

Optimal Monetary and Fiscal Policies in a Search Theoretic Model of Monetary Exchange*

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Abstract

We study optimal monetary and fiscal policies, and the welfare costs of inflation, within the framework of Lagos and Wright (2005). We show how proportional production subsidies and sales taxes associated to trades in decentralized markets can be implemented to alleviate underproduction due to bargaining inefficiencies, while money is still essential. When the government has zero expenditures and negative lump sum money transfers are available, we find multiple combinations of taxes, subsidies and (sometimes strictly positive) nominal interest rates that restore the efficiency of monetary equilibrium allocations. When negative lump sum transfers are not available, the optimality of Friedman rule crucially depends on the bargaining power of the buyer, and equilibria are not first best. With positive government expenditures, the optimal policy must trade-off inflation and production tax distortions. In all these cases, deviations from the Friedman rule may be large, and there are considerable welfare gains of having fiscal and monetary policies in place. Holding fiscal policies constant, the welfare costs of increasing inflation may be as high as 8% of lifetime consumption.

JEL Codes: C70, E40.

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1 Introduction

Since Kiyotaki and Wright (1991, 1993), search theoretic models have been frequently used to formalize the role of money as a medium of exchange, thus providing microeconomic foundations

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for monetary economics. The basic building blocks of this class of models are trading risk and trade in decentralized markets where buyers and sellers are anonymous, no record keeping is possible, and terms of trade are determined by bilateral bargaining. In spite of the wide spread use of search theoretic models of monetary exchange, not much is known about how trading risk and bargaining pricing systems affect the nature of optimal fiscal and monetary policies.

The goal of this paper is to contribute to a better understanding of the properties of optimal fiscal and monetary policy, and of the welfare costs of inflation in the Lagos and Wright (2005) framework. The key features of this model are quasilinear preferences and the possibility of trade in both, a decentralized market, and a centralized market where agents rebalance their money holdings. These features keep the model analytically tractable and easy to quantify. In this framework agents are essentially insured against trading risk, thus, our analysis concentrates on the effects of bargaining on the properties of optimal fiscal and monetary policies.

Lagos and Wright (2005) study optimal monetary policy when lump sum monetary transfers are the only instrument. Their main findings are, first, that equilibrium is not efficient, second, that the welfare costs of inflation are substantially larger than what is found in models where money is introduced with *ad hoc* assumptions, and third, that the Friedman rule is the unique optimal policy. We extend their analysis by considering alternative fiscal instruments. The first question we consider is whether fiscal and monetary policy can restore the efficiency of equilibria. For it, we set government expenditures to zero, and solve for the inflation rate, production tax in the centralized market, as well as the sales tax and production subsidy in the decentralized market that maximize steady state welfare. Two frictions hinder the efficiency of equilibrium in this framework. First, in monetary exchanges agents pay a cost today (production) to receive a future benefit (money that can be used to purchase goods in future trades).¹ The second friction is a direct consequence of the properties of Nash's solution to the bargaining problem.² In particular, unless the buyer has all bargaining power, Nash's solution implies that the buyer's surplus from a given match is not monotone in monetary holdings. Thus, buyers hold too little cash, and there is underproduction in the decentralized market. When lump sum monetary transfers are the only instrument, the Friedman rule eliminates the first friction, and attenuates the impact of the second. However, equilibrium allocations are not efficient.

Our analysis emphasizes the fact that some of the frictions that make money essential, namely, the anonymity of trading partners and the impossibility of record keeping, rule out certain tax instruments. Implementation of fiscal and monetary policy in search models must be consistent with anonymous trades and no record keeping. In our analysis, we require agents to disclose any changes in their money holdings after trading in the decentralized market before being allowed to participate in the centralized market. Agents that lower their money holdings (buyers) are

¹Note that this feature is also present in applied models with *ad hoc* assumptions regarding fiat money.

²For more details see Aruoba, Rocheteau and Waller (2007).

charged a monetary sales tax. Agents that increase their money holdings (producers) are given a monetary subsidy. We allow money printing in order to finance part of the production subsidy, which opens the possibility of a trade-off between two distortions, inflation and sales taxes.

With negative lump sum money transfers available at the centralized market, and no government expenditures to be financed, we find multiple combinations of taxes, subsidies and (sometimes strictly positive) nominal interest rates such that monetary equilibrium allocations are efficient. We then evaluate the welfare costs of inflation. To do so we consider an economy where optimal fiscal and monetary policy is in place, hold fiscal instruments constant, and vary inflation rates via lump sum monetary injections. Since equilibrium under the optimal policy is efficient, the welfare costs of increasing inflation from the Friedman rule rate to 10% are up to 8% of lifetime consumption.

The previous results depend on the availability of costless negative lump sum monetary transfers. Production subsidies, paid in money, can be used to increase production in the decentralized market but they may be inflationary. If costless lump sum negative monetary transfers are available, they can be used to extract the money introduced by the subsidy and thus inflation can be easily contained. Hence, we also evaluate alternative procedures for monetary policy other than costless negative lump sum transfers. Even when no negative lump-sum transfers are available, our instruments make the Friedman rule feasible, by setting sales taxes higher than production subsidies. With zero government expenditures, we find that the optimality of Friedman rule crucially depends on the bargaining power of the buyer. In particular, the Friedman rule is optimal whenever the buyer has little bargaining power. Without costless negative lump-sum transfers, we find that equilibrium allocations are not efficient, nevertheless, the welfare gains of having fiscal and monetary policies in place are substantial.

Finally, we evaluate the case where a positive stream of government expenditures has to be financed. We find that the government must trade-off inflation and production tax distortions, and the Friedman rule is not necessarily optimal. The latter result is standard in models where money is introduced using ad hoc assumptions and agents hold a positive amount of nominal assets, which is the case in our framework since agents hold money. However, optimal inflation rates in our model are considerably larger than what is typically found in the literature.

The closest paper in spirit to ours is Aruoba and Chugh (2006), who study the dynamic Ramsey problem in the Lagos and Wright (2005) framework. These authors are, however, interested in analyzing the business cycle frequency properties of optimal monetary and fiscal policy with positive government expenditures in a model that includes government bonds and capital assets. Fiscal and monetary instruments are restricted to production and capital taxes in the centralized market, and to open market operations. Equilibrium in their model is not efficient, the Friedman rule is typically not optimal and inflation is stable over time. The authors also find that because

capital is under accumulated, the optimal policy includes a subsidy on capital income. In a different environment where buyers and sellers meet at random, where the total number of trade matches is determined by a matching function and where the search intensities are optimally chosen by the households, Ritter (2007) finds that an optimal policy may consist of both a positive tax rate and a positive nominal interest rate. Monetary, but not fiscal, policy alters the agent's bargaining position, leaving a special role for a deviation from the Friedman rule.

As is commonly assumed in search models of monetary exchange, terms of trade in our framework are determined by Nash's bargaining. A recent literature explores alternative allocation procedures for decentralized trades and finds that the resulting equilibrium properties may be qualitatively and quantitatively different in terms of efficiency of monetary equilibria, and for the welfare costs of inflation. Aruoba, Rochetau, and Waller (2007) show that the buyer's surplus may not be monotone under Nash's solution, and that this lack of monotonicity is behind the inefficiency of equilibrium in the Lagos and Wright economy under the Friedman rule. Moreover, these authors show that if terms of trade are instead given by any bargaining solution where the buyer's surplus is strongly monotonic (e.g. the egalitarian rule), and the Friedman rule is followed, then equilibria will be efficient. Within the same spirit, Hu, Kennan, and Wallace (2007) take the mechanism design approach to study the resource allocation problem of two agents that are matched in the decentralized market in a Lagos and Wright (2005) setting. Under the additional assumption that preferences in the centralized market are linear, these authors show that the first best allocation is strongly implementable when people are sufficiently patient. Furthermore, if agents are free to skip the centralized market then negative lump sum taxation to implement the Friedman rule does not help, in the sense that it does not enlarge the set of weakly implementable allocations. Both of these results illustrate that Nash bargaining is a particular solution to the bargaining problem with properties that are far from innocuous in terms of efficiency of equilibria. Our results illustrate, nevertheless, that efficiency of equilibria may be restored if the government has access to certain fiscal and monetary tools, even under Nash bargaining. Finally, we will argue below that, under non-linear preferences for the centralized market good, certain straightforward conditions can be imposed such that agents voluntarily choose to enter the centralized market, where our fiscal and monetary tools are implemented.

