishes that for the type-1 expert, buying a positive amount is always weakly better than inactivity or selling, we characterize equilibria in which the type-1 expert always buys ($\Phi_1(0) = 0$). In this case, any equilibrium buy order must originate with the type-1 expert, so he cannot possibly earn a strictly positive expected trading gain. For ease of exposition, we posit that the the type-1 expert plays a continuous mixed strategy, placing buy orders supported on the entire interval $t \in [-1, 0]$. The assumption that the type-1 expert is always active is not essential for the qualitative results. Henceforth, when we refer to equilibria, we refer to equilibria with the postulated patterns of trade: uninformed inactive, informed buying on positive news, and informed selling on negative news.

Given the dominance arguments made above, the posited trading patterns are intuitive. However, as shown below equilibria are not necessarily unique. Nevertheless, as shown in the Supplemental Materials Section A, introducing small trembles in the technology creates a natural equilibrium refinement which selects a unique equilibrium in which the type-0 expert always sells and the uniformed do not trade, as posited above.

Beliefs. Two classes of on-path order vectors perfectly reveal the economic state. On the equilibrium path, an order vector containing two sell orders only arrives if there is a liquidity shock, the expert outsider exists, and the economic state is bad. Since the revealed economic state is $\omega = 0$, the firm will respond by switching to the safe investment with probability 1. Second, any order vector containing a buy order, with or without a sell order (which would necessarily come from liquidating shareholders), reveals that the economic state is $\omega = 1$. The revelation of this information induces the firm to implement the risky investment with probability 1.

In contrast, complete market inactivity ($T = \vec{0}$) reveals no information about the economic state. After all, the expert outsider is active if he exists. Thus, if there is no activity in the market, i.e. $T = \vec{0}$, then the market maker and firm infer that no expert exists, and their beliefs about the economic state are unaffected. That is, $\chi(\vec{0}) = q$. We summarize this discussion in the following lemma.

Lemma 3.2 *(On Path Revealing Orders and Inactivity). (i) any order vector T containing two sell orders, at least one of which is inside the support of the type-0 expert's strategy, reveals that the* *state is zero,* $\chi(T) = 1$ *and induces a switch to the safe investment,* $\alpha(T) = 1$*; (ii) market inactivity does not affect beliefs* $\chi(\vec{0}) = q$ *and induces the firm to select the risky investment,* $\alpha(\vec{0}) = 0$*; (iii) any order vector* T *containing a single buy order, or a buy order and a single sell order reveals that the state is one,* $\chi(T) = 0$ *and induces the firm to select the risky investment,* $\alpha(T) = 0$.

The preceding lemma describes beliefs for all on-path order vectors except for those containing a single sell order. Consider therefore the nature of beliefs following the arrival of an order vector $(\vec{t})$ containing all zeros and a single sell order of size t. When such an order vector arrives, the firm and market maker consider two possibilities: either (1) the expert outsider does not exist and the sell order is due to a liquidity shock or (2) the expert outsider exists, the economic state is 0, and no liquidity shock arrived. Bayes' rule implies updated beliefs are:

$$\chi(\vec{t}) = \frac{aq(1-l)\phi_0(t) + q(1-a)l}{aq(1-l)\phi_0(t) + (1-a)l}. \tag{5}$$

It is readily verified that $\chi(\vec{t})$ is increasing in $\phi_0(t)$. It follows that if the type-0 expert places the sell order t with higher likelihood, beliefs will be more negative in response to the observation of order vector $\vec{t}$.

The preceding discussion concerned beliefs in response to order vectors on the equilibrium path. However, it is possible that the expert, particularly if he is type-0, will want to deviate to a sell order outside his equilibrium trading support. In this case, it is possible that two sell orders will arrive, with both orders falling outside the equilibrium support of the type-0 expert. Further, if one of the uninformed outsiders chooses to deviate from inactivity, then other off-path order vectors may be observed. These off-path beliefs influence an uninformed agent's expected payoff from deviating from the posited equilibrium. When a single uninformed agent deviates, he may generate a variety of off-path order vectors: three sell orders; two sell orders and one buy order; two buy orders and one sell order; or two buy orders. For the market-reliant firm, we adopt the following simple convention for assigning beliefs to all off-path order vectors (others supporting the same equilibria are possible).

Remark 3.3 *(Market-Reliance: Off-path Beliefs and Actions). Consider an off-path order vector,* T*. If the number of buy orders is greater than or equal to the number of sell orders, then* T *reveals state* 1*,* $\chi(T) = 0$*, and the firm selects the risky investment,* $\alpha(T) = 0$*. Otherwise,* T *reveals state* 0*,* $\chi(T) = 1$*, and the firm selects the safe investment,* $\alpha(T) = 1$.

These beliefs imply that the type-1 expert is indifferent across all possible buy order sizes. These beliefs also imply that each uninformed outsider prefers complete inactivity to either buying or selling. Intuitively, the possibility that trade originates with the informed expert moves prices closer to the fundamental value, generating an adverse price impact. Because an uninformed agent faces this adverse price impact without knowledge of the economic state, he cannot make money by participating in the market—price impact screens out incompetents. We have the following lemma:

Lemma 3.4 *(Market Screening) Given the equilibrium beliefs, an uninformed agent's expected profit from submitting any order to the market is weakly negative.*

The equilibrium beliefs allow us to derive the type-0 expert's expected profit from selling t shares. If the liquidity shock arrives along with his order, then the economic state is revealed to be $\omega = 0$. The firm will then find it optimal to implement the safe investment, with the market maker setting the stock price at $p = 1 - c$, resulting in zero profit for the expert. With probability $1 - l$ the order vector consists of zeros along with the expert's sell order of size t. Consequently, when the type-0 expert submits a sell order t, his expected trading profit is

$$E[u_0(t, T)] = t[1 - \chi(\vec{t})][1 - \alpha(\vec{t})](1 - l). \tag{6}$$

From the preceding equation we see that the type-0 expert properly views his own sell order order size as being the sole determinant of beliefs and the firm's decision provided that no liquidity shock arrives. Again, his trading gain is increasing in the size of his order, holding all else equal. However, he must take into the impact of his order size on beliefs and the firm's decisions.

Key Parameters. We introduce two transformations of the model parameters that simplify subsequent exposition.

$$K \equiv \Big(\frac{aq}{1-a}\Big)\Big(\frac{1-l}{l}\Big), \quad J \equiv \frac{1-q}{1-c}.$$

Parameter $K \in (0, \infty)$ is labeled *market informativeness*, since it captures the information content of an order vector containing a single sell order combined with zeros. The numerator of K is the probability that a single sell order arrives due to the existence of an expert outsider, economic state $\omega = 0$, and no liquidity shock. The denominator of K is the probability that a single sell order instead arrives due to a liquidity shock hitting the firm's shareholders, with the expert outsider non-existent. Thus, the variable K will be high if a is high and l is low. If K is indeed high, then the arrival of a single sell order provides a strong signal to the market maker and to the firm that the true economic state is $\omega = 0$. It is convenient to rewrite the beliefs in equation (5) as a function of K as follows:

$$\chi(\vec{t}) = \frac{K\phi_0(t) + q}{K\phi_0(t) + 1}. \tag{7}$$

Parameter $J \in (1, \infty)$ is labeled *switching difficulty.* Intuitively, as the cost c of implementing the safe investment tends upward to 1, J tends to infinity. In this case, the firm would only find it optimal to switch to the safe investment if it were certain that $\omega = 0$. Conversely, as the cost c of implementing the safe investment tends downward to q, J tends to 1, and the firm would find it optimal to switch to the safe investment in response to even a small negative revision of its beliefs relative to the prior.

It is worth noting that J and K can be adjusted independently through changes in the underlying parameters (a, l, c). Specifically, any market informativeness measure $K \in (0, \infty)$ can be achieved via changes in a or l while holding J fixed. Similarly, any switching difficulty $J \in (1, \infty)$ can be achieved without affecting K by choosing an appropriate value of c in the interval $(q, 1)$.

EQUILIBRIUM WITH INFORMATION RENT

We begin the characterization of equilibria by first analyzing those in which the type-0 expert outsider expects to earn a strictly positive information rent, $u_0^* > 0$.

A number of observations are immediate. First note that if the type-0 expert is to make an information rent, he must use a proper mixed strategy. After all, if he were to submit one particular order with probability 1, then observing this order would reveal the true economic state to be $\omega = 0$. This would induce the firm to implement the safe investment, resulting in zero profit. And given that the liquidity sales have no mass points, the same reasoning implies that the cumulative distribution function Φ_0 cannot contain any mass points.

Second, note that in an equilibrium with an information rent, the minimum sell order size on the support, call it m, must be greater than 0. After all, if m were 0, then the type-0 expert's trading gain would also be 0 . Third, we may note that it must be the case that the mixing density vanishes at m, that is $\phi_0(m) = 0$. After all, equation (5) implies $\chi(\vec{t}) = q$ for any t outside the trading support. If $\phi_0(m)$ were to exceed 0, then order m would entail an adverse price impact $(\chi(\vec{m}) > q)$, and thus the expert would earn a higher expected trading gain by deviating to an order infinitesimally smaller than m. Finally, a similar argument rules out gaps in the trading support, since there would be a gain to deviating to a gap point, given that a gap point t would also have the property that $\chi(\vec{t}) = q$. Thus, in any equilibrium with information rents, the type-0 expert outsider must play a continuous mixed strategy with no mass points or gaps, as we summarize next.

Lemma 3.5 *(Structure of Expert's Strategy). If the type-0 expert expects an information rent in equilibrium, then he plays a continuous mixed strategy with no mass points or gaps supported on interval* $[m, 1]$*, for some* $m > 0$*.*

From equation (6) we have the following indifference condition:

$$t \in [m, 1] \Rightarrow t[1 - \chi(\vec{t})][1 - \alpha(\vec{t})](1 - l) = u_0^*. \tag{8}$$

Ceteris paribus, the type-0 expert's trading gain is increasing in the size of his sell order. To maintain indifference, the gains from trading larger volume must be exactly offset by a combination of greater belief impact (i.e. an increase in $\chi(\cdot)$) and a higher probability of the firm switching to the safe investment.

Small orders have less impact on beliefs, with the smallest order m in the support having no impact at all, implying a stock price $p = 1 - q$ provided no liquidity shock arrives. Thus, the type-0 expert's expected profit from selling m shares is $m(1-q)(1-l)$. Since he must be indifferent among all trades inside the support of his mixed strategy, this must also be the type-0 expert's expected profit u_0^*. We have the following lemma.

Lemma 3.6 *(Expert's Profit). If the type-0 expert expects an information rent in equilibrium, then* $u_0^* = m(1-q)(1-l)$.

Equation (8) also implies that in any equilibrium with an information rent ($u_0^* > 0$), the type-0 expert never submits an order t such that the observation of such an order would induce the firm to switch to the safe investment with probability 1 ($\alpha(\vec{t}) = 1$). After all, such an order would imply an expected trading profit of 0.

To complete the characterization, we must derive the firm's equilibrium investment policy. The appendix establishes the following important results.

Lemma 3.7 *(Structure of Firm's Strategy). In an equilibrium in which the type-0 expert expects an information rent,*

(i) if $m > \frac{1}{J}$*, then* $\alpha(\vec{t}) = 0$ *for all* $t \in (0, 1]$.

(ii) if $m < \frac{1}{J}$*, then* $\alpha(\vec{t}) = 0$ *for all* $t \in (0, Jm]$*, and* $\alpha(\vec{t}) \in (0, 1)$ *for all* $t \in (Jm, 1]$.

The intuition for the preceding lemma is as follows. First the lemma reveals that, ceteris paribus, the firm is more likely to switch to the safe investment the lower the switching difficulty J. Next, note that the impact on beliefs and prices is small for sell orders near the minimum size m.[8] In this neighborhood, the firm will always implement the risky investment (provided no liquidity shock arrives, fully revealing the bad state). The effect on beliefs will then increase with the size of the order. The lemma reveals that if m is small, then beliefs are affected over a wide interval, and orders sufficiently larger than m reveal enough information to induce the firm to mix between the risky and safe strategies. If m is large, then the interval of orders over which beliefs are affected is narrow. In this case, even the largest sell order (it it arrives by itself) does not convey enough information to induce the firm to switch to the safe investment with positive probability.

With the preceding argument in mind, consider the possibility of an equilibrium with large information rents, those with a large minimum sell order $m > 1/J$. Applying Lemma 3.7, in such an equilibrium the firm selects the risky investment following every sell order (that arrives on its own). That is, $\alpha(\vec{t}) = 0$ for all $t \in (0, 1]$. Consequently, the type-0 expert's indifference condition (8) is for all $t \in [m, 1]$,

$$m(1-q)(1-l) = t[1 - \chi(\vec{t})](1-l). \tag{9}$$

In equilibrium, beliefs following a single sell order are given by (7). Substituting this into the preceding indifference condition we find that

$$m(1-q)(1-l) = t\left[1 - \frac{K\phi_0(t) + q}{K\phi_0(t) + 1}\right](1-l) \Rightarrow \phi_0(t) = \frac{t-m}{Km}. \tag{10}$$

[8] Note that m is endogenous and will be determined below.

Thus, the type-0 expert outsider exploits his private information by using a mixing density that increases linearly in the trade size t. To determine the equilibrium value of the minimum sell order size m, note that $\phi_0(t)$ must integrate to 1. We have:

$$\int_m^1 \frac{t-m}{Km}dt = 1 \Rightarrow m = K+1-\sqrt{(K+1)^2-1}. \tag{11}$$

Finally, from the preceding equation it follows that our initial conjecture that $m > 1/J$ is true if and only if the market informativeness measure K is sufficiently low, specifically $K < \underline{K}$, where

$$\underline{K} \equiv \frac{(J-1)^2}{2J}. \tag{12}$$

Intuitively, when the market informativeness measure K is small, which occurs if an informed trader exists with a low probability and/or the probability of a liquidity shock is high, the type-0 expert's trading has a small impact on beliefs. This allows him to use an aggressive trading strategy featuring large sell orders, delivering a high equilibrium trading gain, without ever inducing the firm to switch to the safe investment. We summarize this case below.

Proposition 3.8 *(Low Informativeness). If market informativeness is sufficiently low, $K \in (0, \underline{K})$, then an equilibrium in which the type-0 expert expects an information rent exists, characterized as follows. The type-0 expert submits a sell order drawn from density*

$$\phi_0^L(t) \equiv \frac{t-m_L}{Km_L},$$

supported on interval $[m_L, 1]$, where

$$m_L \equiv K+1-\sqrt{(K+1)^2-1}.$$

The firm selects the risky investment in response to any single sell order, $\alpha(\vec{t}) = 0$. Beliefs and actions for all other order vectors are given in Lemma 3.2 and Remark 3.3.

Second, consider the possibility of an equilibrium with small information rents, one with a small minimum sell order $m < 1/J$. From Lemma 3.7 , the firm stays with the risky investment for sure on the interval $[m, Jm]$. Here the type-0 expert's mixed strategy is derived from equation (10). On the interval $(Jm, 1]$, the firm mixes between safe and risky investments. In order for mixing by the firm to be sequentially rational, it must be that $\chi(\vec{t}) = c$ over this interval. Combining this fact with (7) and (8), we conclude that

$$\text{for } t \in (Jm, 1], \quad m(1-q)(1-l) = t(1-c)\left[1-\alpha(\vec{t})\right](1-l), \quad \text{and} \quad \frac{K\phi_0(t)+q}{K\phi_0(t)+1} = c, \tag{13}$$

and hence,

$$\text{(14)} \qquad \text{for } t \in (Jm, 1], \quad \alpha(\vec{t}) = 1 - \frac{Jm}{t}, \quad \text{and} \quad \phi_0(t) = \frac{J-1}{K}.$$

Notice, orders larger than Jm occur with a constant density. It follows that beliefs and prices are constant on this interval. Since prices are constant on this interval, the probability of the firm switching to the safe investment must increase in the size of the sell order, as shown above, which just offsets the type-0 expert's temptation to submit larger orders. Finally, $\phi_0(t)$ must integrate to one. It follows that:

$$\text{(15)} \qquad \int_m^{Jm} \frac{t-m}{Km} dt + (1 - Jm)(J-1)/K = 1 \Rightarrow m = \frac{2(J-K-1)}{J^2-1}.$$

Thus, our initial supposition, that $m < 1/J$, holds if and only if $K > \underline{K}$. Intuitively, we have a second class of equilibria, featuring lower trading gains if the price informativeness measure exceeds $\underline{K}$. However, if equilibrium information rents are to be strictly positive K cannot be too high. In particular, Lemma 3.5 requires that $m > 0$, and hence it must be the case that $K < \overline{K}$, where:

$$\text{(16)} \qquad \overline{K} \equiv J - 1.$$

We thus have the following proposition.

Proposition 3.9 *(Intermediate Informativeness). For intermediate levels of informativeness, $K \in (\underline{K}, \overline{K})$, an equilibrium in which the type-0 expert expects an information rent exists, characterized as follows. The type-0 expert submits a sell order drawn from density*

$$\phi_0^I(t) \equiv \begin{cases} \frac{t-m_I}{Km_I} & \text{if } t \in [m_I, Jm_I] \\ \frac{J-1}{K} & \text{if } t \in [Jm_I, 1] \end{cases}$$

supported on interval $[m_I, 1]$, where

$$m_I \equiv \frac{2(J-1-K)}{J^2-1}.$$

Following a single sell order, $\alpha(\vec{t}) = 0$ if $t < Jm_I$ and $\alpha(\vec{t}) = 1 - Jm_I/t$ if $t \in [Jm_I, 1]$. Beliefs and actions for all other possible order vectors are given in Lemma 3.2 and Remark 3.3.

Based on the preceding characterization, we can derive a simple expression for the ex ante value of the firm in an equilibrium with information rent, call it V_R. The value of a share ex ante is equal to expected cash flow less the expected trading losses of the shareholders. In turn, since the market maker breaks even in expectation, expected shareholder trading losses are just equal to the ex ante expectation of expert trading gains. Regarding expert trading gains, they only accrue if the expert indeed exists and if the economic state is bad. Thus, the ex ante expectation of shareholder trading losses is aqu_0^*.

Consider next expected cash flow, focusing first on the low market informativeness case. As a benchmark, consider a firm that had zero access to outside information, being forced to rely on prior beliefs. Such a firm would always play the risky strategy, generating expected cash flow $1-q$. In contrast, in the low informativeness case, the firm increases its expected cash flow by correctly shifting to the safe investment in one (but only one) state of nature: the expert exists (probability a); the state is bad (probability q); and a fully revealing liquidity shock occurs (probability l). In this same state of nature, the always-risky strategy would generate a cash flow of 0. In contrast, by following the market and switching to the safe investment in this one state of nature, the firm gains an incremental cash flow equal to $1-c$. It follows that expected cash flow in the low informativeness case is:

$$1 - q + aql(1-c). \tag{17}$$

Consider next the intermediate market informativeness case, in which the firm mixes between safe and risky following some realizations of the order flow vector. Here, the fact that the firm is indifferent between safe and risky for such order vectors implies that the conditional expectation of the cash flow is the same as if it had simply played the risky investment for those order vectors. Thus, expected cash flow is still given by the preceding equation in the intermediate market informativeness case, in which the firm mixes (see the proof of Proposition 3.10 for a formal derivation).

