An Imperfect Storm: Fat-Tailed Hurricane Damages, Insurance, and Climate Policy*

Marc N. Conte          David L. Kelly
Department of Economics Department of Economics
Fordham University       University of Miami
February 2, 2016

Abstract

We perform two tests that estimate the thickness of the tails of the distribution of aggregate US hurricane damages. Both tests reject the hypothesis that the distribution of damages is thin tailed at the 95% confidence level, even after correcting for inflation, population, and per capita income growth. Our point estimates of the shape parameter of the damage distribution indicate that the distribution has finite mean, but infinite variance.

In the second part of the paper, we develop a microfoundations model of insurance and storm size that generates fat tails in aggregate hurricane damages. In the model, the distribution of the number properties within a random geographical area that lies in the path of a hurricane drives fat tails in hurricane damages, and we confirm that the distribution of coastal city population is fat tailed in the US. We show empirically and theoretically that other random variation, such as the distribution of hurricane strength and the distribution of damages across individual properties do not generate fat tails. We consider policy options such as climate change mitigation, policies which encourage adaptation, reducing subsidies for coastal development, and disaster relief policies, which distort insurance markets. Such policies can reduce the thickness of the tail, but do not affect the shape parameter or the existence of the fat tail.

Keywords: natural disasters, fat tails, hurricanes, adaptation, disaster aid, property insurance.
JEL Codes: Q54, H84, Q58, R11

*We would like to thank seminar participants at the 15th Occasional Workshop on Environmental and Resource Economics, the 2015 American Economics Association Meetings, the 2015 Summer Meeting of the Association of Environmental and Resource Economists, Georgia State University, and the University of Miami for useful comments and suggestions.
1 Introduction

Hurricanes are the most costly natural disasters in the US (Smith and Katz 2013). Indeed, several recent storms (e.g. Katrina and Sandy) were catastrophic, causing damages on the order of tens of billions of dollars, as well as loss of life, dislocations, and suppression of economic activity. Despite the large losses, little is known about the ultimate cause of catastrophic hurricane damages and to what extent government policies can reduce the welfare losses caused by these natural disasters.

Although some hurricanes are catastrophic, the majority of storms cause little or no damage. Hence, one might suspect that the distribution of hurricane damages is fat tailed. A growing body of literature shows that the distribution of total damages from natural disasters is often fat tailed. For example, Kousky and Cooke (2009) find evidence of fat-tailed aggregate damages for several varieties of natural disasters, including crop indemnities and National Flood Insurance claims.1

In the first part of this paper, we provide empirical evidence that the distribution of total hurricane damages is indeed fat tailed, even after controlling for inflation and growth in population and per capita income. In the second part of the paper, we develop a microfoundations model that generates a fat-tailed aggregate damage distribution, and show how public policies affect the upper tail of the distribution.

We perform two tests to estimate the thickness of the tails of the damage distribution. The first test estimates the parameters of the generalized Pareto distribution (GPD) using maximum likelihood. The GPD nests both fat and thin tails and is a commonly used test for fat tails (Brazouskas and Kleefeld 2009). The second test estimates the slope of the mean excess function: the expectation conditional on a realization being larger than a threshold. The slope of the mean excess as a function of the threshold is positive if and only if the distribution is fat tailed (Ghosh 2010).

The two tests produce similar estimates of the shape parameter of the damage distribution in the range of 0.66 to 0.8, rejecting the hypothesis that the distribution of damages is thin tailed at the 95% confidence level. The point estimates of the shape parameter suggest that the damage distribution is fat tailed with finite mean but infinite variance.2

---

1In addition, fat-tailed uncertainty has been shown to pose a particular challenge to policy makers in the context of climate change (Weitzman 2014; Kelly and Tan 2015).

2The shape parameter measures the thickness of the tail of the distribution. The mean and variance of the distribution are finite if the shape parameter is less than or equal to one and one half, respectively. See Cooke and Nieboer (2011) for a primer on fat-tailed
A fat-tailed hurricane damage distribution poses a number of challenges. The estimated mean damage depends closely on the damage from catastrophic storms, for which relatively few observations exist. The mean damage, upon which insurance is priced, is more difficult to estimate when damages are drawn from a fat-tailed distribution. Similarly, the expected damage from a catastrophic storm (e.g., the damage from a once-in-a-hundred-years storm), is also more difficult to estimate, meaning that regulators will have difficulty establishing minimum levels of reserve capital, increasing the likelihood of a catastrophic storm for which reserves are inadequate and adding risk to the households. Insurance firms must also hold more costly reserves. Policies that reduce the mass of the upper tail of the damage distribution are potentially welfare increasing. Identifying such policies requires an empirical and theoretical understanding of the determinants of the fat tail in the damage distribution.

Hurricanes that cause catastrophic damages feature extraordinary storm intensity (e.g., Andrew, which had a maximum sustained wind speed of 170 mph), make landfall in areas with an extraordinarily large population and property base (e.g., Sandy, which struck the New York metropolitan area), and cause extraordinary damage to individual properties. Aggregate damages are a function of each of these random variables. A fat tail in the distribution of one or more of these random variables can imply a fat-tailed damage distribution.

One possible cause of fat-tailed hurricane damages is the distribution of storm intensity. However, we show empirically that storm wind speed is thin tailed.\(^3\) This result is intuitive: the expected wind speed conditional on exceeding a threshold should decline with the threshold, because the extra energy required to generate and sustain larger storms is not readily available. Our results show that the wind speed distribution cannot cause a fat tail in the damage distribution.

Aggregate damages are the sum of individual property damages. Because property damages are bounded and therefore have finite variance, the central limit theorems apply. The sum of property damages becomes approximately normally distributed (thin tailed) as the number of properties becomes large; random variation in the individual damage across properties cannot be the source of fat tails in the damage distribution.

Considerable evidence exists that the distribution of city population in the US is fat tailed (see for example, Gabaix 1999; Gabaix and Ibragimov 2011; distributions.

\(^3\)Maximum wind speed is a commonly used measure of storm intensity (Nordhaus 2010). Other measures of storm intensity, such as size, are correlated with wind speed (Holland 1980).
González-Val et al. 2015). Gabaix and Ibragimov (2011) and others find the exponent of the Pareto distribution is close to one (known as Zipf’s law) for US metropolitan areas above a minimum population.\footnote{Gabaix (1999) proposes a theory for city sizes that generates a Pareto distribution. In their model, wages adjust to offset city-specific amenity shocks, which keeps the growth rate of cities independent of the size, which implies a power law steady state distribution.} However, González-Val et al. (2015) considers the GPD and finds a range of potential exponents.

We test the coastal population distribution for fat tails and find that the coastal population distribution is also fat tailed, with a shape parameter in the range of 0.6 to 0.8, very close to the shape parameter of the aggregate damage distribution.\footnote{Our interest is in coastal property, not the US population. Nonetheless, the coastal property distribution is correlated with the coastal population distribution.} Our empirical results indicate that a determinant of fat tails in the damage distribution is fat tails in the geographic distribution of property. Most hurricanes intersect geographic areas with little or no property. However, hurricanes intersect areas with large amounts of property more often than expected given a normal distribution, causing the fat tail in the damage distribution.

Having demonstrated empirically that hurricane damages are fat tailed, we present a model of homeowner behavior that links policies to the mass of the upper tail of the hurricane damage distribution.\footnote{While we are not aware of any other attempt to create a microfoundations model of fat tailed aggregate damages, in a recent paper Jones (2015) creates a model that generates fat tails in the distribution of income. In a theory in finance, fat tails arise in financial return data through random news events (for example, Clark 1973; Haas and Pigorsch 2011).} In the model, households develop both coastal and inland properties using constant marginal cost technologies. The cost of property insurance raises the total cost of ownership for coastal properties, thus discouraging development.\footnote{In practice, households may not have correct risk perceptions, and evidence of homeowners updating their risk perceptions in response to natural disasters is mixed. See Bin and Polasky (2004); Hallstrom and Smith (2005); Kelly et al. (2012) for analyses of changes in risk perceptions following hurricanes. Beron et al. (1997) studies risk perceptions following an earthquake and Gallagher (2014) finds that enrollment in flood insurance programs increases following major flooding events and then subsides until the next such event.} A disaster relief agency exists which reimburses a fraction of household losses.

The total coastal property at risk as determined by household choices is distributed geographically into population centers. Following the empirical results, the size distribution of the population centers has a fat tail. Finally, a model of hurricane size, wind speed, and adaptations (e.g. building codes) determines the area of damage. In the model, a hurricane of random (thin tailed) size and wind speed may intersect a population center of random (fat tailed) size, in which case individual properties experience random, bounded
damages.

We show that, as the number of potentially affected properties becomes large, the damage distribution converges to a distribution that is approximately fat tailed, with tail index equal to the tail index of the property distribution, which is consistent with our close empirical estimates of the shape parameters for damage and coastal population.\(^8\) Although aggregate damages are the sum of individual damages with finite variance, the aggregate damage distribution is not normal because the number of terms in the sum (the number of properties in the population center) is random and fat tailed.

The costs of fat-tailed damages motivate public policies to thin the tail of the damage distribution. In the model, the distribution of hurricane damages is a function of several policies, including climate policy (which affects the frequency of large storms), disaster relief and development subsidies (which affect the total property at risk), and adaptations (which affect the minimum wind speed that causes damage). Each of these policies affects the damage from a catastrophic storm in different ways.

One important policy is disaster relief and other subsides to coastal development. Indeed, the rise in coastal development has led to concern that disaster relief increases hurricane damage (Bunby 2006).\(^9\) After a hurricane, the Federal Emergency Management Agency (FEMA) may declare the affected area a disaster zone and provide relief from uninsured damages. This intervention is meant to reduce the welfare losses from hurricanes.

We show that moral hazard created by disaster relief adds to the total property at risk, both on the extensive (more coastal land is developed versus inland, \textit{ceteris paribus}) and intensive (developed coastal land has more property than developed inland \textit{ceteris paribus}) margins. An increase in the total property at risk implies population centers have more property, increasing the damage from a once-in-a-hundred-years storm.

Adaptations such as stronger building codes affect the damage distribution. Hurricane strength declines with the distance from the eye of the storm (Holland 1980). If a random hurricane path intersects a population center away from the eye of the storm, then little or no damage will result if building codes are sufficiently strong. Much of the literature on adaptations (for example, Kelly et al. 2005; Mendelsohn et al. 1994) shows empirically how adaptations can reduce the point estimate of damages. Our complementary analysis provides theoretical insight into the process whereby adaptation policies affect

---

\(^8\)For the GPD distribution, the tail index is the inverse of the shape parameter.
\(^9\)Populations in coastal communities at risk of hurricane damages are growing more rapidly than populations in the rest of the United States (Blake et al. 2011).
the tail of the damage distribution.

We show that an increase in the strength of adaptations reduces the geographic area of damage in two ways. First, adaptations, such as stronger building codes, reduce the radius of winds that are strong enough to cause damage, as the wind speed beyond a certain distance from the eye is no longer strong enough to cause damage. Second, adaptations decrease the time from landfall during which the hurricane is sufficiently strong to cause damages. The maximum wind speed declines over land, reaching the point where adaptations are strong enough to prevent damage more quickly. In turn, the reduction in the geographic area of damage implies that more hurricanes miss population centers and cause no damage. Because a sample of hurricanes includes more storms with little or no damage, adaptations reduce the damages of a once-in-a-hundred-years storm.

