

Optimal Monetary and Fiscal Policies in a Search Theoretic Model of Monetary Exchange

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Abstract

We study optimal monetary and fiscal policy in a standard random matching model of pairwise trade. In particular, we extend the Lagos-Wright model and solve the Ramsey problem of finding the production subsidies, sales taxes and seignorage that maximize welfare subject to a balanced budget constraint. We find that there are multiple combinations of taxes, subsidies and –sometimes strictly positive- inflation rates that achieve the optimal allocation. The Friedman rule alone is not enough to restore efficiency. Moreover, active fiscal policy can completely eliminate the welfare costs of inflation. Finally, we study costly implementation of fiscal policy. We find that active fiscal policy is optimal even when implementation costs are as high as the total sales tax revenue. In such a case, subsidies are paid with an inflation tax while still achieving the socially optimal level of output.

JEL Codes: C70, E40.

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1 Introduction

The contribution of this paper is to perform a quantitative study of optimal monetary and fiscal policy in models with micro foundations for the existence of and desire to hold

monetary balances. There is a large body of literature devoted to the study of optimal monetary and fiscal policy in frameworks that are not explicit about the frictions that give rise to the existence of fiat money. Chari, Christiano and Kehoe (1996) provide a complete analysis of optimal policies in models characterized by perfect competition, and that abstract from agent heterogeneity. Their main finding is that in such models the Friedman rule is optimal. The reason behind this result is that money is an intermediate good and thus it should not be taxed, as initially suggested by Diamond and Mirlees (1971). In models with heterogeneous agents and perfect competition, da Costa and Werning (2003) have shown that, under certain conditions, the Friedman rule is still optimal. On the other hand, Schmitt-Grohe and Uribe (2004) illustrate that the Friedman rule is not optimal in environments with imperfect competition, particularly if profit taxes are bounded away from 100%, or if taxes are relatively hard to adjust over time. In this environment the positive inflation rate is used as a tax on profits as to help minimize the distortion of imperfect competition. An important shortcoming of this applied literature is that it is built on models with *ad hoc* constructs that rationalize the positive value of fiat money. Hence, it is impossible to determine whether the frictions that make fiat money socially beneficial, and valuable, are likely to affect the nature of optimal fiscal and monetary policy.

Since Kiyotaki and Wright (1991) the fundamental building blocks in the literature exploring the microeconomic foundations of money consist of trading risk and pairwise trade. These frictions, and in particular the nature of the price determination system in pairwise trades, have been shown to have non trivial qualitative and quantitative implications for the design of optimal monetary policy, and for the welfare costs of inflation. The first paper to clearly illustrate the latter connection is Lagos and Wright (2005), who developed a search theoretic model of monetary exchange that is also analytically tractable and easily quantifiable. These authors show that, when monetary policy is the only tool available to the government, the unique optimal policy is the Friedman rule. Moreover, the welfare costs of inflation are large. In particular, a calibrated version of their model illustrates that the gain to reducing inflation from 10% to 0% is worth up to 5% of consumption. The Friedman rule is optimal in such a framework for two different reasons. The first is that in monetary exchanges agents pay a cost today (production) to receive a future benefit (money that can be used to purchase goods in future trades). The Friedman rule serves to eliminate the inefficiency created by such a lag. Notice that this channel is also present in applied models with *ad hoc* assumptions giving value to fiat

money. The second reason for the optimality of the Friedman rule resides in the frictions that make money valuable. In particular, if prices in pairwise trades are determined in a Nash bargaining game then an inefficiency is created. A person who holds cash is making an investment as she has to give up current consumption for future consumption. Then, any difference in the bargaining power of any of the trading partners results in asymmetric splits of the match surplus. Nash bargaining leads potential buyers to hold too little cash. The Friedman rule gives money a positive return, which may counteract this effect. Berentsen and Rocheteau (2003) have shown that in Nash or Kalai-Smorodinsky bargaining the Friedman rule is generally not enough to achieve efficiency. If the terms of trade are determined by an Egalitarian solution, however, then the first best production level can be achieved. Under any of these three different bargaining settings, nevertheless, the welfare costs of inflation have been found to be higher than in models that do not explicitly model the frictions that make fiat money valuable.

As pointed out by Kocherlakota (2005), existing monetary policy analysis in models with microeconomic foundations for the existence of fiat money are incomplete because they have ignored the role of virtually all tax instruments beyond the inflation tax.¹ Moreover, it is not clear whether the high costs of inflation found by Lagos and Wright (2005), or the optimality of the Friedman rule, will be robust once the government has access to different fiscal instruments to cure the bargaining inefficiencies present in these settings.

In this paper, we expand the model of Lagos and Wright (2005) to be able to analyze the interactions of fiscal and monetary policy. In particular, we introduce sales taxes and production subsidies in the decentralized market. It is well known that in models with bilateral exchange, where prices are determined in a bilateral bargaining game, output tends to be less than what would prevail in a frictionless environment, as initially suggested by Trejos and Wright (1993). The introduction of production subsidies is therefore a natural tool to fix such “underproduction”. Production subsidies have been suggested by Trejos and Wright (1993) and Kocherlakota (2005) as a potential way of solving the “underproduction” problem inherent in some models with micro foundations for the exis-

¹A noted exception is Waller (2006), which studies optimal fiscal and monetary policy in a dynamic version of Mirrlees’s model where frictions give rise to money as a medium of exchange. In Waller’s framework, since preference shocks are private information, in a record-keeping economy without money, the planner’s allocation trades off efficient risk sharing against production efficiency in the search market and average consumption when old. For a government to replicate this outcome in a monetary economy without record-keeping, distortionary taxation of money balances is needed.

tence of money.² In our framework, we also allow the government to print money in order to finance part of the production subsidy. Hence, we open the possibility of a trade-off between two distortions, inflation and consumption taxes. Finally, we solve for the Ramsey problem in which a government must maintain a balanced budget and chooses the inflation tax and sales tax rates as well as production subsidies as to maximize welfare. Under the same calibration of Lagos and Wright (2005), we find that there are multiple combinations of welfare maximizing taxes, subsidies and (sometimes strictly positive) inflation rates. More importantly, the combination of fiscal and monetary policy allows for the efficient level of production to be achieved, even when the terms of trade in pairwise trade are given by a generalized Nash bargaining setup. Our findings confirm the conjecture of Kocherlakota (2005) and Wright (2005) that fiscal and monetary policies may have important interactions, particularly in frameworks with micro foundations for the existence of fiat money, and should always be jointly considered in the design of optimal government policy.