Putting this analysis together, we have the following proposition:

Proposition 3.10 *(Ex Ante Firm Value–Eq. With Rent). In an equilibrium in which type-0 expects an information rent, ex ante firm value is*

$$\begin{aligned} V_R &\equiv 1 - q + aql(1-c) - aqu_0^*, \\ &= 1 - q + aql(1-c) - aq(1-q)(1-l)m^*, \end{aligned}$$

where

$$m^* \equiv \begin{cases} m_L & K \in [0, \underline{K}] \\ m_I & K \in [\underline{K}, \overline{K}] \end{cases}$$

To understand the limitations of relying on the market, it is useful to compare the expected firm value to the *first-best benchmark*, in which the firm has direct access to the same information that is available to an expert outsider, should he exist. In this case, the firm would correctly switch to the safe investment whenever the expert outsider exists and the economic state is bad, yielding ex ante firm value

$$V^* \equiv (1-a)(1-q) + aq(1-c) + a(1-q)\cdot 1 = 1 - q + aq(1-c),$$

and hence,

(18) $$\begin{aligned} V_R &= V^* - aq(1-l)(1-c) - aqu_0^* \\ &= V^* - aq(1-l)[(1-c) - m^*(1-q)] \end{aligned}$$

The preceding equations illustrate clearly the two weaknesses associated with market-reliance. First, the firm does not make first-best decisions. In particular, the firm fails to switch to the safe investment if an informed expert exists and the economic state is bad, but no liquidity shock occurs to fully reveal this fact. Second, in this very same state of nature, the type-0 expert makes trading gains at the expense of shareholders, as if he always short-sells m shares at a price $p = 1 - q$ versus a fundamental value of 0.

EQUILIBRIUM WITH NO INFORMATION RENT

We now analyze the possibility of an equilibrium in which the type-0 expert outsider expects no information rent, $u_0^* = 0$.

To begin, note that equations (3) and (6) imply that in such an equilibrium $\alpha(\vec{t}) = 1$ for all $t \in (0, 1]$. In particular, if $u_0^* = 0$, then each sell order t must generate zero expected profit, otherwise the type-0 expert would have a profitable deviation. Hence, for all $t > 0$, either $\chi(\vec{t}) = 1$, or $\alpha(\vec{t}) = 1$. But, from (1) it follows that $\chi(\vec{t}) = 1$ implies $\alpha(\vec{t}) = 1$. Therefore, a no-rent equilibrium exists if and only if it is possible to find a trading strategy $\Phi_0(t)$ such that the firm switching to the safe investment is consistent with (1) for all order vectors consisting of a single sell order.

To characterize the no-rent equilibrium, we derive conditions under which a single sell order—of any size—induces a switch to the safe investment. For this to be the case, the posterior following any single sell order must exceed the cost of switching to the safe investment:

(19) $$c \leq \frac{K\phi_0(t) + q}{K\phi_0(t) + 1}.$$

Therefore, if the firm switches to the safe investment in equilibrium for all sell orders, then for all $t \in (0, 1]$, the type-0 expert's trading density must satisfy:

(20) $$\phi_0(t) \geq \frac{J-1}{K}.$$

Of course, the trading density must also integrate to 1 on the unit interval, which would be impossible if $K < J - 1$. Conversely, if $K \geq \overline{K} = J - 1$, then many feasible mixing densities exist which satisfy the preceding equation. That is, a multiplicity of payoff equivalent no-rent equilibria exist if the market informativeness measure K is sufficiently high. Intuitively, high market informativeness can be understood as resulting from a sufficiently high probability (a) of the expert outsider existing and a sufficiently low probability (l) of a liquidity shock. Under such a parameter configuration, any sell order contains sufficient negative information to induce the firm to switch to the safe investment. But when the firm is on such a hair-trigger the type-0 expert can never trade profitably based upon his private information.

Consider now the ex ante value of the firm in a no-rent equilibrium, again using as a benchmark V^*, the first-best value, i.e. the value of a firm that observes all available information in the economy. To begin, note that since the expert outsider's payoff is zero in both economic states, no adverse selection discount exists in a no-rent equilibrium. That is, ex ante firm value is here equal to the ex ante expectation of realized cash flow.

Next, note that if the expert outsider exists, then the firm selects the risky investment if $\omega = 1$ and the safe investment if $\omega = 0$. That is, if the expert outsider does indeed exist, then the firm always selects the optimal investment for each economic state. However, the firm's decisions are not first-best. After all, in the absence of an informed expert, the only information available is the prior, and under the prior the risky investment is optimal. However, in a no-rent equilibrium, the firm is on a hair-trigger. In particular, if the expert outsider does not exist, but a liquidity shock arrives, then the firm will switch to the safe investment, a departure from the first-best. This scenario occurs with probability $l(1-a)$, reducing the firm's expected cash flow by $(1-q)-(1-c) = c-q$. Compared to the first-best benchmark, in the no-rent equilibrium the firm switches to the safe investment too often, resulting in lower firm value. It follows that:

$$\begin{aligned} V_{NR} &= V^* - l(1-a)(c-q) \\ &= 1 - q + l(1-c)(1-a)\Big(\frac{K}{1-l} - \overline{K}\Big). \end{aligned} \tag{21}$$

As another benchmark, we note that an uninformed firm that has no information other than the prior would always implement the risky investment, with expected cash flow equal to $1-q$. Since here $K \geq \overline{K}$, it follows from the second line above that the market delivers useful information to the firm, increasing firm value relative to the uninformed firm's expected cash flow.

The following proposition summarizes the analysis.

Proposition 3.11 *(High Informativeness). If market informativeness is high, $K \geq \overline{K} \equiv J-1$, then a multiplicity of equilibria in which $u_0^* = 0$ exists. If $K < \overline{K}$, then no such equilibrium exists. In any such equilibrium, $\phi_0(\cdot)$ satisfies condition (20), the firm switches to safe following any single sell order, $\alpha(\vec{t}) = 1$ for all $t \in (0,1]$, and expected firm value is*

$$V_{NR} = 1 - q + aq(1-c) - l(1-a)(c-q).$$

Beliefs and actions for all other possible order vectors are given in Lemma 3.2 and Remark 3.3.

Market-Reliant Firm Value. Figure 1 provides a summary of the equilibria in the market reliance case.

Note that the type-0 expert's minimum sell order size m and his information rent are continuous functions of K. To see this note that at the first threshold, $K = \underline{K}$, the minimum trade sizes are equal, $m_L = m_I = 1/J$. Note further that at the second threshold, $K = \overline{K}$, the minimum trade

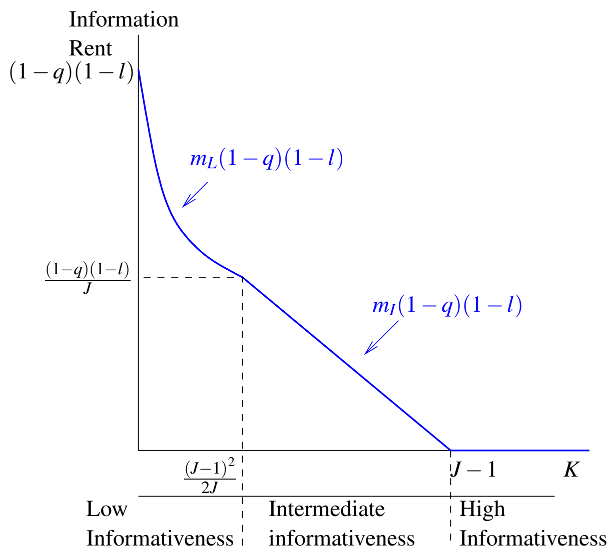

Figure 1: Information rent received by the type-0 expert.

size is zero, $m_I = 0$. Further, it is straightforward to show that m_L and m_I are decreasing functions of K. That is, the expert must trade less aggressively if market informativeness is higher.

Finally, for use later, we summarize the ex-ante value of the market-reliant firm as:

$$
V_{MKT} \equiv \begin{cases} V^* - aq(1-l)(1-c) - aqm_L(1-q)(1-l) & \text{if } K \in [0, \underline{K}] \\ V^* - aq(1-l)(1-c) - aqm_I(1-q)(1-l) & \text{if } K \in [\underline{K}, \overline{K}] \\ V^* - l(1-a)(c-q) & \text{if } K \in (\overline{K}, \infty) \end{cases} \tag{22}
$$

where m_L and m_I are defined in Propositions 3.8 and 3.9, respectively. Note that V_{MKT} is continuous in the underlying parameters.[9]

4 THE CASE FOR MECHANISMS: EXOGENOUS RESERVATION VALUE

This section presents the case for the direct revelation mechanism (DRM) as a device for eliciting private information from an informed agent, when the reservation value is exogenous. In particular, we show that given sufficient bonding capability, the firm can devise a mechanism in which the firm selects the first-best investment, while reducing the expert outsider's payoff to his reservation value. Notably, we show that the firm will be able to achieve this outcome despite the requirement that the firm allocates the mechanism to the first willing agent, and the firm's fiduciary responsibility to act optimally on its information (sequential rationality), both of which limit its commitment power.

Timing. If the firm offers a mechanism, then the game unfolds in the following sequence.

1. *Information State.* The expert's existence (information state) is realized.

[9]To see this note that m is continuous in K and that $K = \overline{K} \Rightarrow aq(1-l)(1-c) = l(1-a)(c-q)$.

2. *Mechanism Offered.* Without observing the expert's existence, the firm publicly offers a mechanism to the countably infinite pool of outsider agents. The mechanism specifies a vector of wages $w_{r\varphi}$, where $w_{r\varphi}$ is the advisor's wage following report $r \in \{0, 1\}$ and realized terminal cash flow $\varphi \in \{0, 1-c, 1\}$. Each outsider decides whether he would like to participate in the mechanism. The mechanism is assigned to the first outsider agent who indicates that he is willing to participate, thereby becoming the firm's advisor, who is prohibited from trading. Acceptance of the mechanism is publicly observed.

3. *Economic State.* The economic state is realized and is privately observed by the expert if he exists.

4. *Reporting.* If any agent agreed to participate, then this advisor privately issues a report of the economic state to the firm. If no agent agreed to participate, then this step is skipped.

5. *Liquidity Shock.* The liquidity shock is realized.

6. *Market.* Orders are anonymously submitted to the market. The market maker and firm observe the countably infinite order vector.

7. *Decision.* The firm chooses an investment, S or R.

8. *Cash flows.* The firm's cash flows are revealed. If the firm hired an advisor, then it makes transfers as specified by the mechanism.

Note that the sequence of events is identical to the game played under market-reliance, except that here a mechanism is publicly posted and accepted/rejected prior to trading in the securities market. Furthermore, these timing assumptions are conservative in the sense of putting the mechanism in a better light. In particular, as we discuss in our model extension section, the mechanism would actually become infeasible if the expert was instead permitted to make his participation decision after observing the economic state. Intuitively, in the bad economic state, the expert could make a rent by deviating, rejecting the mechanism, and trading. To induce expert participation, he must receive a positive wage for reporting negative information. But this would induce incompetents to sign on.

The analysis in this section proceeds as follows. We first derive a set of constraints that are *necessary* for a mechanism to achieve higher ex ante firm value than can be achieved under market-reliance. We next characterize conditions under which it it feasible to satisfy these necessary conditions, and then solve for the optimal mechanism(s) among those satisfying the necessary conditions. In the next section, we derive conditions under which the mechanism dominates the market.

The expert's reservation value is directly relevant for the firm's mechanism design problem. Let $\underline{u}$ represent the expert's continuation payoff from rejecting the posted mechanism. In this section, we treat $\underline{u}$ as exogenous and assume that $\underline{u} > qu_0^*$. In the next section, we derive the expert's endogenous continuation value, showing that $\underline{u}$ is the same for all mechanisms that beat market-reliance and that it satisfies the assumed inequality. In this next section, we will also show that an uninformed outsider's continuation payoff from rejecting the firm's mechanism offer is zero, which we take as given in this section.

Recall, fiduciary duty requires that the firm's behavior is sequentially rational. Since the firm cannot commit to future actions, the Revelation Principle does not apply directly. However, we establish an analogous result in Lemmas 4.1 and 4.3. Some formalities are first necessary. To this end, let χ_r be the firm's belief that the state is $\omega = 0$ following report $r \in \{0, 1\}$. As a normalization, let us label the reports so that $\chi_1 \leq q \leq \chi_0$.[10] Let ρ_r be the probability that the firm selects the risky investment following report r. Let $x \in \{0, 1\}$ denote the expert's participation decision, where $x = 1$ represents the decision to participate.[11] Let γ_ω be the probability that the advisor sends report $r = 1$ in economic state ω. Finally, let d be the probability that each uninformed outsider agrees to participate in the mechanism.

Any mechanism that outperforms market-reliance must have certain properties. First, any such mechanism must be rejected by the uninformed outsiders and accepted by the expert outsider if he exists. After all, the uninformed outsiders are countably infinite, and the mechanism is assigned to the first willing agent. Thus, a mechanism that does not screen out the uninformed will almost surely be accepted by an uninformed outsider, and hence, cannot deliver useful information about the economic state. The firm would therefore watch the market for information, and both the firm and the market maker anticipate that the expert will be active in the market if he exists. Thus, offering a mechanism that fails to screen out incompetents cannot do better than market-reliance. Following the same logic, any mechanism that fails to induce participation by the expert also cannot do better than market-reliance. We have the following lemma.

Lemma 4.1 *(Screening). If a mechanism delivers higher ex ante firm value than market-reliance, then it must screen out uninformed agents and induce participation by the expert if he exists, $d = 0$ and $x = 1$.*

Second, any mechanism that achieves higher ex ante firm value than market-reliance has the property that it grants the expert real authority in the sense that $\rho_1 = 1$ and $\rho_0 = 0$. That is, in equilibrium the firm will follow the "recommendation" of its agent, implementing the risky investment with probability 1 (0) in response to report $r = 1$ ($r = 0$). To see why this must be the case, recall first that report-contingent beliefs are such that $\chi_1 \leq q \leq \chi_0$. Sequential rationality therefore demands $\rho_1 = 1$. Consider next why any mechanism that is value-increasing relative to market-reliance must satisfy $\rho_0 = 0$. To begin, note that any mechanism that induces participation by the expert, as is necessary, features an expected wage bill no less than $a\underline{u} > aqu_0^*$. This exceeds the adverse selection cost under market-reliance. Therefore, any value-increasing mechanism must

[10]The Law of Iterated Expectations requires $\Pr(r = 0)\chi_0 + \Pr(r = 1)\chi_1 = \Pr(\omega = 0) = q$. Therefore one posterior belief must be weakly smaller than the prior and the other weakly larger.

[11]For brevity, we abstract from mixing by the expert in his participation decision in this section. Section 6 considers an extension that is formally equivalent to a setting in which the expert mixes in the participation decision.

lead to a strict increase in expected cash flow relative to market-reliance. With this in mind consider expected cash flow under a mechanism featuring $\rho_0 \in (0,1]$. If $\rho_0 = 1$, the firm always implements the risky investment, and expected cash flow is $1-q$, which is less than the expected cash flow under market-reliance, as shown in the preceding section. If instead $\rho_0 \in (0,1)$, sequential rationality requires the firm to be indifferent between S and R following $r = 0$. But note, mixing implies expected cash flow is the same as if the firm were to always choose the risky investment. To see this formally, note that the firm is willing to mix only if $\chi_0 = c$, and hence:

$$\begin{aligned} E[\varphi] &= \Pr(r=0)[\rho_0(1-\chi_0) + (1-\rho_0)(1-c)] + \Pr(r=1)(1-\chi_1) \\ &= \Pr(r=0)[\rho_0(1-\chi_0) + (1-\rho_0)(1-\chi_0)] + \Pr(r=1)(1-\chi_1) \\ &= \Pr(r=0)(1-\chi_0) + \Pr(r=1)(1-\chi_1) = 1-q. \end{aligned}$$

The last line above follows from the Law of Iterated Expectations. We thus have the following lemma.

Lemma 4.2 *(Delegated Decision). If a mechanism delivers higher ex ante firm value than market-reliance, then it must delegate the decision to the expert, $\rho_0 = 0$ and $\rho_1 = 1$.*

Third, any mechanism that achieves higher ex ante value than under market reliance induces the expert to report truthfully with probability 1. After all, if it is sequentially rational for the firm to follow the expert's advice, with $\rho_0 = 0$ and $\rho_1 = 1$ (see preceding lemma) then it must be that $\chi_0 \geq c > q \geq \chi_1$. Therefore, it must be that the expert tells the truth with positive probability (i.e. he cannot strictly prefer to lie): $\gamma_1 > 0$ and $\gamma_0 < 1$. These conditions imply two constraints on wages. First, to ensure $\gamma_1 > 0$, it must be that $w_{11} \geq w_{01-c}$. Second, to ensure $\gamma_0 < 1$, it must be that $w_{01-c} \geq w_{10}$. Furthermore, as we show in the Appendix, in any mechanism that delivers a higher payoff than market reliance (consistent with Lemma 4.1 and 4.2), these two constraints on wages hold with strict inequality, and so the expert strictly prefers to report truthfully.

Lemma 4.3 *(Truthful Reporting). If a mechanism screens out uninformed agents and induces participation by the expert (as in Lemma 4.1) and delegates the decision to the expert (as in Lemma 4.2), then the expert's unique sequentially rational strategy is to report truthfully with probability 1, $\gamma_0 = 0$ and $\gamma_1 = 1$.*

To understand the preceding lemma, recall that the mechanism must screen out the uninformed and induce expert participation. This has implications for the wage structure. To screen out the uninformed reporting state 0, the firm must not reward a report of state 0. Thus, in order to induce participation by the expert, the firm must reward a correct report of state 1. But then to screen out the uniformed reporting state one, an incorrect report of state 1 must result in a negative wage. If the expert lies in state 1, reporting $r = 0$, he does not receive a positive wage, which is suboptimal. If the expert lies in state 0, he is sure to receive a punishment, which is clearly suboptimal.

Lemma 4.2 allows us to focus on mechanisms in which the advisor's wage depends only on the firm's terminal cash flow, not on his report. To see this, note that if the advisor reports $r = 1$, the firm implements the risky investment with probability 1 which implies wage w_{11-c} is irrelevant. Similarly, if the advisor reports $r = 0$, the firm implements the safe investment which implies wages w_{00} and w_{01} are irrelevant. We thus need only focus on wages $\{w_{01-c}, w_{11}, w_{10}\}$, which can be written as a function only of the realized cash flow. Therefore, in what follows we drop the first subscript (the report) from the agent's wage.