Climate change mitigation is another policy lever that affects the aggregate damage distribution. Some research suggests that a positive link exists between greenhouse gas emissions and hurricane frequency (Holland and Webster 2007; Mann and Emanuel 2006; Emanuel 2005). However, there is not a consensus on this point, as other studies fail to support that conclusion (Vecchi et al. 2008; Vecchi and Knutson 2008; Knutson et al. 2008). One recent study finds that the number of intense storms (category 4 or 5 hurricanes) may double by the end of the 21st century, even as the overall frequency of tropical cyclones decreases (Bender et al. 2010), suggesting that changing climate may primarily affect the tails of the distribution of storm size.10

We show that the effect of climate change mitigation is similar to that of adaptation. Limiting climate change reduces the increase in energy available for storm formation. By reducing the likelihood of intense hurricanes, the geographic area of damage decreases, reducing the damage from a once-in-a-hundred-years storm.

Finally, we show that none of the policies we consider affect the tail index or the existence of the fat tail. While the policies can thin the tail of the distribution, the policies cannot make a fat tailed distribution thin tailed. Specifically, the model predicts that the fat tail is present regardless of the strength of adaptations (e.g. both less and more developed countries should see catastrophic storms more frequently than expected from a normal distribution).

The next section introduces the data used in the empirical analyses. Section

10Pielke et al. (2008), fail to identify an observable linear trend in the mean damage associated with each storm making landfall in the United States from 1900-2008 after controlling for population growth and inflation. In contrast, our focus is on changes in other moments, such as the risk of catastrophic storms.
3 describes the empirical methods. Section 4 presents the empirical results. Section 5 presents a model of property development, insurance, and hurricane strength. Section 6 derives the aggregate damage distribution and the policy implications. Section 7 concludes.

2 Data

The distributions of hurricane damages, hurricane strength, and coastal population are explored using three data sets. The first data set was downloaded from the ICAT insurance firm (http://www.icatdamageestimator.com) website, which provides damages from all storms making landfall in the United States between 1900 and 2012. The second data set is provided by a NASA website, which provides detailed storm-location and characteristic information (e.g., minimum pressure and maximum sustained wind speed) for all storms that originated in the Northern Atlantic Basin between 1850 and 2012. Given changes in satellite technology, only data since 1950 is used in the below analysis of storm strength. The third data set of coastal city populations is from the 2010 Census conducted by the US Census Bureau.

The ICAT data set aggregates data contained in the Monthly Weather Reviews, published since 1872, which provide summaries of the storms that have occurred in the North Atlantic Basin each year. These reports are currently published by employees of the National Hurricane Center, which is an office within the National Weather Service and the National Oceanic and Atmospheric Administration. The raw data includes storm name and the economic damages associated with the storm.\textsuperscript{11} Several adjustments are made to the reported storm-year damages to increase the degree of comparability for damages across different years.

A number of features of coastal communities in the United States have changed through time in ways that might lead the economic damages caused by a storm to depend on the year in which the storm made landfall. Of principal importance is the amount of physical capital located in at-risk coastal communities. The total property in coastal communities has varied through time, though the value of this property is not historically recorded, confounding efforts to directly control for temporal variation in at-risk capital. Other related socioeconomic characteristics, such as per capita income and population are historically available. Using 2005 country-level data from the World Bank, Bakkensen (2014) find that capital scales proportionately with per capita in-

\textsuperscript{11}According to the ICAT website, since 1987, the National Hurricane Center has calculated the total economic damages caused by a storm event as twice the insured damages.
come, estimating that capital stock is 2.65 times the per-capita income in a nation. In the absence of a time series of the capital stock of coastal communities in the United States between 1900 and 2012, we will rely on the population and income level of coastal communities to control for variation in capital stocks across time.

We adopt the methodology of Pielke et al. (2008), who controls for temporal changes in prices due to inflation and the population and wealth of coastal communities, by normalizing damages relative to 2012 prices, income, and population for each storm from 1900-2012 according to:

$$D_{i,2012} = D_{i,y} \times I_{y/2012} \times RWPC_{y/2012} \times P_{2012/y},$$

where $D_{i,2012}$ is the normalized damage of hurricane $i$ in 2012 dollars; $D_{i,y}$ is the reported damage in year $y$ dollars; $I_{2012/y}$ is the inflation adjustment; $RWPC_{2012/y}$ is a real per capita income adjustment; and $P_{2012/y}$ is the coastal county population adjustment. Note that controlling for per capita income assumes that the income elasticity of hurricane damages is one. This assumption implies no changes in adaptation to the threat of hurricane damages as wealth in coastal communities within the United States increases. This assumption is consistent with the finding from Bakkensen (2014), who find no evidence of changes in adaptation with respect to income in the United States over the period 1960 to 2010. If a policy-driven or market response to hurricane damage existed that was influenced only by per capita income, and not storm frequency or intensity, then our approach might overestimate the damages caused by recent storms, when income levels are highest. While a number of catastrophic storms have occurred in recent decades, six of the top 10 storms as measured by damages in 2012 dollars using the above methodology occurred prior to the end of World War II, when per capita income in coastal areas was much lower than modern levels.

Other time-varying characteristics of coastal communities are building codes and materials. Any improvements in building codes and materials over time tend to reduce the damages associated with more recent storms. Changes in building materials include reductions in the use of traditional building material such as wood in favor of reinforced concrete. Building codes in many coastal communities now specify preferred building footprints, materials, and techniques related to both energy efficiency and building stability.\footnote{Since 2000, the International Residential Code has been updated each three years with the latest recommendations regarding construction materials and techniques (International Code Council 2003).}
these techniques, such as tighter nailing patterns for interior panels in high wind regions, increase the cost of construction relative to regions not at risk of storms. Higher construction costs are an additional cost of the risk of hurricanes that is not captured by the ICAT data set.

The distribution of storm strength will play an important role in the theoretical model and in the effects of adaptation, climate change mitigation, and coastal development policies on aggregate damages. Therefore, we also use storm characteristic information on all cyclones formed in the Atlantic basin between 1950 and 2012, gathered from the Tropical Storm Tracks database maintained by the NASA Earth Science Data and Information System Global Hydrology Resource Center using data published by the National Hurricane Center.\(^{13}\) The database provides location, category, minimum pressure, and maximum wind speed data for Atlantic basin storms from 1851.

The distribution of coastal property is also important for the theoretical model. We use coastal population data from the 2010 Census conducted by the US Census Bureau as a proxy for coastal property. The data gives population size within cities located in coastal counties of states in which a storm has led to damages at least once between 1900 and 2012.

3 Empirical Methods

The goal of the empirical analysis is to detect whether or not the distribution of hurricane damages has fat tails, which has implications for the efficacy of climate change mitigation, adaptation, and development policies in coastal communities dealing with the threat of damage-inducing storms. Additional analyses explore the potential sources of the fat tails in the aggregate damages distribution.

The first step in the empirical methodology is the definition of fat tails. No single definition of fat tails exists in the literature. Here we say a distribution is fat tailed if it is asymptotically equivalent to a Pareto distribution (Cooke and Nieboer 2011). That is:

\[
\lim_{D \to \infty} \frac{f(D)}{\alpha D^{-\alpha-1}} \to 1, \quad \alpha > 0.
\]

The above definition particularly convenient because the tail index of a fat-tailed distribution is \(\alpha\). The \(k\)th moment is infinite if and only if \(\alpha < k\).

3.1 Generalized Pareto Distribution

The GPD is a popular choice for extreme value analysis, with applications in a variety of disciplines germane to the current effort, including; actuarial science (e.g., Brazouskas and Kleefeld 2009; Cebrian et al. 2003), climatology (e.g., Nadarajah 2008), and meteorology (e.g., Holmes and Moriarty 1999). The distribution and density functions for the GPD, with shape parameter, $\xi$, and scale parameter, $\sigma$, are:

$$F_{GPD}(D; \sigma, \xi) = \begin{cases} 
1 - (1 + \xi \frac{D}{\sigma})^{-1/\xi} & \text{if } D > 0 , \, \xi \neq 0 \\
1 - e^{-\frac{D}{\sigma}} & \text{if } \xi = 0 
\end{cases}$$ \hspace{1cm} (3)

$$f_{GPD}(D; \sigma, \xi) = \begin{cases} 
(\frac{1}{\sigma})(1 + \xi \frac{D}{\sigma} )^{\frac{1}{\xi} - 1} & \text{if } D > 0 , \, \xi \neq 0 \\
(\frac{1}{\sigma})e^{-\frac{D}{\sigma}} & \text{if } \xi = 0 
\end{cases}$$ \hspace{1cm} (4)

respectively. Note that $0 \leq D < \infty$ if $\xi \geq 0$, and $0 \leq D < -\sigma/\xi$ if $\xi < 0$.\(^{14}\)

The shape parameter affects the tail of the distribution as well as the existence of the moments for the distribution; the tail index is $\alpha = 1/\xi$, so the $k$th moment exists if and only if $\xi < \frac{1}{k}$ (Hosking and Wallis 1987). The mean of the distribution, $\sigma/(1 - \xi)$, is finite if and only if $\xi < 1$. When the mean is infinite, efforts to estimate distribution parameters based on maximized likelihood yield unreliable results. The variance, $\sigma^2/(1 - \xi)^2(1 - 2\xi)$, is finite if and only if $\xi < \frac{1}{2}$. Maximum likelihood estimation of GPD parameters is feasible when the distribution has infinite variance. For $0 < \xi \leq 0.5$, the distribution decays at a rate slower than the normal distribution, but the variance is finite, and the central limit theorems still apply. When $\xi < 0$, the distribution is bounded above by $\frac{1}{\xi}$. When $\xi = 0$, the distribution is the exponential distribution. The area under the tail is monotonically increasing in $\xi$. The probability of observing extreme values is greater than with a normal distribution if and only if $\xi > 0$. Therefore, the GPD nests thin and fat tails depending on $\xi$. Figure 1 illustrates how the area under the tail changes as $\xi$ takes on values between -1 and 1. Figure 1 also gives an example with infinite variance ($\xi = 2/3$) and infinite mean ($\xi = 1$).

\(^{14}\)Some versions of the GPD specify an additional location parameter, which corresponds to the lower bound of the support of the distribution. Here we follow the literature cited above and naturally set the location parameter equal to zero. The estimate of the shape parameter is not affected by the value of the location parameter (Pickands 1975).
3.2 Tests for Fat Tails

Our interest is in estimation of the shape of the tail of the distribution. Most of the literature uses subsets of the data that lie beyond threshold values (see de Zee Bermudez and Kotz 2010, for a review of available techniques). Restricting the data set to values above a threshold is appropriate for two reasons. First, because extreme values are by definition rare, using the entire data set may result in an estimate of $\xi$ that fits most of the empirical distribution well but is a poor fit for the tail (DuMouchel 1983). Second, a fairly general theorem (Pickands 1975) shows that the tail of the GPD approximates the tail of any fat-tailed distribution (in particular, a sufficiently large threshold exists such that the GPD approximates any distribution satisfying a technical condition above the threshold to an arbitrarily small approximation error). Therefore, the GPD is always appropriate for estimation of the tail, but may not be appropriate for estimation of the entire distribution.