2 The model

The basic framework of this paper is the model proposed by Lagos and Wright (2005). We consider two major departures: first, we introduce production subsidies and sales taxes in the decentralized market; and; second, we evaluate the role of a government that chooses the value of taxes, production subsidies, and the inflation tax so as to maximize welfare subject to a balanced budget constraint. We consider this as a first step towards our better understanding of the interactions between monetary and fiscal policies in economic environments with micro foundations for fiat money. This approach is also useful because it permits a direct qualitative and quantitative comparison against well established results in the literature.

2.1 The economic environment

The economy has a continuum of agents who live forever. The representative agent of this economy derives utility from consumption and disutility from labor. Each period is divided in two subperiods labeled day and night. Consumption and production take place

²Trejos and Wright (1995) propose redistribution of direct resources as well as price controls in order to reestablish efficiency.

in both, day and night. Preferences over streams of consumption and labor during the day, denoted by x and h respectively, and during the night, denoted by X and H , are represented by:

$$\sum_{t=0}^{\infty} \beta^t [u(x_t) - h_t + U(X_t) - H_t].$$

where the utility functions $u(\cdot)$ and $U(\cdot)$ are twice continuously differentiable, increasing and strictly concave. Also, we consider preferences such that $u(0) = 0$, and assume there is a \bar{q} such that $u(\bar{q}) = \frac{1}{2}\bar{q}$. The latter two are simple conditions that simplify our theoretical analysis.

All consumption goods in this economy are nonstorable. During the day agents interact, trade and produce in a decentralized market where the only feasible trade is the exchange of goods for money. Moreover, α is the probability of a meeting in the decentralized market. The day good comes in many varieties and each agent produces a good that she does not consume. Total output of the day good generated by agent i is equal to the time she devotes to its production. We further assume that double coincidence of wants is impossible, and that, given two agents, i and j , the probability that agent i consumes what agent j produces (a single coincidence) is σ . Symmetrically, the probability that j consumes what i produces is equal to σ . The probability that neither wants what the other produces is $1 - 2\sigma$.³

During the night subperiod agents trade a general good that everyone can produce and wants to consume in a centralized Walrasian market. Since goods are non-storable the day goods cannot be carried to the night sub-period. Hence, the only feasible trade during the night involves general goods and money.

Money is also available to the agents in this economy, it is perfectly divisible, and storable in any quantity $m \geq 0$. Let m be the money holdings of the agent being considered before trading in the day market, and \tilde{m} the holdings of her partner in a given match. Terms of trade are endogenously determined in a bargaining game and are given by: 1) a production level, $q_t(m, \tilde{m})$, and 2) a monetary payment, $D_t(m, \tilde{m})$, that the producer receives in exchange for her output. Producers also receive a production subsidy equal to $s(q(m, \tilde{m}))D_t(m, \tilde{m})$ units of money; where the subsidy function $s(\cdot)$ is positive, continuous, bounded below by 0 and above by 1, differentiable, and increasing in q . The buyer of

³To better understand the meaning of these probabilities consider an environment where each individual agent has N different random meetings during the day. Clearly, the number of agents, goods, and meetings (N) can be chosen such that the probability that each individual agent meets someone to trade goods for money once in any of her N meetings is σ .

the q units of output pays a total of $(1 + \tau_b)D_t(m, \tilde{m})$ units of money due to a sales tax, at rate τ_b , imposed by the government⁴. The subsidy function and the sales tax are taken as given in this bargaining problem but the reader should keep in mind that the terms of trade functions $D_t(m, \tilde{m})$ and $q(m, \tilde{m})$ will change whenever the sales tax rate or the subsidy function do.

The government must have a balanced budget at each subperiod. Revenue during the day subperiod is obtained from sales taxes and seigniorage. The only government expenditure is the production subsidy. Money printing after pairwise trades serves the purpose of paying for the portion of the subsidy not covered by tax revenues. Hence, money growth between the beginning and the end of the day, market must satisfy:

$$\frac{M_t + [s(q_t(m, \tilde{m})) - \tau_b] D(m, \tilde{m})}{M_t} = 1 + \tau_{m_1}. \quad (1)$$

We denote by $F(\tilde{m})$ the measure of agents entering the day market with money holdings between 0 and \tilde{m} , and by $W(m)$ the value function for an agent that enters the night, centralized, market with m units of money. Hence, the value function of the representative agent at the beginning of the day market is given by:

$$\begin{aligned} V_t(m) = & \alpha\sigma \int \{u[q(m, \tilde{m})] + W_t[m - (1 + \tau_b)D(m, \tilde{m})]\} dF_t(\tilde{m}) + \\ & + \alpha\sigma \int \{-q(\tilde{m}, m) + W_t[m + (1 + s(q(\tilde{m}, m)))D(\tilde{m}, m)]\} dF_t(\tilde{m}) + \\ & + (1 - 2\alpha\sigma)W_t(m). \end{aligned} \quad (2)$$

In turn, the problem of the representative agent when she enters the night centralized market can be summarized by:

$$\begin{aligned} W_t(m_t) = & \max_{X, H, m_{t+1}} \{U(X) - H + \beta V_{t+1}(m_{t+1} + T)\} \\ & s.t. X = H + \phi_t(m_t - m_{t+1}); \end{aligned} \quad (3)$$

where ϕ_t is the value of money balances at the centralized market. Finally, notice that the monetary authority can provide lump-sum monetary transfers, denoted by $T = \tau_{m_2}M^s$,

⁴In order to implement the subsidy we can think of sellers receiving a receipt after trade has occurred. The sellers would take these receipts, before they get to the centralized market, and the government would transfer money to them accordingly.

after all trades in the centralized market of the night subperiod have concluded.