The findings of this paper as well as Aruoba and Chugh (2006) and Ritter (2007) confirm Kocherlakota's (2005) and Wright's (2005) observation that fiscal and monetary policies may have important interactions, particularly in frameworks with micro foundations for the existence of fiat money, and should always be jointly considered in the design of optimal government policy.

The rest of the paper is organized as follows. The model is presented in Section 2. Section 3 contains the main results of the paper and derives the properties of optimal fiscal and monetary policy under different government instruments and expenditures. Each subsection includes a set

of numerical experiments deriving the quantitative implications of the theory, the welfare benefits of optimal fiscal and monetary policy, and the welfare costs of inflation. Section 4 concludes.

2 The model

2.1 The economic environment

The economy has a continuum of agents who live forever. The representative agent of this economy derives utility from consumption and disutility from labor. Each period is divided into two subperiods labeled day and night. Consumption and production take place in both, day and night. Preferences over streams of consumption and labor during the day, denoted by x and h respectively, and during the night, denoted by X and H , are represented by:

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(x_t) - h_t + U(X_t) - H_t]; \quad (1)$$

where the utility functions $u(\cdot)$ and $U(\cdot)$ are twice continuously differentiable, increasing and strictly concave. Finally, we assume that $u(0)=0$ and that all consumption goods are nonstorable. Money is also available to agents in this economy, it is perfectly divisible, and storable in any quantity $m \geq 0$.

The day good comes in many varieties and each agent produces a good that she does not consume. Since no record keeping is possible in the decentralized market, the only feasible trade during the day is the exchange of goods for money. Money is essential in the sense that it makes possible for this economy to reach a higher welfare level than it would be possible without money. In other words, without money in an environment with anonymous trade and no record keeping it would be impossible for trade to take place in the decentralized market.

The probability of a meeting in the decentralized market is α . To simplify our analysis we assume that double coincidence of wants is impossible, and that, given two agents, i and j , the probability that agent i consumes what agent j produces (a single coincidence) is $\sigma \leq \frac{1}{2}$. Symmetrically, the probability that j consumes what i produces is equal to σ . The probability that neither wants what the other produces is $1 - 2\sigma$.

During the night subperiod agents trade a general good that everyone can produce and wants to consume in a centralized Walrasian market. Hence, the only feasible trade during the night involves general goods and money.

The economy we have described up to now is exactly the same as that in Lagos and Wright (2005). Our first departure is to consider a government whose objective is to maximize ex-ante welfare subject to a balanced budget constraint at every period. The government is not capable

of monitoring transactions in the decentralized market but can monitor the change in money holdings between last period's central market (after all trades took place) and the start of the current period's central market (before trades take place). If an agent lowers its money holdings by a D amount (she was a buyer in the decentralized market), then the government collects an additional $\tau_b D$ units of money from the agent. A monetary subsidy at constant rate s is also provided to agents that have increased their real money holdings (when they were producers in the day market).³ It is assumed that $\tau_b \geq 0$ and $s \geq 0$. Note that we can interpret the sales tax as a fee that buyers have to pay in order to enter the centralized market. Moreover, the government may have to print money in order to pay for monetary subsidies in excess of sales tax collection. The trades, timing and redistributions of resources considered in this model are summarized in Figure 1.

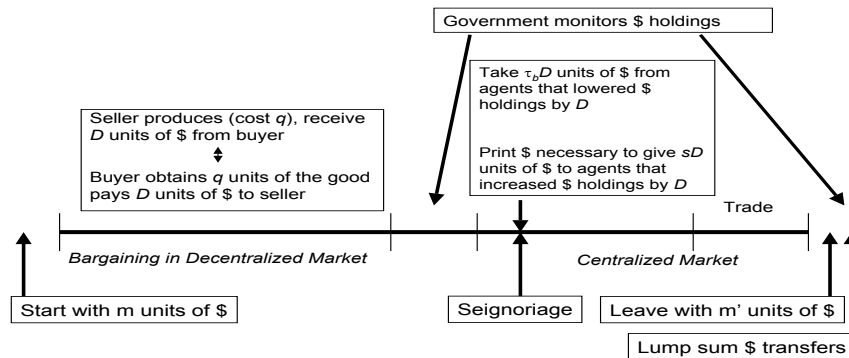


Figure 1: Timing.

The problem of the representative agent consists of maximizing expected utility taking prices, taxes, and the distribution of money holdings of other agents as given. During the day market the agent knows that with probability α that she will trade, and that with probability σ she will be either a buyer or a seller. Consumption (production), q , and money payments (receipts), D , in the decentralized market will be determined by Nash bargaining. The representative agent knows the corresponding functional forms. Hence, if we denote by m the money holdings of the representative agent, by \tilde{m} the holdings of a partner in a given match, and by $F(\tilde{m})$ the distribution of money holdings, then the recursive formulation of the representative agent is characterized, first, by the

³All the redistribution of resources take place in the Walrasian market where we assume there is perfect information, all actions are observable and that the government can collect and redistribute taxes.

value function associated to the day market given by:

$$\begin{aligned}
V_t(m) = & \alpha\sigma \int \{u[q(m, \tilde{m})] + W_t[m - (1 + \tau_b) D(m, \tilde{m})]\} dF_t(\tilde{m}) + \\
& + \alpha\sigma \int \{-q(\tilde{m}, m) + W_t[m + (1 + s)D(\tilde{m}, m)]\} dF_t(\tilde{m}) + \\
& + (1 - 2\alpha\sigma)W_t(m);
\end{aligned} \tag{2}$$

where $W(m)$ represents the value function of an agent that enters the centralized market after monetary taxes and subsidies from the government have been implemented.

Terms of trade in the decentralized market are endogenously determined in a bargaining game. Following Lagos and Wright (2005), we consider the generalized Nash bargaining solution where the buyer has bargaining power $0 \leq \theta \leq 1$, and threat points are given by no trade. Namely,

$$\begin{aligned}
\max_{q,D} \{ & u(q) + W[m - (1 + \tau_b) D] - W[m]\}^\theta \{-q + W[\tilde{m} + (1 + s) D] - W[\tilde{m}]\}^{1-\theta} \\
& s.t. (1 + \tau_b) D \leq m.
\end{aligned} \tag{3}$$

As is well understood, the generalized Nash bargaining solution may not satisfy strong monotonicity. In a version of the model with no taxes or subsidies, Aruoba, Waller and Rocheteau (2007) have shown that the lack of monotonicity of the buyer's surplus causes monetary equilibrium to be inefficient. In particular, there will be underproduction in the decentralized market. As it is clear from (3), taxes and subsidies will impact the optimal D and q resulting from the Nash bargaining game. One of our main objectives is to understand how taxes and subsidies affect the buyer's surplus, and to determine whether the first best can be reached.

The recursive formulation of the representative agent's problem is completed by the following definition for the value function associated to the centralized market:

$$\begin{aligned}
W_t(m_t) = & \max_{X,H,m_{t+1}} \{U(X) - H + \beta V_{t+1}(m_{t+1} + T_t)\} \\
& s.t. X = (1 - \tau_N) H + \phi_t(m_t - m_{t+1});
\end{aligned} \tag{4}$$

where ϕ_t denotes the value of money balances at the centralized market, and τ_N is the tax rate on the production of the centralized good. Finally, as in Lagos and Wright (2005) the monetary authority can provide lump sum monetary transfers, T_t , at the end of all trades in the centralized market.

