Sustainable Banking and Credit Market Segmentation*

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Abstract

We assess the feasibility, optimality, and policy implications of Environmental, Social, and Corporate Governance (ESG)-linked or "green" lending in a credit market where banks incorporate such non-financial data in credit allocation decisions. We identify an asymmetric information problem: borrowers signal low financial risk to banks who are uncertain about borrower risk levels by engaging in green investments. We derive conditions under which banks segment the market into green and brown loan products and evaluate market efficiency. We find borrowers prioritize signaling over the environmental impact of green investments, and the market sustains only limited green lending, since if all borrowers make green investments, no signaling value exists. The optimal carbon tax policy replaces the signaling value of green investments with the marginal damage and outperforms a brown reserve requirement aimed at discouraging brown lending. However, both policies also can sustain only a limited amount of green investments. We conclude that while green lending by banks can enhance welfare relative to an unregulated market, the resulting market segmentation can make the social optimum infeasible, even with carbon tax regulation.

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1 Introduction

Large banks increasingly take into account non-financial information, including Environmental, Social and Corporate Governance (ESG) disclosures when making credit allocation, interest rate, and other lending decisions.[1] Kim et al. (2023) find that lending that accounts for ESG factors grew from $6 billion in 2016 to $322 billion in 2021.[2]

Bank regulators and policy makers are considering policies that account for environmental risks for regulatory decisions that limit risk taking by banks. Acharya et al. (2023) find that regulators in 23 countries have or plan to conduct scenario analysis to measure the effect of climate change on bank portfolios, known as climate stress tests. The idea is that borrowers with poor environmental performance may be at risk of defaulting in scenarios where environmental regulation stringency increases ("transition risk").[3] While some central banks focus solely on financial risks created by climate change,[4] other central banks are using regulation for climate mitigation. For example, the European Central Bank (ECB) is decarbonizing its corporate bond holdings in order to reach the EU's climate neutrality and Paris Agreement goals (ECB, 2022).

We examine the feasibility, optimality, and policy implications of using such non-financial information in the credit allocation process. In our framework, an asymmetric information problem exists where banks are uncertain about the riskiness of a project for which a borrower needs financing. Borrowers receive a private signal that conveys the probability that their project is low risk, which is correlated with the expected cost of an investment technology which reduces an externality (a "green investment"). Borrowers can thus signal to lenders they received a low financial risk signal through observable green investments. We derive conditions under which banks segment the market into a ESG-linked "green" loan product which offers a lower rate but requires the borrower to undertake green investments directly or achieve an ESG score or metric that the borrower can satisfy by undertaking green investments and a "brown" loan product that does not require green investments.[5]

[1]Banks use a variety of terms for these programs, including sustainability-linked loans, green loans, and ESG compliant loans.

[2]An empirical literature has established several reasons for the ESG-linked lending trend, including preferences of investors (e.g. Baker et al., 2002), mandatory disclosure rules for banks (Wang, 2023), and that ESG-linked borrowers have lower credit risk (e.g. Danisman and Tarazi, 2024).

[3]Regulators also consider risks in bank lending to borrowers sensitive to climate impacts (e.g. the property insurance industry), known as "physical risks."

[4]Acharya et al. (2023) notes that the Federal Reserve and Bank of England only focus on limiting climate financial risk.

[5]We use the terms green and brown loan products as the most common ESG-linked loans require borrowers to attain environmental metrics. However, our framework applies to any ESG activity which has social benefits

Our framework applies to any environmental metric or ESG score which is negatively correlated with financial risk, has external benefits, and can be improved with a green investment technology. For example, a firm might install solar panels which improve environmental metrics and reduce financial transition risks. Further, achieving a given emissions metric is more costly for firms that use more energy and such firms also face greater transition risk absent solar panel investments. In another example, community engagement is costly, but raises ESG scores and can reduce financial risks from boycotts or negative press. Conversely, a property insurer might be exposed to physical risks of hurricane damages which creates financial risk. However, in this case no obvious green investment exists for the insurer to improve an observable ESG metric with external benefits and reduce the physical risk.

We derive the equilibrium investment in green investment technologies by unregulated borrowers for signaling purposes. Borrowers undertake green investments if their signal indicates that expected green investment costs (the cost of achieving an environmental metric required for a green loan) are sufficiently low. At the cutoff, the expected cost of green investment equals the difference in interest cost between green and brown loan products. Competitive banks then set green rates lower than brown rates because expected green investment costs are negatively correlated with financial risk, and so the average type applying to green loans is more likely to be low financial risk and thus low expected bankruptcy cost.

We find that (1) green investments are made based on how well such investments signal low financial risk, which is generically not equal to the socially optimal green investment which equates the marginal damage alleviated by green investments to the marginal cost, and (2) the unregulated market can sustain only a limited amount of green lending, since if all borrowers undertake green investments no signaling occurs.

We then consider two policies designed to achieve the social optimum, a standard carbon tax and recent proposals by bank regulators to penalize banks for the higher risk inherent in brown lending, which we model as a reserve requirement for brown loans. We find that the optimal carbon tax is less than the marginal damage and in fact replaces the signaling value of green investments with the marginal damage. The carbon tax can sustain higher levels of green investment vs the unregulated economy but also can only sustain a limited amount of green investment and may not be able achieve the social optimum. In effect, if the marginal damages are too large, the social optimum eliminates the signaling value of green investments since most firms in the regulated economy opt to undertake green investments to avoid the large carbon tax. However, when the equilibrium shifts to pooling, where only a single loan product is available, the optimal carbon tax also shifts as the signaling value no longer needs to

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not captured by the borrower or lender.

be accounted for. Put differently, the optimal carbon tax differs across equilibrium types, and is no longer optimal when the equilibrium switches from separating to pooling equilibrium.

We also show that a Federal Reserve policy that raises the brown lending rate by requiring more reserves cannot sustain as much green investment as the carbon tax, but a combined policy which sets the carbon tax equal to the marginal damage and a *subsidy* for brown loans which eliminates the signaling benefit of green lending can sustain more green lending than either the carbon tax or reserve requirement in isolation. We conclude that unregulated green lending products by banks and bank regulation which encourages green lending can improve welfare relative to an unregulated economy with no green lending. However, unregulated or regulated green lending by banks can be counter productive if the marginal damage is sufficiently high as they cannot sustain enough green investments to maximize welfare. In contrast, a carbon tax in a world where no green lending is possible can always maximize welfare.

2 Related Literature

A nascent literature examines features of the sustainable banking market. Acharya et al. (2023) provides a review but argues that more research is needed, including modeling the response of banks to climate risks and how regulation affects the cost of capital for high emission firms. We show here that banks raise the cost of capital by increasing the brown lending rate, not only because a borrower that applies for a brown loan signals higher risk, but also because market segmentation concentrates high risk firms in the brown lending market. However, a pooling equilibrium with only brown loan products can also arise endogenously if even very low risk borrowers find it costly to signal by investing in green technologies.

Within banking markets, the literature finds a small but significant premium (lower rate) for green loans relative to conventional loans. Ehlers, Packer, and Greiff (2022) finds a firm with carbon intensity one standard deviation above the mean pays a rate about 17 basis points higher than the average firm after the Paris Accords. Similarly, Delis, Greiff, and Ongena (2018) find a 16 basis point higher lending rate for a one standard deviation increase in their measure of climate policy risk exposure and Shin (2021) finds a one standard deviation increase in ESG scores results in a 10.67 basis point decrease in lending rates. Chava (2014) finds that firms with environmental concerns in all categories considered pay 25 basis points higher rates vs firms with an equal number of environmental concerns and strengths. Our results provide theoretical support for this literature. We find that only equilibria with relatively low interest rate spreads and a small green loan market are possible in our framework. If the spread and thus the green market is too large, banks have an incentive to deviate and offer a pooled brown market. Brown borrowers prefer the pooled market versus paying high rates in the segmented market, and most green borrowers also prefer the pooled market over paying for costly green investments to avoid the high brown rate in the segmented market.

Few theoretical papers exist in sustainable banking. Oehmke and Opp (2022) consider a model in which banks are capital constrained and brown borrowers are more profitable. They derive a separating equilibrium in which only some green borrowers receive funding and that higher capital requirements for brown loans can decrease green lending, since banks must allocate more scarce capital to the more profitable brown borrowers. Here we focus instead on equilibria in which two lending rates arise endogenously, as borrowers signal higher profitability by making green investments. Our model thus explains the empirical regularity above of a lower rate for green loans.

In our model banks offer green loans at lower rates because borrowers making green investments signal lower risk. Other authors focus on other motivations for green lending. Baker et al. (2002) argue that the small municipal green bond premium supports the idea that green bonds provide non-pecuniary utility to investors. In Chang, Rhee, and Yoon (2024), non-pecuniary utility is split between the bank and the borrower. Flammer (2021) finds evidence consistent with corporate green bond issuers signaling their *environmental commitments* to customers, employees, and equity investors (a "green halo"). Most likely, green banking markets aggregate all of these motivations as well as the risk-signaling motivation we focus on here. We view the risk-signaling motivation as particularly important as it generates a segmented market with a small rate premium seen in the data as well as a concentration of financial risk in brown loans that is the motivation behind central bank policy proposals.

We focus on how ESG and green lending affects green investments by borrowers. An emerging literature focuses on the ESG scores of the lenders (e.g. Basu et al., 2022). In particular, Wang (2023) shows that mandatory ESG disclosure rules for banks causes banks to increase green lending and reduce brown lending and borrowers to improve ESG scores. Further, Danisman and Tarazi (2024) show that banks with high ESG scores have lower declines in profitability, credit risk, and access to capital during financial crises. In our model, competition ensures that both the green and brown bank have zero expected profits. Nonetheless, the green bank has lower credit risk and therefore smaller losses during periods where defaults are high. However, the green bank has a lower rate and thus lower profits if no defaults occur.

Our model relies on a negative correlation between environmental or ESG performance and financial risk. This idea is supported by evidence primarily in equities and derivative securities markets. Albuquerque, Koskinen, and Zhang (2019) find that firm beta (non-diversifiable risk) is negatively correlated with a measure of ESG performance. Several papers (e.g. Albuquerque et al., 2020; Lins, Servaes, and Tamayo, 2017) find that firms with high ESG scores have better financial performance after negative marcoeconomic shocks, and thus less downside financial risk. The argument is that firms that address concerns of stakeholders such as employees and suppliers are more likely to receive help during a negative shock (higher trust). Hoepner et al. (2024) show that investor engagement in ESG issues reduce downside financial value at risk, especially related to the environment and climate change, and that environmental incidents decrease after investor engagement. Ilhan, Sautner, and Vilkov (2021) find that carbon intensive firms have more tail risk as measured via the cost of protection in the options market.

Our model of banks offering a menu of loan types takes inspiration from a large literature of competitive screening beginning with the seminal paper Rothschild and Stiglitz (1976). In this literature, firms facing adverse selection offer an menu of contracts to induce agents to sort into high and low risk groups. Low risk types use contract provisions such as deductibles to separate from high risk types and obtain lower rates. This is welfare reducing relative to the first best of full insurance. We add to this literature by considering green investments that are privately costly but beneficial from society's perspective as green investment provides benefits that are external to the borrower and lender. We show that the aggregate green investments depends on the signal value, which is generically not equal to the external marginal damages alleviated by green investments. Thus, the market with green lending generically provides too little or too much green investment, relative to the social optimum.

Our model of green lending draws on the foundational work of Burke, Taylor, and Wagman (2012). They describe a banking equilibrium wherein borrowers apply for loans, and banks engage in costly screening to assess borrower profitability. More intensive screening increases the likelihood of identifying and rejecting low-profit applicants. In contrast, in our model high profit borrowers use a signaling mechanism, green investments which improve observable ESG metrics, to obtain favorable interest rates in credit markets.

We adopt a competitive screening approach to show how green investments which improve ESG metrics can create the empirically observed separating equilibrium between green and brown lending. Traditional frameworks mitigate banks' adverse selection problems through mechanisms such as collateral (e.g. Chan and Kanatas, 1985), partial self-financing (e.g. de Meza and Webb, 1987), and bank investigations (e.g. Broecker, 1990; Burke, Taylor, and Wagman, 2012). Banks use these techniques to deny credit to the lowest quality borrowers. In contrast, the highest quality borrowers in our model use green investment to obtain lower rates in a separating equilibrium. Loumioti and Serafeim (2022) find that approximately 17%

of sustainability-linked loans in their sample are collateralized, indicating that banks likely employ a combination of methods to mitigate adverse selection.

The use of ESG criteria in the capital allocation process is also becoming common in non-bank debt financing. Flammer (2021) and Tang and Zhang (2020) study the corporate green bond market and find no significant difference between green and brown rates. Zerbib (2019) finds that green bonds rates are about two basis points lower than similar conventional bonds and Caramichael and Rapp (2002) find green bonds yields are eight basis points lower than conventional bonds, with the difference concentrated in large issuers in developed economies. While we focus on bank lending, most of our results carry over to a segmented market with green and brown bonds with similar intuition.

3 Model

Consider a model in which perfectly competitive banks offer loan products to entrepreneurs or firms (hereafter borrowers) seeking funding for investment projects.[6] The timeline begins with banks posting take it or leave it prices for the loan products. Next, borrowers receive private signals about the value, and thus the default risk, of their investment projects. These signals also convey information about the expected cost of investments that improve the projects' ESG scores (hereafter "green investments"). Borrowers then apply for a single loan product at a single bank. Banks then allocate funding specified by the loan product contract to the borrowers. Borrowers then invest in projects. Investing in the project reveals the true value of their investment projects and the true cost of green investments. Finally, borrowers either default and return the residual project value to the bank, or fulfill the contract returning the interest and principle to the bank and engaging in green investments if the contract requires.[7]

3.1 Borrowers

Borrowers require x dollars to fund their investment projects. One can view the project as a new investment that requires startup funding, or as an established firm investing in a new

[6]The model is loosely related to Burke, Taylor, and Wagman (2012), but adds multiple loan types and an externality, among other features.

[7]An alternative timeline is for borrowers to undertake green investments up front. This formulation is less realistic in most cases as the loan contract typically requires the borrower satisfy ESG metrics after the loan is granted. Up front green investments would not change the set of equilibria we derive, but would make the equilibria exist under much weaker conditions than we find below.

project with uncertain value.[8] Investment projects *ex post* generate profits net of operating expenses which are either high ($v_H - x$) or low ($v_L - x$). Projects are thus either high value or low value types. We assume that $v_L - x < 0 < v_H - x$ so that only high value projects generate positive returns.

Each borrower i receives an unbiased private signal, λ_i, such that the investment project is the low value type with probability λ_i, that is $v_i = v_L$ with probability λ_i. The signal is unobserved by banks and regulators. We assume that $\lambda_i \sim U[0,1]$, where U is the uniform distribution. Each borrower decides which loan product to apply based on the borrower's expected payoff driven by the given λ_i. The signal distribution is common knowledge.