2.2 Analysis of the model

The purpose of this section is to characterize and derive the main equilibrium properties of this model. The first important property of the model is that the value function of the centralized market during the night subperiod is linear in m , with slope ϕ . This can be easily shown by solving for H in the constraint of equation (3) and substituting its value into the objective function. In particular, one obtains the following expression:

$$W_t(m_t) = \max_{X, m_{t+1}} \{U(X) - X + \phi_t(m_t - m_{t+1}) + \beta V_{t+1}(m_{t+1} + T)\}. \quad (4)$$

As in Lagos and Wright (2005), the linearity of the value function associated to trading in the centralized market keeps the model tractable. In particular, it implies that all agents choose m_{t+1} independently of the money balances, m_t , with which they entered into the market.

A full characterization of the representative agent's problem can now be given. The state space of this optimization problem is given by current money holdings m_t , by a subsidy function $s(\cdot)$, and a constant sales tax rate τ_b . The representative agent also takes as given a sequence of distribution functions for money holdings across different agents, $\{F(\tilde{m})\}$, a sequence of lump sum monetary transfers given by the money authority, $\{T\}$, and the sequence of prices in the centralized market $\{1/\phi_t\}$. Given linearity, and after substituting equation (2) in the value function defined by equation (4), we can rewrite the problem of the representative agent as follows:

$$\begin{aligned} W_t(m_t) = & \max_{m_{t+1}, X} \{U(X) - X + (\beta\phi_{t+1} - \phi_t)m_{t+1}\} \\ & + \alpha\sigma\beta \int \{u(q(m_{t+1}+T, \tilde{m})) - \phi_{t+1}D(m_{t+1}+T, \tilde{m})(1 + \tau_b)\} dF_{t+1}(\tilde{m}) + \\ & + \alpha\sigma\beta \int \{-q(\tilde{m}, m_{t+1}+T) + \phi_{t+1}D(\tilde{m}, m_{t+1}+T)(1 + s(q(\tilde{m}, m_{t+1}+T)))\} dF_{t+1}(\tilde{m}) \end{aligned} \quad (5)$$

To keep the analysis tractable, we just consider bargaining settings where the terms of trade depend only on real money balances (and not on actual money holdings), and to stationary equilibria. In a stationary equilibrium prices grow at a constant rate, π , and the distribution of real money balances is constant over time. Moreover, taxes, inflation

and subsidy rates yield allocations that are consistent with a balanced government budget. A formal definition of stationary equilibria is given by:

Definition 1 *A stationary monetary equilibrium is a distribution function F , a subsidy function $s^{eq}(q)$, a sales tax rate τ_b^{eq} , a sequence of money and prices that grow at a time invariant rate $\pi = (1 + \tau_{m_1})(1 + \tau_{m_2})$, a positive consumption level in the Walrasian market, $X^{eq} > 0$, and a positive production level and payment in the decentralized market, q^{eq} and D^{eq} , that solve (5) taking the subsidy function and the sales tax rate as given. Moreover, the government balanced budget constraint is satisfied, and the money market clears, namely*

$$[s^{eq}(q^{eq}) - \tau_b^{eq}] D^{eq} = \max\{0, \tau_{m_1}^{eq} M^s\} \quad (6)$$

$$\frac{M_t^s}{(1 + \pi)^t} = \frac{1}{(1 + \pi)^t} \int m dF \left(\frac{m}{(1 + \pi)^t} \right).$$

The government constraint (6) implies that negative inflation rates are implemented via lump-sum transfers, as is typical in the literature. The implicit subsidy of a negative inflation rate need not be financed with sales tax revenues.

Following the literature we now assume that in single coincidence meetings the terms of trade are given by the generalized Nash solution in which the buyer has bargaining power $\theta > 0$ and threat points are given by the continuation value $W_t(m)$. Hence, given a set of taxes and subsidies, (q, D) are the solution to the following problem:

$$\max_{q, D} \{u(q) + W[m - D(1 + \tau_b)] - W[m]\}^\theta \{-q + W[\tilde{m} + D(1 + s(q))] - W[\tilde{m}]\}^{1-\theta} \quad (7)$$

$$s.t. \quad (1 + \tau_b) D \leq m \quad (8)$$

$$W(m) \leq u(q) + W[m - D(1 + \tau_b)] \quad (9)$$

$$W(\tilde{m}) \leq -q + W[\tilde{m} + D(1 + s(q))], \quad (10)$$

where (9) and (10) denote the participation constraints.

The linearity of $W(\cdot)$ simplifies the above bargaining problem to the following:

$$\max_{q, D} \{u(q) - \phi D(1 + \tau_b)\}^\theta \{-q + \phi D(1 + s(q))\}^{1-\theta} \quad (11)$$

$$s.t. \quad (1 + \tau_b) D \leq m, \quad (12)$$

and the participation constraints simplify to:

$$u(q) - z(1 + \tau_b) \geq 0 \quad (13)$$

$$-q + z(1 + s(q)) \geq 0. \quad (14)$$

After multiplying both sides of the constraint (12) by ϕ , it is clear that the above maximization problem depends only on real monetary balances, $z_t \stackrel{def}{=} \phi_t m_t$. Note that a tax on buyers, together with a transfer to sellers, can affect the redistribution of the match surplus.

If we let $d \stackrel{def}{=} \phi D$, the solution to the generalized Nash bargaining problem (11) is given by the following result:

Proposition 2 *Given the subsidy function $s(q)$ and a tax rate τ_b , the interior solution to the generalized Nash bargaining problem (11) is given by*

$$d(z, \tilde{z}) = \begin{cases} \frac{z}{1+\tau_b} & \text{if } \frac{z}{1+\tau_b} < z^* \\ z^* & \text{if } \frac{z}{1+\tau_b} \geq z^* \end{cases}$$

$$q(z, \tilde{z}) = \begin{cases} \hat{q} & \text{if } \frac{z}{1+\tau_b} < z^* \\ q^* & \text{if } \frac{z}{1+\tau_b} \geq z^* \end{cases}$$

where q^* and z^* are the solutions to the following nonlinear system

$$\theta \{-q^* + z^*(1 + s(q^*))\} u'(q^*) = (1 - \theta) \{u(q^*) - z^*(1 + \tau_b)\} \left(1 - z^* \frac{\partial s}{\partial q}\right) \quad (15)$$

$$\theta \{-q^* + z^*(1 + s(q^*))\} (1 + \tau_b) = (1 - \theta) \{u(q^*) - z^*(1 + \tau_b)\} (1 + s(q^*)), \quad (16)$$

and \hat{q} is given by the solution to the nonlinear equation

$$\theta \left\{ -\hat{q} + \frac{z}{1 + \tau_b} (1 + s(\hat{q})) \right\} u'(\hat{q}) = (1 - \theta) \{u(q) - z\} \left(1 - \frac{z}{1 + \tau_b} \frac{\partial s}{\partial q}\right). \quad (17)$$

Moreover, the solutions to the bargaining problem are bounded.

