Insurance, Consumer Search, and Equilibrium Price Distributions*

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Abstract

We examine a service market with two frictions: search is required to obtain price quotes, and insurance coverage for the service reduces household search effort. While fewer draws from a price distribution will raise a household’s average price, the indirect effect of reduced search on price competition has a much greater impact, accounting for at least 89% of any increase in average expenditures.

We consider how different insurance market structures will address this moral hazard problem. We find that a monopolist insurer offers full insurance, causing all service firms to charge an identical high price. A perfectly competitive insurance market typically results in partial insurance coverage and significant price dispersion. However, the competitive market neglects the indirect effects of search; a second-best contract that internalizes this externality would offer less insurance coverage.

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1 Introduction

In most markets where insurance plays a prominent role (such as medical services or auto repairs), the price of a particular service varies significantly from one firm to another. In a typical market, consumers respond to price dispersion by obtaining price quotes from a number of firms and selecting the lowest price. However, when an insurance company ultimately pays for most of the service, the consumer’s incentive to search is dramatically reduced. This paper studies optimal insurance contracts in an environment with moral hazard due to search.

An insurance contract (or policy) consists of the premium charged to households as well as a coinsurance rate, defined as the percentage of an insurance claim that the household pays out-of-pocket. In offering a particular policy, an insurer must consider the incentives it creates for both households and service providers. We depict this in a general equilibrium model in which the interactions between these agents endogenously determine the insurance policy, the distribution of service prices, and the search intensity of households.

Moral hazard can be measured as the change in the expected total cost of the event due to the presence of insurance. We decompose two effects which contribute to this rise in expected cost. First, a direct effect occurs when consumers request fewer quotes from the same distribution. Second, an indirect effect occurs because requesting fewer quotes results in less price competition among the firms, shifting the distribution toward higher prices. Indeed, we find that regardless of parameter values, the latter effect is at least 8.6 times bigger than the former, or in other words, is responsible for at least 89% of the cost increase caused by moral hazard. Thus, the general equilibrium feedback is a much larger concern in the incentive problem.

We also characterize the second-best insurance policy, which selects a coinsurance rate that balances consumption smoothing against incentives to search. This

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1 Sorensen (2000) provides an empirical investigation of price dispersion in the prescription drug market (pricing the same drug across retailers). He documents that, on average, the highest posted price is over 50 percent above the lowest available price; furthermore, differences in pharmacy characteristics can account for at most one-third of the dispersion.

2 A similar flavor arises in Fershtman and Fishman (1994) and Armstrong, Vickers, and Zhou (2009). Both articles consider the effect of an exogenous price cap in a market with search frictions (but no insurance contracts). While the cap necessarily reduces the high end of the price distribution, it has the unintended consequence of reducing the benefits of search. As fewer consumers seek additional quotes, retailers compete less vigorously and the low end of the price distribution will rise; indeed, consumers can be left worse off after the imposition of the price cap.
is compared to the insurance policy from two different insurance market structures: monopoly and perfect competition. We find that even though a monopolist insurer has the most ability to promote competition among service firms, it has no incentive to do so. Instead, it offers full insurance, resulting in no search, a high service price, and the largest possible risk premium. A perfectly competitive policy includes some coinsurance but still below the second-best policy, due to search externalities. The insurer only optimizes with respect to the policy’s direct effect on its clients, neglecting the indirect benefits of extra search effort.

In our model, there is a continuum of ex-ante identical households and service firms, and an insurance firm. Households face a random event (such as an auto accident or health problem) with some fixed probability. If the event occurs, the household must hire a service firm to fully repair the damage.\(^3\) This service is homogenous across the service firms, but each firm may charge a different price. Households know the distribution of offered prices, but can only learn the price charged by a particular firm through costly search effort.

Households can insure against this event by purchasing a policy offered by the insurance firm. If the event occurs, the policy reimburses a stipulated fraction of the actual price paid. All service firms are within the insurer’s approved network, meaning they have agreed not to charge more than an exogenously-set maximum price.

Search is simultaneous, as in Burdett and Judd (1983); that is, a household receives all quotes at the same time. Unlike sequential search, a simultaneous search environment can generate equilibrium price dispersion even though firms and households are homogeneous.\(^4\) Our model extends Burdett and Judd (1983) in three noteworthy ways: households are risk averse, insurance contracts are incorporated, and quotes are obtained with a probability that depends on search effort. The last feature greatly improves tractability and reduces the number of equilibria.

Our work contributes to a broad literature on moral hazard problems, which occur whenever the presence of insurance distorts incentives for the insured, causing an increase in expected payout. Two other forms of moral hazard are well known. First,

\(^3\)This is purely a monetary loss, then, and ignores any irreparable damage. Ma and McGuire (1997) model health shocks as a monetary loss which can be partially recovered depending on the quantity and quality of health care purchased.

\(^4\)Baye, Morgan, and Scholten (2006) provides a comprehensive and insightful review of models of search that can produce equilibrium price dispersion.
the insured person may exercise less precaution (such as defensive driving), increasing the probability of loss. Second, the insured person may increase his consumption of the covered service (such as medical appointments), increasing the size of loss. These have been extensively studied, beginning with the work of Arrow (1963), Pauly (1968), Smith (1968), Zeuckhauser (1970), and Ehrlich and Becker (1972).

However, moral hazard in search has received much less attention, with the only formal analyses in Dionne (1981, 1984). In a model where the coinsurance rate is taken as exogenous and the distribution of prices is fixed regardless of the number of quotes requested by households, Dionne identifies the negative incentive effect of insurance on search behavior and hence on expected service prices. However, this neglects a crucial (and, as we show, larger) component of the story: the endogenous response of firms to household search.

This paper relates to the optimal insurance literature; in particular, Ma and McGuire (1997) shares the same spirit as our paper, though they examine a different aspect of moral hazard. In their model, health insurance contracts are incomplete because the quantity and quality of health care is not contractible; household or physician effort are hidden to some degree. This is a variation of moral hazard in consumption — households use more services, and physicians provide lower quality care. Our model follows a similar timing of insurance, service firm, and household decisions; but instead, the non-contractible elements are firm pricing and household search effort, leading to moral hazard in search.

Frech and Ginsburg (1975) and Vaithianathan (2006) study moral hazard in consumption (service quantity is non-contractible), but a unique price in the service market is determined by a monopolist or Cournot competition, respectively. In Nell, Richter, and Schiller (2009), a unique service price is determined by spatial competition among providers (location is non-contractible); demand is inelastic as in our model.

All four preceding papers structure the insurance contract in terms of coinsurance,

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5 Arrow (1963) mentions the potential problem: “Insurance removes the incentive on the part of individuals, patients, and physicians to shop around for better prices for hospitalization and surgical care.”


7 Auto insurers sometimes require multiple repair quotes, but this practice is completely absent in medical or prescription drug insurance. Perhaps insurers do not contract on the number of quotes because of the uncertainties in the search process, such as the availability and comparability of alternative service firms.
as we do, and in each case, greater insurance coverage exacerbates the friction in the service market.\footnote{In any of these environments, vertical integration of insurer and service provider would likely eliminate the friction, as would the close analog in which the insurer selects the provider on behalf of the household. While Preferred Provider Organizations (PPOs) may be viewed as a step in that direction, they still leave room for the consumer search that we analyze. Most such insurance plans include a large number of providers for common services, perhaps because of their volume of insured clients. Thus, providers on the list reasonably could compete among themselves for the household’s business.} Thus, an optimal contract must balance the smoothing of risk against the reducing of frictions. The latter two papers also find that a perfectly competitive insurance market provides too much coverage relative to the second-best contract.

A unique contribution of our paper is that search frictions can generate a continuum of prices in the service market. Thus, in addition to the risk of the event occurring, households also face uncertainty about the service price they will find, which expands the role of insurance in smoothing risk. Moreover, it seems the “law of one price” is routinely violated in markets with a heavy insurance presence, and search frictions are a natural candidate to explain this. Also, the preceding papers did not consider a monopolized insurance market; we conjecture that similar results would also occur in those environments, with the monopolist insurer intentionally exacerbating service market frictions so as to increase the risk premium it can extract.

The paper proceeds as follows: Section 2 presents the model in which the insurance contract is exogenous, and characterizes the equilibrium behavior of service firms and households. Insurance contracts are endogenized in Section 3; monopolized and perfectly-competitive insurance contracts are compared to the second-best contract that maximizes household utility. Section 4 provides a measure of moral hazard in search and decomposes this into the direct and indirect effect. Section 5 illustrates equilibrium behavior in a numerical example and comparative statics. We offer conclusions in Section 6. All proofs appear in the appendices.

\section{Exogenous Insurance Contract}

\subsection{Environment}

Three types of agents interact in this economy: households, service firms, and an insurance firm. We assume a continuum (of measure one) of both households and service firms. Within each type, agents are identical ex-ante.
Households face a random event (such as an auto accident or health problem) with probability $\rho$. When the event occurs, the household hires a service firm to fully repair the damage. This service is homogenous across the service firms, but each firm may charge a different price. Households know the distribution of offered prices, $F(p)$, but can only learn the price $p$ charged by a particular firm through costly search.

In particular, the household chooses a search effort $s \in \mathbb{R}_+$ and incurs utility cost $c(s)$. We interpret $s$ as the number of extra quotes (beyond the first) that the household will receive; any fractional part of $s$ indicates the probability of receiving the next quote. The aggregate search effort across all households is denoted $S$.