Borrowers can also make green investments. The bank may specify in the loan contract that the borrower undertake observable green investments or can require borrowers to achieve an observable environmental metric or ESG score. We assume that borrowers that undertake green investments achieve this metric or ESG score. For convenience, we denote such contracts as "green loan" products, noting that the loan contract may additionally or instead be an ESG-linked loan that specifies social and governance metrics or ESG scores. We denote conventional loan contracts with no required green investments as "brown loans." The green loan contract specifies that failure to achieve the specified environmental metrics constitutes a default, in which case the bank claims the residual value of the investment project.[9] Let f_i denote the cost of green investments for borrower i. The cost of green investment satisfies $f_i \in \{f_L, f_H\}$, with $f_H > f_L$.[10] We assume that $v_H > x + f_H$, so that high value projects generate enough profits to fund green investments. This assumption also ensures that it is sequentially rational for a borrower with a high value project to comply with the contract and make green investments rather than default.

The probability that green investments are high cost depends on the project type. In particular, $prob\,(f = f_H|\lambda_i) = \lambda_i$. Because projects that are likely low value (high λ_i) also likely have high green investment costs, a negative correlation exists between project value and green

[8] A variety of funding sources exist, including green bonds, green private equity, and angel investing. We do not model the choice of funding source, noting only that banks are often preferred due to the different tax treatment of debt versus equity, among other considerations.

[9] Kim et al. (2023) defines ESG and sustainability linked loans as specifying environmental or ESG metrics whereas green loans specify that loan funds are used for a green investment. Either fits into our framework.

[10] An equivalent interpretation is that the green investment cost for a borrower receiving signal λ_i is $\lambda_i f_H + (1-\lambda_i) f_L$ with certainty. It is also equivalent to a model in which the cost per unit of externality reduction is known, and borrowers receive signals about how much of the externality will be generated from the project (for example, the cost is $f_i = f \cdot e_i$, $i \in \{H, L\}$, where e_i is either high or low pollution emissions). Kim et al. (2023) find that observable ESG scores are similar for conventional and green loans, prior to the loan. This supports our idea that the cost of complying is private information, especially since only some borrowers with similar ESG scores apply for green loans.

investment costs. In fact, it is straightforward to show the correlation is $-1/3$. This assumption can be modified to allow the correlation to vary without affecting the results.

The assumption that financial risk and the cost of green investments are negatively correlated is motivated by the empirical literature in Section 2 which finds a negative correlation between financial risk and ESG scores (Lins, Servaes, and Tamayo, 2017; Albuquerque, Koskinen, and Zhang, 2019; Albuquerque et al., 2020; Ilhan, Sautner, and Vilkov, 2021; Hoepner et al., 2024). In particular, since firms can reduce financial risk through green investments that improve ESG scores, the existence of firms with low ESG scores indicates that green investments are more costly to such firms. The negative correlation is also consistent with the concern of policy makers of transition risk and stranded assets (Martinez-Diaz and Keenan, 2020, e.g.), where potentially more stringent future environmental regulation can cause low ESG firms to suffer financial distress. Finally, a common argument (e.g. PwC, 2023) is that green lending reduces risk in the value of collateral attached to the loan. For example, green investments might reduce the likelihood of an environmental accident which not only bankrupts the firm, but also reduces the value of the land used for collateral. Although we do not specifically model collateral, this idea supports the idea that green investments lower risk.

3.2 Banks

In the supply side of the market, banks are competitive and make zero expected profits. Banks receive loan applications from borrowers. Loan provision is costly banks. Banks incurs a net dollar cost of funds for all loans equal to c to obtain x dollars for lending. For example, a bank may borrow x dollars in the Federal Funds market repaying $x+c$ after the borrower repays the loan, for a Federal Funds Rate of c/x. Alternatively, c/x could be the cost per dollar of attracting and maintaining deposits used to fund the loan or the interest rate the FED pays on bank reserves (and thus the opportunity cost of the loan). If default occurs the bank incurs a processing cost of k, as well as some loss of principal. We assume that banks always repay depositors and/or creditors in the Fed Funds market. Thus, bank stockholders incur any losses from default through reduced profits.

Initially, banks announce dollar prices, $p_g, p_b \in R_+$, for two different loan products.[11] Loan product g requires the borrower to undertake green investments and loan product b is the brown loan which does not. At these two prices, the bank commits to loan x dollars to a borrower in exchange for $x + p_g$ or $x + p_b$ dollars after the project is completed.[12] Price

[11] Offering the same price, but a higher probability of approval for green loans would work in a similar way.

[12] The implied interest rates offered are thus p_g/x and p_b/x.

announcements are made publicly and simultaneously. Then, borrowers apply to banks for one of the two loan products to start their projects.

Since green investments are more costly in expectation for low value projects, banks will obtain information about signals received by firms by observing which loan products borrowers apply to, which implies different equilibrium interest rates for green and brown loans.

3.3 Externality

We model the external impact of projects through a damage function D. In particular, projects that are funded cause damages of $D(e)$, where e is the level of the externality, and D is strictly increasing. These damages are incurred by third parties, not the bank or borrower. [13] Green investments reduce externality levels to $e_g < e$. We normalize the damage of project with green investments to zero, $D(e_g) = 0$.

Note that projects without green investments that turn out to be low value also cause damage, since borrowers complete the project before deciding whether or not to default. To make the problem non-trivial, let $f_L < D(e) < f_H$. This ensures that it is socially optimal for a project that for certain has low (high) green investment costs to in fact undertake (not undertake) green investments.

For example, let e denote uncontrolled carbon emissions and $D(e)$ damage from higher temperatures and sea level rise. A green investment in solar panels results in $e_g = 0$ and the green loan product is either an ESG-linked loan which requires $e = 0$ or a green loan which requires investment in observable solar panels. Achieving the metric $e = 0$ with solar panels is more costly for firms that use more energy, which are exposed to more transition risk in the absence of solar panels. In another example, e is plastic use and $D(e)$ as the external harm from plastic use. A social example is where e is the probability of an accident from unsafe working conditions and $D(e)$ the expected external health costs associated with unsafe working conditions.

3.4 Payoffs

The payoffs to the banks and borrowers depends on the type of loan and the value of the project. For high value brown loans, borrower profits are $v_H - x - p_b$, and bank profits equal $p_b - c$. For low value brown loans, the borrower defaults, so borrower profits are zero and bank profits are $-c_H \equiv v_L - x - c - k$. That is, the lender incurs a loss of principle equal to

[13] In Chang, Rhee, and Yoon (2024), banks and borrowers split a non-pecuniary benefit, which motivates banks to offer lower green rates.

$x - v_L$ plus the cost of funds and extra processing costs of a default, k. For high value green loans, borrower profits are $v_H - f_j - x - p_b$, $j \in \{L, H\}$, and bank profits equal $p_g - c$. For low value green loans, the borrower defaults so borrower profits are zero and lender profits are $-c_H$. Notice that if the project is low value, the borrower defaults prior to undertaking green investments, as green investments do not improve profits in this state.

Finally, the assumption made earlier that v_H is large enough to fund high cost of green investments, $v_H > x + f_H$, implies $v_H - f_H > x > v_L$. Hence, the payoffs net of green investments costs satisify $v_H > v_H - f_L > v_H - f_H > v_L > v_L - f_L > v_L - f_H$.

4 Unregulated Market Equilibrium

4.1 Borrower Decision

Here we derive a set of conditions under which perfectly competitive and unregulated banks offer both green and brown loan products. We anticipate that the green rate is strictly less than the brown rate, otherwise no borrower would incur the extra cost required for a green loan.

After receiving their signal, a borrower applies for a green loan product if the savings from choosing the lower green rate outweighs the expected green investment cost. Given our assumptions, a borrower defaults if and only if the project is low value. Hence the borrower applies for a green loan if and only if:

$$(1 - \lambda_i)(v_H - p_g - \mathrm{E}[f|\lambda_i]) \geq (1 - \lambda_i)(v_H - p_b). \quad (1)$$

Since the borrower defaults and receives zero for low value projects, the loan decision depends only on the difference in prices and the expected green investment costs given the project turns out to be high value. Condition (1) follows since, conditional on λ_i, v_i and f are independent (unconditionally, v and f are correlated as both distributions depend on the random variable λ_i). Equation (1) implies borrowers apply for green loans if they are sufficiently confident that the green investment cost is low:

$$\lambda_i \leq \lambda^* \equiv \frac{p_b - p_g - f_L}{\Delta f}, \ \Delta f \equiv f_H - f_L. \quad (2)$$

The cutoff type for applying to green loans, λ^*, is increasing in the interest difference. As the differential rises, green loans become more attractive and so firms with marginally higher risk of high green investment costs are willing to apply for green loans to save the interest difference.

All borrowers apply for a loan, even if the expected social value of the loan is negative. Even a borrower with very high λ_i applies since a small probability exists that the project is high value and profitable, whereas the borrower faces no bankruptcy costs if the project is low value.

4.2 Bank Decision

Banks can specialize in either green or brown loans or offer both. We assume N *ex ante* identical banks. Let n_j denote the number of banks offering loan product $j \in \{g, b\}$. If $n_j > 1$ then a borrower in market j applies to the bank offering the lowest rate. If all banks offer the same rate, then the borrower applies randomly to one of the banks and so the probability of a bank of type j receiving an application is $1/n_j$. Given this structure, Bertrand competition between banks of the same type ensures that banks offer the same rate and profits are zero for each type of loan (see Appendix 8.1). Given zero profits we can without loss of generality assume that one bank of each type exists.[14]

Banks anticipate that borrowers that are low risk ($\lambda_i \in [0, \lambda^*]$) will apply for green loans as such borrowers have low expected green investment costs. Hence the zero profit condition for green loans is:

$$\mathrm{E}\left[\pi_G\right] = \int_0^{\lambda^*} \left[\lambda_i \left(-c_H\right) + \left(1-\lambda_i\right)\left(p_g - c\right)\right] d\lambda_i = 0. \tag{3}$$

Solving for the green loan price results in:

$$p_g\left(\lambda^*\right) = c + \frac{\lambda^*}{2-\lambda^*} c_H. \tag{4}$$

The second term on the right hand side spreads the expected losses from the fraction of borrowers that opt for green loans and default, $\lambda^* \cdot (\lambda^*/2)$ across the profits from the fraction of the borrowers that choose green loans and repay, $\lambda^* \cdot (1-\lambda^*/2)$. The second term on the right hand is the default risk premium. An important complication in the model is that λ^* not only affects the probability of default, but also the measure of borrowers over which the default premium is spread.

[14] Thus the green bank has lower risk of default but also a lower rate and thus lower profits if no default occurs. The lower risk of default matches the findings of Danisman and Tarazi (2024), who show that high ESG banks suffer less credit risk declines during financial crises.

The zero profit condition for the brown loan market is identical except borrower quality is lower:

$$\mathrm{E}\left[\pi_B\right] = \int_{\lambda^*}^{1} \left[\lambda_i\left(-c_H\right) + \left(1-\lambda_i\right)\left(p_b - c\right)\right] d\lambda_i = 0. \tag{5}$$

Solving for the brown interest rate results in:

$$p_b\left(\lambda^*\right) = c + \frac{1+\lambda^*}{1-\lambda^*} c_H. \tag{6}$$

The default premium for brown borrowers is larger since brown borrowers are more likely to default.

4.3 Separating Candidate Equilibria

A *candidate equilibrium* consists of prices p_g and p_b and a cutoff signal λ^* which solve equations (1), (4), and (6), and for which $p_b > p_g$. That is, the equilibrium requires optimal borrower decisions and zero profits for both types of loan products. A candidate equilibrium thus accounts for Bertrand competition between *ex ante* identical brown banks and *ex ante* identical green banks. However, zero profits is necessary, but not sufficient, for an equilibrium. In particular, even at zero profits a bank that cuts prices might still increase profits. This could occur if the price cut results in an increase in borrower quality which lowers bankruptcy costs enough to offset the lower margin from the lower price.

It is easiest to proceed in two steps. First, we derive candidate equilibria which have zero profits for both types of banks (and thus banks offering both types of loans). We will then consider whether or not a profitable deviation exists in the form of a price cut that increases borrower quality sufficiently to offset the loss of margin from the price cut. We refer to a candidate equilibrium with no profitable deviations as a *Bertrand equilibrium*. A *Bertrand equilibrium* is thus an equilibrium with zero profits (candidate equilibrium) such that neither a brown or green bank can profitably reduce prices.

Existence of candidate equilibria is best understood intuitively through a graphical analysis. We will also show existence rigorously through a proposition. We begin by showing some properties of the brown and green loan product prices, which are the components of the graphical analysis.

Appendix 8.2 establishes that the green loan price (4) satisfies:

$$p_g(0) = c, \quad p_g(1) = c + c_H, \quad p_g'\left(\lambda^*\right) > 0. \tag{7}$$

First, if only borrowers who never default apply for green loans ($\lambda^* = 0$) then perfect competition drives the loan price equal to the cost of funds, $p_g(0) = c$. If all borrowers apply for green loans ($\lambda^* = 1$), then the loan price equals the cost of funds plus the unconditional default premium, $p_g(1) = c + c_H$. Unconditionally, in expectation half of borrowers default, each costing the bank c_H. This cost is spread over the fraction one half of borrowers that repay, hence the unconditional default premium is $((1/2)c_H)/(1/2)$. Finally, as λ^* increases, expected green borrower quality decreases, causing the bank to increase the price $p'_g(\lambda^*) > 0$.

Appendix 8.2 shows that the brown loan price (6) satisfies:

$$p_b(0) = c + c_H, \lim_{\lambda^* \to 1} p_b(\lambda^*) = \infty, \quad p'_b(\lambda^*) > 0. \tag{8}$$

If all borrowers except the probability zero set of borrowers that never default apply for brown loans ($\lambda^* = 0$), then the loan price equals the cost of funds plus the unconditional default premium. If expected brown borrower quality deteriorates in that the fraction of borrowers applying for brown loans approaches zero ($\lambda^* \to 1$), no borrowers who repay exist to spread the default cost over, and so the default premium, diverges. Finally, as λ^* increases, expected brown borrower quality also decreases, causing the price to increase, $p'_b(\lambda^*) > 0$. Surprisingly, an increase in λ^* decreases borrower quality for both brown and green loans, since the highest quality brown borrower becomes the lowest quality green borrower.

Next, we subtract equation (4) from equation (6):

$$p_b(\lambda^*) - p_g(\lambda^*) \equiv \Delta p_1(\lambda^*) = \frac{2c_H}{(1-\lambda^*)(2-\lambda^*)}. \tag{9}$$

The price difference is the benefit of applying for green loans. The price difference has properties:

$$\Delta p_1(\lambda^*) > 0, \quad \Delta p_1(0) = c_H, \quad \lim_{\lambda^* \to 1} \Delta p_1(\lambda^*) = \infty, \quad \Delta p'_1(\lambda^*) > 0, \quad \Delta p''_1(\lambda^*) > 0. \tag{10}$$

If $\lambda = 0$, all green borrowers repay with probability one, and hence the difference in prices is the unconditional default premium. As the quality of brown borrowers deteriorates, the difference in prices diverges since the default premium for brown borrowers diverges and the default premium for green borrowers is finite. The difference in prices is positive, since green borrowers are higher quality than brown borrowers. The difference in prices increases at an increasing rate with λ^*. Both green and brown prices increase with λ^*, but the increase in prices is less in the green market. A marginal borrower switching from brown to green (higher

λ^*) means there are more borrowers in the green market to spread the default costs across, but less borrowers in the brown market. So the effect on the brown market is larger.

Next, rewriting equation (2) yields:

$$\Delta p_2(\lambda^*) \equiv f_L + \Delta f \lambda^*, \tag{11}$$

$$= \lambda^* f_H + (1-\lambda^*) f_L. \tag{12}$$

Equation (12) gives the expected cost of green investments for a borrower with signal λ^*.