Lemmas 4.1-4.3, along with the agents' liability constraint, imply that any mechanism delivering higher ex ante firm value than market reliance must satisfy the following system of constraints.

$$w_{1-c} \leq 0 \qquad \text{(SC0)}$$
$$qw_0 + (1-q)w_1 \leq 0 \qquad \text{(SC1)}$$
$$qw_{1-c} + (1-q)w_1 \geq \underline{u} \qquad \text{(PC)}$$
$$w_i \geq -B \quad \forall\ i \in \{0, 1, 1-c\}. \qquad \text{(BOND)}$$

Constraint (SC0) ensures that an uninformed outsider prefers to reject the mechanism, rather than accept and report $r = 0$. Similarly, (SC1) rules out an uninformed agent participating and reporting $r = 1$.[12] Constraint (PC) ensures that the expert outsider is willing to participate in the mechanism if he indeed exists, anticipating that he will report the state truthfully; from Lemma 4.3 we know that the expert's only sequentially rational strategy is truthful reporting (with probability 1) in any mechanism that delivers a higher payoff than market-reliance. The constraints in (BOND) reflect the expert's limited liability. We refer to the set of constraints as $\mathcal{S}$. Because (SC0), (SC1), and (PC) are imposed by the mechanism's need to screen out uninformed outsiders and screen in the expert, we refer to $\mathcal{S}$ as the *screening constraints*. If the screening constraints are mutually consistent, we say that *screening is feasible*, and we refer to a mechanism that satisfies $\mathcal{S}$ as a *feasible* mechanism.[13]

The screening constraints demonstrate an important tension between a mechanism's ability to attract the expert and its ability to screen out uninformed outsiders. In order to meet the expert's participation constraint, the firm needs to ensure that a particular linear combination of w_{1-c} and w_1 is sufficiently large. However, increasing w_{1-c} makes it more attractive for an uninformed agent to accept and report $\omega = 0$, while increasing w_1 makes it more attractive for an uninformed agent to accept and report $\omega = 1$. The temptation for an uninformed agent to report $\omega = 1$ can be offset by reducing w_0, thereby generating a punishment for incorrectly reporting that the state is good. However, the firm's ability to punish is restricted by the agent's limited liability, suggesting that screening is not always feasible. This intuition is confirmed by the following proposition.

Proposition 4.4 *(Feasible Screening). If $\underline{u} > qB$, then screening is infeasible and every mecha-*

[12] (SC0) and (SC1) also ensure that an uninformed agent would rather reject than accept and then report randomly.

[13] Note we define feasibility as existence of a mechanism which potentially delivers ex ante value in excess of market-reliance.

nism does no better than market-reliance.

We now find the optimal mechanism assuming liability is large enough that screening is feasible. The firm's objective is to maximize the ex ante value of a share (or equivalently, total firm value) subject to $\mathcal{S}$. In any feasible mechanism, ex ante firm value is

$$(1-a)(1-q) + a[(1-q) + q(1-c)] - a[qw_{1-c} + (1-q)w_1].$$

The first term reflects the fact that if no expert exists, the firm will implement the risky investment, with expected cash flow $1-q$. The second term is the firm's expected cash flow if the expert exists, with Lemmas 4.2 and 4.3 informing us that any feasible mechanism has the property that the firm selects the correct investment in each economic state if the expert exists. The final term is the expected wage bill.[14]

It is worth noting that in contrast to market-reliance, under mechanism-reliance no adverse selection cost is capitalized into the ex ante share value. After all, on the equilibrium path the expert always signs on as the firm's advisor if he exists and is then prohibited from trading in the securities market. With no informed trading, shareholders hit with a liquidity shock are able to sell at fundamental value. In particular, the market maker will set the stock price equal to the true expected cash flow. In turn, expected cash flow is $(1-q)1 + q(1-c)$ if the mechanism offer is accepted and is $1-q$ if the mechanism offer is left sitting.

The next proposition gives a simple condition for the existence of a feasible mechanism and characterizes an optimal mechanism.

Proposition 4.5 *(Optimality). If $\underline{u} \leq qB$, then a feasible mechanism is optimal if and only if (PC) holds with equality. In every optimal mechanism, project selection is first best and ex ante firm value is*

$$\begin{aligned} V_{DRM} &= (1-a)(1-q) + a[(1-q) + q(1-c) - \underline{u}] \\ &= 1 - q + aq(1-c) - a\underline{u} \\ &= V^* - a\underline{u}. \qquad (23) \end{aligned}$$

The following mechanism is feasible and optimal whenever $\underline{u} \leq qB$:

$$(w_0, w_{1-c}, w_1) = \left(-B, 0, \frac{\underline{u}}{1-q}\right).$$

The feasible and optimal mechanism presented in Proposition 4.5 is intuitive. First, since an uninformed outsider can hide ignorance by always reporting $\omega = 0$, thereby inducing the firm to implement the safe investment, the optimal mechanism offers a wage payment of zero if the adviser recommends this course of action. The optimal mechanism also features maximum punishment,

[14] Note that Lemmas 4.2 and 4.3 imply that the firm always selects the correct action in each state whenever the expert exists.

with $w_0 = -B$, since a wrong report reveals the advisor to be uninformed. The wage w_1 is set so that the expert outsider's participation constraint is just binding. Finally, under this mechanism the expert reports truthfully with probability 1, and thus it is sequentially rational for the firm to follow the expert's advice. This mechanism has a natural interpretation as a compensation contract which penalizes the advisor for a low cash flow, neither rewards nor punishes for a moderate cash flow, and rewards for high cash flow.

Notice that the proposed mechanism (as well as every optimal mechanism) delivers the firm the same payoff as a "buyout" by the expert. To see this, note that if the expert owned the entire firm, then he would always select the optimal investment in each economic state. However, by acquiring the firm, the expert would forego his outside option, worth $\underline{u}$. Accounting for the fact that the expert may not exist, the expected sales price of the firm would then be $V^* - a\underline{u}$. Thus, the firm's ex ante value in any optimal mechanism is *as if* every shareholder sells his share to the expert, should he exist, at the largest price that the expert is willing to pay. However, in terms of practical feasibility, the proposed mechanism offers a significant advantage. In particular, with the mechanism, the expert does not need to have wealth sufficient to purchase the entire firm. After all, notwithstanding the fact that the value of assets in place has here been normalized at zero, for most corporations this value is by itself sufficiently large to render the outright purchase of the entire firm prohibitively costly for most potential buyers. In contrast, the proposed mechanism imposes the smallest possible bonding requirement.

The optimal mechanisms are extremely powerful—provided the expert has sufficient bonding capability. Any optimal mechanism allows the firm to implement the first-best, eliminates adverse selection costs, and pins the expert to his outside option value, resulting in ex ante firm value given by (23). The same outcome would arise in an economy without private information, where the firm directly observes whether or not the expert exists, hires the expert just by meeting his outside option, and directly observes the hired expert's signal of the economic state.

5 MARKETS VS. MECHANISMS: ENDOGENOUS RESERVATION VALUE

This section determines conditions under which the firm optimally refrains from posting a mechanism, relying instead on the market to guide its decision. The firm may choose not to offer the mechanism for two distinct reasons. First, it could be that screening is infeasible. Second, it could be that screening is feasible, but ex ante firm value is strictly higher under market-reliance. Both possibilities are analyzed below.

Recall, in the previous section we derived a set of necessary conditions for a mechanism to outperform the market, found a necessary and sufficient condition for these conditions to be mutually consistent, and characterized an optimal mechanism. Critically, the analysis of the preceding section followed the standard mechanism design approach in treating as exogenous $\underline{u}$, the expert's reservation value for participating in the mechanism. The next subsection endogenously determines $\underline{u}$ within a securities market economy. This analysis will allow us to assess the feasibility and optimality of mechanism-reliance.

5.1 ENDOGENOUS RESERVATION VALUE

The expert's reservation value for participating in the mechanism is equal to the expected trading gain he would capture if he were to reject the posted mechanism. This expectation depends upon the nature of price formation after a mechanism has been posted and left sitting. Critically, our analysis of price formation under market-reliance is no longer applicable. The discussion that follows pertains to price formation after a mechanism has been posted and left sitting.

Consider first trading patterns and price formation on the equilibrium path after a mechanism has been posted and left sitting. To begin, we recall that an optimal mechanism has the property that the expert will take up the mechanism in equilibrium if he exists (see Lemma 4.1). Thus, on-path, the expert will not trade in the securities market. We further conjecture, and then verify, that in equilibrium no uninformed outsider has an incentive to trade after a posted mechanism has been left sitting. Thus, on the equilibrium path, after a mechanism has been posted and left sitting, the only possible market participants are the firm's shareholders.

It follows from the preceding discussion that, after a posted mechanism has been left sitting, the only on-path order vectors are $\vec{0}$ and $\vec{t}$. In either case, the order vector is uninformative about the economic state so the firm will implement the risky investment, which is optimal under the prior, and the market maker will set the stock price at $p = 1 - q$. Notice, in stark contrast to price formation under market-reliance, under mechanism-reliance the market maker will set the same stock price in response to the arrival of a single sell order–*regardless of its size.* Thus, the act of posting a mechanism fundamentally alters the nature of beliefs and price formation relative to those under market-reliance.

When the mechanism has been posted and left sitting, the arrival of two sell orders is an off-path event, as is the arrival of a buy order. The beliefs formed in response to such off-path events are relevant since they determine the expert's reservation value ($\underline{u}$) as well as the incentives and reservation value of uninformed outsiders. For the mechanism-reliant firm, we adopt the following simple convention for assigning beliefs to all off-path order vectors, identical in form to those specified under market-reliance.

Remark 5.1 *(Mechanism-Reliance: Off-path Beliefs and Actions). After a posted optimal mechanism has been left sitting, consider an off-path order vector, T. If the number of buy orders is greater than or equal to the number of sell orders, then T reveals state 1, $\chi(T) = 0$, and the firm selects the risky investment, $\alpha(T) = 0$. Otherwise, T reveals state 0, $\chi(T) = 1$, and the firm selects the safe investment, $\alpha(T) = 1$.*

The preceding specification of off-path beliefs is adopted in order to minimize $\underline{u}$, giving the mechanism the best possible chance to beat the market. To see that these beliefs ensure $\underline{u}$ is minimized, consider a unilateral deviation by the expert, rejection of a posted mechanism and subsequent trading on private information. If he attempts to buy based upon positive information two possible order vector possibilities arise, both off-path: one buy order in isolation or one buy order and one sell order. Under the specified off-path beliefs, the expert makes zero profit from such activity since in either case the market maker infers that the state is $\omega = 1$. If the expert attempts to sell based upon negative information he creates two possible order vector possibilities, the former being on-path and the latter off-path: one sell order or two sell orders. In order to minimize $\underline{u}$, we assume the market maker treats the latter off-path order vector as revealing $\omega = 0$.

The preceding specification of off-path beliefs also implies that each uninformed outsider has a preference to remain inactive after a mechanism has been posted and left sitting, and that their reservation value for participation in the mechanism is zero, consistent with our conjectures above. To see this, consider a unilateral deviation by an uninformed outsider. If he places a buy order, he creates two possible order vector possibilities, both off-path: one buy order in isolation or one buy order and one sell order. The uninformed outsider placing a buy order would make an expected loss since the market maker infers $\omega = 1$ and sets $p = 1$ whereas the fundamenal value is only $1 - q$. If an uninformed outsider instead places a sell order he creates two possible order vector possibilities, the former being on-path and the latter off-path: one sell order or two sell orders. In the former case, the firm will implement the risky investment and the market maker sets $p = 1 - q$. In the latter case, the firm implements the safe investment and the market maker sets $p = 1 - c$. In both cases, price is just equal to expected cash flow. Thus, an uninformed outsider placing a sell order would make zero profit.

In light of the preceding discussion consider the optimal strategy of an expert who deviates by rejecting the mechanism. If $\omega = 1$, there is zero profit to be made from any buy order, because a buy order is interpreted as confirmation of $\omega = 1$ and is cleared at price 1. However, if $\omega = 0$, the expert earns maximal expected profits by selling the maximal feasible amount, $t = 1$. After all, if a liquidity shock arrives, then the market maker and firm will infer that $\omega = 0$, the firm will switch to the safe investment, price will be set at fundamental value $(1-c)$, leaving the expert with zero profit. However, if no liquidity shock arrives, then the secondary market price will be set at $p = 1 - q$ for any feasible sell order $t \leq 1$. Thus, if the expert were to deviate by rejecting a posted optimal mechanism, his maximal expected trading profit in state $\omega = 0$ is equal to $(1 - q)(1 - l)$. We thus have the following lemma.

Lemma 5.2 *(Reservation Values). If a mechanism satisfying the screening constraints is posted, then an uninformed outsider's reservation value is 0, and the expert's reservation value is*

$$\underline{u} = q(1 - q)(1 - l). \tag{24}$$

It is readily verified that the expert's endogenous reservation value for participating in the mechanism exceeds the reservation value that would be posited if one were to ignore the effect that posting a mechanism has on trading opportunities. That is, the true reservation value for participating in the mechanism exceeds the trading gain the expert stands to capture in a market when no mechanism has been posted (qu_0^*). In particular:

$$\underline{u} = q \cdot 1(1 - q)(1 - l) > q \cdot m(1 - q)(1 - l) = qu_0^*. \tag{25}$$

By posting a mechanism that screens for expertise, the firm allows the expert to make large trading gains in the market, by giving him an opportunity to convince the market maker and firm that he doesn't exist. In contrast, when no mechanism is posted, the expert is constrained to trade less aggressively, e.g. selling $t \in [m, 1]$ units due to concern over negative price impact. Notice, the preceding inequality also informs us that the expected wage bill under the optimal mechanism ($\underline{u}$) indeed exceeds the adverse selection selection discount under market-reliance (qu_0^*), the value assumed in our analysis of the optimal mechanism in the preceding section.

5.2 MARKETS VS. MECHANISMS: FEASIBILITY

Recall from Proposition 4.4 that screening is feasible only if $qB \geq \underline{u}$. The next proposition follows immediately from (24).

Proposition 5.3 *(Markets vs. Mechanisms I). In equilibrium, screening is feasible only if the expert has bonding capability:*

$$B \geq (1-q)(1-l). \tag{26}$$

Otherwise, the firm offers no mechanism and relies exclusively on the market for information.

To appreciate the significance of the preceding result, suppose outsiders have bonding capability B such that:

$$\underbrace{q(1-q)(1-l)}_{\underline{u}} > qB > \underbrace{qm(1-q)(1-l)}_{qu_0^*}. \tag{27}$$

In this case, with $B > u_0^*$, the feasible set for the mechanism design problem would appear to be non-empty if one were to naively posit that the expert's reservation value is not affected by the posting of the mechanism. However, with an endogenous reservation value, the feasible set is empty due to the increase in (off-equilibrium) informed trading gains that arise from the posting of the mechanism.

In other words, the existence of stock market trading opportunities generates a link between the expert's decision to participate in the mechanism and his reservation value, and this link can cause an otherwise optimal mechanism to become infeasible. Intuitively, if a mechanism is posted, an informed investor can capture especially large trading gains by deviating and rejecting the mechanism. In order to counter this strong temptation to deviate, the mechanism-reliant firm must offer a large reward for correct advice (w_1). But in so doing, the firm also increases the temptation of incompetents to take up the mechanism. If the trading gains following a deviation become sufficiently high (equation 24), accomplishing the dual tasks of eliciting expert participation in the mechanism and the screening out of incompetents becomes impossible.

5.3 MARKETS VS. MECHANISMS: OPTIMALITY

The remainder of the analysis assumes that the bonding capability of the outsider agents (B) is sufficiently large to ensure that screening is feasible.

Combining (23) and (24), we find that the ex ante share price of a firm posting a mechanism is

$$V_{DRM} = V^* - aq(1-q)(1-l), \tag{28}$$

while equation (22) gives the ex ante share price of a market-reliant firm. Therefore, we turn next to a case-by-case comparison of the ex ante stock prices attained by mechanism-reliant versus market-reliant firms.

No-Rent Equilibrium. When the market informativeness parameter is sufficiently high relative to the switching cost, specifically when $K > \overline{K} = J - 1$, the equilibrium under market-reliance entails no rent for the expert outsider. Further, in equilibrium, the firm switches to the safe investment if a single sell order arrives without an accompanying buy order. Equation (22) implies the ex ante value of the firm in this equilibrium is

$$V_{MKT} = V_{NR} = V^* - l(1-a)(c-q).$$

Comparing the expressions for firm value, we find that

$$\begin{aligned} V_{MKT} \geq V_{DRM} &\iff l(1-a)(c-q) \leq aq(1-q)(1-l) \\ &\iff \frac{J-1}{J} \leq K \end{aligned} \tag{29}$$

The preceding inequality illustrates the fundamental tradeoff between the market and mechanism when the market informativeness measure is high. Specifically, the right side of the first line of (29) captures the expected wage bill for a firm posting a mechanism, which reflects the high reservation value of an expert outsider. The left side of the inequality captures the cost of relatively less efficient production of a market-reliant firm, which mistakenly switches to the safe investment in response to an uninformative sell order generated by a liquidity shock. Since here $K > J - 1$ and $J > 1$, the second line of (29) is necessarily satisfied. Thus, when parameters are such that the no-rent equilibrium obtains, ex ante firm value is always higher under market-reliance, $V_{MKT} > V_{DRM}$. Thus, even if screening for expertise using a mechanism is feasible, doing so is suboptimal in the case of the no-rent equilibrium.

Proposition 5.4 *(Markets vs. Mechanisms II: High Informativeness). If $K > \overline{K}$, then the ex ante value of a market-reliant firm is strictly larger than the ex ante value of a mechanism-reliant firm.*

Before proceeding to the cases of intermediate and low K, it is worth discussing why, at an intuitive level, the market looks especially attractive relative to the mechanism when K is high.

Recall, that K is high when there is a high probability (a) of an expert outsider existing and/or a low probability of a liquidity shock (l). In this case, the firm's investment decision under market-reliance is efficient, except that it incorrectly switches to the safe investment in the event of a liquidity shock combined with non-existence of the expert. But this inefficient investment decision only occurs with probability $l(1-a)$, which is low when a is high or l is low (i.e., when K is high). Thus, the first advantage of the mechanism, better information about the economic state, is not large. Furthermore, when K is large, the expert's trading profit under market-reliance (qu_0^*) is equal to zero and so there is zero adverse selection cost under market-reliance. Phrased differently, the *endogenous increase in the reservation value generated by the mechanism posting is largest in this case*. Thus, the cost of posting the mechanism is as large as possible, and the slightly better information provided by the mechanism is not enough to overcome it. Indeed, we can interpret (29) as:

$$V_{MKT} \geq V_{DRM} \iff \text{ Cash Flow Loss } \leq \underline{u} - qu_0^* = \text{ Change in Reservation Value } .$$

As described above, the left hand side of the preceding inequality is small, and the right hand side is large when K is large.