Proof. Observe that (15) and (16) are the first order conditions of (11), ignoring the constraint $\frac{z}{1+\tau_b} \geq d$. Thus, z^* , q^* are the solutions to (11) when $\frac{z}{1+\tau_b} \geq z^*$. Moreover, if $\frac{z}{1+\tau_b} < z^*$ the constraint $\frac{z}{1+\tau_b} \geq d$ binds. In such a case $d = \frac{z}{1+\tau_b}$, while q is given by the first order condition of (11) with respect to q , i.e. (17). It follows that functions d and q

yield the interior solutions to (11). To prove that functions d and q are bounded it is useful to consider the participation constraints of the bargaining game. Given a current value for the real money holdings of any given agent, each participant of the bargaining problem (11) has a reservation utility of $z = \phi m$. Hence, the following participation inequalities must be satisfied by any incentive feasible solution to the bargaining game:

$$u(q) - z(1 + \tau_b) \geq 0 \quad (18)$$

$$-q + z(1 + s(q)) \geq 0. \quad (19)$$

These inequalities imply that $z = 0$ iff $q = 0$. The previous two incentive compatibility constraints reduce to $u(q) \geq \frac{1+\tau_b}{1+s(q)}q$, which paired with the strict concavity of u imply that $u(q)$ is strictly less than $\frac{1}{2}q$ and strictly decreasing for all $q > \bar{q}$. Given that $s(q)$ is bounded by 1 and that $0 \leq \tau_b \leq 1$ it follows that $u(q) < \frac{1}{2}q \leq \frac{1+\tau_b}{1+s(q)}q$ for all $q > \bar{q}$, which violates the incentive compatibility constraint $u(q) \geq \frac{1+\tau_b}{1+s(q)}q$. Given that q is bounded by \bar{q} then $\bar{z} = \frac{u(\bar{q})}{1+\tau_b}$ bounds the values of z that are compatible with (18). ■

A key property of the solution of this bargaining game is that the terms of trade do not depend on the real money holdings of the seller. As in Lagos and Wright (2005), this property comes from the fact that quasi-linearity eliminates wealth effects on the demand for money and, as our following result illustrates, transforms the problem of the representative consumer into a much simpler one.

Corollary 3 *Under the conditions of Proposition 2, the problem of the representative consumer can be written as follows:*

$$W_t(m_t) = \max_{z', X} \left\{ \frac{U(X) - X}{\beta} + \left(1 - \frac{(1 + \pi)}{\beta}\right)z' \right\} + \alpha\sigma \{u(q(z')) - d(z')(1 + \tau_b)\}. \quad (20)$$

The problem of the representative agent can be written as above, where real money holdings are chosen and q is determined in the bargaining game. Proposition 4 below imposes an additional interiority assumption, which makes it possible to think of the problem of the representative agent as that of choosing q . This characterization of the representative agent's problem will be useful later on in establishing existence of equilibrium with a degenerate distribution of money holdings.

Proposition 4 *Consider any given subsidy function $s(q)$, sales tax rate τ_b , and a sequence of prices that grows at rate $(1 + \pi)$. If the solutions to the bargaining problem are interior,*

then the constraint $\frac{z}{1+\tau_b} \geq d$ binds and an optimal solution to the production level of the representative agent,

$$q^e = \arg \max_{[0, \bar{q}]} \left(\left(1 - \frac{1+\pi}{\beta} \right) z(q) + \alpha \sigma \{u(q) - z(q)\} \right), \quad (21)$$

always exists.

Proof. It can be shown that the buyer's surplus, the term $u(q(z')) - d(z')(1 + \tau_b)$ in the representative agent's problem (20), decreases as z approaches z^* from the left; i.e., $\frac{\partial u_b}{\partial z} < 0$ for all q near q^* . This property then implies that the solution to the representative agent's problem satisfies $z' < z^*$. Thus, the constraint $\frac{z}{1+\tau_b} \geq d$ is always binding, $d(z') = \frac{z'}{1+\tau_b}$ and q is given by the solution to equation (17). Hence, the representative agent's problem is equivalent to choosing q knowing that z' must solve (17). Existence of a solution to the maximization problem (21) follows directly from the definition of $d(z)$ in an interior solution, and from the fact that the previous maximization problem involves continuous functions over a compact space. ■

Proposition 4 explicitly shows the trade-off of holding real balances and consuming. In particular, if a seller could turn the proceeds from her production into immediate consumption, as in a static or frictionless model, then she would produce until marginal utility equals marginal cost, which in our model is given by the condition $u'(q) = 1$. In a monetary exchange economy, however, the proceeds from production consist of cash that can only be spent in the future. Since she discounts the future, a seller is willing to produce less than the amount that satisfies $u'(q) = 1$ as is observed in the standard Lagos and Wright (2005). The fact that in this paper we consider fiscal policies that may redistribute resources between buyers and sellers may make the efficient allocation possible.

The main result of this section establishes existence for a monetary equilibrium for a commonly used class of utility functions, and the subsidy function $s(q) = A \left(1 - \frac{1}{1+q} \right)$. We have chosen this subsidy function because it makes it easy to verify the conditions of our theorems. Intuitively, all that matters is that the subsidy function increases at a decreasing rate, to avoid breaking the concavity of the bargaining problem. As our quantitative experiments illustrate later on, the main conditions of the theorem are easy to verify in numerical experiments.

