Pigouvian Taxes and Technology Adoption under Imperfect Competition

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March 10, 2020

Abstract

It is well known that a Pigouvian tax on emissions (equal to the marginal damages) provides incentives for firms to adopt the most efficient emissions-savings technologies. However, this result relies in part on perfect competition to drive firms with inefficient technologies out of the market. With imperfect competition, excess profits can allow inefficient technologies to remain in the market. In a duopoly, if one firm is both less productive and more emissions intensive —a case we show occurs frequently in practice— then the regulator faces a tradeoff. By increasing the emissions tax, the regulator can drive the inefficient firm out of the market. However, the resulting monopoly then reduces production since competition is weaker. Conversely, the regulator can reduce the emissions tax, resulting in lower productivity and production which is more emissions intensive, but with more competition and therefore more production. We show that the regulator maximizes welfare in general equilibrium by driving out the inefficient firm unless preferences for environmental quality are very weak. In extreme cases, the optimal emissions tax may even exceed the Pigouvian level, contrary to the well known opposite result for Cournot competition and identical firms.

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1 Introduction

In perfectly competitive markets, it is well known that the Pigouvian tax, which is equal to the marginal damage, maximizes welfare. In addition to resolving the externality by forcing firms to account for the social costs of pollution, the Pigouvian tax provides incentives to adopt emissions savings technologies. Further, since the tax does not prescribe a particular method of reducing emissions, firms use whatever technology reduces emissions at the lowest cost. Perfect competition then ensures that firms using inefficient emissions savings technologies earn negative profits and therefore either adopt efficient technologies or (if the technology is proprietary) exit the market.

However, whether this result holds with imperfect competition is less clear. Suppose a duopoly in which one firm is both more productive and less emissions intensive.[1] Excess profits from imperfect competition may allow the firm using an inefficient emissions savings technology to remain in the market. In this case, the regulator faces a certain tradeoff. Keeping the pollution tax low allows the inefficient firm to remain in the market increasing competition and therefore production, but at a cost of more emissions and more input costs. These costs occur not only because production is higher, but also because the inefficient technology is in use. Conversely, if the pollution tax is high enough the inefficient firm must adopt a more efficient technology to remain in the market. If the more efficient firm uses a technology which is proprietary, then sufficiently high pollution taxes cause the inefficient firm to exit the market. The resulting monopolist responds to the lack of competition by reducing production. However, because all production occurs with the more efficient technology, industry wide productivity increases and the emissions intensity falls.

We consider an economy where households have preferences over consumption of a dirty good, a clean numeraire, and pollution. Households derive income from renting capital, supplying labor, receiving profits from the production of the dirty good and transfers from the government. On the production side, two firms engage in imperfect competition (à la Cournot) in the output market and behave competitively in the input markets. Firms are asymmetric with respect to their total factor productivities (TFP) and their emission intensities. We derive conditions under which the regulator maximizes welfare by driving the inefficient firm out of the market. In general, the regulator drives the inefficient firm from the market unless preferences for the environment are weak enough. This result requires a proper general equilibrium accounting of input usage. As the emissions tax rises, the more efficient firm accounts for a larger share of production. Input usage falls both because higher emissions costs reduces overall output, but also because the remaining production is concentrated in the more productive firm. In turn, in general equilibrium, lower input demand leads to higher production of the numeraire clean good. Further, pollution falls both because production falls and because production concentrates in the less emissions intensive firm. We derive the optimal tax which balances these welfare gains against the welfare cost of lower production of the dirty good resulting from less competition. Whether or not optimal tax

[1] We argue below this case is common in practice.

is high enough to drive the inefficient firm from the market depends on the relative strength of environmental preferences.

Our results extend the literature on optimal emissions taxes with imperfect competition. This literature focuses on how reductions in output by monopolies (Buchanan 1969, Barnett 1980, Martin 1986) or duopolies (Simpson 1995) affect optimal emissions taxes. A Pigouvian tax (i.e. a tax equal to the marginal damage) achieves the socially efficient production of a dirty good in perfect competition. However, imperfect competition allows firm to restrict production, and so a Pigouvian tax results in less dirty goods production than is efficient. The efficient production can be achieved by an emissions tax that is below the Pigouvian level (Barnett 1980).

In some cases, however, the optimal emissions tax can exceed the marginal damage. For example, Martin (1986) shows that if a monopolist has access to an abatement technology which reduces emissions independent of reducing output, then the monopolist may simultaneously produce too little output and too much emissions, leading to an optimal tax which exceeds the marginal damage. Simpson (1995) shows that the optimal tax may exceed the Pigouvian level if one firm is more efficient than the other, as the emissions tax tends to concentrate production in the more efficient firm.

Our results build upon this literature in several ways. First, in our framework firms have heterogeneous technologies, and market changes from duopoly to monopoly depending on policy. That is, a sufficiently high emissions tax causes the inefficient firm to exit the market, leading to a tradeoff between a monopolist with more efficient production and lower emissions but less competition, and a duopoly with more competition but less efficient production and more emissions. Second, in our framework, one firm is both less productive and more emissions intensive. This combination strongly favors an emissions tax that exceeds the marginal damage and causes the less efficient firm to exit the market.

A large literature shows that large differences exist in firm productivity exists, even for goods that are relatively homogeneous (see for example Foster, Haltiwanger, and Syverson 2008). Reasons include differences in management practices (e.g. Bloom and Reenen 2010), increased competition arising from entering export markets (e.g. De Loecker 2007, Wagner 2007), and family ownership (. Bertrand and Schoar 2006) Regardless of the reason, this literature emphasizes that large productivity differences arise in imperfectly competitive markets. Given the apparent pervasiveness of productivity differences in imperfectly competitive markets, it is important from a policy perspective to consider firms with heterogeneous productivity levels when determining optimal emissions tax policies.

Emissions intensity also varies across firms, often for similar reasons. For example, Forslid, Okubo, and Ulltveit-Moe (2018) show that exporting firms tend to be less emissions intensive. Barrows and Ollivier (2018) show that manufacturers in India that are less efficient are also more emissions intensive. Our results show that when less efficient firms are also more emissions intensive, the optimal emissions tax under imperfect competition is greater than the marginal damage, unless preferences for the environment are sufficiently weak.

Governmental support of low productivity firms is a common practice that leads to a market in which firms have heterogeneous productivity levels and emissions intensities. Many empirical studies confirm the extent of awarded subsidies in national economies and in sectors that have significant environmental footprint. van Beers and van den Bergh (2001) estimate world subsidies to account for 3.6% of global GDP. Barde and Honkatukia (2004) find agriculture, fishing, water, transport, manufacturing, and energy are industries with significant environmental impact that are heavily subsidized. The authors find that subsidies have a technology lock-in effect in that inefficient firms that otherwise would have exited remain in the market, which directly increases emissions. Bajona and Kelly (2012) and Heutel and Kelly (2016) show that subsidized firms in China are less productive and more emissions intensive than firms which are not subsidized. The present paper builds upon this literature by considering the case where competition between heterogeneous firms is imperfect.

Finally, our results relate to a literature that studies the effect of environmental policy on firm entry and exit.[2] Fowlie, Reguant, and Ryan (2016) show that, in the cement industry, exit following the imposition of a cap and trade system with auctioned permits leads to reduced competition and associated welfare loss. Boom and Dijkstra (2009) study intensity standards (with trading) under imperfect competition and find that such a policy leads to higher output and a larger number of firms than a cap and trade system. We build upon this literature by showing the importance of heterogeneity in firm productivity and emissions intensity. If firms that exit are low productivity and high emissions intensity, then an emissions tax greater than the marginal damage which causes firms to exit can produce economy wide gains in terms of higher productivity and lower emissions intensity, which can offset welfare losses from the concentration of market power.

In what follows, Section 2 provides the basic setup of households, firms, and the role of the government. Section 3 describes the equilibrium concept. Section 4 provides comparative statics when the environmental tax is a given parameter, and Section 5 derives the optimal tax. Section 6 concludes.

2 The model

2.1 Households

We consider a continuum of households that have quasilinear preferences.[3] Households derive utility from consumption of a dirty good, c, and a clean numeraire c_0. Households receive disutility from

[2]See Requate (2007) for a survey of environmental policy given imperfect competition more generally.

[3]This assumption has been widely used in the non-linear taxation literature (see for example Boone and Bovenberg 2007), and it allows the derivation of closed form solutions in our model.

pollution emissions, E, caused by the production of the dirty good. Preferences are:

$$U = c_0 + \alpha c - \frac{\beta}{2}c^2 - \frac{\zeta}{2}E^2, \tag{2.1}$$

where α and β are strictly positive demand parameters and ζ is a strictly positive parameter representing the strength of disutility from pollution.

Households derive income from renting capital K and labor L at rental rates r and wage w respectively. The supply of capital is inelastic, and household labor supply is normalized to unity. Households also receive profits π from the Cournot-competitive firms producing the dirty good, and receive lump sum net transfers from the government, NT. Let p be the price of the dirty good, then the household budget constraint is:

$$c_0 + pc = rK + w + NT + \pi. \tag{2.2}$$

Households are atomistic and take prices as given. Maximization of household utility (2.1) subject to the budget constraint (2.2) leads to linear inverse demand for the dirty good:

$$p = \alpha - \beta c.$$

Equating demand c with supply Q in the inverse demand for the dirty good gives:

$$p = \alpha - \beta Q. \tag{2.3}$$

2.2 Dirty Good Production

A Cournot-competitive duopoly supplies dirty goods. Subscripts G and P denote the two firms, where P will be the higher productivity firm. The duopoly refers to the output market of the dirty good. All producers of both goods are competitive price takers in the input markets. Total production of the dirty good, Q, is the sum of production from the two firms.

$$Q = Q_P + Q_G \tag{2.4}$$

Both firms have access to production technologies that differ only in their total factor productivity (TFP) and emission intensities. Firm P has TFP equal to A_P whereas firm G has TFP $A_G < A_P$. Without loss of generality, we set $A_G = 1$ and $A_P = A > 1$. We are motivated here by the common observation of newer technologies which are both more productive and more emissions efficient. If a firm with a new technology enters the market, existing firms may not be able to immediately copy it, in which case new and older technologies coexist until older technologies are unprofitable to operate.