A tradeoff exists regarding the choice of threshold value: thresholds that are too high result in small sample sizes and high variance for the estimated parameters,\(^\text{15}\) while thresholds that are too low include ranges of the distribution for which the GPD may not be a good fit, leading to biased parameter estimates (Smith 1987). Although a number of techniques based on peaks-over-threshold approaches exist, no clear rule exists for the selection of the threshold (Davison and Smith 1990).\(^\text{16}\)

The analysis explores the presence of fat tails in the hurricane damage distribution using two methods. The first method directly estimates of the shape parameter of the GPD via maximum likelihood. The second method estimates the slope of the mean-excess function via weighted least squares (WLS), which is a function of the shape parameter (Ghosh 2010). The tests utilize damage data from storms that made landfall between 1900 and 2012.

3.2.1 Direct MLE estimation

Consistent and asymptotically normal maximum likelihood estimators (MLEs) of the shape and scale parameters exist for $-1/2 < \xi < 1$. Of course, the mag-

\(^{15}\)Giles et al. (2011) derives analytic expressions for the small-sample bias of MLE of the parameters of the GPD. Giles et al. (2011) also finds that, for positive shape parameters, the small-sample bias decreases as the true value of the shape parameter increases, for samples of 50 and 100 observations.

\(^{16}\)In a sense, the empirical dilemma of choosing a threshold mirrors the difficulty of pricing insurance when the damage distribution may have fat tails: a dearth of extreme values and sensitivity of the shape parameter to the choice of threshold makes estimating the probability of extreme events difficult.
magnitude and sign of the shape parameter are not known \textit{a priori}, making estimation of the GPD parameters via maximum likelihood estimation challenging in practice. The estimation uses storm level damage data.\footnote{Given that some insurance contracts are annual, MLE of annual damages may be theoretically more relevant than data at the individual storm level. However, evidence exists that prospective homeowners may make choices about property purchases and insurance coverage in response to the damages caused by a single storm (Gallagher 2014; Bin and Polasky 2004; Beron et al. 1997).}

### 3.2.2 Mean-Excess Estimation

The distinguishing feature of a fat-tailed distribution, relative to a thin-tailed distribution, is that the expected size of a draw larger than any draw yet observed is much larger than the largest draw to date. Given a threshold $u$, the mean excess function is:

$$M(u) \equiv E[X - u | X > u],$$

as long as $E[X_+] < \infty$. A distribution is fat tailed if $M(u)$ is increasing in $u$. The implication is that previous realizations from a fat-tailed distribution are poor predictors of the magnitude of observations from the tails of the distribution.\footnote{In contrast, new records in, for example, athletic events tend to be close to old records, indicating that results from athletic events are thin-tailed.} Distributions can be identified as fat tailed through the mean-excess function.

The empirical estimate of the mean-excess is:

$$\hat{M}(u) = \frac{\sum_{i=1}^{n}(X_i - u)I_{X_i > u}}{\sum_{i=1}^{n}I_{X_i > u}},$$

The slope of the mean-excess function can be used to estimate $\xi$, as the mean-excess function of a GPD is linear in $u$ (Ghosh 2010):

$$M(u) = \frac{\sigma}{1 - \xi} + \frac{\xi}{1 - \xi}u.$$
equation (6). The maximum threshold must not be too large, as data points with too few terms result in the average (6) being a noisy estimate of the true mean excess (Ghosh 2010).

The number of data points greater than the threshold is monotonically decreasing in the threshold, so the mean excess of smaller thresholds are more precisely estimated. The standard approach using WLS is to assign weights based on the precision of the estimated mean-excess. Then the weights monotonically decrease as the threshold increases, which means that WLS puts little emphasis on fitting the tail. Therefore, we use two weighting schemes to deal with the heteroskedasticity in the mean excess data. The first weights by the number of observations used to calculate the mean excess, while the second weights by the inverse of the number of observations.

4 Results

4.1 Fat-tailed Damages

Tables 1 and 2 report estimates of the shape parameter for storm level damage data, during the period 1900 to 2012, using MLE and WLS, respectively.

Given the lack of guidance about the appropriate threshold for use in estimation of the shape parameter, Table 1 reports MLEs of $\xi$ for several thresholds. Table 1 indicates the MLE of the shape parameter is sensitive to the threshold. Nonetheless, all point estimates are in the region indicating a fat tailed distribution with finite mean but infinite variance, $0.5 < \xi < 1$. Further, a thin-tailed distribution ($\xi \leq 0$) is rejected at the 95% confidence level for all threshold choices.

For lower thresholds, more data is available, increasing the precision of the estimates. However, the additional data implies the shape parameter is chosen to fit a wider range of the tail of the damage distribution. In fact, a larger shape parameter fits the middle part of the damage distribution best. In contrast, a smaller shape parameter fits the rightmost part of the tail of the damage distribution best. Restricting the data set to larger thresholds comes at a price of reducing the number of observations. Therefore, the standard errors widen and finite variance cannot be rejected at the 95% confidence level for thresholds equal to the 60th-80th percentiles. Nonetheless, the point estimates continue to indicate fat tails with finite mean but infinite variance, even at the 80th percentile.

The results in Table 2 provide even stronger support for the conclusion that the distribution of hurricane damages is fat tailed. When the upper threshold is chosen at the 70th or 75th percentile, so that each mean excess is calculated
using at least 49 observations, the point estimates of $\xi$ lie in a tight range, from 0.69 to 0.74. These point estimates indicate the distribution is fat tailed, with a finite mean and infinite variance. Further, the bootstrapped 95% confidence intervals are all between 0.5 and 1, indicating thin tails, infinite mean, and finite variance are all rejected at the 95% confidence level.

Table 2 also reports results generated assuming the upper threshold equals the second largest data point. For the second largest data point, the largest mean excess is estimated using a single observation and the larger mean excess values have very few data points. The resulting additional noise in the mean excess estimates increases the standard errors by almost a factor of 10 and causes the point estimates to vary across specifications. Nonetheless, a thin tail is rejected at the 95% confidence level for both specifications. However, both point estimates indicate that the variance is finite when the upper threshold equals the second largest data point.

### 4.2 Distribution of Windspeed

We next perform empirical tests for fat tails identical to section 4.1, but using data on hurricane strength, measured by maximum wind speed,$^{19}$ rather than damages. Testing the wind speed data provides a convenient falsification test. Given the underlying physical system changes at most slowly over time, the wind speed distribution is likely to be thin-tailed. The mean wind speed conditional on the speed exceeding a threshold is likely to be constant or decreasing in the threshold. More importantly, showing that the wind speed distribution is thin tailed eliminates one possible cause of fat tails in the theoretical model of Section 5.

Table 3 reports the results from WLS estimation of the mean excess (the MLE results are similar). Table 3 shows that, regardless of the lower and upper thresholds, the point estimate of the tail index is always negative indicating the wind speed distribution has thin tails. In all specifications, a fat tail is rejected at the 95% confidence level.

---

$^{19}$Maximum wind speed is a commonly used measure of the strength of storms (see, for example Nordhaus 2010). Other measures of hurricane strength, such as size, are correlated with wind speed. Further, we find that the distributions of storm surge, storm tides, and the combination of storm surge and tide are all thin-tailed using the mean-excess methodology on a Gulf of Mexico storm surge data set (Needham and Keim 2012). Details are available on request.
4.3 Coastal Population Distribution

Another possible cause of fat tailed damages in the theoretical model of section 5 is a fat tail in the property distribution. Table 4 reports the results of WLS estimation of the coastal property data. Table 4 reports shape parameter estimates between 0.66 and 0.73, again rejecting thin tails at the 95% confidence level. Further, the shape parameter estimates are very close to the shape parameter estimates for the aggregate damages. In section 6, we show that the theoretical model predicts the fat tail in the damage distribution arises from a fat tail in the property distribution, and that the shape parameter estimates of damages and the property distributions are equal.

5 Theoretical Model

We consider a theoretical model of aggregate damages which consists of three major parts. The first part consists of a model of household behavior. Households decide whether or not to live on the coast, and how much insurance and property to purchase. In the second part of the model, households living on the coast organize into population centers. Finally, a model of hurricane strength relative to the strength of adaptations determines the probability that a population center is in the path of a hurricane. Figure 3 illustrates the three parts of the model.

5.1 Household Behavior

A continuum of households with measure one have preferences for consumption, \(c\), and property. “Property” here refers to the structures on the land, not the land itself. We denote the quantity of property for household \(i \in [0, 1]\) as \(A_{j,i}\), where \(j \in \{K, I\}\) denotes the property type (coastal, \(K\), or inland, \(I\)). Households have heterogeneous preferences for property. The period utility function for household \(i\) is:

\[
U_i = u[c_i] + v[\phi(i)A_{K,i} + A_{I,i}].
\]  (8)

Equation (8) assumes property types are perfect substitutes, which implies households will either buy coastal or inland properties, but not both. Property is homogeneous except for type. Without loss of generality, we order households from least to strongest preference for the coastal property, so that \(\phi(i)\) is increasing. We assume \(\phi(0) = 0\) and \(2q_r \leq \phi(1) < \infty\), where \(q_r = q_K/q_I\) is the price ratio (coastal over inland property). The assumptions on \(\phi\) ensure
that the household with the smallest preference for coastal property lives inland and the household with the strongest preference lives on the coast. We further assume the Inada conditions hold for $u$ and $v$, so that total property (coastal plus inland) is positive for each household.

5.2 Hurricane Damage and Insurance

Only the coastal property is at risk of damage from hurricanes. Let $h$ be a random variable which equals one if a unit of coastal property suffers a loss from a hurricane and zero otherwise. Conditional on $h = 1$, the fractional loss of property the household suffers is $d \sim f_d(d)$. We assume that $d$ and $h$ are independent.\(^{20}\)

Let $q_K$ and $q_I$ denote the price per unit of coastal and inland properties, respectively. We assume both types of property can be produced at constant marginal cost. The cost of coastal property includes any adaptation costs such as the cost of compliance with stricter building codes. Perfect competition among suppliers of property implies $q_K = MC_K$ and $q_I = MC_I$.

The total value of assets at risk is therefore $q_K \int_0^1 A_{k,i} \, di = q_K A_K \equiv q_K n$. It is convenient to treat each unit of property (as opposed to each individual) as potentially suffering a loss. That is, some units of an individual’s property may suffer a loss, while other units do not. Let $\tilde{N} \in \{0, 1, \ldots, n\}$ be the (random) total units of property which suffer any loss. If $\tilde{N}$ has a continuous distribution, then for any realization, $N$, we take the highest integer less than or equal to $N$. Total losses are then:

$$D = q_K \sum_{i=1}^{\tilde{N}} d_i. \quad (9)$$

According to equation (9), each of $\tilde{N}$ units of coastal property experience a loss, and the losses are realizations $d_i$. One could equivalently (given independence) sum over households and let the distribution of $d$ grow as households acquire more property. However, the above formulation simplifies the limiting arguments presented later.