Equilibrium With Information Rent. We now turn to the case of $K < \overline{K}$, implying the market-reliant equilibrium delivers the expert an information rent. From equation (22), the expected value of the market reliant firm is equal to

$$V_{MKT} = V_R = V^* - aq(1-l)(1-c) - aq(1-q)(1-l)m^*,$$

where m^* is the minimum trade size in the support of the type-0 expert's equilibrium mixed strategy. Comparing the expressions for ex ante firm value, we find that

$$V_{MKT} \geq V_{DRM} \iff aq(1-l)(1-c) \leq aq(1-l)(1-q)(1-m^*). \tag{30}$$

The preceding equation again reveals the fundamental tradeoff between markets and mechanisms, a tradeoff between investment efficiency and relative implementation costs. The left side of the equation reflects the fact that even if the investor is informed, the market-reliant firm incorrectly fails to switch to the safe investment in the bad state absent a fully revealing liquidity shock, with the output loss equal to $1-c$. The right side of the equation reflects the difference in relative implementation costs. Specifically, the type-0 expert is forced to trade less aggressively if the firm does not post a mechanism, with his minimum sell size equal to m shares. In contrast, if a mechanism is posted, a type-0 expert anticipates the possibility of being able to deviate and trade aggressively, selling one share with zero price impact. As argued above, this deviation gain represents the informed investor's opportunity cost of participating in the mechanism and is reflected in the expected wage bill for the mechanism-reliant firm. Thus, the right hand side again represents the change in the reservation value resulting from posting a mechanism.

Dividing both sides by the expected wage bill under mechanism-reliance, $aq(1-q)(1-l)$, we find that

$$V_{MKT} \geq V_{DRM} \iff \frac{1}{J} \leq 1 - m^*(K). \tag{31}$$

Here the right hand side is the change in the reservation value and the left hand side is the loss of cash flow from market-reliance, both normalized by the wage. Recall that m^* is a continuous and decreasing function of K, satisfying $m(0) = 1$ and $m(\overline{K}) = 0$. The next result follows immediately.

Proposition 5.5 *(Markets vs. Mechanisms III: Low and Intermediate Informativeness). Suppose $K < \overline{K}$. For each J, there exists a threshold for market informativeness $\widehat{K}(J) < \overline{K}$ such that the ex ante value of the market-reliant firm is strictly larger than the ex ante value of a mechanism-reliant firm if and only if $K > \widehat{K}(J)$. Furthermore, $\widehat{K}(J)$ is decreasing in J.*

Recall that the market informativeness parameter K is the likelihood ratio that a sell order (cum zero order) originates with type-0 expert as opposed to arising from the liquidity shock. A larger value of K increases the attractiveness of market-reliance since trades by the type-0 expert then reveal more information, reducing adverse selection costs to the atomistic shareholders. Proposition 5.5 further indicates that, as the switching cost parameter J increases, the region of K values for which the market-reliant firm value exceeds the mechanism reliant firm value expands. Recall that in the intermediate and low informativeness cases, the market-reliant firm does not switch to the safe investment often enough relative to the mechanism-reliant firm. In particular, if the expert exists, the economic state is bad, and no liquidity shock arrives, the mechanism-reliant firm switches to the safe investment while the market-reliant firm either does not switch at all, or mixes. Because the firm would only mix if it's payoff from safe and risky investment is identical, its payoff is the same as if it always played risky. Thus, by not switching to the safe investment, the firm's expected cash flow is 0 in the bad state, instead of $1 - c$. Parameter J is high precisely when the probability of the bad state is low and when the switching cost is high. In this case, the expected cash flow loss from not switching is relatively small.

Surprisingly, market reliance can be optimal in a wide variety of circumstances. Proposition 5.5 shows that even when incorrect decisions result in large reductions in expected cash flow, market-reliance is still optimal if the endogenous change in reservation value is sufficiently high. In turn, the change in the reservation value is high when trades are informative and the market has low adverse selection costs. Conversely, even in markets plagued by adverse selection, market-reliance is optimal if expected cash flow under market-reliance is not too much lower than under mechanism-reliance.

Figure 2 summarizes our discussion of the trade-off between mechanism and market-reliance. Recall equation (31) normalized differences in expected cash flow and implementation costs by the expected wage bill under the mechanism. We found that the tradeoff between market-reliance and mechanism reliance depends on the relationship between the normalized difference in expected cash flows, $1/J$, and the normalized change in the expert's reservation value, $1 - m^*$. The left panel in Figure 2 shows that the expected wage bill under the mechanism exceeds the expected adverse

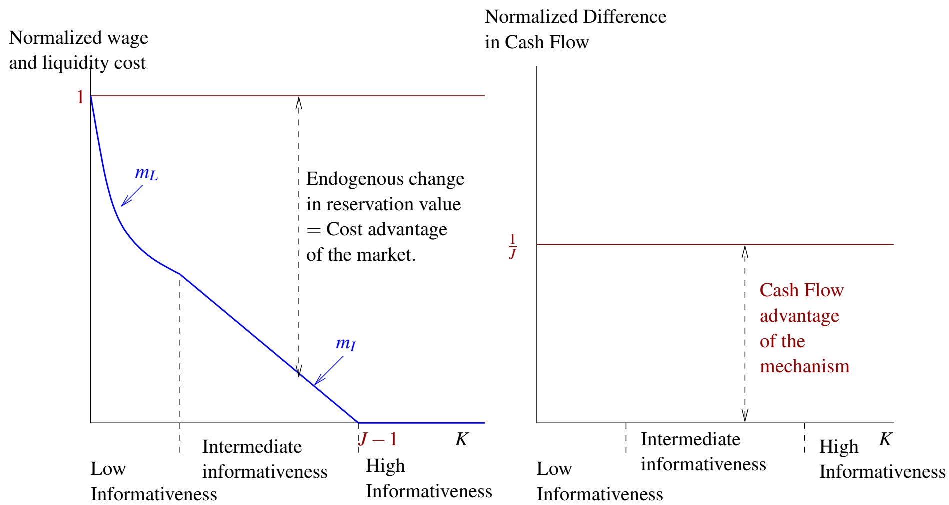

Figure 2: Left panel: Normalized expected reservation wage of the mechanism and adverse selection cost of market reliance. Right panel: normalized difference in expected cash flow between the mechanism and market-reliance.

selection cost of market-reliance. The difference between the two lines is the endogenous increase in the reservation wage arising from posting the mechanism. The right panel is the normalized difference in expected cash flows. The mechanism delivers higher cash flows. As K increases, expected cash flows under both mechanism and market reliance increase, but the normalized difference is constant. As shown in the figure, for K sufficiently small the mechanism delivers greater firm value, and for K sufficiently close to $J-1$, market-reliance delivers greater firm value.

6 SEARCH FRICTION

In the preceding sections we revealed a weakness of mechanisms as a source of information in an economy with a securities market: in order to elicit useful advice, a mechanism must induce an expert to participate with probability one if he exists, but such a mechanism results in a high endogenous reservation value for the expert. Indeed, rejecting such a mechanism allows the expert to trade in the market with no price impact. This line of reasoning suggests that it might be possible to improve upon the original mechanism by designing an alternative mechanism that leaves some doubt regarding the expert's existence, if no one accepts the contract.

Suppose now that the firm has the ability to introduce a search friction which limits the expert's ability to observe the posted mechanism. In particular, suppose the firm can determine the probability $\pi \in [0,1]$ that the contract will be observed by the expert if he exists, with the firm's choice being common knowledge.[15] For example, by advertising the mechanism less widely or for less time, the firm can reduce the probability the expert will see it. Notice, this technology subsumes

[15]The model in this section is formally equivalent to the setting of the preceding sections, except that the expert plays a mixed strategy in his acceptance decision. That is, if the expert is indifferent between accepting and rejecting, he accepts with probability $\pi \in (0,1)$.

the market-reliant and mechanism-reliant firm as special cases in which, respectively, $\pi = 0$ and $\pi = 1$. That is, in our previous analysis, the firm's only choices were $\pi \in \{0, 1\}$, and we now allow the firm to choose a value of π between these two extremes. To highlight this difference, we refer to $\pi = 0$ and $\pi = 1$ as "pure market-reliance" and "pure mechanism-reliance", respectively.

As before, the market maker can observe whether the mechanism is accepted. However, in contrast to the preceding analysis, if the firm offers a mechanism and this mechanism has been left sitting, the market maker cannot infer that the expert does not exist. After all, there is a positive probability that the expert did not see the mechanism offer.

Anticipating, we will show that the firm *always* prefers to add a search friction: pure mechanism-reliance ($\pi = 1$) can always be improved upon by introducing a search friction ($\pi < 1$).

Mechanism Design. To begin, note that the choice of π affects the market maker's belief about the expert's existence if the mechanism is not accepted, which is described in more detail below. Therefore, the choice of π can change the pricing rule used by the market maker if the mechanism is not accepted, which, in turn, affects the expert's reservation value for participating in the mechanism. Therefore, we now express the expert's reservation value for participating in the mechanism as $\underline{u}(\pi)$.

Given the reservation value $\underline{u}(\pi)$, the mechanism design problem facing the firm is identical in form to the one considered in Section 4. In particular, Lemmas 4.1, 4.2, and 4.3 are not affected by the firm's choice of π.[16] Because the choice of π only affects the expert's reservation value, we can apply the results of Section 4 to draw three conclusions. First, a mechanism that does better than pure market-reliance ($\pi = 0$) must simultaneously screen out incompetents and induce the expert to accept if he observes the offer, thereby inducing truthful reporting of the economic state. Second, a mechanism satisfying $\mathcal{S}$ exists if and only if $qB \geq \underline{u}(\pi)$. A sufficiently large value of B implies that screening is feasible for all possible π, and we maintain this assumption throughout this section. Third, any feasible mechanism that also satisfies the expert's participation constraint with equality maximizes ex ante firm value subject to $\mathcal{S}$. Thus, if the firm offers such a mechanism *and it is accepted*, then the firm expects to make the correct production decision in each economic state, while paying the expert an expected wage equal to $\underline{u}(\pi)$. Therefore, the expected value of the firm, conditional upon the mechanism offer being accepted is

$$V_{ACC}(\pi) = (1 - q)1 + q(1 - c) - \underline{u}(\pi). \tag{32}$$

Securities Market. We next turn to an analysis of outcomes if the posted mechanism is not accepted. Anticipating, if the mechanism is not accepted, the firm essentially becomes market-reliant. However, our analysis of equilibrium outcomes under pure market-reliance ($\pi = 0$), as presented in Section 3, must be modified to reflect that, when the mechanism is not accepted, the market maker and firm will revise downward their probability assessment of the expert's existence.

[16] The term "market-reliance" used in the lemmas refers to $\pi = 0$.

The posted mechanism can be left sitting for one of two reasons: either the expert does not exist or the expert exists but did not observe the offer. Therefore, the conditional probability of the expert's existence, given that the posted mechanism has not been accepted is given by

$$\widehat{a}(\pi) = \left[\frac{1-\pi}{a(1-\pi)+(1-a)}\right] a. \tag{33}$$

It is readily verified that $\widehat{a}(0) = a$, with $\widehat{a}$ decreasing and tending to 0 as π approaches 1. Intuitively, the higher the likelihood of the mechanism offer being observed, the lower is the conditional probability of the expert's existence if the mechanism has not been accepted.

Next, consider how beliefs about the economic state respond to trade in the market if the mechanism has been left sitting. First, note that Lemma 3.2 continues to hold along this path. In particular, for all π: the arrival of two sell orders reveals that the economic state is 0; a buy order reveals the economic state is 1;[17] and the absence of market activity causes the market maker and firm to revert to their prior beliefs.

However, the formula for beliefs following a single sell order given in (7) must be modified. For a given value of π, the Bayesian update that the economic state is bad following the arrival of a single sell order is

$$\chi_\pi(\vec{t}) = \frac{\widehat{a}(\pi)q(1-l)\phi_0(t) + (1-\widehat{a}(\pi))ql}{\widehat{a}(\pi)q(1-l)\phi_0(t) + (1-\widehat{a}(\pi))l}. \tag{34}$$

Following the same procedure as in Section 3, we divide through by $(1-\widehat{a})l$, which allows us to rewrite this belief as

$$\chi_\pi(\vec{t}) = \frac{K_\pi\phi_0(t) + q}{K_\pi\phi_0(t) + 1}, \tag{35}$$

where

$$K_\pi \equiv \frac{\widehat{a}(\pi)q}{1-\widehat{a}(\pi)}\frac{1-l}{l} = K(1-\pi). \tag{36}$$

The preceding equation reveals that an increase in the probability of the posted mechanism being observed is equivalent to a reduction in the market informativeness parameter. Intuitively, if the mechanism offer is more likely to be observed, then the expert is less likely to exist and trade if the mechanism is not accepted, reducing market informativeness.

With this modification to the beliefs following a single sell order, the equilibrium conditions are identical to those in Section 3. In particular: the firm's sequentially rational investment after a posted mechanism has been left sitting is described by (1); the secondary market stock price is still given by (2); and the type-0 expert's expected payoff from submitting a sell order of size t is

[17] Strictly speaking in the case of $\pi = 1$, i.e. when the expert always accepts the mechanism if he exists, the arrival of two sell orders or a buy order are off-path. The convention for off-path beliefs described in Remark 5.1 nevertheless implies that such order vectors reveal the economic state as zero or one, respectively.

still given by (6). It follows that for a given value of $\pi \in [0,1)$, if the posted mechanism has not been accepted, the continuation-game equilibrium is identical to the one characterized under pure market-reliance (Section 3) with K_π replacing K.[18]

In order to establish the main result of this section—that pure mechanism-reliance is never optimal—we only need to consider values of π above a certain threshold. In particular, note that

$$\pi > \max\{1 - \underline{K}/K, 0\} \Rightarrow K_\pi < \underline{K}. \tag{37}$$

Thus, for any K, a π sufficiently close to 1 ensures that $K_\pi < \underline{K}$, and we focus on such π for the rest of the section. In this case, the securities market equilibrium is the low informativeness case, characterized by Proposition 3.8 with K_π replacing K. From the equilibrium characterization we know the type-0 expert's expected trading profit is given by

$$u_0^*(\pi) = m_L(K_\pi)(1-q)(1-l), \tag{38}$$

where Proposition 3.8 defines $m_L(\cdot)$. Thus, if the expert exists but fails to observe the mechanism, an on-path event for all $\pi < 1$, his expected trading profit is equal to $qu_0^*(\pi)$. But note, this same expected trading profit is also available to the expert if he deviates by failing to take up the mechanism despite observing it. Thus, the expert's reservation value for participating in the mechanism is

$$\underline{u}(\pi) = qu_0^*(\pi). \tag{39}$$

Because π is sufficiently large so that $K_\pi < \underline{K}$, the expert expects a positive information rent from trading in the market, and therefore his reservation value for participating in the mechanism is also positive. Furthermore, since K_π is decreasing in π and $m_L(\cdot)$ is decreasing, the expert's reservation value $\underline{u}(\pi)$ is *increasing* in π. Intuitively, if the expert is more likely to observe the mechanism, he is less likely to be trading in the market; consequently, his trades have smaller price impact, increasing his expected trading profit.

Next, we use the results in Section 3 to calculate expected firm value conditional upon the mechanism not being accepted. With $K_\pi < \underline{K}$, the expert earns a positive information rent, and firm value is given by Lemma 3.10 (with $\widehat{a}(\pi)$ replacing a). Thus, if the mechanism is not accepted, expected firm value is

$$V_{REJ}(\pi) = 1 - q + \widehat{a}(\pi)ql(1-c) - \widehat{a}(\pi)qu_0^*(\pi). \tag{40}$$

To understand the preceding equation, note that in the present setting, market informativeness is very low, with $K_\pi < \underline{K}$. Thus, the arrival of a single sell order is never sufficient to induce a switch to the safe investment. Rather, the firm only switches to the safe investment following two sell orders. Two sell orders arrive if the expert exists (probability $\widehat{a}(\pi)$), the state is bad (probability

[18] If $\pi = 1$ this continuation-game is never reached on-path.

q), and a liquidity shock occurs (probability l). In this case, switching generates an incremental cash flow equal to $1 - c$ over what would have been generated under a strategy of never switching to the safe investment. Here the firm expects to pay an adverse selection cost of $qu_0^*(\pi)$ if the expert exists. Given that the mechanism was not accepted, the expected adverse selection cost is $\widehat{a}(\pi)qu_0^*(\pi)$.

Ex Ante Firm Value. We decompose the ex ante value of the firm into two parts, based on whether the mechanism is accepted or not:

$$V(\pi) = a\pi V_{ACC}(\pi) + [a(1-\pi) + 1 - a]V_{REJ}(\pi).$$

The first term is the probability that the mechanism is accepted in equilibrium multiplied by expected firm value conditional on acceptance, while the second term is the probability that the mechanism is not accepted multiplied by expected firm value conditional on the mechanism not being accepted. Substituting in the preceding expressions for V_{ACC} and V_{REJ}, we find

$$V(\pi) = 1 - q + qa[\pi + (1-\pi)l](1-c) - am_L(K_\pi)q(1-q)(1-l). \tag{41}$$

To understand this expression, note that if the expert exists, then he expects a rent equal to $qu_0^*(\pi)$, whether or not he sees the mechanism. If he doesn't observe the mechanism, then this payoff comes in the form of trading gains in the market, and if he does, then this payoff comes in the form of an expected wage payment. Conversely, regardless of whether the expert observes the mechanism, if the expert exists, then the firm expects to pay $qu_0^*(\pi)$ either as a wage or as an adverse selection cost. Thus, from an ex ante perspective, offering a mechanism with search friction π costs the firm $aqu_0^*(\pi)$.

Consider next expected cash flow. The economic state is good with probability $1 - q$. Here the firm always selects the risky investment, whether or not the mechanism is taken up. This generates cash flow equal to 1. If the state is bad, the firm switches to the safe investment in two circumstances: 1) the expert exists and observes the mechanism, or 2) the expert exists but fails to observe the mechanism, with a liquidity shock arriving to reveal the negative information held by the expert trader. In both circumstances, switching from the risky to the safe investment yields an incremental cash flow of $1 - c$. Combining these observations yields the expected cash flow embedded in (41).

The expression for ex ante firm value in (41) reveals a fundamental tradeoff inherent in the firm's choice of the mechanism observation probability π. An increase in π results in an increase in the probability that the firm selects the correct investment in each economic state. However, increasing π results in an endogenous decline in price impact, making informed trading more profitable. This effect simultaneously increases the expected wage bill if the mechanism offer is accepted and the expected adverse selection cost borne by shareholders if the mechanism is not accepted.

Differentiating $V(\pi)$, we find that

$$V'(\pi) = qa(1-l)(1-c) + am'_L(K_\pi)q(1-q)(1-l)K.$$

From the preceding equation if follows that:

$$V'(\pi) < 0 \iff m'_L(K_\pi) < -\frac{1}{KJ}. \tag{42}$$

A straightforward calculation reveals that for all $K_\pi > 0$, $m'_L(K_\pi)$ is a continuous function of K_π. Moreover, $m'_L(0) = -\infty$.[19] Hence, for K_π sufficiently close to zero, or equivalently, for π sufficiently close to one, (42) is satisfied. Therefore, ex ante firm value is decreasing in π for π sufficiently close to one. We therefore have the following result.

Proposition 6.1 *(Search Friction). If the firm has the ability to choose the probability $\pi \in [0,1]$ with which the expert observes the mechanism if he exists, then pure mechanism-reliance, $\pi = 1$, is suboptimal for all possible model parameters.*

In other words, even if the model parameters are such that pure mechanism-reliance generates greater ex ante firm value than pure market-reliance, firm value increases further by introducing a search friction, which sometimes causes the firm to rely on the market for information. The search friction reduces the expert's equilibrium reservation value by more than the drop in the firm's expected cash flow resulting from the expert sometimes not seeing the mechanism, forcing the firm to rely on the imperfect information provided by the market.