Emissions are proportional to output. The emission intensities are σ_i, $i = P, G$, and so the total emissions are

$$E = \sigma_P Q_P + \sigma_G Q_G. \tag{2.5}$$

The government imposes an emissions tax τ per unit of emissions. Because the firms have different emissions intensities, the tax raises costs non-uniformly across firms, which changes the relative competitive advantage of the two firms. In general, our focus is on the case where $\sigma_G > \sigma_P$, matching the common empirical observation that older technologies are both less productive and more emissions intensive. We therefore refer to firm G as the "high cost" or "inefficient" firm (and the reverse for firm P). However, some results hold even if $\sigma_G < \sigma_P$. Therefore, we specify emissions intensity assumptions in each proposition.

2.2.1 Production Costs

Dirty good producers use capital, K_i, and labor, L_i, for $i = G, P$, to produce the homogeneous dirty good. We assume a Cobb-Douglas production function

$$Q_i = A_i K_i^{\gamma} L_i^{1-\gamma},\ i = G, P. \tag{2.6}$$

Cost minimizing yields cost functions and input demands of:

$$c_i\left(Q_i\right) = \left(\frac{r}{\gamma}\right)^{\gamma} \left(\frac{w}{1-\gamma}\right)^{1-\gamma} \frac{Q_i}{A_i} \equiv c_i' Q_i,\ i = G, P, \tag{2.7}$$

$$K_i = \frac{\gamma}{r} c_i' Q_i, \tag{2.8}$$

$$L_i = \frac{1-\gamma}{w} c_i' Q_i. \tag{2.9}$$

2.2.2 Cournot Duopoly

The dirty producers maximize profits accounting for the effect of their production on the price, taking the other firm's action, input prices, and the emissions tax as given:

$$\pi_i = p\left(Q_P + Q_G\right) Q_i - c_i\left(Q_i\right) - \tau \sigma_i Q_i,\ i = G, P. \tag{2.10}$$

The first order condition is:

$$p'\left(Q_P + Q_G\right) Q_i + p\left(Q_P + Q_G\right) = c_i' + \tau \sigma_i,\ i = G, P.$$

The linear form of the inverse demand (2.3) implies:

$$\alpha - 2\beta Q_P - \beta Q_G = c_i' + \tau \sigma_i,\ i = G, P. \tag{2.11}$$

Combining the reaction function (2.11) for $i = G, P$ gives the Cournot-Nash output levels:

$$Q_P = \frac{\alpha - 2c'_P + c'_G + (\sigma_G - 2\sigma_P)\,\tau}{3\beta}, \tag{2.12}$$

$$Q_G = \frac{\alpha - 2c'_G + c'_P - (2\sigma_G - \sigma_P)\,\tau}{3\beta}. \tag{2.13}$$

$$Q = \frac{2\alpha - c'_P - c'_G - (\sigma_G + \sigma_P)\,\tau}{3\beta}, \tag{2.14}$$

The emissions tax raises production costs for both firms. Therefore the emissions tax tends to decrease production at both firms. However, if the cost increase at the cleaner firm is small enough relative to the cost increase at the dirty firm, then the cleaner firm will increase output to take strategic advantage of the increase in the cost differential.

In the partial equilibrium context, taxes affect firm decisions directly by changing their absolute costs, and strategically through changes in the firms relative costs. In our model, taxes also have general equilibrium effects through changes in factor demands.

2.3 Clean Good Production

A continuum of competitive firms with mass equal to one produce the clean good (Q_0) using inputs K_0 and L_0 according to:

$$Q_0 = A_0 K_0^{\gamma} L_0^{1-\gamma}. \tag{2.15}$$

Profit maximization by the competitive clean good producers implies:

$$r = A_0 K_0^{\gamma-1} L_0^{1-\gamma}, \tag{2.16}$$

$$w = A_0 K_0^{\gamma} L_0^{-\gamma}. \tag{2.17}$$

2.4 The government

The government collects emissions taxes and distributes the proceeds to the household lump sum. The government constraint is then:

$$NT = (\sigma_P Q_P + \sigma_G Q_G)\,\tau \tag{2.18}$$

3 Equilibrium

In the equilibrium, the competitive clean sector determines the factor prices. The emissions tax and relative differences in TFP and emissions intensity then determine the outputs of the duopoly and their factor demands, while the clean sector uses the rest of the input supply.

3.1 Rental Rates

Dividing equation (2.17) by equation (2.16), the clean good industry has capital to labor ratio:

$$\frac{K_0}{L_0} = \frac{w}{r}\frac{\gamma}{1-\gamma}. \tag{3.1}$$

Equations (2.8) and (2.9) imply both dirty goods producers have the same capital to labor ratio as the clean firm:

$$\frac{K_0}{L_0} = \frac{K_P}{L_P} = \frac{K_G}{L_G}. \tag{3.2}$$

The market clearing conditions set input demands equal to supply:

$$K = K_P + K_G + K_0, \tag{3.3}$$

$$1 = L_P + L_G + L_0. \tag{3.4}$$

Equations (3.2), (3.3), and (3.4) imply that the capital to labor ratio in the dirty industry is identical to the individual firm ratios:

$$\frac{K_G + K_P}{L_G + L_P} = \frac{K_P}{L_P} = \frac{K_G}{L_G}. \tag{3.5}$$

This fact, along with (3.2), (3.3) and (3.4) imply that all firms will have the same capital to labor ratio as the economy wide capital to labor ratio:

$$\frac{K_0}{L_0} = \frac{K_P}{L_P} = \frac{K_G}{L_G} = K. \tag{3.6}$$

Therefore, equilibrium rental rates are:

$$r = A_0 K^{\gamma-1}, \tag{3.7}$$

$$w = A_0 K^{\gamma}. \tag{3.8}$$

The competitive clean sector effectively fixes the interest rate and wage. Therefore, higher emissions taxes in the dirty sector cause the demand for capital and labor to fall in the dirty sector, leading to greater clean production. The interest rate and wage remain unchanged.[4]

[4] The emissions costs are proportional to output at both firms. Therefore, the emissions tax does not alter the optimal capital to labor ratio.

3.2 Equilibrium Costs, Production, Price, and Profits

Market clearing in the factor markets, (3.7) and (3.8), implies in equilibrium the cost function (2.7) becomes:

$$c_i' = \frac{A_0}{A_i} \; i = G, P. \tag{3.9}$$

Equation (3.9) can be thought of as a marginal rate of transformation between dirty goods and clean goods at each firm. Since $A > 1$, firm P has lower costs, and therefore less dirty goods need to be sacrificed to produce a clean good, relative to firm G.

Equilibrium production of the dirty good is:

$$Q_P = \frac{\alpha - 2\frac{A_0}{A} + A_0 + (\sigma_G - 2\sigma_P)\,\tau}{3\beta}, \tag{3.10}$$

$$Q_G = \frac{\alpha - 2A_0 + \frac{A_0}{A} - (2\sigma_G - \sigma_P)\,\tau}{3\beta}. \tag{3.11}$$

$$Q = \frac{2\alpha - \frac{A_0}{A} - A_0 - (\sigma_G + \sigma_P)\,\tau}{3\beta}, \tag{3.12}$$

Cournot competition implies that firms consider both their own costs, and the relative cost differential when choosing production levels. The cost differential consists of both the TFP difference, which affects input costs per unit of output, and the emissions intensity and tax, which affect emissions cost per unit of output. Overall production is unambiguously decreasing in the emissions tax, despite that production might increase at the low cost firm due to the change in strategic advantage.

When the high cost sector is more than twice as emissions intensive, $\sigma_G > 2\sigma_P$, the strategic advantage the low cost firm gains when the emissions tax rises is so strong that the low cost firm increases production in response to the emissions tax.[5]

The price (2.3) of the dirty good at equilibrium becomes:

$$p = \frac{\alpha + A_0 + \frac{A_0}{A} + \tau\,(\sigma_G + \sigma_P)}{3}. \tag{3.13}$$

The emissions tax decreases total production and therefore raises the price of the dirty good.

Equilibrium profits are:

$$\pi_P = \frac{\left(\alpha + A_0 - 2\frac{A_0}{A} + \tau\,(\sigma_G - 2\sigma_P)\right)^2}{9\beta}, \tag{3.14}$$

[5]The sharp result that the cutoff is exactly twice as emissions intensive depends on the assumption of quadratic preferences. Other specifications could imply production at the low cost firm is increasing in the tax at even lower emissions intensities.

$$\pi_G = \frac{\left(\alpha - 2A_0 + \frac{A_0}{A} - \tau\left(2\sigma_G - \sigma_P\right)\right)^2}{9\beta}, \tag{3.15}$$

$$\pi = \frac{\left(\alpha - 2A_0 + \frac{A_0}{A} - \tau\left(2\sigma_G - \sigma_P\right)\right)^2}{9\beta}, \tag{3.16}$$

The tax might be high enough to drive the high cost firm from the market, creating a monopoly. The critical tax, τ_c, for which equilibrium production by the high cost firm becomes zero, is:

$$\tau_c = \frac{\alpha - 2A_0 + \frac{A_0}{A}}{2\sigma_G - \sigma_P}. \tag{3.17}$$

The optimal emissions tax policy will depend on preferences for the environment, but because of the externality, the critical tax does not. The firms care only about the environment through the tax. Many of the cases in the paper occur because, depending on preferences for the environment, the optimal emissions tax may be either in the monopoly or duopoly region.