To close the model notice that the money supply is determined by the government. Money

supply must always equal the money demand and thus

$$M_t = \int m dF_t(m) \forall t,$$

where M_t denotes the money supply. Moreover, we impose a balanced budget constraint for the government before trades in the centralized market take place. Recall that in the decentralized market only a fraction $2\alpha\sigma$ of the population actually trade goods for money. Hence, the balanced budget constraint that requires total money subsidies equal total sales tax revenue plus money printing can be written as follows:

$$2\alpha\sigma(s - \tau_b) \int \int D(m, \tilde{m}) dF_t(m) dF(\tilde{m}) = \tau_{m_1,t} M_t; \quad (5)$$

where we denote $\tau_{m_1,t}$ as the growth rate of the money supply required to satisfy the above equality for all t . To determine the total growth in the money supply it is important to recall that the government has lump sum monetary transfers at the end of the decentralized market.

The government must also balance its budget by the end of the centralized market where government consumption expenditures, denoted by G , are financed by the sum of total production taxes plus seigniorage. We consider two situations regarding the costs of extracting money. In the first case, we assume, as in Lagos and Wright (2005), that extracting money using negative lump sum transfers is costless. Hence, the government constraint is

$$\tau_N H_t + \max[(M_t - M_{t-1})\phi_t, 0] \geq G_t. \quad (6)$$

In the second case we consider negative lump sum transfers are either costly, or simply not possible. With costly lump sum monetary extraction the max operator is dropped from (6). In this case total government expenditure, which we denote by \tilde{G} , equals G plus the real resource cost of extracting money from the economy.

In what follows, we will denote by $\tau_{m_2,t}$ the growth rate in the money supply in the centralized market. Note that there are two ways of extracting money from the economy. The first is to set $\tau_b > s$, the second is to set $\tau_{m_2} < 0$. These two alternative monetary instruments have important consequences for the implementation of the Friedman rule.

Finally, we assume available time for production in the centralized market is bounded above by \bar{H} , and impose the market clearing condition that total demand must equal the available supply, namely,

$$H = \tilde{G} + X. \quad (7)$$

2.2 Equilibrium

An equilibrium for this economy consists of sequences of prices, money holding distributions, and lump-sum money transfers $\{\phi_t, F_t, T_t\}$, production and money payment functions in the decentralized market (q, D) , and production, consumption, and money carried for future purchases from the centralized market $\{H_t, X_t, m_{t+1}\}$ such that:

1. Functions (q, D) solve the Nash Bargaining problem (3).
2. $\{X_t, H_t, m_{t+1}\}$ solve the representative agent's problem taking transfers, taxes, government expenditures, and the distribution of money holdings as given.
3. The government constraints (5) and (6) hold at all $t \geq 0$.
4. The aggregate resource constraint (7) is satisfied at all $t \geq 0$.
5. There is consistency of beliefs and the actual distribution of money.

2.3 Analysis of the model

Our analysis of equilibrium will proceed as follows. We start by studying how taxes and subsidies affect the properties of the bargaining solution. Then we analyze the intertemporal problem of the representative agent, where the determination of the optimal amount of money to carry forward to the decentralized market is key. One of our key contributions is to show that taxes and subsidies can restore efficiency of equilibrium.

The first important property of equilibrium in this model is that the value function of the centralized market during the night subperiod is linear in m , with slope $\frac{\phi}{1-\tau_N}$. As in Lagos and Wright (2005), the linearity of the value function associated to trading in the centralized market keeps the model tractable. In particular, it implies that all agents choose m_{t+1} independently of the money balances, m_t , with which they entered into the market. This result is easily derived by solving for H in the constraint of equation (4) and substituting its value into the objective function.

The linearity of $W(\cdot)$ also simplifies the bargaining problem in the decentralized market as follows:

$$\max_{q,D} \left\{ u(q) - \frac{\phi}{1-\tau_N} (1+\tau_b) D \right\}^\theta \left\{ -q + \frac{\phi}{1-\tau_N} D (1+s) \right\}^{1-\theta} \quad (8)$$

$$s.t. \quad (1+\tau_b) D \leq m. \quad (9)$$

After multiplying both sides of constraint (9) by ϕ , it is clear that the above maximization problem depends only on real monetary balances, $z_t \stackrel{def}{=} \phi_t m_t$. If we let $d \stackrel{def}{=} \phi D$, the solution to the generalized Nash bargaining problem (8) is given by the following result.

Proposition 1 *Given a production tax τ_N , a subsidy s , and a sales tax rate τ_b , the interior solution to the generalized Nash bargaining problem (8) is given by:*

$$d(z, \tilde{z}) = \begin{cases} \frac{z}{1+\tau_b} & \text{if } \frac{z}{1+\tau_b} < z^* \\ z^* & \text{if } \frac{z}{1+\tau_b} \geq z^* \end{cases}$$

$$q(z, \tilde{z}) = \begin{cases} \hat{q} & \text{if } \frac{z}{1+\tau_b} < z^* \\ q^* & \text{if } \frac{z}{1+\tau_b} \geq z^* \end{cases}$$

where q^* and z^* are the solutions to the following nonlinear system:

$$\theta \left\{ -q^* + z^* \frac{(1+s)}{1-\tau_N} \right\} u'(q^*) = (1-\theta) \left\{ u(q^*) - z^* \frac{(1+\tau_b)}{1-\tau_N} \right\} \quad (10)$$

$$\theta \left\{ -q^* + z^* \frac{(1+s)}{1-\tau_N} \right\} (1+\tau_b) = (1-\theta) \left\{ u(q^*) - z^* \frac{(1+\tau_b)}{1-\tau_N} \right\} (1+s), \quad (11)$$

and \hat{q} is given by the solution to the nonlinear equation:

$$\theta \left\{ -\hat{q} + z \frac{(1+s)}{(1-\tau_N)(1+\tau_b)} \right\} u'(\hat{q}) = (1-\theta) \left\{ u(\hat{q}) - \frac{z}{1-\tau_N} \right\}. \quad (12)$$

Proof. Observe that equations (10) and (11) are the first order conditions of (8) ignoring the constraint $\frac{z}{1+\tau_b} \geq d$. Thus, z^* , q^* are the solutions to (8) when $\frac{z}{1+\tau_b} \geq z^*$. Moreover, if $\frac{z}{1+\tau_b} < z^*$ the constraint $\frac{z}{1+\tau_b} \geq d$ binds. In such a case $d = \frac{z}{1+\tau_b}$, while q is given by the first order condition of (8) with respect to q , i.e. equation (12). It follows that functions d and q yield the interior solutions to (8). ■

As Proposition 1 illustrates, a key feature of the model is that the functions characterizing the bargaining game do not depend on the real money holdings of the seller. The latter is a key property of Lagos and Wright (2005) and carries over to our version of the model with taxes and subsidies. What is new here is that the function determining production in the decentralized marketing is positively related to the subsidy rate, and to the production tax in the centralized market.

Corollary 2 For each value of the real money holdings of the buyer we have the following results:

(i) given $(1 - \tau_N)$, $q(z)$ is increasing in $\frac{1+s}{1+\tau_b}$; (ii) given $\frac{1+s}{1+\tau_b}$, $q(z)$ is decreasing in $(1 - \tau_N)$.