Households insure against this event by purchasing a policy offered by the insurance firm, which specifies a premium $\theta$ as well as a coinsurance rate $\gamma$. We initially consider this insurance contract as exogenously given and assume that all households insure; in Section 3, both the insurance policy and the decision to purchase it are endogenously determined. If the event occurs, the policy reimburses a fraction $1 - \gamma$ of the actual price paid. All service firms are within the insurer’s approved network, meaning they have agreed not to charge more than an exogenously-set maximum price $M$. Even though service firms precisely know this price cap, they are permitted to charge less; indeed, quoting a lower price could undercut quotes from other firms and thus win more sales. This pricing decision is fully described in Section 2.2.

Note that demand for the service is perfectly inelastic; fraction $\rho$ of the population will always purchase one unit of service from some firm. The only question is what price they will pay for it. This assumed demand is needed to isolate the effect of moral hazard in search. If consumers had any elasticity in their demand, then the presence of insurance would encourage them to consume more units of service, which is moral hazard in consumption.

Decisions occur simultaneously: the event is realized for some of the households, who must decide their search effort. At the same time, firms simultaneously set their

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9 e.g. when $s = 0.25$, the household receives one quote for sure and a second with probability $\frac{1}{4}$, but when $s = 1$, the household is guaranteed two quotes. This probabilistic approach captures the uncertainty in being able to learn prices before committing to use a given service firm. In the authors’ own experiences with filling drug prescriptions, pharmacies are not particularly consistent in being willing or able to look up prices in advance of the sale. Sometimes a phone call will suffice, but often one must visit in-person and nearly fill the prescription.

10 Though we take this upper bound on prices as given, the rest of the price distribution is endogenously determined. We discuss how this maximum price may be endogenously determined in Section 6.
prices. After the number of quotes is realized, households select the lowest price among them. We refer to the time prior to search but after the event as *ex-interim*.

### 2.2 Service Firms

The individual service firms are able to repair a household’s loss at constant marginal cost \( r \). We assume \( r < M \). Each firm sets a price \( p \), taking as given the distribution of prices among other firms and the aggregate search behavior of consumers, represented by \( F(\cdot) \) and \( S \).\(^{11}\) Only \( \rho \) percent of the population will be in the market for their service, and among those customers, a firm will only make the sale if its quoted price is lower than all other quotes requested by that customer. Thus, a firm considers not only the profit per sale, \( p - r \), but also the probability of making the sale.

As shown in Burdett and Judd (1983), if all customers obtain at least two quotes, all firms must charge \( p = r \) in equilibrium. This occurs because whichever firm charges the highest price will almost surely lose to another bid, and hence earn zero profit. This leads to Bertrand-like competition, driving the price down to marginal cost. Furthermore, this cannot be consistent with equilibrium, since a household will never choose to incur the cost of extra quotes if all firms charge the same price.

Thus, we restrict our analysis to \( S \in [0, 1) \). The service firm’s expected profit is thus:

\[
\max_p \pi(p; S) \equiv \begin{cases} 
\rho(p - r) \left(1 - S + 2(1 - F(p))S\right) & \text{if } p \leq M \\
0 & \text{if } p > M.
\end{cases}
\]

Note that \( 1 - S \) indicates the fraction of households that obtain only one quote; this fraction of the firm’s customers will always accept the quoted price. The other fraction \( S \) will have obtained two quotes; the firm provides \( 2S \) quotes to this group, and has a probability \( 1 - F(p) \) that offered price \( p \) will be lower than the alternative and hence be accepted. Since firms are identical, they can only select different prices if they are indifferent among them. Hence, for a given aggregate search effort \( S \), we require that for any \( p \) in the support of \( F(p) \), \( \pi(p; S) \geq \pi(\hat{p}; S) \) for all \( \hat{p} \in \mathbb{R}_+ \).

The price distribution for a given \( S \) can be derived from the requirement that all

\(^{11}\)In particular, an individual service firm does not expect that raising its price will change the search behavior of consumers, since it is only one of the continuum of firms and cannot affect the price distribution. This would only occur if a positive mass of firms raised their prices.
prices in the support be equally profitable.\textsuperscript{12} In particular, one compares the profit from charging price $M$ to the profit from any other price $p$, namely $(M - r)(1 - S) = (p - r)(1 - S + 2(1 - F(p))S)$, and solves for $F(p)$:

$$F(p; S) = 1 - \frac{(M - p)(1 - S)}{2(p - r)S} \text{ for } p \in [p, M].$$

(2)

The lower bound of the support, $\bar{p} \equiv r + \frac{(M - r)(1 - S)}{1 + S}$, is derived such that $F(p) = 0$. Also, $dF(p; S) = \frac{(M - r)(1 - S)}{2S(p - r)^2}$. Each service firm has an ex-ante expected profit of $\pi = \rho(M - r)(1 - S)$. Thus, firm behavior and the resulting price distribution are entirely determined by $S$, the fraction of people who obtain two quotes. Furthermore, a smaller $S$ results in prices more concentrated on the right tail of the distribution, as established in the following lemma.

\textbf{Lemma 1.} If $\hat{S} < \tilde{S}$ then $F(p; \hat{S}) < F(p; \tilde{S})$ for each $p \in [p, M]$.

\textit{Proof.} Since $\hat{S} < \tilde{S}$, $\frac{1 - \hat{S}}{\hat{S}} > \frac{1 - \tilde{S}}{\tilde{S}}$. Hence $F(p; \hat{S}) = 1 - \frac{(M - p)(1 - \hat{S})}{2(p - r)\hat{S}} < 1 - \frac{(M - p)(1 - \tilde{S})}{2(p - r)\tilde{S}} = F(p; \tilde{S}).$

In other words, when fewer people obtain second quotes, firms have lower probability of being undercut; as a consequence, they can charge higher prices. In particular, the new distribution will first-order stochastically dominate the original distribution.

\section*{2.3 Household Search Effort}

Only households who experience the event have a choice to make: search effort $s$. In doing so, they treat the distribution of service prices as fixed; equivalently, aggregate search effort $S$ is taken as given. The quotes are all received simultaneously, after which the household will choose the lowest among them.\textsuperscript{13}

Household utility is assumed additively-separable with respect to search costs; that is, a form: $u(w) - c(s)$. To create a role for insurance, we assume that households are risk averse: $u' > 0$ and $u'' < 0$ for all wealth $w$. The choice of $s$ is made so as to maximize ex-interim expected utility:

\textsuperscript{12}This distribution coincides with that in Burdett and Judd (1983), with the change of variable $S = 1 - q$.

\textsuperscript{13}In particular, there is no option to seek another set of quotes, as in sequential search. Manning and Morgan (1982) and Morgan and Manning (1985) offer several scenarios in which simultaneous search dominates sequential search, such as when the repair or surgery must take place within a short timeframe.
max \( s \in [0,1] \) \( V(s; \theta, \gamma, S) \equiv \int_{p}^{M} u(w - \theta - \gamma p) \left( 1 - s + 2(1 - F(p; S))s \right) dF(p; S) - c(s). \)

(3)

If \( u(w) = w \) and \( c(s) = c \cdot s \), the objective function is linear with respect to \( s \), and taking the first order condition \( V'(s) = 0 \) results in the same indifference condition as in Burdett and Judd (1983). Here, we consider strictly concave utility functions, and impose \( c(s) = c \cdot s^2 \) throughout. The most important features of this functional form are the strictly increasing marginal cost of search effort, which ensures a unique solution for \( s \), and \( c'(0) = 0 \), which ensures some search effort will occur so long as the price distribution is not degenerate. The linearity of marginal search cost is analytically convenient, but not critical to the qualitative properties of equilibrium.

**Lemma 2.** Suppose \( c(s) = c \cdot s^2 \). Then Equation 3 has a unique solution, denoted \( \zeta(\theta, \gamma, S) \). Moreover, \( \zeta = 0 \) whenever \( S = 0 \).

Since households are identical and their effort has a unique solution, individual effort \( s \) must match aggregate effort \( S \) in equilibrium. Thus, an Insured Search Equilibrium for a given insurance policy \((\theta, \gamma)\) is a price distribution \( F^*(\cdot) \), aggregate search effort \( S^* \), individual search effort \( s^* \), and service firm profit \( \pi^* \) such that:

1. For all \( p \) in the support of \( F^*(p) \), \( \pi^* = \pi(p; S) \).
2. For any \( \hat{p} \in \mathbb{R}_+ \), \( \pi^* \geq \pi(\hat{p}; S) \).
3. \( s^* = \zeta(\theta, \gamma, S^*) \).
4. \( s^* = S^* \).

If the equilibrium \( F^*(\cdot) \) is a non-degenerate distribution, we refer to it as a dispersed price equilibrium.

### 2.4 Insured Search Equilibrium Characterization

We now characterize some of the equilibrium properties, including existence, uniqueness, and comparative statics. To do this, we can reduce the equilibrium requirements to a single equation, used to solve for \( S^* \). The first two equilibrium requirements are
uniquely satisfied by utilizing the solution for \( F(p) \) given in Equation 2. This is substituted into the first-order condition for a maximum of Equation 3 to satisfy the third requirement, and then \( S \) is substituted for \( s \) to satisfy the fourth. This results in:

\[
\Delta(\theta, \gamma, S) \equiv \int_{\bar{p}}^{M} u(w - \theta - \gamma p) \frac{(1 - S)(M - r)((M - r)(1 - S) - p + r)}{2S^2(p - r)^3} dp - 2cS.
\]

(4)

The equilibrium \( S^* \) is found by solving \( \Delta(\theta, \gamma, S) = 0 \) for \( S \). For any policy \((\theta, \gamma)\), let \( \sigma(\theta, \gamma) \equiv \{ S \in [0, 1) : \Delta(\theta, \gamma, S) = 0 \} \) denote the set of equilibrium search effort.