The primary interest of the paper is the case where the regulator decides between a low tax which increases competition at the expense of using some inputs inefficiently at the high cost firm, and a monopoly which has less competition but uses inputs more productively. Therefore, we assume that $\tau_c > 0$, so that in the absence of regulation, the industry is a duopoly. Hence:

Assumption 1 *The parameter values satisfy:*

1a. $\sigma_G/\sigma_P \equiv R \geq \frac{1}{2}$.

1b. $\alpha - 2A_0 + \frac{A_0}{A} > 0$.

The first assumption ensures that the inefficient firm drops out for $\tau > \tau_c$.

From equation (3.11), the second assumption assumes that production at the inefficient firm is strictly positive in the absence of the tax and the first assumption implies the tax raises relative costs at the inefficient firm enough to decrease production. Given these two assumptions, a critical value τ_c exists such that for $\tau > \tau_c$, the inefficient firm exits the market.

For $\tau \geq \tau_c$ the market becomes a monopoly with production levels:

$$Q^m = Q_P^m = \frac{\alpha - \frac{A_0}{A} - \tau\sigma_P}{2\beta}. \tag{3.18}$$

The monopoly price is therefore:

$$p^m = \frac{\alpha + \frac{A_0}{A} + \tau\sigma_P}{2\beta}. \tag{3.19}$$

For $\tau \geq \tau_c$, monopoly profits are:

$$\pi_P^m = \frac{\left(\alpha - \frac{A_0}{A} - \tau\sigma_P\right)^2}{4\beta}, \tag{3.20}$$

3.3 Equilibrium Input Demands

Appendix A derives closed form solutions for the input demands for all three firms, for both the monopoly and duopoly cases. These are straightforward functions of the Cournot and monopoly outputs.

3.4 Equilibrium Clean Good Production

Assigning labor and capital according to (A.10) and (A.11) to the clean sector, equation (2.15) implies clean production is:

$$Q_0 = A_0 K^\gamma - \frac{A_0}{A} \frac{\alpha\left(1+A\right) + 2A_0\left(1 - A + \frac{1}{A}\right) - \left(\sigma_G\left(2A-1\right) + \sigma_P\left(2-A\right)\right)\tau}{3\beta}. \tag{3.21}$$

For $\tau \geq \tau_c$, clean production is:

$$Q_0^m = A_0 K^\gamma - \frac{A_0}{A} \cdot \frac{\alpha - \frac{A_0}{A} - \sigma_P \tau}{2\beta}. \tag{3.22}$$

3.5 Equilibrium Emissions

Equilibrium production of the dirty good (3.10) and (3.11) imply emissions (2.5) at equilibrium are:

$$E = \frac{\alpha\left(\sigma_G + \sigma_P\right) - \frac{A_0}{A}\left(\sigma_G\left(2A-1\right) + \sigma_P\left(2-A\right)\right) - 2\tau\left(\left(\sigma_G - \sigma_P\right)^2 + \sigma_G\sigma_P\right)}{3\beta}, \tag{3.23}$$

If $\tau \geq \tau_c$, the monopolist emits:

$$E^m = \frac{\sigma_P\left(\alpha - \frac{A_0}{A} - \tau\sigma_P\right)}{2\beta}. \tag{3.24}$$

3.6 Summary

Definition 1 defines the Cournot-Nash equilibrium.

Definition 1 *Given a government policy $\tau < \tau_c$, a* Cournot-Nash Equilibrium *consists of a set of firm input demands, $[K_i, L_i]$, firm production levels Q_i, profits π_i, and emissions E_i for $i = G, P, 0$, prices $[p, r, w]$, household allocations $[c, c_0]$, total profits π, total dirty production Q, total emissions E, and government transfers NT such that:*

1. *Households maximize utility by choosing c, given prices, profits, and net transfers.*
2. *Each firm in the dirty sector strategically chooses production levels taking as the given the rival firm's production levels, input prices, and government policy, accounting for changes in the output good price caused by changes in the firm's own production.*

3. *Each firm chooses input demands to minimize costs taking as given input prices and government policies.*

4. *Equations (3.14) and (3.15) determine firm profits, with* $\pi = \pi_G + \pi_P$.

5. *Emissions are given by (3.23), and total output is (3.12).*

6. *The household and government budgets balance.*

7. *The input markets clear.*

8. *The goods markets clear.*

Equations (3.7), (3.8), (3.10)-(3.16), (A.2)-(A.7), and (3.21) determine 16 unknowns r, w, p, Q, π, π_i for $i = G, P$, and Q_i, L_i and K_i for $i = P, G, 0$. In addition $c = Q$, $NT = \tau E$ and (3.23), determine c, NT, and E. Finally, the equilibrium budget constraint determines $c_0 = Q_0$.

If the tax rate is sufficiently high, the equilibrium is a monopoly.

Definition 2 *Given a government policy* $\tau \geq \tau_c$, *a* Monopoly Equilibrium *consists of a set of firm input demands,* $[K_i, L_i]$, *firm production levels* Q_i, *profits* π_i, *emissions* E_i, $i = P, 0$, *prices* $[p, r, w]$, *household allocations* $[c, c_0]$, *total profits* π, *total production* Q, *total emissions* E, *and government transfers* NT *such that:*

1. *Households maximize utility by choosing c, given prices, profits, and net transfers.*

2. *The low cost firm in the dirty sector strategically chooses production levels taking as given input prices and government policy, but accounting for changes in the output good price caused by changes in the firm's own production.*

3. *Each firm chooses input demands to minimize costs taking as given input prices and government policies.*

4. *Equation (3.20) determines firm profits, with* $\pi = \pi_P$.

5. *Emissions are given by (3.24), and total output is (3.18).*

6. *The household and government budgets balance.*

7. *The input markets clear.*

8. *The goods markets clear.*

Equations (3.7), (3.8), (3.18)-(3.20), (A.8)-(A.11), and (3.22) determine 10 unknowns r, w, p^m, π_P^m, and Q_i^m, L_i^m and K_i^m for $i = P, 0$. Further, $c = Q^m = Q_P^m$, $NT = \tau E^m$ with E^m determined from (3.24). Finally, the equilibrium budget constraint determines $c_0 = Q_0^m$.

Notice that in our equilibrium concept, dirty firms behave strategically in that they account for the effect of their production on the price of the dirty good. However, firms are perfectly competitive in the input markets. These assumptions match the idea that firms have more control over the price of their output good, than over the cost of capital and labor, which are determined by economy wide asset and labor markets. We also make the standard assumption in perfectly competitive models that firms cannot affect government policies (for example, firms cannot lobby for a lower emissions tax rate).

4 Comparative Statics

Proposition 1 summarizes the comparative statics with respect to the tax rate.

Proposition 1 *Suppose assumptions (1a) and (1b) hold, then the Cournot-Nash and monopoly equilibrium comparative statics with respect to τ are:*

1. *The rental rates r and w are independent of the tax rate τ. The equilibrium price p is increasing in τ.*
2. *Equilibrium emissions E are decreasing in τ.*
3. *Firm G production, profits, and use of both inputs are decreasing in τ for $\tau < \tau_c$ and independent of τ for $\tau \geq \tau_c$.*
4. *Firm P production, profits, and use of both inputs are increasing in τ if and only if $R > 2$ for $\tau < \tau_c$ and strictly decreasing in τ for $\tau \geq \tau_c$.*
5. *Total dirty production Q is decreasing in τ.*
6. *Clean production and input usage Q_0, K_0, and L_0 are all increasing in τ.*

The emissions tax is effectively a tax on dirty output, and so does not affect the capital labor ratio. Thus, the rental rates are independent of τ.

An important condition is $R \geq 2$. The emissions tax hurts both firms, but hurts the inefficient firm more. Therefore, the emissions tax has two countervailing effects on the efficient firm. On one hand production decreases as costs rise at the efficient firm, but on the other hand costs rise more at the inefficient firm, and so the efficient firm takes strategic advantage by increasing production. For $R \geq 2$, the strategic effect is stronger and production *increases* at the efficient firm with the emissions tax. However, for τ high enough, the high cost firm drops out and the strategic effect vanishes. Therefore, for $R \geq 2$, production at the efficient firm is non-monotonic in τ.

Overall dirty production and overall input use in the dirty sector is always decreasing in the tax, however. Therefore, the price is increasing in the tax. Further, the clean sector uses inputs no longer in use in the dirty sector to increase clean production. Therefore, clean production and input use are increasing in the emissions tax.

The model has no fixed costs of entry or exit. We can therefore show that dirty production $Q(\tau)$ is continuous at τ_c. Equations (3.12) and (3.18) imply dirty production is linear in τ, with the slope increasing (becoming more flat) at τ_c. In the duopoly range, increases in production by the low cost firm partially offset decreases in production by the high cost firm as τ increases. Therefore, one might suspect that the slope becomes more steep in the monopoly range as changes in production no longer partially offset. However, since the high cost firm is more emissions intensive, it's production is more sensitive to the tax. Therefore, in the monopoly range, production is less sensitive to the tax as the more sensitive (high cost) firm is out of the market.