A candidate separating equilibrium is a price difference $\Delta p = \Delta p_1 = \Delta p_2$ and λ^* that satisfy (9) and (11). These two equations equate the price difference (benefit of a green loan), with the expected green investment cost of a green loan for a borrower who is indifferent ($\lambda_i = \lambda^*$) between the two loan products.

Although conceptually straightforward, existence is complicated because an increase in λ^* has a nonlinear effect on the price difference, because the default premium must be spread over fewer borrowers. In addition, corner solutions can result if investment costs are too small or too large.

Existence of candidate equilibria with two loan products depends on two conditions:

$$c_H \leq f_L + g_1(f_L, f_H), \tag{13}$$

where $g_1 > 0$ is a function defined in Appendix (8.3) and

$$f_H > \frac{5}{2} f_L. \tag{14}$$

Appendix 8.3 shows:

Proposition 1. *Existence of candidate equilibria.*

1. *If $f_L > c_H$ then a unique candidate separating equilibrium exists with both brown and green lending, $\lambda^* \in (0,1)$.*
2. *If $f_L = c_H$ then a unique candidate pooling equilibrium exists with both brown and green lending, $\lambda^* \in (0,1)$, if and only if condition (14) holds, otherwise no equilibrium exists with both brown and green lending.*
3. *If $f_L < c_H$ and*
   - *condition (13) holds strictly and condition (14) holds, then two candidate equilibria exist, both with brown and green lending.*

- *condition (13) holds with equality and condition (14) holds, then a candidate pooling equilibrium exists with both brown and green lending.*
- *if either (13) or (14) does not hold, no equilibrium exists with both brown and green lending.*

The function g_1 is defined in the appendix and is increasing in f_H. Therefore, although three cases exist, Proposition 1 implies at least one candidate separating equilibrium exists with both brown and green lending for f_H sufficiently large.

Figure 1 shows the equilibrium geometry.

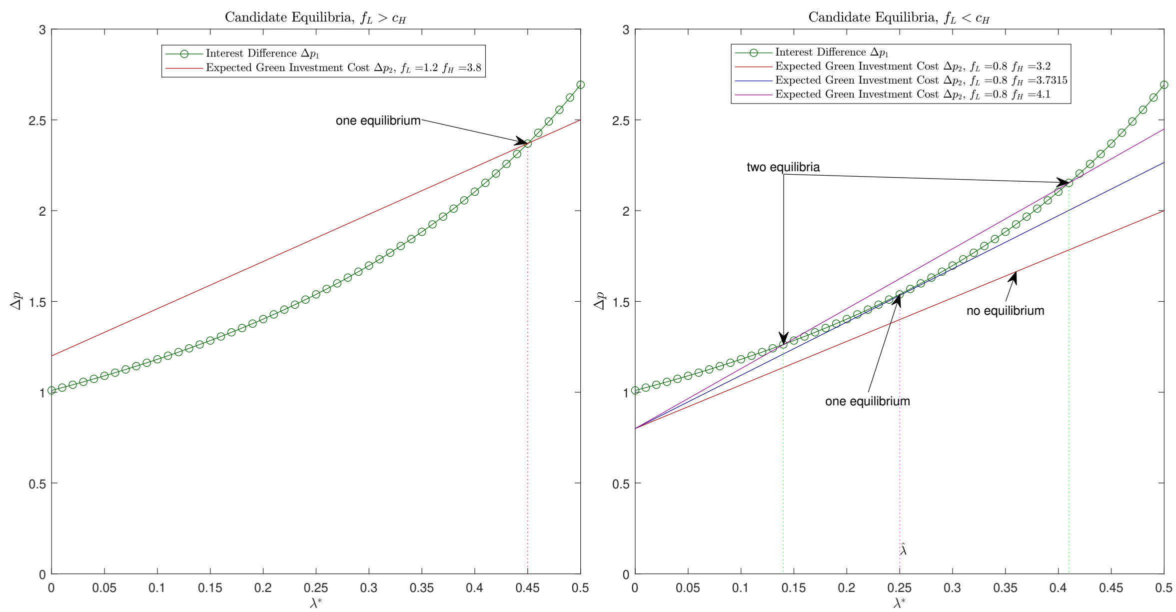

Figure 1: Equilibrium Geometry. The three lines in the right panel are for increasing values of f_H.

In Figure 1, the curve with circles is the difference in prices between brown and green loans, which is the benefit of choosing green loans for the borrower. The convexity reflects that the interest charged on brown loans increases as the default costs are spread across relatively few repaid loans when λ^* is high. The lines are the expected cost of green investments for a borrower with $\lambda_i = \lambda^*$. Equilibrium requires the price difference to be equal to the expected cost of green investment for a borrower with $\lambda_i = \lambda^*$.

The right panel shows that for $f_L < c_H$, if f_H is large enough, two candidate equilibria exist, both with green and brown loan products. In the first separating equilibrium, λ_1^*, a small price difference exists and so only very high quality borrowers have expected green investment costs low enough to justify applying for green loans. Since borrowers of moderate quality apply for brown loans, the average quality of brown borrowers is good and the brown price is relatively low. Thus, the price difference is also relatively low. In turn, only high quality borrowers apply for green loans given the small price difference.

If f_H is large, a second candidate separating equilibrium also exists where the price difference is large enough to attract moderate quality borrowers to green loans. Because only low quality borrowers are applying for brown loans, the brown loan price is high, and thus so is the price difference. Because the price difference is large, low and moderate quality borrowers have expected green investment costs which are low enough to apply for green loans.

As f_H decreases, the right panel shows that eventually the economy reaches a knife edge with only one candidate pooling equilibrium (condition 13 holds with equality) and then further decreases result in no equilibrium (condition 13 does not hold). In this case, the expected cost of green investments increases more slowly than the price difference. Therefore, a borrower with $\lambda_i = \lambda^*$ strictly prefers a green loan for any λ^*, violating the separating equilibrium indifference condition.

The left panel shows that if $f_L > c_H$, a single candidate separating equilibrium exists with both brown and green lending. The equilibrium is similar to the separating equilibrium with $f_L < c_H$ where low and moderate quality borrowers apply for green loans and the price difference is large. However, no equilibrium exists where only high quality borrowers apply for green loans. Since f_L is large, even high quality borrowers prefer brown loans if the price difference is small.

4.4 Pooling Candidate Equilibria

Additional candidate equilibria where all borrowers apply for only one type of loan product also exist. Consider first a pooling equilibrium where all borrowers apply for brown loan products. In such an equilibrium,

$$p_b(0) = c + c_H, \tag{15}$$

is the brown loan price since all borrowers are applying for brown loans. Further, $\lambda^* \leq 0$ so that even borrowers who are certain their green investment costs are low ($\lambda_i = 0$) apply for brown loans because the price difference does not cover the lowest cost of green investments:

$$\lambda^* \leq 0 \rightarrow \frac{p_b - p_g - f_L}{\Delta f} \leq 0, \tag{16}$$

$$p_g \geq c + c_H - f_L. \tag{17}$$

We restrict the equilibrium set to where equation (17) holds with equality, since all values of p_g which satisfy (17) result in the same equilibrium allocations with probability one.[15]

Hence, a pooling candidate equilibrium with single brown satisfies:

$$\begin{aligned} p_b(0) &= c + c_H, \\ p_g &= c + c_H - f_L, \\ \lambda^* &= 0 \end{aligned} \tag{18}$$

Using identical logic, a candidate pooling equilibrium with a single green loan product satisfies:

$$\begin{aligned} p_b &= c + c_H + f_H, \\ p_g &= c + c_H, \\ \lambda^* &= 1. \end{aligned} \tag{19}$$

In this case, the price difference, f_H, is large enough so that even a firm that for certain has high green investment costs applies for a green loan.

4.5 Bertrand Equilibria

Multiple candidate equilibria exist in the model because the relative prices affects the quality of loan applicants. If higher quality borrowers apply for a loan product, the likelihood of default falls, which lowers the price, which justifies more borrowers applying for the loan product. To refine the set equilibria, we impose Bertrand competition so that a bank cannot deviate by lowering a loan price and make positive profits. Bertrand competition is reasonable in a modern banking market, since typical features that lead to non-Bertrand competition are absent. Borrowers can shop a loan via internet service providers at low search cost and capacity constraints are minimal given that banks have large excess reserves and can borrow from other banks easily to increase capacity if necessary. Brown and green loan products are homogeneous across banks.

[15]Throughout the paper, we consider an equilibrium with $\lambda^* = 0$ to be a pooling equilibrium with a single brown loan product as the probability of a green loan application is zero, given only a borrower with $\lambda_i = 0$ is indifferent between brown and green loans and no borrower prefers green loans.

4.5.1 Green Pooling Candidate Equilibria

Consider first the candidate pooling equilibrium with only green loan products. Suppose a brown bank deviates by lowering the price to:

$$\tilde{p}_b = p_b - \varepsilon \Delta f, \tag{20}$$

$$\tilde{\lambda} = \frac{\tilde{p}_b - p_g - f_L}{\Delta f} = 1 - \varepsilon. \tag{21}$$

A bank making such a deviation will have a lower price than the brown banks offering the equilibrium price and will thus capture all green borrowers of quality $[1-\varepsilon, 1]$ as the lower brown price makes green loans less attractive. As ε increases, the deviating brown bank captures more borrowers and borrower quality improves.

In fact, Appendix 8.6 shows that the gain in market share and borrower quality outweighs the lower price and so a brown bank which deviates by setting $\varepsilon = 1$ captures the entire market with positive profits. Hence, no Bertrand equilibrium exists with only green loan products.

4.5.2 Brown Pooling Candidate Equilibria

Consider next a candidate pooling equilibrium with only brown loan products. A green bank with zero market share deviates by lowering the price to:

$$\tilde{p}_g = p_g - \varepsilon \Delta f, \tag{22}$$

$$\tilde{\lambda} = \frac{p_b - \tilde{p}_g - f_L}{\Delta f} = \varepsilon. \tag{23}$$

As the deviating price falls, the green bank begins to attract brown borrowers starting with the highest quality brown borrowers.

Increasing ε attracts more borrowers, but lowers both the price and average borrower quality. Hence, the best chance of a profitable deviation occurs for small ε, where the price decrease is small and only the highest quality brown borrowers switch to the deviating green bank. In fact, Appendix 8.7 shows that if:

$$f_L \geq c_H, \tag{24}$$

then no profitable deviation exists. Further, if condition (24) does not hold then a profitable deviation exists near $\varepsilon = 0$. Hence, a pooling equilibrium with only brown loan product exists if and only if condition (24) holds.

4.5.3 Separating Candidate Equilibria

Proposition 1 shows that for separating equilibria with both green and brown loan products, either zero, one, or two candidate equilibria exist, depending on the parameter values. The next proposition establishes which of the candidate equilibrium are Bertrand equilibria. Two conditions are required so that a bank cannot deviate and earn positive profits.

$$c_H \leq f_L + g_2(f_L, f_H), \tag{25}$$

where $g_2 < g_1$ is a function defined in Appendix (8.8) and

$$f_H > 3f_L. \tag{26}$$

Proposition 2. *Existence of Bertrand equilibria.*

1. *If $f_L \geq c_H$ then the unique candidate separating equilibrium that exists with both green and brown loan products is not a Bertrand equilibrium.*

2. *If $f_L < c_H$ then*

   (a) *if conditions (25) and (26) hold, two candidate equilibria exist, both with green and brown loan products, λ_1^* and λ_2^*, such that $0 < \lambda_1^* < \lambda_2^* < 1$ and λ_1^* is a Bertrand equilibrium and λ_2^* is not.*

   (b) *if condition (25) and/or (26) do not hold, then no Bertrand equilibrium exists with separating equilibrium.*

Proposition 2 shows that, under Bertrand competition, if an equilibrium exists it is always unique. Both pooling and separating candidate equilibria exist, but at most one equilibrium has no profitable deviations. The appendix shows that g_2 is increasing in f_H. Thus, for f_H sufficiently large we have only two cases. Either $f_L \geq c_H$ and the unique Bertrand equilibrium is a pooling equilibrium with only brown lending, or $f_L < c_H$ and the unique Bertrand equilibrium is a separating equilibrium with both green and brown lending.

Corollary 1. *For f_H sufficiently large:*

1. *if $f_L \geq c_H$ then in the unique Bertrand equilibrium, banks offer a single brown loan product.*

2. *if* $f_L < c_H$ *then the unique Bertrand equilibrium,* $\lambda_1^* \in (0,1)$*, has both green and brown loan products.*

If f_H is large relative to f_L, a Bertrand equilibrium always exists and is unique. The result that, for some parameter values, the Bertrand equilibrium consists of a single brown bank is in contrast to Rothschild and Stiglitz (1976), where pooling equilibria never exist. The difference is that in their paper insurance companies are free to adjust deductibles to cream skim low risk types. Here, the cost of a green investment like a solar panel cannot be manipulated by the bank and so parameter values exist where the signaling cost for low risk types is too high for cream skimming to take place.

Corollary 1 (see also Figure 8.2) provides a theoretical explanation for the empirical result that the green banking market has small interest rate premia and comprises a relatively small share of the overall banking market (Chava, 2014; Delis, Greiff, and Ongena, 2018; Ehlers, Packer, and Greiff, 2022; Shin, 2021): a large interest rate differential creates the opportunity for a brown bank to deviate and capture the market as brown borrowers prefer the lower rate and moderate quality green borrowers prefer not to undertake expensive green investments.

5 Welfare

Welfare in the model depends on the surplus accruing to banks and borrowers as well as green investment costs and environmental damage. Here we show that welfare in the unregulated market is not socially optimal, as the signaling value of green investments does not equal the net benefit from alleviating the externality.

Ex-ante expected welfare equals the sum of borrower surplus (WB) and bank surplus, less external damages to third parties. Bertrand competition ensures bank surplus is zero in expectation (ex-post surplus can be positive or negative depending on if the investment project turned out to be high or low value) and all expected private surplus from the projects accrues to the borrowers. Thus:

$$\begin{aligned} W &= \mathrm{E}\left[\textit{borrower surplus}\right] + \mathrm{E}\left[\textit{bank surplus}\right] - \mathrm{E}\left[\textit{environmental damages to households}\right] \\ &= \mathrm{E}\left[WB\right] + 0 - \mathrm{E}\left[D\right]. \end{aligned} \tag{27}$$

Parameter changes affect welfare directly, and potentially indirectly if the type of Bertrand equilibrium changes (pooling with only brown loan product vs separating with both products).

5.1 Welfare, Brown Loan Pooling Equilibrium

Corollary 1 shows that a region of the parameter space exists where banks offer a single brown loan product that doesn't require any green investment. Banks accept all loan applicants at $p_b = c + c_H$ (refer to equation 18), and borrowers have no incentive to make green investments. Therefore, damage $D(e)$ is generated by all projects.

The borrower surplus in the region of the parameter space with a single brown loan product, WB_b, is:

$$WB_b = \int_0^1 [\lambda_i \cdot 0 + (1-\lambda_i)(v_H - x - p_b)] f(\lambda_i) d\lambda_i, \tag{28}$$

$$= \frac{1}{2}(v_H - x - p_b) \tag{29}$$

In equation (28), with probability λ_i the project is low value and so the borrower defaults and gets zero. With probability $1-\lambda_i$ the project is high value, in which case the borrower repays the loan principle and interest with the income generated from the project. Given that all loan applications are accepted and the signals are uniform on the unit interval, the project is unconditionally high value with probability $1/2$, and so the borrower expected welfare in (29) is the probability that the project is high value times the borrower surplus for a high value project.