**Proposition 1.** Suppose \( c(s) = c \cdot s^2 \).

1. There always exists a degenerate equilibrium (with \( S^* = 0 \)).

2. There can exist at most one dispersed equilibrium (where \( 0 < S^* < 1 \)).

3. If \( \gamma < \frac{2c}{(M-r)u(w-\theta-M)} \), only the degenerate equilibrium occurs.

4. In a dispersed equilibrium, \( S^* \) is increasing in \( \gamma \) and decreasing in \( c \).

As in most search models, a degenerate equilibrium in which no one searches always exists. Here, this means \( S^* = 0 \) and all service firms charge a price \( p = M \). Firms have no incentive to offer lower prices because no household obtains a second quote, and hence there is no chance of being undercut. Households have no reason to choose an \( s > 0 \), because additional quotes will yield the same price.

Exactly one dispersed price equilibrium may also exist.\(^{14}\) This cannot exist for \( \gamma \) near 0, but typically will for moderate to high levels of coinsurance. That is to say, households must see enough direct savings (on out-of-pocket expenses) to make search worthwhile.

Dispersed price equilibria must be solved for numerically — analytic solutions are not possible even in the case of linear utility, and risk aversion only increases the complexity of Equation 4. In Section 5, we provide a numerical example that

\(^{14}\)This is in contrast with the model of Burdett and Judd (1983), which can have two dispersed price equilibria, in addition to the degenerate equilibrium that always exists. In an extension of that model, Fershtman and Fishman (1992) argue that one of the two dispersed equilibria is not stable; here, such a case is altogether eliminated. The key reason for this difference is that, instead of having a constant marginal search cost of \( c \), here \( c'(0) = 0 \), which encourages search even when there is little (but some) dispersion.
demonstrates the typical equilibrium behavior. One of the features of greatest interest can be derived through implicit differentiation: how changes in the coinsurance rate affect search effort. In equilibrium, greater coinsurance encourages households to search more, which (by Lemma 1) leads to lower service prices.

3 Endogenous Insurance Contracts

We next consider the decisions of the insurer in setting the terms of the insurance policy. We will examine the contracts produced under two insurance market structures: monopoly and perfect competition. These are compared to the utility-maximizing contract. For these evaluations, it is necessary to introduce \( \text{ex-ante expected utility} \), evaluated before the event is realized:

\[
EU(s; \theta, \gamma, S) \equiv (1 - \rho)u(w - \theta) + \rho V(s; \theta, \gamma, S).
\] (5)

In either market structure, we add an initial stage in which the insurance firm selects a policy \((\theta, \gamma)\) and households decide whether or not to insure. Thereafter, the service firms and households behave as depicted in the insured search equilibrium.

3.1 Monopolist Insurance Firm

Suppose there is a single insurer with market power. This insurer proposes a policy \((\theta, \gamma)\) in a take-it-or-leave-it offer to households, who have an outside option to remain uninsured (possibly increasing their search effort). In making this decision, the household treats aggregate search effort \(S\) as given (which fixes the price distribution). In particular, he might choose a higher search effort \(s > S\) if uninsured, but as an infinitesimal participant of the market, he does not expect this choice to alter the price distribution.

The insurance firm is risk neutral and seeks to maximize expected profit. The insurer understands that household search behavior can be affected by the policy terms, which in turn determines the service firms’ price distribution. In other words, the insurer recognizes that it can influence \(S\) through its choice of \(\theta\) and \(\gamma\), due to its market power. This is directly relevant for its profits, as indicated in the next lemma.
Lemma 3. For a given policy \((\theta, \gamma)\) and aggregate search effort \(S\), insurance profits will be: 
\[
\Pi(\theta, \gamma, S) = \theta - \rho(1 - \gamma)(M(1 - S) + rS).
\]

The proof is a straightforward computation. Similarly, the household’s expected out-of-pocket costs (i.e. unreimbursed service expense) is \(\text{OP} \equiv \rho\gamma(M(1 - S) + rS)\). We then define a Monopoly-insured Search Equilibrium as a policy \((\theta^*, \gamma^*)\) and search effort \(S^*\) such that:

\[
(\theta^*, \gamma^*, S^*) \in \arg \max_{\theta \in [0, M], \gamma \in [0, 1], S \in \sigma(\theta, \gamma)} \theta - \rho(1 - \gamma)(M(1 - S) + rS)
\text{ s.t. } EU(S; \theta, \gamma, S) \geq \max_s EU(s; 0, 1, S).
\]

This definition fits the principal-agent framework, where the insurer (principal) must choose a contract that is both incentive compatible and individually rational for the household (agent). The latter is represented in the participation constraint, since households can remain uninsured. The former is embodied by \(S \in \sigma(\theta, \gamma)\); that is, the aggregate search effort must be consistent with individual incentives for the given contract. By imposing this requirement, the principal is forced to anticipate not only the direct effect of the contract on search behavior \((s)\) but also its indirect impact on the price distribution \((F, \text{via } S)\).

The remarkable consequence of monopolization is that the insurer prefers service firms to have a degenerate price distribution. The insurance firm offers full insurance, even knowing that this results in the highest possible service firm prices. The intuition for this result is that the monopolist’s profit is precisely the risk premium it can extract from the household. By discouraging search, the insurer increases the size of loss from the negative event and hence the variance in the household’s wealth. Thus, households are willing to pay a larger risk premium (in addition to the insurer’s expected payout). The claim is formalized in the following proposition.

Proposition 2. Suppose \(\rho \leq \frac{1}{2}\) and \(\frac{u(w-M)}{u(w-M)} < \frac{11}{M-r}\). A pure monopolist insurer will maximize profits by setting \(\gamma^* = 0\) and \(\theta^*\) such that \(EU(0; \theta^*, \gamma^*, 0) = EU(0; 0, 1, 0)\).

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15Recall that there are potentially two incentive-compatible \(S\) for any contract: a degenerate equilibrium \((S = 0)\) and a dispersed equilibrium. The maximization problem above literally allows the monopolist to choose which equilibrium emerges; however, this is not crucial to Proposition 2. Indeed, even when given this freedom, the monopolist strictly prefers \(\gamma = 0\), producing a unique insured search equilibrium of \(S = 0\). Thus, other contracts are dominated regardless of which equilibrium emerges under them.
We note that the imposed assumptions are only sufficient conditions. While those bounds are not as tight as possible, the used assumptions provide the simplest expression and have intuitive content. The first condition ensures that the negative event is not too likely. If violated in the extreme (e.g. $\rho \approx 1$), the contract becomes more about pre-payment than insurance, altering how households are impacted by changes in coinsurance.

The second condition is related to the curvature of the utility function.\(^{16}\) This is sufficient for households to find insurance more valuable after a decrease in $S$. There are two sources of variance in household wealth: whether the event occurs (i.e. the variance between $w - \theta$ and $w - \theta - OP$) and the actual price paid when it does (i.e. the variance around $OP$ in actual out-of-pocket expenses, $\gamma p$). Under this assumption, households derive greater value from smoothing the first source of risk, while the second is a lower-order concern.

### 3.2 Utility-Maximizing Insurance

The preceding section showed that a profit-maximizing monopolist would always offer full insurance; but this will not typically be in the best interest of households. To set up this contrast, we consider a hypothetical market in which a single insurer offers an actuarially-fair premium, chosen so as to maximize the ex-ante utility of the households.\(^{17}\)

The insurance firm’s profit is calculated the same as in Lemma 3; since this must be zero, the competitive premium is $\Theta(\gamma, S) \equiv \rho(1 - \gamma)(M(1 - S) + rS)$. We define a \textit{Utility-Maximizing Insured Search Equilibrium (UM)} as a policy $(\theta^*, \gamma^*, S^*)$ and search effort $S^*$ such that:

$$(\theta^*, \gamma^*, S^*) \in \arg \max_{\theta \in [0, M], \gamma \in [0, 1]} \EU(S; \theta, \gamma, S) \text{ s.t. } \theta = \Theta(\gamma, S).$$

\(^{16}\)With constant absolute risk aversion utility $u(w) = -e^{-aw}$, then $\frac{u'(w-M)}{u(w-M)} = -a$, so the second condition would be satisfied for any degree of risk aversion, $a > 0$, regardless of $M$ or $r$. With constant relative risk aversion, $u(w) = \frac{w^{1-a}}{1-a}$, then $\frac{u'(w-M)}{u(w-M)} = \frac{1-a}{w-M}$, which will always hold if $a \geq 1$.

\(^{17}\)This establishes the second-best outcome. The first-best outcome would have vertical integration of the insurer and service firms, offering the service at marginal cost $r$ so no search would be necessary, and providing full insurance of the service.
The constraint imposes an actuarially-fair insurance premium, while requiring $S \in \sigma(\theta, \gamma)$ ensures incentive compatibility.

It is straightforward to show that if $S^* = 0$ in a competitively-insured search equilibrium, then $\gamma^* = 0$. As long as there is a degenerate price distribution, coinsurance serves no purpose, and only interferes with the efficient transfer of risk from the household to the insurer.