A useful parameter is the difference in non-environmental costs between the two firms. In particular, let

$$Y \equiv A_0 - \frac{A_0}{A} \tag{4.1}$$

measure the difference in the marginal rate of transformation between clean and dirty goods at the inefficient firm (A_0) and the marginal rate of transformation at the efficient firm (A_0/A). In addition, let

$$T \equiv \alpha - \frac{A_0}{A} > 0 \tag{4.2}$$

measure the difference between the marginal rate of substitution and the marginal rate of transformation at the efficient firm when production is zero. The parameter T measures demand relative to costs at the efficient firm. Therefore, $T - Y$ measures demand when additional production comes from the inefficient firm, and $T - 2Y$ includes the strategic effect where a large cost difference reduces the incentive to produce at the inefficient firm. Assumption 1b is equivalent to $T - 2Y > 0$.

Most equations in the equilibrium for a given τ depend on only three parameters: the non-environmental cost differential Y, the environmental cost differential R, and the demand parameter T. Proposition 2 gives comparative statics for these parameters.

Proposition 2 *Suppose assumptions (1a) and (1b) hold, then the Cournot-Nash and monopoly equilibrium comparative statics with respect to R, T, and Y are:*

1. *The rental rates r and w are independent of R, T, and Y.*

2. *The equilibrium price p is increasing in T, and increasing in R for $\tau < \tau_c$ and independent of R for $\tau > \tau_c$.*

3. *Firm G production is decreasing in R and Y and increasing in T for $\tau < \tau_c$, and independent of R, T, and Y for $\tau > \tau_c$.*

4. *Firm P production is increasing in R, T, and Y for $\tau < \tau_c$, and independent of R and Y and increasing in T for $\tau > \tau_c$.*

5. *Total production Q is increasing in T and decreasing in R and Y for $\tau < \tau_c$, and increasing in T and independent of R and Y for $\tau > \tau_c$.*

6. *Clean production is decreasing in T and increasing in R for $\tau < \tau_c$ and independent of R for $\tau > \tau_c$.*

The clean sector determines the rental rates, which are therefore independent of the cost structure of the dirty industry. An increase in T corresponds to an increase in demand for the dirty good, which causes an increase in the price and the quantity produced by both firms. Dirty firms use more inputs, which reduces inputs available to the clean sector, causing clean production to fall. An increase in the non-environmental cost differential gives a strategic advantage to the low cost firm, causing production to rise in the low cost firm and fall in the high cost firm. Overall, dirty production falls as a larger cost differential effectively reduces competition between firms. Once the high cost firm exits the market, increasing the environmental or non-environmental cost differential has no effect on the monopolist.

5 Optimal environmental policy

In this section, we consider how environmental policy should respond in an environment with heterogeneous firms and imperfect competition. In general, the regulator faces a tradeoff whereby higher emissions taxes drive the unproductive firm from the market and improve environmental quality, which raises welfare, which is offset by the reduction in competition which reduces production and welfare. We assume throughout this section that the subsidized firm is more emissions intensive. Then, in a first best setting, the social planner uses only the private firm's technology, which is more efficient, and sets output to balance the marginal utility of consumption with social cost of production. A competitive market achieves first best because competition drives the inefficient technology out of the market. An emissions tax is necessary only to make firms account for the social cost of production. However, with a duopoly, competition alone is not enough to drive the inefficient firm from the market.

5.1 First best

Assume $\sigma_G > \sigma_P$. Then the low cost firm achieves any level of output with lower emissions and with less inputs than the high cost firm. Therefore, optimality implies use of only the low cost technology. The social planning problem becomes:

$$\max_{K_i, L_i,\ i=P,0} U = Q_0 + \alpha Q - \frac{\beta}{2} Q^2 - \frac{\zeta}{2} (\sigma_P Q)^2 , \tag{5.1}$$

subject to: (2.6) for $i = P$, (2.15), (3.3), and (3.4).

The first order conditions of (5.1) indicate that the capital labor ratio in the clean and dirty industries both equal the economy wide capital to labor ratio:

$$\frac{K_P}{L_P} = \frac{K_0}{L_0} = K. \tag{5.2}$$

Therefore, although competition is imperfect, the economy maintains the socially optimal capital to labor ratio.

Given the optimal capital to labor ratio, the marginal rate of transformation between clean and dirty goods becomes A_0/A. The social optimum therefore chooses dirty goods production to equate the marginal rate of transformation with the social marginal rate of substitution, which includes the disutility from emissions:

$$\frac{A_0}{A} = \alpha - \beta Q - \zeta\sigma_P^2 Q, \tag{5.3}$$

$$Q^{**} = \frac{\alpha - \frac{A_0}{A}}{\zeta\sigma_P^2 + \beta}. \tag{5.4}$$

First best emissions, production, and welfare are:

$$E^{**} = \sigma_P Q^{**}, \tag{5.5}$$

$$Q_G^{**} = 0,\ Q_P^{**} = Q^{**},\ Q_0^{**} = A_0K^\gamma - \frac{A_0}{A}Q^{**}, \tag{5.6}$$

$$U^{**} = A_0K^\gamma + \frac{\left(\alpha - \frac{A_0}{A}\right)^2}{2\left(2\zeta\sigma_P^2 + \beta\right)}. \tag{5.7}$$

5.2 Perfect Competition Benchmark

Before determining the optimal emissions tax under imperfect competition, we derive the optimal emissions tax under the benchmark of perfect competition, and show that the Pigouvian tax achieves the socially optimal production, emissions, and utility.

It is straightforward to show that under perfect competition with $\sigma_G > \sigma_P$, firm G earns negative profits and therefore exits the market. Further, profit maximization by firms P and 0 imply capital to labor ratios are identical across industries and equal to the economy wide capital to labor ratio K. The price of the dirty good equals the marginal cost:

$$p^{pc} = \frac{A_0}{A} + \tau\sigma_P. \tag{5.8}$$

Since the equilibrium consumer first order condition (2.3) is unchanged, the competitive equilibrium results in dirty good production equal to:

$$Q^{pc} = \frac{\alpha - \frac{A_0}{A} - \tau\sigma_P}{\beta}. \tag{5.9}$$

Therefore, from (5.4), the tax which achieves the social optimum levels of production of both goods and emissions is:

$$\tau^{pc,*} = \frac{\alpha - \frac{A_0}{A}}{\sigma_P} \cdot \frac{\zeta\sigma_P^2}{\zeta\sigma_P^2 + \beta}. \tag{5.10}$$

The marginal damages, MD, at the optimum are:

$$MD = \frac{\partial U}{\partial E} = \zeta E = \zeta\sigma_P Q^{**} = \tau^{pc,*}. \tag{5.11}$$

Hence under perfect competition the regulator achieves first best by setting the emissions tax equal to the marginal damages, which is the Pigouvian level.

5.3 Imperfect Competition

Through the emissions tax, the regulator controls not only the ultimate output of the dirty good, but also whether the industry structure is a duopoly or monopoly. The regulator's problem with imperfect competition is:

$$\max \left\{ \begin{array}{c} \max_{\tau\in[\tau_c,\infty]} Q_0^m(\tau) + \alpha Q^m(\tau) - \frac{\beta}{2}Q^m(\tau)^2 - \frac{\zeta}{2}\sigma_P^2 Q^m(\tau)^2, \\ \max_{\tau\in[0,\tau_c]} Q_0(\tau) + \alpha Q(\tau) - \frac{\beta}{2}Q(\tau)^2 - \frac{\zeta}{2}\sigma_P^2 Q(\tau)^2 \end{array} \right\}, \tag{5.12}$$

subject to (3.12), (3.21), (3.18), and (3.22).

Therefore, the regulator determines first the optimal monopoly tax, $\tau^{m,*}$, within the range of $\tau \geq \tau_c$, and the optimal duopoly tax, $\tau^{D,*}$, in the range $\tau \leq \tau_c$. The regulator then effectively decides whether the market is a duopoly or a monopoly (chooses $\tau^* \in \{\tau^{D,*}, \tau^{m,*}\}$, based on whichever industry structure gives the highest welfare).

Notice that all Q's in (5.12) are linear functions of τ. Therefore, (5.12) is the union of two quadratic programming problems (further, the second order conditions imply both quadratic programming problems are concave). We also know that $Q^m(\tau_c) = Q(\tau_c)$, and that the unconstrained monopoly problem must have higher welfare than the unconstrained duopoly problem (which uses an inferior technology). Figure 1 shows that the solution to (5.12) breaks down into at most three cases.