Total environmental damages to households are $D(e)$, because no green investments occur and all projects generate damages, as the project must be implemented to learn the true value.

Substituting in the price (18), borrower surplus (29), and the externality cost into the welfare function implies welfare for a pooling equilibrium with single brown loan product, W_b, is:

$$W_b = \frac{1}{2}(v_H - x) - \frac{1}{2}(c + c_H) - D(e), \tag{30}$$

$$= \frac{1}{2} \cdot \underbrace{(v_H - x - c)}_{\text{gain from high value}} - \frac{1}{2} \cdot \underbrace{(x - v_L - c)}_{\text{loss from low value}} - \underbrace{\frac{k}{2}}_{\text{transaction cost}} - \underbrace{D(e)}_{\text{environment damage}} \tag{31}$$

Here the second equation follows from the definition of c_H. The assumption $v_L - x < 0 < v_H - x$ ensures banks generate enough profits from good projects to cover the default and bank funding costs, otherwise the market would not exist. However, W_b might be negative if the project has negative net social benefits (if the external damages exceed the private net benefits).

5.2 Separating Equilibrium

With brown and green lending, green borrowers who do not default make green investments, which reduce total expected damages. The expected bank surplus is zero for both loan types and borrower surplus, WB_{gb}, is the sum of expected green and brown borrower surplus:

$$\begin{aligned} WB_{gb} = & \int_0^{\lambda^*} \left[(1-\lambda_i)(v_H - E[f|\lambda_i] - x - p_g)\right] d\lambda_i + \int_{\lambda^*}^1 \left[(1-\lambda_i)(v_H - x - p_b)\right] d\lambda_i \\ = & \frac{1}{2}\lambda^*(2-\lambda^*)(v_H - x - p_g) + \frac{1}{2}(1-\lambda^*)^2 (v_H - x - p_b) \\ & - (\lambda^*)^2 \left(\frac{1}{2} - \frac{\lambda^*}{3}\right) f_H - \lambda^* \left(\lambda^{*,2} - 3\lambda^* + 3\right) f_L. \end{aligned} \tag{32}$$

The second equality in equation (32) states that borrower surplus is the probability the loan is green and high value, times the borrower profits from a high value green loan excluding green investment costs, plus the probability that the loan is brown and high value, times the borrower profits from a high value brown loan, less the probability that the loan is green and high value, and then either higher or low green investment costs are realized. The complications arise since all of these probabilities depend crucially on λ^*.

Next, using the separating equilibrium green (4) and brown (6) prices, equation (32) simplifies to:

$$\mathrm{E}\left[WB_{gb}\right] = \frac{1}{2}(v_H - x - c - c_H) - (\lambda^*)^2 \left(\frac{1}{2} - \frac{\lambda^*}{3}\right) f_H - \frac{\lambda^*}{3}\left(\lambda^{*,2} - 3\lambda^* + 3\right) f_L. \tag{33}$$

Recall, all loan applications are eventually funded and the unconditional probability that the loan is high value is $1/2$. Thus, with probability $1/2$ we have a high value loan generating profits exclusive of green investment costs equals $v_H - x - c$. Conversely, with probability $1/2$ we have a low value loan which generates profits of $-c_H$. The last two terms of (33) reflect the green investment costs which only occur given a green loan that turns out to be high value.

Total environmental damages to households from both loan types are:

$$\begin{aligned} \mathrm{E}[D] = & \int_0^{\lambda^*} \left[(1-\lambda_i)\cdot 0 + \lambda_i D(e)\right] d\lambda_i + \int_{\lambda^*}^1 \left[(1-\lambda_i) D(e) + \lambda_i D(e)\right] d\lambda_i \\ = & D(e)\left[\frac{\lambda^{*,2}}{2} + 1 - \lambda^*\right]. \end{aligned} \tag{34}$$

Damages occur in all cases except for a high value green loan. Total welfare when banks offer two loan products is expected borrower surplus (33) less expected damages (34).

$$W_{gb}(\lambda^*) = \frac{1}{2}(v_H - x - c - c_H) - (\lambda^*)^2\left(\frac{1}{2} - \frac{\lambda^*}{3}\right) f_H - \frac{\lambda^*}{3}\left(\lambda^{*,2} - 3\lambda^* + 3\right) f_L$$
$$- D(e)\left[\frac{\lambda^{*,2}}{2} + 1 - \lambda^*\right]. \tag{35}$$

Welfare in the two loan product case reflects the private benefits of lending plus the expected green investment costs and environmental damages. The green investment costs arise from borrowers sending signals to banks. In a typical screening model (Rothschild and Stiglitz, 1976, e.g.), low risk agents do not fully insure to signal low risk, which is welfare reducing relative to full information. Here signaling has a private cost but a social benefit in reducing damages. The amount of green lending in equilibrium depends on the signaling value of green investment. However, here the net social benefit of signaling depends on the expected reduction in damages and the expected green investment costs.

5.3 Optimal Green Lending

A benevolent social planner that possesses perfect information about the borrower signals could choose which borrowers undertake green investment. Note that if it is socially optimal for a borrower with signal λ_i to make green investments, then it is optimal for any borrower with signal $\lambda_j < \lambda_i$ to also make green investments as the expected cost is lower while the expected avoided damages is the same. Thus, given perfect information about the signals, the planner would set a cutoff $\bar{\lambda}$ and require green investment for all borrowers with signals $\lambda_i \leq \bar{\lambda}$. In the next section, we show how such a welfare maximum can be implemented via government policy (e.g. a carbon tax). Even though the regulator does not in fact know the borrower signals, it can alter the payoffs of green vs brown loans to borrowers and then rely on the signaling equilibrium created by banks to get the correct borrowers to make green investments.

A social planner with perfect information might not allow borrowers with sufficiently high λ_i to invest in their projects at all, in particular if the expected damages are high enough to make the project have negative expected social value. In fact, this over-investment problem is worse than in de Meza and Webb (1987), since some borrowers have expected positive financial return, and yet have negative social return due to the externality. However, absent knowledge of λ_i, the planner cannot prevent these loans.[16] Therefore, we will compute the

[16]In de Meza and Webb (1987), borrowers must use some of their own wealth in the project. Thus, in their

value $\bar{\lambda}$ which maximizes social welfare and show this optimum is implementable via policy in the next section.

Let $\bar{\lambda}$ denote the second best optimum fraction of borrowers that make green investments. Maximizing welfare (35) results in:

$$W\left(\bar{\lambda}\right)' = \left(1-\bar{\lambda}\right)\left(D(e) - f_L - \Delta f \bar{\lambda}\right) = 0 \tag{36}$$

To find the local maximum value of $\bar{\lambda}$, we calculate second order condition:

$$W\left(\bar{\lambda}\right)'' = 2\Delta f \bar{\lambda} - D(e) + f_L - \Delta f \tag{37}$$

Here, $W(1)'' < 0$ holds only if $f_H < D(e)$, which we have ruled out by assumption.[17] Next, $W\left(\frac{D(e)-f_L}{\Delta f}\right)'' < 0$ since $f_H > D(e)$. Thus, the social optimum satisfies:

$$D(e) = \bar{\lambda} f_H + \left(1-\bar{\lambda}\right) f_L \quad \leftrightarrow \bar{\lambda} = \frac{D(e) - f_L}{\Delta f}. \tag{38}$$

The social planner maximizes welfare by incentivizing borrowers who receive a signal such that expected green investment costs are less than damages to undertake green investments. The assumption $f_L < D(e) < f_H$ also implies $\bar{\lambda} \in (0,1)$.

Since $\bar{\lambda} > 0$, any regulation which induces the social optimum generates a separating equilibrium with both brown and green lending. Therefore, the pooling equilibrium with only brown lending does not maximize welfare. In the next section we show how regulation can cause the equilibrium to generate the optimal amount of green investment.

6 Optimal Regulation

Many regulation options exist including Pigouvian taxation (e.g. a carbon tax), subsidizing green lending, or taxing brown lending. Here we focus on carbon taxes and penalizing brown lending by requiring more reserves.

framework regulators can raise the safe rate of interest, so that low expected return borrowers invest their wealth at the safe rate of interest and do not apply for loans. Here we assume borrowers do not have any wealth in the project to focus on the externality that is important for green lending.

[17]Recall $f_L < D(e) < f_H$, so that it is socially optimal for a firm with for sure low (high) green investment costs to undertake (not undertake) green investments.

6.1 Carbon Tax

Consider a tax τ per unit of e such that total tax payments are τe. To fix ideas, we will refer to τ as a carbon tax and e as carbon emissions. An important assumption is what priority carbon tax payments receive as a creditor during bankruptcy. Although the law is not settled on the matter and often bankruptcy judges have some discretion, Appelbaum (2021) and others argue that the government is an unsecured creditor and thus ranks somewhere below secured creditors and even bankruptcy fees and costs. We therefore treat the bank as a secured creditor with first claim given bankruptcy. Since we have assumed low value projects generate revenues that are insufficient to make the secured creditor (bank) whole, no carbon taxes are paid in the event of bankruptcy. The borrower decision (1) becomes:

$$(1-\lambda_i)\left(v_H - p_{g,t} - \mathrm{E}\left[f|\lambda_i\right]\right) \geq (1-\lambda_i)\left(v_H - p_{b,t} - \tau e\right), \tag{39}$$

and so the condition for applying for a green loan becomes:

$$\lambda_i \leq \lambda_\tau \equiv \frac{p_b - p_g - f_L + \tau e}{\Delta f}. \tag{40}$$

Since banks do not directly pay the tax, the loan prices at which banks earn zero profit is unchanged up to λ. Equations (4) and (6) imply:

$$p_{g,t} = c + \frac{\lambda_\tau}{2-\lambda_\tau} c_H, \tag{41}$$

$$p_{b,t} = c + \frac{1+\lambda_\tau}{1-\lambda_\tau} c_H, \tag{42}$$

$$\Delta p_1 = \frac{2c_H}{(1-\lambda_\tau)(2-\lambda_\tau)} \tag{43}$$

$$\Delta p_{2,\tau} = f_L - \tau e + \Delta f \lambda_\tau. \tag{44}$$

So, in Figure 1, the line Δp_2 shifts down if the tax is positive. For small shifts starting from a region of the parameter space where two equilibria exist with both loan products, $\lambda_{1,\tau} > \lambda_1^*$ and $\lambda_{2,\tau} < \lambda_2^*$. The equilibrium with the smaller green lending share continues to be a Bertrand equilibrium. But if the shift is too much, a single brown bank can profitably deviate and undermine the equilibrium. If the shift is larger still, the economy can enter a region where no equilibrium exists.

The equilibrium with a carbon tax is a solution to:

$$\Delta p_1 \left(\lambda_\tau\right) = \Delta p_{2,\tau} \left(\lambda_\tau\right). \tag{45}$$

Further, an equilibrium with taxes is socially optimal if $\lambda_\tau = \bar{\lambda}$.[18]

$$\begin{aligned} \Delta p_1 \left(\bar{\lambda}\right) &= \Delta p_{2,\tau} \left(\bar{\lambda}\right), \\ \frac{2c_H}{\left(1-\bar{\lambda}\right)\left(2-\bar{\lambda}\right)} &= f_L - \bar{\tau} e + \Delta f \bar{\lambda}, \end{aligned} \tag{46}$$

$$\bar{\tau} e = f_H \bar{\lambda} + \left(1-\bar{\lambda}\right) f_L - \frac{2c_H}{\left(1-\bar{\lambda}\right)\left(2-\bar{\lambda}\right)}, \quad \bar{\lambda} = \frac{D(e) - f_L}{\Delta f}. \tag{47}$$

The optimal tax first removes the signaling incentives, which is the third term after the equality in (47) and equals the price difference. Then, the first two terms of the optimal tax makes a borrower with expected green investment costs equal to expected damages indifferent between a green loan which has no tax costs and a brown loan which does.[19]

Equation (47) can also be written as:

$$\bar{\tau} = \frac{D(e) - 0}{e - 0} - \frac{1}{e} \frac{2c_H}{\left(1-\bar{\lambda}\right)\left(2-\bar{\lambda}\right)}. \tag{48}$$

The tax equals the equivalent, in our discrete framework, to the marginal damage less the correction which removes the signaling value.

Note from Figure 1, $\Delta p_1 > (<,=) \Delta p_2$ for $\lambda < (>,=) \lambda_1^*$. Hence:

$$\begin{aligned} \bar{\lambda} > \lambda_1^* \quad & \bar{\tau} e > 0 \\ \bar{\lambda} = \lambda_1^* \quad & \bar{\tau} e = 0 \\ \bar{\lambda} < \lambda_1^* \quad & \bar{\tau} e < 0. \end{aligned} \tag{49}$$

That is, if the signaling value generates too little (too much) green investment, the optimal carbon tax is positive (negative) to create more (less) green investment.

It remains to determine whether profitable deviations exist at $\lambda_\tau = \bar{\lambda}$. As shown in Proposition 2, if the separating equilibrium fraction of green lending (in this case $\bar{\lambda}$) is too large, then a brown bank can deviate and make positive profits. Consider the following change of

[18] We assume that tax revenue is rebated back to households so that tax revenue does not affect welfare. Only the change in incentives created by the tax matter.

[19] Further, the optimal loan product price difference depends only on the external damages, not the signal value.

variables:

$$f_{L,\tau} \equiv f_L - \tau e, \tag{50}$$

$$f_{H,\tau} \equiv f_H - \tau e, \tag{51}$$

$$\Delta f_\tau \equiv f_{H,\tau} - f_{L,\tau} = \Delta f. \tag{52}$$

Then:

$$\lambda_\tau \equiv \frac{p_b - p_g - f_{L,\tau}}{\Delta f_\tau}. \tag{53}$$

The problem with a tax is identical to the unregulated problem except f_H and f_L change by identical amounts. So can use all of the results in Propositions 1 and 2. In particular, λ_τ is a Bertrand equilibrium with two loan products if and only if:

$$f_{H,\tau} > 3 f_{L,\tau}, \tag{54}$$

$$f_{L,\tau} < c_H \leq f_{L,\tau} + g_2\left(f_{H,\tau}, f_{L,\tau}\right). \tag{55}$$

Suppose that the unregulated signaling economy generates too little green investment and the optimal tax is positive, so that $f_{L,\tau} < f_L$ and $f_{H,\tau} < f_H$. Then condition (54) and the first inequality in condition (55) become less restrictive. Further, in the right inequality of (55) the first term decreases and g_2 increases relative to the unregulated problem. The tax means it is more difficult for a brown bank to deviate and profitably offer a lower rate given that a borrower switching from a green to brown loan must now pay the tax (g_2 increases). However, the term $f_{L,\tau}$ also decreases, which reflects that the optimal tax creates more green lending, and too much green lending creates an opportunity for a brown bank to profitably deviate.

Next, suppose that either (54) or (55) is violated at the optimal tax, so that the optimal tax does not result in a Bertrand equilibrium. From equations (36) and (37), welfare is increasing (decreasing) in λ for $\lambda < (>) \bar{\lambda}$. The best the regulator can do is choose a tax to get the highest λ such that no deviations are possible.