Among dispersed price equilibria, increases in the coinsurance rate have competing effects, reflected in the following total derivative:

$$
\frac{dEU}{d\gamma} = \frac{\partial EU}{\partial \gamma} + \frac{\partial EU}{\partial S} \frac{\partial S}{\partial \gamma} + \frac{\partial EU}{\partial \theta} \left( \frac{\partial \theta}{\partial \gamma} + \frac{\partial \theta}{\partial S} \frac{\partial S}{\partial \gamma} \right). 
$$

In Proposition 1, we established that $\frac{\partial \gamma}{\partial S} > 0$, and with the actuarially-fair insurance premium, it is readily apparent that $\frac{\partial \theta}{\partial S} < 0$ and $\frac{\partial \theta}{\partial \gamma} < 0$. The direct effects on expected utility are fairly straightforward and intuitive: $\frac{\partial EU}{\partial \gamma} < 0$, as this reduces wealth in all states; $\frac{\partial EU}{\partial \gamma} < 0$, as this exposes the household to more risk; and $\frac{\partial EU}{\partial S} > 0$, as this improves the (aggregate) price distribution enough to offset the (individual) cost of search.

Returning to the total derivative, then, the first term is always negative: increased coinsurance increases risk exposure. The second term is always positive: greater coinsurance encourages more search effort and hence more competition among service firms, thereby lowering the price distribution and improving expected outcomes. The final term is also positive: increased coinsurance will reduce the insurance premium, both because of lower expected service prices and because insurance covers less of the service expense.

Thus, the UM coinsurance will balance the value from the transfer of risk with the value of encouraging greater competition among service firms. This tradeoff typically results in a concave function with an interior maximizer (i.e. partial insurance);\(^\text{18}\) but this non-linear system of equations makes the maximization of $EU$ analytically intractable. One must numerically compute the dispersed equilibrium associated with

---

\(^\text{18}\)In contrast, total welfare (the sum of the household’s utility in certainty equivalent terms, service firm profits, and insurer profits) is always maximized with full insurance and no search effort. Since demand for the repair service is perfectly inelastic, price increases above marginal cost do not create deadweight loss, but are simply a transfer from households to firms. Yet higher coinsurance reduces welfare in two ways: it places more risk on households rather than risk neutral insurers, and it encourages (costly) search effort.
each coinsurance rate and find the maximizer. In Section 5, we illustrate a typical dispersed equilibrium with a numerical example.

Typically, the dispersed price equilibrium with partial coverage will dominate the degenerate equilibrium with full coverage, giving a remarkably different outcome than a profit-maximizing monopolist would. In extreme circumstances, however, it is possible that UM insurance provides full coverage. Most often, this is simply because a dispersed equilibrium does not exist for any coinsurance rate. But with deliberately chosen parameters, one can find outcomes where dispersed equilibria exist for some coinsurance rates, and yet are dominated by full insurance. In such a case, insurance protection is more valuable to the household than the lower prices obtained via search.

3.3 Perfectly Competitive Insurance Market

Competition among insurers introduces another crucial element into play: the search externality. The aggregate search effort of all households determines the distribution of prices, yet any one household has negligible effect. Moreover, if a given insurance firm serves only a fraction of households in the economy, that insurer will have limited impact on aggregate search effort, only being able to influence its own clients.

In the preceding two environments, the insurer effectively internalized the search externality. A utility-maximizing insurer explicitly knows that changes in the coinsurance will encourage greater search from all households; thus, the insurer acts as a social planner coordinating their aggregate search effort indirectly via $\gamma$. A profit-maximizing monopolist also recognizes how coinsurance rates determine the price distribution, but uses this to maximize risk for the uninsured and thus extract the largest risk premium.

We consider a continuum of insurers, each choosing their own insurance contract. We focus on symmetric equilibria where every insurer offers the same contract. Each insurer will recognize the direct effect of coinsurance on the search of his clients, but as measure zero of the households, they have literally no impact on the aggregate search of others. Thus, the indirect benefits of search (increasing competition and thus improving the price distribution) are completely neglected.\footnote{One could also consider an insurance oligopoly, in which a finite number of insurance firms cover the households. On the one hand, insurers would internalize some of the indirect effect since their clients make a non-negligible contribution to aggregate search effort. On the other hand, insurers could try to extract consumer surplus, though less aggressively than the monopolist. The nature...}
In equilibrium, each firm’s contract must provide the same expected utility; otherwise, clients would abandon the less desirable contracts. This requirement has the immediate consequence of preventing insurers from charging more than an actuarially-fair premium; otherwise, a competitor could offer the same coinsurance with a lower premium to provide a higher expected utility.

In choosing a contract to offer, a given insurer does not need to know the details of every other contract on the market. The relevant information is fully summarized in \( S \), the average search intensity of the rest of the market, since this determines the price distribution of service firms, \( F(\cdot) \). Moreover, \( S \) (and therefore \( F(\cdot) \)) is taken as given.

Note that a given insurer’s actuarially-fair premium is affected by the chosen coinsurance rate \( \gamma \), as well as the resulting search effort \( s \) of its clients. Greater search effort will result in more second quotes, thereby reducing the average price obtained even though the distribution of offered prices is unchanged. The resulting premium is calculated through a minor alteration of Lemma 3:

\[
\Theta(s, \gamma, S) = \rho(1 - \gamma) \int_{p}^{M} \left( p(1 - s + 2(1 - F(p; S)))s \right) dF(p; S) \tag{7}
\]

\[
= \rho(1 - \gamma) \left( r + \frac{(1 - S)(M - r)}{S} \left( s + \frac{S - s}{2S} \ln \left( \frac{1 + S}{1 - S} \right) \right) \right). \tag{8}
\]

Thus, for a given level of aggregate search \( S \), a competitive insurer offers a contract \( \gamma^* \) such that:

\[
\gamma^* \in \arg \max_{\gamma \in [0,1]} \text{EU}(s, \theta, \gamma, S) \text{ s.t. } \theta = \Theta(s, \gamma, S) \text{ and } s = \zeta(\theta, \gamma, S). \tag{9}
\]

Note in the constraints that \( \theta \) depends on \( s \) and \( s \) depends on \( \theta \). This pair of non-linear equations cannot be analytically solved, since in the latter constraint \( \theta \) occurs inside the utility function; but for a given \( \gamma \) and \( S \), they will have a unique solution (as shown in Lemma 4). Thus, fixing \( S \), one finds \( \gamma^* \) by numerically solving these two equations over a grid of possible coinsurance rates, computing the expected utility associated with each \( \gamma \) and \( (\theta, s) \) pair, then selecting the \( \gamma \) that produces the highest
Lemma 4. For any $\gamma$ and $S$, a unique solution $(\theta^*, s^*)$ solves $\theta = \Theta(s, \gamma, S)$ and $s = \zeta(\theta, \gamma, S)$.

The insurance market reaches equilibrium when no firm wants to deviate from its offered coinsurance, given the aggregate search effort. In all numerical computations we have performed, the utility-maximizing coinsurance for any given $S$ is unique. Thus, since our households are identical, requiring no deviations is equivalent to requiring $S = s$. That is, in equilibrium, the contract selected by any insurance firm must induce its clients to search the same as the aggregate effort.\(^{20}\)

We thus define a competitively-insured search equilibrium (PC) as a contract $\gamma^*$ and search intensity $S^*$ such that:

1. $\gamma^*$ solves Equation 9
2. $S^* = \zeta(\Theta(S^*, \gamma^*, S^*), \gamma^*, S^*)$.

We offer two analytical results.

Proposition 3. Assuming $c(s) = c \cdot s^2$,

1. $\gamma^* = 0$ and $S^* = 0$ always constitutes a competitively-insured search equilibrium.
2. If a competitively-insured search equilibrium has positive search effort, then the utility-maximizing contract will always provide a higher coinsurance and search effort.

The first states that the full-insurance, degenerate-price equilibrium always exists, even if this provides much lower utility than partial insurance and dispersed prices. This illustrates the search externality in extreme: since the rest of the market is not searching and lower prices cannot be found, an insurer would only harm its clients by increasing coinsurance. Yet all households could be better off if all of the insurers encouraged additional search (if this led to a dispersed equilibrium).

The second result shows a less extreme form of the externality. Even when competitive insurance results in a dispersed equilibrium, the coinsurance is lower than

---

\(^{20}\)Suppose all other insurers offer coinsurance $\bar{\gamma}$, inducing search effort $S$. If an insurer offers a $\gamma$ such that for its clients, $s \neq S$, then it must be that $EU(s, \Theta(s, \gamma, S), \gamma, S) > EU(S, \Theta(S, \bar{\gamma}, S), \bar{\gamma}, S)$, since $\bar{\gamma}$ was in the insurer’s choice set. Thus, the equilibrium requirement that $s = S$ exactly ensures that there are no utility-increasing deviations.
its second-best level. This is because each insurer only considers the direct effect (of more expected draws) of its contract on its client’s search. The insurer neglects the indirect benefits (of a better price distribution) that spill over if all households were to increase their search effort.

Both results illustrate the difficulty of relying on individual action to encourage greater price competition. Indeed, this can explain the tepid results of some real world experiments. For instance, in early 2007, New Hampshire created a program to report average prices for medical procedures at various facilities in hopes of encouraging greater price competition (effectively lowering search costs). However, this seems to have had little effect on prices in the succeeding years, as most households have very low coinsurance rates (See Tu and Lauer, 2009).

4 Measuring Moral Hazard

Proposition 3 established that a perfectly-competitive insurance market will offer a lower coinsurance than what would maximize ex-ante expected utility. This reduces search effort and hence drives up service prices. We quantify the effect of moral hazard in search using the ex-ante expected total cost of repair, which includes all expenditures, whether paid for by insurance or out-of-pocket.