Households may purchase an insurance contract which reimburses the household in the event of a loss from a hurricane. Let $p(x)$ be the insurance premium per unit of insured property, where $x$ is the co-insurance rate. A large number

\(^{20}\)Presumably both the number of properties damaged and the amount of damage increase with the strength of the storm. Positive correlation between $d$ and $h$ would strengthen the results, but at a cost of considerable complexity.
of risk neutral firms provide insurance in a competitive market. Total expected profits equal premiums less expected losses:

$$E[\pi] = p(x)n - xE[D].$$  \hspace{1cm} (10)

Applying Wald’s lemma to (9) gives:

$$E[\pi] = p(x)n - xq_KE[\tilde{N}]E[d] = p(x)n - xq_Kn\bar{h}\bar{d}. \hspace{1cm} (11)$$

Here $\bar{d}$ is the expected fraction of loss a damaged property receives. Equation (11) assumes that the expected fraction of total properties which suffer damage ($\bar{h}$) is independent of $n$. \footnote{That is, adding more property does not change the probability that another existing property is damaged by a hurricane. If $\bar{h}$ was a decreasing function of $n$, then property insurance becomes less expensive as additional properties are added. This creates a feedback mechanism, where disaster relief encourages the purchase of additional coastal property, which decreases the cost of insurance, which encourages still further coastal property building. Such a feedback mechanism is not likely to be large, but adds considerable complexity to the problem.}

Perfect competition implies profits are zero, in which case the premium equals the expected reimbursement per unit of property:

$$p(x) = xq_K\bar{h}\bar{d}. \hspace{1cm} (12)$$

5.3 Government

A government agency exists which reimburses all coastal property owners a fraction $\tau$ of their losses. The government agency is funded by a lump sum tax on all households (irrespective of whether the households own coastal property). In addition, the government subsidizes the cost of coastal property at rate $\lambda$ per dollar.

The government budget constraint then sets tax revenue, $T$, equal to total reimbursement expenditures, $\tau D$ plus total coastal property subsidies $sq_KA_K$:

$$T = \tau D = \tau q_K\sum_{j=1}^{\tilde{N}}d_j + \lambda q_KA_K. \hspace{1cm} (13)$$

The reimbursement of losses can be thought of as a disaster relief agency, such as FEMA. The loss subsidy is equivalent to direct government provision of insurance with co-insurance rate $\tau$ at no cost. Examples of government
provision of property insurance include the National Flood Insurance Program, NFIP and Florida’s Citizen’s Property Insurance. Jenkins (2005) estimates some NFIP premiums are 35-40% below actuarially fair levels. The property subsidy captures development subsidies.

Total reimbursement is a random variable, so equation (13) implies tax payments are random. In practice, FEMA keeps a reserve fund to smooth taxes over time. Further, the government can borrow (and has in fact borrowed) to smooth tax payments in the event the fund is exhausted. Here we assume FEMA can perfectly smooth tax payments over time. Therefore, taxes equal the mean of total disaster reimbursements plus total coastal property subsidies:

\[ T = \tau E[D] = \tau nq_K \bar{h} \bar{d} + \lambda q_K A_K. \] (14)

5.4 Household Problem

The budget constraint sets household endowment income \( \omega \) plus disaster relief equal to expenses, which are consumption, the tax, the uninsured hurricane losses, the insurance premium, and the purchase of property.

\[
\omega + \tau q_K \sum_{j=1}^{A_{K,i}} h_j d_j = c_i + T + (1 - x) q_K \sum_{j=1}^{A_{K,i}} h_j d_j + p(x) A_{K,i} + q_K (1 - \lambda) A_{K,i} + q_I A_{I,i}. \] (15)

Note that the property subsidy applies only to the initial purchase, not the repair cost paid by either the insurance company or household after the damage.\(^{22}\) The timing of the problem is such that at the beginning of the period the household purchases insurance and enjoys property. Then losses are realized. The household must spend endowment income replacing any damaged property not already reimbursed by insurance or disaster relief.\(^{23}\) At the end of the period, the household consumes the remaining endowment.

Substituting the actuarially fair insurance premium (12) into the budget constraint and the budget constraint into the utility function (8) results in the

\(^{22}\)The model allows for subsidizing repair costs by appropriately modifying \( \tau \).

\(^{23}\)In practice, a household might elect not to replace all damaged property.
household problem:

$$\max E[U] = \max_{x,A_{K,i},A_{I,i}} \mathbb{E}_{h_j,d_j} \left[ u \left( \omega - T + (\tau - (1-x)) q_K \sum_{j=1}^{A_{K,i}} h_j d_j - x q_K \bar{h} \bar{d} ight. \right. \\
\left. \left. - q_K (1 - \lambda) A_{K,i} - q_l A_{I,i} \right) \right] + v \left[ \phi (i) A_{K,i} + A_{I,i} \right].$$  \hspace{1cm} (16)

In problem (16) each unit of coastal property is subject to an iid distributed loss. Hence the households take expectations with respect to each unit of property at risk. Problem (16) implies the disaster relief agency effectively subsidizes the acquisition of coastal property.

An equilibrium given a government policy $\tau$ is a set of household decisions $[x,A_{K,i},A_{I,i}]$, prices $[p,q_K,q_I]$, and a government tax rate $T$ such that households and firms optimize, the government budget constraint holds, and supply equals demand in the property markets: $q_K = MC_K$ and $q_I = MC_I$.

## 5.5 Results: Household Problem

We first derive a set of preliminary results for the household problem. The government insures part of the risk through the disaster relief agency. We first show that the household buys enough insurance to remove any remaining risk. The first order condition for the optimal co-insurance rate is:

$$\mathbb{E}_{h_j,d_j} \left[ u' \left( \omega - T + (\tau - (1-x)) q_K \sum_{j=1}^{A_{K,i}} h_j d_j - x q_K \bar{h} \bar{d} - q_K (1 - \lambda) A_{K,i} ight) \right] = 0.$$  \hspace{1cm} (17)

We hypothesize that the household insures all risk not covered by the government agency. That is, we hypothesize that $x = 1 - \tau$ satisfies the first order condition (17) and is therefore the optimal co-insurance rate. Substituting the
proposed solution into the first order condition results in:

$$u' \left[ \omega - T - q_K (1 - \tau) A_K,\bar{h}d - q_K (1 - \lambda) A_K,i - q_I A_I,i \right] \cdot E_{h_j,d_j} \left[ \sum_{j=1}^{A_{K,i}} h_j d_j - \bar{h}d A_{K,i} \right] = 0,$$

(18)

because marginal utility is not stochastic under the proposed solution. Further, $h_j$ and $d_j$ are iid, so equation (18) simplifies to:

$$\sum_{j=1}^{A_{K,i}} E[h_j] E[d_j] = \bar{h}d A_{K,i}.$$  

(19)

Equation (19) holds, which verifies the assumed behavior is optimal. Therefore, the household insures all risk not covered by the disaster relief agency.

We next derive the purchase of coastal versus inland properties. Substituting the optimal insurance rate $x = 1 - \tau$ into the problem (16) results in:

$$\max_{A_{K,i},A_{I,i}} u \left[ \omega - T - (1 - \tau) \bar{h}d q_K A_{K,i} - q_K (1 - \lambda) A_{K,i} - q_I A_{I,i} \right] + v \left[ \phi(i) A_{K,i} + A_{I,i} \right].$$

(20)

The next proposition shows that a cutoff household exists such that all households with stronger preferences for coastal property than the cutoff household will buy only coastal property. All other households will buy only inland property.

**Proposition 1.** Assume the Inada conditions hold for $u$ and $v$, $\phi(0) = 0$, $2q_r < \phi(1) < \infty$, and $\phi$ is continuous and increasing. Then there exists a cutoff household $i^* \in (0,1)$ such that:

$$\phi(i^*) = q_r \left( 1 - \lambda + (1 - \tau) \bar{h}d \right).$$

(21)

Further,

1. $A_{I,i} = 0$ for all $i \in (i^*,1]$.
2. $A_{K,i} = 0$ for all $i \in [0,i^*)$.

Given that each household holds only one type of property, the first-order
conditions become:

\[
\begin{align*}
  u' [\omega - T - (1 - \tau) \bar{h}\bar{d}q_K A_{K,i} - q_K (1 - \lambda) A_{K,i}] & = \\
  \phi (i) v' [\phi (i) A_{K,i}] & , \ i > i^*, \\
  u' [\omega - T - q_I A_{I,i}] & = v' [A_{I,i}] , \ i < i^*. 
\end{align*}
\]

(22)

(23)

For \( i < i^* \), the first-order condition is independent of \( i \). Therefore, households purchase identical amounts of inland property: \( A_I = i^* A_{I,i} \). The demand for coastal property may vary by household. General comparative statics and analytical solutions for special cases are available for equation (22):

**Proposition 2.** Let the assumptions of Proposition 1 hold. Then:

2.1. The fraction of households living on the coast, \( 1 - i^* \), is increasing in \( \tau \) and \( \lambda \).

2.2. The quantity of coastal property, \( A_{K,i} \), is increasing in \( \tau \) and \( \lambda \) for all \( i \in [i^*, 1] \).

2.3. The total replacement cost of coastal property at risk, \( q_K n = q_K \int A_{K,i} di \), is increasing in \( \tau \) and \( \lambda \).

2.4. For the special case of \( U_i = \log C_i + \log (\phi (i) A_{K,i} + A_{I,i}) \), total property at risk is:

\[
  n = \frac{(w - T) (1 - i^*)}{2 (1 - \lambda + (1 - \tau) \bar{h}\bar{d})}. 
\]

(24)

2.5. Per household purchase of inland property, \( A_{I,i} \), is independent of \( \tau \) and \( \lambda \).

Proposition 2 shows that disaster relief causes households living along the coast to buy more property, which directly increases total property at risk. Further, disaster relief causes more households to locate on the coast, which also increases total property at risk. Nonetheless, disaster relief, and indeed all parameters governing the insurance market, do not affect the damage distribution of an individual property. Adding property at risk does not change the damage distribution for another property already at risk.

Most of the assumptions are relatively innocuous. For example, we assume independence both across \( h \) and \( d \) and across properties. Adding positive correlation would not, for sufficiently large \( n \), affect the distribution to which \( D \) converges, in the same way that the central limit theorems hold when observations are correlated.
5.6 Population Centers

We let all property be divided into $S$ population centers. Following the results of section 4.3, we assume each population center $s$ contains a quantity of property, $n_s$, drawn from a truncated GPD. In particular, we let $n_s \sim f_{GPD}(n_s; \sigma n, \xi) / F_{GPD}(n; \sigma n, \xi)$. Here equations (3) and (4) define $F_{GPD}$ and $f_{GPD}$, respectively. If $\sigma$ satisfies:

$$n \left( \frac{\sigma}{1 - \xi} - \Gamma(\sigma) \right) = \frac{n}{S}, \quad (25)$$

the mean total property in a population center is $n/S$ and the mean total amount of property is $n$. The truncation of the distribution ensures that the maximum amount of property in a population center is $n$.