Consider now a carbon tax where the equilibrium consists of a single brown loan product. The borrower still decides whether or not to pay the carbon tax. The borrower considers only the tax vs the cost of reducing carbon emissions. The borrower reduces if and only if:

$$\mathrm{E}\left[f|\lambda_i\right] \leq \tau e,$$

$$\lambda_i \leq \frac{\tau e - f_L}{\Delta f}. \tag{56}$$

Next, in any equilibrium where $p_b \geq p_g$, any borrower with λ_i satisfying (56) will also apply for a green loan since they are making green investments already. Thus, any pooling equilibrium with one brown loan product satisfying $p_b \geq p_g$ must also satisfy $\tau e \leq f_L$, so that no borrower chooses to reduce and therefore also chooses a green loan. Modifying (18) results in:

$$\begin{aligned} p_b &= c + c_H, \\ p_g &= c + c_H - f_L + \tau e = c + c_H - f_{L,\tau} \\ \lambda_\tau &= 0. \end{aligned} \tag{57}$$

The equilibrium results in no green lending and has the same no deviation condition, up to f_L, $f_{L,\tau} \geq c_H$. Such an equilibrium is sub-optimal since it is efficient for at least some borrowers to make green investments (recall $D > f_L$). The regulator can improve welfare by increasing τe above f_L and shifting to a separating equilibrium with two loan products.

Proposition (3) summarizes the above analysis:

Proposition 3. *Regulated economy with a tax τ per unit of e.*

1. *If $f_{L,\tau} \geq c_H$ (which implies $\tau e < f_L$), then (57) is a sub-optimal Bertrand equilibrium with no green investment and only brown lending.*

2. *If (54) and (55) hold, a Bertrand equilibrium $\lambda_{1,\tau}$ with two loan products exists satisfying (45).*

3. *Equation (47) gives the socially optimal tax, which is less than the marginal damage.*

4. *The socially optimal tax results in a Bertrand equilibrium if (54) and (55) hold, where equation (47) gives the tax.*

5. *If the socially optimal tax does not result in a Bertrand equilibrium, the highest welfare that is a Bertrand equilibrium, the constrained social optimum, is a tax which creates the highest possible λ_τ such that no deviations are possible.*

Proposition 3 points to several subtleties with regard to carbon tax policy given a green banking market. First, if in the absence of a carbon tax the parameter values generate a pooling brown market, then a tax smaller than the marginal damage is optimal. The tax shifts the equilibrium to a separating equilibrium which provides private motivation for green investment. In this sense, a small carbon tax can have a relatively large effect by shifting the equilibrium from pooling to separating. Although the US does not have a national carbon tax, carbon is priced in some regions through cap and trade systems. Thus, one could empirically test whether or not the green banking market developed in response to an increasing price of carbon.

Surprisingly, Proposition 3 states that green lending can create a problem: if the optimum requires too much green investment, an an equilibrium does not exist. Notice that if no green lending were possible, then the optimal carbon tax equals the marginal damage and the social optimum results.

6.2 Reserve Requirement

Central banks have proposed including climate risks as part of their overall assessment of financial risk at banks and in the banking system (Acharya et al., 2023). Often, central banks specify that such regulation be designed to reduce financial risk at banks related to climate change. The environmental benefits are ancillary. However, the ECB is aligning policy with "the objectives of the Paris Agreement and the EU's climate neutrality objectives" (ECB, 2022). Taking this statement as given, here we consider bank regulation designed to induce the welfare maximizing level of green investment. A straightforward way to model such regulation is to impose a reserve requirement for brown loans.[20]

In particular, suppose regulators impose a required reserve ratio of $rr = \alpha(1-\lambda_r)/2 = R/x$ on brown loans, where α is a policy parameter, λ_r is the cutoff such that borrowers with $\lambda_i > \lambda_r$ apply for brown loan given the reserve requirement, and R is the dollar amount of required reserves.

The functional form of the required reserve ratio is chosen for ease of analysis but in fact is not a restriction, because we will show the optimal required reserve ratio depends on $\lambda_r = \bar{\lambda}$.[21] The bank now requires $x+R$ dollars to make a brown loan of x. The rate of interest the bank pays the depositor or other funding source is c/x, and so the dollar cost of funds for a brown loan becomes $(1+rr)\,c$.

In the event of bankruptcy by the borrower, the bank has some reserves. We assume that the bank losses are paid by the bank stockholders through negative profits, rather than having the bank default and not repay the depositors. In this case, bank profits on a defaulted brown

[20] A number of other options are possible but all would generate similar results, including subsidizing green loans through establishment of a green bank that provides credit enhancements for green loans (recommended in a CFTC report, see Martinez-Diaz and Keenan, 2020, p. 109) or prioritizing green bonds in central bank corporate bond holdings (announced by the European Central Bank, ECB, 2022).

[21] The optimal carbon tax also depends on $\bar{\lambda}$.

loan are $-c_{H,r} = v_L - x - (1+rr)c - k = -c_H - rrc$, and bank profits on a successful brown loan are $p_b - (1+rr)\,c$.

The equilibrium outcomes in the model with a reserve requirement are:

$$p_{g,r} = c + \frac{\lambda_r}{2-\lambda_r} c_H = p_g, \tag{58}$$

$$\begin{aligned} p_{b,r} &= c\,(1+rr) + \frac{1+\lambda_r}{1-\lambda_r} c_{H,r} \\ &= p_b + \frac{2rr \cdot c}{1-\lambda_r}, \end{aligned} \tag{59}$$

$$\lambda_r = \frac{p_{b,r} - p_{g,r} - f_L}{\Delta f} \tag{60}$$

The price difference rises to:

$$\begin{aligned} \Delta p_{1,r} &= \frac{2c_H}{(1-\lambda_r)\,(2-\lambda_r)} + \frac{2rr \cdot c}{1-\lambda_r}, \\ &= \frac{2c_H}{(1-\lambda_r)\,(2-\lambda_r)} + \alpha \end{aligned} \tag{61}$$

and the line governing borrower behavior is unchanged up to λ:

$$\Delta p_{2,r} = f_L + \Delta f \lambda_r. \tag{62}$$

In Figure 1, the line is unchanged and the curve which is the price difference shifts up by α. The functional form for the reserve ratio was in fact chosen so that α shifts the price difference upward equally for all λ. Assuming the new candidate separating equilibrium with two banks remains a Bertrand equilibrium, we have $\lambda_{1,r} > \lambda_1^*$ and $\lambda_{2,r} < \lambda_2^*$.

Combining equations (58)-(60) generates the equilibrium condition for the economy with a required reserve ratio:

$$\Delta p_{1,r} = \frac{2c_H}{(1-\lambda_r)\,(2-\lambda_r)} + \alpha = \Delta p_{2,r} = f_L + \Delta f \lambda_r. \tag{63}$$

Further, $\bar{\lambda}$ is an equilibrium in the economy with a reserve requirement if:

$$\alpha = D(e) - \frac{2c_H}{\left(1-\bar{\lambda}\right)\left(2-\bar{\lambda}\right)}, \tag{64}$$

which is identical to the optimal carbon tax ($\tau e = \alpha$ at the optimum).[22] As with the tax, the optimal reserve policy replaces the signaling value of green investments with the marginal damage. The implied optimal required reserve ratio is:

$$rrc = \frac{1-\bar{\lambda}}{2}\left(D(e) - \frac{2c_H}{(1-\bar{\lambda})(2-\bar{\lambda})}\right). \tag{65}$$

As damage and $\bar{\lambda}$ increase, the price difference widens as brown borrowers are increasingly likely to default, and so the brown rate becomes large to maintain zero profits. The optimal reserve requirement decreases as borrowers already have a strong signaling incentive for green loans.

It remains to find the range of parameter values such that a Bertrand equilibrium exists. Consider the change of variables:

$$f_{L,r} \equiv f_L - \alpha, \tag{66}$$

$$f_{H,r} \equiv f_H - \alpha, \tag{67}$$

$$\Delta f_r \equiv f_{H,r} - f_{L,r} = \Delta f. \tag{68}$$

The equilibrium condition then becomes:

$$\frac{2c_H}{(1-\lambda_r)(2-\lambda_r)} = f_{L,r} + \Delta f_r \lambda_r. \tag{69}$$

Thus the equilibrium equation and conditions for the existence of candidate equilibria are unchanged up to f_H and f_L.

The conditions for existence of a candidate equilibrium are identical under tax and required reserve regulations, but the no-deviation conditions are different. To see which regulation system has the largest range of λ with no profitable deviations, consider the case where the reserve requirement and the tax are set so that the equilibrium is the same: $\lambda_r = \lambda_\tau$. Then:

$$\lambda_r = \frac{P_{b,r} - p_{g,r} - f_L}{\Delta f} = \lambda_\tau = \frac{P_{b,\tau} - p_{g,\tau} - f_L + \tau e}{\Delta f},$$

$$\Leftrightarrow p_{b,r} = p_{b,\tau} + \tau e \tag{70}$$

The brown loan product price is higher with the reserve requirement versus the tax. The tax

[22] Note that the cost of holding reserves is accounted for in the welfare function. The borrowers pay for the cost of holding extra reserves through the higher brown rate, which accounts for the welfare loss of the opportunity cost of using these funds elsewhere, c.

is placed on the borrowers which discourages brown borrowing. In contrast, the reserve requirement relies on increasing the price difference to discourage brown borrowing. However, the high price difference given required reserve regulation makes deviating more attractive. A deviating brown bank induces green borrowers to switch which is more profitable when the brown price is already high and very few repaying brown borrowers exist to spread the default costs over.[23]

Graphically, Figure 1 shows that the price difference increases in a convex way with λ. As the quality of borrowers decreases in the brown loan market, an increasingly high brown rate is required to spread the default costs over the relatively few brown borrowers that repay their loan. As λ increases and the price difference widens, it becomes increasingly attractive for a brown bank to lower the rate slightly, gaining green borrowers who, if they repay, generate large profits. This is why in the unregulated economy high values of λ^* are not Bertrand. The same principle holds in the regulated economy. The higher reserve requirement increases the price difference and makes deviations more attractive.

In fact, we show in the appendix that the new no deviation conditions become:

$$f_{H,r} > 3f_{L,r} + \alpha \tag{71}$$

$$f_{L,r} < c_H < f_{L,r} + g_{2,r}\left(f_{L,r}, f_{H,r}, \alpha\right) \tag{72}$$

Here $g_2\left(f_{L,r}, f_{H,r}\right) = g_{2,r}\left(f_{L,r}, f_{H,r}, 0\right)$ and $g_{2,r}$ is decreasing in α. Then if the optimal policy is to increase green lending ($\alpha > 0$), parameter values exist such that $\bar{\lambda}$ is a Bertrand equilibrium given the carbon tax, but is not an equilibrium with the optimal reserve policy.

Proposition 4 summarizes the above analysis:

Proposition 4. *Required reserve regulation. Suppose a required reserve ratio* $rr = \alpha\left(1-\lambda_r\right)/2$ *per dollar of brown loans and* $\alpha > 0$.

1. *If* $f_{L,\tau} \geq c_H$ *(which implies* $\alpha < f_L$*), then a sub-optimal Bertrand equilibrium exists with no green investments and only brown lending.*

2. *If (71) and (72) hold, a Bertrand equilibrium* $\lambda_{1,r}$ *with two loan products exists satisfying (58), (59), and (63).*

3. *Equation (65) gives the socially optimal required reserve policy, which is less than the marginal damage.*

[23]The deviating brown bank also loses more if the borrower that switches from green to brown does not repay, since the reserve requirement must also be repaid. However, this is outweighed by the higher price if the borrower does repay and the spreading of the default costs.

4. *The socially optimal required reserve policy results in a Bertrand equilibrium if (71) and (72) hold, where equation (65) gives the required reserve ratio.*

5. *The optimal reserve policy parameter α equals the optimal tax policy.*

6. *There exists an open set of parameter value such that the optimal tax policy is a Bertrand equilibrium and the optimal reserve policy is not.*

Both the carbon tax and the reserve requirement can be set so that the socially optimal fraction of green investment, $\bar{\lambda}$ is a candidate equilibrium. However, if the social optimum is too large, brown banks may deviate and enter the market, causing $\bar{\lambda}$ to be not a Bertrand equilibrium.

6.3 Combined Regulation

If the efficient amount of green investment is not too high, either a carbon tax or a reserve requirement on brown loans can induce the efficient allocation by replacing the signaling incentive with the marginal damage. However, one of the virtues of the traditional carbon tax is that the efficient carbon tax is independent of the economic financing structure. The regulator need only set the carbon tax equal to the marginal damage to get the efficient outcome.[24] The information requirement of the carbon tax rises considerably if the tax varies depending on if the project is bank financed or not.[25]

An appealing alternative is to set the carbon tax equal to the marginal damage, and then use the reserve requirement on brown loans to remove the signaling benefit of green loans in the lending market. This reduces the information required for the carbon tax. The required reserve ratio on brown loans can then be set at the Federal Reserve where banks submit information that might allow the FED to learn the signaling value (e.g. c_H).

Consider then a combined optimal policy:

$$\begin{aligned} \tau e &= D(e), \\ \alpha &= -\frac{2c_H}{\left(1-\bar{\lambda}\right)\left(2-\bar{\lambda}\right)}, \end{aligned} \tag{73}$$

[24] Note that other market characteristics which vary by industry may affect the optimal carbon tax. For example, Simpson (1995) shows that the optimal carbon tax in a duopoly is less than the optimal carbon tax given perfect competition.

[25] Indeed, the expected bankruptcy costs, c_H, and thus the optimal carbon tax, likely varies from borrower to borrower.

Policy (73) yields the efficient allocation, assuming no profitable deviations exist. Notice that the optimal required reserve policy *subsidizes* brown loans. Given the signaling value of green investments, the reserve requirement must be negative to equalize the attractiveness of brown versus green lending, after which a carbon tax equal to the marginal damage resolves the externality.

To determine whether profitable deviations exist, consider the change of variables:

$$f_{L,\tau r} \equiv f_L - \alpha - \tau e, \tag{74}$$

$$f_{H,\tau r} \equiv f_H - \alpha - \tau e, \tag{75}$$

$$\Delta f_{\tau r} \equiv f_{H,r} - f_{L,r} = \Delta f. \tag{76}$$

The conditions for existence are unchanged, up to $f_{L,\tau r}$ and $f_{H,\tau r}$. Further, the condition for no-deviations is unchanged up to $f_{L,\tau r}$, $f_{H,\tau r}$, and α. Thus an equilibrium exists with no profitable deviations if:

$$f_{H,\tau r} > 3 f_{L,\tau r} + \alpha \tag{77}$$

$$f_{L,\tau r} < c_H < f_{L,\tau r} + g_{2,r}\left(f_{L,\tau r}, f_{H,\tau r}, \alpha\right) \tag{78}$$

Finally, at the efficient allocation, $f_{H,r} = f_{H,\tau r} = f_{H,\tau}$ and the same for f_L. Thus, the no-deviation conditions given the combined regulation (77) and (78), differ from the no-deviation conditions given the required reserve ratio regulation (71) and (72) only because the reserve requirement on brown loans is larger $\alpha_r > \alpha_{\tau r}$. Recall, $g_{2,r}$ is decreasing in α so the range of values c_H such that no-deviations are possible is larger when using the combined regulation. Further, since $\alpha = 0$ when the carbon tax is used in isolation, we have $\alpha_\tau = 0 > \alpha_{\tau r}$ and so:

$$\begin{aligned} f_{L,\tau r} < c_H &< f_{L,\tau r} + g_{2,r}\left(f_{L,\tau r}, f_{H,\tau r}, \alpha_r\right) \\ &< f_{L,\tau} + g_2\left(f_{L,\tau}, f_{H,\tau}, 0\right) \\ &< f_{L,\tau r} + g_{2,r}\left(f_{L,\tau r}, f_{H,\tau r}, \alpha_{\tau r}\right). \end{aligned} \tag{79}$$

Further, the condition $f_{H,\tau r} > 3 f_{L,\tau r} + \alpha$ is most strict with required reserve regulation and least strict with the combined regulation.