Proof. To show output is increasing in $\frac{1+s}{1+\tau_b}$, notice that equations (10) and (11) deliver

$$u'(q^*) = \frac{1 + \tau_b}{1 + s}.$$

Hence, the concavity of $u(\cdot)$ yields that q^* is increasing in $\frac{1+s}{1+\tau_b}$. Using equation (12) and the implicit function theorem we can also derive the following:

$$\frac{\partial \hat{q}}{\partial \left(\frac{1+\tau_b}{1+s}\right)} = - \frac{\theta z u'(\hat{q})}{\theta \left(-u' + u'' \left(-\hat{q} + \frac{z(1+s)}{(1+\tau_b)(1-\tau_N)}\right)\right) - (1-\theta) u'}.$$

Thus, given the monotonicity of $u(\cdot)$, the participation constraint $-\hat{q} + \frac{z}{1+\tau_b}(1+s) \geq 0$, and the concavity of $u(\cdot)$, \hat{q} is increasing in $\frac{1+s}{1+\tau_b}$. The monotonicity of q^* and \hat{q} in $\frac{1+s}{1+\tau_b}$ establish (i). Similarly,

$$\frac{\partial \hat{q}}{\partial (1 - \tau_N)} = \frac{\left[\theta z \frac{1+s}{1+\tau_b} u'(q) + (1 - \theta) z\right] (1 - \tau_N)^{-2}}{\theta \left(-u' + u'' \left(-\hat{q} + \frac{z(1+s)}{(1+\tau_b)(1-\tau_N)}\right)\right) - (1 - \theta) u'},$$

together with the fact that q^* is independent of $(1 - \tau_N)$, delivers (ii). ■

As can be seen in the proof of Corollary 2, the unconstrained (by the money holdings of the buyer) production level q^* , and the constrained production level \hat{q} resulting from Nash's bargaining solution are increasing in $\frac{1+s}{1+\tau_b}$. As it is well known from public finance, subsidies to production tend to increase it. Moreover, Corollary 2 also shows that $(1 - \tau_N)$ is a relevant instrument for increasing production in the decentralized market. This result highlights the fact that by distorting choices in the centralized market, by making it more costly to consume, agents increase consumption in the decentralized market. In particular, by changing τ_N the government is effectively changing the cost of consuming at the centralized market, and thus the outside option of agents in the decentralized market. Hence, the government has two alternative fiscal instruments to increase production in the decentralized market.

The structure of the bargaining solution for this model simplifies greatly the problem of the representative agent, as Corollary 3 illustrates.

Corollary 3 *Under the conditions of Proposition 1, the problem of the representative consumer can be written as follows:*

$$\begin{aligned}
W_t(m_t) = \max_{m_{t+1}, X} & \left[\left\{ U(X) - \frac{X}{1 - \tau_N} + \frac{\beta\phi_{t+1} - \phi_t}{1 - \tau_N} m_{t+1} \right\} \right. \\
& \left. + \alpha\sigma\beta \left\{ u(q(m_{t+1})) - \frac{(1 + \tau_b)}{1 - \tau_N} \phi_{t+1} D(m_{t+1}) \right\} \right] \\
& + \alpha\sigma\beta \int \left\{ -q(\tilde{m}) + \frac{1 + s}{1 - \tau_N} \phi_{t+1} D(\tilde{m}) \right\} dF_{t+1}(\tilde{m}).
\end{aligned} \tag{13}$$

As is well understood now, additional regularity conditions on $u(\cdot)$ can be imposed so that the solution to m_{t+1} to the above problem is unique. The latter yields a degenerate distribution of money holdings, which keeps the model analytically tractable.

To keep our analysis simple, we will focus now on stationary monetary equilibrium. Stationary monetary equilibria are equilibria where prices grow at a constant rate and real money holdings are strictly positive. Finally, for ease of presentation we will also constraint our analysis to the utility functions

$$\begin{aligned}
u(c) &= \frac{(q + b)^{1-\eta} - b^{1-\eta}}{1 - \eta}, \\
U(X) &= B \frac{(X + a)^{1-\epsilon} - a^{1-\epsilon}}{1 - \epsilon}, \\
&\text{with } B, a, b > 0, \text{ and } 0 \leq \eta, \epsilon \leq 1.
\end{aligned}$$

The case $a = 0$ corresponds to the utility functions used by Lagos and Wright (2005) for quantitative analysis.

The problem of the representative agent can be written as above, where money holdings are chosen and q is determined in the bargaining game. Notice, however, that it is also possible to think of the problem of the representative agent as that of choosing q , with real money holdings given by $z(q)$. This alternative characterization of the representative agent's problem will be useful later on in establishing the welfare properties of equilibrium.

Proposition 4 *Consider any given subsidy, sales and production tax rates (s, τ_b, τ_N) and a sequence of prices that grows at rate $(1 + \pi)$, then the representative agent's problem can be written as:*

$$(X^e, q^e) = \arg \max_{X, q} U(X) - \frac{X}{1 - \tau_N} + \alpha\sigma \left(\left(1 - \frac{1 + \pi}{\beta} \right) \frac{z(q)}{1 - \tau_N} + \left\{ u(q) - \frac{z(q)}{1 - \tau_N} \right\} \right) \tag{14}$$

Moreover, the solution to the above problem satisfies $q^e < q^*$ and $\frac{z}{1 - \tau_b} < z^*$.

Proof. From Proposition 1 we know that if $\frac{z}{1+\tau_b} > z^*$ then $q = q^*$ for all z . Moreover, D is also fixed at z^* and for monetary equilibrium to exist we must satisfy $\left(1 - \frac{1+\pi}{\beta}\right) \leq 0$. If the latter term is equal to zero then the objective of the representative household is constant for all $\frac{z}{1+\tau_b} > z^*$, and is strictly decreasing if $\left(1 - \frac{1+\pi}{\beta}\right) < 0$. Thus, it suffices to consider the range $\frac{z}{1+\tau_b} \leq z^*$ and to study problem (14). From the first order condition, equation (12), it is possible to define the output that solves the bargaining problem as a function of z . This function is invertible so we have that

$$z(q) \equiv \frac{(1-\theta)u(q) + \theta u'(q)q}{\theta \frac{1+s}{(1-\tau_N)(1+\tau_b)} u'(q) + \frac{(1-\theta)}{1-\tau_N}}.$$

It is easy to verify that $-z'(q) < 0$ for all q . To evaluate the monotonicity properties of the buyer's surplus $\left\{u(q) - \frac{z(q)}{1-\tau_N}\right\}$, we follow Aruoba, Rocheteau and Waller (2007). In particular, notice that the first order condition of the bargaining problem with respect to q yields

$$\frac{\theta}{(1-\theta)} \left\{ -q + z \frac{(1+s)}{(1-\tau_n)(1+\tau_b)} \right\} u'(q) = \left\{ u(q) - \frac{z}{1-\tau_N} \right\}.$$

Then, we substitute z into the left hand side of the above equation to get

$$\frac{\theta(1-\theta)u'}{\theta(1+s)u' + (1-\theta)(1+\tau_b)} [u(q)(1+s) - q(1+\tau_b)] = \left\{ u(q) - \frac{z}{1-\tau_N} \right\}. \quad (15)$$

Taking derivatives, one finds that the left hand side of equation (15) is non-monotone and that it is negative as $q \nearrow q^*$. The latter fact paired with $-z'(q) < 0$ implies that in any optimum $q^e < q^*$, so that $\frac{z}{1+\tau_b} < z^*$. ■

As we can see, Proposition 4 explicitly shows the trade-off of holding real balances and consuming. In particular, if a seller could turn the proceeds from her production into immediate consumption, as in a static or frictionless model, then she would produce until marginal utility equals marginal cost. In a monetary exchange economy, however, the proceeds from production consist of cash that can only be spent in the future.

It is important to mention that our analysis assumes that the government is capable of denying access to the central market to agents that do not pay sales taxes. We have also implicitly assumed that buyers in the decentralized market are willing to pay sales taxes in order to enter the centralized market. In principle, however, buyers might want to opt out from the centralized market and go directly to the decentralized market next period. Participation in the centralized market can be easily guaranteed by assuming a sufficiently high value of the marginal utility of central market goods at zero consumption, which is trivially satisfied in our framework. Producers in the decentralized market, on the other hand, have incentives to reveal their money holdings since this entails them to a production subsidy.