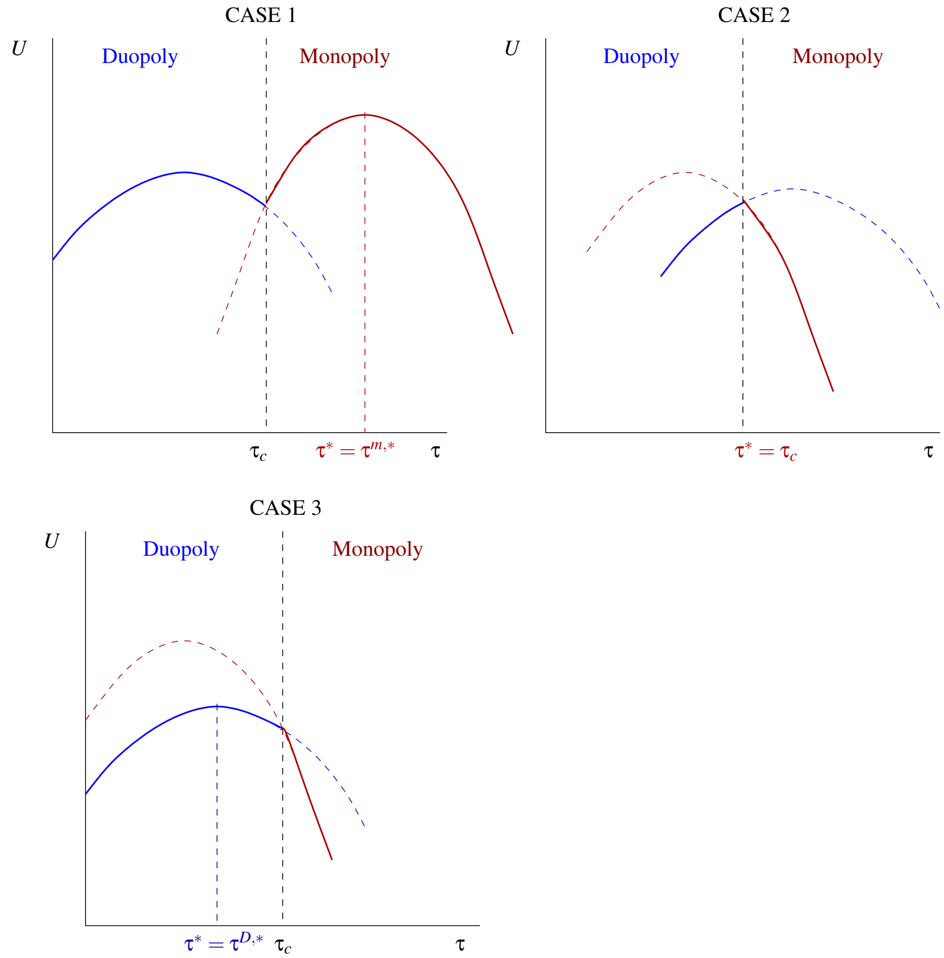

Figure 1: Intuition for the regulator's problem. The solid lines are feasible. That is, points on the duopoly curve are feasible only if $\tau < \tau_c$ and the reverse for the monopoly.

Suppose first that preferences for environmental quality are very strong. Let $z \equiv \zeta\sigma_P^2/\beta$ index relative preferences for the environment. Then for z sufficiently high, the social optimum calls for relatively little production of the dirty good. Therefore, the regulator sets an emissions tax which induces a monopoly, because the decline in production that results from the lack of competition only helps to keep dirty good production low. In particular:

Proposition 3 *Let $R > 1$ and:*

$$z \geq z_2 \equiv \frac{R \cdot T - Y}{(R-1)\, T + Y}. \tag{5.13}$$

Then the optimal tax, $\tau^* = \tau^{m,*}$, has the following properties:

1. *The regulator drives the high cost firm from the market: $\tau^* \geq \tau_c$.*

2. *The regulator induces first best welfare: $Q^m(\tau^*) = Q^{**}$, $Q_0(\tau^*) = Q_0^{**}$, and $E^m(\tau^*) = E^{**}$.*

3. *Optimal production is below the critical level which induces the monopoly:* $Q^{**} \leq Q^m(\tau_c)$.

4. *The optimal tax is less than the Pigouvian level* ($\tau^* < \tau^{pc,*}$).

Proposition 3, corresponding to case 1 in Figure 1, shows that if socially optimal dirty goods production is sufficiently low, or if the relative costs of the two firms are sufficiently different so that even a low tax drives the inefficient firm from the market, then the regulator achieves first best with a tax which induces a monopoly. By driving the low cost firm from the market, the regulator reduces the industry average emissions intensity and increases the industry average TFP, both of which increase welfare. The cost in terms of reduced output of the dirty good is zero here, because even after just driving the inefficient firm from the market, dirty production is still too high (Proposition 3, result 3). Therefore, the regulator increases the tax above τ_c until output is first best. Since monopoly production is less than that of perfect competition, the optimal tax is smaller than perfect competition, as in Simpson (1995).

The optimal tax in the monopoly ($z \geq z_2$) range is:

$$\tau^* = \frac{T}{\sigma_P} \cdot \frac{z-1}{z+1} < \tau^{pc,*} = \frac{T}{\sigma_P} \cdot \frac{z}{z+1}. \tag{5.14}$$

The monopolist is already reducing production to reduce output, so an emissions tax less than Pigouvian is sufficient to reduce output to the social optimum.

Figure 2 shows welfare under imperfect and perfect competition for case 1. Notice that welfare under perfect competition achieves the same maximum utility as the unconstrained monopoly (first best). Further, with perfect competition the tax rate that maximizes welfare must be greater than the optimal unconstrained monopoly tax. Therefore, for case 1, the optimal tax with imperfect competition is less than the marginal damages.

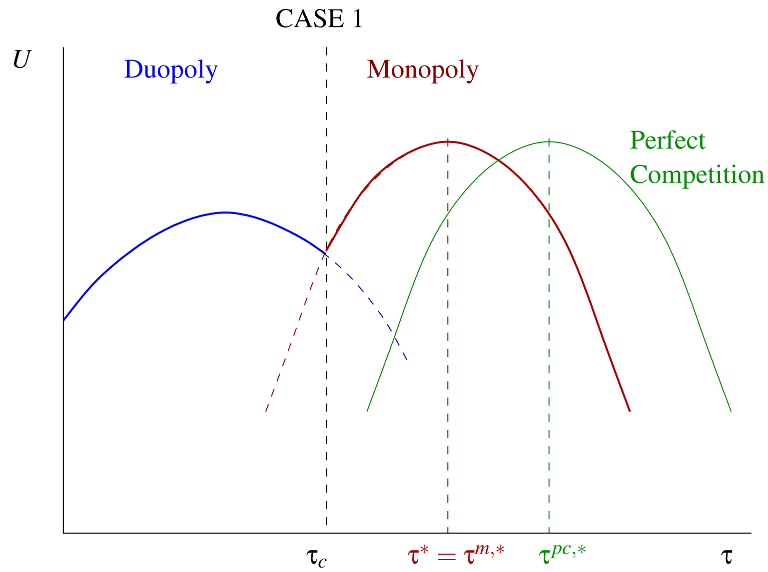

Figure 2: Imperfect and perfect competition for case 1. The solid lines are feasible.

When (5.13) is violated, the regulator faces a tradeoff between decreasing the market share of the high cost firm, which increases TFP and decreases the emissions intensity and too little production of the dirty good. For an intermediate range of environmental preferences, it remains optimal to drive the high cost firm from the market:

Proposition 4 *Let $R > 1$ and:*

$$z_1 \equiv \frac{R(1+R)(T-2Y) - Y(R-2)(2R-1)}{2(R^2-R+1)((R-1)T+Y)} \leq z \leq z_2. \tag{5.15}$$

Then the optimal tax, $\tau^* = \tau_c$, has the following properties:

1. *The regulator just drives the high cost firm from the market: $\tau^* = \tau_c$.*
2. *The regulator cannot achieve first best welfare.*
3. *Production of the dirty good is less than first best $Q^m(\tau_c) < Q^{**}$, production of clean goods exceeds first best, and emissions is less than first best.*
4. *The optimal tax is greater than the Pigouvian if and only if:*

$$z < z_P \equiv \frac{T-2Y}{2(T(R-1)+Y)}. \tag{5.16}$$

Proposition 4 (case 2 in Figure 1) shows that as environmental preferences fall into in an intermediate range, more dirty goods production becomes optimal. The regulator would therefore like to decrease the emissions tax below τ_c. However, doing so would cause the inefficient firm to enter the market, causing the industry wide TFP to fall and the emissions intensity to rise. Thus, decreasing τ below τ_c affects the tradeoff between clean and dirty goods. As the industry emissions intensity rises and TFP falls, more clean goods becomes optimal. Therefore, the tax which maximizes utility is as low as possible while still maintaining a monopoly (τ_c). In summary, for z in the intermediate range given by (5.15), a marginal unit of capital and labor generates highest marginal welfare by producing a good at the efficient dirty firm, intermediate marginal welfare by producing a clean good, and lowest welfare by producing a dirty good at the inefficient firm. Since the regulator can effectively choose between only the last two options, we have a corner solution with over-production of clean goods and under-production of dirty goods, relative to the social optimum. Interestingly, emissions in this case is actually *below* first best. Even though preferences for the environment are only intermediate, emissions are low because the regulator lacks a way to induce the economy to produce dirty goods efficiently.

Whether or not the emissions tax is less than the Pigouvian is more complex.[6] If preferences for the environment are sufficiently weak or the cost difference is sufficiently small, the emissions tax exceeds the Pigouvian level. This counterintuitive result occurs because in this range, decreasing environmental preferences does not affect the optimal tax: the regulator is stuck at the corner solution of τ_c. However, as preferences for the environment decrease, the welfare gain from keeping out the inefficient firm is falls. The optimal tax under perfect competition strictly decreases as preferences for the environment decreases. Eventually, if $z_P > z_1$, the optimal tax under perfect competition decreases below the Pigouvian level. Figure 3 shows both the case where the optimal tax is less than Pigouvian (2A), and the case where the optimal tax exceeds the Pigouvian level (2B).