Theorem 5 Let $u(q) = \frac{(q+b)^{1-\eta} - b^{1-\eta}}{1-\eta}$ with η and b strictly positive, $s(q) = A \left(1 - \frac{1}{1+q} \right)$, $1 -$

$\frac{1+\pi}{\beta} \leq 0$, and $0 \leq s(q) \leq 1$ for all q . Also, assume that $\alpha\sigma u(q) - (\alpha\sigma + \frac{1+\pi}{\beta} - 1)z(q)$ is strictly concave and that the solutions to the bargaining problem are interior. Then, for any small enough $b \geq 0$ and any given τ_{m_2} , τ_b and A there exists a money growth rate τ_{m_1} that yields a stationary monetary equilibrium where the distribution of real money balances is degenerate.

Proof. Notice first that optimization problem in the centralized market is independent of the level of taxes in the decentralized market, and that, under our assumptions on $U(\cdot)$, a solution always exists. Regarding the decentralized market, and given a set of taxes and subsidies, a solution to the representative agent's problem always exists, as Proposition 4 illustrates. Moreover, our concavity assumption together with the Maximum Theorem imply that this solution must vary continuously with A , τ_b , τ_{m_2} , and τ_{m_1} . We denote this solution by function $q(A, \tau_b, \tau_{m_1}, \tau_{m_2})$. Strict concavity implies that, for a given set of taxes, all agents choose the same production and real money balances, which delivers a degenerate distribution for real money balances. Consider now the government budget constraint:

$$\begin{aligned} \tau_b \frac{z^s}{2(1+\tau_b)} + \tau_{m_1} z^s &= \frac{z^s}{2(1+\tau_b)} A \left(1 - \frac{1}{1+q}\right) \text{ if } \tau_{m_1} \geq 0 \\ \tau_b \frac{z^s}{2(1+\tau_b)} &= \frac{z^s}{2(1+\tau_b)} A \left(1 - \frac{1}{1+q}\right) \text{ otherwise} \end{aligned}$$

where z^s denotes the real value of the money supply. Also note that the above inequalities are consistent with a positive inflation rate during the day and a negative one not financed by sales taxes, respectively. As previously discussed, every agent chooses to arrive to the day market with the same stock of money. Therefore, buyers and the sellers split the total money supply in half. Our task now is to determine whether there exists a τ_{m_1} , given arbitrary values for τ_{m_2} , τ_b and A , that is consistent with stationary equilibrium. Start at the balanced budget constraint of the government above and solve for τ_{m_1} to define the following function:

$$G(\tau_{m_1}) := \max\left\{\frac{A \left(1 - \frac{1}{1+q(A, \tau_b, \tau_{m_1}, \tau_{m_2})}\right) - \tau_b}{2(1+\tau_b)}, 0\right\}.$$

Proposition 2 established that $q^e \in [0, \bar{q}]$. Thus for each τ_{m_1} the government may possibly choose there will be an optimal level of output $q(A, \tau_b, \tau_{m_1}, \tau_{m_2})$ and a continuous function

$G(\tau_{m_1}) \in \left[0, \frac{A(1-\frac{1}{1+q})-\tau_b}{2(1+\tau_b)}\right]$. It follows that if one only considers values for τ_{m_1} in the domain of G then Brouwer's fixed point theorem establishes the existence of $\tau_{m_1}^{eq} : G(\tau_{m_1}^{eq}) = \tau_{m_1}^{eq}$. Therefore, for the given values for τ_{m_2}, τ_b and A there is a money growth parameter $\tau_{m_1}^{eq}$ that satisfies the balanced budget constraint. If we let $q^{eq} = q(A, \tau_b, \tau_{m_1}^{eq}, \tau_{m_2})$ then the last detail we have to show is that $q^{eq} > 0$. To do so we solve $z(q)$ explicitly and find, after some algebra, that the limit of the objective (21) as q converges from the right to 0 converges to infinity as $b \rightarrow 0$. Then, for any small enough $b > 0$, $q^e > 0$. Hence, we have found a stationary monetary equilibrium with positive inflation rates. A symmetric argument can be easily constructed to proof existence of equilibria for negative inflation rates. In such a case the inflation rate can be taken as given, and one can establish the existence of a subsidy rate (or a tax rate) consistent with stationary equilibria. ■

To conclude this section we establish a positive relation between the production of the unconstrained bargaining problem and the subsidy rate in the decentralized market.

Proposition 6 *Let a sales tax τ_b be given and let $s(q) > 0$, $s'(q) > 0$ for all q and $0 < \theta < 1$. Then, the unconstrained solution for the level of output in the Nash bargaining problem, q^* , is larger than the one that emerges under $s(q) \equiv 0$.*

Proof. Notice that equations (15) and (16) imply

$$\frac{1}{u'(q^*)} = \frac{1 - \phi m^* s'(q^*)}{(1 + s(q^*)) (1 + \tau_b)}. \quad (22)$$

Then, if $s(q) \equiv 0$ then $q_{s=0}^*$ satisfies $u'(q_{s=0}^*) = 1 + \tau_b$. If $s(q) > 0$, $s'(q) > 0$ then the right hand side of (22) is strictly less than $(1 + \tau_b)$. Given that $u'(q)$ is decreasing in q , the solutions to (15) and (16) must have $q^* > (1 + \tau_b)$. ■

This proposition highlights that when subsidies are present it is possible to overproduce. Since it is well known that in models with bilateral exchange, where prices are determined in a bilateral bargaining game, output tends to be less than what would prevail in a frictionless environment, this proposition opens the possibility that a combination of fiscal and monetary policies could fix such “underproduction” problem. By introducing production subsidies and sales taxes, we allow for the possibility that the outcome of the Nash bargaining can directly affect the extensive margin, the quantity traded in a match, so that production efficiency is possible even when the bargaining is non monotonic. This

is in line with Rocheteau and Waller's (2005) results that highlight how alternative bargaining solutions affect the extensive margin of trading and that deviations from the Friedman rule may be optimal over a wide range of parameter values.