Our analysis should be seen as a first step towards a better understanding of the properties of optimal monetary and fiscal policy in search models. We have abstracted from important considerations such as the possibility of hiding money, and from alternative pricing mechanisms in the decentralized market. Hu, Kennan, and Wallace (2007) discuss such abstractions and suggest that these may have important implications for the welfare properties of monetary equilibrium, and thus for the properties of optimal policy. In particular, these authors show that in a world with linear utility for the centralized market good, negative lump sum taxes do not enlarge the set of weakly implementable allocations. This result questions then the beneficial role of monetary and fiscal policy in the Lagos and Wright framework when agents are allowed to opt out from the centralized market. The important issue of determining the conditions under which fiscal policy matters for the efficiency of equilibria from the perspective of implementation theory, under our assumption of non-linear utility for the central market good, is thus left as a subject of future research.

3 Optimal fiscal and monetary policy

In this section we consider the government's problem of choosing taxes, transfers and monetary instruments as to maximize social welfare with full commitment. Thus we are contemplating an environment in which the government sets an inflation and fiscal plan that will not change over time. As a result, lags in fiscal policies are not considered in this paper since we abstract from the process that makes the design of fiscal policy lengthier than the monetary one. Moreover, as our analysis focuses on the comparison of different long-run stationary equilibria, temporary delays in fiscal policy will have no impact on the figures we report.

Definition 5 *A sales tax rate, a production tax, an inflation rate, and a production subsidy $(\tau_b^*, \tau_N^*, \pi^*, s^*)$ are optimal if they solve the following problem:*

$$\max_{(\tau_b, \tau_N, \tau_{m_2}, s)} U(B(1 - \tau_N)) - H + \alpha\sigma(u(q^e) - q^e) \quad (16)$$

$$s.t. \quad q^e \in \arg \max_q \left(\left(1 - \frac{1 + \pi}{\beta} \right) \frac{z(q)}{1 - \tau_N} + \alpha\sigma \left\{ u(q) - \frac{z(q)}{1 - \tau_N} \right\} \right) \quad (17)$$

$$\tau_N H + \max \left[\frac{\tau_{m_2, t}}{(1 + \tau_{m_2, t})} z(q^e), 0 \right] = G \quad (18)$$

$$H = \tilde{G} + B(1 - \tau_N) \quad (19)$$

$$\tau_{m_1} \equiv \frac{(s - \tau_b) \alpha \sigma}{1 + \tau_b} \quad (20)$$

$$(1 + \pi) = (1 + \tau_{m_2})(1 + \tau_{m_1}) \quad (21)$$

$$\left(1 - \frac{1 + \pi}{\beta}\right) \leq 0. \quad (22)$$

According to Definition 5, the problem of the government consists of choosing sales taxes, inflation and subsidy rates that maximize social welfare subject to the constraint that production and consumption in both markets are stationary monetary equilibria.⁴

Observe that taxes and subsidies have to be consistent with the government balanced budget constraint during the decentralized market, equation (5), and that the availability of lump sum monetary transfers in the centralized market can neutralize any impact of τ_{m_1} in the growth of prices at the centralized market (where we measure inflation). Thus, lump sum money transfers, and the implied τ_{m_2} , can be used to set inflation at any desired level. In a later section we study the case where negative lump sum monetary transfers are either costly, or not available, $\tau_{m_2} \geq 0$. Even under such circumstances, our model provides a way to implement the Friedman rule by just setting $\tau_{m_2} = 0$, and $\tau_b > s$. Finally, notice that equation (18) implies that inflation may be used as a source of revenue, but we assume deflation is not costly.

Molico (2006), Bhattacharya, Haslag, and Martin (2005), and Deviatov and Wallace (2001), among others, have provided examples in which a policy that consists of increasing the money supply through lump sum transfers induces some redistribution across individuals. Our paper also emphasizes the importance of distributional effects when examining the Friedman rule. The source of our heterogeneity is the asymmetric fiscal treatment of buyers and sellers. Through sales taxes, production subsidies and lump sum injections/withdrawals of money in the decentralized market redistribution of resources between buyers and sellers is possible. To illustrate the importance of distributional effects let's suppose a society is inflating faster than the Friedman rule. If it lowers the growth rate of the money supply, there are two effects. All agents are made better off because the monetary inefficiency is reduced. However, there is a transfer of wealth from agents who hold little money to those who hold more. This effect may make those with little money worse off. If we allow transfers, then society can undo the latter effect and therefore make all agents better off.

⁴The constraint in (??) deserves further explanation. At the beginning of each agent in a pairwise trade arrives, in equilibrium, with M units of money. The buyer pays $\frac{M}{1+\tau_b}$ units of money to the seller. Taxes and subsidies are based on changes in money holdings. The buyer is thus charged $\tau_b \frac{M}{1+\tau_b}$ and ends up with zero cash. The government needs to give the seller $s \frac{M}{1+\tau_b}$. Thus, $(s - \tau_b) \frac{M}{1+\tau_b}$ units of money are printed (or retired from circulation). The seller ends up with $M + s \frac{M}{1+\tau_b}$ units of cash. Relative to the two total units of money that the buyer and the seller brought to the trade, the change in money is $M + s \frac{M}{1+\tau_b} - 2M = \frac{s - \tau_b}{1 + \tau_b} M$.

Without transfers, society cannot undo the latter effect, and the Friedman rule is not necessarily Pareto optimal.

One of the important results of this paper, established by Proposition 6 below, illustrates that fiscal and monetary policies can restore the efficiency of equilibria in spite of the non-monotonicity of the buyer surplus implied by the Nash bargaining solution.

Proposition 6 *Let $G = 0, \tau_N^* = 0, X^* = B^{1/\epsilon} - a$ be given. Then, for any given $\tau_b^* > 0$, and $\beta - 1 \leq \pi^* \leq 0$ there exists $s^*, \tau_{m_2}^*$ such that $(\tau_b^*, \tau_N^*, \tau_{m_2}^*, s^*)$ solve the optimal taxation problem. Moreover, the resulting equilibrium allocations are first best in both markets.*

Proof. Efficiency in the decentralized market requires $q^e = 1$. Since $q^* \geq q^e$, and $q^* = \left(\frac{1+s}{1+\tau_b}\right)^{\frac{1}{\eta}} - b$ a necessary condition for efficiency is $s^* \geq \tau_b^*$. Now, given that $\beta - 1 \leq \pi^* \leq 0$ then (??) implies $\tau_{m_2} \leq 0$. Furthermore, the government budget constraint is trivially satisfied. Now, given τ_b^* it is possible to solve for s^* in the first order condition (17) at $q^e = 1$. Moreover, we show in the appendix that the first order condition characterizes the solution to the households problem q^e . Hence, efficiency in decentralized trades can be achieved. Finally, notice that $X^* = B^{1/\epsilon} - a$ satisfies the first order condition of the household in the central market, and is also first best, establishing the desired result. ■

Proposition 6 shows that, when $G = 0$, increasing production in the decentralized market through subsidies is less distorting than through the production tax in the centralized market. This Proposition implies that there are multiple combinations of taxes and subsidies (and thus redundant tax instruments) that can restore the efficiency of monetary equilibrium outcomes. Recall that Corollary 2 shows that the unconstrained (by the money holdings of the buyer) production level q^* , and the constrained production level \hat{q} resulting from Nash's bargaining solution are increasing in $\frac{1+s}{1+\tau_b}$. As a result, we can have multiple combinations of sales taxes and subsidies that will be able to yield efficiency in the decentralized market. Given the fact that the government has access to negative lump sum transfers it can always undo, in a non distorting fashion, part of the inflation generated to provide subsidies to production. Finally, some of the properties of the bargaining solution derived in Corollary 2 are useful to think about the trade-offs of the optimal taxation problem. Since q is increasing in $\frac{1+s}{1+\tau_b}$, lower inflation, through $\tau_b > s$, tends to lower equilibrium output in the bargaining stage. However, Corollary 2 also establishes that increasing production taxes in the centralized market, τ_N , increases production in the decentralized market. In principle, then, lower output in the decentralized market due to lowering inflation implemented by sales taxes and subsidies, can be undone by an increase in production taxes in the centralized market. Since the term $\alpha\sigma(u(q^e) - q^e)$, in the government's stationary equilibrium maximization problem, is increasing in τ_N but $U(1 - \tau_N) - B(1 - \tau_N)$ is decreasing in τ_N , the government faces

a trade-off. Thus, optimal policy must equate the marginal gains to the marginal losses in the social welfare objective function.