Proposition (5) summarizes the above analysis:

Proposition 5. *Consider a regulated economy with a required reserve ratio of* $rr = \alpha\left(1-\lambda_r\right)/2$ *per dollar of brown loans and a tax* τ *per unit of carbon emissions.*

1. *If (77) and (78) hold, a Bertrand equilibrium* $\lambda_{1,\tau r}$ *with two loan products exists.*

2. *Equation (73) gives the socially optimal policy.*

3. *If (77) and (78) hold at the socially optimal policy (73), then the social optimum is a Bertrand equilibrium.*

4. *There exists an open set of parameter value such that the optimal combined policy is a Bertrand equilibrium and the optimal reserve and tax policies in isolation are not.*

The combined policy thus has two advantages. First, the tax is set equal to the marginal damage and so the tax does not have to vary by funding source or by borrower, and can be set by the EPA or other regulator with knowledge of the damages. The required reserve ratio does require knowledge of the borrower risk (c_H), but is set by the bank regulator who presumably has better access to such information. Second, the combined policy can support a higher level of green investment with no-deviations than either the carbon tax or required reserve ratio in isolation. Thus, the combined policy is particularly attractive if damages are large.

7 Conclusions

The Federal Reserve and other public and private organizations argue that bank lending to borrowers with poor environmental metrics, or more generally poor ESG scores, have risks. These risk include transition risk (the risk of stricter environmental regulation imposing additional compliance costs), reputation risk, and others. In turn, banks have responded by offering lower lending rates to borrowers that satisfy environmental or ESG performance metrics. Here we take these risks as given, and examine the implications for welfare and environmental policy.

We find that borrowers engage in costly effort to improve environmental performance (green investments) based their ability to signal low risk. Therefore, green lending may increase welfare relative to an unregulated economy, but will not result in the social optimum as the marginal signaling value does not equal the marginal value of environmental damage alleviated. A carbon tax or a reserve requirement on brown loans can achieve the social optimum by replacing the signaling value with the marginal damage. However, if the optimal green lending is too large so that most or all borrowers make green investments, then the signaling value vanishes and the equilibrium unravels as banks are no longer willing to give lower rates, which in turn changes the optimal regulation. Thus, if the marginal damage is too high, carbon taxes or reserve requirements are unable to achieve the social optimum. Finally, a combined carbon tax equal to the marginal damage and a subsidy for brown loans which removes the incentive to signal with green investments achieves the highest level of green investments.

Our results come with several caveats. First, our model assumes banks rely only on public information to set loan product prices. An interesting extension is to allow banks to investigate borrowers for credit worthiness. We leave this possibility for future research, noting that investigating can result in more efficient lending. We have assumed only a single motive for green lending, which is the risk associated with poor environmental metrics. Other motivations, including stockholder and employee preferences, likely also increase green lending, but do not incentivize socially optimal green lending. We also leave other motivations for future research.

Across the economy, firms are undertaking activities to improve environmental, social, and governance metrics. Firms have a variety of private motivations for these activities which align only partially with the public interest. Regulators seeking to achieve welfare improvements by regulating activities that cause external harm, must now consider the interaction between regulation and private ESG activities. Given the trend in the size of green investments by firms, this coordination will only become more important in the future.

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8 Appendix

8.1 Zero Profits

Here we show that Bertrand competition ensures banks of the same type offer identical rates and earn zero profits. Consider any set of prices such that a bank l' offers a rate $p_{j,l'} > p_{j,l}$ in market $j \in \{g,b\}$, which exceeds that of n_j bank(s) which offer the lowest rate in market j, p_j. Then bank l' has no customers and earns zero profits. Further, banks offering price p_j earn non-negative profits $\left(1/n_j\right)\pi_j \geq 0$ to offer a loan in market j. Thus, bank l' can weakly improve profits from $\pi_{l'} = 0$ to $\left(1/\left(n_j+1\right)\right)\pi_j \geq 0$ by lowering its price to match the lowest price among competing banks. Thus all banks in the same market offer the same rate in equilibrium.

Next, suppose that there exists $N > 1$ banks offering the lowest price in market j, p_j, with corresponding profits $(1/N)\pi_j\left(p_j\right) > 0$. Note that if only $n_j < N$ banks are in market j, then a bank that is not participating can enter at the same price and increase profits from zero to $\left(1/(n_j+1)\right)\pi_j\left(p_j\right) > 0$. Suppose that participating bank l' deviates and offers a slightly lower rate, $p_j - \varepsilon$. Since bank l' now has the lowest price, it captures the entire market. The deviation is profitable if and only if:

$$(1/N)\pi_j\left(p_j\right) < \pi_j\left(p_j - \varepsilon\right), \tag{80}$$

$$1/N < \frac{\pi_j\left(p_j - \varepsilon\right)}{\pi_j\left(p_j\right)}. \tag{81}$$

Since the profit function is continuous (refer to equations 5 and 3) and we have assumed $\pi_j(p_j) > 0$, the right hand side approaches one for ε small and the left side is at most $1/2$. It follows that (81) holds for ε sufficiently small and thus an incentive to deviate and cut prices exists whenever $\pi_j(p_j) > 0$.

Thus, we have shown that profits are zero and all banks in market $j \in \{g,b\}$ charge the same price.

8.2 Graphical Analysis

A candidate equilibrium is a solution $[\lambda^*, p_g, p_b]$ that satisfies $p_b > p_g$ and solves equations (1), (4), and (6). To find the equilibrium we show each equation satisfies certain properties.

The properties of p_g and p_b follow from substituting in $\lambda^* = 0$ and $\lambda^* = 1$, and by taking the derivatives of equations (4) and (6) to get:

$$p_g' = \frac{2c_H}{(2-\lambda^*)^2} > 0,$$
$$p_b' = \frac{2c_H}{(1-\lambda^*)^2} > 0, \tag{82}$$

Next, we combine equations (4) and (6) to form:

$$\Delta p_1(\lambda^*) \equiv p_b - p_g = \frac{2c_H}{(1-\lambda^*)(2-\lambda^*)}. \tag{83}$$

Equation (83) implies $\Delta p_1(\lambda^*) > 0$, $\Delta p_1(0) = c_H$, and $\lim_{\lambda^* \to 1} \Delta p_1(\lambda^*) = \infty$. The remaining properties follow from:

$$\Delta p_1'(\lambda^*) = \frac{2(3-2\lambda^*)c_H}{(1-\lambda^*)^2(2-\lambda^*)^2} > 0, \tag{84}$$

$$\Delta p_1''(\lambda^*) = \frac{4\left(7-9\lambda^*+3\lambda^{*2}\right)c_H}{(1-\lambda^*)^3(2-\lambda^*)^3}. \tag{85}$$

Denote the numerator of (85) as $M(\lambda^*)$. Then:

$$M(\lambda^*) \equiv 7-9\lambda^*+3\lambda^{*2},$$
$$M'(\lambda^*) = -9+6\lambda^* < 0,$$
$$M(0) = 7\,,\, M(1) = 1. \tag{86}$$

Hence the numerator of (85) is function which strictly decreases from seven to one over the interval $[0,1]$, and so the second derivative of Δp_1 is positive.

Next, rewriting equation (2) results in:

$$\Delta p_2(\lambda^*) \equiv p_b - p_g = f_L + \Delta f \lambda^*, \tag{87}$$

which is an increasing function which satisfies $\Delta p_2(0) = f_L$ and $\Delta p_2(1) = f_H$.

8.3 Proof of First Proposition

It is convenient here to work with a different version of the equilibrium condition. For any interior (both brown and green lending) candidate equilibrium λ^*:

$$DP_1(\lambda^*) = DP_2(\lambda^*), \tag{88}$$

$$\frac{2c_H}{(1-\lambda^*)(2-\lambda^*)} = f_L + \Delta f \lambda^*, \tag{89}$$

which holds for interior λ^* if and only if:

$$H(\lambda^*) \equiv \Delta f \lambda^{*,3} - (3f_H - 4f_L)\lambda^{*,2} + (2f_H - 5f_L)\lambda^* - 2(c_H - f_L) = 0. \tag{90}$$

Any equilibrium with both brown and green lending λ^* satisfies $H(\lambda^*) = 0$. We now establish several properties of H:

$$H(0) = -2(c_H - f_L). \tag{91}$$

$$H(1) = -2c_H < 0. \tag{92}$$

$$H'(\lambda) = 3\Delta f \lambda^2 - 2(3f_H - 4f_L)\lambda + 2f_H - 5f_L. \tag{93}$$

$$H'(0) = 2f_H - 5f_L. \tag{94}$$

$$H'(1) = -f_H < 0. \tag{95}$$

$$H''(0) = -2(3f_H - 4f_L). \tag{96}$$

$$H''(1) = 2f_L > 0. \tag{97}$$

Consider first the quadratic derivative H'. Since $H'(1) < 0$, H' is increasing at $\lambda = 1$, and $H''' > 0$, H' has a global minimum which is less than one. Further, if $H'(0) > 0$, then the global minimum is on the $(0,1)$ interval and there exists a unique $\tilde{\lambda} \in (0,1)$ such that $H'(\lambda) > (=,<) 0$ for $\lambda < (=,>) \tilde{\lambda}$. Note further that $H'(0) > 0$ if and only if $2f_H > 5f_L$ from (94). If $2f_H \leq 5f_L$, then $H'(\lambda) \leq 0$ for all $\lambda \in [0,1]$ (either $H''(0) < 0$ and the minimum is negative, or $H''(0) > 0$ and the minimum is positive).

Figure 2 shows the possible cases.

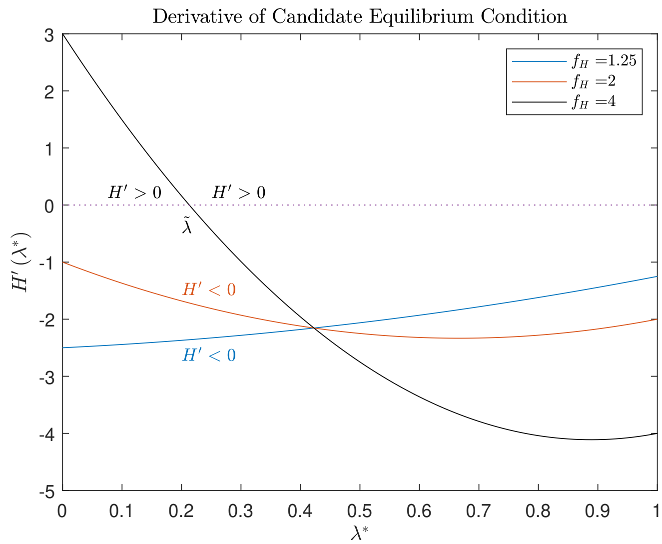

Figure 2: Derivative of candidate equilibrium condition, with $f_L = 1$ and values of f_H which correspond to the three possible cases.

Consider now the equilibrium condition (90). We consider three cases, $f_L > (<,=) c_H$.

Case 1: $f_L > c_H$

Consider first $f_L > c_H$, so that $H(0) > 0$ from (91). From equations (92) and (95), H is negative and decreasing at $\lambda = 1$. Thus, since H is continuous on $\lambda \in (0,1)$ there exists at least one candidate equilibrium λ^* such that $H(\lambda^*) = 0$.

Next, H' is either strictly negative or is negative on the interval $(\tilde{\lambda}, 1)$ (refer to Figure 2). If H' is strictly negative, it is immediate that λ^* is unique because H is monotonically decreasing for $\lambda \in (0,1)$. If H' is negative on the interval $(\tilde{\lambda}, 1)$ then H has a unique maximum on $(0,1)$ at $\tilde{\lambda}$. Since H is positive and increasing at 0 and negative and decreasing at 1, it follows that a unique $\lambda^* \in (\tilde{\lambda}, 1)$ exists such that $H(\lambda^*) = 0$. Thus, for the case $f_L > c_H$, a unique candidate equilibrium exists on the interval $(0,1)$.

Case 2: $f_L = c_H$

For the knife-edge case where $f_L = c_H$, equation (90) implies $\lambda^* = 0$ is a candidate equilibrium. However, $\lambda^* = 0$ means no green lending exists. Dividing equation (90) by λ when $f_L = c_H$ yields:

$$\tilde{H} = H(\lambda; f_L = c_H)/\lambda = \Delta f \lambda^2 - (3f_H - 4f_L)\lambda + (2f_H - 5f_L) = 0. \tag{98}$$

$$\tilde{H}(1) = -2f_L < 0, \tag{99}$$

$$\tilde{H}(0) = 2f_H - 5f_L, \tag{100}$$

$$\tilde{H}' = 2\Delta f \lambda - 3f_H + 4f_L \rightarrow \lambda_{min} = \frac{3f_H - 4f_L}{2\Delta f}, \tag{101}$$

$$\tilde{H}'(1) = -f_H + 2f_L, \tag{102}$$

$$\tilde{H}'(0) = -3f_H + 4f_L, \tag{103}$$

All of these conditions depend on how large is f_H relative to f_L:

1. $f_L < f_H < 4/3 f_L$: $\tilde{H}$ is negative and increasing at zero and negative and increasing at one, and the global minimum is negative. Thus, $\tilde{H}$ is negative on $(0,1)$, and no root (candidate equilibrium) exists.

2. $4/3 f_L < f_H < 2f_L$: $\tilde{H}$ is negative and decreasing at zero and negative and increasing at one, and the global minimum is positive. Thus, $\tilde{H}$ is negative on $(0,1)$ and no root (candidate equilibrium) exists.

3. $2f_L < f_H < 5/2 f_L$: $\tilde{H}$ is negative and decreasing at zero and negative and decreasing at one, and the global minimum is positive. Thus, $\tilde{H}$ is negative on $(0,1)$ and no root exists.

4. $5/2 f_L < f_H$: $\tilde{H}$ is positive and decreasing at zero and negative and decreasing at one. By continuity, a unique root exists on $(0,1)$.

Thus we see that for the knife edge case of $f_L = c_H$, a unique equilibrium with brown and green lending exists if and only if $f_H > 5/2 f_L$.

Case 3: $f_L < c_H$

Consider next the case where $f_L < c_H$. The above properties then show that $H(1) < H(0) < 0$. Thus if $H' < 0$ on $(0,1)$, H is strictly negative on the interval $(0,1)$ and thus no equilibrium exists with both brown and green lending. It follows that $2f_H > 5f_L$ is necessary for existence in this case. Given $2f_H > 5f_L$, we have $H'(0) > 0$ and $H'(1) < 0$. It follows that H has a unique maximum exists at $\hat{\lambda}$ on the interval $(0,1)$. Thus H has two roots on the interval $(0,1)$ if and only if $H\left(\hat{\lambda}\right) > 0$.