3 Optimal Fiscal and Monetary Policy

In this section we consider the Ramsey approach to optimal taxation by finding solutions to the problem of choosing optimal taxes and transfers given that only distortionary tax instruments are available. In other words, we consider optimal fiscal and monetary policies with commitment by choosing taxes, transfers and monetary instruments as to maximize social welfare. Implicitly we are contemplating an environment in which the government sets an inflation and fiscal plan that will not change over time. As a result, lags in fiscal policies are not considered in this paper since we abstract from the process that makes the design of fiscal policy lengthier than the monetary one. Moreover, as our analysis focuses on the comparison of different long-run stationary equilibria temporary delays in fiscal policy will have no impact on the figures we report.

Proposition 7 *The optimal (Ramsey) taxes and transfers for this economy are given by a set of vectors of the form $(\tilde{A}, \tilde{\tau}_b, \tilde{\tau}_{m_1}, \tilde{\tau}_{m_2})$ that solve the following constrained maximization problem:*

$$\max_{(q, A, \tau_b, \tau_{m_1}, \tau_{m_2})} u(q) - q \quad (23)$$

$$s.t. (1 + \tau_{m_1})(1 + \tau_{m_2}) = \beta \left[\alpha \sigma \frac{u'(q)}{z'(q)} + (1 - \alpha \sigma) \right] \quad (24)$$

$$\tau_{m_1} = \max \left\{ \frac{(s(q) - \tau_b)}{2(1 + \tau_b)}, 0 \right\} \quad (25)$$

$$(1 + \tau_{m_1})(1 + \tau_{m_2}) \geq \beta, \quad (26)$$

Proof. The objective function of the maximization problem (23) is to achieve the socially optimal level of production taking as constraints, 1) The first order condition of the representative agent's problem (24), 2) The government budget constraint (25), and 3) a necessary condition for existence of a monetary equilibrium (26). ■

The objective of the government is then to design a combination of fiscal and monetary policies such that the efficient allocation is reached. As we can see from equation (17), the

efficient allocation for this economy, q_e , satisfies $u'(q_e) = 1$. A straightforward Corollary of Theorem 5 is the following

Corollary 8 *If an efficient solution to the Ramsey problem exists then it can be achieved by a continuum of values of money growth rates, τ_{m_1}, τ_{m_2} , sales taxes, τ_b and production subsidies, A .*

A very important implication of Corollary 8 is that the Friedman rule may not be the only optimal policy rule available to this economy that can yield efficiency. In particular, observe that we allow for lump-sum monetary taxes-transfers after the night period. Thus, a feasible government policy consists of printing money during the day to pay for production subsidies and then reversing the implied growth in the money supply via lump sum money taxes after the night market up to the Friedman rule. However, such a policy is at most as good as a policy where efficiency is reached at positive inflation rates. Finally, we would like to mention that although we have abstracted from inflation smoothing across subperiods Corollary 8 suggests that such policy is not superior to other policies that achieve efficiency.

3.1 Quantitative analysis

This section illustrates some of the quantitative and qualitative properties of the solution to the Ramsey problem. In particular, we show that different operating procedures for monetary and fiscal policy are possible. That is, there are different mixes of fiscal and monetary policies that achieve the efficient allocation. Such policies may involve strictly positive inflation rates.

For ease of comparison with existing results in the literature, we parameterize the model as in Lagos and Wright (2005), Table 1, column 4. Thus, we set

$$u(q) = \frac{(q + b)^{1-\eta} - b^{1-\eta}}{1 - \eta},$$

with b arbitrarily close to zero, which yields an efficient level of output $q_e = 1$. For the centralized market the utility we consider $U(X) = B \log(X)$ so that output in the centralized market equals B . The annual rate of time preference is set at $r=0.04$. We can normalize α to 1 as only the product $\alpha\sigma$ matters in the model. Finally, we set $\sigma \approx 0.5$,

$\theta=0.343$, $\eta=0.39$ and $B=1.78$. This calibration is such that the model with taxes and subsidies set at zero provides the best fit of the model to the annual "money demand" data of the United States. In this calibration a period is interpreted as one year (over which the day and the night markets occur). Notice that the standard velocity equation

$$MV = PY,$$

where M is the money demand, P is the price level and Y is output, can be easily mapped into the variables of this model. First of all, the price level corresponds to the prices of goods in a centralized market, $\frac{1}{\phi}$. Real output Y in units of the centralized market equals $B + \sigma\phi M$, and real money balances equal $\phi z(q)$, which equal ϕM in equilibrium. Hence,

$$V = \frac{B + \sigma z(q)}{z(q)}.$$

Finally, notice that equation (21) can be used to determine q , and therefore V , as a function of the nominal interest rate $\frac{1+\pi}{\beta} - 1$. The latter can be used then to derive the money demand implied by the model. The interest rate data employed in this exercise is the annual commercial paper rate while M is measured by $M1$. The sample period considered was 1900-2000. Further details can be found in Lagos and Wright (2005).

In Table 1 below we present two different cases (out of the continuum) of fiscal and monetary policies that solve the Ramsey problem with the resulting inflation, subsidy and sales tax rates, as well as the optimal output level for the day market. These two experiments assume that money printing is only used to pay for production subsidies and reports the optimal rate of inflation, subsidies and tax rates that are compatible with an efficient level of production. Table 1 also shows, in its last row, the Lagos-Wright (2005) experiment where fiscal policy is not allowed and where the Friedman rule is the unique optimal policy.

Different Policies	π	$s(q^e), \tau_b$	q^e
High Inflation during the day	10%	34.4%, 26.5%	1
Friedman rule during the day	-3.8%	4.8%, 4.8%	1
Friedman Rule with no Fiscal Policy (Lagos-Wright benchmark)	-3.8%	0%, 0%	0.6

Table 1: Different taxes, subsidies and inflation rates achieve the efficient allocation.

Monetary policy alone cannot.

As we can see from Table 1, if we allow fiscal policy to be active, the Friedman rule is one of the possible policy options that the government has to attain efficiency. As pointed out by Berentsen and Rocheteau (2003) the different reservation values in out-of-equilibrium matches are the reason why in the Lagos and Wright (2005) economy is not able to achieve efficiency under the Friedman rule since a buyer who brings an additional unit of money into a match cannot extract the whole surplus that this unit provides to the match, lowering the marginal value of money. In our set up once sales taxes and production subsidies are introduced this is not the case anymore.

Finally, the output costs of abstracting from optimal fiscal policy are significant, output during the pairwise trade period falls by 40% relative to first best, which can be attained with a combination of monetary and fiscal policies.