In Sections 3.1 and 3.2, we derived expressions for expected out-of-pocket expenses and competitive premiums, respectively. Their sum, the ex-ante total cost, is \( \rho(M(1-S) + rS) \). Note that this is not directly affected by \( \gamma \); holding \( S \) fixed, an increase in \( \gamma \) would reduce the insurance premium, but that would be exactly offset by increases in expected out-of-pocket costs. Of course, any change in \( \gamma \) that induces greater search effort will drive down total cost.

Let \( \bar{S} \) denote aggregate search effort under the perfectly competitive (PC) contract \( \bar{\gamma} \), while \( S^* \) denotes aggregate search effort under the utility-maximizing (UM) contract \( \gamma^* \). We measure moral hazard, then, as the difference between expected cost of the third-best (PC) and second-best (UM) contracts, which is: \( \rho(M - r)(S^* - \bar{S}) \).

In our model, two effects contribute to the moral hazard problem. The direct effect is that, by requesting more quotes, the household has more draws from the distribution, giving additional chances to obtain a lower quote. This effect was the sole focus of Dionne (1981), for instance, and is the only effect that perfectly competitive insurers consider in setting their insurance policy. There is also an indirect (or
general equilibrium) effect: as more households request a second quote, they encourage greater price competition among the firms, actually lowering the distribution of prices. This is accounted for in the UM contract. Our model is the first to incorporate both effects — and the latter effect is much larger than the former.

Note that in moving from $\bar{\gamma}$ to $\gamma^*$, both effects enter to reduce the expected cost. Households receive more quotes, and service firm competition is intensified. We would like to decompose the two effects.

To isolate the direct effect on total cost, we consider what would happen to total cost (off the equilibrium path) if the service firm price distribution were fixed, with $S = \bar{S}$, but the search effort of households increased to $S^*$. The total expected cost in this scenario is easily derived from the PC insurance premium in Equation 8, only removing the coinsurance term:

$$EC(S^*, \bar{S}) \equiv \frac{\Theta(S^*, 0, \bar{S})}{1 - \gamma} = \rho \left( r + \frac{(1 - \bar{S})(M - r)}{\bar{S}} \left( S^* + \frac{\bar{S} - S^*}{2\bar{S}} \ln \left( \frac{1 + \bar{S}}{1 - \bar{S}} \right) \right) \right).$$

The direct effect of moral hazard is then measured as the reduction in cost solely due to the additional $S^* - \bar{S}$ quotes received by households: $\rho(M(1 - \bar{S}) + r\bar{S}) - EC(S^*, \bar{S})$. Indeed, this measure simplifies to:

$$\frac{\rho(S^* - \bar{S})(1 - \bar{S})(M - r) \left( \ln \left( \frac{1 + \bar{S}}{1 - \bar{S}} \right) - 2\bar{S} \right)}{2\bar{S}^2}.$$

An even more informative measure is the fraction of the total moral hazard effect that is due to the direct effect: $\frac{\rho(M(1 - \bar{S}) + r\bar{S}) - EC(S^*, \bar{S})}{\rho(M - r)(S^* - \bar{S})}$. This simplifies to:

$$H \equiv \frac{(1 - \bar{S}) \left( \ln \left( \frac{1 + \bar{S}}{1 - \bar{S}} \right) - 2\bar{S} \right)}{2\bar{S}^2}. \quad (10)$$

Surprisingly, the fraction $H$ only depends on the fixed reference point $\bar{S}$. $H$ is concave in $\bar{S}$, approaches 0 as $\bar{S}$ approaches either 0 or 1, and reaches a maximum of $H \approx 0.104$ when $\bar{S} \approx 0.635$. Thus, the direct effect cannot account for more than 10.4% of the change in total cost; the remaining 89.6% must be due to the indirect effect. Put another way, the indirect effect is at least $\frac{0.896}{0.104} = 8.6$ times as big as the direct effect, and (if other $\bar{S}$ are used) potentially more. Note that this theoretical bound holds for all parameter values.
5 A Numerical Example

Since the model is not analytically solvable (even with linear utility), we now provide a numerical solution. Our aim is to illustrate equilibrium behavior and to offer a sense of the magnitude of the problem that moral hazard in search creates in a perfectly-competitive insurance market.

We proceed using CRRA preferences $u(w) = \frac{w^{1-a}}{1-a}$, setting $a = 4$. Other parameters are set to $w = 40,000$, $M = 4,000$, $r = 2,000$, $\rho = 0.05$, and $c = 1.566 \cdot 10^{-17}$. The search cost is expressed in utils, and is equivalent to spending $40 to obtain a second quote for sure.

The behavior described in this section is qualitatively robust to changes in the parameters. In numerous alternative parameterizations, the only key difference seems to be in whether the UM contract reaches a corner solution. Our chosen values for $M$ and $r$ are reasonable in settings such as auto repairs following a “fender bender,” a significant outpatient medical procedure, or annual expenditures on a patent-protected prescription drug.

We begin by illustrating the equilibrium relationship between the coinsurance rate and search effort. Figure 1 presents the insured search equilibrium $S$ for any particular $\gamma \in [0, 1]$, with $\theta$ set to the competitive premium. Note that the degenerate equilibrium ($S^* = 0$) exists for any coinsurance level, but is the only equilibrium for $\gamma^* < 11\%$. For higher levels of coinsurance, a dispersed price equilibrium exists, with search effort rapidly increasing as households pay more out of pocket.

Also depicted are the equilibrium pairs under the various insurance market structures. A monopolist insurer produces full insurance and no search, a competitive market yields a modest 14% coinsurance rate with some search ($S^* = 0$), while the UM contract offers 65.5% coinsurance and induces far more search effort ($S^* = 0.95$).

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21 The computer code used for this section is available in a technical appendix (as a Mathematica file) at http://economics.byu.edu/Documents/BrennanPlatt/TechnicalAppendix.nb.

22 This provides a moderate amount of risk aversion while still being within the range of values that are commonly accepted for individuals. Lower values for $a$ reduces the importance of insurance, and if small enough, result in a utility-maximizing coinsurance rate of 100%.

23 For example, http://www.nhhealthcost.org/costByProcedure.aspx provides average procedure costs at various New Hampshire facilities. Outpatient colonoscopies in the Nashua area (within 20 miles of zip code 03060 on a CIGNA PPO plan) are in this range and display similar variance.

24 Lipitor in a 40 mg dose, for instance, has similar mean and variance in annual expenditures, as recorded in the 2005 Medical Expenditure Panel Survey, compiled by the US Department of Health and Human Services. Sorensen (2000) documents similar price dispersion in other prescription drugs.
Figure 1: Insured search equilibrium pairs of coinsurance rate, $\gamma$, and search effort, $S$. Points indicate the monopolist ($\gamma = 0$), perfectly-competitive ($\gamma = 0.14$), and utility-maximizing ($\gamma = 0.655$) equilibria.

Of course, each of these contracts result in a different service firm price distribution. Figure 2 plots the cumulative distribution function $F(p)$ for each. For low levels of search effort, the CDF is close to linear, i.e. nearly uniformly distributed. As effort increases, the lower bound of the support falls, and more density is shifted to the lowest prices. For instance, over 80% of firms charge less than $\$2,250$ under the UM contract.

This dramatic change in the price distribution has a stark effect on the insurance premiums and out-of-pocket costs paid in each insured search equilibrium. These are illustrated in Figure 3. In the degenerate equilibrium range, an increase in the coinsurance rate simply transfers responsibility from the insurance firm to the individual; the total expected cost remains constant. When the dispersed price equilibrium emerges at $\gamma = 11\%$, the reduction in the price distribution is reflected in the steep drop in both insurance premium and total costs.

Note that total expected cost falls from $\$200$ to $\$178$ in moving from full insurance to the PC contract, but falls to $\$105$ under the UM contract. Consistent with the decomposition in Section 4, the direct effect of search only accounts for $\$4.30$ (or 5.9\%) of the cost reduction between the PC and UM contracts.

Clearly higher coinsurance results in lower expected prices for households; yet it also leaves the household exposed to more risk from the negative event. The net effect of these two factors on ex-ante utility is represented in Figure 4. The price reductions are more important initially, but eventually are not sufficient to
Figure 2: Equilibrium distribution of prices under the monopolist (solid), perfectly-competitive (dash), and utility-maximizing (dotted) contracts.

Figure 3: Ex-ante expected costs for each coinsurance rate, $\gamma$: actuarially-fair insurance premium (solid), out-of-pocket (dotted), and total costs (dashed). The kink at $\gamma = 11\%$ indicates where a dispersed equilibrium first emerges.
Figure 4: Equilibrium expected utility, expressed as certainty-equivalent wealth, for each coinsurance rate. Points indicate the degenerate, perfectly-competitive, and utility-maximizing outcomes. The monopolist contract would produce a certainty-equivalent wealth of $39,755, as the monopolist extracts the full risk premium of $45.

compensate for additional risk. For a simple interpretation of utility, we translate expected utility into dollar terms by finding the certainty equivalent wealth for each level of coinsurance; that is, the wealth $w$ such that $\frac{w^{1-a}}{1-a} = EU(\theta, \gamma, S)$.

Since we must numerically solve this model, an important question is how the equilibrium contracts change under different parameters. In Table 1, we report the sign of the comparative statics. We repeated these computations for a variety of parameter values, and as long as the UM coinsurance was interior, the comparative statics maintain the same sign.

For the first three parameters, the PC and UM coinsurance rates move in the same direction, for essentially the same reasons. Absent any change in the policy, households would have less incentive to search after an increase in $w$ or $c$, or a decrease in $M$. This would cause upward pressure on prices; an increase in coinsurance can offset this reluctance to search. In fact, the net effect is positive for $S_{PC}^*$.