[6]The Pigouvian tax is the marginal damage, which varies according to the amount of dirty production. Hence, some ambiguity exists in the definition. Here we define the Pigouvian tax as equal to the marginal damages that would occur at the socially optimal level of production. The results are similar if the definition of the Pigouvian tax is equal to the marginal damages at the second best level of production $MD = \zeta E = \zeta \sigma_P Q(\tau_c)$.

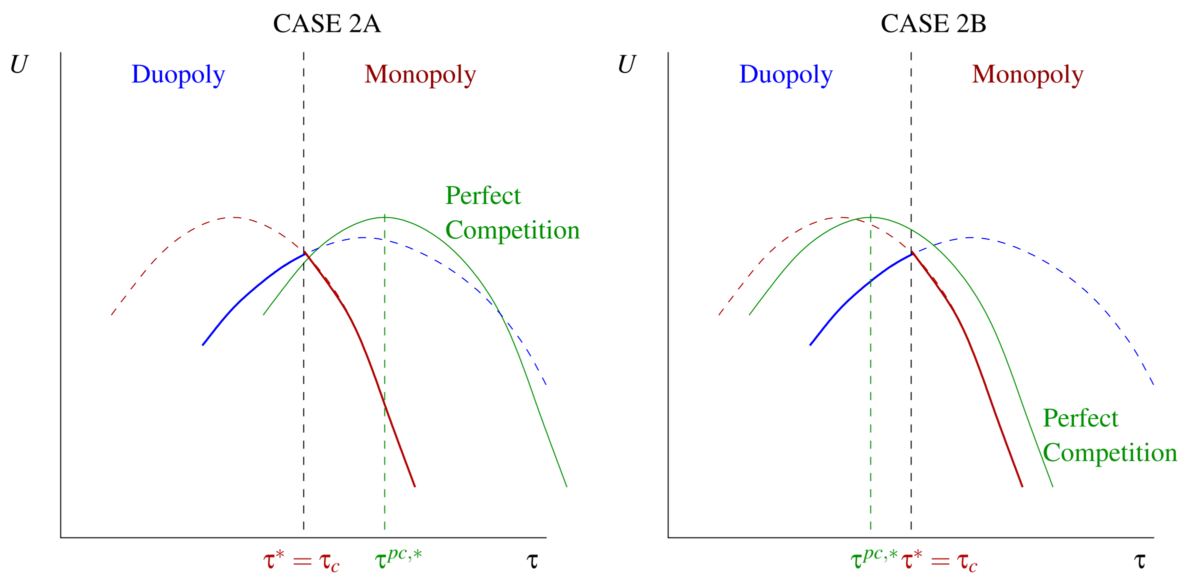

Figure 3: Imperfect and perfect competition for case 2. The solid lines are feasible. In case 2A the optimal tax is less than the marginal damages, whereas the reverse occurs in case 2B.

A small cost difference implies that the regulator must increase the emissions tax to drive out the inefficient firm, eventually above the Pigouvian level. In contrast, the cost differential does not affect the tax under perfect competition, because competition drives out the inefficient firm regardless of τ. However, the cost difference cannot be so small that it becomes optimal to let in the inefficient firm. That is, $z_P > z_1$ implies that a range $z_1 < z < z_P < z_2$ exists for which the emissions tax exceeds the Pigouvian level. The condition for $z_1 < z_P$ is:

$$\frac{T}{R} < Y < \frac{T}{2}. \tag{5.17}$$

Condition (5.17) requires $R > 2$: if the difference in emissions intensities is strong enough such that low cost production is increasing in the emissions tax, then a range of preferences exists such that the emissions tax exceeds the Pigouvian level.

If preferences for the environment are sufficiently low, a duopoly results:

Proposition 5 *Let $R > 1$ and $z < z_1$. Then the optimal tax, $\tau^* = \tau^{D,*}$, has the following properties:*

1. *The regulator allows both firms in the market: $Q_G > 0$.*
2. *The regulator cannot achieve first best welfare.*
3. *Production of the dirty good is less than first best $Q(\tau^*) < Q^{**}$.*

4. The optimal tax is less than the Pigouvian for $R \leq 2$. For $R > 2$, there exists a region with positive measure such that the optimal tax exceeds the Pigouvian tax.

For $z < z_1$, preferences for environmental quality are sufficiently weak, and therefore socially optimal production of the dirty good is sufficiently high, so that the regulator allows the inefficient firm in the market. When the inefficient firm enters the market, however, the industry wide average TFP falls and the industry wide emissions intensity rises. Because the dirty industry is less efficient, producing dirty goods becomes less attractive, and we have a second best optimum where production of dirty goods is below first best.

Although the regulator allows the inefficient firm to remain in the market to increase production of the dirty good, it is optimal to use the emissions tax to discourage production to some degree by the inefficient firm. In extreme cases, this requires the emissions tax to exceed the Pigouvian level. In particular, if the inefficient firm is sufficiently emissions intensive, $R > 2$, the emissions tax discourages production at the inefficient firm while increasing production at the efficient firm. Therefore, for $R > 2$ a region exists such that the emissions tax exceeds the Pigouvian level.[7]

Figure 4 shows the case for which the optimal tax exceeds the Pigouvian level (case 3B). In the unconstrained monopoly case the optimal tax would be less than Pigouvian, but this requires a tax rate low enough such that the high cost firm remains in the market. The regulator is unable to achieve first best (the maximum utility with perfect competition).

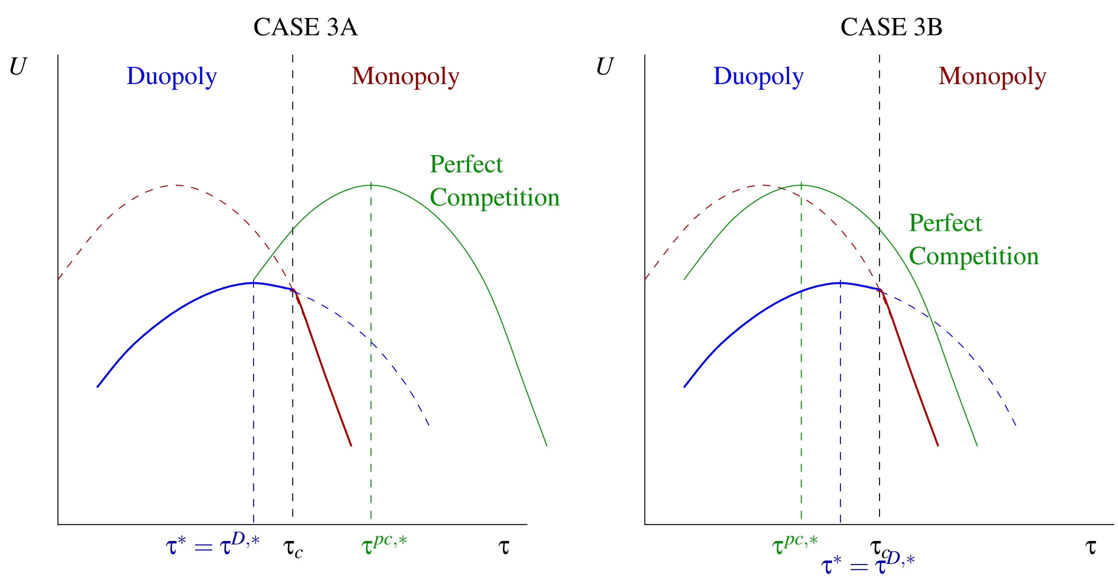

Figure 4: Imperfect and perfect competition for case 3. The solid lines are feasible. In case 3A the optimal tax is less than the marginal damages, whereas the reverse occurs in case 3B.

[7] We have an analytical solution for the region for which the emissions tax exceeds the Pigouvian level. The solution, which is complex and adds little intuition, is not reported.

6 Conclusions

We have shown that, unless preferences for environmental quality are sufficiently weak, the optimal emissions tax in an imperfectly competitive market exceeds the marginal damages. Intuitively, with perfect competition, the competitive market eliminates firms with inefficient technologies. With imperfect competition the regulator uses the pollution tax as a substitute for the market, reducing the market share of the inefficient firm by setting an optimal pollution tax which is above the Pigouvian level.

Our results show that policy makers should pay close attention to the distribution of productivity and emissions intensity across firms when choosing an emissions tax. If productivity and emissions intensity differences are sufficiently large, an emissions tax that exceeds the marginal damage improves welfare as firms are incentivized to adopt better technologies or exit the market.

In our model, the only policy instrument available to the regulator is the emissions tax. The reader might be concerned that if another regulatory instrument was available which could ensure only the high TFP/low emissions intensity technology existed in the market, then the optimal emissions tax would revert to the standard monopoly case in which the emissions tax is below the Pigouvian level and play no role in determining the market structure. However, in our context with heterogeneous firms and technologies, additional regulatory instruments are unlikely to be more effective than the emissions tax. For example, an output subsidy is often prescribed with a Pigouvian tax to induce a monopolist to produce at the socially optimal level (Martin 1986). Here, however, an output subsidy would only encourage the high cost firm to stay in the market (Heutel and Kelly 2016). A subsidy for adoption of the efficient firm's technology would be helpful, but in practice the government typically has considerable difficultly identifying which technologies are lowest cost. In contrast, the emissions tax is not technology or firm specific, and so requires less information. Another possible instrument is a production or input tax. Production or input taxes would strengthen the strategic advantage of the low cost firm in a way related to how the emissions tax favors the low cost firm. However, even if a production or input tax was available, the emissions tax might be a more effective way of increasing the strategic advantage of the low cost firm, given the negative correlation between emissions intensity and productivity.