The fact that the Friedman rule is not necessarily optimal when a positive stream of government expenditures need to be financed is a well known result in traditional models of money. In models where money is introduced using ad hoc assumptions and agents hold a positive amount of nominal assets, the Friedman rule is typically not optimal.⁵ Since in our model agents hold a positive amount of money, it seems natural to explore whether the non-optimality of the Friedman rule extends to models with microfoundations of money. With $G > 0$, it is possible to show that there are production subsidies such that efficiency in the decentralized market is attained.⁶ Production subsidies entail inflation, which can be undone freely using lump sum negative money transfers. Notice, however, that this strategy forces all government expenditure to be financed via production taxes in the centralized market. If the marginal utility gain in the central market from reducing production taxes is larger than the marginal loss in the decentralized market from setting $\tau_{m_2} > 0$, then the Friedman rule may not be optimal. An analytical characterization of optimal policy under these conditions is hard to obtain. However, quantitative experiments in the next sections illustrate that for parameterizations found in the literature the Friedman rule is not optimal when $G > 0$.

The remainder of this paper is devoted to deriving the quantitative implications of the theory for the efficiency of equilibrium allocations, measuring the welfare gains of optimal fiscal and monetary policy, and estimating the welfare costs of inflation when no government expenditures need to be financed. Finally, we will explore the properties of optimal fiscal and monetary policy in an environment with positive government expenditures.

3.1 Calibration

The parameterization of the model we choose for our quantitative analysis comes from Lagos and Wright (2005) and it is such that the model with taxes and subsidies set at zero provides the best fit of the model to the annual “money demand” data of the United States. In this calibration a period is interpreted as one year (over which the day and the night markets occur). Notice that the standard velocity equation

$$M\hat{V} = PY,$$

where M is the money demand, P is the price level, \hat{V} is velocity and Y is output, can be easily mapped into the variables of this model. First of all, the price level corresponds to the prices of goods in a centralized market, $\frac{1}{\phi}$. Real output Y in units of the centralized market equals

⁵See for example, Lucas (2000).

⁶The method of proof of Proposition 6 can be applied to deliver this result.

$B + \sigma\phi M$, and real money balances equal $\phi z(q)$, which equal ϕM in equilibrium. Hence,

$$\hat{V} = \frac{B + \sigma z(q)}{z(q)}.$$

Finally, notice that equation (14) can be used to determine q , and therefore \hat{V} , as a function of the nominal interest rate $\frac{1+\pi}{\beta} - 1$. The latter can be used then to derive the money demand implied by the model. The interest rate data employed in this exercise is the annual commercial paper rate while M is measured by $M1$. The sample period considered was 1900-2000.

For our quantitative analysis the annual rate of time preference is set at $r=0.04$. Moreover, we normalize α to 1 and $\sigma=0.5$, which means that every agent always has an opportunity to either buy or sell in each meeting of the decentralized market. Finally, we set $\theta=0.343$, $\eta=0.39$, $a=b \approx 0$, $\epsilon=1$ and $B=1.78$. Lagos and Wright (2005) show that parameter θ is hard to identify and report results for three different values of this parameter. From their parameter specifications, we pick the value of θ that yields the largest welfare costs of inflation. Hence, our quantitative analysis of the welfare costs of inflation can be taken as a measure of the maximum gains that can be obtained from having active monetary and fiscal policy.

3.2 Quantitative implications of optimal fiscal and monetary policy

This section illustrates some of the quantitative and qualitative properties of the solution to the government's problem. We first consider an environment where the government does not face government expenditures. Our numerical findings are reported in Table 1 which presents different cases of fiscal and monetary policies that solve the government's problem with the resulting inflation, the optimal output level for decentralized and centralized market, and the subsidy. Given that output in the decentralized market is increasing in $\frac{1+s}{1+\tau_b}$ we set $\tau_b=0$ in all experiments reported. Thus we are ruling out multiple combinations of taxes and subsidies that yield the same inflation rate. Table 1 also shows, in its first row, the Lagos and Wright (2005) experiment where the Friedman rule is the only optimal policy.

Table 1: Welfare Maximizing Policies for $G=0$ and $\tau_b=0$

π	q	X	s	τ_N
-3.8%	0.56	1.78	-	-
0.0%	1.0	1.78	61%	0%
-3.8%	1.0	1.78	25%	0%

As we can see from the previous table, the output costs of abstracting from optimal fiscal policy are significant, output during the pairwise trade period falls by 40% relative to first best,

which can be attained with a combination of monetary and fiscal policies. Notice that money printing after the decentralized market would imply at least a total growth rate in the money supply of 12%. However, the inflation rates we report are always lower than implied by money printing used to pay the subsidy. Here, the government is effectively undoing most of the money printing by extracting money in the night market via lump sum (negative) transfers. Moreover, positive nominal rates of interest are optimal given that subsidies and costless negative lump sum transfers are allowed. These fiscal and monetary policies can yield an equilibrium where efficiency is attained. Finally, we recall that Corollary 2 shows that q is increasing in $\frac{1+s}{1+\tau_b}$ and τ_N . Thus since the optimal production tax is zero, we can conclude that the inflation tax is the least distorting policy that can increase production in the decentralized market.

To further understand the economic intuition of our results, we present in Figure 2, a plot of the representative agent's objective when inflation follows the Friedman rule and sales tax rates are set at zero. Recall that the proof of Corollary 2 shows that q^* increases as the subsidy, s , increases. Given that we consider the Nash's solution for the bargaining game in the decentralized market, the buyer's surplus is non-monotone in q^* so that the maximum is attained to the left of q^* . Nevertheless, we can see that higher subsidies push both, q^* and the representative household's optimal output, q^e , to the right.

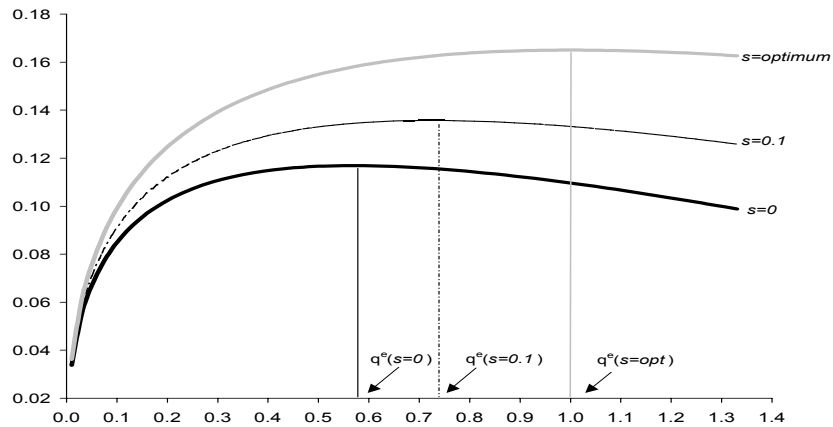


Figure 2: Objective of the representative agent as function of the subsidy rate and q . [Note: $q^*(s = 0) = 1$, $q^*(s = 0.1) = 1.28$, $q^*(s = opt) = 1.77$]

3.2.1 The Welfare costs of inflation

Assessing the welfare costs of inflation requires a sound understanding of the benefits of monetary exchange. Recently, Craig and Rocheteau (2007) have shown that the estimates for the welfare cost of inflation in Lagos and Wright (2005) coincide with those provided by the Bailey method

whenever money holders can appropriate the marginal social return of their real balances. This condition is satisfied when buyers have all the bargaining power to set prices in bilateral trades, or when pricing is competitive. If this condition does not hold, then the welfare cost of inflation is larger than what traditional estimates predict. In Craig and Rocheteau’s (2007) analysis fiscal policy was ignored, in this section we try to fill the gap.