For the last three parameters, the PC and UM coinsurance rates move in opposite directions. These parameters have a stronger impact on the value of risk smoothing. This is the primary concern driving down the UM coinsurance rate following an increase in $r$ or $a$, or a decrease in $\rho$. It does result in less search effort and higher
Table 1: Comparative statics on the perfectly-competitive and utility-maximizing equilibria.

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prices, but since these prices are generally close to $r$ already, this extra cost is minor. In the PC equilibrium, however, the prices are concentrated closer to $M$, so reductions in search effort and the resulting price increases are much more damaging, and takes precedence over risk smoothing.\(^{25}\)

Two results are surprising: First, higher coinsurance is needed precisely when it is more costly to obtain price quotes ($c$ increases). The insurance contract must provide incentives to keep consumers searching. Second, under the UM contract, a higher maximum price is actually better for consumers, resulting in higher expected utility.\(^{26}\) A higher maximum price naturally gives households more incentive to search, thus coinsurance can be reduced while still obtaining more search effort on net. This does not hold true for the PC contract, though; search effort falls and households are left worse off after $M$ rises.

6 Conclusion

Our model of moral hazard in search has allowed us to study optimal insurance contracts when service prices are endogenously determined. Accounting for the firms’ response to consumer search behavior significantly worsens the moral hazard problem. Higher insurance coverage motivates consumers to search less, which in turn reduces

\(^{25}\)We also briefly consider when corner solutions arise in the UM coinsurance rate. For instance, if $\gamma_{UM}^* = 1$, households are best off remaining uninsured; in a sense, they self-insure by exerting high levels of search effort should the event occur. This arises when $w$, $c$, or $\rho$ are rather high, or $M$, $r$, or $a$ are rather low. Moving to the opposite extreme (e.g. if $r \geq 3,465$, ceteris paribus) will eventually result in full insurance, $\gamma_{UM}^* = 0$.

\(^{26}\)This effect diminishes as $M$ increases, but is still positive when $M = w$. 24
competition among service firms. We find that the indirect effect of search (i.e., greater price competition among firms) is at least 8.6 times as large as the direct effect (i.e., more quotes evaluated by households).

The theory also allows us to comment on the industrial organization of the insurance market and its impact on the equilibrium contract. When the insurance firm is a monopolist, the equilibrium contract results in full insurance and no price dispersion among service firms. When the insurance market is perfectly competitive and prices are dispersed, the equilibrium coinsurance rate is still lower than the utility-maximizing rate. This inefficient outcome arises because each insurer only considers the direct effect of its contract on its own clients, thereby neglecting the indirect benefits that spill over to other households.

From a policy point of view, our work has particular relevance to the debate regarding health savings accounts (HSAs). HSAs have high deductibles (essentially a 100% coinsurance rate) coupled with full coverage for catastrophic events. Advocates have cited increased price competition among health providers as one of the benefits of HSAs, since households covered by such plans will have incentive to shop around (at least for services that aren’t urgent or likely to exceed the full-coverage threshold). This paper offers some theoretical foundation for that claim. At the same time, our analysis suggests that neither a monopolized nor a perfectly-competitive insurance market is likely to encourage optimal HSA participation. For the former, more price competition will reduce profits, and for the latter, search externalities are neglected by the insurer.

Our purpose here was to build a parsimonious model of insurance to show its potential effects on consumer search and prices. However, several extensions to this work would make the set-up applicable to a broader range of markets.

First, to isolate the search friction as the only source of service price variation, we have assumed homogeneity in households, service firms, and insurers. However, the model could be enriched by allowing differences within one (or more) of these groups of agents. Heterogeneity among service firms, for instance, would likely pin down which firms offer low or high prices, and could produce a very different price distribution. However, since we generate a continuum of prices in equilibrium, heterogeneity can render the problem intractable even for numerical solutions.\textsuperscript{27}

\textsuperscript{27}If households are well-informed of and agree upon differences in service firm quality, our model could be used to explain residual price differentials; that is, dispersion above and beyond what can
Second, a multi-period environment would allow future premiums or coinsurance rates to be conditioned on claims paid in the past, as is the case with most auto insurance. This would encourage additional search and might reduce our estimates of the moral hazard problem.\(^{28}\)

Third, we have treated the maximum service price, \(M\), as exogenous.\(^{29}\) However, one would typically expect this price cap to be negotiated between the insurers and service firms. The bargaining power of each would determine how their joint surplus is split; for instance, an insurer with many subscribers would have a stronger position than a smaller insurance group (see Sorensen, 2003). If the insurer has all of the bargaining power, the clear outcome would be to set \(M = r\); but if service firms have any bargaining power, they cannot be pushed to their threat point of zero profit.\(^{30}\) To understand the complex dynamics by which \(M\) is pinned-down, we would need detailed administrative micro-data on service prices set between insurance firms and service providers of different sizes.

Finally, we have restricted insurance firms to a particular class of contracts (i.e. those which reimburse a constant percentage of realized expenses). A natural extension of the model would be to allow a broader class of contracts, including: copays or deductibles (where the household pays for the first \(x\) dollars of a claim), policy limitations (where the household is responsible for 100% of expenses above some amount), fixed payments (where the insurer pays a fixed amount on any claim, as well as coinsurance on actual expenses), or maximum out-of-pocket clauses (where coinsurance drops to 0% on any expenses beyond some threshold). In numerical computations, we have incorporated each of these, but found that none had a significant effect on search behavior beyond what coinsurance already provides. All of the above extensions warrant careful analyses, which we leave for future work.

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\(^{28}\)See Lewis and Ottaviani (2008) for an example of dynamic optimal contracting between a principal and an agent (with full commitment) in the presence of moral hazard.

\(^{29}\)In Armstrong, Vickers, and Zhou (2009), price caps played a similar role and were also taken to be exogenous. They suggested interpreting this cap as an exogenous maximum value that the consumer places on the service, being willing to forgo the service if charged more. Even so, if \(M\) is to be a reservation price, it ought to be determined through an optimal search process (as in Janssen and Moraga-González, 2004), which would greatly complicate our model.

\(^{30}\)This issue is even more nuanced in light of our results from Section 3. For instance, a monopolist insurer benefits from a higher \(M\), as it increases the risk premium they can charge. Moreover, in Section 5, we noted that a higher \(M\) increases expected utility under the utility-maximizing contract. Thus, it is not clear that the insurer would always want to negotiate a lower \(M\).
A Proofs

A.1 Lemma 2

Proof. The objective function in Equation 3, after substituting for $F(p)$ and $F'(p)$, simplifies to:

$$\int_{p}^{M} u(w - \theta - \gamma p) \frac{(1 - S)(M - r)((M - r)(1 - S)s + (p - r)(S - s))}{2S^2(p - r)^3} dp - cs^2.$$

After taking the first order condition, $s$ drops out of the integral and is linear in the marginal cost of search:

$$\int_{p}^{M} u(w - \theta - \gamma p) \frac{(1 - S)(M - r)((M - r)(1 - S) + r - p)}{2S^2(p - r)^3} dp = 2cs,$$

which always yields a unique solution for $s$. Also, as $S \rightarrow 0$, the integral approaches 0, and hence $\zeta$ also approaches 0. Intuitively, the expected advantage from a second quote evaporates as the price distribution collapses to be degenerate at $p = M$; hence, it is optimal to exert no effort. \qed

A.2 Proposition 1

Claim 1:

Proof. Note that $\Delta(\theta, \gamma, 0) = 0$. This is because, when $S = 0$, the integrals are replaced with the degenerate distribution, concentrated on $p = M$. Thus, $V(s; \theta, \gamma, 0) = (1 - s)u(w - \theta - \gamma M) + su(w - \theta - \gamma M) - cs^2$. After taking the derivative w.r.t. $s$ and setting $s = S$, we obtain $\Delta(\theta, \gamma, 0) = 0$. This confirms the existence of the degenerate equilibrium under all parameter values. \qed

Claim 2:

Proof. A dispersed equilibrium is characterized by $S^* \in (0, 1)$ such that $\Delta(\theta, \gamma, S^*) = 0$. We will show that there can be no more than one such equilibrium.

Define $\beta_0(S) \equiv \frac{S^2}{1-S} \Delta(\theta, \gamma, S)$. Note that $\beta_0(S) = 0$ if and only if $\Delta(\theta, \gamma, S) = 0$.
and shares the same sign for \( S \in [0, 1) \). The first derivative of \( \beta_0(S) \) w.r.t. \( S \) is:

\[
\beta'_0(S) = \frac{S}{(1 - S)^2} \cdot \left( u(w - \theta - \gamma p) - 2S(3 - 2S)c \right.
\]

\[
- \int_{p}^{M} u(w - \theta - \gamma p) \frac{(M - r)^2(1 - S)^2}{2S(p - r)^3} dp \right).
\]

and its second derivative is:

\[
\beta''_0(S) = -\frac{2S}{(1 - S)^3} \left( 2c(3 - 3S + S^2) - \frac{(M - r)\gamma(1 - S)}{(1 + S)^2} u'(w - \theta - \gamma p) \right).
\]

Note that \( \beta'_0(0) = 0 \) (in particular, the integral evaluates to \( u(w - \theta - \gamma M) \) in the limit); as well as \( \beta''_0(0) = 0 \).