Figure 3 shows the possible cases.

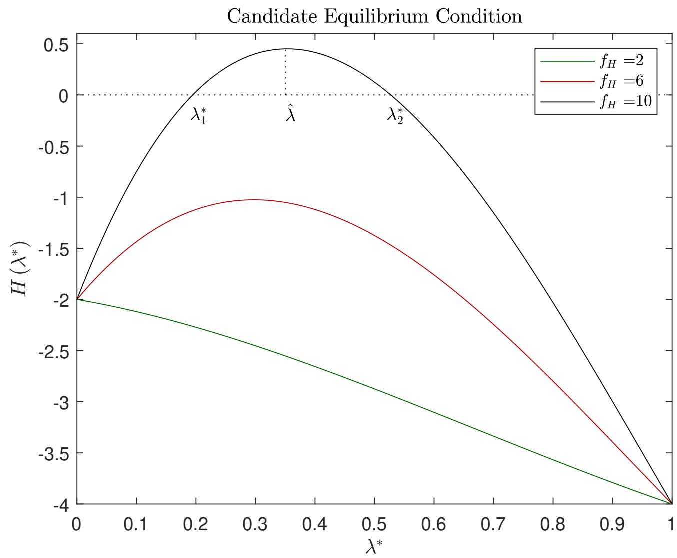

Figure 3: Candidate equilibrium condition, with $f_L = 1$, $c_H = 2$, and values of f_H which correspond to the three possible cases. The black line is positive at the maximum ($H\left(\hat{\lambda}\right) > 0$) and thus has two equilibrium candidates with both brown and green lending (λ_1^* and λ_2^*). The other two lines are negative at the maximum, ($H\left(\hat{\lambda}\right) < 0$) and thus have no candidate equilibria with both brown and green lending.

It remains to derive a condition such that $H\left(\hat{\lambda}\right) > 0$. Note also that $H\left(\hat{\lambda}\right) = 0$ is the knife edge case with a single equilibrium. We must show that:

$$\Delta f \hat{\lambda}^3 - (3f_H - 4f_L)\hat{\lambda}^2 + (2f_H - 5f_L)\hat{\lambda} - 2(c_H - f_L) > 0, \tag{104}$$

given:

$$3\Delta f \hat{\lambda}^2 - 2(3f_H - 4f_L)\hat{\lambda} + 2f_H - 5f_L = 0. \tag{105}$$

Multiplying the above two equations by 3 and λ and then subtracting simplifies the condition to:

$$-(3f_H - 4f_L)\hat{\lambda}^2 + 2(2f_H - 5f_L)\hat{\lambda} - 6(c_H - f_L) > 0 \tag{106}$$

Note also that the solution to (105) is:

$$\hat{\lambda} = \frac{3f_H - 4f_L - \sqrt{3f_H^2 - 3f_H f_L + f_L^2}}{3\Delta f}. \tag{107}$$

Substituting the solution for $\hat{\lambda}$ into (106) and extensively simplifying gives:

$$\frac{2\left(3f_H^2-3f_Hf_L+f_L^2\right)^{\frac{3}{2}}-f_L\left(3f_H-4f_L\right)\left(15f_H-13f_L\right)}{9\Delta f^2}>6\left(c_H-f_L\right). \tag{108}$$

$$c_H<f_L+\frac{2\left(3f_H^2-3f_Hf_L+f_L^2\right)^{\frac{3}{2}}-f_L\left(3f_H-4f_L\right)\left(15f_H-13f_L\right)}{54\Delta f^2}\equiv f_L+g_1\left(f_H,f_L\right). \tag{109}$$

Since in Case 3 $c_H>f_L$, the set of parameter values with two candidate equilibria is non-empty if $g_1\left(f_H,f_L\right)>0$. This can be verified by squaring both sides and simplifying extensively. It follows that two candidate equilibria with green and brown lending exist if and only if condition (109) and $2f_H-5f_L>0$ hold. If either does not hold, then no candidate equilibria with both brown and green lending exist.

8.4 Candidate Equilibrium, Single Brown Bank

Consider the candidate equilibrium for a single brown bank. We show that the prices in (18) must hold. By definition, a single brown bank services the entire market, thus $\lambda^*=0$. Thus, from (5) brown bank profits are:

$$\mathrm{E}\left[\pi_B\right]=\int_0^1\left[\lambda_i\left(-c_H\right)+\left(1-\lambda_i\right)\left(p_b-c\right)\right]d\lambda_i=0. \tag{110}$$

Solving for the brown interest rate results in:

$$p_b\left(0\right)=c+c_H. \tag{111}$$

Next, from (2):

$$0\geq\lambda^*=\frac{p_b\left(0\right)-p_g-f_L}{\Delta f}. \tag{112}$$

$$p_g\geq c+c_H-f_L. \tag{113}$$

Any green loan price satisfying (113) is large enough so that even the highest quality borrowers prefer brown loans. Given $\lambda^*=0$, from (3), the green bank has no customers and thus earns zero profits. Thus, the prices satisfying (18) result in a one brown bank candidate equilibrium.

8.5 Candidate Equilibrium, Single Green Bank

Consider the candidate equilibrium for a single green bank. We show that the prices in (19) must hold. By definition, a single green bank services the entire market, thus $\lambda^* = 1$. Thus, from (3) green bank profits are:

$$\mathrm{E}\left[\pi_G\right] = \int_0^1 \left[\lambda_i\left(-c_H\right) + \left(1-\lambda_i\right)\left(p_g - c\right)\right] d\lambda_i = 0. \tag{114}$$

Solving for the green interest rate results in:

$$p_g\left(1\right) = c + c_H. \tag{115}$$

Next, from (2):

$$1 \leq \lambda^* = \frac{p_b - p_g\left(1\right) - f_L}{\Delta f}. \tag{116}$$

$$p_b \geq c + c_H + f_H. \tag{117}$$

Any brown loan price satisfying (117) is large enough so that even the lowest quality borrowers prefer green loans. Given $\lambda^* = 1$, from (5), the brown bank has no customers and earns zero profits with probability one. Thus, the prices satisfying (19) result in a one green bank candidate equilibrium.

8.6 Bertrand Equilibrium, Single Green Loan Product

We first derive a general deviation condition that can be used in both one product and two product equilibria, then apply the condition to the single green loan product candidate equilibrium. Consider any candidate equilibrium p_b, p_g, and λ^*. A brown bank that deviates by lowering the price to $\tilde{p}_b = p_b - \Delta f \varepsilon$ captures all of the brown borrowers and some green borrowers according to:

$$\tilde{\lambda} = \frac{\tilde{p}_b - p_g - f_L}{\Delta f} = \lambda^* - \varepsilon. \tag{118}$$

Profits of the deviating brown bank are:

$$\begin{aligned}\tilde{\pi}_b &= -c_H \int_{\lambda^*-\varepsilon}^1 \lambda d\lambda + \left(\tilde{p}_b - c\right)\int_{\lambda^*-\varepsilon}^1 \left(1-\lambda\right) d\lambda, \\ &= \frac{1}{2}\left(1-\lambda^*+\varepsilon\right)\left[-c_H\left(1+\lambda^*-\varepsilon\right) + \left(\tilde{p}_b - c\right)\left(1-\lambda^*+\varepsilon\right)\right].\end{aligned} \tag{119}$$

The single green loan product equilibrium (19) satisfies $\lambda^* = 1$ and $\tilde{p}_b = c + c_H + f_H - \Delta f \varepsilon$. Imposing these conditions results in:

$$\tilde{\pi}_b = \frac{1}{2}\varepsilon \left[-c_H (2-\varepsilon) + (c_H + f_H - \Delta f \varepsilon)\varepsilon \right]. \tag{120}$$

Choosing for example $\varepsilon = 1$ results in:

$$\tilde{\pi}_b = \frac{1}{2} \left[-c_H + (c_H + f_H - \Delta f) \right] = \frac{1}{2} f_L > 0. \tag{121}$$

Hence in any single green loan product equilibrium, a brown bank can deviate by offering a price low enough to attract the entire market and make positive profits. Therefore, no single green loan product equilibrium candidate is a Bertrand equilibrium.

8.7 Bertrand Equilibrium, Single Brown Bank

We first derive a general deviation condition that can be used in both one product and two product equilibria, then apply the condition to the single brown loan product candidate equilibrium. Consider any candidate equilibrium p_b, p_g, and λ^*. A green bank that deviates by lowering the price to $\tilde{p}_g = p_g - \Delta f \varepsilon$ captures all of the green borrowers and some brown borrowers according to:

$$\tilde{\lambda} = \frac{p_b - \tilde{p}_g - f_L}{\Delta f} = \lambda^* + \varepsilon. \tag{122}$$

Profits of the deviating green bank are:

$$\begin{aligned} \tilde{\pi}_g &= -c_H \int_0^{\lambda^*+\varepsilon} \lambda d\lambda + (\tilde{p}_g - c) \int_0^{\lambda^*+\varepsilon} (1-\lambda)\, d\lambda, \\ &= \frac{1}{2} (\lambda^* + \varepsilon) \left[-c_H (\lambda^* + \varepsilon) + (\tilde{p}_g - c)(2 - \lambda^* - \varepsilon) \right]. \end{aligned} \tag{123}$$

The single brown loan product equilibrium (18) satisfies $\lambda^* = 0$ and $\tilde{p}_g = c + c_H - f_L - \Delta f \varepsilon$. Imposing these conditions results in:

$$= \frac{\varepsilon}{2} \left(-\varepsilon c_H + (c_H - f_L - \varepsilon \Delta f)(2-\varepsilon) \right). \tag{124}$$

The second term of condition (124) is decreasing in ε since $\tilde{p}_g > c$, so it is sufficient to show (124) is non-positive for ε small. Consider a deviation of $\varepsilon \to 0$. If the limit of green bank profits remains positive as $\varepsilon \to 0$, then by continuity an interval $0 < \varepsilon < \hat{\varepsilon}$ exists such that profits are positive and a profitable deviation exists. The second term in (124) approaches $2\left(c_H - f_L\right)$ as $\varepsilon \to 0$. Thus no profitable deviations exist if and only if:

$$c_H \leq f_L \tag{125}$$

8.8 Bertrand Equilibrium with Green and Brown Lending

Green Bank Deviations

We first show that no green bank can profitably deviate by lowering the green price in a separating equilibrium. Equation (123) gives the profits of a deviating green bank regardless of the equilibrium type. Evaluating the deviation profits of a green bank (123) in a separating equilibrium given by equations (4) and (6) results in:

$$\begin{aligned}\tilde{p}_g &= \frac{1}{2}\left(\lambda^* + \varepsilon\right)\left[-c_H\left(\lambda^* + \varepsilon\right) + \left(\frac{\lambda^*}{2-\lambda^*}c_H - \varepsilon\Delta f\right)\left(2-\lambda^*-\varepsilon\right)\right], \\ &= \frac{\varepsilon}{2}\left(\lambda^* + \varepsilon\right)\left[-\frac{2c_H}{2-\lambda^*} - \left(2-\lambda^*-\varepsilon\right)\Delta f\right] < 0. \end{aligned} \tag{126}$$

Hence any decrease in price by a green bank at zero profits decreases profits below zero. Hence no profitable deviation exists for the green bank in a separating equilibrium with two loan products.

Brown Bank Deviations

Next, we derive conditions such that no brown bank can profitably deviate by lowering the brown price in a two loan product candidate equilibrium. Equation (119) gives the profits of a deviating brown bank regardless of the equilibrium type. Evaluating the deviation profits of a deviating brown bank (119) in a two bank equilibrium given by equations (4) and (6) results in:

$$\begin{aligned}\tilde{\pi}_b &= \frac{1}{2}\left(1-\lambda^* + \varepsilon\right)\left[-c_H\left(1+\lambda^* - \varepsilon\right) + \left(\frac{1+\lambda^*}{1-\lambda^*}c_H - \Delta f\varepsilon\right)\left(1-\lambda^*+\varepsilon\right)\right], \\ &= \frac{\varepsilon}{2}\left(1-\lambda^* + \varepsilon\right)\left[\frac{2c_H}{1-\lambda^*} - \Delta f\left(1-\lambda^*+\varepsilon\right)\right] \end{aligned} \tag{127}$$

The term inside the large brackets is decreasing in ε. Hence deviation profits increase as ε approaches zero. Therefore, if the term inside the large brackets is non-positive at $\varepsilon = 0$, no profitable deviations exist for all ε. Therefore no profitable deviations exist if and only if:

$$2c_H \leq \Delta f\left(1-\lambda^*\right)^2. \tag{128}$$

Condition (128) is in terms of the endogenous variable λ^*. In the remaining analysis, we use the equilibrium condition to convert condition (128) to a condition that depends only on the parameters. Recall that any equilibrium $\lambda^* \in (0,1)$ satisfies (90). Substituting this equilibrium condition into (128) to eliminate c_H results in:

$$2c_H = \Delta f \lambda^3 - (3f_H - 4f_L)\lambda^2 + (2f_H - 5f_L)\lambda - 2f_L \leq \Delta f (1-\lambda)^2, \tag{129}$$

$$\Delta f \lambda^3 - (3f_H - 4f_L)\lambda^2 + (2f_H - 5f_L)\lambda - 2f_L \leq \Delta f (1-\lambda)^2, \tag{130}$$

$$(1-\lambda)\left(\Delta f \lambda^2 - (3f_H - 4f_L)\lambda + f_H - 3f_L\right) \geq 0. \tag{131}$$

Since we are considering interior solutions, we need only show the second term is non-negative at λ^* to ensure no profitable deviations exist:

$$J(\lambda^*) \equiv \Delta f (\lambda^*)^2 - (3f_H - 4f_L)\lambda^* + f_H - 3f_L \geq 0. \tag{132}$$

The key properties are:

$$J(0) = f_H - 3f_L, \tag{133}$$

$$J(1) = -f_H < 0, \tag{134}$$

$$J'(\lambda) = 2\Delta f \lambda - 3f_H + 4f_L, \ \rightarrow \ \lambda_{\min} = \frac{3f_H - 4f_L}{2\Delta f}, \tag{135}$$

$$J'(0) = -3f_H + 4f_L, \tag{136}$$

$$J'(1) = -f_H + 2f_L, \tag{137}$$

Note that $f_H < 3f_L$ means $J(0) < 0$ and since $J(1) < 0$ and J is quadratic, it follows that $f_H < 3f_L$ implies $J < 0$ for all $\lambda \in (0,1)$, including λ^*. Thus profitable deviations exist in this case and $f_H > 3f_L$ is necessary for no deviations. Given $f_H > 3f_L$, we have three cases.

Case 1 ($f_L > c_H$)

From Proposition (1), a single candidate equilibrium exists. Recall, $f_H > 3f_L$ is necessary, in which case $J'(0) < 0$, $J'(1) < 0$, $J(0) > 0$, and $J(1) < 0$. Since further the global minimum is greater than one, J is monotonically decreasing over $(0,1)$ with a single root $\lambda^{**} \in (0,1)$. Further, J is positive for $\lambda^* < \lambda^{**}$ Thus, no deviations exist if $\lambda^* \leq \lambda^{**}$.

Given case 1, $H(0) > 0$, $H(1) < 0$, and $H(\lambda^*) = 0$. Since exactly one root exists, it follows from continuity and $H' < 0$ that $\lambda^* < \lambda^{**}$ if and only if $H(\lambda^{**}) < H(\lambda^*) = 0$. Thus we need only show that $H(\lambda^{**}) < 0$ to establish that $\lambda^* < \lambda^{**}$. If so, then λ^* is a Bertrand equilibrium.