7 DISCUSSION/ALTERNATIVE ASSUMPTIONS

We further discuss a number of the model's assumptions.

Exclusivity. In the interest of realism, and in order to give the mechanism the best possible hearing, we adopted the assumption that an agent who agrees to participate in the mechanism is barred from trading the firm's securities (exclusivity). However, it is readily apparent that the firm cannot improve upon the DRM by posting a mechanism that does not impose exclusivity. To see this, note that the firm makes first-best production decisions under the DRM, so any alternative incentive scheme cannot increase expected cash flow. Consider next the expected cost of information. Without exclusivity, the costs of information are equal to the expected wage bill plus the expected trading gains of the firm's informed insider who has now been freed from exclusivity. But note, in order to induce the expert outsider to accept this new incentive scheme, the sum of his expected wages and trading gains must not fall below his reservation value. But this reservation value is just equal to the trading gains an expert outsider can expect if all agents form the belief

[19] Note that $m'_L(x) = \frac{-m_L(x)}{\sqrt{(x+1)^2-1}}$, $m_L(x)$ is continuous for $x > 0$, and $m_L(0) = 1$. Hence, $m'_L(x)$ is continuous for $x > 0$ and $\lim_{x\to 0} m'_L(x) = -\infty$.

that no informed agent exists. This is just equal to $\underline{u}$ (equation 24), which is the cost of information (expected wage bill) under the DRM with exclusivity imposed.

Public vs. Private Corporations. Our modeling assumptions most naturally fit publicly traded corporations. However, our argument is not confined to such firms. Rather, the necessary assumption for our argument is that there exists some security or market that allows the expert outsider to place bets on the performance of the firm. In advanced market economies, such bets can be placed on many private firms. For example, private firms often have traded debt claims, as well as credit default swaps written on them. Further, recently, platforms have been created for the express purpose of trading the equity of private firms.

Another conclusion following from the preceding discussion is that it will prove difficult for firm's in economies with advanced securities markets to shut down trading connected with their performance. However, for the sake of argument, let us suppose the firm could simply shut down the secondary stock market (and all other markets related to its performance). Under what conditions would it do so? To address this question, assume shareholders hit with a liquidity shock face a carrying cost that consumes a fraction ξ of the value of the their terminal dividend if they fail to liquidate. In this case, the expert cannot expect to make any trading gains and so the firm could offer a DRM with an expected wage payment of 0 and still achieve first-best decisions. The ex ante value of the private firm would be $(1 - l\xi/2)V^*$. Thus, the private firm structure would only be preferred if the probability and intensity of liquidity shocks is sufficiently low. But note, if the probability of liquidity shocks is very low, firm value under market-reliance is also relatively high. After all, with low volumes of noise-trading, the market-reliant firm comes close to first-best decision making, with low adverse selection costs.

Contract Timing. It was also assumed that the expert outsider must make his decision regarding whether to participate in the mechanism before observing the realization of the economic state. If instead the firm can only post a mechanism after the expert observes the economic state, it cannot improve upon pure market-reliance. To see this, first note that the mechanism-attempting firm would be forced to rely on the market in state 0 because no mechanism could screen for the type-0 expert. After all, the type-0 would have a positive reservation value. But paying him a positive value in return for recommending the safe investment would induce uninformed outsiders to accept the contract.

Next note that any mechanism screening for the type-1 expert replicates the payoffs of the market-reliant firm. After all, in state 1 the market reliant firm will implement the risky investment and the expert receives zero rent since buy orders are fully revealing. Similarly, the mechanism-reliant firm would pay the expert zero in return for his recommending the risky strategy.

Monopolist Expert. The baseline model assumes at most one informed expert exists. This assumption is adopted for simplicity of exposition, with results being robust to competition amongst experts. In particular, in the appendix we consider an extension of the model in which: a single expert exists with probability $(1-\psi)a$; a finite number $N \geq 3$ of experts exists with probability ψ;

and an expert does not observe whether other experts exist.[20] Critically, Supplemental Materials Section B shows the baseline model conditions for the dominance of markets over mechanisms, (29) and (30), remain robust to the introduction of such competition amongst experts.

The intuition is as follows. Consider first firm value under the mechanism. First, expected cash flow increases since at least one expert is more likely to exist ($(1-\psi)\,a+\psi>a$), and so the firm is more likely to execute the optimal state-contingent investment rule. Moreover, the expected wage bill under the mechanism falls. After all, the possibility that other experts exist reduces the expected gain to deviating and rejecting a posted mechanism since trading by other experts will reveal the true state. Effectively, the possibility of multiple experts scales the off-path trading gain by the factor $1-\psi$, since an expert can only profit on his trading if other experts do not exist.

Competition has a symmetric effect on firm value under market reliance. First, expected cash flow increases since with probability ψ multiple experts will exist and their trading will fully reveal the state, thus moving the market-reliant firm to the optimal state-contingent investment rule with higher probability, just as under the mechanism. Moreover, the increased probability of fully revealing trade under market-reliance reduces expected adverse selection costs. Intuitively, the possibility of multiple experts scales the on-path expert trading gain under market-reliance by the factor $1-\psi$, since an expert can only profit on his trading if other experts do not exist. The net effect of these symmetric forces is to leave unaltered the conditions for the optimality of market-reliance.

8 CONCLUDING REMARKS

When awarding the Nobel Prize in Economic Sciences, the Royal Swedish Academy generally takes great pains to highlight the numerous real-world applications to which a given contribution has been put, or to trumpet the successes of a given framework in helping to better understand empirical regularities or existing institutional arrangements. The 2007 Prize, given for mechanism design theory, was notable in that here the Academy went to some effort to explain that the theory is not intended to be positive: "While direct mechanisms are not intended as descriptions of real-world institutions, their mathematical structure makes them relatively easy to analyze." In a similar vein, in his Nobel lecture, Eric Maskin (2008) positioned mechanism design theory primarily as a normative theory:

> The theory of mechanism design can be thought of as the "engineering" side of economic theory. Much theoretical work, of course, focuses on *existing*[his italics] economic institutions. The theorist wants to explain or forecast the economic or social outcomes that these institutions generate. But in mechanism design theory, the direction of inquiry is reversed. We begin by identifying our desired outcome or social goal. We then ask whether an appropriate institution (mechanism) could be designed to attain that goal.

[20] For $N=2$, see Supplementary Materials Section B.4.

In this paper we showed how securities markets may impose limits on the usage of mechanisms. After all, posting a DRM meeting an informed agent's participation constraint generates a high endogenous reservation value since rejecting said DRM (deviating) convinces markets no informed agent exists, allowing aggressive informed trading sans price impact. The DRM-reliant firm must pay expected wages equal to this high outside option value, implying high costs of information. For the market-reliant firm, information acquisition costs (paid via uninformed shareholder trading losses) are necessarily lower, since price impact naturally limits informed trading gains when agents know informed parties have been left outside firm boundaries, and left free to trade. However, this reduction in information acquisition costs must be weighed against the concomitant reduction in information quality associated with reliance on noisy securities prices.

In our framework the firm considers two alternative sources of information: the mechanism and the market. One might expect similar results to apply in other mechanism design settings where a market provides an alternative source of information. Regardless, the use of securities prices and hired consultants for information is ubiquitous, and so a theory of markets versus mechanisms in this setting is an important step forward.

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APPENDICES

A PROOFS FROM SECTION 3

Proof. (Lemma 3.4). Suppose an uninformed agent deviates from the equilibrium by submitting order t. Let T_{-} be the order vector submitted by the other market participants, i.e. T_{-} is formed from the overall order vector T by deleting the uninformed agent's order. If the uninformed agent submits order t and the realization of the overall order vector is T, then the uninformed agent's belief just before the order is executed is $\chi(T_{-})$, as in Lemma 3.2 and equation (7). That is, after the overall order vector is realized, the uninformed agent updates his beliefs based on the trading activity of the other market participants. Meanwhile, the market maker's belief is $\chi(T)$, as in Lemma 3.2, equation (7), and Remark (3.3). That is, the market maker updates his beliefs based on the entire vector of submitted orders. The market price is set as in equation (2). Therefore, if the uninformed agent submits order $t \in [-1, 1]$ and the overall order vector is T, then the uninformed agent expects profit $u_N(t, T) = t(p(T) - \chi(T_{-})) = t(\chi(T_{-}) - \chi(T))(1 - \alpha(T))$. We show that for any $t \neq 0$, and any realization of the expert's trade and the liquidity shock, $u_N(t, T) \leq 0$.

Suppose an uninformed agent deviates from the equilibrium and submits a buy order, $t_B < 0$.

Case 1: the uninformed agent's buy order is the only one submitted. The overall order vector T consists of a single buy order, t_B. From Lemma 3.2, $\chi(T) = 0$ and $\alpha(T) = 0$. Meanwhile, $T_{-} = 0$. From Lemma 3.2 it follows that $\chi(T_{-}) = q$. Hence, $u_N(t_B, T) = t_B q$. Because $t_B < 0$ it follows that $u_N(t_B, T) < 0$.

Case 2: the uninformed agent's buy order $t_B < 0$ and a single sell order, t_S arrive. The overall order vector T consists of a single buy order and a single sell order. From Lemma 3.2 it follows that $\chi(T) = 0$ and $\alpha(T) = 0$. Meanwhile, $T_{-} = t_S$. Hence, $\chi(T_{-}) = \chi(\vec{t_S}) \geq q$ (consult equation (7)). Hence, $u_N(t, T) = t_B \chi(\vec{t_S})$. Because $t_B < 0$ it follows that $u_N(t_B, T) < 0$.

Case 3: two buy orders arrive. From Remark 3.3 it follows that $\chi(T) = 0$ and $\alpha(T) = 0$. Meanwhile, T_{-} is a single buy order. Hence, $\chi(T_{-}) = 0$ (consult Lemma (3.2)). Hence, $u_N(t, T) = 0$.

Case 4: the uninformed agent's buy order $t_B < 0$ and two sell orders arrive. The overall order vector T consists of a single buy order and two sell orders. From Lemma 3.2 it follows that $\chi(T) = 1$ and $\alpha(T) = 1$. Hence, $u_N(t, T) = 0$.

Case 5: two buy orders and a single sell order arrive. From Remark 3.3 it follows that $\chi(T) = 0$ and $\alpha(T) = 0$. Meanwhile, T_{-}, consists of one buy order and one sell order. Hence, $\chi(T_{-}) = 0$ (consult Lemma (3.2)). Hence, $u_N(t, T) = 0$.

This exhausts the possible cases.

Suppose an uninformed agent deviates from the equilibrium and submits a sell order, $t_S > 0$.

Case 1: the uninformed agent's sell order is the only one submitted. The overall order vector T consists of a single sell order, t_S. From equation 7, $\chi(T) \geq q$ and $\alpha(T) \in [0, 1]$. Meanwhile, $T_{-} = 0$. From Lemma 3.2 it follows that $\chi(T_{-}) = q$. Hence, $u_N(t_S, T) = t_S(q - \chi(T))(1 - \alpha(T))$. Hence, $u_N(t_S, T) \leq 0$.

Case 2: the uninformed agent's sell order $t_S > 0$ and a single buy order arrive. The overall order vector T consists of a single buy order and a single sell order. From Lemma 3.2 it follows that $\chi(T) = 0$ and $\alpha(T) = 0$. Meanwhile, T_{-} is a single buy order. Hence, $\chi(T_{-}) = 0$ and $\alpha(T) = 0$ (consult Lemma (3.2)). Hence, $u_N(t, T) = 0$.

Case 3: two sell orders arrive. From Lemma 3.2 it follows that $\chi(T) = 1$ and $\alpha(T) = 1$. Hence, $u_N(t, T) = 0$.

Case 4: three sell orders arrive. From Remark 3.3 it follows that $\chi(T) = 1$ and $\alpha(T) = 1$. Hence, $u_N(t, T) = 0$.

Case 5: two sell orders and a single buy order arrive. From Remark 3.3 it follows that $\chi(T) = 1$ and $\alpha(T) = 1$. Hence, $u_N(t, T) = 0$.

This exhausts the possible cases.

Thus, if an uninformed agent deviates from the equilibrium by buying or selling in the market, his expected profit is weakly negative for any possible realization of the order vector. ■

Proof. (Lemma 3.5). *Step 1*: We show that $\Phi_0(\cdot)$ has no mass points. Because the order flow generated by the liquidity shock is uniformly distributed, if $\Phi_0(\cdot)$ has a mass point at t, then $\chi(\vec{t}) = 1$. Consequently, the expert's expected payoff of submitting this order given in (6) is zero, and hence, the expert's equilibrium payoff in state zero must also be zero, contradicting $u_0^* > 0$.

Step 2: We show that the support of $\Phi_0(\cdot)$ has no gaps. In particular, we show that if t_1 is inside the support of $\Phi_0(\cdot)$ and $t_2 \in (t_1, 1]$, then t_2 is also inside the support of $\Phi_0(\cdot)$. To derive a contradiction, suppose not. Because t_2 is outside the support of $\Phi_0(\cdot)$, $\chi(t_2) = q$ and hence $\alpha(\vec{t_2}) = 0$. Meanwhile, t_1 is inside the support of $\Phi_0(\cdot)$, and hence $\chi(\vec{t_1}) \geq q$. Therefore, $t_2(1 - \chi(\vec{t_2}))(1 - l) > t(1 - \chi(\vec{t_1}))(1 - \alpha(\vec{t_1}))(1 - l)$, a contradiction.

Step 3: We show that $m > 0$. Suppose $m = 0$. For any $\epsilon > 0$, type-0 expert's indifference condition (8) requires $u_0^* = \epsilon(1 - \chi(\vec{\epsilon}))(1 - \alpha(\vec{\epsilon}))(1 - l)$. Because $(1 - \chi(\vec{\epsilon}))(1 - \alpha(\vec{\epsilon}))(1 - l) < 1$, it follows that $u_0^* < \epsilon$ for any $\epsilon > 0$, contradicting $u_0^* > 0$. ■

Proof. (Lemma 3.6). *Step 1.* We show that $u_0 \geq m(1 - q)(1 - l)$. Lemma 3.5 implies that for $t < m$, $\phi_0(t) = 0$, and thus equation (7) implies that $\chi(t) = q$ and thus $\alpha(t) = 0$ for $t < m$. Consequently, the expected payoff from submitting order $t \in (0, m)$ is $t(1 - q)(1 - l)$. Thus, the expert does not prefer to deviate to an order outside the support and $u_0 \geq m(1 - q)(1 - l)$.

Step 2. We show that $u_0 \leq m(1 - q)(1 - l)$. Consider $t = m + \epsilon$. Because this order is inside the support, $u_0 = (m + \epsilon)(1 - \chi(m + \epsilon))(1 - \alpha(m + \epsilon))(1 - l)$. From Lemma 3.5 and equation (7), $\chi(m + \epsilon) \geq q$. Also, $\alpha(m + \epsilon) \geq 0$. Hence, $u_0 \leq (m + \epsilon)(1 - q)(1 - l)$ for all $\epsilon > 0$. It follows that $u_0 \leq m(1 - q)(1 - l)$.

Combining Steps 1 and 2, we find that $u_0 = m(1 - q)(1 - l)$. ■

Proof. (Lemma 3.7)

Proof of part (i). For $t < m$, $\chi(\vec{t}) = q$, and hence $\alpha(\vec{t}) = 0$. From (8) and Lemma 3.6,

$$t \in [m, 1] \Rightarrow \chi(\vec{t}) = 1 - \frac{m(1-q)}{t(1-\alpha(\vec{t}))}.$$

Because $m > 1/J$ and $t(1-\alpha(\vec{t})) \leq 1$,

$$1 - \frac{m(1-q)}{t(1-\alpha(\vec{t}))} < 1 - \frac{1-c}{t(1-\alpha(\vec{t}))} \leq c.$$

Hence, $\chi(\vec{t}) < c$ for all $t \in [0,1]$. Thus, (1) implies $\alpha(\vec{t}) = 0$ for all $t \in [0,1]$.

Proof of part (ii). For $t < m$, $\chi(\vec{t}) = q$, and hence $\alpha(\vec{t}) = 0$. For $t \in [m, Jm)$, equation (A) holds and because $1 - \alpha(\vec{t}) \in [0,1]$,

$$\chi(\vec{t}) = 1 - \frac{m(1-q)}{t(1-\alpha(\vec{t}))} \leq 1 - \frac{m(1-q)}{t},$$

and because $t < Jm$,

$$1 - \frac{m(1-q)}{t} < 1 - \frac{1-q}{J} = c.$$

Hence, if $t \in [m, Jm)$, then $\chi(\vec{t}) < c$. Thus, (1) implies $\alpha(\vec{t}) = 0$ for $t \in [m, Jm)$. Consider $t \in (Jm, 1]$ (note that by assumption $Jm < 1$). To derive a contradiction, assume that $\alpha(\vec{t}) = 0$. From (8),

$$\chi(\vec{t}) = 1 - \frac{m(1-q)}{t}.$$

Because $t > Jm$,

$$\chi(\vec{t}) = 1 - \frac{m(1-q)}{t} > 1 - \frac{(1-q)}{J} = c.$$

Hence, if $\alpha(\vec{t}) = 0$, then $\chi(\vec{t}) > c$. But (1) implies that $\alpha(\vec{t}) = 1$, a contradiction. Hence, for $t \in (Jm, 1]$, it must be that $\alpha(\vec{t}) > 0$. Next, note that if $\alpha(\vec{t}) = 1$ for $t \in (Jm, 1]$, then $E[u_0(t, T)] = 0$. Because t is inside $(Jm, 1] \subset [m, 1]$, it follows that $u_0^* = 0$, a contradiction. Hence, if $t \in (Jm, 1]$, then $\alpha(\vec{t}) < 1$. ■

Proof. (Proposition 3.10). We provide a calculation for the ex ante expected cash flow. The rest of the argument is in the text. Conditional on order vector T, the firm's expected cash flow is

$$(1 - \chi(T))(1 - \alpha(T)) + \alpha(T)(1 - c) = 1 - \chi(T) + \alpha(T)(\chi(T) - c).$$

Hence, ex ante firm value is

$$E[1 - \chi(T)] + E[\alpha(T)(\chi(T) - c)],$$

where the expectation is taken with respect to the distribution of the equilibrium order flow vector. Because $1 - \chi(T)$ is the probability of economic state one conditional on order vector T, the Law of Iterated Expectations implies that $E[1-\chi(T)] = 1-q$. From Lemma 3.2, $\alpha(T) = 0$ if a buy order arrives or the market is inactive, and hence, in these cases $\alpha(T)(\chi(T)-c) = 0$. From Lemma 3.7 when a single sell order arrives, the firm either selects risky $\alpha(T) = 0$ or it mixes, $\alpha(T) \in (0,1)$. If the firm mixes, then (1) requires $\chi(T) = c$. Thus, whenever a single sell order arrives, $\alpha(T)(\chi(T)-c) = 0$. Finally, if two sell orders arrive, then $\chi(T) = 1$ and $\alpha(T) = 1$, and hence, $\alpha(T)(\chi(T)-c) = 1-c$. Therefore, $E[\alpha(T)(\chi(T)-c)] = aql(1-c)$. ■

B PROOFS FROM SECTION 4

Proof. (Lemma 4.2) (i) $\rho_1 = 1$ follows from $\chi_1 \leq q$ and the firm's sequential rationality, equation (1). (ii). Note that any mechanism that beats remaining unadvised must induce the expert to participate and screen out uninformed agents and has $\rho_1 = 1$. Hence, the expected payoff to the firm in any such mechanism is

$$\begin{aligned}(1-a)(1-q)+&a\Pr(r=0)\{(1-\chi_0)\rho_0+(1-\rho_0)(1-c))\}+\\ &a\Pr(r=1)(1-\chi_1)-aU,\end{aligned}$$

where $U \geq \underline{u}$ is the expert's expected wage. Suppose $\rho_0 = 1$. Using the Law of Iterated Expectations, the firm's payoff simplifies,

$$\begin{aligned}(1-a)(1-q)+a\Pr(r=0)(1-\chi_0)+a\Pr(r=1)(1-\chi_1)-aU &=\\ (1-a)(1-q)+a(1-q)-aU = 1-q-aU.\end{aligned}$$

Therefore, $\rho_0 = 1$ is inferior to market reliance.