Next, define \( \beta_1(S) = \frac{(1 + S)^2(1 - S)^2}{2S} \beta''_0(S) \). Again, \( \beta_1 \) shares the same signs and zeros as \( \beta'_0 \) for \( S \in (0, 1) \). Its derivative is:

\[
\beta'_1(S) = -2c \left( 2 - 3S^2 + \frac{4}{(1 - S)^2} \right) + \frac{2(M - r)^2\gamma^2}{(1 + S)^2} u''(w - \theta - \gamma p).
\]

The first term is always negative, and since \( u'' < 0 \), the second term is as well. Thus, \( \beta'_1(S) < 0 \). Since \( \beta_1(S) \) is strictly decreasing in \( S \), there can be at most one \( \hat{S} \in (0, 1) \) such that \( \beta_1(\hat{S}) = 0 \), which implies the same for \( \beta''_0(S) \).

It is helpful to recall Rolle’s Theorem: if \( g \) is a continuous, differentiable function on \([a, b]\) and \( g(a) = g(b) \), there exists a point \( c \in (a, b) \) such that \( g'(c) = 0 \). As previously noted, \( \beta'_0(0) = 0 \). Suppose there exists an \( \tilde{S} \in (\hat{S}, 1) \) such that \( \beta'_0(\tilde{S}) = 0 \). Then \( \tilde{S} \) is unique; otherwise, Rolle’s Theorem would require that \( \beta''_0(S) = 0 \) for more than one \( S \).

This same logic is applied to \( \beta_0(S) \). Again, \( \beta_0(0) = 0 \), and if there exists an \( S^* \in (\tilde{S}, 1) \) such that \( \beta_0(S^*) = 0 \), then it is unique. But this gives us the uniqueness of the dispersed equilibrium, since the zeros of \( \beta_0(S) \) coincide with those of \( \Delta(\theta, \gamma, S) \).

One should note that if no such \( \hat{S} \) or \( \tilde{S} \) occurs, then there cannot be a dispersed equilibrium \( S^* \). The \( \beta'_0(S) \) and \( \beta_0(S) \) functions would be strictly increasing or strictly decreasing throughout, equaling 0 only at \( S = 0 \).

\[ \square \]

\textbf{Claim 3:}

\textit{Proof.} Consider \( \beta'_0(S) \) from Claim 2. If we replace \( u(w - \theta - \gamma p) \) with \( u(w - \theta - \gamma M) \),
this increases \( \beta'_0(S) \) (since the integrand is positive but the integral is multiplied by \(-1\)). After making this substitution and moving the utility function outside the integral, the latter evaluates to 1:

\[
\beta'_0(S) < \frac{S}{(1-S)^2} \left( u(w - \theta - \gamma p) - 2S(3-2S)c - u(w - \theta - \gamma M) \right).
\]

Moreover, because of diminishing marginal utility, \( u(w - \theta - \gamma p) - u(w - \theta - \gamma M) < \gamma(M - p)u'(w - \theta - \gamma M) \). Moreover, \( u'(w - \theta - \gamma M) < u'(w - \theta - M) \). With this substitution and replacing \( \bar{p} \) with its endogenous value, this becomes:

\[
\beta'_0(S) < \frac{2S^2}{(1+S)(1-S)^2} \left( (1 + S)(2S - 3)c + \gamma(M - r)u'(w - \theta - M) \right).
\]

Note that \((1 + S)(2S - 3) \leq -2\). Thus, if \( \gamma < \frac{2c}{(M - r)u(w - \theta - M)} \), then \( \beta'_0(S) < 0 \) for all \( S \). Since \( \beta_0(0) = 0 \) and is strictly decreasing, there cannot be an \( S^* > 0 \) such that \( \beta_0(S^*) = 0 \). Since \( \beta_0 \) shares the same zeros as \( \Delta \), there cannot be a dispersed price equilibrium in that case.

\( \Box \)

Claim 4:

Proof. We place signs on the two comparative statics via implicit differentiation.

First, note that for a dispersed price equilibrium \( S^* \in (0,1) \), \( \Delta_S(\theta, \gamma, S^*) < 0 \). This is because \( \Delta(\theta, \gamma, 0) = 0 \), \( \Delta(\theta, \gamma, 1) = -2c \), and \( \Delta(\theta, \gamma, S) \) only crosses 0 once for \( S \in (0,1) \). Thus the derivative must be negative when it crosses.

It is also readily apparent that \( \Delta_c(\theta, \gamma, S) < 0 \) for any \( S > 0 \). Thus, \( \frac{\partial S^*}{\partial c} = -\frac{\Delta_c}{\Delta_S} < 0 \).

To do the same for \( \gamma \) requires a few more steps. The derivative of \( \Delta \) with respect to \( \gamma \) is:

\[
\Delta_\gamma = \int_p^M \frac{p(1-S)(M-r)(p-M + (M-r)S)}{2S^2(p-r)^3} u'(w - \theta - \gamma p) dp.
\]

Let \( \tilde{p} = M - (M-r)S \). The integrand is negative for all \( p \in [\tilde{p}, \bar{p}) \), and positive elsewhere. Thus, we split the integral at \( \tilde{p} \), then create a lower bound by substituting for \( u' \) at its largest value in the interval for the first term, and its smallest value for the second term. Since \( u'' < 0 \), this occurs at \( \tilde{p} \) in both cases.
\[ \Delta_\gamma > u'(w - \theta - \gamma \tilde{p}) \int_{\tilde{p}}^{p} \frac{p(1 - S)(M - r)(p - M + (M - r)S)}{2S^2(p - r)^3} dp + u'(w - \theta - \gamma \tilde{p}) \int_{\tilde{p}}^{M} \frac{p(1 - S)(M - r)(p - M + (M - r)S)}{2S^2(p - r)^3} dp = \frac{(M - r)(1 - S)(-2S + \ln \left(\frac{1 + S}{1 - S}\right))}{2S^2} u'(w - \theta - \gamma \tilde{p}). \]

For all \( S \in (0, 1) \), \( \frac{(M - r)(1 - S)(-2S + \ln \left(\frac{1 + S}{1 - S}\right))}{2S^2} > 0 \) (and equal to 0 at \( S = 0 \) or 1). Thus, \( \frac{\partial \Delta}{\partial \gamma} > 0 \) and by implicit differentiation, \( \frac{\partial S^*}{\partial \gamma} > 0 \).

**A.3 Lemma 3**

*Proof.* For a given \( S \), the expected price from a single quote request is:

\[ \int_{\tilde{p}}^{M} \frac{(1 - S)(M - r)p}{2S(p - r)^2} dp = r + \frac{(1 - S)(M - r) \ln \left(\frac{1 + S}{1 - S}\right)}{2S}. \]

Similarly, the expected price after two quote requests is:

\[ \int_{\tilde{p}}^{M} \frac{(1 - S)^2(M - r)(M - p)p}{2S^2(p - r)^3} dp = r + 2S(1 - S)(M - r) - (1 - S)^2(M - r) \ln \left(\frac{1 + S}{1 - S}\right) \frac{1}{2S^2}. \]

Fraction \( \rho \) of the population incurs the loss. Of those, fraction \( 1 - S \) obtain a single quote, while \( S \) obtain two. The insurance company pays \( 1 - \gamma \) of the price they are quoted. Thus, the expected profit is:

\[ \Pi_I(\theta, \gamma, S) = \theta - (1 - \gamma)\rho \left[ (1 - S) \left( r + \frac{(1 - S)(M - r) \ln \left(\frac{1 + S}{1 - S}\right)}{2S} \right) + S \left( r + \frac{2S(1 - S)(M - r) - (1 - S)^2(M - r) \ln \left(\frac{1 + S}{1 - S}\right)}{2S^2} \right) \right] \]

\[ = \theta - \rho(1 - \gamma)(M(1 - S) + rS). \]

\[ \square \]
A.4 Proposition 2

Proof. We proceed by examining profits for any \((\gamma, S)\) pair (whether it constitutes an insured search equilibrium or not). We find that \(S = 0\) and \(\gamma = 0\) provides the globally maximal profit. Note that \(\sigma(\theta, 0) = 0\), since there is never incentive to request multiple quotes under full insurance. For partial insurance contracts, the following applies regardless of which \(S\) is selected from \(\sigma(\theta, \gamma)\).

Let \(\Theta(\gamma, \Pi, S) \equiv \rho(1 - \gamma)(M(1 - S) + rS) + \Pi\), and define the consumer surplus from being insured as:

\[
CS(\gamma, \Pi, S) \equiv (1 - \rho) (u(w - \theta) - u(w)) + \rho (V(S; \theta, \gamma, S) - V(S; 0, 1, S))
+ \rho \int_p^M \frac{(1 - S)^2(M - r)^2}{2S(p - r)^3} (u(w - \theta - \gamma p) - u(w - p)) dp,
\]

where the arguments of \(\theta = \Theta(\gamma, \Pi, S)\) are omitted for notational simplicity. The monopolist extracts this consumer surplus by solving for the \(\Pi\) for which \(CS(\gamma, \Pi, S) = 0\), i.e. makes the household ex-ante indifferent between insuring and not.

Note that this consumer surplus definition assumes the individual search effort equals aggregate search effort \((s = S)\), even if this is not the optimal response of a household to \((\Theta(\gamma, \Pi, S), \gamma, S)\). We show that even these out-of-equilibrium situations (as well as all triples that do constitute an equilibrium) would not generate more profit for the insurer than providing full insurance.

Also, this definition assumes that individual search effort is the same whether insured or not, which is reflected in the first argument of \(V(S; 0, 1, S)\). Of course, if \(S > 0\), an uninsured household would typically request \(s > S\) if they requested \(S\) when insured, resulting in a higher utility \(V(s; 0, 1, S) > V(S; 0, 1, S)\) (and less surplus available to extract). In that sense, \(\Pi\) is an upper bound on what the monopolist may charge, though it is the exact monopolist profit when \(S = 0\) (since the household will not want to deviate in a degenerate equilibrium).