Our results have interesting implications for other types of regulation. In industries such as banking, the number of firms as decreased as the cost of regulation compliance has caused firms to exit the market. Our results imply such consolidation can be welfare increasing. Indeed, a general trend in recent years is one of industry consolidation and increasing firm size. Understanding how emissions taxes work in imperfectly competitive markets is becoming more important.

Appendices

A Equilibrium Input Usage

Production of the dirty good implies:

$$Q_i = A_i \left(\frac{K_i}{L_i}\right)^{\gamma} L_i = A_i K^{\gamma} L_i, \tag{A.1}$$

Hence, using (3.10) and (3.11), equilibrium labor input usage is:

$$L_P = \frac{\alpha + A_0 - 2\frac{A_0}{A} + \tau\left(\sigma_G - 2\sigma_P\right)}{3\beta A K^{\gamma}} \tag{A.2}$$

$$L_G = \frac{\alpha - 2A_0 + \frac{A_0}{A} - \tau\left(2\sigma_G - \sigma_P\right)}{3\beta K^{\gamma}} \tag{A.3}$$

Using the capital to labor ratio, equilibrium capital input usage is:

$$K_P = \frac{\left(\alpha + A_0 - 2\frac{A_0}{A} + \tau\left(\sigma_G - 2\sigma_P\right)\right) K^{1-\gamma}}{3\beta A} \tag{A.4}$$

$$K_G = \frac{\left(\alpha - 2A_0 + \frac{A_0}{A} - \tau\left(2\sigma_G - \sigma_P\right)\right) K^{1-\gamma}}{3\beta} \tag{A.5}$$

Finally, equilibrium in the capital and labor markets (3.3) and (3.4) determines clean input usage:

$$L_0 = 1 - \frac{\alpha\left(1+A\right) - 2A_0\left(A^2 - A + 1\right) - A\tau\left(\sigma_G\left(2A-1\right) + \sigma_P\left(2-A\right)\right)}{3\beta A^2 K^{\gamma}} \tag{A.6}$$

$$K_0 = K - \frac{\left(\alpha\left(1+A\right)A - 2A_0\left(A^2 - A + 1\right) - A\tau\left(\sigma_G\left(2A-1\right) + \sigma_P\left(2-A\right)\right)\right) K^{1-\gamma}}{3\beta A^2} \tag{A.7}$$

For $\tau \geq \tau_c$, firm G drops out and uses no inputs. The remaining firms have input usage of:

$$L_P^m = \frac{\alpha - \frac{A_0}{A} - \tau\sigma_P}{2\beta A K^{\gamma}} \tag{A.8}$$

$$K_P^m = \frac{\left(\alpha - \frac{A_0}{A} - \tau\sigma_P\right) K^{1-\gamma}}{2\beta A} \tag{A.9}$$

$$L_0^m = 1 - \frac{\alpha - \frac{A_0}{A} - \tau\sigma_P}{2\beta A K^{\gamma}} \tag{A.10}$$

$$K_0^m = K - \frac{\left(\alpha - \frac{A_0}{A} - \tau\sigma_P\right) K^{1-\gamma}}{2\beta A} \tag{A.11}$$

B Proofs of Propositions

Proof of Proposition 1.

The comparative statics follow from differentiating the equilibrium conditions (3.7), (3.8), (3.10)-(3.16), (A.2)-(A.7), (3.21), and (3.23) for the duopoly case and the equilibrium conditions (3.7), (3.8), (3.18)-(3.20), (A.8)-(A.11), (3.22) and (3.24) for the monopoly region. ■

Proof of Proposition 2. Writing the equilibrium conditions in terms of R, T, and Y results in:

$$Q_P = \frac{T + Y + (R-2)\,\sigma_P \tau}{3\beta}, \tag{B.1}$$

$$Q_G = \frac{T - 2Y - (2R-1)\,\sigma_P \tau}{3\beta}, \tag{B.2}$$

$$Q = \frac{2T - Y - (R+1)\,\sigma_P \tau}{3\beta}, \tag{B.3}$$

$$Q_0 = A_0 K^\gamma - \frac{A_0}{A}\frac{T + Y + (T-2Y)\,A + (R - 2 - (2A-1)\,A)\,\sigma_P \tau}{3\beta}. \tag{B.4}$$

$$p = \frac{T + Y + 3\frac{A_0}{A} + (R+1)\,\sigma_P \tau}{3}. \tag{B.5}$$

Notice that for Q_0 and p, the marginal rates of transformations enter the equations, but variables cannot be reduced to a function of the difference between the marginal rates of transformation. Therefore, for these variables we consider only the comparative statics with respect to T. The comparative statics then follow from the signs of the derivatives of (B.1)-(B.5), using that $R > \frac{1}{2}$ and $A > 1$.

For the monopoly case, rewriting the monopoly equilibrium in terms of T, Y, and R results in:

$$Q^m = Q_P^m = \frac{T - \sigma_P \tau}{2\beta}. \tag{B.6}$$

$$Q_0^m = A_0 K^\gamma - \frac{A_0}{A} \cdot \frac{T - \sigma_P \tau}{2\beta}. \tag{B.7}$$

$$p^m = \frac{T + 2\frac{A_0}{A} + \sigma_P \tau}{2}. \tag{B.8}$$

The environmental and non-environmental cost differentials are now irrelevant as only one firm is in the market. The comparative statics for T then follows from the signs of the derivatives of (B.6)-(B.8). ■

Proof of Proposition 3. We first solve a relaxed version of the monopoly part of (5.12), which ignores the constraint that $\tau \geq \tau_c$. We then construct a range of parameter values such that the solution to the relaxed problem in fact satisfies the constraint. We then argue that the solution to the relaxed problem results in higher welfare than the duopoly problem.

The first order condition of (5.12) implies the solution to the relaxed monopoly problem is:

$$\tau^{m,*} = \frac{T}{\sigma_P} \cdot \frac{z-1}{z+1}. \tag{B.9}$$

Next, from equation (3.17), the solution to the relaxed problem satisfies the constraint $\tau^{m,*} \geq \tau_c$ if and only if:

$$\frac{T}{\sigma_P} \cdot \frac{z-1}{z+1} \geq \frac{T-2Y}{(2R-1)\,\sigma_P}. \tag{B.10}$$

Solving (B.10) for z results in:

$$z \geq z_2 \equiv \frac{R \cdot T - Y}{(R-1)\,T + Y}. \tag{B.11}$$

Therefore, for $z \geq z_2$, the imposing $\tau^{m,*}$ results in a monopoly and the solution to the relaxed problem also solves the constrained problem.

Next, imposing $\tau^{m,*}$ in equation (3.18) generates dirty production equal to Q^{**}, which is first best. Emissions and clean production are also first best since the economy is using only the most efficient technology, which the social planner also uses. Therefore, the unconstrained monopoly problem achieves first best. The duopoly has lower average TFP and higher average emissions intensity, so the duopoly cannot achieve first best welfare. Therefore, for $z \geq z_2$, the unconstrained solution (B.9) is the solution to (5.12). This is case 1 in Figure 1.

Finally, equation (5.14) implies the optimal tax is less than Pigouvian. ■

Proof of Proposition 4. Case 2 occurs when both $\tau^{m,*} < \tau_c$ and $\tau^{D,*} \geq \tau_c$ (see Figure 1): the constraint that $\tau \geq \tau_c$ binds in the monopoly case, and the constraint that $\tau < \tau_c$ binds in the duopoly case. The proof of Proposition 3 implies the constraint binds in the monopoly problem whenever the unconstrained solution satisfies $\tau^{m,*} < \tau_c$, which occurs when $z < z_2$.

For $\tau^{D,*} \geq \tau_c$, note that the first order condition of the relaxed version (ignoring the constraint that $\tau < \tau_c$) of the duopoly part of (5.12) implies the unconstrained solution is:

$$\tau^{D,*} = \frac{(5R-4)\,Y + (1+R)\left(2z\left(R^2-R+1\right)-1\right)T - 2\left(R^2-R+1\right)(2R-1)\,zY}{\left((R+1)^2 + 4\left(R^2+R-1\right)^2 z\right)\sigma_P}. \tag{B.12}$$

The constraint binds if and only if $\tau^{D,*} \geq \tau_c$, which from (3.17), occurs if and only if:

$$\frac{(5R-4)\,Y + (1+R)\left(2z\left(R^2-R+1\right)-1\right)T - 2\left(R^2-R+1\right)(2R-1)\,zY}{\left((R+1)^2 + 4\left(R^2+R-1\right)^2 z\right)\sigma_P} \geq \frac{T-2Y}{(2R-1)\,\sigma_P}. \tag{B.13}$$

After some algebra, the condition simplifies to:

$$z \geq z_1 \equiv \frac{TR\,(1+R) - Y\left(4R^2-3R+2\right)}{2\left(R^2-R+1\right)\left(T\,(R-1)+Y\right)}. \tag{B.14}$$

Simple algebra shows that $z_1 < z_2$ for $R \geq 1$. Therefore, case 2 arises for $z_1 \leq z < z_2$. Since both the unconstrained monopoly and duopoly problems are quadratic programming problems, both achieve a corner solution at τ_c. Therefore, τ_c is optimal, at which point the high cost firm just exits the market.

Since the constrained monopoly problem gives strictly lower welfare than the unconstrained monopoly problem (first best). Therefore case 2 does not achieve first best welfare.