Population centers have equal geographic area, and are distributed randomly along the coast, which has total area $J_c$. If a hurricane path over land covers an area equal to $J_D$ (determined below), then the probability that a randomly placed population center is in the hurricane path equals $\rho = J_D / J_c$. For simplicity, we assume at most one population center is in the hurricane path. If a population center is in the hurricane path, then each property in the population center suffers the random loss $d_i$. Therefore:

$$\tilde{N} = \begin{cases} 
    n_i \sim f_{GPD}(n_i; \sigma n, \xi) / F_{GPD}(n; \sigma n, \xi) & \text{w.p. } \rho \\
    0 & \text{w.p. } 1 - \rho
\end{cases}, \quad (26)$$

---

24 Gabaix (1999) proposes a theory for city sizes that generates a Pareto distribution. In their model, wages adjust to offset city-specific amenity shocks, which keeps the growth rate of cities independent of the size, which implies a power law steady state distribution. Here the property distribution is exogenous, since most of the policies of interest affect total property or the relationship between hurricane strength and damage, not the property distribution.

25 However, the total amount of property is not equal to $n$ with probability one. Small differences between $\sum_{s=1}^{S} n_s$ and $n$ do not affect the results for large $n$ and scaling the realizations of the population center sizes so that the total property equals $n$ would vastly complicate the distribution of $n_s$.

26 If instead the assumption was that a hurricane could intersect multiple population centers, then $\tilde{N}$ would equal the sum of a random number of GPD random variables. Since the sum of Pareto random variables converges to a stable distribution with the same tail index, the restriction that the hurricane may intersect only a single population center is unlikely to alter the tail index of $\tilde{N}$, or the conclusion that $\tilde{N}$ is fat tailed.
\[ N \sim \begin{cases} \rho f_{\text{GPD}} \left( \tilde{N}; \sigma n, \xi \right) / F_{\text{GPD}} \left( n; \sigma n, \xi \right) & \text{if } \tilde{N} > 0 \\ 1 - \rho & \text{if } \tilde{N} = 0 \end{cases} \]  

(27)

Therefore \( \tilde{N} \) is a GPD random variable, conditional on a population center being in the hurricane path.

### 5.7 Hurricane Properties

A variety of hurricane-specific factors influence total damages, including wind speed, the size of the storm, and the path of the storm over land. We model these influences in a simple way. Let \( r \) denote the distance from the center of a storm, with \( t \) the time since landfall. Holland (1980) studies a model of the wind speed, \( w_r(t) \), of a hurricane:

\[ w_r(t) = r^\theta \frac{w_{r_m}(t)}{r^\theta} \quad r \geq r_m. \]  

(28)

Here \( r_m \) is the distance between the center of the storm and the edge of the eye of the hurricane, where the maximum wind speed occurs. Holland (1980) cites estimates of \( \theta \in [0.4, 0.6] \). The wind speed decreases as the distance from the eye of the storm increases.

We assume that building codes and adaptations are such that no damage occurs if the wind speed is less than \( \bar{w} \).\(^{27}\) Therefore, the radius of the storm beyond which no damage occurs, \( \bar{r} \), is:

\[ \bar{r}(t) = r_m \left( \frac{w_{r_m}(t)}{\bar{w}} \right)^{\frac{1}{\theta}}. \]  

(29)

Hurricanes lose energy over land, causing wind speed to decline. Following Kaplan et al. (2007), we assume maximum winds decay exponentially with time since landfall:

\[ w_{r_m}(t) = w_{r_m}(0) \exp(-\delta t). \]  

(30)

The hurricane causes no further damage when \( w_{r_m}(t) = \bar{w} \), that is, when the maximum winds are no longer strong enough to cause damage. This occurs at

\(^{27}\)Nordhaus (2010) finds support for a similar assumption, based on the idea that some building materials may withstand a maximum amount of wind with little or no damage, and then break all at once when the wind speed crosses a threshold. An alternative is to let the distribution of damage \( d \) be a function of the wind speed.
Further, combining equations (29) and (30) yields:

\[
\tilde{r}(t) = r_m \left( \frac{w_{rm}(0)}{\tilde{w}} \right)^{\frac{1}{\theta}} \exp \left( -\frac{\delta}{\theta} t \right).
\]  

Equation (32) gives the radius of damage as a function of the wind speed at landfall and the time since landfall.

Assume that the hurricane travels at a constant velocity \( \nu \), which has units of distance, \( m \), per unit of time. Then the area of damage is:

\[
J_D = \int_{0}^{m(t)} 2\tilde{r}(t(m)) \, dm = \int_{0}^{\nu t} 2\tilde{r} \left( \frac{m}{\nu} \right) \, dm,
\]  

\[
= 2 \int_{0}^{\tilde{v} t} r_m \left( \frac{\tilde{w}}{\tilde{w}} \right)^{\frac{1}{\theta}} \exp \left( -\frac{\delta}{v\theta} m \right) \, dm.
\]

Here \( \tilde{w} = w_{rm}(0) \), the maximum wind speed at landfall, which we have estimated in Section 4.2. Therefore, the probability that the hurricane intersects a population center is:

\[
\rho = \int_{\tilde{w}} J_D(\tilde{w}) \frac{f_{GP\Gamma}(\tilde{w}, \sigma_w, \xi_w)}{f_c} \, d\tilde{w}.
\]

The empirical evidence that climate change increases the number of hurricanes is mixed, but the frequency of high wind (category 4 or 5) storms may increase (Bender et al. 2010). Therefore, the model takes \( \sigma_w \) as the policy variable, with a decrease in \( \sigma_w \) interpreted as the result of climate change mitigation. According to the empirical results, hurricane strength is thin tailed (\( \xi_w < 0 \)). The next section derives the distribution of the aggregate damages.

6 Theoretical Results

6.1 Fat Tails

The theoretical model has several sources of random variation: hurricane wind speed, the path of the hurricane, the quantity of property in an area affected by a hurricane, and the damage to individual properties in the area affected
by a hurricane. We show here that the distribution for \( \tilde{N} \) largely determines the distribution for the aggregate damages \( D \). Because \( d_i \) is bounded on the unit interval, the variance of \( d_i \) is also bounded. Therefore, the central limit theorems imply that, regardless of the distribution of \( d_i \), if \( \tilde{N} \) is non-stochastic (for example \( \tilde{N} = \beta n \) with \( 0 < \beta < 1 \)), then \( D \) has a normal (thin-tailed) limiting distribution as \( \tilde{N} \to \infty \). Increasing the total property at risk increases the mean and variance of \( D \), but the distribution remains thin tailed.

However, Section 5.6 shows that \( \tilde{N} \) is random. When \( \tilde{N} \) is random, standard central limit theorems no longer hold and the distribution of \( D \) will not in general converge to a normal distribution. Instead, we will show that the limiting distribution of \( D \) will inherit properties of the limiting distribution of \( \tilde{N} \): if \( \tilde{N} \) is thin (fat) tailed then \( D \) converges to a thin (fat) tailed distribution.

The definition of fat tails poses a difficult problem for modeling the distribution of damages from a natural disaster. As the total number of properties at risk is always finite, for any \( n \) the distribution of damages is bounded and therefore fails to satisfy (2). To reconcile the apparent bound on total damages with empirical tests in Section 3, which indicate total damages are fat tailed, we construct a sequence of bounded distributions which converge to a fat-tailed distribution as \( n \to \infty \). For any finite \( n \), the distribution is bounded and not fat tailed, but, as \( n \) becomes large, a fat-tailed distribution becomes an increasingly accurate approximation.

We let \( \tilde{N}_n = \tilde{N} \) to emphasize that \( \tilde{N} \) depends on \( n \). To see how \( D \) inherits the tail properties of \( \tilde{N} \), assume for example, that \( \tilde{N}_n \) has a thin-tailed binomial distribution: \( \tilde{N}_n \sim \text{Binomial}(n, \rho) \) for any \( n \) (as would be the case if each property had a uniform probability of locating at each geographical point on the coast). Then from Robbins (1948), example (ii) as \( n \to \infty \):

\[
D \overset{d}{\to} N \left[ n\rho \bar{d}, n\rho \left( \sigma_d^2 + (1 - \rho) \bar{d}^2 \right) \right].
\]  

Similarly, if \( \tilde{N} \) has a generalized inverse Gaussian distribution, then \( D \) converges to a heavy-tailed (i.e. not fat tailed, but with a decay rate slower than the normal distribution) generalized hyperbolic distribution (Haas and Pigorsch 2011).

Here, conditional on the hurricane intersecting a population center, the distribution of \( \tilde{N} \) is a truncated GPD.

**Proposition 3.** Let \( d_j \sim f_d \) be iid with finite mean \( \bar{d} \) and finite variance \( \sigma_d^2 \). Let:

\[
\tilde{N}_n|\tilde{N}_n > 0 \sim \frac{f_{\text{GPD}}\left( \tilde{N}; \sigma n, \xi \right)}{F_{\text{GPD}}(n; \sigma n, \xi)}, \quad 0 \leq \tilde{N}_n < n,
\]
where $F_{\text{GPD}}$ and $f_{\text{GPD}}$ are given by (3) and (4), respectively. Then $D$ converges in distribution to a normal variance mean mixture with Pareto mixing density as $n \to \infty$. That is:

$$D|\bar{N}_n > 0 \to q_K \bar{d} \bar{N} + q_K \sigma_d \bar{N}^{\frac{1}{2}} \bar{z},$$

where $\bar{z}$ is a standard normal and $\bar{N}$ is distributed as a GPD with parameters $\sigma_n$, and $\xi$. Further, $D$ has tail index $\alpha = 1/\xi$ and mean:

$$E[D|\bar{N}_n > 0] = q_K \bar{d} n \left( \frac{\sigma}{1 - \xi \Gamma} \right), \quad \xi < 1, \quad \Gamma \equiv \frac{1 - F_{\text{GPD}}(1, \sigma, \xi)}{F_{\text{GPD}}(1, \sigma, \xi)} \frac{1}{1 - \xi}.$$

(39)

Proposition 3 shows that if the number of properties that experience a loss follows a GPD, then for large $n$, the distribution of damages is approximately a mixture of a normal distribution and a GPD. The form of $D$ given in equation (38) is known as a normal variance mean mixture, with GPD mixing distribution. Proposition 3 shows that $D$ inherits the tail index of $\bar{N}$. The theory is in alignment with the empirical results of section 4, in which the point estimates of the tail index of $D$ and $\bar{N}$ are similar.

From equation (39), the mean damage depends on the tail index. As seen in Section 4.1, the tail index is difficult to estimate given the lack of data in the tail. However, equation (39) may also be interpreted so that the mean damages equal the mean damage per property times the mean amount of property in a population center. That is, by choosing $\sigma$ as in equation (25), the mean damages equal $q_K \bar{d} n / S$, which may be estimated using insurance data on the total property under risk and the average claim size, rather than by working with the aggregate damage data directly.

In Proposition 3, the number of properties affected is drawn from a distribution bounded by $n$. At any point in time, the total amount of property $n$ remains finite. However, as the total property at risk grows, the bound expands and so does the upper bound for $\bar{N}$. Because $\bar{N}_n$ converges to a fat-tailed distribution, for large $n$ the distribution of $\bar{N}_n$ and $D$ will appear to have fat tails, even though for any fixed $n$ the distribution is bounded and therefore not fat tailed.