To consider the effect of \(\gamma\) and \(S\) on \(\Pi\), we perform implicit differentiation on \(CS(\gamma, \Pi, S) = 0\). First, higher profits clearly reduce consumer surplus.

\[
\frac{\partial CS}{\partial \Pi} = - \left(\rho \int_p^M \frac{(1 - S)^2(M - r)^2}{2S(p - r)^3} u'(w - \theta - \gamma p) dp\right) < 0.
\]
Higher coinsurance has the same effect:

\[
\frac{\partial CS}{\partial \gamma} = \rho(1 - \rho)(M(1 - S) + rS)u'(w - \theta) + \rho \int_p^M \frac{(1 - S)^2(M - r)^2(\rho(M(1 - S) + rS) - p)}{2S(p - r)^3} u'(w - \theta - \gamma p) dp.
\]

By the assumption that \( \rho \leq \frac{1}{2} \), \( \rho(M(1 - S) + rS) < r + \frac{(M - r)(1 - S)}{1 + S} = p \) for all \( S \). Thus, the integrand is always negative. Since \( u'(w - \theta - \gamma p) > u'(w - \theta) \) for all \( p \), we may substitute as follows:

\[
\frac{\partial CS}{\partial \gamma} < \rho(1 - \rho)(M(1 - S) + rS)u'(w - \theta) + \rho \int_p^M \frac{(1 - S)^2(M - r)^2(\rho(M(1 - S) + rS) - p)}{2S(p - r)^3} u'(w - \theta) dp = \rho(1 - \rho)(M(1 - S) + rS) (u'(w - \theta) - u'(w - \theta)) = 0.
\]

Thus, implicit differentiation gives us that \( \frac{\partial \Pi}{\partial \gamma} = -\left( \frac{\partial CS}{\partial \gamma} / \frac{\partial CS}{\partial \Pi} \right) < 0 \). In other words, for any \( S \), profits are maximized by setting \( \gamma = 0 \).

Next, let \( \gamma = 0 \) and examine how \( \Pi \) responds to changes in \( S \). \( CS \) simplifies to:

\[
CS(0, \Pi, S) \equiv u(w - \theta) - (1 - \rho)u(w) - \rho \int_p^M \frac{(1 - S)^2(M - r)^2}{2S(p - r)^3} u(w - p) dp.
\]

As before, \( \frac{\partial CS}{\partial \Pi} < 0 \). The derivative with respect to \( S \) is:

\[
\frac{\partial CS}{\partial S} = \rho \left( (M - r)u'(w - \theta) - \frac{1 + S}{(1 - S)S} \left( u(w - p) + \int_p^M \frac{(1 - S)^2(M - r)^2}{2S(p - r)^3} u(w - p) dp \right) \right).
\]

Note that \( u(w - p) \geq u(w - M) \), has a positive integrand, and is multiplied by a negative number, \(-\frac{1 + S}{(1 - S)S}\). Similarly, \( u(w - p) > u(w - M) \). Thus, we can substitute as follows:

\[
\frac{\partial CS}{\partial S} < \rho \left( (M - r)u'(w - \theta) - \frac{1 + S}{(1 - S)S} \left( u(w - M) + \int_p^M \frac{(1 - S)^2(M - r)^2}{2S(p - r)^3} u(w - M) dp \right) \right) = \rho \left( (M - r)u'(w - \theta) - \frac{2(1 + S)}{(1 - S)S} u(w - M) \right).
\]
Next, note that $EU(s; 0, 1, S) \geq u(w - M)$ for any $S \in [0, 1]$ and any $s \geq 0$, since $w - M$ is the worst possible outcome in the probability distribution. Thus, $\theta \leq M$; if an insurer charged more than $M$, the household would surely reject the insurance contract. As a consequence, $u'(w - \theta) < u'(w - M)$, which yields:

$$\frac{\partial CS}{\partial S} < \rho \left( (M - r)u'(w - M) - \frac{2(1 + S)}{(1 - S)} u(w - M) \right).$$

Finally, $\frac{2(1 + S)}{(1 - S)^2}$ is smallest on $S \in [0, 1]$ when $S = \sqrt{2} - 1$, producing a value of $6 + 4\sqrt{2} > 11$. So

$$\frac{\partial CS}{\partial S} < \rho \left( (M - r)u'(w - M) - 11u(w - M) \right) < 0,$$

where the last inequality holds by assumption.

Thus, implicit differentiation yields $\frac{\partial \Pi}{\partial S} < 0$; the monopolist can charge a higher risk premium $\Pi$ when $S$ is lower. Thus, $(\gamma, S) = (0, 0)$ produces the globally maximal profit — strictly more than any other $(\gamma, S)$ pair (including all insured search equilibrium pairs).

\section*{A.5 Lemma 4}

\textit{Proof.} The function $s = \zeta(\theta, \gamma, S)$ is simply:

$$s = \frac{1}{2c} \int_{\bar{p}}^{M} u(w - \theta - \gamma p) \left( 1 - S \right) \left( M - r \right) \left( (1 - S)(1 - S) + r - p \right) \frac{dp}{2S^2(p - r)^3}.$$  \hspace{1cm} (11)

Clearly the left hand side is strictly increasing. We can substitute in $\theta = \Theta(s, \gamma, S)$, then take the first derivative of the right hand side with respect to $s$. This yields:

$$\zeta_s(\Theta(s, \gamma, S), \gamma, S) = \frac{\rho(1 - \gamma)}{2c} \left( \frac{1}{2} \ln \left( \frac{1 + S}{1 - S} \right) - S \right) \cdot \int_{\bar{p}}^{M} u'(w - \theta - \gamma p) \left( 1 - S \right)^2 \left( M - r \right)^2 \left( 1 - S - p + rS \right) \frac{dp}{2S^4(p - r)^3}.$$  

Note that $\frac{1}{2} \ln \left( \frac{1 + S}{1 - S} \right) > S$ for all $S \in (0, 1)$. Let $\bar{p} = M - (M - r)S$. The integrand is positive for all $p \in [\bar{p}, \bar{p})$, and negative elsewhere. Thus, as in the proof of Claim 4 of Proposition 1, we split the integral at $\bar{p}$, then replace $p$ with $\bar{p}$ in $u'$. Since $u'' < 0,$
this forms an upper bound on the derivative. Hence,

\[
\zeta_s < \frac{\rho(1-\gamma)}{2c} \left( \frac{1}{2} \ln \left( \frac{1+S}{1-S} \right) - S \right) u'(w - \theta - \gamma \hat{\rho}) \\
\cdot \int_p^M \frac{(1-S)^2(M-r)^2(M(1-S)-p+rS)}{2S^4(p-r)^3} dp = 0,
\]

because the integral evaluates to 0. Thus, the right hand side of Equation 11 is strictly decreasing in \( s \). Either they will intersect for a unique \( s \in [0,1) \), or \( s = 0 \) will be the corner solution.

\[\square\]

A.6 Proposition 3

Claim 1:

Proof. In Proposition 1, Claim 1, we established that \( s^* = 0 \) is optimal given \( S^* = 0 \). In section 3.2, we noted that \( \gamma^* = 0 \) is utility maximizing given \( S^* = 0 \). Thus, with the actuarially-fair premium \( \theta = \rho M \), this satisfies the requirements for a competitively-insured search equilibrium.

\[\square\]

Claim 2:

Proof. Suppose that \((\bar{\gamma}, \bar{\theta}, \bar{S})\) is a perfectly-competitive (PC) insured search equilibrium with \( \bar{S} > 0 \). If one considers the total derivative of expected utility w.r.t. \( \gamma \), evaluated at \( \gamma = \bar{\gamma}, S = \bar{S}, \) and \( s = \bar{S} \), the coinsurance rate \( \bar{\gamma} \) must be optimal:

\[
\frac{dEU_{PC}}{d\gamma} = \frac{\partial EU}{\partial \gamma} + \frac{\partial EU}{\partial s} \frac{\partial s}{\partial \gamma} + \frac{\partial EU}{\partial \theta} \left( \frac{\partial \theta}{\partial \gamma} + \frac{\partial \theta}{\partial s} \frac{\partial s}{\partial \gamma} \right) = 0.
\]

Note that in taking these derivatives, \( S \) is held fixed at \( \bar{S} \).

Compare this to the problem for the utility-maximizing (UM) contract, which additionally considers the effect of \( \gamma \) on \( S \). Therefore, if evaluated at \( \gamma = \bar{\gamma}, S = \bar{S}, \) and \( s = \bar{S} \),

\[
\frac{dEU_{UM}}{d\gamma} = \frac{dEU_{PC}}{d\gamma} + \frac{\partial EU}{\partial S} \frac{\partial s}{\partial \gamma} + \frac{\partial EU}{\partial \theta} \frac{\partial \theta}{\partial S} \frac{\partial s}{\partial \gamma}.
\]

It is trivial to show that \( \frac{\partial EU}{\partial S} > 0 \), since an increase in \( S \) results in a lower prices but no added search cost for the individual household (\( s \) is held constant). Likewise,
\[ \frac{\partial EU}{\partial \theta} < 0 \] since this increases wealth in all states, and \[ \frac{\partial \theta}{\partial S} < 0 \] since the average accepted price will be lower as service prices fall. Finally, \[ \frac{\partial S}{\partial \gamma} > 0 \] from Proposition 1, Claim 4.

Therefore, \[ \frac{dEU_{U,M}}{d\gamma} > 0 \] when evaluated at the perfectly competitive outcome, which neglects the positive spill-overs from increased competition via increased search. \[ \square \]
References