Suppose $\rho_0 \in (0,1)$. Sequential rationality by the firm (1) requires $\chi_0 = c$, and hence, the firm's payoff simplifies to

$$\begin{aligned}(1-a)(1-q)+a\Pr(r=0)\{(1-\chi_0)\rho_0+(1-\rho_0)(1-\chi_0))\}+a\Pr(r=1)(1-\chi_1)-aU &=\\ (1-a)(1-q)+a\Pr(r=0)(1-\chi_0)+a\Pr(r=1)(1-\chi_1)-aU &=\\ (1-a)(1-q)+a(1-q)-aU &=\\ (1-q)-aU.\end{aligned}$$

Note that the transition from the second to the third line uses the Law of Iterated Expectations. Note that $U \geq \underline{u} > qu_0^*$. Thus, the cost of offering the mechanism aU exceeds the adverse selection cost under market-reliance, aqu_0^*. Finally, consider the firm's cash flow under market-reliance. If $K < \overline{K}$, then equation (17) gives the firm's cash flow of $1-q+aql(1-c) > 1-q$. If $K > \overline{K}$, then from equation (21) it is $1-q+l(1-c)(1-a)\left(K/(1-l)-\overline{K}\right) > 1-q$. Thus, the firm's expected cash flow is larger under market reliance. Simultaneously, the adverse selection cost under market reliance is smaller than the expected wage bill under the mechanism. Therefore, $0 < \rho_0 < 1$, is inferior to market reliance. ■

Proof. (Lemma 4.3). Claim 1: *If a mechanism delivers the firm a higher expected payoff than market-reliance, then it cannot be the case that* $\gamma_0 = \gamma_1 = 1$. If $\gamma_0 = \gamma_1 = 1$, then $\chi_1 = q$ and any value of χ_0 is consistent with Bayes' rule. Because the expert always reports $r = 1$ in equilibrium, the firm always implements the risky action, and hence expected firm value is $1 - q - aU$, where $U \geq \underline{u}$ is the expert's expected wage. This is smaller than expected firm value under market-reliance, as shown in the proof of Lemma 4.2.

Claim 2: *If a mechanism delivers the firm a higher expected payoff than market-reliance, then* $w_{11} \geq w_{01-c}$ *and* $w_{01-c} \geq w_{10}$. From Lemma 4.2, $\rho_0 = 0$, and hence, $\chi_0 \geq c$. From Bayes' Rule,

$$\chi_0 = \frac{q(1-\gamma_0)}{q(1-\gamma_0) + (1-q)(1-\gamma_1)}.$$

From Claim 1, χ_0 is well defined. Hence,

$$\chi_0 \geq c \iff q(1-c)\gamma_0 + c - q \leq c(1-q)\gamma_1. \tag{43}$$

First we show $w_{11} \geq w_{01-c}$. Note that

$$\begin{aligned} q(1-c)\gamma_0 + c - q \leq c(1-q)\gamma_1 &\Rightarrow \\ c - q \leq c(1-q)\gamma_1 &\Rightarrow \\ \gamma_1 \geq \frac{c-q}{c(1-q)} &> 0. \end{aligned}$$

Thus, $\gamma_1 > 0$, which implies the expert must report truthfully in state 1 with positive probability. Thus, the expert's expected payoff of reporting truthfully in state 1 must be at least as large as his expected payoff of lying, and hence $w_{11} \geq w_{01-c}$.

Next we show $w_{01-c} \geq w_{10}$. Suppose that $\gamma_0 = 1$. Substituting into (43),

$$q(1-c) + c - q \leq c(1-q)\gamma_1 \Rightarrow \gamma_1 \geq 1.$$

Hence, $\gamma_0 = 1$ implies $\gamma_1 = 1$, contradicting Claim 1. Hence, $\gamma_0 < 1$, which implies that the expert must report truthfully in state 0 with positive probability. Thus, the expert's expected payoff of reporting truthfully in state 0 must be at least as large as his expected payoff of lying, and hence $w_{01-c} \geq w_{10}$.

Claim 3: *If a mechanism delivers the firm a higher expected payoff than market-reliance, then* $w_{11} > w_{01-c}$ *and* $w_{01-c} > w_{10}$. From Lemma 4.1, any mechanism which achieves higher value than market reliance screens out uninformed outsiders and requires participation of the expert. These constraints are:

(SC0) $\qquad w_{01-c} \leq 0$

(SC1) $\qquad qw_{10} + (1-q)w_{11} \leq 0$

(PC) $\qquad q[\gamma_0 w_{10} + (1-\gamma_0)w_{01-c}] + (1-q)[\gamma_1 w_{11} + (1-\gamma_1)w_{01-c}] \geq \underline{u}$

Constraint (SC0) ensures that uninformed outsiders prefer to reject the mechanism over accepting and reporting $r = 0$, (SC1) ensures that uninformed outsiders prefer to reject the mechanism over accepting and reporting $r = 1$, and (PC) ensures that an expert prefers to participate (if he exists).

Next, note that Claim 2 ensures $w_{11} \geq w_{01-c}$. Therefore, either $w_{11} > w_{01-c}$ which implies $\gamma_1 = 1$, or $w_{11} = w_{01-c}$. In either case (PC) reduces to:

$$q[\gamma_0 w_{10} + (1-\gamma_0)w_{0,1-c}] + (1-q)w_{11} \geq \underline{u}$$

Analogously, either $\gamma_0 = 1$ or $w_{01-c} = w_{10}$ in which case (PC) reduces further to

(PC$'$) $$qw_{01-c} + (1-q)w_{11} \geq \underline{u}.$$

Hence:

$$w_{11} \geq \frac{\underline{u}}{1-q} - \frac{q}{1-q} w_{01-c} > 0,$$

where the last inequality follows because $\underline{u} > 0$ and $w_{01-c} \leq 0$. Hence, $w_{11} > 0 \geq w_{01-c}$.

Note further that subtracting (SC1) from (PC$'$) yields

$$qw_{01-c} + (1-q)w_{11} - (qw_{10} + (1-q)w_{11}) \geq \underline{u} \Rightarrow w_{01-c} \geq w_{10} + \frac{\underline{u}}{q} \Rightarrow w_{01-c} > w_{10},$$

where the last inequality follows from $\underline{u} > 0$.

Claim 4: *The expert's unique sequentially rational reporting strategy is* $\gamma_0 = 0$ *and* $\gamma_1 = 1$. Follows immediately from Claim 3. ■

Proof. (Proposition 4.4). We show that (SC0), (SC1), (PC), and (BOND) imply $\underline{u} \leq qB$. Subtracting (SC0) from (PC$'$) yields $(1-q)w_1 \geq \underline{u}$. Substituting into (SC1) we find that $w_0 \leq -\underline{u}/q$. Hence, (BOND) implies that $\underline{u}/q \leq B$, and hence $\underline{u} \leq qB$. ■

Proof. (Proposition 4.5). In the text, we argued that in any feasible mechanism, expected firm value is

$$(1-a)(1-q) + a[(1-q) + q(1-c)] - a[qw_{1-c} + (1-q)w_1].$$

Thus, the firm would like to minimize expected compensation, $qw_{1-c}+(1-q)w_1$, but (PC) requires $qw_{1-c}+(1-q)w_1 \geq \underline{u}$. Hence, any feasible mechanism in which (PC) holds with equality is optimal, yielding payoff

$$(1-a)(1-q)+a[(1-q)+q(1-c)]-a\underline{u}=1-q+aq(1-c)-a\underline{u}.$$

Via direct substitution, it is readily verified that the proposed mechanism is feasible and optimal if $\underline{u}\leq qB$. ■

SUPPLEMENTAL MATERIAL

A PERTURBATION AND EQUILIBRIUM SELECTION

In this section we provide a perturbation argument which allows us to sharpen the equilibrium characterization and provides further support for the equilibrium trading patterns that we consider in the paper. We consider a straightforward perturbation of the main model, described in detail below; as in the text, we focus on deriving equilibria in which uninformed outsiders do not trade, and we maintain the assumption that the type-1 expert always trades. Thus, all of our results below pertain to equilibria with this property.

We establish three results. First, we characterize all such equilibria of the perturbed market-reliance game, showing that for any (J, K) only one such equilibrium exists. Second, we show that in the unique equilibrium, uninformed outsiders strictly prefer not to trade, thereby sharpening the result of Lemma 3.4. We also show that in the unique equilibrium of the perturbed game the type-0 expert always has a strictly positive equilibrium payoff and strictly prefers to be active in the market. Thus, the infinite number of no-rent equilibria encountered in Proposition 3.11 are eliminated by the perturbation. Third, we analyze the limit of the perturbed equilibrium as the perturbation vanishes. In the low and intermediate informativeness cases, i.e. $K < \overline{K}$, the unique equilibrium with perturbations converges to the equilibrium characterized in Propositions 3.8 and 3.9, respectively. In the high informativeness case, the unique equilibrium with perturbations converges to a particular equilibrium in the multiplicity of payoff-equivalent equilibria characterized in Proposition 3.11.

Perturbation. In particular, we consider the possibility that the firm may be unable to switch to the safe investment despite its desire to do so. That is, an attempt by the firm to switch to the safe investment fails with some small probability $\epsilon > 0$, and thus, the probability of switching to the safe investment cannot exceed $1 - \epsilon$. Hence, the firm's sequentially rational switching strategy is

$$\alpha(T) = \begin{cases} 0 & \text{if} \quad \chi(T) < c \\ \in [0,1] & \text{if} \quad \chi(T) = c \\ 1-\epsilon & \text{if} \quad \chi(T) > c \end{cases} \tag{44}$$

Note that this is the only equilibrium condition that is directly affected by the perturbation. In particular, the expressions for the expert's trading profit and the equilibrium beliefs are identical to the ones in the text.

Analysis. We first establish that in equilibrium the type-0 expert submits a sell order with probability one and expects a strictly positive equilibrium payoff, which in turn implies the structure of the type-0 expert's mixed strategy.

Lemma A.1 *If $\epsilon > 0$, then in equilibrium* *(i) type-0 expert expects a strictly positive payoff, $u_0^* > 0$.*

(ii) the type-0 expert plays a continuous mixed strategy with no mass points or gaps supported on interval $[\overline{m}, 1]$, for some $\overline{m} > 0$.

Proof. Claim (i). First, note that the type-0 expert's equilibrium payoff u_0^* cannot be strictly negative; otherwise the type-0 expert would have a strictly beneficial deviation to inactivity. Therefore, we must rule out the possibility that the type-0 expert's equilibrium payoff is 0. To derive a contradiction, suppose that the type-0 expert's equilibrium payoff is 0. Note that in the perturbed model, the price is given by equation (2), and hence, when the type-0 expert submits order t and the realization of the order flow vector is T, the expert's profit is still given by equation (3). As in Lemma 3.1, the type-0 expert's trading profit from submitting sell order t is non-negative, for any realization of the order vector, $u_0(t, T) \geq 0$ for all $t > 0$. Hence, if the type-0 expert's equilibrium payoff is 0, then $E[u_0(t, T)] = 0$ for all $t > 0$; otherwise, the expert would have a beneficial deviation to some sell order $t > 0$. Next, decompose the expert's expected payoff into two parts by conditioning on the possibility that the sell expert's order is the only one that arrives:

$$E[u_0(t, T)] = (1 - l)u_0(t, \vec{t}) + lE[u_0(t, T)|T \neq \vec{t}].$$

Because $u_0(t, T) \geq 0$ and $E[u_0(t, T)] = 0$ for $t > 0$, it follows that $u_0(t, \vec{t}) = 0$ for all $t > 0$. That is, $t(1 - \chi(\vec{t}))(1 - \alpha(\vec{t})) = 0$ for all $t > 0$. From (44) $1 - \alpha(\vec{t}) \geq \epsilon > 0$. Hence, $\chi(\vec{t}) = 1$ for all $t > 0$; any sell order that arrives on its own reveals state 0. Because the distribution of sell shocks has no mass points, any order t for which $\chi(\vec{t}) = 1$ must be a mass point of the type-0 expert's mixed strategy, i.e. if $\chi(\vec{t}) = 1$, then $\Phi_0(t)$ has a jump discontinuity at t. Therefore, $\Phi_0(t)$ must have a jump discontinuity for all $t > 0$, and therefore the number of jump discontinuities is uncountable. Because $\Phi_0(t)$ is weakly increasing, it must have a countable number of jump discontinuities (Froda's Theorem), yielding a contradiction. Hence, type-0's equilibrium expected payoff must be strictly positive. Proof of Claim (ii) is analogous to the proof of Lemma 3.5. ■

Next, we incorporate the perturbation into Lemma 3.4, which yields the following stronger version of the result.

Lemma A.2 *If $\epsilon > 0$, then in equilibrium an uninformed outsider's expected payoff of submitting any order $t \neq 0$ is strictly negative.*

Proof. Suppose an uninformed agent deviates from the equilibrium by submitting order t. Let T_- be the order vector submitted by the other market participants, i.e. T_- is formed from the overall order vector T by deleting the uninformed agent's order. If the uninformed agent submits order t and the realization of the overall order vector is T, then the uninformed agent's belief just before the order is executed is $\chi(T_-)$, as in Lemma 3.2 and equation (7). That is, after the overall order vector is realized, the uninformed agent updates his beliefs based on the trading activity of the other market participants. Meanwhile, the market maker's belief is $\chi(T)$, as in Lemma 3.2, equation (7), and Remark (3.3). That is, the market maker updates his beliefs based on the entire vector of submitted orders. The market price is set as in equation (2). Therefore, if the uninformed agent submits order $t \in [-1, 1]$ and the overall order vector is T, then the uninformed agent expects profit $u_N(t, T) = t(p(T) - \chi(T_-)) = t(\chi(T_-) - \chi(T))(1 - \alpha(T))$. We show that for any $t \neq 0$, and any realization of the expert's trade and the liquidity shock, $u_N(t, T) \leq 0$, and that the probability that $u_N(t, T) < 0$ is non-zero.

Suppose an uninformed agent deviates from the equilibrium and submits a buy order, $t_B < 0$.

Case 1: the uninformed agent's buy order is the only one submitted. The overall order vector T consists of a single buy order, t_B. Hence, beliefs are evaluated using Lemma 3.2, which implies $\chi(\vec{t_B}) = 0$ and $\alpha(\vec{t_B}) = 0$. Meanwhile, $T_- = 0$. From Lemma 3.2 it follows that $\chi(T_-) = q$. Hence, $u_N(t_B, T) = t_B q$. Because $t_B < 0$, it follows that $u_N(t_B, T) < 0$.

Case 2: the uninformed agent's buy order $t_B < 0$ and a single sell order, $t_S > 0$ arrive. The overall order vector T consists of a single buy order and a single sell order. From Lemma 3.2 or Remark 3.3 it follows that $\chi(T) = 0$ and $\alpha(T) = 0$. Meanwhile, $T_- = t_S$. Hence, $\chi(T_-) = \chi(\vec{t_S}) \geq q$ (consult equation (7)). Hence, $u_N(t, T) = t_B \chi(\vec{t_S})$. Because $t_B < 0$ it follows that $u_N(t_B, T) < 0$.

Case 3: the uninformed outsider's buy order and a second buy order arrive. From Remark 3.3 it follows that $\chi(T) = 0$ and $\alpha(T) = 0$. Meanwhile, T_- is a single buy order. From Lemma 3.2, $\chi(T_-) = 0$. Hence, $u_N(t_B, T) = 0$.

Case 4: the uninformed outsider's buy order $t_B < 0$ and two sell orders arrive. The overall order vector T consists of a single buy order and two sell orders. From Lemma 3.2 it follows that $\chi(T) = \chi(T_-) = 1$ and $\alpha(T) = 1 - \epsilon$. Hence, $u_N(t_B, T) = 0$.

Case 5: two buy orders and a single sell order arrive. From Remark 3.3 it follows that $\chi(T) = 0$ and $\alpha(T) = 0$. Meanwhile, T_- consists of one buy order and one sell order. Hence, $\chi(T_-) = 0$ (consult Lemma (3.2) and Remark (3.3)). Hence, $u_N(t_B, T) = 0$.

Thus, if an uninformed outsider deviates by submitting a buy order, then for any realization of the overall order vector, his trading profit is weakly negative. Furthermore, in cases 1 and 2, which occur with positive probability following the uninformed outsider's deviation, the uninformed outsider's trading profit is strictly negative. Hence, the uninformed outsider's overall expected trading profit is negative.

Suppose an uninformed outsider deviates from the equilibrium and submits a sell order, $t_S > 0$.

Case 1: the uninformed agent's sell order is the only one submitted. The overall order vector T consists of a single sell order, t_S. From equation (7), $\chi(T) \geq q$ and $\alpha(T) \leq 1 - \epsilon$. Meanwhile, $T_- = 0$. From Lemma 3.2 it follows that $\chi(T_-) = q$. Hence, $u_N(t_S, T) = t_S(q - \chi(T))(1 - \alpha(T))$. Hence, $u_N(t_S, T) \leq 0$.

Case 2: the uninformed agent's sell order $t_S > 0$ and a single buy order arrive. The overall order vector T consists of a single buy order and a single sell order. From Lemma 3.2 it follows that $\chi(T) = 0$ and $\alpha(T) = 0$. Meanwhile, T_- is a single buy order. Hence, $\chi(T_-) = 0$ and $\alpha(T) = 0$ (consult Lemma (3.2)). Hence, $u_N(t, T) = 0$.