As in Bhattacharya, Haslag, and Martin (2005) our paper emphasizes the basic intertemporal inefficiency underlying the Friedman rule which ignores distributional effects. As opposed to Bhattacharya, Haslag, and Martin's work the source of our heterogeneity is ex post due to asymmetric fiscal policies between buyers and sellers. Through taxes and subsidies redistributions are possible. To illustrate this point better let's suppose a society is inflating faster than the Friedman rule. If it lowers the growth rate of the money supply, there are two effects. All agents are made better off because the monetary inefficiency is reduced. However, there is a transfer of wealth from agents who hold little money to those who hold more. This effect may make those with little money worse off. If we allow transfers, then society can undo the latter effect and therefore make all agents better off. Without transfers, society cannot undo the latter effect, and the Friedman rule is not necessarily Pareto optimal. In our current framework, we exploit the same mechanism but use fiscal and monetary policies to implement those transfers while reducing the intertemporal inefficiency.

3.2 The Welfare costs of inflation

Assessing the welfare costs of inflation requires a sound understanding of the benefits of monetary exchange. Recently, Craig and Rocheteau (2006) have shown that the estimates for the welfare cost of inflation provided by the basic version of the search model of Lagos and Wright coincide with those provided by the Bailey method whenever money holders

can appropriate the marginal social return of their real balances. This condition is satisfied when buyers have all the bargaining power to set prices in bilateral trades, or when pricing is competitive. If this condition does not hold, then the welfare cost of inflation is larger than what traditional estimates predict. This discrepancy arises because of a rent-sharing externality associated with noncompetitive pricing mechanisms. In Craig and Rocheteau's (2006) fiscal policy analysis was ignored. Here, we try to fill this gap.

Within the same spirit, we now recast the classical analysis of the welfare costs of inflation in a setting where the existence and need for money is based on micro foundations and fiscal policies are considered. Following Lucas (2000), we report in Table 2 the percentage of consumption that an agent living in an economy where the optimal fiscal and monetary policy is followed will be willing to give up in order to avoid going to an economy where the inflation rate varies as in the different rows of Table 2, while fiscal policies are held fixed. Given that a multiplicity of solutions to the Ramsey problem exist, we consider the welfare costs of inflation departing from three different optimal policies (columns two through four in Table 2). To compare with the results of the previous literature, we also reproduce the welfare costs of inflation in the Lagos and Wright (2005) case, where fiscal policy is not considered.

π	Welf. loss From $\pi=7\%$ $A = 0.55, \tau_b = 0.22$	Welf. loss From $\pi=3\%$ $A = 0.37, \tau_b = 0.16$	Welf. loss From $\pi=-3.8\%$ $A = 0.1, \tau_b = 0.05$	Welf. loss From $\pi=-3.8\%$ No fiscal P.
-2%	-	-	0.5%	1.2%
0%	-	-	1.6%	2.5%
4%	-	0.1%	3.9%	4.6%
6%	-	0.9%	4.9%	5.5%
8%	0.1%	2.0%	5.8%	6.2%
10%	0.7%	3.1%	6.5%	6.8%
12%	1.7%	4.2%	7.2%	7.3%
14%	2.7%	5.2%	7.7%	7.8%

Table 2: The Welfare costs of inflation, in terms of percentage consumption, departing from different base line fiscal and monetary policies.

Table 2 shows that the welfare costs of inflation of our benchmark economy, where the optimal policies are able to achieve efficiency, can be quite large. These results are in

line with Rocheteau and Waller (2005) that show that monotonic bargaining solutions are associated with lower welfare costs of inflation near the Friedman rule than non-monotonic bargaining solutions. However, the costs are very similar for inflation rates sufficiently far away from it.

The results of Table 2 can also imply that inflation costs may be high but only when fiscal policy is not simultaneously adjusted. In particular, inflation in the first column of Table 2 is more than ten percentage points higher than the Friedman rule and yet efficiency is attained. Thus, if an economy switched from a -3.8% inflation rate to 7% inflation and fiscal policy was also adjusted inflation costs would be exactly zero.

In summary, our findings suggest that ignoring active fiscal policies can be quite costly. Thus knowing the empirical “money demand” curve is not enough: one really needs to understand the micro foundations, and especially how the terms of trade are determined and affected by policy actions, in order to correctly estimate the welfare cost of inflation. Moreover, note that as we increase the inflation rate we keep moving away from efficiency, thus welfare costs tend to increase quite dramatically after a certain inflation threshold is reached. This finding is in line with the empirical findings of Bullard and Keating (1995) of the nonlinear relationship between inflation and output.

3.3 Costly Fiscal Implementation

One fundamental task of the Economic science consists in determining the fiscal and monetary policies that maximize welfare given a set of government constraints. Sometimes the optimal policy is costly to implement. Then, it becomes crucial to measure the welfare costs associated to deviations from such optimal policy. If the costs of implementing the optimal policy are larger than the welfare losses associated to the use of a simple policy rule then the government should choose the simple policy. In this paper we ask ourselves the question if fiscal policy, instead of monetary policy or along side, should be used to correct the inefficiency in the extensive margin found in the Lagos and Wright model. In the previous section we have shown that fiscal and monetary policies can induce efficient outcomes. But in order to correctly answer to this question one has to consider how well fiscal policy can be implemented.

In the previous section we have implicitly assumed that implementation of fiscal policy is costless. But taxes are likely to be costly to collect, because the exchange occurs in separated matches, thus requiring record-keeping. One way to model the cost of imple-

menting fiscal policy is to assume that the fiscal authority must spend a given amount of monetary resources for record-keeping. We explicitly model this cost assuming to be proportional to the money holdings agent have. This implementation cost can be viewed as resulting from a positive and constant marginal monetary cost equal to g . Then the corresponding government budget constraint is then given by:

$$\tau_{m_1} = \frac{s + g - \tau_b}{2(1 + \tau_b)} \quad (27)$$

Implementation of fiscal policy in this framework requires transfers of fiat money rather than goods. Hence, lags in fiscal policy are not present in this paper.