The expected damage from a catastrophic storm is implicit from the distribution of damages given by Proposition 3. Let $D^*$ denote the minimum damage from the $1 - 1/T$ upper quantile of the damage distribution. We then

---

say a storm is catastrophic if the storm results in damage large enough so that such storms occur with probability $1/T$. For example, if $f$ storms occurred per year and $T = 100f$, then a catastrophic storm would be a once-in-a-hundred-years storm. Higher values of $D^*$ imply that a catastrophic storm, which occurs once every $T$ observations, is more damaging. Higher values of $D^*$ also imply that more mass of the distribution resides in the tail (the tail is thicker). Because $\tilde{z}$ and $\tilde{N}$ are independent, equation (38) implies that $D^*$ is the solution to:

\[
\begin{align*}
\text{Prob}(D \geq D^*) &= \frac{1}{T}, \\
\text{Prob}(D \leq D^*) &= 1 - \frac{1}{T}, \\
1 - \rho + \rho \text{Prob}(D \leq D^* | \tilde{N} > 0) &= 1 - \frac{1}{T}, \\
F_D(D) &= \text{Prob}(D \leq D^* | \tilde{N} > 0) = 1 - \frac{1}{\rho T}, \\
\int_0^\infty \int_{-\infty}^{z(\tilde{N}, D^*)} f_{\tilde{z}}(\tilde{z}) d\tilde{z} f_N\left(\tilde{N}\right) d\tilde{N} &= 1 - \frac{1}{\rho T},
\end{align*}
\]

where $f_{\tilde{z}}$ is the standard normal density, $f_N$ is the GPD with parameters $\sigma_N$ and $\xi$, and:

\[
z(\tilde{N}, D^*) = \frac{D^* - q_K \tilde{D} \tilde{N}}{q_K \sigma_d \tilde{N}^{1/2}}.
\]

From equations (39) and (45), the policy variables affect both the mean damage and the damage from a $T$ observation storm in different ways. We examine these effects in the next section.

### 6.2 Policy Implications

Four policy variables exist in the model. First, the government may set the disaster relief policy $\tau$. Second, the government can subsidize the cost of coastal property by reducing $q_K$. Third, the government can choose the relative strength of adaptations, $\bar{w}$. Fourth, the government could, through climate change regulation, attempt to reduce the mean or the mass of the upper quantiles of the storm strength distribution.

**Proposition 4.** Let the assumptions of Proposition 3 hold. Then an increase in the strength of adaptations ($\bar{w}$), a decrease in the disaster relief subsidy ($\tau$), a decrease in the property subsidy ($\lambda$), and a decrease in the scale parameter
of the hurricane strength distribution ($\sigma_w$) all cause a decrease in the damage from a $T$ observation storm ($D^*$) and a decrease in the mean damages ($E[D]$). The tail index of the damage distribution is not affected by $\tau$, $\lambda$, $\bar{w}$, or $\sigma_w$.

While each policy variable affects the mean and catastrophic damages, the channels differ substantially. First, an increase in the disaster relief subsidy adds to the total property at risk, both through the intensive and extensive margins. In turn, an increase in the total property at risk has two effects. First, each population center has more property at risk, which causes the mean damage ($n/S$) to increase. Second, large population centers have more property at risk, so that large damages will occur more often.

To ensure that a catastrophic storm occurs only once every $T$ observations, the amount of damage required for a storm to be considered catastrophic must increase. Second, an increase in the total property at risk extends the right tail of the truncated GPD. However, this effect vanishes for large $n$, because as $n$ increases, a realization of $\tilde{N}$ at the truncation point becomes less and less likely. That is, as the number of properties increases, the likelihood that a storm affects all coastal property at risk approaches zero.

As with the disaster relief subsidy, the property subsidy increases the total property, along both the intensive and extensive margins. Therefore, the disaster relief subsidy increases the damage from all storms, including catastrophic storms.

An increase in the strength of adaptations, $\bar{w}$, reduces the geographic area of damage in two ways. First, the radius of winds which are strong enough to cause damage decreases, as the wind speed at a certain range of distances from the eye are no longer strong enough to cause damage. Second, the time since landfall at which the hurricane causes no further damage decreases, because $\bar{w}$, the minimum wind speed which causes damage, increases. In turn, the reduction in the geographic area of damage implies that fewer hurricanes will intersect a population center and cause damage. Therefore, the unconditional mean damage decreases. In addition, in a sample of $T$ storms, fewer storms cause damage meaning that there will be fewer storms that cause damage greater than a given level, $D$. So, $D^*$ must decrease to bring the unconditional probability of a storm causing damage greater than $D^*$ back to $1/T$. Alternatively, in a 100-year sample, fewer storms cause damage, so fewer storms are catastrophic, which decreases the likelihood that a catastrophic storm causes damage above a given level.

We model climate change mitigation as a decrease the scale parameter of the wind distribution, which results in hurricanes with lower (in the sense of first order stochastic dominance) wind speeds. As a result, the geographic
area of damage decreases both because the radius of the hurricane is smaller and because the hurricane dissipates more quickly. Climate change mitigation reduces the number of hurricanes that intersect a population center, reducing both the mean damage and the damage from a catastrophic storm.

Proposition 3 indicates that disaster relief policy and the structure of the insurance market increase the mean of $D$ by increasing $n$. However, disaster relief and the structure of the insurance market do not affect the tail index, $\alpha$, which determines the tail behavior of $D$. Similarly, adaptation and climate change mitigation can affect the probability that a storm causes damage, but do not affect the tail index.

7 Conclusion

Tropical cyclones can impose significant welfare losses on coastal communities. The empirical analysis shows that hurricane damages from 1900-2012 follow a fat-tailed distribution, so much so that the parameter estimates indicate infinite variance. A fat-tailed distribution can result in additional costs: the mean damages are difficult to estimate, leading to mispricing of insurance. The probability of a catastrophic loss is also difficult to estimate, leading to the possibility that reserves will not be adequate. Past storm events are not good predictors of the damages of future catastrophes.

A theoretical model of homeowner behavior, population centers, and hurricane strength is presented. The model reveals that homeowners will fully insure their properties from risk of damage, paying premiums to recoup all losses not covered by the disaster-relief program. In this way, the disaster relief program is equivalent to an insurance premium subsidy, which tends to increase the quantity of coastal property along both the extensive and intensive margins. A subsidy to the cost of coastal property also increases the quantity of coastal property along both margins. These subsidies increase the mean damage and the damage from a catastrophic storm. However, subsidies do not create a fat tail. Surprisingly, the fat tail results from the distribution of households in population centers. The empirical results agree with the prediction of the theoretical model that the tail index of the population center distribution equals the tail index of the damage distribution.

In the model, adaptations and climate change mitigation offer the possibility to reduce the coastal area over which winds are strong enough to result in damage. More hurricanes miss population centers, which lowers the average damage and also implies storms intersect large population centers less often.

In reality, the increase in coastal property continues, implying that catastrophic storms will become increasingly important from a welfare perspective.
Thus, the need for policies that can minimize the occurrence of, or damage from, catastrophic storms will only become more acute.

References


Appendix A  Proof of Theorems

A.1  Proof of Proposition 1

The Kuhn-Tucker conditions for problem (20) require $\mu_{ji}A_{ji} = 0$, where $\mu_{ji}$ is the multiplier which ensures property choices are non-negative and $j \in [K, I]$. Eliminating the multipliers from the first order conditions for optimal property choice results in:

$$(-u'[c_i] q_K (1 - \lambda + (1 - \tau) \bar{h}\bar{d}) + \phi(i) v' [\phi (i) A_{K,i} + A_{I,i}]) A_{K,i} = 0, \quad (46)$$

$$(-u'[c_i] q_I + v' [\phi (i) A_{K,i} + A_{I,i}]) A_{I,i} = 0. \quad (47)$$

We first show that $A_{I,i} > 0$ implies $A_{K,i} = 0$ for all $i \neq i^*$. Since $A_{I,i} > 0$, $\mu_{I,i} = 0$. Therefore, equation (47) reduces to:

$$u'[c_i] q_I = v' [\phi (i) A_{K,i} + A_{I,i}], \quad (48)$$

Combining (48) and (46) results in:

$$(-u'[c_i] q_K (1 - \lambda + (1 - \tau) \bar{h}\bar{d}) + u'[c_i] \phi(i) q_I) A_{K,i} = 0. \quad (49)$$

Therefore either $A_{K,i} = 0$ or:

$$\phi(i) = q_r (1 - \lambda + (1 - \tau) \bar{h}\bar{d}). \quad (50)$$

However, since the right hand side of (21) is independent of $i$ and $\phi$ is strictly increasing, equation (21) holds for at most one $i$, denoted $i^*$. Therefore, for all $i \neq i^*$ for which $A_{I,i} > 0$, we have $A_{K,i} = 0$. Reversing the argument implies that $A_{K,i} > 0$ implies $A_{I,i} = 0$ for all $i \neq i^*$. For the converse, we must show that $A_{K,i} = 0$ implies $A_{I,i} > 0$. Suppose not, suppose $A_{K,i} = A_{I,i} = 0$. Then total housing is zero which cannot hold by the Inada conditions. Therefore $A_{K,i} = 0$ if and only if $A_{I,i} > 0$ and the reverse.

Next from (48) and (49), the range of $i$ for which $A_{K,i} = 0$ and $A_{I,i} > 0$ satisfies:

$$u'[c_i] q_K (1 - \lambda + (1 - \tau) \bar{h}\bar{d}) \geq u'[c_i] \phi(i) q_I, \quad (51)$$
\[ \phi(i) \leq q_r \left( 1 - \lambda + (1 - \tau) \bar{h}\bar{d} \right). \]  

(52)

Since \( \phi(i) \) is increasing, the above inequality is satisfied for all \( i < i^* \), where \( i^* \) satisfies (21). Reversing the argument shows that \( A_{K,i} > 0 \) and \( A_{I,i} = 0 \) for all \( i \geq i^* \).

It remains to show that \( i^* \in (0, 1) \). Since \( \phi(0) = 0 \) by assumption, condition (52) holds with strict inequality for \( i = 0 \). Therefore \( i^* > 0 \). Since \( \phi(1) > 2q_r \) and \( \tau, \bar{h}, \) and \( \bar{d} \), are all between zero and one, condition (52) is not satisfied for \( i = 1 \). Therefore \( i^* < 1 \). □

A.2 Proof of Proposition 2

Propositions 2.1, 2.2, and 2.5 follow from applying the implicit function theorem to equations (21), (22), and (23) respectively. For 2.3, note that:

\[
q_K n = q_K \int_{i^*(\tau)}^1 A_{K,i}\left(\tau\right) \, di. \tag{53}
\]

Applying Leibniz’s rule results in:

\[
\frac{\partial q_K n}{\partial \tau} = q_K \left( \int_{i^*}^1 \frac{\partial A_{K,i}}{\partial \tau} \, di - A_{K,i} \frac{\partial i^*}{\partial \tau} \right). \tag{54}
\]

The result then follows since Propositions 2.1 and 2.2 show that \( i^* \) is decreasing in \( \tau \) and \( A_{K,i} \) is increasing in \( \tau \).