Within the same spirit, we now recast the classical analysis of the welfare costs of inflation in a setting where the existence and need for money is based on microfoundations and fiscal policies are considered. Following Lucas (2000), we report in Table 2 the percentage of consumption that an agent living in an economy where the optimal fiscal and monetary policy is followed will be willing to give up in order to avoid going to an economy where the inflation rate varies, while fiscal policies are held fixed. To compare with the results of the previous literature, we also reproduce the welfare costs of inflation in the Lagos and Wright (2005) case, where only lump sum taxes are considered.

The inflation tax introduces a wedge in the decision to invest in real balances. The extent of this distortion depends crucially on the assumed pricing mechanism. The basic intuition behind this large welfare cost of inflation is the notion of a “hold-up” problem as we move away from the Friedman rule. In other words, an agent that carries a dollar into next period is making an investment with cost equal to the value of money. When the agent spends the money he reaps all of the returns to his investment if and only if $\theta=1$; otherwise the seller “steals” part of the surplus. Thus $\theta < 1$ reduces the incentive to invest, which lowers the demand for money and hence production in the decentralized market. This phenomena becomes more important once fiscal instruments are in place such that a higher level of production can be achieved.

Table 2: The Welfare costs of inflation for $G = 0$ and $\tau_b = 0$.

π	From $\pi = -3.8\%$ $s = 0.1$	From $\pi = -3.8\%$ No Fiscal P.
-2%	0.5%	0.6%
0%	1.6%	1.9%
4%	3.9%	4.8%
6%	4.9%	6.2%
8%	7.4%	6.2%
10%	8.4%	6.8%

Table 2 shows that the costs of inflation can be considerably larger than what Lagos and Wright originally found. The welfare costs of inflation of our benchmark economy, where the optimal policies are able to achieve efficiency, can be as large as 8% of consumption.⁷

The previous findings then suggest that ignoring active fiscal policies can be quite costly. Thus

⁷Similar results are obtained when we consider positive government expenditures.

knowing the empirical “money demand” curve is not enough, one really needs to understand the micro foundations, and especially how the terms of trade are determined and affected by policy actions, in order to correctly estimate the welfare cost of inflation.

3.2.2 Alternative operating procedures for monetary policy

Given that our previous results crucially depend on the availability of costless lump sum monetary transfers at the of the centralized market, we would like to ask ourselves: What happens when costless negative lump sum monetary transfers are not available? In this subsection we also want to explore the robustness of the Friedman rule once fiscal policies are in place. In the previous section, we have shown that the Friedman rule is one of the optimal policies that maximize welfare when the government has no expenditures. We now want to determine how optimal policies change as the bargaining power of the buyer increases. As is well understood now, the magnitude of the inefficiencies caused by Nash’s bargaining solution on equilibrium output in the decentralized market are decreasing in the bargaining power of the buyer. In fact, equilibrium is efficient when $\theta = 1$ and the Friedman rule can be implemented using lump sum money transfers, as shown by Lagos and Wright (2005). Hence, we expect that for values of θ close to one the positive impact on decentralized production of reducing inflation should be smaller than in the case where θ is lower.

Once costless negative lump sum monetary transfers are not available, the first thing to note is that taxes and subsidies cannot be chosen independently of the inflation rate. Increasing production in the decentralized market, by setting production subsidies above sales taxes, generates growth in money balances, since $\tau_{m_1} \equiv \frac{(s-\tau_b)\alpha\sigma}{1+\tau_b}$, and this is now costly to counteract. In this new environment, in order to withdraw money from the centralized market, the government can either set $\tau_b > s$ and/or $\tau_{m_2} < 0$, at a cost $-\min\left\{\frac{\tau_{m_2}}{(1+\tau_{m_2})}z, 0\right\}$. Moreover, it can also increase the production tax in the centralized market “far beyond” its government expenditures and sell “these extra” goods for money, which will then be retired from circulation. In this paper we want to determine which available policy is the least distorting.

For a given parameterization, the government’s problem can be easily solved with standard numerical methods. Table 3 summarizes our findings. To determine the importance of fiscal policies, we first consider the case where they are not active, first row of Table 3, and then the second of this table constrains production subsidies and sales taxes to zero.

As we can see above, given that lump-sum transfers are costly the government finds it optimal not to use them in any of the cases we consider. In our case, sales taxes and production subsidies make the Friedman rule an optimal strategy. In particular, it is possible to set $\tau_{m_2} = 0$, and $\tau_b > s$ such that $\pi = \tau_{m_1} = \beta - 1$. However, this implementation of the Friedman rule cannot achieve the

Table 3: Welfare maximizing policies with buying/selling goods for $G=0$

θ	π	s	τ_b	τ_N	τ_{m_2}	q^e	X	Govmt. surplus
0.343	0.0%	-	-	-	0.0%	0.29	1.78	0.0
0.343	0.0%	-	-	11.51%	0.0%	0.90	1.58	0.18
0.343	-3.84%	0.01%	8.42%	5.32%	0.0%	0.81	1.70	0.08
0.95	-0.66%	0.00%	1.24%	3.79%	0.0%	0.87	1.71	0.07

efficient outcome since $\tau_b > s$ implies

$$q^* = \left(\frac{1+s}{1+\tau_b} \right)^{1/\eta} < 1,$$

and Proposition 4 established $q^e < q^*$. Similarly, the government always trades off the lower utility in the centralized market that results from positive production taxes, τ_N , with the positive impact that such taxes have on production at the decentralized market, as found in Corollary 2. Finally, when the government has a surplus, the monetary authority is purchasing goods in exchange for money that is taken away from circulation.

In the optimum with production taxes but no subsidies or sales taxes, production in decentralized trades triples relative to having no fiscal instruments. The third row of Table 3 summarizes our results when all instruments are available. We find that, for the benchmark value for $\theta = 0.343$, the Friedman rule is optimal. Here sales taxes are used to implement the Friedman rule instead of costly lump sum negative money transfers. With all instruments available, the government surplus (which here is pure resource waste) is half as big as in the case where no subsidies and sales taxes are available.

We also explore the robustness of the Friedman rule as the bargaining power of the buyer increases, fourth row of Table 3. Once the bargaining power of the buyer increases to $\theta = 0.95$, the buyer reaps more of the returns to his investment, thus increasing the incentive to invest, which increases the demand for money and hence production in the decentralized market. As a result the Friedman rule is not optimal anymore. This example then illustrates how the optimality of Friedman rule crucially depends on the bargaining power of the buyer, and equilibria are not first best.

Given that the optimal growth rate for money during the centralized market, τ_{m_2} , is always zero not allowing negative lump sum money transfers is not-binding.

3.2.3 Optimal policy with positive government expenditures

In this new environment we follow the calibration proposed by Aruoba and Chugh (2006) who set $G=0.4$ so that $G/GDP=18\%$. These authors find that real activity is distorted by ex-post

inflation because inflation affects relative prices of goods. They also consider a sales tax in the centralized market and find that when the bargaining power of the buyer, θ , is high enough then the solution to the Ramsey problem must imply deviations from the Friedman rule.

We first consider the case where the government faces no restrictions on τ_{m_2} , see Table 4. As previously discussed, it is well known, in models with ad hoc assumption regarding money, that the Friedman rule is not necessarily optimal when a positive stream of government expenditures need to be financed. In fact, even for the benchmark parameterization with a relatively low bargaining power for the buyer, $\theta=0.343$, we find that the Friedman rule is not optimal.