Case 3: the uninformed agent's sell order $t_S > 0$ and a second sell order, $t'_S > 0$, arrive. From Lemma 3.2 it follows that $\chi(T) = 1$ and $\alpha(T) = 1 - \epsilon$. Meanwhile, $T_- = t'_S$, and hence,

$\chi(T_{-}) = \chi(\vec{t'_S})$. Because the type-0 expert's mixed strategy contains no mass points, $\chi(\vec{t'_S}) < 1$. Hence, $u_N(t, T) = t_s(0 - \chi(\vec{t'_S}))(1 - \epsilon) < 0$.

Case 4: three sell orders arrive. From Remark 3.3 it follows that $\chi(T) = \chi(T_{-}) = 1$ and $\alpha(T) = 1 - \epsilon$. Hence, $u_N(t, T) = 0$.

Case 5: two sell orders and a single buy order arrive. From Remark 3.3 it follows that $\chi(T) = \chi(T_{-}) = 1$ and $\alpha(T) = 1 - \epsilon$. Hence, $u_N(t, T) = 0$.

Thus, if an uninformed outsider deviates by submitting a sell order, then for any realization of the overall order vector, his trading profit is weakly negative. Furthermore, in case 3, which occurs with positive probability following the uninformed outsider's deviation, the uninformed outsider's trading profit is strictly negative. Hence, the uninformed outsider's overall expected trading profit is negative. ■

It remains to characterize are the type-0 expert's trading strategy and the firm's switching strategy. With the obvious modification of replacing $\alpha(T) = 1$ with $\alpha(T) = 1 - \epsilon$, the beliefs and firm strategy following inactivity or multiple orders are identical to the ones in Lemma 3.2 and Remark 3.3. We present them here for completeness.

Lemma A.3 *In equilibrium, (i) any order vector T containing two sell orders such that at least one is inside the support of the type-0 expert's mixed strategy reveals that the state is zero, $\chi(T) = 1$ and induces a switch to the safe strategy with largest feasible probability, $\alpha(T) = 1 - \epsilon$; (ii) market inactivity does not affect beliefs $\chi(\vec{0}) = q$, and induces the firm to select the risky strategy, $\alpha(\vec{0}) = 0$; (iii) any order vector T containing a single buy order, or a buy order and a single sell order reveals that the state is one, $\chi(T) = 0$, and it induces the firm to select the risky strategy, $\alpha(T) = 0$.*

The preceding lemma gives the belief for all on-path order vectors,

Remark A.4 *(Off-path Beliefs and Actions). Consider an off-path order vector, T. If the number of buy orders is greater than or equal to the number of sell orders, then T reveals state 1, $\chi(T) = 0$, and the firm selects the risky investment, $\alpha(T) = 0$. Otherwise, T reveals state 0, $\chi(T) = 1$, and the firm selects the safe investment with the largest feasible probability, $\alpha(T) = 1 - \epsilon$.*

Thus, to complete the characterization, we must derive the type-0 expert's trading strategy and the firm's switching strategy following the arrival of a single sell order. Recall that, when the type-0 expert submits a sell order and a second sell order arrives, the market maker infers that the economic state is zero, and the expert's trading profit is 0. Hence, the type-0 expert's expected trading profit is

$$E[u_0(t, T)] = t(1 - \chi(\vec{t})(1 - \alpha(\vec{t}))(1 - l).$$

Furthermore, Lemma 3.6 continues to hold, and hence, the expert's indifference condition is simply

$$\overline{m}(1 - q)(1 - l) = t(1 - \chi(\vec{t})(1 - \alpha(\vec{t}))(1 - l), \tag{45}$$

as in the main text. We characterize the structure of the firm's strategy, extending Lemma 3.7.

Lemma A.5 *In any equilibrium of the perturbed game,*

(i) if $t \in [0,1]$ and $t \leq J\overline{m}$, then $\alpha(\vec{t}) = 0$.

(ii) if $t \in [0,1]$ and $J\overline{m} < t < J\overline{m}/\epsilon$, then $\alpha(\vec{t}) \in (0, 1-\epsilon)$.

(iii) if $t \in [0,1]$ and $t \geq J\overline{m}/\epsilon$, then $\alpha(\vec{t}) = 1-\epsilon$.

Proof. From the type-0 expert's incentive constraint (45), we have

$$\chi(\vec{t}) = 1 - \frac{\overline{m}(1-q)}{t(1-\alpha(\vec{t}))}. \tag{46}$$

Claim (i). Condition $t < Jm$ and (46) imply

$$\chi(\vec{t}) < 1 - \frac{\overline{m}(1-q)}{J\overline{m}(1-\alpha(\vec{t}))} \leq 1 - \frac{1-q}{J} = c.$$

From (44), $\alpha(\vec{t}) = 0$. Consider $t = J\overline{m}$, and suppose $\alpha(\vec{t}) > 0$. (46) implies $\chi(\vec{t}) < c$, and hence, (44) implies $\alpha(\vec{t}) = 0$, a contradiction.
Claim (ii). Condition $Jm < t < Jm/\epsilon$ and (46) imply

$$1 - \frac{\overline{m}(1-q)}{J\overline{m}(1-\alpha(\vec{t}))} < \chi(\vec{t}) < 1 - \frac{\overline{m}(1-q)}{\frac{J\overline{m}}{\epsilon}(1-\alpha(\vec{t}))},$$

and hence,

$$1 - \frac{1-c}{1-\alpha(\vec{t})} < \chi(\vec{t}) < 1 - \epsilon\frac{1-c}{1-\alpha(\vec{t})}.$$

Therefore, if $\alpha(\vec{t}) = 0$, then $\chi(\vec{t}) > c$, and (44) implies $\alpha(\vec{t}) = 1-\epsilon > 0$, a contradiction. Similarly, if $\alpha(\vec{t}) = 1-\epsilon$, then $\chi(\vec{t}) < c$, which implies $\alpha(\vec{t}) = 0$. Hence, it must be that $\alpha(t) \in (0, 1-\epsilon)$.
Claim (iii). Condition $t > Jm/\epsilon$ and (46) imply

$$\chi(\vec{t}) > 1 - \frac{\overline{m}(1-q)}{\frac{J\overline{m}}{\epsilon}(1-\alpha(\vec{t}))} = 1 - \epsilon\frac{1-c}{1-\alpha(t)} \geq 1 - \epsilon\frac{1-c}{\epsilon} = c,$$

and hence, $\alpha(\vec{t}) = 1-\epsilon$ from (44). ∎

Based on the preceding lemma, we can consider three cases separately, based on a conjectured value for $\overline{m}$.

Case A. If $\overline{m} > 1/J$, then point (i) of the preceding lemma applies for all $t \in [0,1]$, and hence, $\alpha(\vec{t}) = 0$ for all $t \in [0,1]$. The analysis of the type-0 expert's trading strategy is identical to the one given in the text for the low informativeness case. Briefly, the type-0 expert's indifference condition is (10), which implies that the mixing density must be of the form $(t-\overline{m})/(K\overline{m})$. That the density integrates to 1 implies in turn that $m = m_L$ as in the text. The initial conjecture that $\overline{m} > 1/J$ holds true if and only if the market informativeness measure $K < \underline{K}$, as in the text.

Proposition A.6 *(Low Informativeness, Perturbed). If $K \in (0, \underline{K})$, then the unique equilibrium is characterized as follows. The type-0 expert submits a sell order drawn from density $\phi_0^L(t)$, supported on interval $[m_L, 1]$. The firm selects the risky action in response to any single sell order, $\alpha(\vec{t}) = 0$. Beliefs and actions for all other order vectors are given in Lemma A.3 and Remark A.4. As $\epsilon \to 0$ this equilibrium converges to the one in Proposition 3.8.*

Note that the only difference between the perturbed and unperturbed equilibrium is that in the perturbed equilibrium, the probability of switching to the safe investment following two sell orders is $1-\epsilon$, rather than 1. The convergence of the perturbed equilibrium to the unperturbed equilibrium is immediate.

Intuitively, in the equilibrium with low market informativeness (which is characterized in Proposition 3.8), the firm never switches to the safe strategy following a single sell order, i.e. $\alpha(\vec{t}) = 0$ for all $t > 0$. Therefore, upper bound on the switching probability imposed by the perturbation is satisfied in this equilibrium following any single sell order, and therefore, neither the expert's trading strategy nor the firm's switching investment following a single sell order is affected by the perturbation.

Case B. If $\frac{\epsilon}{J} < \overline{m} < 1/J$, then point (i) of the preceding lemma applies for all $t \in [\overline{m}, J\overline{m}]$ and point (ii) applies for all $t \in (J\overline{m}, 1]$. Therefore, $\alpha(\vec{t}) = 0$ for all $t \in [\overline{m}, J\overline{m}]$ and $\alpha(\vec{t}) \in (0, 1-\epsilon)$ for all $t \in [J\overline{m}, 1]$. The analysis of the type-0 expert's trading strategy is identical to the one given in the text for the intermediate informativeness case. Briefly, the type-0 expert's indifference condition implies that the mixing density must be of the form $(t - \overline{m})/(K\overline{m})$ for $t \in (\overline{m}, J\overline{m})$ and $(J-1)/K$ for $t \in (J\overline{m}, 1)$. That the density integrates to 1 implies in turn that $m = m_I$ uniquely, as in the text. The initial conjecture that $\epsilon/J < \overline{m} < 1/J$ holds true if and only if the market informativeness measure $K \in (\underline{K}, \overline{K})$ and $\epsilon < Jm_I$.

Proposition A.7 *(Intermediate Informativeness, Perturbed). If $K \in (\underline{K}, \overline{K})$ and $0 < \epsilon < Jm_I$ then the unique equilibrium is characterized as follows. The type-0 expert submits a sell order drawn from density $\phi_0^I(t)$ supported on interval $[m_I, 1]$. Following a single sell order, $\alpha(\vec{t}) = 0$ if $t < Jm_I$ and $\alpha(\vec{t}) = 1 - Jm_I/t$ if $t \in [Jm_I, 1]$. Beliefs and actions for all other possible order vectors are given in Lemma A.3 and Remark A.4. As $\epsilon \to 0$ this equilibrium converges to the one in Proposition 3.9.*

Note that the only difference between the perturbed and unperturbed equilibrium is that in the perturbed equilibrium, the probability of switching to the safe investment following two sell orders is $1-\epsilon$, rather than 1. The convergence of the perturbed equilibrium to the unperturbed equilibrium is immediate.

The intuition is similar to the preceding case. In the unperturbed equilibrium, the probability of switching following a single sell order is bounded from above, $\alpha(\vec{t}) \leq 1 - Jm_I$, and hence, for $0 < \epsilon < Jm_I$, the upper bound imposed by the perturbation is again satisfied by the unperturbed equilibrium.

Case C. If $J\overline{m} < \epsilon$, then point (i) of the preceding lemma applies for all $t \in [\overline{m}, J\overline{m}]$, point (ii) applies for all $t \in (J\overline{m}, J\overline{m}/\epsilon)$, and point (iii) applies for all $t \in (J\overline{m}/\epsilon, 1]$. Therefore, $\alpha(\vec{t}) = 0$ for all $t \in [m, J\overline{m}]$, $\alpha(\vec{t}) \in (0, 1-\epsilon)$ for all $t \in [0, J\overline{m}]$, and $\alpha(\vec{t}) = 1-\epsilon$ for all $t \in (J\overline{m}, 1]$. Substituting into the type-0 expert's indifference condition and solving for $\phi_0(t)$, we find

$$\begin{aligned}
\phi_0(t) &= \frac{t-\overline{m}}{K\overline{m}} \quad \text{for} \quad t \in (\overline{m}, J\overline{m}) \\
\phi_0(t) &= \frac{J-1}{K} \quad \text{for} \quad t \in (J\overline{m}, J\overline{m}/\epsilon) \\
\phi_0(t) &= \frac{t\epsilon-\overline{m}}{K\overline{m}} \quad \text{for} \quad t \in (J\overline{m}/\epsilon, 1].
\end{aligned}$$

To find $\overline{m}$, we require that the density integrates to one,

$$\int_{\overline{m}}^{J\overline{m}} \frac{t-\overline{m}}{K\overline{m}} dt + \int_{J\overline{m}}^{J\overline{m}/\epsilon} \frac{J-1}{K} dt + \int_{J\overline{m}/\epsilon}^{1} \frac{t\epsilon-\overline{m}}{K\overline{m}} dt = 1,$$

and hence, two candidates for $\overline{m}$ exist:

$$\begin{aligned}
\overline{m}_- &= \frac{\epsilon}{J^2-\epsilon(J^2-1)}(K+1-\sqrt{(K+1)^2-J^2+\epsilon(J^2-1)} \\
\overline{m}_+ &= \frac{\epsilon}{J^2-\epsilon(J^2-1)}(K+1+\sqrt{(K+1)^2-J^2+\epsilon(J^2-1)}.
\end{aligned}$$

Note that for either candidate solution to be real for all sufficiently small perturbations, i.e. $\epsilon \to 0$ we must have $K+1 > J$, or equivalently $K > \overline{K}$; otherwise, for sufficiently small ϵ the term inside the square root is negative. That is, if such an equilibrium exists with small ϵ, then it must be that $K > J-1 = \overline{K}$. Next, note that for all $\epsilon \in (0,1)$ both candidate solutions are positive. Thus, the candidate value for $\overline{m}$ only needs to satisfy our initial conjecture, $J\overline{m} < \epsilon$.

First, we show that if $K > \overline{K}$, then $\overline{m}_+$ fails to satisfy the required condition, i.e. $J\overline{m}_+ > \epsilon$ for any $\epsilon > 0$.

$$\begin{aligned}
\frac{J\overline{m}_+}{\epsilon} &= \frac{J}{J^2-\epsilon(J^2-1)}(K+1+\sqrt{(K+1)^2-J^2+\epsilon(J^2-1)} > \frac{1}{J}(K+1+\sqrt{(K+1)^2-J^2}) \\
&= \frac{K+1}{J} + \sqrt{\left(\frac{K+1}{J}\right)^2 - 1} > 1,
\end{aligned}$$

where the last inequality follows from $K > \overline{K} = J-1$.

Second, we show that if $K > \overline{K}$, then $\overline{m}_-$ satisfies the required condition for all $\epsilon \in (0,1)$.

$$\frac{J\overline{m}_-}{\epsilon} = \frac{J}{J^2-\epsilon(J^2-1)}(K+1-\sqrt{(K+1)^2-J^2+\epsilon(J^2-1)}.$$

Solving,

$$\frac{J\overline{m}_-}{\epsilon} < 1 \iff \epsilon \in (-\frac{2J}{J^2-1}(K-(J-1)), \frac{J^2}{J^2-1}).$$

Note that the lower bound of this interval is negative because $K > \overline{K}$, and the upper bound exceeds one. Hence, the required inequality holds for all $\epsilon \in (0,1)$.

Proposition A.8 *(High Informativeness, Perturbed). If $K > \overline{K}$ and $\epsilon > 0$ then the unique equilibrium is characterized as follows. The type-0 expert submits a sell order drawn from density $\phi_0^H(t)$ supported on interval $[m_H, 1]$, where*

$$\phi_0^H(t) \equiv \begin{cases} \frac{t-m_H}{Km_H} & \text{if } t \in [m_H, Jm_H] \\ \frac{J-1}{K} & \text{if } t \in [Jm_H, Jm_H/\epsilon] \\ \frac{t\epsilon-m_H}{Km_H} & \text{if } t \in [Jm_H/\epsilon, 1] \end{cases}$$

and

$$m_H \equiv \frac{\epsilon}{J^2 - \epsilon(J^2-1)}(K + 1 - \sqrt{(K+1)^2 - J^2 + \epsilon(J^2-1)})$$

Following a single sell order, $\alpha(\vec{t}) = 0$ if $t < Jm_H$ and $\alpha(\vec{t}) = 1 - Jm_H/t$ if $t \in [Jm_H, Jm_H/\epsilon]$ and $\alpha(\vec{t}) = 1 - \epsilon$ if $t \in (Jm_H/\epsilon, 1]$. Beliefs and actions for all other possible order vectors are given in Lemma A.3 and Remark A.4.

As in the preceding cases, one difference between the perturbed and unperturbed equilibrium for $K > \overline{K}$ is that in the perturbed equilibrium, the probability of switching to the safe investment following two sell orders is $1 - \epsilon$, rather than 1. However, unlike the previous cases, this is not the only difference. First, the firm's switching strategy following a single sell order is considerably different. In the unperturbed equilibrium, the firm always switches to safe following a single sell; in the perturbed equilibrium, the maximum possible probability of switching is $1 - \epsilon$, and even this maximum switching probability is attained only on a range of large sell orders, $[Jm_H/\epsilon, 1]$. For smaller sell orders the firm either mixes (without attaining the maximum probability) or does not switch at all. Thus, there is a question of whether the firm's switching strategy converges to the unperturbed switching strategy as the perturbation vanishes. Furthermore, the type-0 expert's strategy is considerably different. Recall that in the unperturbed equilibrium, any mixing density in which $\phi_0(t) \geq (J-1)/K$ for all $t > 0$ could be part of an equilibrium. In the perturbed equilibrium, the type-0 expert's mixed strategy is determined uniquely, and it exceeds $(J-1)/K$ only for orders $t > Jm_H$. Therefore, it is not apparent that the type-0 expert's mixed strategy in the perturbed equilibrium converges to *one of* the multiplicity of unperturbed equilibria. To answer these questions, the following result is helpful.

Lemma A.9 *Suppose $K > \overline{K}$.*

(i) As $\epsilon \to 0$, $m_H \to 0$.

(ii) As $\epsilon \to 0$, $m_H/\epsilon \to \widetilde{m}$, where

$$\widetilde{m} \equiv \frac{1}{J}\Big(\frac{K+1}{J} - \sqrt{\Big(\frac{K+1}{J}\Big)^2 - 1}\Big).$$

(iii) $J\widetilde{m} \in (0,1)$.

(iv) *For* $\epsilon > 0$, $m_H/\epsilon > \widetilde{m}$.

Next we consider the convergence of the perturbed equilibrium in the high informativeness case.

Proposition A.10 *(Convergence of firm's strategy). Consider* $K > \overline{K}$. *As* $\epsilon \to 0$ *the firm's strategy in the unique equilibrium of the perturbed game converges to the firm's equilibrium strategy in the unperturbed game for every possible order vector.*

Proof. The result is obvious for every possible order vector that does not consist of a single sell order. Therefore, we prove that $\alpha(\vec{t}) \to 1$ as $\epsilon \to 0$ for all $t > 0$.

Consider any $t > J\tilde{m}$. For ϵ sufficiently small, Lemma A.9(ii) implies $t > Jm_H/\epsilon$ as well. Therefore, Proposition A.8 implies $\alpha(t) = 1 - \epsilon$ and $\alpha(t) \to 1$ as $\epsilon \to 0$.

Consider next any $t < J\tilde{m}$. For ϵ sufficiently small, Lemma A.9(ii) implies $J\frac{m_H}{\epsilon}\epsilon < t < Jm_H/\epsilon$. Therefore, Proposition A.8 implies $\alpha(t) = 1 - J\frac{m_H}{\epsilon}\epsilon$ which converges to one as $\epsilon \to 0$.