For production, the high cost firm just exits the market, so total production is $Q^m(\tau_c)$. From (3.18) and (5.4), $Q^m(\tau_c) < Q^{**}$ if and only if:

$$\frac{\left(T - \sigma_P \frac{T-2Y}{(2R-1)\sigma_P}\right)}{2\beta} < \frac{T}{(z+1)\beta}, \tag{B.15}$$

$$z < \frac{RT - Y}{(R-1)T + Y} = z_2, \tag{B.16}$$

which holds by assumption since $z < z_2$ for case 2. Since in case 2 the high cost firm exits from the market and production is less than first best, emissions are also less than first best, and clean production is greater than first best.

For the relation between the optimal tax and the Pigouvian tax, equations (3.17) and (5.10) $\tau_c > \tau^{pc,*}$ if and only if:

$$\frac{M - 2Y}{\sigma_P (2R-1)} > \frac{M}{\sigma_P} \cdot \frac{z}{z+1}. \tag{B.17}$$

Rewriting in terms of z results in:

$$z < z_P \equiv \frac{M - 2Y}{2(M(R-1) + Y)}. \tag{B.18}$$

Next, clearly $z_P < z_2$ since $R > 1$. Further, using (5.15) and $R > 1$, $z_P > z_1$ if and only if $T < RY$. Assumption (1b) has $T > 2Y$, so for $2Y < T < RY$ (which requires $R > 2$), $z_1 < z_P$ and a range $z \in [z_1, z_P]$ exists such that the optimal tax exceeds the Pigouvian tax (see Figure 3). ■

Proof of Proposition 5. The proofs of Propositions 3 and 4 imply that for $z < z_1$, the constraint binds for the monopoly problem and the constraint does not bind for the duopoly problem. Further, because the problems give identical welfare at τ_c, the unconstrained duopoly problem gives the highest welfare (see case 3 in Figure 1).

Since the high cost firm exits at τ_c, and $\tau^{D,*} < \tau_c$, the high cost firm has positive production. Since first best requires only the best technology to be used, welfare is below first best.

Next, the first order condition of the relaxed version of the duopoly part of (5.12) implies the optimal duopoly tax is:

$$\tau^{D,*} = \frac{-(T-2Y)(R+1)+3Y(R-2)+2\left(R^2-R+1\right)((T-2Y)(R+1)+3Y)z}{\left((R+1)^2+4\left(R^2-R+1\right)^2 z\right)\sigma_P}. \tag{B.19}$$

Hence equation (5.10) implies $\tau^{D,*} > \tau^{pc,*}$ if and only if:

$$\begin{aligned} ch(z) \equiv & -(T-2Y)(R+1)+3Y(R-2) \\ & + \Big[R(R+1)(2R-3)(T-2Y)+(R-2)(4R+1)Y\Big] z \\ & - 2\left(R^2-R+1\right)\left((R-1)(2R-1)(T-2Y)+(2R-1)^2 Y\right) z^2 > 0. \end{aligned} \tag{B.20}$$

Condition (B.20) is quadratic in z. Further, since $R > 1$ and $T > 2Y$, $ch'' < 0$, so the polynomial is concave. From this point, the proof is divided into several cases:

1. **Case A:** $R \leq 3/2$**.** If $R \leq 3/2$, then $ch(0) < 0$ and $ch'(0) < 0$. Therefore, since ch is quadratic and concave, both roots of ch are negative. Therefore, $ch(z) < 0$, which implies $\tau^{D,*} < \tau^{pc,*}$.

2. **Case B:** $3/2 < R \leq 2$**.** For this case, $ch(0) < 0$. Further, $ch'(0) > 0$ if and only if:

$$\frac{M-2Y}{Y} \equiv X > X_B \equiv \frac{(2-R)(4R+1)}{R(R+1)(2R-3)}. \tag{B.21}$$

Therefore, for $X \leq X_B$, $ch'(0) < 0$ and the case is identical to case 1, so $ch < 0$. For $X > X_B$, note that:

$$\begin{aligned} ch(z_1) = & -\frac{(2R-1)(T-RY)}{\left(R^2-R+1\right)(T(R-1)+Y)} \cdot \\ & \left((R+1)\left(R^2+1\right)(T-RY)+\left(R^2-R+1\right)\left(R^2-2R+3\right)Y\right). \end{aligned} \tag{B.22}$$

Since $R \leq 2$, we have $T > 2Y > RY$. Therefore, $ch(z_1) < 0$.

Further,

$$ch'(z_1) = -R(R+1)(2R+1)(T-2Y)+(R-2)\left(8R^2-4R+3\right). \tag{B.23}$$

Therefore, $ch'(z_1) \leq 0$ for $R \leq 2$. Therefore, in this case both roots of ch are between zero and z_1. For $ch < 0$, both roots must be imaginary. After some tedious algebra, the following facts can established for the discriminant of ch:

- The discriminant is quadratic in X.

- For any $R \in [3/2, 2]$, the discriminant is concave in X.
- For any $R \in [3/2, 2]$, the discriminant is negative at $X = X_B$.
- For any $R \in [3/2, 2]$, the derivative of the discriminant with respect to X is negative at $X = X_B$.

These facts are sufficient to establish that the discriminant is negative for all parameters such that $R \in [3/2, 2]$ and $X \geq X_B$. Therefore, the roots of ch are imaginary, which implies $ch < 0$, which implies $\tau^{D,*} < \tau^{pc,*}$. Cases A and B together imply that $\tau^{D,*} < \tau^{pc,*}$ for $R \leq 2$ as claimed.

3. **Case C:** $R > 2$. Note that:

$$ch\,(0) = -\,(R+1)\,(T-2Y) + 3\,(R-2)\,Y. \tag{B.24}$$

Therefore, $ch\,(0) > 0$ if and only if:

$$X > X_A \equiv \frac{3\,(R-2)}{R+1}. \tag{B.25}$$

For $X > X_A$, $ch\,(0) > 0$ and so there exists a region around $z = 0$ such that $ch > 0$, which implies a region exists such that $\tau^{D,*} > \tau^{pc,*}$.[8]

For $X \leq X_A$, $ch\,(0) < 0$. Then since $X_A < R-2$, $X < R-2$ or equivalently $T < RY$. Thus (B.22) implies $ch\,(z_1) > 0$ if and only if:

$$\left((R+1)\left(R^2+1\right)(T-RY) + \left(R^2-R+1\right)\left(R^2-2R+3\right)Y\right) > 0, \tag{B.26}$$

$$\left((R+1)\left(R^2+1\right)(T-2Y) - \left(2R^3-7R^2+4R-5\right)Y\right) > 0. \tag{B.27}$$

Therefore, $ch\,(z_1) \geq 0$ if and only if:

$$X \geq X_C \equiv \frac{2R^3-7R^2+4R-5}{(R+1)\,(R^2+1)}. \tag{B.28}$$

Note that $X_C < X_A$. Therefore, for $X_C \leq X \leq X_A$, $ch\,(0) < 0$ and $ch\,(z_1) > 0$. Therefore, ch has a root $z^* \in (0, z_1)$ such that $ch > 0$ for $z > z^*$. Therefore a region exists such that $\tau^{D,*} > \tau^{pc,*}$ for parameters such that $X_C \leq X \leq X_A$ and $R > 2$.

Finally, note that if $z_1 < 0$, then case 3 does not exist, and the regulator always drives the high cost firm out of the market. From (5.15), the condition for $z_1 > 0$ is:

$$R\,(1+R)\,T - Y\left(4R^2-3R+2\right) > 0, \tag{B.29}$$

[8] Although establishing that a region exists is sufficient for the proof, the region might be much larger. For example, if $X > R-2$, then $\tau^{D,*} > \tau^{pc,*}$ for all $z \in [0, z_1]$.

$$R(1+R)X-(2R-1)(R-2)>0, \tag{B.30}$$

$$X>X_D\equiv\frac{(2R-1)(R-2)}{R(1+R)}. \tag{B.31}$$

Next, note that $X_A > X_D > X_C$, for $R > 2$. Therefore, no other cases exist. That is, if $X \leq X_C$, then Proposition 5 does not apply and instead Proposition 4 applies for all $z \in (0, z_2)$.

It remains to show $Q^{**} > Q\left(\tau^{D,*}\right)$. Using (5.4), (B.3), and (B.19), $Q^{**} > Q\left(\tau^{D,*}\right)$ if and only if:

$$\begin{aligned} ch_2(z) = &(R+1)(2R-1)Y+ \\ &\left((R-1)\left(2R^3+3R-1\right)T-\left(2R^3-6R^2+3R-1\right)Y\right)z- \\ &2(R-1)\left(R^2-R+1\right)\left(T(R-1)+Y\right)z^2>0. \end{aligned} \tag{B.32}$$

It is immediate that $ch_2'' < 0$ and $ch_2(0) > 0$. Therefore $ch_2(z_1) > 0$ is sufficient for $ch(z) > 0$ for all $z < z_1$. After some algebra, $ch_2(z_1) > 0$ if and only if:

$$X>X_E\equiv\frac{2R^3-7R^2+4R-5}{(R+1)\left(R^2+1\right)}. \tag{B.33}$$

It is straightforward to show that $X_D > X_E$. Therefore, existence of case 3 requires $X > X_D > X_E$. Therefore, $X > X_E$, which implies $ch_2(z) > 0$ for all $z < z_1$. Therefore, $Q^{**} > Q\left(\tau^{D,*}\right)$. ■

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