Our following experiment determines the maximum cost that makes the agents indifferent between having only monetary policy and having active fiscal and monetary policy. In particular, we consider different values for the subsidy parameter A and the maximum costs of implementation, g^{\max} , such that the optimal tax rate is below confiscation levels (say 90%) and such that, in equilibrium, the level of output in the decentralized market is equal to an economy with no fiscal policy that follows the Friedman rule. For the examples we report in Section 3.2, we find that $g^{\max} = 1$. Table 3 reports the different levels of inflation and subsidy rates that are consistent with this new equilibrium.

	$s = 20.6\%, A = 0.55$	$s = 13.9\%, A = 0.37$	$s = 3.7\%, A = 0.1$
π	33%	32%	29%

Table 3: Optimal taxes, inflation and subsidy rates for g^{\max}

As we can see from Table 3, since the per-unit implementation cost that yields the output of the Friedman rule without fiscal policy is 100%, the costs of ignoring fiscal policy even when it is costly to implement are huge. In other words, for any g lower than 100% having fiscal policy results in higher welfare than just considering the Friedman rule.

4 Conclusions

The double coincidence problem is at the heart of modern monetary theory and, since bilateral meetings and random matching provide a natural framework to study double coincidence problems, random matching models have become widespread in the literature exploring the microeconomic foundations of money. Price determination is no longer

Walrasian in such type of models. Moreover, the nature of the pricing mechanism has been found to have important implications for the design of monetary policy, and for welfare analysis. However, relatively little is known about the properties and interactions of optimal fiscal and monetary policy in random matching models.

In this paper we have delved into the foundations of optimal monetary and fiscal policy in a framework where the frictions that make money valuable are explicitly modeled. In particular, we followed the Ramsey approach and derived the optimal sales tax rate, production subsidy, and monetary injections for a standard search theoretic model of monetary exchange. Our analysis indicates that the Friedman rule is not a necessary future of optimal policy in monetary economies. The economic intuition behind this result works as follows: The basic inefficiency associated to the existence of money is that an agent that holds money is making a sacrifice today in order to obtain a future benefit. A further inefficiency is typically added in models with microfoundations for the existence of money as different pricing mechanisms in bilateral trades give rise to asymmetric splits of the surplus of the match. These two inefficiencies translate into production and money holdings that are typically lower than efficiency considerations would dictate. However, when the government has access to monetary and fiscal instruments it can undo the inefficiencies inherent to monetary exchanges directly. A combination of fiscal and monetary tools can restore economic efficiency even with positive inflation. An important corollary of this result is that the high costs of inflation found in models with microfoundations for the existence of money vanish once the government is capable of responding with different fiscal tools.

In a second stage of our analysis we explored the implications of costly fiscal policy implementation. Since bilateral meetings and monetary exchanges in random matching models take place in decentralized markets, monetary and fiscal policies may be costly to implement. Some type of monitoring or record-keeping may be necessary to achieve the optimal policy rules. As a first approximation to the study of the costly implementation problem we assumed that implementation costs can be paid in cash. We found that fiscal policy may be desirable even when the per-unit monetary costs of implementation are as high as the taxes or subsidies to be implemented.

Our findings confirm the conjecture of Kocherlakota (2005) and Wright (2005) that fiscal and monetary policies have important interactions, particularly in frameworks with micro foundations for the existence of fiat money, thus they should always be jointly considered in the design of optimal government policy.

References

- [1] Berentsen, A., and G. Rocheteau (2003): "On the Friedman Rule in Search Models with Divisible Money," *Contributions to Macroeconomics*, Vol. 3: No. 1, Article 11.
- [2] Bhattacharya, J., J. Haslag, and A. Martin. (2005): "Heterogeneity, Redistribution, and the Friedman Rule," *International Economic Review*, Vol. 46:437-54.
- [3] Bullard, J., and J. Keating. (1995): "The Long-run Relationship Between Inflation and Output in Postwar Economies," *Journal of Monetary Economics*, 36:477-496.
- [4] da Costa, C. and I. Werning (2003): "On the Optimality of the Friedman rule with Heterogeneous agents and Non-linear Income Taxation," Manuscript, M.I.T.
- [5] Craig, B., and G. Rocheteau (2006): "Inflation and Welfare: A Search Approach," *Policy Discussion Paper*, no. 12. Federal Reserve Bank of Cleveland.
- [6] Diamond, P. and J. Mirrlees (1971): "Optimal Taxation and Public Production I: Production Efficiency," *American Economic Review*, Vol. 61: 8-27.
- [7] Kalai, E. (1977): "Proportional solutions to bargaining situations: Interpersonal utility comparisons," *Econometrica*, Vol. 45:1623-1630.
- [8] Kalai, E., and M. Smorodinsky (1975): "Other solutions to Nash's bargaining problem," *Econometrica*, 43:513-518.
- [9] Kiyotaki, N. and R. Wright (1991): "A Contribution to the Pure Theory of Money," *Journal of Economic Theory*, Vol. 53: 215-235.
- [10] Kocherlakota, N. (2005): "Optimal Monetary Policy: What we Know and What we don't Know," *International Economic Review*, Vol. 46(2):305-16.
- [11] Lagos, R., and R. Wright (2005): "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, Vol. 113(31):463-484.
- [12] Lucas, R. (2000): "Inflation and Welfare," *Econometrica*, Vol. 68:247-274.
- [13] Nash, J. (1950): "The bargaining problem," *Econometrica*, Vol. 18:155-162.
- [14] Rocheteau, G. ,and C. Waller (2005): "Bargaining and the Value of Money," *Working Paper FRB Cleveland*.

- [15] Schmitt-Grohe, S. and M. Uribe (2004): "Optimal Fiscal and Monetary Policy Under Sticky Prices," *Journal of Economic Theory*, Vol. 114: 183-209.
- [16] Trejos, A. and R. Wright (1993): "Search, Bargaining, Money and Prices: Recent Results and Policy Implications," *Journal of Money Credit and Banking*, Vol. 25(3):558-76.
- [17] Wright, R. (2005): "Introduction to 'models of monetary economies II: The next Generation,'" *International Economic Review*, Vol. 46(2):715-729.
- [18] Waller, C. (2006): "Dynamic taxation, private information and money search," *manuscript*.