For 2.4, for the special case where both \( u \) and \( v \) are logarithmic, equation (22) reduces to:

\[
\frac{q_K \left( 1 + (1 - \tau) \bar{h}\bar{d} \right)}{w - T - q_K A_{K,i} \left( 1 + (1 - \tau) \bar{h}\bar{d} \right)} = \frac{1}{A_{K,i}}. \tag{55}
\]

Solving for \( A_{K,i} \) results in equation (24). □

A.3 Proof of Proposition 3

The following lemma is Theorem 3 from Clark (1973), which we will apply directly.

Lemma 5. Let \( d_i \sim f_d \) be iid with mean zero and unit variance and \( \bar{N}_n \) be a
random variable independent of $d_i$ such that:

$$\frac{\tilde{N}_n}{n} \overset{p}{\rightarrow} \tilde{M} \text{ as } n \to \infty.$$  \hspace{1cm} (56)

Then:

$$D = n^{-\frac{1}{2}} \sum_{i=1}^{\tilde{N}_n} d_i \overset{d}{\rightarrow} f_D,$$  \hspace{1cm} (57)

where $f_D$ is the distribution of a normal variance mean mixture. That is: $D \rightarrow \tilde{M}^{\frac{1}{2}} \tilde{z}$, where $\tilde{z}$ is distributed as a standard normal.

The proof applies Lemma 5. First, note that $\tilde{M} \equiv \tilde{N}_n/n$ has distribution:

$$\tilde{N}_n/n \sim f_{\tilde{N}_n} \left( \tilde{N} (\tilde{M}) \right) \frac{\partial \tilde{N}_n}{\partial \tilde{M}},$$  \hspace{1cm} (58)

$$= \frac{1}{\sigma_N n} \left( 1 + \frac{\xi}{n\sigma_N} (\tilde{M} n) \right)^{-1 - \frac{1}{\tau}} n,$$  \hspace{1cm} (59)

$$= \frac{1}{\sigma_N} \left( 1 + \frac{\xi}{\sigma_N \tilde{M}} \right)^{-1 - \frac{1}{\tau}},$$  \hspace{1cm} (60)

which is independent of $n$. Therefore, $\tilde{N}_n/n$ converges trivially to $\tilde{M}$.

The next step converts the problem into one that satisfies the assumptions of Lemma 5. Let $y_i = (d_i - \bar{d}) / \sigma_d$, then by Lemma 5, conditional on $\tilde{N}$,

$$n^{-\frac{1}{2}} \sum_{i=1}^{\tilde{N}} y_i \overset{d}{\rightarrow} \text{N} \left[ 0, \tilde{M} \right],$$  \hspace{1cm} (61)

$$n^{-\frac{1}{2}} \left( \sum_{i=1}^{\tilde{N}} d_i - \bar{d}\tilde{N} \right) \overset{d}{\rightarrow} \text{N} \left[ 0, \sigma_d^2 \tilde{M} \right].$$  \hspace{1cm} (62)

Therefore for $n$ large:

$$\sum_{i=1}^{\tilde{N}} d_i - \bar{d}\tilde{N} \overset{d}{\rightarrow} \text{N} \left[ 0, \sigma_d^2 \tilde{M} n \right],$$  \hspace{1cm} (63)
\[
\sum_{i=1}^{\hat{N}} d_i \xrightarrow{d} N \left[ \bar{d} \hat{N}, \sigma_{\bar{d} \hat{N}}^2 \right],
\]
\[
D \xrightarrow{d} N \left[ q_K \bar{d} \hat{N}, q_K^2 \sigma_{\bar{d} \hat{N}}^2 \right],
\]
which is the normal variance mean mixture (38) with the parameters given in the proposition. Since the limit of \( \hat{N}_n \) is the GPD, the mixing distribution is the GPD.

The moments and tail behavior follow from the properties of the normal variance mean mixture.

**Lemma 6.** Let \( D \) be a normal variance mean mixture, i.e.:
\[
D = \mu \hat{N} + \sigma \hat{N}^{\frac{1}{2}} \tilde{z},
\]
where \( \tilde{z} \) is a standard normal independent of \( \hat{N} \), and \( \hat{N} \) has finite mean. Then:

1. \( E[D] = \mu E[\hat{N}] \).
2. The tail index of \( D \) equals the tail index of \( \hat{N} \).

We prove each result separately.

1. Since \( \hat{N} \) and \( \tilde{z} \) are independent:
\[
E[D] = \mu E[\hat{N}] + \sigma E \left[ \hat{N}^{\frac{1}{2}} \right] E[\tilde{z}].
\]
Next Jensen’s inequality implies \( E \left[ \hat{N}^{\frac{1}{2}} \right] \leq E \left[ \hat{N} \right]^{\frac{1}{2}} < \infty \). Therefore, since \( \tilde{z} \) has mean zero, the second term in (67) is zero and the result follows.

2. The \( k \)th moment of \( D \) is finite if and only if:
\[
E[D^k] = E \left[ \left( \mu \hat{N} + \sigma \hat{N}^{\frac{1}{2}} \tilde{z} \right)^k \right] < \infty.
\]
The binomial theorem implies:
\[
E[D^k] = \sum_{i=0}^{k} \left[ \begin{array}{c} k \\ i \end{array} \right] E \left[ \left( \hat{d} \hat{N} \right)^{k-i} \left( \sigma \hat{N}^{\frac{1}{2}} \tilde{z} \right)^i \right].
\]
A property of the GPD is that if the $i$th moment exists, then so do all moments $j \leq i$. The largest exponent on $\tilde{N}$ in (69) occurs when $i = 0$. Therefore it sufficient to show $E\left[\tilde{N}^k\right]$ exists. But since $\tilde{N}$ has tail index equal to $\alpha$, $E\left[\tilde{N}^k\right]$ exists if and only if $\alpha > k$. Therefore the $k$th moment of $D$ exists if and only $\alpha > k$. Therefore, $\alpha$ is the tail index of $D$. □

A.4 Proof of Proposition 4

Equations (27) and (39) imply the unconditional mean damage is:

$$\bar{D} = \rho q_K \bar{d}_n \left( \frac{\sigma}{1 - \xi} - \Gamma \right). \quad (70)$$

From equation (70), the unconditional mean damage is increasing in $n$. Further, Proposition 2.3 shows that $n$ is increasing in $\tau$ and $\lambda$. Therefore, $E[D]$ is increasing in $\tau$ and $\lambda$. The unconditional mean damage is also increasing in $\rho$. Further, equations (34) and (35) imply that $\rho$ is decreasing in $\bar{w}$. Therefore, $E[D]$ is decreasing in $\bar{w}$. Finally, for $\sigma_w$, combining equations (34) and (35) gives:

$$\rho = \int_{\tilde{w}} 2 \int_{0}^{\nu^i} r_m \left( \frac{\bar{w}}{\tilde{w}} \right)^{\frac{1}{\theta}} \exp \left( -\frac{\delta}{v^\theta} m \right) dm f_{GPD} (\tilde{w}, \sigma_w, \xi_w) d\tilde{w}, \quad (71)$$

$$\rho = \int_{\tilde{w}} 2 r_m \theta \nu \frac{J_c}{\delta} \left( \left( \frac{\bar{w}}{\tilde{w}} \right)^{\frac{1}{\theta}} - 1 \right) f_{GPD} (\tilde{w}, \sigma_w, \xi_w) d\tilde{w} = \int_{\tilde{w}} h (\tilde{w}) f_{GPD} (\tilde{w}, \sigma_w, \xi_w) d\tilde{w}. \quad (72)$$

Note that $F_{GPD}$ is decreasing in $\sigma_w$ and therefore $\tilde{N}_1 \sim F_{GPD} (\sigma_{w1})$ first order stochastic dominates $\tilde{N}_2 \sim F_{GPD} (\sigma_{w2})$ for $\sigma_{w1} > \sigma_{w2}$. Therefore, since $h$ is an increasing function, $\rho$ is increasing in $\sigma_w$. Therefore, the unconditional mean damage is increasing in $\bar{w}$.

For the damage from a $T$ observation storm, equation (44) defines an implicit function $D^* (\bar{w}, \tau, \lambda, \sigma_w)$. Equation (44) simplifies to:

$$\int_{0}^{\infty} F_z \left( \frac{D^* - q_K d\tilde{N}}{q_K \sigma_d N_{\tilde{w}}} \right) f_N \left( \tilde{N}, \sigma_n \right) d\tilde{N} = 1 - \frac{1}{\rho T}. \quad (73)$$

Consider first the derivative with respect to $n$. Taking the total derivative of
\begin{equation}
\int_0^\infty f_z \left( \frac{D^*-q_Kd\tilde{N}}{q_K\sigma_d\tilde{N}^{3/2}} \right) \frac{\partial}{\partial D^*} \frac{\partial}{\partial n} f_N \left( \tilde{N}, \sigma n \right) d\tilde{N} + \int_0^\infty F_z \left( \frac{D^*-q_Kd\tilde{N}}{q_K\sigma_d\tilde{N}^{3/2}} \right) \frac{\partial f_N}{\partial n} d\tilde{N} = 0. \tag{74}
\end{equation}

\begin{equation}
\frac{\partial D^*}{\partial n} = \frac{-\int_0^\infty F_z \left( \frac{D^*-q_Kd\tilde{N}}{q_K\sigma_d\tilde{N}^{3/2}} \right) \frac{\partial f_N}{\partial n} d\tilde{N}}{\int_0^\infty f_z \left( \frac{D^*-q_Kd\tilde{N}}{q_K\sigma_d\tilde{N}^{3/2}} \right) \frac{\partial}{\partial D^*} f_N \left( \tilde{N} \right) d\tilde{N}}. \tag{75}
\end{equation}

Next, the derivative of the GPD with respect to \( n \) follows from equation (4):

\begin{equation}
\frac{\partial f_N \left( \tilde{N}, \sigma n \right)}{\partial n} = \frac{\tilde{N} - \sigma n}{\sigma^2 n^3} \left( 1 + \frac{\xi}{\sigma n} \tilde{N} \right)^{-2-\frac{4}{\gamma}}. \tag{76}
\end{equation}

Therefore:

\begin{equation}
\int \frac{\partial f_N \left( \tilde{N}, \sigma n \right)}{\partial n} d\tilde{N} = -\frac{\tilde{N}}{\sigma n^2} \left( 1 + \frac{\xi}{\sigma n} \tilde{N} \right)^{-1-\frac{4}{\gamma}} = -\frac{n}{\tilde{N}} f_N \left( \tilde{N} \right) d\tilde{N}. \tag{77}
\end{equation}

Next, the numerator of (75) simplifies using (77) and integration by parts:

\begin{equation}
-\int_0^\infty F_z \left( \frac{D^*-q_Kd\tilde{N}}{q_K\sigma_d\tilde{N}^{3/2}} \right) \frac{\partial f_N}{\partial n} d\tilde{N} = -F_z \left( \frac{D^*-q_Kd\tilde{N}}{q_K\sigma_d\tilde{N}^{3/2}} \right) \frac{\tilde{N}}{n} f_N \left( \tilde{N} \right) \bigg|_0^\infty + \\
\int_0^\infty f_z \left( \frac{D^*-q_Kd\tilde{N}}{q_K\sigma_d\tilde{N}^{3/2}} \right) \frac{\partial}{\partial n} \frac{\tilde{N}}{n} f_N \left( \tilde{N} \right) d\tilde{N}.
\tag{78}
\end{equation}

Simple calculations show the first term on the right hand side of (78) is zero at both integration end points. The derivative (75) thus simplifies to:

\begin{equation}
\frac{\partial D^*}{\partial n} = \frac{\int_0^\infty f_z \left( \frac{D^*-q_Kd\tilde{N}}{q_K\sigma_d\tilde{N}^{3/2}} \right) \frac{\partial}{\partial n} \frac{\tilde{N}}{n} f_N \left( \tilde{N} \right) d\tilde{N}}{\int_0^\infty f_z \left( \frac{D^*-q_Kd\tilde{N}}{q_K\sigma_d\tilde{N}^{3/2}} \right) \frac{\partial}{\partial D^*} f_N \left( \tilde{N} \right) d\tilde{N}}. \tag{79}
\end{equation}
\[
\frac{\partial D^*}{\partial n} = \int_0^\infty f_z \left( \frac{D^* - q_K d\tilde{N}}{q_K \sigma d\tilde{N}^2} \right) f_N \left( \tilde{N} \right) \tilde{N}^{-\frac{1}{2}} \left( D^* + q_K d\tilde{N} \right) \frac{1}{2n} d\tilde{N}.
\]

(80)

Both the numerator and denominator of (80) are positive, and so the derivative is positive. Proposition 2.3 shows that \( n \) is increasing in \( \tau \) and \( \lambda \). Therefore, \( D^* \) is increasing in \( \tau \) and \( \lambda \).