Table 4: Welfare maximizing policies with buying/selling goods $G=0.4$

π	s	τ_b	τ_N	τ_{m_2}	q^e	X
4.4%	-	-	19.2%	4.4%	0.981	1.44
3.5%	5.17%	11.8%	17.9%	6.7%	0.85	1.46
3.5%	1.11%	7.55%	17.9%	6.7%	0.85	1.46

As we can see from Table 4, the benefits of having sales taxes and production subsidies on top of inflation and production taxes are not easy too see from the numbers reported in Table 4 (production in decentralized trades is lower, but consumption in the central market is higher). However, we can compute how much lifetime consumption has to be taken away from an agent in a world where sales taxes and production subsidies are optimally set (either the second or third rows in Table 4) so that her welfare equals that of an equilibrium with $\tau_b = s = 0$. This computation suggests that agents are willing to pay up to 2.5% of their lifetime consumption in order to have subsidies and sales taxes available. Notice that deviations from the Friedman rule are larger than what is typically found, and will become even larger if the buyer's bargaining power increased.

Finally, note that since the optimal τ_{m_2} is positive the different operating procedures for monetary policy considered in this paper are going to yield the same answer.

4 Conclusions

The objective of this paper is to have a better understanding of the interactions between monetary and fiscal policies in an economic environment with micro foundations for fiat money. In particular, this paper derives the optimal monetary and fiscal policies in a standard search theoretic model of monetary exchange where production subsidies, sales taxes and the possibility of injecting fiat money at different times of the day are possible.

One of the main results in this paper is to show that even when Nash bargaining is considered, in the decentralized market, some of the inefficiencies in the Lagos and Wright framework can be restored when appropriate fiscal policies are considered. In particular, we find that production

subsidies and sales taxes are able to increase the production possibilities resulting from the Nash's bargaining game in the decentralized market. Changing the money supply at different times of the day can increase output in the decentralized market whenever production subsidies are in place.

When the government has zero expenditures and negative lump sum monetary transfers are possible, we show that there are multiple combinations of taxes, subsidies and (sometimes strictly positive) nominal interest rates that yield the efficient allocation. Thus the Friedman rule is one of the possible policy options that the government has at its disposal that yields efficiency. When we consider alternative operating procedures for monetary policy, other than negative lump sum transfers, we find that the optimality of Friedman rule crucially depends on the bargaining power of the buyer. In particular, for low enough values of the bargaining power of the buyer the Friedman rule is the unique optimal policy. Irrespective of this bargaining power, the efficient allocation cannot be achieved.

With positive government expenditures, the optimal policy must trade-off inflation and production tax distortions. Regardless of the operating procedures for monetary policy, having positive nominal interest rates is an optimal policy but the efficient allocation cannot be achieved. These results emphasize the importance of examining the interactions between the implementation of monetary policy with the available fiscal policies.

Finally, under any of the operating procedures for monetary policy considered in this paper and the level of government expenditures, we find that there are large welfare gains of having fiscal and monetary policies in place. Thus ignoring active fiscal policies can be quite costly.

The findings of this paper confirm the conjecture of Kocherlakota (2005) and Wright (2005) that fiscal and monetary policies have important interactions, in frameworks with micro foundations for the existence of fiat money, thus they should always be jointly considered in the design of optimal government policy.

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Appendix

Interiority for Corollary 7

Recall that the objective of the consumer, relative to q , can be written as

$$\left(\left(1 - \frac{1 + \pi}{\beta} \right) \frac{z(q)}{1 - \tau_N} + \alpha \sigma \left\{ u(q) - \frac{z(q)}{1 - \tau_N} \right\} \right).$$

It is easy to show that $z(q)$ is monotone in q (for a given set of taxes and subsidies). Thus the first term is monotone decreasing in q . The only relevant term to evaluate is the buyer's surplus. From the proof of Proposition 4, however, we know the buyer's surplus is, in fact, given by

$$\frac{\theta(1-\theta)u'}{\theta(1+s)u' + (1-\theta)(1+\tau_b)} [u(q)(1+s) - q(1+\tau_b)]$$

Given our assumption on the functional form of u , as $b \rightarrow 0$, the derivative of the above is given by

$$\begin{aligned} & \frac{-\eta\theta(1-\theta)q^{1-\eta} \left[\frac{q^{1-\eta}}{1-\eta}(1+s) - q(1+\tau_b) \right]}{q^2 [\theta(1+s)q^{-\eta} + (1-\theta)(1+\tau_b)]} \\ & - \frac{\theta(1-\theta)q^{1-\eta}(1-\eta) \left[\frac{q^{1-\eta}}{1-\eta}(1+s) - q(1+\tau_b) \right] \left(-\eta \frac{\theta(1+s)q^{1-\eta}}{q^2} \right)}{q [\theta(1+s)q^{-\eta} + (1-\theta)(1+\tau_b)]^2} \\ & + \frac{\theta(1-\theta)q^{1-\eta}(1-\eta) \left[\frac{q^{-\eta}}{1-\eta}(1+s) - (1+\tau_b) \right]}{q [\theta(1+s)q^{-\eta} + (1-\theta)(1+\tau_b)]} \end{aligned}$$

The sign of the above expression is given by (after eliminating $\theta(1-\theta)$, and a little algebra)

$$\begin{aligned} & \frac{-\eta \left[\frac{q^{1-\eta}}{1-\eta}(1+s) - q(1+\tau_b) \right]}{q^2 [\theta(1+s)q^{-\eta} + (1-\theta)(1+\tau_b)]} \\ & + \frac{\left[q^{1-\eta}(1+s) - q(1+\tau_b) \right]}{q [\theta(1+s)q^{-\eta} + (1-\theta)(1+\tau_b)]} \\ & - \frac{\left[\frac{q^{1-\eta}}{1-\eta}(1+s) - q(1+\tau_b) \right] [\theta(1+s)q^{1-\eta}(1-\eta) - \theta(1+s)q^{1-\eta}]}{q^2 q [\theta(1+s)q^{-\eta} + (1-\theta)(1+\tau_b)]^2} \end{aligned}$$

Given we care only about the sign, we can multiply by a positive term, $[\theta(1+s)q^{-\eta} + (1-\theta)(1+\tau_b)]^2$, throughout. We also divide, each of the terms in brackets by q . Hence, the sign of the derivative of the buyer's surplus is given by

$$\begin{aligned} & -\eta(1-\theta)(1+\tau_b) \left[\frac{q^{-\eta}}{1-\eta}(1+s) - (1+\tau_b) \right] + \\ & \left[q^{-\eta}(1+s) - (1+\tau_b) \right] \left[\theta q^{-\eta}(1+s) + (1-\theta)(1+\tau_b) \right], \end{aligned}$$

which after further simplifications can be written as

$$(1 + \tau_b)(1 + s)q^{-\eta} \left[(1 - \theta) \left[1 - \frac{\eta}{1 - \eta} \right] - \theta \right] + (1 + \tau_b)^2 (1 - \theta) [\eta - 1] + \quad (23)$$

$$+ q^{-2\eta} (1 + s)^2 \theta.$$

Notice that the second term in (23) is negative. The third term is positive and decreasing in q at rate 2η . The first term may be positive or negative. Nevertheless, it is decreasing in q at rate η . It is also easy to show that at q^* , defined by $q^{*\eta} = \frac{1+\tau_b}{1+s}$, the above expression is negative. Notice that (23) goes to $+\infty$ as q goes to zero, and thus, by continuity, there is a \hat{q} such that (23) equals zero. Finally, given the rates of decrease in q of the positive, and of the possibly negative term, it follows that (23) is negative for all $q > \hat{q}$. We conclude that a maximum q^e exists, that it is interior, and that it satisfies $q^e < q^*$ for any given s, π, τ_b .