Consider finally $t = J\tilde{m}$. Lemma A.9(iv) implies that $t = J\tilde{m} < Jm_H/\epsilon$. Therefore, for ϵ sufficiently small, $\alpha(t) = 1 - J\frac{m_H}{\epsilon}\epsilon$, which converges to one. ■

Next, we consider the convergence of the type-0 expert's mixed strategy. We consider convergence in distribution, i.e. pointwise convergence of the CDF. To this end, we calculate the CDF of the type-0 expert's mixed strategy for a given perturbation by integrating the density $\phi_0^H(t)$:

$$\Phi_0^H(t|\epsilon) = \begin{cases} \frac{(t-m_H)^2}{2Km_H} & \text{if } t \in [m_H, Jm_H] \\ \frac{(J-1)^2}{2K}m_H + \frac{J-1}{K}(t - Jm_H) & \text{if } t \in [Jm_H, Jm_H/\epsilon] \\ \frac{(J-1)^2}{2K}m_H + \frac{J-1}{K}Jm_H\frac{1-\epsilon}{\epsilon} + \frac{1}{2K}\left((2J - J^2)\frac{m_H}{\epsilon} + t^2\frac{\epsilon}{m_H} - 2t\right) & \text{if } t \in [Jm_H/\epsilon, 1] \end{cases}$$

Calculating the pointwise limit as $\epsilon \to 0$, we find the following result.

Proposition A.11 *(Convergence of type-0 expert's strategy). Consider* $K > \overline{K}$. *(i) As* $\epsilon \to 0$ *the type-0 expert's mixed strategy converges in distribution,* $\Phi_0^H(t|\epsilon) \to \Phi_0^*(t)$, *where*

$$\Phi_0^*(t) = \begin{cases} \frac{J-1}{K}t & \textit{if } t \in [0, J\widetilde{m}] \\ \frac{(J-1)}{K}(J\widetilde{m}) + \frac{1}{2K}\left((2J - J^2)\widetilde{m} + \frac{t^2}{\widetilde{m}} - 2t\right) & \textit{if } t \in [J\widetilde{m}, 1]. \end{cases}$$

(ii) The limiting distribution $\Phi_0^*(t)$ *is inside the multiplicity of distributions that are possible in the equilibrium of the unperturbed game.*

Proof. *Claim (i).* Consider the behavior of $\Phi_0^H(t|\epsilon)$ as $\epsilon \to 0$. Note first that $\Phi_0^H(0|\epsilon) = 0$ for all $\epsilon > 0$, and hence, $\lim_{\epsilon\to 0}\Phi_0(0|\epsilon) = 0$.

Next, consider $t \in (0, J\widetilde{m}]$. Because $Jm_H \to 0$ as $\epsilon \to 0$, an $\overline{\epsilon}$ exists such that, $t > Jm_H$ for $\epsilon > \overline{\epsilon}$. Furthermore, for any $\epsilon > 0$, $t \leq J\widetilde{m} < Jm_H/\epsilon$, where the last inequality follows from point

(iv) of Lemma A.9. Therefore, $t \in (Jm_H, Jm_H/\epsilon)$ for any $\epsilon \in (0, \overline{\epsilon})$. Thus, for any $t \in (0, J\widetilde{m}]$,

$$\lim_{\epsilon \to 0} \Phi_0^H(t|\epsilon) = \lim_{\epsilon \to 0} \Big\{ \frac{(J-1)^2}{2K} m_H + \frac{J-1}{K}(t - Jm_H) \Big\} = \frac{J-1}{K} t,$$

where use has been made of point (i) of Lemma A.9.

Finally, consider $t \in (J\widetilde{m}, 1]$. Because $Jm_H/\epsilon \to J\widetilde{m}$ as $\epsilon \to 0$, there exists $\overline{\overline{\epsilon}}$ such that $t \in (Jm_H/\epsilon, 1]$ for $\epsilon < \overline{\overline{\epsilon}}$. Thus, for any $t \in (J\widetilde{m}, 1]$,

$$\begin{aligned} \lim_{\epsilon \to 0} \Phi_0^H(t|\epsilon) &= \lim_{\epsilon \to 0} \Big\{ \frac{(J-1)^2}{2K} m_H + \frac{J-1}{K} J m_H \frac{1-\epsilon}{\epsilon} + \frac{1}{2K} \Big((2J - J^2) \frac{m_H}{\epsilon} + t^2 \frac{\epsilon}{m_H} - 2t \Big) \Big\} \\ &= \frac{(J-1)}{K}(J\widetilde{m}) + \frac{1}{2K} \Big((2J - J^2)\widetilde{m} + \frac{t^2}{\widetilde{m}} - 2t \Big), \end{aligned}$$

where use has been made of points (i) and (ii) of Lemma A.9.

Claim (ii). We show that the limiting distribution satisfies condition (20), and it is therefore an element of the multiplicity of mixing distributions that are part of the equilibria of the unperturbed game. Note that $\Phi_0^*(t)$ is continuous, differentiable, and

$$\frac{d\Phi_0^*(t)}{dt} = \begin{cases} \frac{J-1}{K} & \text{if } t \in [0, J\widetilde{m}] \\ \frac{t - \widetilde{m}}{K\widetilde{m}} & \text{if } t \in [J\widetilde{m}, 1]. \end{cases}$$

Thus, for all $t \in (0, 1]$ we have $\frac{d\Phi_0^*(t)}{dt} \geq \frac{J-1}{K}$, as required by (20). ■

B MULTIPLE INFORMED TRADERS

Consider an extension of the model in which $3 \leq N < \infty$ informed traders exist with probability ψ.[21] Thus, the distribution of the number of informed traders, n, is:

$$n = \begin{cases} N & \text{w.p. } \psi \\ 1 & \text{w.p. } (1-\psi)\, a \\ 0 & \text{w.p. } (1-\psi)(1-a) \end{cases}.$$

Thus, conditional on the lack of multiple informed traders, the model reverts back to the one in the text. Traders initially do not know if they are the only informed trader or if others exist.

B.1 MARKET RELIANCE

We posit an equilibrium identical to the one described in the text: the uninformed do not trade, the type-1 informed traders buy one share, and the type-0 informed traders sell using a mixed density $\phi_0(t)$. The response of the market maker and firm are identical, except that four additional order

[21] If $N = \infty$ the screening constraints may no longer be necessary, and in any event the information is not private if known by infinitely many traders. We also rule out $N = 2$ since, as shown below, the mechanism fails to remove adverse selection from the market in this case, severely reducing benefits of the mechanism.

vector types, N and $N+1$ buy and sell orders, are on path. Clearly N or $N+1$ sell (buy) orders reveal state 0 (1), in which case the market maker sets a price of $1-c$ (1), and the firm executes the safe (risky) strategy. The market maker and firm respond to all other order vector types, on and off path, as in the text.

An informed trader has no incentive to deviate from this equilibrium: if an informed trader deviates and multiple informed traders exist, then Remark 3.3 implies the market maker and firm will follow the $N-1$ traders who are not deviating. Since their trades perfectly reveal the state, the informed trader who deviates earns zero profits if multiple traders exist. If one trader exists then the model reduces to the one in the text and the gains from deviating are negative. In other words, each informed trader behaves as if he is the only informed trader since the existence of multiple informed traders reduces profits to zero regardless of the trader's strategy.

An uninformed trader also has no incentive to deviate. If multiple informed traders exist, then the market maker will follow the $N > 2$ informed sell or buy orders, so the state is revealed and profits of the uninformed trader are zero. If multiple traders do not exist, then the model reduces to the one in the text and Lemma 3.4 applies.

Finally, the market maker and firm also do not deviate because if less than three trades are observed, the model reduces to the model in the text. Note that K is implicitly defined conditional on the existence of at most one trader, and J is independent of the number of traders. Thus the regions for which each type of equilibrium holds does not change.

The payoffs under market reliance change when multiple informed traders is a possibility. In particular, since multiple informed traders immediately reveals the state, profits for the informed traders and adverse selection costs of the market are lower, and project selection is more accurate. Consider first the payoff to the informed trader, u_0^*, which is defined in the text as conditional on the existence of the type-0 expert. Modifying the definition to the existence of a single type-0 expert, we see that u_0^* is unchanged, since the model reverts to the one in the text in this case. The firm value under market reliance in the low and intermediate informativeness cases changes as follows. With probability $1-\psi$ the model reverts to the one in the text and the firm correctly switches to safe if an informed type-0 expert exists and the a liquidity shock occurs. The firm also correctly switches to safe if multiple type-0 traders exist. Thus:

$$V_R = 1 - q + a\left(1-\psi\right)ql\left(1-c\right) + q\psi\left(1-c\right) - aq\left(1-\psi\right)u_0^*.$$

The first best value of the firm changes since the probability of information existing in the economy changes. We have:

$$V^* = 1 - q + aql\left(1-\psi\right)\left(1-c\right) + \psi q\left(1-c\right).$$

Combining the above two equations and using the definition of u_0^* we have that:

$$V_R = V^* - aq\left(1-\psi\right)\left(1-l\right)\left(1-c+m\left(1-q\right)\right).$$

The firm makes better project decisions under market reliance since with probability ψq multiple traders exist and reveal the state is bad. Further, adverse selection costs are lower because the state is revealed more often.

Consider the payoffs now in the high informativeness case. Recall the firm only incorrectly switches when no informed trader exists and a liquidity shock occurs. This now occurs with probability $(1-\psi)(1-a)\,l$. Thus:

$$V_{NR} = V^{*} - (1-a)(1-\psi)\,l\,(c-q)\,.$$

B.2 MECHANISM RELIANCE

For mechanism reliance, we assume first that if the firm hires an advisor, then the firm has no credible way of reporting the recommendation of the advisor before the market opens. In this case, if $N=2$ and one type-0 expert accepts the mechanism, the other could still trade in the market without fully revealing his information. It follows that the mechanism does not eliminate adverse selection costs, severely reducing the benefit of the mechanism. With $N>2$ however, if one type-0 expert accepts the mechanism, then the other experts all sell, revealing the bad state, resulting in zero profits for the experts and no adverse selection costs.

Next we posit an equilibrium in which the firm offers the mechanism in the text with the wage modified below, and all experts play a pure strategy which is to accept the mechanism if offered.[22] If another informed agent takes up the mechanism, then the remaining informed traders play an identical strategy as in market reliance. Without loss of generality, we assume off path that if the advisor and the market recommend opposite decisions, the firm follows the advisor.

Consider now potential deviations by an informed expert who is not first to be offered the mechanism. Such a trader earns zero profits in all states, since multiple trades perfectly reveal the state. Deviations have no effect if $N \geq 4$ since the market maker will follow the trades of the two or more traders who did not take up the mechanism and did not deviate. If $N=3$ then if one trader takes up the mechanism, and one of the two remaining informed traders deviates, then the profits from deviating are as follows. If the news is bad and no liquidity shock occurs, and one informed expert deviates to buy, then the market maker sets a price equal to one, but the firm follows the advisor and plays safe. The firm value is $1-c$ and so profits from deviating are $-c<0$. If a liquidity shock does occur, state 0 is revealed and profits are zero. Thus a type-0 expert who is not first to be offered the mechanism will not deviate to buy. If the state is good and no liquidity shock occurs and an informed trader deviates to sell, then state 1 is revealed and the deviation earns zero profits. If the state is good and a liquidity shock occurs and an informed trader deviates by selling, then the market maker incorrectly sets the price to $1-c$. However, the firm follows the advisor by playing risky, yielding a firm value of 1. The profits from this deviation are therefore $-c<0$. It follows that the traders who do not take up the mechanism have no profitable deviations.

Consider now potential deviations by the expert who is offered the mechanism. Suppose the expert deviates and does not accept the mechanism. Any order vector with more than one trade

[22] Mixed strategy equilibria may also exist.

is then off path. Such order vectors always reveal the state, and so profits from deviating are zero if multiple trades occur. Similarly, one buy order is also off path, and reveals the state to be one, yielding zero profits. If a single sell order occurs, then the market maker sets a price equal to $1-q$ and the deviation earns a positive payoff.

To compute the expected profit from deviating, the expert trader must update the probability that he is the only informed trader, conditional on his own existence. We have:

$$Pr\left(1 \text{ informed } | 1 \text{ known trader }\right) = \frac{(1-\psi)\, a}{(1-\psi)\, a + \psi}.$$

With the above probability, an informed trader who deviates and does not take up the mechanism earns profits $(1-q)$, if the state is bad, no liquidity shock occurs, and he is the only informed trader. Therefore, the expected profit from deviating is:

$$\underline{u} = \frac{(1-\psi)\, a}{(1-\psi)\, a + \psi} q\,(1-l)\,(1-q)\,.$$

The deviation gain reduces to the one in the text as $\psi \to 0$. For $\psi > 0$, the range of B for which the mechanism is feasible becomes:

$$B \geq \frac{(1-\psi)\, a}{(1-\psi)\, a + \psi} l\,(1-q)\,.$$

Thus the mechanism is feasible for a wider range of B, since deviating is less attractive with multiple traders.

Consider now the mechanism reliant firm value. The firm follows the advisor and so earns V^* cash flows, and pays $\underline{u}$ whenever an informed trader exists. Thus:

$$\begin{aligned} V_{DRM} &= V^* - (\psi + (1-\psi)\, a)\, \underline{u}. \\ &= V^* - (1-\psi)\, aq\,(1-l)\,(1-q) \end{aligned}$$

The value of the mechanism relative to perfect information increases with multiple traders because the wage is lower, since the gains from deviating are lower.

B.3 MARKETS VERSUS MECHANISMS

Comparing V_{MKT} with V_{DRM}, we see that adding multiple traders increases the second term of each by a factor $1-\psi$. Thus the conditions given the text for which markets dominate mechanisms remain unchanged. The mechanism value increases since the gains from deviating are lower. But the gains from the market also increase, because project selection is better and adverse selection costs are lower.

B.4 THE CASE OF $N = 2$

Consider here the extension to multiple traders, where now $N = 2$ with probability ψ. It is clear that firm value under market reliance does not change. However, we will show that the firm value decreases under mechanism reliance, relative to the case where $N \geq 3$.

If $N = 2$ and the firm posts a mechanism which the first informed trader accepts, the second informed trader is free to trade in the market. Because the firm has no credible way of communicating the information provided by the first expert, the second expert has an information advantage. The market maker is aware that the firm has hired an expert, and so knows the firm value is either 1 or $1 - c$. Therefore, the spread narrows and the trading gains of the second informed trader are less. Denote these trading gains as $\tilde{u}$. Since the trading gains of the second informed trader equal the adverse selection costs under mechanism reliance, we have that firm value under the mechanism is:

$$V_{DRM} = V^* - (\psi + (1 - \psi) a) \underline{u} - a (1 - \psi) q\tilde{u}. \tag{47}$$

First consider trading gains of an expert who deviates and does not sign the contract. If a second expert does not exist, the expert convinces the market he does not exist and trades will no price impact as in the case where $N \geq 3$. If a second expert does exist, that trader will sign the contract, and the expert will achieve trading gains of $\tilde{u}$. Thus, the ex ante value of deviating is:

$$\underline{u} = \frac{(1 - \psi) a}{(1 - \psi) a + \psi} q (1 - l) (1 - q) + \frac{\psi}{(1 - \psi) a + \psi} \tilde{u}. \tag{48}$$

Notice that the wage when $N = 2$ is larger than if $N \geq 3$. If $N \geq 3$ and more than one expert exists, trading gains are zero since the expert(s) who do not sign the contract will trade and reveal the state, but with $N = 2$ the expert who deviates gets still gets trading gains of $\tilde{u}$, since he is the only trader.

It remains to determine $\tilde{u}$. First note that if the expert is type-0 and another expert has accepted the mechanism, then the expert who trades knows that the firm is worth $1 - c$ as the firm will follow the advice of the expert who signed the contract. Therefore, the trader knows that his trade size has no effect on α, which is always one. However, the trader still faces the price discipline imposed by the market maker who does not know the true state. Thus:

$$u_0 (t, T) = t (p (T) - (1 - c)). \tag{49}$$

The market maker knows the firm has hired an expert who revealed the state. Therefore, the market maker sets a price of:

$$p (T) = (1 - \chi (T)) \cdot 1 + \chi (T) (1 - c). \tag{50}$$

With probability χ the market maker believes the state is bad and the firm will execute the safe strategy, leading to a payoff of $1-c$. With probability $1-\chi$, the market maker believes the expert has truthfully told the firm to play the risky strategy, leading to a firm value of 1. Since the firm is now following the advice of the hired expert, trades no longer have an impact on firm decisions. Substituting the price into the trading profits of the type-0 expert, we see that:

$$u_0(t) = tc(1-\chi(t))(1-l)\,. \tag{51}$$

At the minimum trade size m, $\chi(t) = q$, and so the trader's indifference condition implies that the type-0 expert's profits are:

$$\tilde{u} = mc(1-q)(1-l)\,. \tag{52}$$

To compute the minimum trade size m, note that the market maker knows that at least one expert exists, so beliefs evolve according to:

$$\chi(t) = \frac{q\psi(1-l)\phi_0(t) + (1-\phi)alq}{q\psi(1-l)\phi_0(t) + (1-\psi)al}. \tag{53}$$

The market maker updates the probability of state zero by considering the probability of observing a single sell order conditional on state zero and conditional on the existence of at least one informed trader. Introducing a change of variables results in:

$$\chi(t) = \frac{\tilde{K}\phi_0(t) + q}{\tilde{K}\phi_0(t) + 1}, \quad \tilde{K} \equiv \frac{\psi}{a(1-\psi)}\frac{q(1-l)}{l}. \tag{54}$$

Here $\tilde{K}$ has a sightly different interpretation since it reflects market informativeness conditional on at least one expert existing.

Next the type-0 trader's indifference condition $u_0(t) = u_0^*$ and equations (58) and (54) imply:

$$u_0(t) = tc(1-\chi(t))(1-l) = u_0^* = mc(1-q)(1-l)\,, \tag{55}$$

$$t\left(1 - \frac{\tilde{K}\phi_0(t) + q}{\tilde{K}\phi_0(t) + 1}\right) = m(1-q)\,, \tag{56}$$

$$\phi_0(t) = \frac{t-m}{m\tilde{K}}, \tag{57}$$

which is identical to the text up to the parameter $\tilde{K}$. Since the density must integrate to one, it follows that $m = m_L\left(\tilde{K}\right)$. Notice that the low informativeness case is the only case here, since the other cases in the text are the result of the type-0 trader seeking to prevent the firm from switching to the safe strategy with probability one. Thus,

$$\tilde{u} = m_L \left(\tilde{K}\right) c \left(1 - q\right) \left(1 - l\right). \tag{58}$$

Finally, we combine equations (47) and (58) to find the firm value under the DRM when $N = 2$:

$$V_{DRM} \left(N = 2\right) = V^* - q \left(1 - q\right) \left(1 - l\right) \left(\left(1 - \psi\right) a + \psi 2 c m_L \left(\tilde{K}\right)\right). \tag{59}$$

It is straightforward to verify that $V_{DRM} \left(N = 2\right) < V_{DRM} \left(N \geq 3\right)$. Therefore, the region of the parameter space for which mechanisms yield higher firm value than markets is smaller when $N = 2$ versus $N \geq 3$.