Consider now the derivative of \( D^* \) with respect to \( \rho \). Following a similar logic, equation (73) implies:

\[
\frac{\partial D^*}{\partial \rho} = \int_0^\infty f_z \left( \frac{D^* - q_K d\tilde{N}}{q_K \sigma d\tilde{N}^2} \right) f_N \left( \tilde{N} \right) \tilde{N}^{-\frac{1}{2}} \frac{1}{T \rho^2} d\tilde{N}.
\]

(81)

Therefore, the derivative of \( D^* \) with respect to \( \rho \) is positive. Since \( \rho \) is decreasing in \( \bar{w} \) from equations (34) and (35), \( D^* \) is decreasing in \( \bar{w} \). For \( \sigma_w \), note that we have shown that \( \rho \) is increasing in \( \sigma_w \) and therefore \( D^* \) is increasing in \( \sigma_w \).

**Appendix B  Figures and Tables**
Figure 1: Generalized Pareto distribution for different values of the tail index $\alpha = 1/\xi$.

Figure 2: Mean excess for GPD with shape parameter $\xi = 1/\alpha = 2/3$. In the Figure, $M(1) = 5$ and $M(2) = 7$, indicating that the mean-excess function $M(u)$ is increasing.
Figure 3: Illustration of the model. In the figure, seven households choose to live on the coast, organizing into three population centers. If, for example, the hurricane took the west path, the hurricane would damage the two houses in the second population center.

<table>
<thead>
<tr>
<th>Threshold</th>
<th>$\hat{\xi}$</th>
<th>Confidence Interval</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>45th percentile</td>
<td>0.946</td>
<td>0.593 - 1.299</td>
<td>133</td>
</tr>
<tr>
<td>Median</td>
<td>0.873</td>
<td>0.52 - 1.227</td>
<td>119</td>
</tr>
<tr>
<td>55th percentile</td>
<td>0.902</td>
<td>0.512 - 1.292</td>
<td>108</td>
</tr>
<tr>
<td>60th percentile</td>
<td>0.806</td>
<td>0.42 - 1.192</td>
<td>95</td>
</tr>
<tr>
<td>65th percentile</td>
<td>0.751</td>
<td>0.356 - 1.146</td>
<td>84</td>
</tr>
<tr>
<td>70th percentile</td>
<td>0.668</td>
<td>0.274 - 1.061</td>
<td>72</td>
</tr>
<tr>
<td>75th percentile</td>
<td>0.764</td>
<td>0.271 - 1.258</td>
<td>60</td>
</tr>
<tr>
<td>80th percentile</td>
<td>0.618</td>
<td>0.141 - 1.095</td>
<td>48</td>
</tr>
</tbody>
</table>

Table 1: MLE of the shape parameter for the hurricane damage distribution.
<table>
<thead>
<tr>
<th>Lower Threshold</th>
<th>Upper Threshold</th>
<th>Weighting</th>
<th>$\hat{\xi}$</th>
<th>Std. Err.</th>
<th>Confidence Interval</th>
<th>Minimum Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>30th percentile</td>
<td>70th percentile</td>
<td>obs.</td>
<td>0.73</td>
<td>0.011</td>
<td>0.71 - 0.75</td>
<td>57</td>
</tr>
<tr>
<td>30th percentile</td>
<td>70th percentile</td>
<td>inverse obs.</td>
<td>0.71</td>
<td>0.011</td>
<td>0.69 - 0.73</td>
<td>57</td>
</tr>
<tr>
<td>35th percentile</td>
<td>70th percentile</td>
<td>obs.</td>
<td>0.70</td>
<td>0.010</td>
<td>0.68 - 0.72</td>
<td>57</td>
</tr>
<tr>
<td>35th percentile</td>
<td>70th percentile</td>
<td>inverse obs.</td>
<td>0.70</td>
<td>0.010</td>
<td>0.68 - 0.72</td>
<td>57</td>
</tr>
<tr>
<td>30th percentile</td>
<td>75th percentile</td>
<td>obs.</td>
<td>0.72</td>
<td>0.008</td>
<td>0.70 - 0.73</td>
<td>49</td>
</tr>
<tr>
<td>30th percentile</td>
<td>75th percentile</td>
<td>inverse obs.</td>
<td>0.70</td>
<td>0.009</td>
<td>0.68 - 0.71</td>
<td>49</td>
</tr>
<tr>
<td>35th percentile</td>
<td>75th percentile</td>
<td>obs.</td>
<td>0.69</td>
<td>0.008</td>
<td>0.68 - 0.71</td>
<td>49</td>
</tr>
<tr>
<td>35th percentile</td>
<td>75th percentile</td>
<td>inverse obs.</td>
<td>0.69</td>
<td>0.009</td>
<td>0.68 - 0.72</td>
<td>49</td>
</tr>
<tr>
<td>35th percentile</td>
<td>n/a</td>
<td>obs.</td>
<td>0.24</td>
<td>0.09</td>
<td>0.06 - 0.40</td>
<td>1</td>
</tr>
<tr>
<td>35th percentile</td>
<td>n/a</td>
<td>inverse obs.</td>
<td>0.43</td>
<td>0.054</td>
<td>0.33 - 0.54</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: WLS estimation of the shape parameter using the mean-excess function for the hurricane damage distribution. The confidence intervals were generated by bootstrapping. In the second column, “n/a” corresponds to setting the maximum threshold equal to the second largest data point, the largest data point for which calculation of the mean excess is possible. The third column indicates that weighting is done either by the number of data points used to calculate the mean excess, or the inverse.
<table>
<thead>
<tr>
<th>Lower Threshold Percentile</th>
<th>Upper Threshold Percentile</th>
<th>Weighting</th>
<th>( \hat{\xi} )</th>
<th>Std. Err.</th>
<th>Confidence Interval</th>
<th>Minimum Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>30th percentile</td>
<td>70th percentile</td>
<td>obs.</td>
<td>-0.27</td>
<td>0.005</td>
<td>-0.28 - -0.26</td>
<td>192</td>
</tr>
<tr>
<td>30th percentile</td>
<td>70th percentile</td>
<td>inverse obs.</td>
<td>-0.28</td>
<td>0.005</td>
<td>-0.29 - -0.27</td>
<td>192</td>
</tr>
<tr>
<td>35th percentile</td>
<td>70th percentile</td>
<td>obs.</td>
<td>-0.24</td>
<td>0.011</td>
<td>-0.26 - -0.22</td>
<td>192</td>
</tr>
<tr>
<td>35th percentile</td>
<td>70th</td>
<td>inverse obs.</td>
<td>-0.27</td>
<td>0.011</td>
<td>-0.29 - -0.25</td>
<td>192</td>
</tr>
<tr>
<td>30th percentile</td>
<td>75th</td>
<td>obs.</td>
<td>-0.29</td>
<td>0.005</td>
<td>-0.30 - -0.28</td>
<td>172</td>
</tr>
<tr>
<td>30th percentile</td>
<td>75th</td>
<td>inverse obs.</td>
<td>-0.31</td>
<td>0.006</td>
<td>-0.32 - -0.30</td>
<td>172</td>
</tr>
<tr>
<td>35th percentile</td>
<td>75th</td>
<td>obs.</td>
<td>-0.28</td>
<td>0.011</td>
<td>-0.30 - -0.25</td>
<td>172</td>
</tr>
<tr>
<td>35th percentile</td>
<td>75th</td>
<td>inverse obs.</td>
<td>-0.30</td>
<td>0.012</td>
<td>-0.33 - -0.30</td>
<td>172</td>
</tr>
</tbody>
</table>

Table 3: WLS estimation of the shape parameter using the mean-excess function for the wind speed distribution.
<table>
<thead>
<tr>
<th>Lower Threshold Percentile</th>
<th>Upper Threshold Percentile</th>
<th>Weighting</th>
<th>$\hat{\xi}$</th>
<th>Std. Err.</th>
<th>Confidence Interval</th>
<th>Minimum Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>35th</td>
<td>70th</td>
<td>obs.</td>
<td>0.729</td>
<td>0.0007</td>
<td>0.728 - 0.730</td>
<td>934</td>
</tr>
<tr>
<td>35th</td>
<td>70th</td>
<td>inverse obs.</td>
<td>0.726</td>
<td>0.0007</td>
<td>0.725 - 0.727</td>
<td>934</td>
</tr>
<tr>
<td>40th</td>
<td>70th</td>
<td>obs.</td>
<td>0.726</td>
<td>0.0008</td>
<td>0.725 - 0.728</td>
<td>934</td>
</tr>
<tr>
<td>40th</td>
<td>70th</td>
<td>inverse obs.</td>
<td>0.724</td>
<td>0.0008</td>
<td>0.722 - 0.726</td>
<td>934</td>
</tr>
<tr>
<td>35th</td>
<td>90th</td>
<td>obs.</td>
<td>0.669</td>
<td>0.005</td>
<td>0.660 - 0.679</td>
<td>313</td>
</tr>
<tr>
<td>35th</td>
<td>90th</td>
<td>inverse obs.</td>
<td>0.674</td>
<td>0.005</td>
<td>0.664 - 0.684</td>
<td>313</td>
</tr>
<tr>
<td>40th</td>
<td>90th</td>
<td>obs.</td>
<td>0.660</td>
<td>0.005</td>
<td>0.650 - 0.670</td>
<td>313</td>
</tr>
<tr>
<td>40th</td>
<td>90th</td>
<td>inverse obs.</td>
<td>0.668</td>
<td>0.006</td>
<td>0.657 - 0.679</td>
<td>313</td>
</tr>
</tbody>
</table>

Table 4: WLS estimation of the shape parameter using the mean-excess function for the coastal city population distribution.