We wish to show:

$$H(\lambda^{**}) = \Delta f(\lambda^{**})^3 - (3f_H - 4f_L)(\lambda^{**})^2 + (2f_H - 5f_L)\lambda^{**} - 2(c_H - f_L) < 0, \tag{138}$$

where

$$J(\lambda^{**}) = \Delta f(\lambda^{**})^2 - (3f_H - 4f_L)\lambda^{**} + f_H - 3f_L = 0. \tag{139}$$

Multiplying J by λ^{**} and subtracting results in:

$$(f_H - 2f_L)\lambda^{**} - 2(c_H - f_L) < 0, \tag{140}$$

Next, equation (139) has solution:

$$\lambda^{**} = \frac{3f_H - 4f_L - \sqrt{5f_H^2 - 8f_H f_L + 4f_L^2}}{2\Delta f}. \tag{141}$$

Notice we use the smaller root since given the necessary condition that $f_H > 3f_L$, the smaller root satisfies $\lambda^{**} \in (0,1)$. Substituting the solution into (140) results in:

$$c_H > f_L + g_2(f_H, f_L), \tag{142}$$

$$g_2(f_H, f_L) \equiv (f_H - 2f_L)\frac{3f_H - 4f_L - \sqrt{5f_H^2 - 8f_H f_L + 4f_L^2}}{4\Delta f}. \tag{143}$$

We have already assumed $f_L > c_H$ however, and it is straightforward to verify that $g_2 > 0$. It follows that $H(\lambda^{**}) > 0$ and thus $\lambda^{**} < \lambda^*$ and thus $J(\lambda^*) < 0$ and so a profitable deviation exists. Thus no Bertrand equilibrium exists for $f_L > c_H$.

Case 2 ($f_L = c_H$)

The analysis here is identical as case 1. Since $H(0) = 0$, we have $H(\lambda) > (<)0$ for $\lambda < (>)\lambda^*$. As in case one, $H(\lambda^{**}) > 0$, and so $\lambda^{**} < \lambda^*$ and the deviation results in positive profits. The candidate equilibrium is not a Bertrand equilibrium.

Case 3 ($f_L < c_H$)

Given that $f_H > 3f_L$ is a necessary condition, $J(0) > 0$, $J'(0) < 0$, $J'(1) < 0$, Since the global minimum is occurs at $\lambda_{\min} > 1$, J has a single root λ^{**} on $(0,1)$, with $J(\lambda^*) \geq 0$ for $\lambda^* \leq \lambda^{**}$. Thus, we need to show whether or not the two candidate equilibria λ_1^* and λ_2^* are less than λ^{**}.

In this case, $H(0) < 0$, $H(1) < 0$, $H(\lambda_1^*) = 0$, and $H(\lambda_2^*) = 0$. Since exactly two roots exist, it follows from continuity that $H(\lambda) > 0$ if and only if $\lambda \in (\lambda_1^*, \lambda_2^*)$. Thus we need only show that $H(\lambda^{**}) > 0$ to establish that $\lambda_1^* < \lambda^{**} < \lambda_2^*$. If so, then λ_1^* is a Bertrand equilibrium and λ_2^* is not.

Using an identical logic as the case 1, we must show:

$$(f_H - 2f_L)\lambda^{**} - 2(c_H - f_L) > 0, \tag{144}$$

where equation (141) gives λ^{**}. Substituting equation (141) into (144) results in:

$$c_H < f_L + g_2(f_H, f_L), \tag{145}$$

$$g_2(f_H, f_L) \equiv (f_H - 2f_L)\frac{3f_H - 4f_L - \sqrt{5f_H^2 - 8f_H f_L + 4f_L^2}}{4\Delta f}. \tag{146}$$

We have already made a similar assumption for existence. We now show this condition is more restrictive than the previous assumption, $c_H < f_L + g_1(f_H, f_L)$. That is, we show that $g_2 < g_1$, or:

$$(f_H - 2f_L)\frac{3f_H - 4f_L - \sqrt{5f_H^2 - 8f_H f_L + 4f_L^2}}{4\Delta f} < g_1 \tag{147}$$

$$= \frac{2\left(3f_H^2 - 3f_H f_L + f_L^2\right)^{\frac{3}{2}} - f_L(3f_H - 4f_L)(15f_H - 13f_L)}{54\Delta f^2}, \tag{148}$$

where the equality follows from (109). Multiplying all this out verifies that $g_2 < g_1$ and so the condition for a Bertrand equilibrium is stricter than the condition for existence.

Thus we have three cases:

1. $c_H < f_L + g_2 < f_L + g_1$: two candidate equilibria exist and λ_1^* is a Bertrand equilibrium and λ_2^* is not.

2. $c_H > f_L + g_1$: no candidate equilibria exist.

3. $f_L + g_2 < c_H < f_L + g_1$. We show below that in this case $\lambda^{**} < \lambda_1^* < \lambda_2^*$ and so two candidate equilibria exist but neither is a Bertrand equilibrium.

First, it is straightforward to verify that $\lambda^{**} < \hat{\lambda}$ and neither depends on c_H. Now consider the boundary case where $c_H = f_L + g_1$. In this case $H\left(\hat{\lambda}\right) = 0$ and $\hat{\lambda}$ is the unique candidate equilibrium (that is, the maximum of H over $(0,1)$ is zero and occurs at $\hat{\lambda}$). In this case, $\hat{\lambda}$ is not a Bertrand equilibrium since $\lambda^{**} < \hat{\lambda}$.

Next, consider an arbitrarily small decrease in c_H. It follows from the above calculation of g_1 that $H\left(\hat{\lambda}\right) > 0$ and 2 roots exist. Given the arbitrarily small decrease, $\lambda_1^* < \hat{\lambda} < \lambda_2^*$ is an arbitrarily small interval, and so $\lambda^{**} < \lambda_1^* < \hat{\lambda} < \lambda_2^*$ and neither of the candidate equilibria are Bertrand equilibria.

Next, continuing to decrease c_H until $c_H = f_L + g_2$, we see that $\lambda^{**} = \lambda_1^* < \lambda_2^*$. At this point, λ_1^* becomes a Bertrand equilibrium. Thus, over the interval $f_L + g_2 < c_H < f_L + g_1$, both roots are not Bertrand equilibria.

The figure below summarizes these cases.

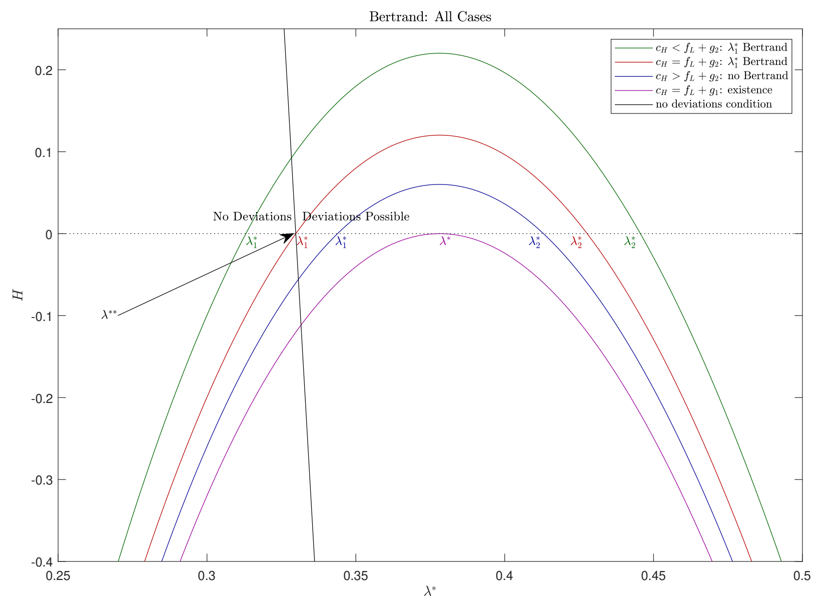

Figure 4: Stability Geometry. Increasing c_H moves H from the green to the magenta curve. The black curve is positive when no deviations are possible ($\lambda \leq \lambda^{**}$ or $2c_H/(1-\lambda^*)^2 \leq \Delta f$). The green curve is a value of c_H for which the smaller candidate equilibrium, λ_1^*, is a Bertrand equilibrium since no profitable deviations exist. The red curve satisfies $c_H = f_L + g_2$, the maximum value of c_H for which the smaller candidate equilibrium has no profitable deviations. The blue curve is a value of c_H for which neither of the candidate equilibria are Bertrand equilibria. The magenta curve is the maximum value of c_H for which a candidate equilibrium exists, $c_H = f_L + g_1$. At this knife edge, only a single candidate equilibrium exists which is not a Bertrand equilibrium.

This completes the proof as all cases have been analyzed.

8.9 Proof of No-Deviation Condition with Required Reserves

We begin by repeating the analysis starting with equation (119), while adding in the reserve requirement. The deviating brown bank profits become:

$$\tilde{\pi}_{b,r} = \frac{1}{2}(1-\lambda_r+\varepsilon)\left[-(c_H+rrc)(1+\lambda_r-\varepsilon)+(\tilde{p}_b-c(1+rr))(1-\lambda_r+\varepsilon)\right]. \quad (149)$$

The analysis differs only in added opportunity cost of holding reserves and that the equilibrium fraction of borrowers that chooses green loans changes. Substituting in the two bank equilibrium given by equations (58) and (59) results in:

$$\begin{aligned}\tilde{\pi}_b &= \frac{1}{2}(1-\lambda_r+\varepsilon)\left[-(c_H+rrc)(1+\lambda_r-\varepsilon)+\left(\frac{1+\lambda_r}{1-\lambda_r}(c_H+rrc)-\Delta f\varepsilon\right)(1-\lambda_r+\varepsilon)\right],\\ &= \frac{\varepsilon}{2}(1-\lambda_r+\varepsilon)\left[\frac{2(c_H+rrc)}{1-\lambda_r}-\Delta f(1-\lambda_r+\varepsilon)\right] \end{aligned} \quad (150)$$

Deviating profits are decreasing in ε and so the largest deviation profits occur as ε approaches zero. Thus deviation profits are non-positive if and only if:

$$2(c_H+rrc) \leq \Delta f(1-\lambda_r)^2. \quad (151)$$

Note that $\Delta f = \Delta f_r$, and the equilibrium condition is unchanged up to $f_{H,r}$ and $f_{L,R}$. Hence, substituting the equilibrium condition into (151) and repeating the same simplification gives a no deviation condition that is identical except for a single term:

$$J(\lambda_r) \equiv \Delta f(\lambda_r)^2-(3f_H-4f_L)\lambda_r+f_H-3f_L+2rrc \geq 0. \quad (152)$$

Next, recall $2rrc = (1-\lambda)\alpha$, which implies:

$$J(\lambda_r;\alpha) \equiv \Delta f(\lambda_r)^2-(3f_H-4f_L+\alpha)\lambda_r+f_H-3f_L+\alpha \geq 0. \quad (153)$$

Analyzing the modified quadratic function reveals similar cases as in the unregulated case. In particular J has a positive region $(0,\lambda_r^{**})$ if and only if $f_H-3f_L+\alpha>0$. Note that this necessary condition is identical to $f_{H,r}-3f_{L,r}-\alpha>0$, which is condition (71) in Proposition 4. Note also that λ^{**} is the smaller root of (151).

Next, we need to derive the condition under which $\lambda_r \in (0,\lambda_r^{**})$. Analogous to the unregulated case, we have $\lambda_r < \lambda_r^{**}$ if and only if $H(\lambda_r^{**}) < H(\lambda_r) = 0$, where H is defined using

$f_{H,r}$ and $f_{L,r}$. Following the steps in the unregulated case, this holds if and only if:

$$c_H \leq f_{L,r} + (f_{H,r} - 2f_{L,r} + \alpha)\,\lambda_r^{**}. \tag{154}$$

Substituting in for λ_r^{**} as the smaller root of (151) gives the second condition:

$$c_H > f_L + g_{2,r}(f_{H,r}, f_{L,r}, \alpha), \tag{155}$$

$$g_2(f_{H,r}, f_{L,r}, \alpha) \equiv (f_{H,r} - 2f_{L,r} + \alpha)\,\frac{3f_{H,r} - 4f_{L,r} + \alpha - \sqrt{5f_{H,r}^2 - 8f_{H,r}f_{L,r} + 4f_{L,r}^2 + 4\alpha\Delta f_r}}{4\Delta f_r}. \tag{156}$$

The above condition is (72) in Proposition 4.

8.10 Remaining Parts of the Proof of Proposition 4

For the first result in the proposition, the case is $f_{L,r} \geq c_H$. Following the steps in Section 8.9, the no deviation conditions become $f_{H,r} - 3f_{L,r} - \alpha > 0$ and:

$$c_H \geq f_{L,r} + g_{2,r}(f_{H,r}, f_{L,r}, \alpha). \tag{157}$$

Given in this case $f_{L,r} > c_H$, condition (157) can hold only if $g_{2,r} < 0$.

Imposing $\alpha = 0$ implies $f_{H,r} = f_H$ and $f_{L,r} = f_L$, and equation (156) implies $\alpha = 0$ implies $g_{2,r} = g_2 > 0$, where $g_2 > 0$ is shown below equation (146). Further, taking the derivative of $g_{2,r}$ with respect to α holding $f_{H,r}$ and $f_{L,r}$ fixed and extensively simplifying reveals the derivative is negative. Thus, $g_{2,r} < 0$ is possible only if $\alpha > \alpha^*$ where α^* is such that $g_{2,r}(f_{H,r}, f_{L,r}, \alpha^*) = 0$. However, solving for α^* using equation (156) reveals that either $\alpha^* = -f_{H,r} + 2f_{L,r}$ or $\alpha^* = f_{H,r} - 3f_{L,r}$. The first solution is not possible since $\alpha^* < 0$ and the Proposition assumes $\alpha > 0$. The second solution implies any $\alpha \geq \alpha^*$ violates the stability condition $f_{H,r} - 3f_{L,r} - \alpha > 0$. Therefore, $g_{2,\alpha} > 0$ for any feasible α. Thus, the region satisfying both case 1 and (157) is empty. Thus, the candidate equilibrium with both brown and green lending is not Bertrand in case 1.

An identical analysis reveals that case 2, $c_H = f_{L,r}$ also has a candidate equilibrium which is not Bertrand. We have thus shown result 1 in Proposition 4.

Result 2 is shown in Section 8.9 and results 3-5 were shown in the text.

For result 6, for $\alpha = \tau e > 0$, $g_{2,r}(f_{H,r}, f_{L,r}, \alpha) < g_2(f_{H,r}, f_{L,r})$ since we have shown $g_{2,r}$ is decreasing in α. Therefore, any policy $\tau e = \alpha$ such that $g_{2,r} < c_H < g_2$ is Bertrand given tax regulation but is not Bertrand given a reserve requirement. Note also that the other condition $f_{H,r} > f_{L,r} + \alpha$ is more restrictive when $\alpha > 0$. Thus, any policy $\tau e = \alpha$ such that $3f_{L,r} < f_{H,r} < 3f_{L,r} + \alpha$ is also a Bertrand equilibrium with a tax policy but not with a required reserve policy.